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Publication Date
1999-07-01

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California PATH Research Report
UCB-ITS-PRR-99-23

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration.

Report for MOU 318

July 1999

ISSN 1055-1425

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October 16, 1997

Abstract

Normal Communication Networks require the Sender to specify the address of the Receiver before communication can take place. In problems that arise in an Automated Vehicle Highway System, the Sender can identify the Receiver by its physical location only (i.e. a car wanting to communicate with the car immediately in front of it). Normal Communication Procedures that use the Receiver's Address to establish communication must be preceded by an Address Finding Protocol. An Address Finding Protocol which uses the Absoulate Location of vehicles has been proposed in [2] and is reviewed in Section 2. We consider the question of whether it is possible to build an Address Finding Protocol which uses less information such as only the inter-vehicle distances. We prove that no such protocol can exist. Thus, the proposed protocol which uses the Absoulate Location of Vehicles may be considered to be information-wise optimal.

1 Introduction

In a normal communication network such as a telephone network or a computer network, each entity wanting to engage in communication is assigned a unique address (i.e. Telephone Number, IP Address). The sender uses this address to identify the receiver of its message. For example, we have to dial the telephone number of the person we want to talk with before communication can be established. In case we do not know the address of the receiver, we can identify the receiver by some other unique identification which can be translated into an address. For example we may get the telephone number of someone by calling the operator and telling the operator their name. The physical location at which the sender or receiver are located is not used in identifying the address of the receiver.

In problems that arise in coordination among vehicles in an automated highway system [1] [2] [3], the physical location of sender and receiver play an important role. In fact, the sender may know the physical location of the receiver (as in a car on an automated highway system wanting to communicate with the car in front), but may not know the receiver's address. Normal communication procedures which use the receiver's address to establish communication must be preceded by an address finding procedure. In this paper, we consider problems where the sender can identify the receiver by its physical location but does not know the receiver's address. Furthermore the medium of communication is assumed to be radio broadcast.

In Section 2, we review a protocol that uses the absoulate location of the Sender and Receiver for finding the address of the receiver. In Section 3, we present a model for computation and communication which we use to show that some problems cannot be solved. In Section 4, we prove that it is not possible to build a protocol which functions only using the inter-vehicle distances. In Section 5, we present a protocol which uses global communications and the relative x- and y- distances between vehicles to find the address of the receiver in a system with finite number of vehicles. In Section 6, we show that in a system with a large number of vehicles (potentially infinite) using local communication, the protocol which worked in Section 4 no longer works; furthermore, we prove that there is no protocol which can use the relative x- and y- distances between vehicles to find the address of the receiver under these conditions. Because of our impossiblity results, the protocol developed by [2] and presented in Section 2 maybe considered to be information-wise optimal.

2 Finding the Receiver Using Location Coordinates

The problem of finding the address of a neighboring vehicle (i.e. vehicle in front or the vehicle on the side of the sender) arises in the design of an automated highway system. A protocol which uses the coordinates of the location of the vehicles to solve this problem has been proposed [2]. The protocol requires each vehicle j to maintain its location coordinates (L_j, Y_j) where L_j is the lane number, and Y_j is the distance from a fixed origin for vehicle j. A sender wanting to find the address of a neighboring receiver broadcasts a request. Every neighbor which hears the request replies back with $(Addr_i, L_i, Y_i)$ where $Addr_i$ is the Address of the neighbor i and (L_i, Y_i)

are the location coordinates of the neighboring vehicle i. The sender, by comparing this information with its own location coordinates identifies the address of the intended neighboring receiver.

This protocol requires each vehicle to maintain its location coordinates with sufficient accuracy. Thus the question arises as to whether it is possible to build a protocol which solves the problem using information that may be more easily available or maintainable. An example would be a protocol which uses only local information such as inter-vehicle distances. These problems are discussed in Section 4 - 6.

3 A Model For Coordination and Computation

Each vehicle engaged in the coordination protocol begins with some information. Using this information, the vehicles coordinate to come up with the result. We model this coordination as being carried out by programs running synchronously in each vehicle. At each step, a program broadcasts some messages to other vehicles, and it updates its state based upon the messages it receives during that step. At some point the program knows the result and terminates.

More formally, a program $P = (Q, q_0, \Sigma, \delta, \Gamma, F)$, where q_0 is the initial state, Σ is the set of messages that can be broadcast, δ is the state transition function ($q_{i+1} = \delta(q_i, \sigma)$ is the new state of the program when it receives the message σ in state q_i), $\Gamma : Q \rightarrow \Sigma$ is the broadcast function (i.e $\Gamma(q_i)$ is the message that is broadcast when the program is in state i), and F is the set of termination state (i.e. program stops when it reaches the termination state).

A state of a program is determined by the value of its variables and the Program Counter. The initial state of a car is determined by the value of the variables before the protocol begins; in our problem, this would represent the information each car uses to participate in the protocol. We show that some problems are impossible to solve with the given information. *We do this by giving two different instances of a problem which have the same initial state (i.e. vehicles in each instance have the same information) but the protocol should give different answers for them. We then show that under any protocol the state trajectory for the two instances will be the same, and we will get the same answer for the two instances - thereby we conclude that no protocol can solve the problem.*

4 Intervehicle Distances are Insufficient

We consider the problem of finding a neighboring vehicle when each vehicle knows the intervehicle distances between itself and other vehicles and it can also check whether there are vehicles in certain directions. We also assume that all broadcasts are heard by every vehicle. More formally, each vehicle has an unordered list of distances from itself to other vehicles $d_1, \ldots, d_k$, $d_i > 0$, and a function $D : [0, 2\pi) \rightarrow \{True, False\}$ where $D(\theta) = True$ provided there is a vehicle in the direction θ. The problem is to design a protocol which will find the nearest vehicle in front of a given vehicle. We show that there is no protocol to solve this problem with the given information.

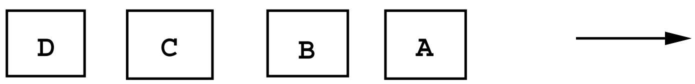

Figure 1

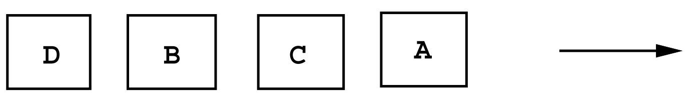

Figure 2

In the configurations in Figure 1 and Figure 2, Vehicle C would like to find the address of the vehicle in front of it. Each vehicle in the two configurations has the same initial information (i.e. the initial state of car_i in Figure 1 is the same as in Figure 2). For any protocol, it follows by induction that the state trajectories will be the same in the two configurations. So the protocol will give the same answer in both configurations, but that is incorrect since the answer is Vehicle B in Figure 1 and Vehicle A in Figure 2. Thus there is no protocol to solve this problem with this limited information.

5 Relative Coordinates are Sufficient

In this section we consider the problem of finding a neighboring vehicle in a finite system when each vehicle knows the relative x- and y- coordinates between itself and other vehicles. More formally, in a system with k vehicles each vehicle has an unordered list $\langle (x_1, y_1), \ldots, (x_k, y_k) \rangle$ where (x_i,y_i) are the relative x- and y- coordinates to some vehicle. We assume that all broadcasts are heard by every vehicle. The problem, as in Section 4 is to find the nearest vehicle in front of a given vehicle. We show that this problem can be trivially solved by an exchange of two messages.

Given the list of relative x- and y- coordinates, each vehicle i assigns to itself the id $\langle F_i, R_i \rangle$ where F_i is the number of vehicles in front of it and R_i is the number of vehicles to the right of it. This id uniquely identifies the vehicle. A vehicle wanting to find the address of a receiver vehicle uses the list of relative coordinates to find the id $\langle F, R \rangle$ of the receiver vehicle and broadcasts it. The vehicle with the id replies back with its address.

6 Relative Coordinates with Local Communication are Insufficient

In Section 5, we assumed that the problem contained a finite number of vehicles, each vehicle had global information and used global communication. In this Section, we assume that the system has a large number of vehicles (potentially infinite), and each vehicle only has local information and communicates only locally. In particular, we assume that the each vehicle has the relative x- and y- coordinates of vehicles in a circle around it, and the communication is local and is not biased in any direction. We show that the problem of finding the vehicle in front again becomes unsolvable under these conditions.

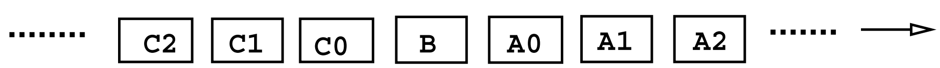

Figure 3

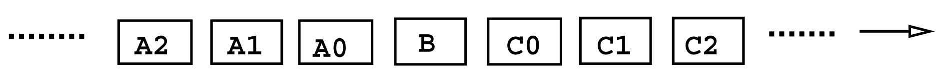

Figure 4

Again, as in Section 4, we assume that there is a protocol that can solve the problem and thus should give different answers to the two configurations in Figure 3 and Figure 4. But as in Section 4, we note that the initial state of each car will be the same and it can be shown by induction that the state evolution will be the same for the two configurations. Thus the protocol will give the same answer for the two configurations - a contradiction.

References

[1] A. Hsu, F. Eskafi, S. Sachs and P. Varaiya. Protocol Design for an Automated Highway System. *Discrete Event Dynamic Systems,* 2: 183-206, 1993

[2] S.Sachs and P. Varaiya. A Communication System for the Control of Automated Vehicles. *PATH Technical Memorandum*

[3] P.Varaiya. Smart Cars on Smart Roads: Problems of Control. *IEEE Transactions on Automatic Control*, 38(2):195-207, February 1993.