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Homothetic Preferences, Homothetic Transformations, and the Law of Demand in Exchange Economies

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There are four parts to this working paper. In the first part, we show that in an exchange economy where all agents have homothetic preferences, and where preferences and endowments are independently distributed, the economy's demand may be represented by a single agent. The proof involves extending an earlier theorem by Eisenberg. In the second part, we show that with additional smoothness assumptions, the economy will satisfy the Restricted Monotonicity Property (RMP), and so have, amongst other virtues, only one equilibrium price. Note that RMP could be thought of as a restricted form of the Law of Demand, hence the title (see Hildenbrand and Kirman, 1988). RMP also guarantees that the equilibrium price will be stable under some plausible tatonnement processes, these will be discussed in the third part. Finally, we will be considering a sequence of exchange economies, $\mathcal{E}_n$, without homothetic preferences, but with "increasing heterogeneity", this heterogeneity captured through the use of homothetic transformations, somewhat similar to the use of affine transformations in Grandmont (1992). We show that increasing heterogeneity will guarantee that the sequence $\mathcal{E}_n$ eventually satisfies RMP.

1 Introduction

Modern General Equilibrium Theory began with the proofs by Arrow and Debreu (1954), and McKenzie (1954) on the existence of general equilibrium. Once the issue of existence of such equilibria was settled, the programme of research quite naturally moved on to a more thorough investigation of its properties. It has since been shown that they satisfy the First and Second Welfare Properties and are stable in the sense of being contained in, and almost equal to the core

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when the economy is large (the language here is necessarily vague, for a survey see Anderson (1986)). Work in these areas has by and large been fruitful. However, general results on the uniqueness of the price equilibrium, or its dynamic stability (in the sense of being the limit of some plausible tatonnement) proved a lot more elusive. There are results in these areas, but they could not be obtained without making seemingly strong assumptions on the market demand function. For example, it was shown that when the market demand function satisfies gross substitubility, the equilibrium price is unique and dynamically stable. Results of this sort left an uncomfortable gap: It is not clear what assumptions (besides exceedingly restrictive ones) could lead to the market demand having these desired properties.

It also became evident by the mid-70s, with the results of Sonnenschein (1973, 1974), Debreu (1974), Mantel (1976) and others that the assumptions made to prove existence are not sufficient to endow the market demand function with sufficient structure to prove uniqueness and dynamic stability of the price equilibrium. Loosely speaking, the result says that given a continuous function f defined on the price simplex that satisfies Walras' Law, and a compact subset of the simplex, an excess demand function can be constructed that will agree with f on the given compact set. In other words, these results show that properties one might reasonably assume about *individual* demand functions, like the weak axiom of revealed preference, get lost in the process of aggregation. Mantel's result was especially striking since the economy he constructed for the purpose had only homothetic preferences.

So to arrive at stronger conclusions about the shape of the market demand function, it is necessary to make assumptions beyond those required for existence. It could, for example, be assumptions made on the distribution of endowments and preferences. A decisive step in this direction was made by Hildenbrand: he showed that when income is independent of prices, the Law of Demand is satisfied when the density of the income distribution is downward sloping. The market demand function $F(p)$ satisfies the law of demand when

$$(p-q)\cdot(F(p)-F(q))<0$$

for prices p, q where $p \neq q$.

In the context of an exchange economy, income is, of course, price-dependent. However, let us consider an economy in which all agents have collinear endowments, i.e., some fraction of the total endowment $\bar{x}$. Then any price change from p to q that leaves total income unchanged, $p\cdot\bar{x} = q\cdot\bar{x}$, will not change the income of any of the agents. Hildenbrand's result on the law of demand then implies that market demand $\hat{F}(p)$ of the exchange economy satisfies the following:

$$(p-q)\cdot(\hat{F}(p)-\hat{F}(q))<0$$

for all prices $p \neq q$, with $p\cdot\bar{x} = q\cdot\bar{x}$, provided the density of the endowment distribution is downward sloping. It turns out that this property, which we shall call the restricted monotonicity property (RMP) is sufficient to guarantee uniqueness and dynamic stability. We say "restricted" because we have restricted the property to prices that generate the same total income. Such a restriction is necessary as the law of demand *cannot* be true for every pair of prices in an exchange economy (see page 215 in Hildenbrand and Kirman, 1988).

Hildenbrand's result is somewhat surprising because he had found a property of the market demand function that need not be true of the individual demand function. That the law of demand may be violated at the level of the individual agent is well known to any neophyte student of micro-economics, and the graphical decomposition normally performed to explain this is essentially correct: the problem lies with the income effect. We shall henceforth refer to this as the intrinsic income effect since it arises from a change in prices even when income is held fixed. When one studies an exchange economy, an extrinsic income effect also arises since income actually changes with prices; unless one assumes collinearity, this occurs even when the price change leaves total (or mean) income unchanged.

This paper is devoted to finding sufficient conditions for RMP to hold in an exchange economy. We begin in section 2 by looking at the specific case of homothetic preferences. We show, in contrast to Mantel's result, that when one assumes preferences and endowments to be independently distributed in the economy, a representative consumer with a homothetic preference exists. In this case, we show (at the end of section 2, and again in Section 3) that RMP holds. The existence of a representative consumer is a consequence of putting together and generalising two existing results: Antonelli's theorem which guarantees the existence of a representative consumer in the case when all agents have the same homothetic preference, and Eisenberg's theorem, which guarantees the existence of a representative consumer in the case when all agents have homothetic, but not necessarily identical preferences, provided the income distribution is proportionately fixed (see Shafer and Sonnenschein, 1982). Section 4 is devoted to tatonnement processes under the weak axiom (the weak axiom is a simple consequence of RMP). We show that the equilibrium is stable under two tatonnement processes, one of which, to the author's knowledge, is new to the literature.

In section 5, we introduce the notion of a homothetic transformation. Given a utility function $u(x)$, a homothetic t-transformation $u_t(x)$ of $u(x)$ is another utility function defined by

$$u_t(x) = u(e^{-t}x).$$

In connection with the demand aggregation issues we are interested in, they were first introduced by Grandmont (1987), but these transformations, along with the more general affine transformations, have been put to other uses before (see Grandmont (1992) for a thorough listing). The nice thing about these transformations, is that it gives us a convenient way to describe and manipulate the distribution of preferences by specifying the distribution of t. We will show, through the use of homothetic transformations, that sufficient heterogeneity, corresponding roughly to a sufficiently flat density for t will lead to RMP. There are actually two results in this section: one result places all the assumptions on the preferences, while the other mixes assumptions on the preferences with a fairly mild restriction on the endowment distribution. In the latter theorem, endowments need not be collinear, and its density can be increasing at some points, provided the part where it is increasing is "smaller" than the part where it is decreasing. In contrast to Hildenbrand's model, the burden of the assumptions in both our results, rest on the preferences rather than on the endowments.

Finally, a word on the differences between this model and Grandmont's (1992). Grandmont's paper, like ours, makes crucial use of the idea of heterogeneity of preferences. He describes this heterogeneity through the use of affine transformations. Given a utility function $u(x)$, and a vector $s \in R^l$, an affine s-transformation $u_s(x)$ of $u(x)$ is another utility function where

$$u_s(x) = u(e^{-s_1}x_1, e^{-s_2}x_2, .., e^{-s_l}x_l).$$

Clearly, affine transformations include homothetic transformations: when $s_1 = s_2 = ... = s_l$, an affine transformation is just a homothetic transformation. We observe that if u is a Cobb-Douglas utility function, all affine transformations of u will leave the preference induced unchanged. Analogously, all homothetic preferences will be invariant under homothetic transformations. These simple observations furnish a clue to what happens when we aggregate across a set of consumers.

Let $f(u_s, p, w)$ be the demand of an agent with utility u_s, income w, and facing price p. Then

$$F(p, w) = \int_{R^l} f(u_s, p, w)g(s)ds$$

is the mean demand of the affine class of consumers with utility functions that are affine transformations of u, distributed according to some density g. At the heart of Grandmont's paper is the fact that when g is sufficiently flat, $F(p, w)$ will display characteristics approximating those of a Cobb-Douglas demand function. Now, it is not difficult to see that in an economy where all agents have Cobb-Douglas preferences, market demand will satisfy gross substitubility. So it is fairly reasonable to believe that in an economy where agents are divided into sufficiently heterogenous affine classes, each of which displays Cobb-Douglas characteristics, market demand will again satisfy gross substitubility. This turns out to be true. With this property, the uniqueness and dynamic stability of the price equilibrium is guaranteed. It is also notewothy that in Grandmont's model, no assumption of individual rationality need be made. In this case, he considers the affine transformations of an individual *demand* *function* as the primitive description of heterogeneity, without assuming that the demand function is generated by some underlying utility.

In this paper, we consider preferences induced by the homothetic transformations of some utility u. Let

$$\tilde{F}(p,w) = \int_R f(u_t, p, w)h(t)dt$$

be the demand of the homothetic class of consumers with utility functions that are homothetic transformations of u, distributed according to some density h. We show that when h is sufficiently flat, $\tilde{F}(p,w)$ will behave like a homothetic demand function. This is not quite as strong as saying that it displays Cobb-Douglas characteristics, which is unsurprising since the class of homothetic transformations of u is a subset of the affine transformations. Nonetheless, it is sufficient for our purpose. As we shall see in Section 3, an economy in which all agents have homothetic preferences, distributed independently of endowments, will have a market demand that satisfies RMP. So it is reasonable to guess that in an economy where agents could be divided into sufficiently heterogenous, homothetic classes, each of which displays homothetic properties, market demand will again satisfy RMP. Assuming individual rationality, we prove that this guess is good.

2 Homothetic Preferences and the Representative Consumer

We begin this section with the description of a distribution economy. After that, we prove an extension of Eisenberg's Theorem and use it to show that an exchange economy where preferences are homothetic and distributed independently of endowments has a representative consumer with a homothetic preference. Lastly, we prove using revealed preference arguments, that a consumer with a demand function that is homogenous of degree one with respect to income must obey the law of demand.

2.1 The Distribution Economy $\mathcal{D}$

Let (A,d) be a complete metric space, and $\mathcal{F}^*$ the completion of its Borel σ-field $\mathcal{F}$ under some measure μ. $(A, \mathcal{F}^*, \mu^*)$ will serve as the probability space of economic agents. Let U be the set of continuous utility functions $u : R^l_{++} \rightarrow R$. We endow U with the topology of uniform convergence on compacta (see page 70 of Mas-Colell, 1985),and denote the topological (in fact, metric) space by $(U, \Im)$. This topology in turn induces a topology on the set of continuous preferences on R^l_{++} that coincides with the topology of closed convergence(see remark on page 74 in Mas-Colell, 1985).

Let $K \subset (U, \Im)$ be a compact set of utility functions that are homogenous of degree one, strongly monotone, strongly convex and satisfy the boundary condition, i.e., for every $x \in R^l_{++}$, the set $y \in R^l_{++} : u(y) \geq u(x)$ is closed in R^l_{++}. Let $m : \mathcal{A} \rightarrow R_{++}$ be a measurable function which is bounded above and also uniformly bounded away from zero, with $\int_{\mathcal{A}} m(a)d\mu = 1$. The expression $m(a)M$ represents the agent a's share of some total wealth M. The Distribution Economy $\mathcal{D}$ is a measurable function $\mathcal{D} : \mathcal{A} \rightarrow U \times R_{++}$, mapping each agent a to $(u(a), m(a))$, where $u(a)$ is the agents utility function and $u(a)$ is in K for all a. The σ-field on $U \times R_{++}$ is the one generated by its product topology.

At each price $p \in R^l_{++}$, the agent a has a budget set $B_p(m(a)M) = \{y \in R^l_{++} : p.y \leq m(a)M\}$. We define $y_p : \mathcal{A} \rightarrow R^l_{++}$ by $y_p(a)$ = Demand of the agent a with the budget set $B_p(m(a)M)$. Strong convexity guarantees that the demand of each agent is a singleton, the Boundary condition guarantees that it exists and is in the interior of the positive orthant. It is well-known that y_p is measurable (see Debreu. 1982). Mean market demand function at M is

$$Y_M : R^l_{++} \rightarrow R^l_{++},$$

$$Y_M(p) = \int_{\mathcal{A}} y_p(a)d\mu \gg 0$$

2.2 Eisenberg's Theorem

In this section, we shall prove that in the economy $\mathcal{D}$, market demand can be generated by a representative consumer. In the case when the set of agents is finite, this theorem was proved by Eisenberg (1961) and Chipman and Moore (1972, 1974). We generalize it to the case when the set of agents is not finite, but a probability space instead. We will first consider the case when $\mathcal{A} = R$ and $\mathcal{F}^* = \mathcal{B}^*$, the completion of the Borel σ-field $\mathcal{B}$ under μ. An agent will be denoted by $r \in R$.

THEOREM 2.1 (Eisenberg, Chipman and Moore)

In the economy $\mathcal{D} : R \rightarrow K \times R$, mean market demand $Y_M(p)$ can be generated by a representative consumer with a price-income situation (p, M) and a homogenous of degree one utility function,

$$V : R^l_{++} \longrightarrow R$$

$$\log V(\bar{y}) = \max_{S(\bar{y})} \int_R \log u(r, y(r))m(r)d\mu$$

where $S(\bar{y})$ is the set of measurable functions $y : R \rightarrow R^l_{++}$ with $\int_R y(r)d\mu = \bar{y}$ and y bounded above and uniformly bounded away from zero. Note that we have denoted $u(r)$ evaluated at $y(r)$ by $u(r, y(r))$.

Before we prove the theorem, we need the following lemma:

LEMMA 2.2

Let $F : R \rightarrow R_{++}$ be a measurable function, bounded above and also bounded uniformly away from zero, satisfying $\int_R F d\mu = M$. Then

$$\int_R \log[F(r)]m(r)d\mu \leq \int_R \log[m(r)M]m(r)d\mu.$$

Proof

Let us assume to the contrary that for some function F,

$$\int_R \log[F(r)]m(r)d\mu > \int_R \log[m(r)M]m(r)d\mu. \quad (1)$$

We will now find *continuous* functions F_2 and m_2 that also satisfy the inequality (1). To do this, we employ Littlewood's Second Principle. This principle is usually mentioned in the context of the Lebesgue measure on compact intervals (see, for example, Royden (1968)), its extension here should be fairly obvious. Suppose that $b_F \leq F \leq B_F$. Then the principle says that for any $\epsilon > 0$, there exists a continuous function F_1, $b_F \leq F_1 \leq B_F$, such that $|F_1 - F| < \epsilon$ except on some set S_ϵ, with $\mu(S_\epsilon) < \epsilon$.

The function

$$F_2 = M\frac{F_1}{\int_R F_1(r)d\mu}$$

will satisfy $\int_R F_2(r)d\mu = M$. Furthermore,

$$|\int_R \log[F(r)]d\mu - \int_R \log[F_2(r)]m(r)d\mu|$$

$$\leq |\int_R \log[F(r)]m(r) - \log F_1(r)m(r)d\mu| + |\log[\frac{M}{\int_R F_1(r)d\mu}]|$$

which can be made arbitrarily small provided ϵ is small enough. Therefore

$$\int_R \log[F_2(r)]m(r)d\mu > \int_R \log[m(r)M]m(r)d\mu$$

Now suppose that $b_M \leq m \leq B_M$. Using Littlewood's Second Principle again, we can find a continuous function m_1, $b_M \leq m_1 \leq B_M$, such that $|m_1 - m| < \epsilon$ except on some set S_ϵ, with $\mu(S_\epsilon) < \epsilon$. We define

$$m_2 = \frac{m_1}{\int_R m_1(r)d\mu},$$

so $\int_R m_2(r)d\mu = 1$. Then

$$\int_R \log[F_2(r)]m_1(r)d\mu - \int_R \log[m_2(r)M]m_1(r)d\mu$$

$$= \int_R \log[F_2(r)]m_1(r)d\mu - \int_R \log[m_1(r)M]m_1(r)d\mu + [\log \int_R m_1(r)d\mu] \int_R m_1(r)d\mu$$

If ϵ is sufficiently small,

$$\int_R \log[F_2(r)]m_1(r)d\mu - \int_R \log[m_1(r)M]m_1(r)d\mu > 0$$

and $\log[\int_R m_1(r)d\mu]$ will be close to zero, so that

$$\int_R \log[F_2(r)]m_1(r)d\mu - \int_R \log[m_2(r)M]m_1(r)d\mu > 0$$

which implies that

$$\int_R \log[F_2(r)]m_2(r)d\mu - \int_R \log[m_2(r)M]m_2(r)d\mu > 0 \qquad (2)$$

Remember also that $\int_R F_2(r)d\mu = M$, $\int_R m_2(r)d\mu = 1$ and that both F_2 and m_2 are bounded above and away from zero.

We now construct ν_n, atomic measures with finitely many atoms satisfying $\nu_n \to \mu$ weakly: Let $q_1, q_2, ...$ be a countable dense set in R. Define $Q_n = \{q_1, .., q_n\}$. We rearrange the elements of Q_n so that $q_{n1} < q_{n2} < ... q_{nn}$ and define

$$\nu_n\{q_{n1}, .., q_{nk}\} = \nu(-\infty, q_{nk}]$$

for $k = 1, 2, ..., n-1$ and $\nu_n\{q_{n1}, .., q_{nn}\} = 1$. Then it is fairly easy to show that $\nu_n \to \mu$. Furthermore, when $2 \leq k \leq n-1$, $\nu_n\{q_{nk}\} = \mu(q_{n(k-1)}, q_{nk}]$, $\nu_n\{q_{nn}\} = \mu(q_{n(n-1)}, \infty)$ and $\nu_n\{q_{n1}\} = \mu(-\infty, q_{n1}]$. We define the function

$$F_n' : R \to R_{++}$$

by

$$\begin{aligned} F_n'(r) &= \tfrac{1}{\mu(q_{n(k-1)}, q_{nk}]} \int_{(q_{n(k-1)}, q_{nk}]} F_2(r)d\mu \quad && q_{n(k-1)} < r \leq q_{nk}, \\ & && 2 \leq k \leq n-1 \\ &= \tfrac{1}{\mu(q_{n(n-1)}, \infty)} \int_{(q_{n(n-1)}, \infty)} F_2(r)d\mu && q_{n(n-1)} < r \\ &= \tfrac{1}{\mu(-\infty, q_{n1}]} \int_{(-\infty, q_{n1}]} F_2(r)d\mu && r \leq q_{n1} \end{aligned}$$

It is easy to check that $\int_R F_n'(r)d\nu_n = \int_R F_2(r)d\mu = M$, and that $b_F \leq F_n'(r) \leq B_F$. Furthermore, on any compact set $F_n' \to F_2$ uniformly. Repeating the procedure exactly with m_2 we can define m_n' with $\int_R m_n'(r)d\nu_n = \int_R m_2(r)d\mu \doteq 1$, $b_m \leq m_n'(r) \leq B_m$ and $m_n' \to m_2$ uniformly on any compact set. For the atomic measure ν_n, one may verify that

$$\int_R \log[F_n'(r)]m_n'(r)d\nu_n - \int_R \log[m_n'(r)M]m_n'(r)d\nu_n \leq 0.$$

Now if we can somehow justify taking limits as n tends to infinity, we will obtain

$$\int_R \log[F_2(r)]m_2(r)d\mu - \int_R \log[m_2(r)M]m_2(r)d\mu \leq 0.$$

This contradicts (2), which in turn means that (1) is not possible. So we shall now justify taking limits. Choose $K = [a, b]$, a compact interval large enough so that $\mu(R \setminus K) < \epsilon$; a and b we choose to be non-atoms. Then

$$\begin{aligned} & |\int_R \log[F_n'(r)]m_n'(r)d\nu_n - \int_R \log[F_2(r)]m_2(r)d\mu| \\ \leq \; & |\int_R \log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)d\nu_n| \\ & +|\int_R \log[F_2(r)]m_2(r)d\nu_n - \int_R \log[F_2(r)]m_2(r)d\mu| \\ \leq \; & |\int_K \log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)d\nu_n| \\ & +|\int_{R\setminus K} \log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)d\nu_n| \\ & +|\int_R \log[F_2(r)]m_2(r)d\nu_n - \int_R \log[F_2(r)]m_2(r)d\mu| \end{aligned}$$

On K, $\log[F_n'(r)]m_n'(r) \to \log[F_2(r)]m_2(r)$ uniformly, so for n sufficiently large,

$$|\int_K \log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)d\nu_n| < \epsilon.$$

On R,

$$|\log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)|$$

is bounded, say by T, so

$$|\int_{R\setminus K} \log[F_n'(r)]m_n'(r) - \log[F_2(r)]m_2(r)d\nu_n| \leq T\nu_n(R \setminus K) < 2T\epsilon$$

when n is sufficiently large. Lastly, standard theory tells us that

$$|\int_R \log[F_2(r)]m_2(r)d\nu_n - \int_R \log[F_2(r)]m_2(r)d\mu| < \epsilon$$

if n is sufficiently large. Therefore, we conclude that

$$\int_R \log[F_n'(r)]m_n'(r)d\nu_n \to \int_R \log[F_2(r)]m_2(r)d\mu$$

Similarly, we can prove that

$$\int_R \log[m_n'(r)]m_n'(r)d\nu_n \to \int_R \log[m_2(r)]m_2(r)d\mu.$$

Therrefore taking limits is justified. QED.

Proof of THEOREM 2.1

The proof is a modification of the one given by Shafer and Sonnenschein (1982) for the case where the set of agents is finite. Readers familiar with that proof will see that the principal difficulty here comes from the fact that the set $S(\bar{y})$ in this case is no longer compact. Hence, we cannot immediately conclude that V is always finite or that the supremum is always achieved.

So we must first show that V makes sense. The map $r \to \log u(r, y(r))$ is measurable, because we may decompose it to

$$r \to (r, y(r)) \to (u(r), y(r)) \to u(r, y(r)) \to \log u(r, y(r))$$

For the measurability (in fact, continuity) of the evaluation map $(u, y) \to u(y)$ see Prop. K.1.2 in Mas-Colell (1985). The continuity of this evaluation map, together with the compactness of K ensure that, when $y(r)$ is a bounded function, $\log u(r, y(r))$ is a bounded function, so the integral is certainly finite. We will show next that these integrals are uniformly bounded in the set $S(\bar{y})$. To accomplish that, we must first demonstrate the following: At any (p, M),

$$\int_R \log u(r, y^*(r))m(r)d\mu \quad < \quad \int_R \log u(r, y_p(r))m(r)d\mu \tag{3}$$

for any measurable function $y^* : R \to R^l_{++}$, $y^* \neq y_p$, y^* bounded above and uniformly bounded away from zero, with

$$p. \int_R y^*(r)d\mu = M$$

Remember that $y_p(r)$ is the demand of the agent r at (p, M), and so it clearly satisfies the condition we impose on y^*.

Observe also that $u(r, y_p(r))$ is bounded above and uniformly bounded away from zero, and so the right hand side of (3) is always finite. The fact that m is bounded above guarantees that $y_p(r)$ and so $u(r, y_p(r))$ is bounded above. Note also that m is uniformly bounded away from zero, which implies that for a given price p, there exists λ small enough such that $\lambda(1, 1, ..., 1)$ is in the budget set of all the agents in R. This guarantees that

$$u(r, y_p(r)) \geq u(\lambda(1, 1, .., 1)) \geq \lambda\tau,$$

where $\tau = \inf\{u(1, 1, .., 1) : u \in K\}$. We claim that $\tau > 0$, so $u(r, y_p(r)) > 0$. If this is not true, there exists $u_n \in K$, with $u_n \to u_0 \in K$ such that $u_n(1, 1, .., 1) \to 0$. However this implies that $u_0(1, 1, .., 1) = 0$ which cannot happen since u_0, being in K, is strongly monotone.

Suppose the inequality to be false for some y^*. Define

$$\theta(r) = \frac{m(r)M}{p.y^*(r)}$$

then $\theta(r)y^*(r) \in B_p(m(r)M)$, which implies that $u(r, y_p(r)) > u(r, \theta(r)y^*(r))$ by strong convexity. Therefore

$$\begin{aligned}\int_R \log u(r, y^*(r))m(r)d\mu &\geq \int_R \log u(r, y_p(r))m(r)d\mu \\ &> \int_R \log u(r, \theta(r)y^*(r))m(r)d\mu \\ &= \int_R \log[\theta(r)]m(r)d\mu + \int_R \log u(r, y^*(r))m(r)d\mu\end{aligned}$$

The last step is justified since θ is bounded above and bounded away from zero, and therefore the integral is finite. This in turn is a consequence of the fact that y^* and m are both bounded above and uniformly bounded away from zero. So we obtain

$$\int_R \log[m(r)M]m(r)d\mu - \int_R \log[p.y^*(r)]m(r)d\mu$$

$$= \int_R \log[\theta(r)]m(r)d\mu < 0$$

or

$$\int_R \log[p.y^*(r)]m(r)d\mu > \int_R \log[m(r)M]m(r)d\mu$$

By Lemma 2.2 this is impossible, and so we have proven (3).

From (3) we conclude that at the point $\bar{y}$,

$$\sup_{S(\bar{y})} \int_R \log u(r, y(r))m(r)d\mu \quad \leq \quad \int_R \log(r, y_p(r))m(r)d\mu \tag{4}$$

where y_p is the market demand function at $(p, M = p.\bar{y})$ (p may be arbitrarily chosen).

We will now show that the supremum is actually achieved. The trick is to appeal to the theorem on the existence of general equilibrium in an exchange economy (see Debreu, 1982), which guarantees that for any $\bar{y} \gg 0$, there exists a price p_0 satisfying $\int_R y_{p_0}(r)d\mu = \bar{y}$, $M = p_0.\bar{y}$. Inequality (4) then implies that

$$V(\bar{y}) = \int_R \log u(r, y_{p_0}(r))m(r)d\mu$$

In general, we conclude that

$$V(\int_R y_p(r)d\mu) = \int_R \log u(r, y_p(r))m(r)d\mu.$$

Let $\bar{z} \neq \int_R y_p(r)d\mu$ satisfy $p.\bar{z} = M$. Then $V(\bar{z}) = \int_R \log u(r, z^{**}(r))m(r)d\mu$ for some z^{**} satisfying $\int_R z^{**}(r)d\mu = \bar{z}$. Inequality (3) says that

$$V(\bar{z}) = \int_R \log u(r, z^{**}(r))m(r)d\mu$$

$$< \int_R \log u(r, y_p(r))m(r)d\mu$$

$$= V(\int_R y_p(r)d\mu)$$

Therefore, given (p, M), the mean market demand $\int_R y_p(r)d\mu$ is equal to the demand of the representative agent at (p, M). QED.

Next we shall extend the result beyond $A = R$ to A a complete separable metric space. The probability space is $(A, \mathcal{F}^*, \mu^*)$, the completion (in the measure theoretic sense) of $(A, \mathcal{F}, \mu)$, where $\mathcal{F}$ is the σ-field of Borel sets. The space $(A, \mathcal{F}, \mu)$ is called nice (or standard Borel) because there is a 1-1 map ϕ from $(R, \mathcal{B})$ onto $(A, \mathcal{F})$ where both the function and its inverse are measurable (see Chapter 1, Theorem 4.10 in Durrett (1991)). The map ϕ induces a measure ν on $(R, \mathcal{B})$ in the usual way. One may easily verify that ϕ and ϕ^{-1} remain measurable when considered as maps between $(R, \mathcal{B}^*, \nu^*)$ and $(A, \mathcal{F}^*, \mu^*)$. Given the economy

$$\mathcal{D} : (A, \mathcal{F}^*, \mu^*) \longrightarrow U \times R$$

we may contruct the economy

$$\mathcal{D} \circ \phi : (R, \mathcal{B}^*, \nu^*) \longrightarrow U \times R.$$

For a given (p, M), the demand of the agent a in $\mathcal{D}$ is the same as the demand of the agent $\phi^{-1}(a)$ in $\mathcal{D} \circ \phi$, since $y_p(a) = y_p \circ \phi(\phi^{-1}(a))$. Mean market demand in the two economies are equal, i.e.,

$$\int_A y_p(a)d\mu^* = \int_R y_p \circ \phi(r)d\nu^*$$

Hence, we have the following corollary to Theorem 2.1:

COROLLARY 2.3

Mean market demand for the economy $\mathcal{D} : (A, \mathcal{F}^*, \mu^*) \longrightarrow U \times R$ at (p, M) could be generated by a representative consumer with the homogenous of degree one utility function

$$V : R^l_{++} \longrightarrow R$$

$$\log V(\bar{y}) = max_{S(\bar{y})} \int_R \log u(\phi(r), y(r))(m \circ \phi)(r)d\nu^*$$

where the set $S(\bar{y})$ consists of those measurable functions bounded above and uniformly bounded away from zero, $y : R \longrightarrow R^l_{++}$, with $\int_R y(r)d\nu^* = \bar{y}$.

2.3 The Representative Consumer in an Exchange Economy with Homothetic Preferences

Given the probability spaces $(R^l_+, \mathcal{B}, \mu_1)$ and $(K, \mathcal{F}, \mu_2)$ where K is defined as in Section 2.1 and $\mathcal{B}$, $\mathcal{F}$ are the Borel σ-fields generated by the topologies on R^l_+ and K respectively, we define the exchange economy $\mathcal{E}$ as the product space

$$(K \times R^l_+, \mathcal{F} \times \mathcal{B}, \mu_2 \times \mu_1).$$

The element $(u, x) \in \mathcal{E}$ represents an agent with preference u and endowment x. We assume that $proj_2(u, x)$ is integrable with respect to $\mu_2 \times \mu_1$, and that

$$\int_{K \times R^l_+} proj_2(u, x) d(\mu_2 \times \mu_1) \gg 0$$

From these assumptions we may conclude that:

1. By Fubini's Theorem, the economy's mean demand

$$\begin{aligned}\int_{K \times R^l_+} proj_2(u, x) d(\mu_2 \times \mu_1) &= \int_K \int_{R^l_+} proj_2(u, x) d\mu_1 d\mu_2 \\ &= \int_K \int_{R^l_+} x d\mu_1 d\mu_2 \\ &= \int_{R^l_+} x d\mu_1 \gg 0\end{aligned}$$

2. We denote the demand of agent (u, x) at price $p \in \Delta^l_{++}$ where $\Delta^l_{++} = \{p \in R^l_{++} : \sum p_i = 1\}$ by $\hat{y}_p(u, x)$ (the agent (u, x) has an income of $p.x$. It is known that $\hat{y}_p(u, x)$ is a continuous function in $K \times R^l_{++}$ (see remark on page 74 in Mas-Colell (1985)). Furthermore,

$$\hat{y}_p(u, x) \le (\frac{p.x}{p_1}, .., \frac{p.x}{p_2})$$

so the mean market demand at price p,

$$\begin{aligned}Y(p) &= \int_{K \times R_+ l} \hat{y}_p(u, x) d(\mu_2 \times \mu_1) \\ &\le (\frac{p. \int_{R^l_+} x d\mu_1}{p_1}, \ldots, \frac{p. \int_{R^l_+} x d\mu_1}{p_l})\end{aligned}$$

and is therefore finite.

3. $Y(p)$ is also continuous with respect to p. This follows easily from the fact that $\hat{y}_p(u,x)$ is continuous with respect to (p,u,x) (see Prop. 2.4.8 in Mas-Colell (1985)) and the fact that K is compact; while R^l_+ is not compact, we may effectively restrict our attention to some compact set E such that $\int_{R^l_+\setminus E} y_p(u,x)d(\mu_2\times\mu_1)$ is arbitrarily small in some neighbourhood of p. The details are left to the reader.

4. $Y(p)$ satisfies the following Desirability Property: Let p_0 be a price satisfying $\sum p_i = 1$ and $p_i = 0$ for some i. Fatou's Lemma says that

$$\int_{K\times R^l_+} \liminf_{p\to p_0} |\hat{y}_p(u,x)| d(\mu_2\times\mu_1) \leq \liminf_{p\to p_0} \int_{K\times R^l_+} |\hat{y}_p(u,x)| d(\mu_2\times\mu_1)$$

Because of monotonicity,

$$\liminf_{p\to p_0} |\hat{y}_p(u,x)| = \infty$$

so

$$\liminf_{p\to p_0} \int_{K\times R^l_+} |\hat{y}_p(u,x)| d(\mu_2\times\mu_1) = \lim_{p\to p_0} |Y(p)| = \infty$$

(1), (2), (3) and (4) together with the obvious fact that the excess demand $Y(p) - \int_{R^l_{++}} x d\mu_1$ satisfies Walras' Law guarantee that an equilibrium price for the economy exists (see Debreu, 1982).

THEOREM 2.4

If in the exchange economy $\mathcal{E}$, the utility functions in the set K are all homogenous of degree one, then

(a) the market demand $Y(p)$ may be generated by a representative consumer with a homogenous of degree one utility function and an endowment equal to the economy's mean endowment,

(b) Y satisfies the weak RMP, i.e., $(p-q).(Y(p)-Y(q)) \leq 0$ for all prices p, q satisfying $p.\int_{R^l_{++}} x d\mu_1 = q.\int_{R^l_{++}} x d\mu_1$, and

(c) the set of price equilibria is convex.

Remark. All the measure theoretic technicalities should not obscure the fact that the proof of (a) is really just a simple two step procedure. Firstly, we add up the demand of agents having the same homothetic preference, and see that this demand is representable by a single consumer (a theorem essentially due to Antonelli (1886)); secondly, we represent the demand of these representative consumers with an overall representative consumer, through Eisenberg's theorem.

Proof of (a)

We denote by $y_p(u,w)$ the demand of an agent with preference u and wealth w. Remember that for an agent with homothetic preferences, $y_p(u,w) = y_p(u,1)w$.

And so, by Fubini's Theorem,

$$\begin{aligned}
\int_{K\times R^l_+} \hat{y}_p(u,x)d(\mu_2\times\mu_1) &= \int_K\int_{R^l_+} \hat{y}_p(u,x)d\mu_1 d\mu_2 \\
&= \int_K\int_{R^l_+} y_p(u,1)p.x d\mu_1 d\mu_2 \\
&= \int_K y_p(u,1)p.(\int_{R^l_+} x d\mu_1)d\mu_2 \\
&= \int_K y_p(u,p.(\int_{R^l_+} x d\mu_1))d\mu_2
\end{aligned}$$

This is the demand of the distribution economy

$$\mathcal{D} : (K, \mathcal{F}, \mu) \longrightarrow U \times R_{++},$$

where $\mathcal{D}(u) = (u, 1)$. Corollary 2.2 tells us that this demand can be represented by a consumer facing price p and income $M = p.(\int_{R^l_+} x d\mu_1)$, with utility V and $m : (K, \mathcal{F}, \mu_2) \longrightarrow R_+$, $m \equiv 1$. We do not need $\mathcal{F}$ to be complete here because it was needed in Theorem 2.1 only to guarantee the measurability of the demand function in that more general context, whereas over here y_p is in fact continuous in K. Note also that K, being a compact subset of a metric space is complete and separable. QED.

Theorem 2.4(b) is a straightforward consequence of the next proposition. (c) in turn follows from (b) (for a proof see Proposition 6.2 in Hildenbrand and Kirman (1988)).

PROPOSITION 2.5

Let $F(p, w)$ be the demand of an agent at price $p \gg 0$ and income w. Assume that

(a) $p.F(p, w) = w$

(b) F satisfies the weak axiom of revealed preference, i.e.,
if $F(\tilde{p}, \tilde{w}) \neq F(\bar{p}, \bar{w})$ and $\tilde{p}.F(\bar{p}, \bar{w}) \leq \tilde{p}.F(\tilde{p}, \tilde{w}) = \tilde{w}$
then $\bar{w} = \bar{p}.F(\bar{p}, \bar{w}) < \bar{p}.F(\tilde{p}, \tilde{w})$

(c) $F(p, kw) = kF(p, w)$

then F satisfies the weak law of demand, i.e., for all prices p, q

$$(F(p, w) - F(q, w)).(p - q) \leq 0$$

Proof

The consumption bundle

$$\frac{w}{q.F(p,w)}F(p, w) = F(p, \frac{w^2}{q.F(p, w)})$$

by (c). If

$$\frac{w}{q.F(p,w)}F(p,w) = F(q,w)$$

then

$$\frac{w}{q.F(p,w)}p.F(p,w) = p.F(q,w)$$

so

$$F(q,w).qF(p,w).p - F(q,w).pF(p,w).q = 0$$

On the other hand, if

$$\frac{w}{q.F(p,w)}F(p,w) = F(p,\frac{w^2}{q.F(p,w)}) \neq F(q,w),$$

then the fact that

$$\frac{wq.F(p,w)}{q.F(p,w)} = w = F(q,w).q$$

will lead us to conclude through the weak axiom that

$$p.[\frac{w}{q.F(p,w)}F(p.w)] < p.F(q,w)$$

$$F(q,w).qF(p,w).p - F(q,w).pF(p,w).q < 0.$$

Combining the two cases, we conclude that

$$F(q.w).qF(p,w).p - F(q,w).pF(p,w).q \leq 0$$

so

$$F(q,w).q(F(p,w).p - F(q,w).p) + F(q,w).p(F(q,w).q - F(p,w).q) \leq 0$$

We shall henceforth refer to the above inequality as $(\star)$. Notice that the expressions in the two brackets add up to $(p-q).(F(p,w)-F(q,w))$. We will try to "cancel away" their coefficients.

Suppose $F(p,w).p \geq F(q,w).p$. This implies by the weak axiom of revealed preference that $F(p,w).q \geq F(q,w).q$ or $F(p,w).q - F(q,w).q \geq 0$. Using $(\star)$, we see that

$$F(q,w).q(F(p,w).p - F(q,w).p) + F(p,w).p(F(q,w).q - F(p,w).q)$$

$$\leq F(q,w).q(F(p,w).p - F(q,w).p) + F(q,w).p(F(q,w).q - F(p,w).q) \leq 0.$$

Dividing by $q.F(q,w) = p.F(p,w) = w$ gives us what we want, i.e.,

$$F(p,w).p - F(q,w).p + F(q,w).q - F(p,w).q$$

$$= (p-q).(F(p,w) - F(q,w)) \leq 0.$$

Suppose now that $F(q,w).q = F(p,w).p < F(q,w).p$, then using $(\star)$ again, we obtain

$$F(q,w).p(F(p,w).p - F(q,w).p) + F(q,w).p(F(q,w).q - F(p,w).q)$$

$$\leq F(q,w).q(F(p,w).p - F(q,w).p) + F(q,w).p(F(q,w).q - F(p,w).q) \leq 0.$$

Dividing by $F(q,w).p$ will give us what we want:

$$F(p,w).p - F(q,w).p + F(q,w).q - F(p,w).q$$

$$= (p-q).(F(p,w) - F(q,w)) \leq 0.$$

QED

The reader may wonder if the assumptions in Proposition 2.5, ostensibly weaker than assuming the existence of a homothetic preference, are nonetheless sufficient to imply it. The answer is no (see Kihlstrom, Mas-Colell and Sonnenschein (1976) under the section "A related conjecture"). It is also easy to check that if we had assumed that the support price of every consumption bundle is unique, then we will get the law of demand, and not just the weak form of the law. Finally, note that the prove of the proposition does not require the continuity of the demand function.

3 Smooth, Homothetic Preferences and RMP

In this section, we shall show that by adding smoothness and regularity assumptions to the exchange economy in Section 2, we may prove that RMP holds. We begin with a complete description of the economy.

3.1 The Economy $\mathcal{E}^2$

We define U^2 to be the set of C^2 utility functions on R^l_{++} with no critical points. We endow U^2 with the topology of C^2 uniform convergence on compacta. This makes U^2 into a metric space which we shall denote as $(U^2, \Im^2)$ (see page 70 of Mas-Colell (1985)). Let K be a compact subset of U^2 that consists of differentiably strictly convex preferences that satisfy the Boundary condition (Definition 2.6.1 in Mas-Colell (1985)).

We assume that the exchange economy's endowments come from some compact subset E of R^l_{++}, with E bounded away from the origin. The economy $\mathcal{E}^2$ is defined in our usual way, as

$$(K \times E, \mathcal{F}^2 \times \mathcal{B}, \mu_2 \times \mu_1)$$

where $\mathcal{F}^2$ and $\mathcal{B}$ are the Borel σ-fields and μ_2 and μ_1 are the measures on K and E respectively. We assume that the economy's mean endowment

$$\int_{K \times E} proj_2(u,x) d(\mu_2 \times \mu_1)$$

$$= \int_E x d\mu_1$$

is finite and in R^l_{++}. Henceforth, we shall denote $\int_E x d\mu_1$ by $\bar{x}$, with $\bar{x} \gg 0$.

We define

$$f : K \times R^l_{++} \times R_{++} \longrightarrow R^l_{++}$$

by $f(u,p,w)$ = Demand of an agent facing price p, with wages w, and utility u. It is well known that with our assumptions f, $\frac{df_i}{dp_j}$, and $\frac{df_i}{dw}$ all exist and are continuous (see Prop. 2.7.2 in Mas-Colell (1985)). This fact in turn guarantees that the demand of the agent (u, x),

$$\hat{f} : K \times R^l_{++} \times E \longrightarrow R^l_{++}$$

where $\hat{f}(u,p,x) = f(u,p,p.x)$, and $\frac{d\hat{f}}{dp_i}$ are continuous for all i. On any set $K \times P \times E$ where P is a compact set of prices in R^l_{++}, these functions are also bounded (we make use here of our assumption that E is bounded away from the origin). Using this fact, we may prove the following:

LEMMA 3.1

The economy $\mathcal{E}^2$'s mean demand at price p,

$$F(p) = \int_{K \times E} \hat{f}(u,p,x) d(\mu_2 \times \mu_1)$$

is continuously differentiable for all $p \in R^l_{++}$, with

$$\frac{dF_i}{dp_j}(p) = \int_{K \times E} \frac{d\hat{f}_i}{dp_j}(u,p,x) d(\mu_2 \times \mu_1).$$

Proof

$$\frac{dF_i}{dp_j}(p^*) = \lim_{p_j \to p_j^*} \frac{F_i(p) - F_i(p^*)}{p_j - p_j^*}$$

$$= \lim_{p_j \to p_j^*} \int_{K \times E} \frac{\hat{f}_i(u,p,x) - \hat{f}_i(u,p^*,x)}{p_j - p_j^*} d(\mu_2 \times \mu_1)$$

For each $p = (p_1^*, p_2^*, .., p_j, .., p_l^*)$, the mean value theorem tells us that

$$\frac{\hat{f}_i(u,p,x) - \hat{f}_i(u,p^*,x)}{p_j - p_j^*} = \frac{d\hat{f}_i(u,q,x)}{dp_j}$$

for some $q = (p_1^*, .., q_j, ..p_l^*)$ with $p_j < q_j < p_j^*$. Since $\frac{d\hat{f}_i}{dp_j}$ is uniformly bounded on the set $K \times N(p^*) \times E$, where $N(p^*)$ is some closed neighbourhood around p^*, we may apply the dominated convergence theorem, which says that

$$\frac{dF_i}{dp_j}(p^*) = \lim_{p_j \to p_j^*} \int_{K \times E} \frac{\hat{f}_i(u,p,x) - \hat{f}_i(u,p^*,x)}{p_j - p_j^*} d(\mu_2 \times \mu_1)$$

$$= \int_{K\times E} \lim_{p_j \to p_j^*} \frac{\hat{f}_i(u,p,x) - \hat{f}_i(u,p^*,x)}{p_j - p_j^*} d(\mu_2 \times \mu_1)$$

$$= \int_{K\times E} \frac{d\hat{f}_i}{dp_j}(u,p,x) d(\mu_2 \times \mu_1)$$

The continuity of $\frac{dF_i}{dp_j}$ follows, via the dominated convergence theorem again, from the continuity of $\frac{d\hat{f}_i}{dp_j}(u,p,x)$. QED

3.2 The Restricted Monotonicity Property

Let C be a convex cone of prices in R^l_{++}, with its apex at the origin. We say that the economy's mean demand $F(p)$ satisfies the Restricted Monotonicity Property (RMP) in C if whenever we have $p, q \in C$, $p \neq q$, with $p.\bar{x} = q.\bar{x}$ ($\bar{x}$ is the economy's mean endowment)

$$(p-q).(F(p) - F(q)) < 0$$

When we say that F satisfies RMP without saying where, we mean $C = R^l_{++}$. A sufficient condition for F to satisfy RMP in C is for the Jacobian $dF(p)$ to be negative definite on the plane $V(\bar{x}) = \{v \in R^l : v.\bar{x} = 0\}$ for $p \in C$. Note that since $F(p) = F(kp)$ for any positive multiple k, $dF(p) = kdF(kp)$, so we need only check that $dF(p)$ is negative definite on the prices in $C^* = \{\frac{p}{|p|} : p \in C\} \subset \Delta^l_{++}$. It is also clear, that if we know, a priori, that the equilibrium prices of an economy are confined to C (or equivalently, C^*) and that RMP holds in C, then there is only one equilibrium price, up to multiples of course. The Slutsky decomposition of $dF(p)$ will give us some indication of the work we need to do to prove that it is negative definite.

$$dF(p)_{ij} = \int_{K\times E} \frac{d\hat{f}_i}{dp_j}(u,p,x) d(\mu_2 \times \mu_1)$$

$$= \int_{K\times E} s_{ij}(u,p,p.x) - \frac{df_i}{dw}(u,p,p.x) f_j(u,p,p.x) + x_j \frac{df_i}{dw}(u,p,p.x) d(\mu_2 \times \mu_1)$$

Since we already know that the substitution matrix, $S(p)$, is negative semi-definite, our real work lies in showing that the two income effect matrices,

$$A(p)_{ij} = \int_{K\times E} \frac{df_i}{dw}(u,p,p.x) f_j(u,p,p.x) d(\mu_2 \times \mu_1)$$

$$B(p)_{ij} = \int_{K\times E} \frac{df_i}{dw}(u,p,p.x) x_j d(\mu_2 \times \mu_1)$$

are, respectively, positive and negative definite on $V(\bar{x})$, or at least that they will not be so misbehaving as to overwhelm the negative definiteness of the substitution matrix.

3.3 RMP in the Exchange Economy $\mathcal{E}^2$

THEOREM 3.2

Assuming that all the preferences in K are homothetic, the market demand F of the exchange economy $\mathcal{E}^2$ satisfies RMP.

Proof

We will show that $B(p)v = 0$ for all $v \in V(\bar{x})$, and that $A(p)$ is positive semi-definite (for all v, not just v in $V(\bar{x})$), these together with the fact that $S(p)$ is negative semi-definite will guarantee that $dF(p)$ is negative semi-definite. Lastly, we will prove that $dF(p)$ is negative definite on $V(\bar{x})$ and not just negative semi-definite. The crucial property of homothetic preferences we will be using throughout the prove is the fact that when u is homothetic, $\frac{df_i}{dw}(u,p,w)$ may depend on p but is independent of w. The reader might want to bear in mind that the method of proof which he will encounter in Section 5 is essentially a more complicated version of what follows here.

We will first look at $B(p)$.

$$
\begin{aligned}
B(p)_{ij} &= \int_{K\times E} \frac{df_i}{dw}(u,p,p.x)x_j d(\mu_2 \times \mu_1) \\
&= \int_K \int_E \frac{df_i}{dw}(u,p,1)x_j d\mu_1 d\mu_2 \\
&= \int_K \frac{df_i}{dw}(u,p,1)[\int_E x_j d\mu_1]d\mu_2 \\
&= \bar{x}_j \int_K \frac{df_i}{dw}(u,p,1)d\mu_2
\end{aligned}
$$

so clearly $B(p)v = 0$ since $v.\bar{x} = 0$.

Now let us look at $A(p)$.

$$
\begin{aligned}
v.A(p)v &= \int_E \int_K \sum_i \sum_j v_i v_j \frac{df_i}{dw}(u,p,p.x)f_j(u,p,p.x)d\mu_2 d\mu_1 \\
&= \int_E \int_K \sum_i \sum_j v_i v_j \frac{df_i}{dw}(u,p,1)\frac{df_j}{dw}(u,p,1)p.x d\mu_2 d\mu_1 \\
&= \int_E p.x \int_K \sum_i \sum_j v_i v_j \frac{df_i}{dw}(u,p,1)\frac{df_j}{dw}(u,p,1)d\mu_2 d\mu_1 \\
&= \int_E p.x[\int_K (v.\frac{df}{dw}(u,p,1))^2 d\mu_2]d\mu_1 \\
&= [\int_E p.x d\mu_1][\int_K (v.\frac{df}{dw}(u,p,1))^2 d\mu_2] \\
&\geq 0
\end{aligned}
$$

So we have shown that $dF(p)$ is negative semi-definite. Now to see why it is not possible for $v.dF(p)v = 0$ if $v \neq 0$.

Since K consists only of differentiably strictly convex preferences, the Slutsky matrix satisfies $v.S(u,p,p.x)v < 0$ for every v not collinear with p (see Prop. 2.7.8 in Mas-Colell (1985), and Kihlstrom, Mas-Colell, and Sonnenschein (1976)). Integrating, we obtain

$$\begin{aligned} v.[\int_{K\times E} S(u,p,p.x)d(\mu_2\times\mu_1)]v &= v.S(p)v \\ &< 0 \end{aligned}$$

If $v = \lambda p$ then

$$\begin{aligned} p\cdot f(u,p,w) &= w \\ p\cdot w\frac{df}{dw}(u,p,1) &= w \\ p\cdot \frac{df}{dw}(u,p,1) &= 1 \\ v\cdot \frac{df}{dw}(u,p,1) &= \lambda \end{aligned}$$

This implies that

$$\begin{aligned} v.A(p)v &= \int_E p.x\int_K \lambda^2 d\mu_2 d\mu_1 \\ &= (\int_E p.x d\mu_1)\lambda^2 = (p.\bar{x})\lambda^2 \\ &> 0 \end{aligned}$$

Therefore $dF(p)$ is negative definite on $V(\bar{x})$ for all $p\in R^l_{++}$. QED.

As a consequence of Theorem 3.2, the economy $\mathcal{E}^2$ has a unique price equilibrium. If there were two different prices, p^* and p^{**} with $p^*.\bar{x} = p^{**}.\bar{x}$ and $F(p^*) = F(p^{**}) = \bar{x}$, then

$$(p^* - p^{**}).(F(p^*) - F(p^{**})) = 0$$

which contradicts RMP. Observe also that because of the core equivalence theorem of Aumann (1964), if μ_1 and μ_2 are atomless, the core too will be unique. RMP also guarantees that the price equilibrium is stable under various tatonnement processes. We discuss this next: but before we end this section, we verify that the result of Section 2 applies to the economy $\mathcal{E}^2$.

PROPOSITION 3.3

In the economy $\mathcal{E}^2$, market demand $F(p)$ can be generated by a representative consumer with a homogenous of degree one utility function, and an endowment equal to the economy's mean endowment $\bar{x}$.

Proof

Following the same steps in Theorem 2.4 we see that

$$F(p) = \int_K f(u, p, p.\bar{x}) d\mu_1.$$

This is the demand of the distribution economy

$$\mathcal{D} : (K, \mathcal{F}^2, \mu_2) \rightarrow (U \times R_{++}, \mathcal{F}^1 \times \mathcal{B}, \mu_2 \times \mu_1)$$

where $\mathcal{D}(u) = (u, 1)$, $M = p.\bar{x}$, and $m : (K, \mathcal{F}^2, \mu_2) \rightarrow R_{++}$, $m \equiv 1$.

To apply Corollary 2.3, $\mathcal{D}$ must be measurable: this is certainly true as a set in U^2 that is closed in $(U, \Im)$ is also closed in $(U^2, \Im^2)$, so $\mathcal{D}$ is in fact continuous. Furthermore, the set K, compact in $(U^2, \Im^2)$ must also be compact in $(U, \Im)$. So the conditions of Corollary 2.3 are fulfilled and we conclude that market demand is representable by a single consumer with a homogenous of degree one utility function. QED

4 Tatonnement under the Weak Axiom

The word "tatonnement" is French for "to proceed by trial and error". It refers to a process by which a price vector in the economy is adjusted according to some rule dependent on the price and demand conditions. The aim is that such a rule will lead the price to converge to an equilibrium price. Since the assumption of perfect competition has all agents taking prices as given, the actual job of changing the price is traditionally assigned to an auctioneer. His job is to announce the price, receive feedback from the agents on demand at those prices, adjust the price according to some rule, announce prices again, etc. Notice that no trading takes place until the equilibrium price is reached (see Chapter 11 in Arrow and Hahn (1971)).

One need hardly point out that this is a rather unsatisfactory model of market dynamics. Though there are models of trading off equilibrium (see, for example, Keisler (1986,1990)) none, for now, have gained wide acceptance. Mathematically, the tatonnement process is usually represented by a differential equation system. Differential equations with solutions that converge almost always to an equilibrium are known to exist (see Smale (1976) and Kamiya (1990)). However, they suffer from the defect that the auctioneer requires too much information to implement the tatonnement rule. Not only must he know the excess demand, $Z(p)$, he must also know the Jacobian $dZ(p)$. Unfortunately, unless $Z(p)$ is assumed to satisfy strong properties like gross substitutability, there is little hope of finding a convergent tatonnement that requires significantly less information (see Saari and Simon, 1978).

In this section, we consider tatonnement processes under the assumption that $Z(p)$ satisfies the weak axiom at equilibrium (WAE), i.e., if p^* is the unique equilibrium price in the economy, then for all $p \neq p^*$, $p^*.Z(p) > 0$ (we will show later that this is a consequence of RMP). The two tatonnement processes we will be considering are

(I)

$$\frac{dp_i}{dt} = \alpha_i z_i(p)$$

where $\alpha_i > 0$, $i = 1, 2, .., l$, and

(II)

$$\frac{dp_i}{dt} = \beta_i p_i z_i(p)$$

where $\beta_i > 0$, and $i = 1, 2, .., l$.

These two tatonnement processes agree with the intuitive (or traditional) notions of how markets operate: when a good is in excess demand, its price rises, when it is in excess supply, its price falls, and, secondly, that price changes for a particular good obey rules dependent primarily on demand for that good, with little reference to demand in other markets. So when prices do converge to an equilibrium, they do so without the benefit of an over-arching intelligence. Returning to the story of the auctioneer, in the case of tatonnement processes like (I) and (II) we may imagine, if we like, not one, but l auctioneers: each auctioneer is in charge of changing the price of one good based on p and the demand for that good, with a view to equalising demand and supply for that one good. No co-ordination with the other auctioneers is necessary, except agreement on when to begin each round of price announcements. Furthermore, each auctioneer, need not know anything about the demand in other markets.

We want to show that with the assumption of WAE, tatonnement processes (I) and (II) converge to the equilibrium price, no matter what is the initial price condition. Before we do that, we prove that WAE is a consequence of RMP. This result is not new (see Prop. 5.7.3 in Mas-Colell (1985)), but the proof is short and so we repeat it here for completeness.

PROPOSITION 4.1

Suppose the excess demand function $Z : R^l_{++} \to R^l$ satisfies RMP. Then Z satisfies the weak axiom of revealed preference (WA), i.e.,

if $p.Z(q) \leq 0$ then $q.Z(p) > 0$, provided $Z(p) \neq Z(q)$.

In particular, for all prices $p \neq \lambda p^*$, where λ is a positive scalar and p^* the equilibrium price, $p^*.Z(p) > 0$ (WAE).

Proof

Suppose $p.Z(q) \leq 0$. Choose λ such that $p.\bar{x} = \lambda q.\bar{x}$. Then

$$(p - \lambda q).(Z(p) - Z(\lambda q)) < 0$$

provided $Z(p) \neq Z(\lambda q) = Z(q)$, so

$$-p.Z(\lambda q) - \lambda q.Z(p) < 0$$

Since $p.Z(\lambda q) = p.Z(q) \leq 0$, this means that $\lambda q.Z(p) > 0$, which is what we want. When Z satisfies RMP, the equilibrium price, if it exists is unique; so then $p.Z(p^*) = 0$ will imply that $p^*.Z(p) > 0$ for all $p \neq \lambda p^*$. QED.

Before we prove our principle claims, let us define stability a little more carefully. Suppose p^* is an equilibrium price. Let $p(t)$ be a solution to

$$\frac{dp_i}{dt} = H_i(p),$$

for $i = 1, 2, .., l$. We assume that for any given initial condition $p(0)$, the solution to $p(t)$ is unique and is defined on $[0, \infty)$. Then

DEFINITION

The price p^* is asymptotically stable on the set $\mathcal{S}$ if $\lim_{t\to\infty} p(t) = \lambda p^*$, for any solution $p(t)$ with $p(0) \in \mathcal{S}$. Here λ is a positive scalar; obviously in an exchange economy λp^* is also an equilibrium price. We say that p^* is globally, asymptotically stable if $\mathcal{S} = R^l_{++}$.

We first prove a theorem on the tatonnement process (I). The fact that solutions to (I) under the assumptions we will be making last forever is a rather well- known fact. The proof we give here is a more elementary variation of Proposition 5.1 in Dierker(1974). It is also known that if the eqilibrium price is unique and that excess demand satisfies WAE then the equilibrium price is globally asymptotically stable. On the other hand, stability under (II) appears to be less well known, though stability under a more general class of tatonnement processes has been proven before, with the assumption of gross substitubility (see Section 19.2 in Nikaido (1968)). As it turns out, the proof of stability under (II) is particularly simple when WAE is assumed. For another result on stability under WAE see Nikaido and Uzawa (1960).

THEOREM 4.2

Suppose that $Z : R^l_{++} \to R^l$ is a C^1 function satisfying:

(a). $Z(p) = Z(\lambda p)$ for all $\lambda > 0$ (homogeneity),

(b). $p.Z(p) = 0$ (Walras' Law),

(c). $Z(p)$ is bounded below,

(d). for any $p_n \to p_0$, where p_0 has some component equal to zero,

$$|Z(p_n)| \to \infty,$$

(Desirability Property)

then for any initial condition $p(0)$, there is a unique solution to (I) defined for $t \in [0, \infty)$. If, in addition,

(e). there exists a unique equilibrium price p^* with $p^*.Z(p) > 0$ for all $p \neq \lambda p^*$ (WAE), then p^* is globally, asymptotically stable under (I).

Proof

It is well known that given an initial condition $p(0) \in R^l_{++}$, the solution to (I) exists and is unique (see any text on Ordinary Differential Equations). What we need to prove is that the solution $p(t)$ exists for $t \in [0, \infty)$. We first note that

$$\frac{d}{dt}[\alpha_1^{-1}p_1^2 + \alpha_2^{-1}p_2^2 + ...\alpha_l^{-1}p_l^2] = 2(p_1 z_1(p) + ...p_l z_l(p)) = 0$$

so that $p(t)$ lives in the set

$$\Omega = \{p \in R^l_{++} : \alpha_1^{-1}p_1^2 + ...\alpha_l^{-1}p_l^2 = \alpha_1^{-1}p_1^2(0) + ...\alpha_l^{-1}p_l^2(0)\}$$

The solution $p(t)$ will last forever if we can find a compact set $\Omega'' \subset \Omega$ such that $p(0) \in \Omega''$ and $p(t)$ never leaves Ω'' (see Chapter 8, Section 5 in Hirsch and Smale (1974)). We now set out to construct Ω''. The idea is to construct a sort of buffer around the boundary of Ω beyond which $p(t)$ will not cross. Define

$$\begin{aligned}
W_{k_1}(\delta_1) &= \{p \in \Omega : \sum_{i \neq k_1} p_i \leq \delta_1\} \\
W_{k_1,k_2}(\delta_2) &= \{p \in \Omega : \sum_{i \neq k_1,k_2} p_i \leq \delta_2, p_{k_1} \geq \frac{1}{l}\delta_1, p_{k_2} \geq \frac{1}{l}\delta_1\} \\
&\vdots \\
W_{k_1,\ldots,k_{l-1}}(\delta_{l-1}) &= \{p \in \Omega : \sum_{i \neq k_1,\ldots,k_{l-1}} p_i \leq \delta_{l-1}, p_{k_1} \geq \frac{1}{l}\delta_{l-2}, .., p_{k_{l-1}} \geq \frac{1}{l}\delta_{l-2}\}
\end{aligned}$$

where $\delta_1, \delta_2, ..., \delta_l$ will be chosen in the way we now outline.

For any price $p \in W_1(\delta_1)$, the price of 1, p_1 is bounded away from zero. Walras' law, together with the fact that Z is bounded below means that z_1 is bounded above, so we may conclude from (d), that for δ_1 sufficiently small,

$$\sum_{i \neq 1} z_i(p)\alpha_i \geq a > 0,$$

for all $p \in W_1(\delta_1)$. Indeed, δ_1 can be chosen so small that if $p \in W_k(\delta_1)$, then

$$\sum_{i \neq k} z_i(p)\alpha_i \geq a > 0.$$

Next, we choose a suitable δ_2 for $W_{1,2}(\delta_2)$. For a price $p \in W_{1,2}(\delta_2)$, p_1 and p_2 are bounded away from zero. Walras' law, together with the fact that Z is bounded below means that z_1 and z_2 are bounded above, so we may conclude from (d), that for δ_2 sufficiently small,

$$\sum_{i \neq 1,2} z_i(p)\alpha_i \geq a > 0,$$

for all $p \in W_{1,2}(\delta_2)$. Once again, δ_2 can be chosen so small that whenever $p \in W_{k_1,k_2}(\delta_2)$,

$$\sum_{i \neq k_1,k_2} z_i(p)\alpha_i \geq a > 0.$$

In addition, we require that $\delta_2 + \frac{1}{l}\delta_1 < \delta_1$. We proceed along these lines to choose $\delta_3, ..., \delta_{l-1}$ with

$$\delta_s + \frac{1}{l}\delta_{s-1} < \delta_{s-1}$$

and such that

$$p \in W_{k_1,..,k_s}(\delta_s)$$

will guarantee that

$$\sum_{i \neq k_1,..,k_s} z_i(p)\alpha_i \geq a > 0.$$

Note also that $\delta_1, .., \delta_{l-1}$ can be chosen with

$$p(0) \notin W = \bigcup W_{k_1,k_2,..,k_s}(\delta_s)$$

We define $\Omega' = \Omega \setminus W$ and make the following claims:
Claim 1

$$\Omega' = \{p \in \Omega : \sum_{i \neq k_1,k_2,..,k_s} p_i > \delta_s, \forall k_1, .., k_s, 0 \leq s \leq l-1\} = \Omega \setminus W$$

That $\Omega' \subset \Omega \setminus W$ is obvious; for the other direction, let us suppose that there exists $p \in \Omega \setminus W$ such that $\sum_{i \neq 1,2,3} p_i \leq \delta_3$ (we use 1,2,3, instead of $k_1, .., k_s$ for notational convenience, no loss of generality is involved). Since $p \notin W_{1,2,3}(\delta_3)$, we assume that $p_1 < \frac{1}{l}\delta_1$, in which case $\sum_{i \neq 2,3} p_i < \delta_3 + \frac{1}{l}\delta_2 < \delta_2$. Now $p \notin W_{2,3}(\delta_2)$ either, so we assume that $p_2 < \frac{1}{l}\delta_1$, in which case $\sum_{i \neq 3} p_i < \delta_2 + \frac{1}{l}\delta_1 < \delta_1$. But this implies that $p \in W_3(\delta_1) \subset W$: we have a contradiction.
Claim 2

$$intW = \bigcup intW_{k_1,k_2,..k_s}(\delta_s)$$

where "int" refers to the interior of a set.
That

$$\bigcup intW_{k_1,k_2,..k_s}(\delta_s) \subset intW$$

is obvious; we prove the other inclusion by contradiction. Suppose $p \in intW$, and p is on the boundary of $W_{k_1,..,k_s}$ for all $k_1, .., k_s$. Remember that from the definition of $W_{k_1,..,k_s}(\delta_s)$, if p is such that $\sum_{i \neq k_1,..,k_s} p_i = \delta_s$ for all $k_1, .., k_s$, then p is on the boundary of $\Omega' = \Omega \setminus W$ (see claim 1) so p is on the boundary of W: a contradiction. Therefore, without loss of generality, we assume that $\sum_{i\neq 1,2,3} p_i < \delta_3$ and $p_1 = \frac{1}{l}\delta_2$, in which case $\sum_{i\neq 2,3} p_i < \delta_3 + \frac{1}{l}\delta_2 < \delta_2$. Since $p \notin intW_{2,3}(\delta_2)$, $p_2 = \frac{1}{l}\delta_1$, and it follows that $\sum_{i\neq 3} p_i < \delta_2 + \frac{1}{l}\delta_1 < \delta_1$. This is a contradiction since it implies that $p \in W_3(\delta_1)$.

We define the compact set Ω'' to be the closure of Ω'. We will now proof that the solution to (I) does not leave Ω''. Suppose that it does, then there exists a time t' such that $p(t') \in intW = \bigcup intW_{k_1,k_2,..k_s}(\delta_s)$. Suppose that

$$t^0 = \inf\{t : p(t) \in intW_{k'_1,...,k'_{s'}}(\delta_{s'})\}$$

$$\leq \inf\{t : p(t) \in intW_{k_1,...,k_s}(\delta_s)\}$$

for all $W_{k_1,...,k_s}(\delta_s)$, i.e., $W_{k'_1,...,k'_{s'}}(\delta_{s'})$, is the first of the sets in W the path of $p(t)$ enters. For $t < t^0$, $p(t) \in \Omega''$, and $p(t^0) \in W_{k'_1,...,k'_{s'}}(\delta_{s'})$. As Ω'' is the closure of $\Omega' = \Omega \setminus W$, for $t = t^0$, $\sum_{i\neq k'_1,...,k'_{s'}} p_i(t) \geq \delta_{s'}$. In addition,

$$\frac{d}{dt} \sum_{i\neq k'_1,...,k'_{s'}} p_i(t)|_{t=t^0} = \sum_{i\neq k'_1,...,k'_{s'}} \alpha_i z_i(p(t^0))$$

$$\geq a > 0$$

This means that there exists $\epsilon > 0$ such that for $t^0 \leq t \leq t + \epsilon$,

$$\sum_{i\neq k'_1,..,k'_{s'}} p_i(t) \geq \delta_{s'},$$

contradicting $t^0 = \inf\{t : p(t) \in intW_{k'_1,...,k'_{s'}}(\delta_{s'})\}$. Therefore, $p(t)$ never leaves Ω''. This completes the proof that any solution to (I) lasts forever.

We will now show that p^* is globally, asymptotically stable under (I). We arrange to have $\alpha_1^{-1}p_1^{*2}+..+\alpha_l^{-1}p_l^{*2} = \alpha_1^{-1}p_1^2(0)+..+\alpha_l^{-1}p_l^2(0)$. The Lyapounov function is the (modified) Euclidean distance, defined by

$$L : \Omega \longmapsto R_+$$

$$L(p) = \sum_{i=1}^{l} \frac{(p_i - p_i^*)^2}{2\alpha_i}$$

Clearly $L(p^*) = 0$ and $L(p) > 0$ for $p \neq p^*$. Furthermore,

$$\begin{aligned} \frac{dL}{dt} &= \sum_{i=1}^{l} \frac{2(p_i - p_i^*)}{2\alpha_i}\frac{dp_i}{dt} \\ &= \sum_{i=1}^{l}(p_i - p_i^*)z_i(p_i) \\ &= -p^*.Z(p) < 0 \end{aligned}$$

This suffices as proof that (I) is globally, asymptotically stable. QED.

The next theorem deals with stability under (II).

THEOREM 4.3

Suppose that $Z : R^l_{++} \longrightarrow R^l$ is a C^1 function satisfying:

(a). $Z(p) = Z(\lambda p)$ for all $\lambda > 0$ (homogeneity),

(b). $p.Z(p) = 0$ (Walras' Law), and

(c). there exists a unique equilibrium price p^* with $p^*.Z(p) > 0$ for all $p \neq \lambda p^*$ (WAE),

then p^* is globally, asymptotically stable under (II).

Proof

Once again, it is well-known that under our assumptions, existence and uniqueness of the solution, given the initial condition $p(0)$ is guaranteed. To show that the solution lasts forever, we first observe that

$$\frac{d}{dt}[\beta_1^{-1}p_1 + \ldots + \beta_l^{-1}p_l] = \sum_{i=1}^{l} p_i z_i(p) = 0$$

so

$$p(t) \in \Psi = \{p \in R^l_{++} : \beta_1^{-1}p_1 + \ldots + \beta_l^{-1}p_l = \beta_1^{-1}p_1(0) + \ldots + \beta_l^{-1}p_l(0)\}$$

Assuming p^* to be in Ψ, the Lyapounov function we need is

$$\mathcal{L} : \Phi \longrightarrow R_+,$$

$$\mathcal{L}(p) = \sum_{i=1}^{l} \beta_i^{-1}(p_i - p_i^* \log p_i - p_i^* + p_i^* \log p_i^*).$$

It is easy to check that for $p \in \Psi$, $p \neq p^*$, $\mathcal{L}(p) > 0$ and, clearly $\mathcal{L}(p^*) = 0$. Furthermore,

$$\begin{aligned} \frac{d\mathcal{L}}{dt} &= \sum_{i=1}^{l} \beta_i^{-1}(1 - \frac{p_i^*}{p_i})\beta_i p_i z_i(p(t)) \\ &= \sum_{i=1}^{l}(p_i - p_i^*)z_i(p(t)) \\ &= -p^*.Z(p(t)) < 0 \end{aligned}$$

if $p(t) \neq p^*$. Lastly, we observe that if $p_n \longrightarrow p_0 \in R^l_+ \setminus R^l_{++}$, then $\mathcal{L}(p_n) \longrightarrow \infty$; as such, $\mathcal{L}^{-1}[0, \mathcal{L}(p(0))]$ is a compact subset in Ψ. With $\frac{d\mathcal{L}}{dt}(p(t)) < 0$, we see that $p(t) \in \mathcal{L}^{-1}[0, \mathcal{L}(p(0))]$. The solution is always contained in a compact set, therefore, it lasts forever and converges to p^*. QED.

Lastly, we re-visit the economy $\mathcal{E}^2$.

COROLLARY 4.4

Assuming that all the preferences in K are homothetic, the exchange economy $\mathcal{E}^2 = (K \times E, \mathcal{F} \times \mathcal{B}, \mu_2 \times \mu_1)$ (as defined in Section 3.1) has the following properties:

(a). its market (excess) demand satisfies RMP,

(b). an equilibrium price p^* exists, and is unique,

(c). p^* is globally, asymptotically stable under (I) and (II).

Proof

Put together Theorems 3.2, 4.2 and 4.3. QED.

5 Heterogenous Consumers without Homothetic Preferences

In this section, we shall show how we may obtain RMP in a sequence of exchange economies $\mathcal{E}_n$ without assuming homothetic preferences. In its place, however, we need to make heterogeneity assumptions on the preferences of agents. This heterogeneity is described in terms of homothetic transformations of preferences. We will define homothetic transformations and give a description of the economies $\mathcal{E}_n$ in Section 5.1. Section 5.2 gives an overview of the proof we wish to make. After this, we examine the intrinsic and extrinsic income effects, and show how on any given closed cone of prices, RMP will hold in the cone for the economy $\mathcal{E}_n$, n sufficiently large.

5.1 Homothetic Transformations and $\mathcal{E}_n$

Let $u : R^l_+ \to R$ be a utility function. We assume that u satisfies the following properties:

(i). the preference it induces is differentiably strictly convex,

(ii). if $f(u, p, w)$ is the demand function associated with u, then for any compact set, K, of strictly positive prices, $|\frac{df_i}{dw}(u, p, w)|$ is uniformly bounded for all $(p, w) \in K \times R_{++}$.

A word on assumption (ii). Since

$$p_1 f_1 + \dots p_l f_l = w$$

$$p_1 \frac{df_1}{dw} + \dots p_l \frac{df_l}{dw} = 1,$$

so the assumption will be true if we assume that the $\frac{df_i}{dw}(p, w)$ is bounded below for all $w > 0$ on any compact set of prices. In particular, this will be true if there are no inferior goods, i.e., $\frac{df_i}{dw}(p, w) \geq 0$ for all i, p, and w. The absence of inferior goods is not unreasonable provided we think of goods in sufficiently broad categories. We emphasize though that our assumption is considerably weaker.

The homothetic t-transformation of u is another utility function, defined by $u_t(x) = u(e^{-t}x)$. The following observations follow easily:

$$f(u^t, p, w) = e^t f(u, p, e^{-t}w).$$

Hence,

$$\frac{df_i}{dw}(u_t, p, w) = \frac{df_i}{dw}(u, p, e^{-t}w)$$

for $w > 0$, and

$$\frac{df_i}{dp_j}(u_t, p, w) = e^t \frac{df_i}{dp_j}(u, p, e^{-t}w)$$

for any i, j. When u is homothetic, a transformation leaves demand unchanged. We denote by $H(u)$ the class of homothetic transformations of u, i.e.,

$$H(u) = \{u_t : t \in R\}.$$

Let $U^* = H(u^0) \cup H(u^1) \cup ...H(u^s)$ where u^0 is homothetic. A sequence of probability measures (ν_n) on U^* is given by $\nu_n(H(u^r)) = \phi^r$ where $\Sigma\phi^r = 1$, and

$$\nu_n(\{u_t^r : t_1 \le t \le t_2\}) = \phi^r \int_{t_1}^{t_2} h_n^r(t)dt$$

where the density functions $h_n^r, r = 0, 1, 2, .., s$, are assumed to have compact support. We denote by μ_n^r the measure on $H(u^r)$ (and R) generated by h_n^r so

$$\nu_n(\{u_t^r : t_1 \le t \le t_2\}) = \phi^r \mu_n^r(\{u_t^r : t_1 \le t \le t_2\})$$

With the sequence of probability spaces $(U^*, \mathcal{F}, \nu_n)$ we may construct a sequence of exchange economies $\mathcal{E}_n = (U^* \times E, \mathcal{F} \times \mathcal{B}, \nu_n \times \mu)$. We assume that the endowment space E is compact and bounded away from zero. The mean market demand is

$$\hat{F}_n : R_{++}^l \to R_+^l$$

$$\hat{F}_n = \sum_{r=0}^{s} \int_{H(u^r)\times E} f(u_t^r, p, p \cdot x) d(\nu_n \times \mu)$$

$$= \sum_{r=0}^{s} \phi^r \int_{R\times E} f(u_t^r, p, p \cdot x) d(\mu_n^r \times \mu)$$

$$= \sum_{r=0}^{s} \phi^r F_n^r(p)$$

where $F_n^r(p)$ is the mean demand of class $H(u^r)$. The following result should be fairly obvious; the proof is essentially the same as Lemma 3.1.

LEMMA 5.1

The functions $\hat{F}_n(p)$ and $F_n^r(p)$, $r = 0, 1, 2, ...s$ are continuously differentiable with

$$\frac{d\hat{F}_n(p)_i}{dp_j} = \sum_{r=0}^{s} \phi^r \int_{R\times E} [\frac{df_i}{dp_j}(u_i^r, p, p.x) + x_j \frac{df_i}{dw}(u_i^r, p, p.x)] d(\mu_n^r \times \mu)$$

and

$$d\hat{F}_n(p) = \sum_{r=0}^{s} \phi^r dF_n^r(p).$$

5.2 The Strategy Reviewed

Our objective is the same as in Section 3: To try to make $d\hat{F}_n(p)$ a negative definite matrix on $V(\bar{x})$ (where $\bar{x}$ is the mean endowment) for n sufficiently large and so establish RMP. In particular, we would like to impose conditions on the sequence h_n^r of density functions that will allow us to reach such a conclusion. Notice that u^0 is homothetic, and so we know that $F_n^0(p)$ is the same for all n and that its Jacobian is negative definite on $V(\bar{x})$. The existence of a segment of each economy $\mathcal{E}_n$ with agents having u^0 as its utility function also guarantees that the equilibrium prices of all the economies are contained in the interior of a compact, convex set of prices in Δ_{++}^l which we shall call P. We shall show that $d\hat{F}_n(p)$ is eventually negative definite on $V(\bar{x})$ for all prices in P. This guarantees that RMP holds for all prices in the cone $R_{++} \times P$, which in turn guarantees the uniqueness of the equilibrium price of $\mathcal{E}_n$ for n sufficiently large, since the equilibria of all the economies in the sequence were, a priori, restricted to P.

We want to conclude that $v.d\hat{F}_n(p)v < 0$ for v in $V(\bar{x})$ and p in P. We may restrict our attention further by confining v to $|v| = 1$. This set has the virtue of being compact. Because $v.dF_n^0(p)v$ is jointly continuous on the set $P \times \{v : v \in V(\bar{x}), |v| = 1\}$ (henceforth to be called Q) we may conclude that there is some $c > 0$ such that $v.dF_n^0(p)v \leq -c < 0$ for $(p, v) \in Q$. We shall show that when n gets big, $v.dF_n^r(p)v$ can be made arbitrarily small on Q, i.e., given $\epsilon > 0$, there exists N such that whenever $n \geq N$, $v.F_n^r(p)v \leq \epsilon$ for $r = 1, 2, .., s$. This will then guarantee that, eventually, $v.d\hat{F}_n(p)v < 0$ on Q which implies that

$$v.d\hat{F}_n(p)v < 0$$

on $P \times V(\bar{x})$, which is what we want. Without loss of generality, we may confine our study to the Jacobian of a single homothetic class, dF_n^r. Warning: We shall henceforth drop the superscript r, and when it is convenient, write $f(p, w)$ when we mean $f(u, p, w)$.

Once again we may split the Jacobian into three matrices: the Slutsky matrix which we know to be negative semi-definite and the two Income Effect matrices,

$$A_n(p)_{ij} = \int_{R\times E} f_j(u_t, p, p.x)\frac{df_i}{dw}(u_t, p, p.x)d(\mu_n \times \mu)$$

$$= \int_E [\int_R f_j(u_t, p, p.x)\frac{df_i}{dw}(u_t, p, p.x)h_n(t)dt]d\mu$$

and

$$B_n(p)_{ij} = \int_{R\times E} x_j \frac{df_i}{dw}(u_t, p, p.x)d(\mu_n \times \mu)$$

$$= \int_E [\int_R x_j \frac{df_i}{dw}(u^t, p, p.x)h_n(t)dt]d\mu$$

We will have to find conditions under which we could guarantee that $v.A_n(p)v$ and $v.B_n(p)v$ are well behaved on the set Q for n sufficiently large. The idea, in a nutshell, is to use homothetic transformations to re-capture some of the virtues of homothetic preferences, so that we may carry out an approximate version of what we did in Section 3.

Before we begin studying matrix $B_n(p)$, we must give a precise meaning to "increasing heterogeneity". We have two definitions.

(1) Dispersion Property (1)

Suppose that the density functions h_n satisfy the following: Let I be a compact interval. Define $\Phi_n : I \rightarrow R$ by

$$\Phi_n(k) = \int_{-\infty}^{\infty} |h_n(t) - h_n(t+k)|dt.$$

Then $\Phi_n \rightarrow 0$ uniformly on I. If this is true for *all* compact intervals I, we say that the sequence $\{h_n\}$ has Dispersion Property (1).

(2) Dispersion Property (2)

The sequence $\{h_n\}$ satisfies Dispersion Property (2) if $|h'_n|$ is integrable, with $\int_R |h'_n(t)|dt \rightarrow 0$ as $n \rightarrow 0$.

Dispersion property (2) is similar to that employed by Grandmont (1992). It is stronger than dispersion property (1).

PROPOSITION 5.2[1]

If the sequence of density functions $\{h_n\}$ satisfies dispersion property (2), then it satisfies dispersion property (1).

Proof

We observe that

$$\Phi_n(k) \quad = \quad \int_{-\infty}^{\infty} |h_n(t) - h_n(t+k)|dt$$

[1] I owe this proof to Professor Anderson.

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} | \int_0^k h_n'(t+s)ds|dt \\
&\leq \int_{-\infty}^{\infty} \int_0^k |h_n'(t+s)|dsdt \\
&= \int_0^k \int_{-\infty}^{\infty} |h_n'(t+s)|dtds \\
&= \int_0^k \int_{-\infty}^{\infty} |h_n'(t)|dtds \\
&= k \int_{-\infty}^{\infty} |h_n'(t)|dt
\end{aligned}
$$

so $\Phi_n - 0$ uniformly on compact intervals if $\{h_n\}$ satisfies dispersion property (2). QED.

An example of a sequence which satisfies dispersion property (1) is the following: $h_n(t) = \frac{1}{2n}$ if $t \in [-n. n]$ and $h_n(t) = 0$ otherwise. More generally, the following intuitive procedure to "flatten" a density function implies the dispersion properties (see also Grandmont. 1992).

PROPOSITION 5.3

Given a density function h. we may construct the sequence of density functions

$$h_n(t) = \frac{1}{n} h(\frac{t}{n})$$

(a). If h is Lipschitz on compact intervals. $\{h_n\}$ satisfies dispersion property (1).

(b). If h is differentiable. and $|h'(t)|$ is integrable. $\{h_n\}$ satisfies dispersion property (2).

Proof

(a)

$$
\begin{aligned}
\Phi_n(k) &= \int_R |h_n(t) - h_n(t+k)|dt \\
&= \int_R |\frac{1}{n}h(\frac{t}{n}) - \frac{1}{n}h(\frac{t}{n} + \frac{k}{n})|dt \\
&= \int_R |h(s) - h(s + \frac{k}{n})|ds \\
&= \int_{-\infty}^{-L} |h(s) - h(s+\frac{k}{n})|ds + \int_L^{\infty} |h(s) - h(s+\frac{k}{n})|ds \\
&\quad + \int_{-L}^{L} |h(s) - h(s+\frac{k}{n})|ds
\end{aligned}
$$

For L sufficiently large,

$$\Phi_n(k) \leq \epsilon + \int_{-L}^{L} \frac{k}{n} B ds$$

where B is the Lipschitz constant on $[-2L, 2L]$. Therefore, for n sufficiently large $\Phi_n(k) \leq 2\epsilon$ for all $k \in I$.

(b) Since $h'_n(t) = n^{-2}h'(n^{-1}t)$,

$$\int_R |h'_n(t)|dt = \frac{1}{n^2}\int_R |h'(\frac{t}{n})|dt = \frac{1}{n}\int_R |h'(y)|dy$$

which tends to zero as n tends to infinity. QED.

5.3 The Extrinsic Income Effect

LEMMA 5.4

Suppose that the density functions h_n have dispersion property (1). Then given an interval $[w_L, w_U], 0 < w_L < w_U < \infty$,

$$\int_{-\infty}^{\infty} (\frac{df_i}{dw}(u_t, p, w') - \frac{df_i}{dw}(u_t, p, w''))h_n(t)dt$$

$$= \int_{-\infty}^{\infty} (\frac{df_i}{dw}(p, w'e^{-t}) - \frac{df_i}{dw}(p, w''e^{-t}))h_n(t)dt \longrightarrow 0$$

as $n \to \infty$, uniformly for all (p, w', w'') in $P \times [w_L, w_U] \times [w_L, w_U]$.

Remark. The lemma says that with increasing heterogeneity, the average slope of the Engle curves

$$\int_{-\infty}^{\infty} \frac{df_i}{dw}(u_t, p, w)h_n(t)dt$$

do not differ significantly across different levels of w on the interval $[w_L, w_U]$.

Proof

Using the substitution $s = w'e^{-t}$ we obtain

$$\int_{-\infty}^{\infty} \frac{df_i}{dw}(p, w'e^{-t})h_n(t)dt = \int_0^{\infty} \frac{df_i}{dw}(p, s)h_n(\log w' - \log s)(\frac{1}{s})ds.$$

We assume that

$$|\frac{df_i}{dw}(p, w)| < M$$

for all p in $P, w > 0, i = 1, 2, ...l$ (refer to our definition of u in Section 5.1). Then

$$|\int_{-\infty}^{\infty} (\frac{df_i}{dw}(p, w'e^{-t}) - \frac{df_i}{dw}(p, w''e^{-t}))h_n(t)dt|$$

$$\leq \int_0^\infty |\frac{df_i}{dw}(p, s)||h_n(\log w' - \log s) - h_n(\log w'' - \log s)|(\frac{1}{s})ds$$

$$\leq M \int_0^\infty |h_n(\log w' - \log s) - h_n(\log w'' - \log s)|(\frac{1}{s})ds$$

$$= \int_{-\infty}^{\infty} |h_n(t) - h_n(\log w'' - \log w' + t)|dt$$

which tends to zero uniformly for all (p, w', w'') in $P \times [w_L, w_U] \times [w_L, w_U]$, as n goes to infinity because of dispersion property (1). QED.

With Lemma 5.4, we are able to prove the following proposition:

PROPOSITION 5.5

Suppose the density functions h_n satisfy dispersion property (1). Then for any $\epsilon > 0$, there exists N^* such that for all $n > N^*$, $|v.B_n(p)v| < \epsilon$ for (p, v) in Q.

Proof

Remember that

$$B_n(p)_{ij} = \int_E x_j [\int_R \frac{df_i}{dw}(p, (p.x)e^{-t})h_n(t)dt]d\mu$$

Choose $w_U = \sup\{w : w = p.x, p \in P, x \in E\}$ and $w_L = \inf\{w : w = p.x, p \in P, x \in E\}$. Since E is compact $w_U < \infty$; $w_L > 0$ because E is bounded away from zero. We choose w_0, with $w_L < w_0 < w_U$. We may split the matrix $B_n(p)$ into two matrices by

$$B_n(p)_{ij} = \int_E x_j \int_R [\frac{df_i}{dw}(p, (p.x)e^{-t}) - \frac{df_i}{dw}(p, w_0 e^{-t})]h_n(t)dtd\mu$$

$$+ \int_E x_j \int_R \frac{df_i}{dw}(p, w_0 e^{-t})h_n(t)dtd\mu.$$

We name the matrices B_n^1 and B_n^2 respectively. Now, we see that

$$B_n^2(p)_{ij} = \int_E x_j \int_R \frac{df_i}{dw}(p, w_0 e^{-t})h_n(t)dtd\mu$$

$$= (\int_R \frac{df_i}{dw}(p, w_0 e^{-t})h_n(t)dt)(\int_E x_j d\mu).$$

Therefore $B_n^2(p)v = 0$ as we have assumed that $v.\bar{x} = 0$.

So we are left with having to bound the matrix B_n^1. By Lemma 5.4, we know that given any $\delta > 0$, there exists N^* such that for all n greater than N^*,

$$|\int_{-\infty}^{\infty} (\frac{df_i}{dw}(p, w'e^{-t})h_n(t)dt - \int_{-\infty}^{\infty} (\frac{df_i}{dw}(p, w_0 e^{-t})h_n(t)dt| < \delta$$

for all p in P and w' in $[w_L, w_U]$. Therefore the entries of the matrix $B_n^1(p)$ can made arbitrarily small, and we have shown that $|v.B_n(p)v| = |v.B_n^1(p)v| < \epsilon$ for (p, v) in Q when n is sufficiently large. QED.

5.4 The Intrinsic Income Effect

Our next task is to find conditions that will guarantee that $v.A_n(p)v$ is positive (or rather not too negative) on Q for n sufficiently large. We will adopt two approaches. In the first approach, we will strengthen the assumptions on the sequence $\{h_n\}$. This method can be thought of as an approximate version of Grandmont (1987). In the second approach, we will not strengthen the dispersion property assumption on $\{h_n\}$, instead we will impose a fairly mild distributional assumption on the endowments that is reminiscent of Hildenbrand's (1983).

PROPOSITION 5.6[2]

Suppose that the density functions h_n satisfy dispersion property (2). Then for any $\epsilon > 0$, there exists N^* such that for all $n > N^*$,

$$v.A_n(p)v > -\epsilon$$

for all $(p, v) \in Q$.

Proof

Since each h_n has compact support, we may assume that h_n and its derivative h'_n are zero outside some compact interval $[a_n, b_n]$, with $h_n(a_n) = h_n(b_n) = 0$. As P is a compact subset of Δ^l_{++}, $|\frac{f_i(p,w)}{w}|$ is bounded for all $w > 0$ and $i = 1, 2, .., l$. This implies that there exists L such that

$$[v.\frac{f(p,w)}{w}]^2 < L$$

for $(p, v) \in Q$.

$$A_n(p)_{ij} = \int_E [\int_R f_j(u_t, p, p.x)\frac{df_i}{dw}(u_t, p, p.x)h_n(t)dt]d\mu$$

$$v.A_n(p)v = \int_E [\int_R \sum_i \sum_j v_i v_j f_j(u_t, p, p.x)\frac{df_i}{dw}(u_t, p, p.x)h_n(t)dt]d\mu$$

We focus on the inner integral,

$$\int_{a_n}^{b_n} \sum_i \sum_j v_i v_j f_j(u_t, p, p.x)\frac{df_i}{dw}(u_t, p, p.x)h_n(t)dt]$$

$$= \int_{a_n}^{b_n} \frac{1}{2}\frac{d}{dt}[v.f(p, p.xe^{-t})]^2(-p.x)^{-1}e^{2t}h_n(t)dt]$$

$$= \int_{a_n}^{b_n} \frac{1}{2}[v.f(p, p.xe^{-t})]^2(p.x)^{-1}[2e^{2t}h_n(t) + e^{2t}h'_n(t)]dt$$

[2] I am grateful to Professor Grandmont for suggesting this line of inquiry.

$$\geq \int_{a_n}^{b_n} \frac{1}{2}[v.\frac{f(p,p.xe^{-t})}{(p.x)e^{-t}}]^2(p.x)h'_n(t)]dt$$

Therefore

$$v.A_n(p)v \geq \frac{1}{2}\int_E \int_{a_n}^{b_n}[v.\frac{f(p,p.xe^{-t})}{(p.x)e^{-t}}]^2(p.x)h'_n(t)]dt]d\mu$$

The right hand side of the last inequality has an absolute value less than

$$\frac{1}{2}L\int_E \int_{a_n}^{b_n} |h'_n(t)|dt(p.x)d\mu,$$

which will be less than ϵ for n sufficiently large, because of dispersion property (2). So we obtain

$$v.A_n(p)v \geq -\epsilon$$

for all $(p, v) \in Q$, when n sufficiently large. QED.

Re-capitulating our results on the matrices $A_n(p)$ and $B_n(p)$, we obtain the following theorem:

THEOREM 5.7

Let $\mathcal{E}_n$ be a sequence of economies as defined in Section 5.1. Suppose that the sequences $\{h_n^r\}$, $r = 1, 2, .., l$ satisfy dispersion property (2). Then

(a). all equilibrium prices for the economies $\mathcal{E}_n$ are contained in the interior of a compact, convex set $P \subset \Delta_{++}^l$.

Given such a P, for n sufficiently large,

(b). market demand $\bar{F}_n(p)$ satisfies RMP in the cone $R_{++} \times P$.

(c). the economy $\mathcal{E}_n$ has a unique equilibrium price, and

(d). the equilibrium price is asymptotically stable, i.e., it is the limit point of the tatonnement processes (I) and (II), provided the initial price $p(0)$ is in $R_{++} \times P$.

Proof

(a) is a trivial observation we made in Section 5.2. (b) follows from Propositions 5.2, 5.5 and 5.6. (c) is a trivial consequence of (b). So only (d) needs to be proved.

We begin with tatonnement (I). Remember that we have shown in Theorem 4.2 that if the market demand satisfies RMP for all prices, then the unique price equilibrium is also globally stable. In this case, we have a slight difficulty because RMP holds only in the cone $R_{++} \times P$. We therefore cannot immediately apply the Lyapounov type argument we used in that theorem since there is no guarantee that a solution to (I), $p(t)$, will not drift out of $R_{++} \times P$. We begin by constructing a convex cone containing $R_{++} \times P$ which will also contain all solutions $p(t)$ beginning in $R_{++} \times P$ for all the economies $\mathcal{E}_n$.

We define $||.||$ as the modified norm, $||x|| = \sum_{i=1}^{l} \alpha_i^{-1} x_i^2$, and $\Omega = \{p \in R_{++}^l : ||p|| = 1\}$. If $p(t)$ is a solution to (I), then $\frac{p(t)}{||p(0)||}$ is the projection of the solution onto Ω, and $\frac{p(\|p(0)\|t)}{\|p(0)\|}$ is the solution to (I) with initial condition

$$\frac{p(0)}{\|p(0)\|} \in P' = \{\frac{p}{\|p\|} : p \in R_{++} \times P\} \subset \Omega.$$

We now construct a compact set Ω'' such that $P' \subset int(\Omega'') \subset \Omega'' \subset \Omega$ and for which all solutions to (I) beginning in P', for all the economies $\mathcal{E}_n$, will be contained in Ω''. This also implies, by our reasoning above, that all solutions beginning in $R_{++} \times P$ will stay in $R_{++} \times \Omega''$ for all the economies $\mathcal{E}_n$. Ω'' will have to be constructed in a way that does not depend on n. Remember that we have assumed that each $\mathcal{E}_n$ contains contains a homothetic class (see Section 5.1); indeed, market demand

$$\hat{F}_n = \phi^0 F^0(p) + \phi^1 F_n^1(p) + ... + \phi^l F_n^l(p).$$

Let $Z^0(p) = \phi^0 F^0(p) - \bar{x}$, where $\bar{x}$ is the mean endowment. Then the excess demand of $\mathcal{E}_n$, $Z_n(p) \geq Z^0(p)$. $Z^0(p)$ satisfies the weak Walras Law, $Z^0(p).p \leq 0$, is bounded below, and also satisfies the desirability property, i.e., $|Z^0(p)| \to \infty$ whenever $p_n \to p_0 \in R^l_+ \setminus R^l_{++}$.

Now referring to the proof of Theorem 3.2, we may construct W, Ω' and Ω'' with $Z^0(p)$ in place of $Z(p)$. If $\delta_1, .., \delta_{l-i}$ are chosen small enough $P' \subset \Omega''$. Then the fact that $Z_n(p) \geq Z^0(p)$ now guarantees that for all $\mathcal{E}_n$, a solution to (I) with $p(0) \in P'$ will always remain in Ω'', or more generally, a solution to (I) with $p(0) \in R_{++} \times P$ will always be contained in $R_{++} \times \Omega''$.

By taking the convex hull if necessary, we may assume that $R_{++} \times \Omega''$ is a closed convex cone in R^l_{++}. By part (b) of this theorem, RMP holds in $R_{++} \times \Omega''$ for market demand $\hat{F}_n$ of the economies $\mathcal{E}_n$, $n > N$. Since we also know that no solutions to (I) leaves $R_{++} \times \Omega''$, we conclude (using Lyapounov's method, as in the proof of Theorem 4.2) that the equilibrium price of the economies $\mathcal{E}_n$, $n > N$, is asymptotically stable, i.e., all solutions to (I) in $\mathcal{E}_n$ with $p(0) \in R_{++} \times P$ converges to the equilibrium price of $\mathcal{E}_n$.

We now consider tatonnement process (II). As in the case with (I), we have to allow for the possibility that a solution to (II) with $p(0) \in P$ may drift out of P, and so if we are to use the Lyapounov function in the proof of Theorem 4.3, we must ensure that RMP holds on some bigger set of prices, which always contains $p(t)$.

We define the norm $|.|_\beta$ by $|p|_\beta = \sum_{i=1}^l \beta_i^{-1}|p_i|$ and $\Psi = \{p \in R^l_{++} : |p|_\beta = 1\}$. We observe that $p(t)$ is the solution to (II) with initial condition $p(0)$ if and only if $\frac{p(t)}{|p(0)|_\beta}$ is the solution to (II) with initial condition $\frac{p(0)}{|p(0)|_\beta}$. So without loss of generality, we may confine our study to solutions with $p(0) \in \Psi \cap (R_{++} \times P)$. We note that $\Psi \cap (R_{++} \times P)$ is a compact subset of Ψ. Defining

$$\mathcal{L}^0(p, q) = \sum_{i=1}^{l} \beta_i^{-1}(p_i - q_i \log p_i - q_i + q_i \log q_i),$$

we see that $\mathcal{L}^0$ is a continuous function so there is a number M such that $\mathcal{L}^0(p,q) \leq M$ for all p and q in $\Psi \cap (R_{++} \times P)$. As n varies, so does the equilibrium price of $\mathcal{E}_n$, nonetheless it will always stay within $\Psi \bigcap (R_{++} \times P)$. The fact that $\mathcal{L}^0(p,q) \leq M$ guarantees that the value of the Lyapounov function $\mathcal{L}$ at $p(0)$ as defined in Theorem 4.3, will always be bounded by M, for all the economies $\mathcal{E}_n$.

We define $S(p') = \{p \in \Psi : \mathcal{L}(p,p') \leq M\}$ and

$$S = \bigcup_{p' \in \Psi \cap (R_{++} \times P)} S(p').$$

It is fairly trivial to show that S is compact. Then the convex hull of S, $con(S)$, is a compact, convex subset of Ψ. By part (b) of this theorem, there exists N_0, such that for all $n > N_0$, the demand function of $\mathcal{E}_n$ satisfies RMP in the cone $R_{++} \times con(S)$. By the proof of Theorem 4.3, a solution $p(t)$ to (II) in the economy $\mathcal{E}_n$, $n > N_0$, with $p(0) \in R_{++} \times P$ will never leave $con(S)$, and will always converge to the equilibrium price of $\mathcal{E}_n$. QED.

For our next result we need the following lemma:

LEMMA 5.8

Suppose that $h_n(t)$ has dispersion property (1). Then, given any interval $[w_L, w_U]$, $0 < w_L < w_U < \infty$,

$$\int_R [f_i(u_t,p,w)f_j(u_t,p,w) - f_i(u_t,p,w')f_j(u_t,p,w')(\frac{w^2}{w'^2})]h_n(t)dt \longrightarrow 0$$

as $n \longrightarrow \infty$, uniformly for all (p,w,w') in $P \times [w_L, w_U] \times [w_L, w_U]$

Remark: Clearly the result implies that

$$\int_R [\frac{f_i(u_t,p,w)}{w}\frac{f_j(u_t,p,w)}{w} - \frac{f_i(u_t,p,w')}{w'}\frac{f_j(u_t,p,w')}{w'}]h_n(t)dt \longrightarrow 0$$

In other words, when h_n is sufficiently dispersed,

$$\int_R [\frac{f_i(u_t,p,w)}{w}\frac{f_j(u_t,p,w)}{w}]h_n(t)dt$$

will not differ significantly across income w; again, this is an approximation of what would, in the homothetic case, be identical.

Proof

By the substitution $v = we^{-t}$, we obtain

$$\int_{-\infty}^{\infty} f_i(u_t,p,w)f_j(u_t,p,w)h_n(t)dt = \int_{-\infty}^{\infty} f_i(p,we^{-t})f_j(p,we^{-t})e^{2t}h_n(t)dt$$

$$= \int_0^{\infty} f_i(p,v)f_j(p,v)w^2v^{-3}h_n(\log w - \log v)dv$$

Similarly,

$$\int_{-\infty}^{\infty} f_i(u_t,p,w')f_j(u_t;p,w')h_n(t)dt = \int_0^{\infty} f_i(p,v)f_j(p,v)w'^2v^{-3}h_n(\log w' - \log v)dv.$$

We choose L satisfying

$$|\frac{f_i(p,v)}{v}| \leq L$$

for all $i = 1,2,..l$. L exists because P is a compact subset of Δ^l_{++}. Therefore,

$$|\int_{-\infty}^{\infty}[f_i(u_t,p,w)f_j(u_t,p,w) - \frac{w^2}{w'^2}f_i(u_t,p,w)f_j(u_t,p,w)]h_n(t)dt| =$$

$$|\int_0^{\infty} w^2\frac{f_i(p,v)}{v}\frac{f_j(p,v)}{v}[h_n(\log w - \log v) - h_n(\log w' - \log v)]v^{-1}dv|$$

$$\leq w^2L^2\int_0^{\infty}|h_n(\log w - \log v) - h_n(\log w' - \log v)|v^{-1}dv$$

$$= w^2L^2\int_{-\infty}^{\infty}|h_n(t) - h_n(\log w' - \log w + t)|dt \longrightarrow 0$$

as $n \to \infty$, uniformly for all (p,w,w') in $P \times [w_L, w_U] \times [w_L, w_U]$ by dispersion property (1). QED.

Description of the endowment set E:

Suppose we write E as $\{rx^* : \sum_{i=1}^{l} x_i^{*2} = 1, x^* \in R^l_+, 0 \leq r \leq T < \infty\}$. Let $S = \{x^* \in R^l_+ : \sum x_i^{*2} = 1\}$. B_s is the Borel σ-field generated by the subspace topology on S. Let ν be a probability measure on (S, B_s). The conditional probability of r given x^* is given by the density function $g(r,x^*)$, so,

$$\mu(\{rx^* : x^* \in S^0, r_1 \leq r \leq r_2\}) = \int_{S^0}\int_{r_1}^{r_2} g(r,x^*)drd\nu.$$

PROPOSITION 5.9

Suppose that g has the following properties:

(1) The derivative $\frac{dg}{dr}(r,x^*)$ exists and is uniformly bounded for all (r,x^*) (from now on to save some space, we will denote $\frac{dg}{dr}(r,x^*)$ by $g'(r,x^*)$),

(2) for all x^* in S

$$\int_0^T r^2g'(r,x^*)dr \leq 0$$

(3) the sequence $\{h_n\}$ satisfies dispersion property (1).

Then for any $\epsilon > 0$, there exists N such that for all $n > N$,

$$v.A_n(p)v > -\epsilon$$

for (p, v) in Q.

Proof

We know that

$$A_n(p)_{ij} = \int_R \int_S \int_0^T f_j(u_t, p, rp.x^*) \frac{df_i}{dw}(u_t, p, rp.x^*) g(r, x^*) dr d\nu d\mu_n$$

Making the substitution $w = rp.x^*$, we obtain

$$A_n(p)_{ij} = \int_R \int_S \int_0^{Tp.x^*} f_j(u_t, p, w) \frac{df_i}{dw}(u_t, p, w) g(\frac{w}{p.x^*}, x^*) \frac{1}{p.x^*} dw d\mu_n d\nu$$

and so $v.A_n(p)v$

$$\begin{aligned}
&= \int_S \frac{1}{p.x^*} \int_R \int_0^{Tp.x^*} \sum_i \sum_j v_i v_j f_j(u_t, p, w) \frac{df_i}{dw}(u_t, p, w) g(\frac{w}{p.x^*}, x^*) dw d\mu_n d\nu \\
&= \int_S \frac{1}{2p.x^*} \int_R \int_0^{Tp.x^*} \frac{d}{dw}(f(u_t, p, w).v)^2 g(\frac{w}{p.x^*}, x^*) dw d\mu_n d\nu \\
&= \int_S \frac{1}{2p.x^*} [\int_R g(T, x^*)(f(u_t, p, Tp.x^*).v)^2 d\mu_n \\
&\quad - \frac{1}{p.x^*} \int_R \int_0^{Tp.x^*} (f(u_t, p, w).v)^2 g'(\frac{w}{p.x^*}, x^*) dw d\mu_n] d\nu
\end{aligned}$$

Notice that the first term is always positive, so we shall focus only on the second.

$$\int_S \frac{1}{2(p.x^*)^2} [\int_R \int_0^{Tp.x^*} (f(u_t, p, w).v)^2 g'(\frac{w}{p.x^*}, x^*) dw d\mu_n] d\nu$$

Now, in the absence of any assumptions on the preferences we will have to stop and conclude (as Hildenbrand (1983) did) that $v.A_n(p)v$ is positive when g' is negative. With dispersion property (1) we may go further: Firstly, let us note that since E is bounded away from zero, there exists $\bar{r}$ such that for all $r < \bar{r}$ and for all $x^* \in S$, $g'(r, x^*) = g(r, x^*) = 0$. In other words, there exists Y such that

$$0 < Y < \min\{Tp.x^* : p \in P, x^* \in S\},$$

and

$$g'(\frac{w}{p.x^*}, x^*) = g(\frac{w}{p.x^*}, x^*) = 0$$

for all $w \le Y$., $x^* \in S$ and $p \in P$. As such,

$$\int_{\frac{Y}{p.x^*}}^T r^2 g'(r, x^*) dr = \int_0^T r^2 g'(r, x^*) dr \le 0$$

for all $x^* \in S$, $p \in P$.

Therefore, we will be done if we can show that

$$\int_S \frac{1}{2(p.x^*)^2} \int_R \int_Y^{Tp.x^*} (f(u_t, p, w).v)^2 g'(\frac{w}{p.x^*}, x^*) dw d\mu_n d\nu$$

is not too positive for n sufficiently large, for all $(p, v) \in Q$. We choose a constant w', with

$$Y < w' < \max\{Tp.x^* : p \in P, x^* \in S\},$$

so the expression can be written as.

$$\int_S \frac{1}{2(p.x^*)^2}[\int_Y^{Tp.x^*} \int_R ((f(u_t, p, w).v)^2 - w^2(\frac{v.f(u_t, p, w')}{w'})^2) h_n(t) dt g'(\frac{w}{p.x^*}, x^*) dw] d\nu$$

$$+ \int_S \frac{1}{2(p.x^*)^2} \int_Y^{Tp.x^*} \int_R w^2(\frac{v.f(u_t, p, w')}{w'})^2 h_n(t) dt g'(\frac{w}{p.x^*}, x^*) dw d\nu$$

By Lemma 5.8 and the uniform boundedness of g', the first term in the sum can be made arbitrarily small, say, with an absolute value less than ϵ, provided n is sufficiently large. As for the second term.

$$\begin{aligned}
& \int_S \frac{1}{2(p.x^*)^2} \int_Y^{Tp.x^*} \int_R w^2(\frac{v.f(u_t, p, w')}{w'})^2 h_n(t) dt g'(\frac{w}{p.x^*}, x^*) dw d\nu \\
= & \int_S \frac{1}{2(p.x^*)^2}[\int_R (\frac{v.f(u_t, p, w')}{w'})^2 h_n(t) dt][\int_Y^{Tp.x^*} w^2 g'(\frac{w}{p.x^*}, x^*) dw] d\nu \\
= & \int_S \frac{p.x^*}{2}[\int_R (\frac{v.f(u_t, p, w')}{w'})^2 h_n(t) dt][\int_{\frac{Y}{p.x^*}}^T r^2 g'(r, x^*) dr] d\nu \\
\leq & \ 0.
\end{aligned}$$

Therefore we have shown that $v.A_n(p)v > -\epsilon$ for all $(p, v) \in Q$. QED.

Weaving together Proposition 5.5 on $B_n(p)$ and the last proposition, we obtain the following theorem:

THEOREM 5.10

Let $\mathcal{E}_n$ be a sequence of economies as defined in Section 5.1. Suppose that the sequences $\{h_n^r\}$, $r = 1, 2, .., l$ satisfy dispersion property (1). Writing E as $\{rx^* : \sum_{i=1}^l x_i^{*2} = 1, x^* \in R_+^l, 0 \leq r \leq T < \infty\}$, we assume that the conditional probability of r given x^* is given by the density function $g(r, x^*)$, where $g(r, x^*)$ has the following properties:

(1) $\frac{dg}{dr}(r, x^*)$ exists and is uniformly bounded for all (r, x^*), and

(2) for all x^* in S,

$$\int_0^T r^2 g'(r, x^*) dr \leq 0.$$

Then

(a). all equilibrium prices for the economies $\mathcal{E}_n$ are contained in the interior of a compact, convex set $P \subset \Delta^l_{++}$.

Given such a P, for n sufficiently large,

(b). market demand $\hat{F}_n(p)$ satisfies RMP in the cone $R_{++} \times P$,

(c). the economy $\mathcal{E}_n$ has a unique equilibrium price, and

(d). the equilibrium price of $\mathcal{E}_n$ is asymptotically stable, i.e., it is the limit point of the tatonnement processes (I) and (II), provided the initial price $p(0)$ is in $R_{++} \times P$.

Proof

(a) is a trivial observation we made in Section 5.2. (b) follows from Propositions 5.5 and 5.9. (c) is a trivial consequence of (b). (d)'s proof is found in Theorem 5.7. QED.

We end with a few remarks. Comparing Theorem 5.10 with that of Hildenbrand (1983), we see that our assumptions on the endowment distribution are relatively mild. They need neither be collinear nor have a downward sloping density; in its place, we have assumption (2), which will be satisfied if g is unimodal and the upward sloping part of g is not "too big". As an illustration, supppose that $g(T, x^*) = g(0, x^*) = 0$ for all x^* and let $\mathcal{R}(x^*)$ be the maximum of $g(\cdot, x^*)$. Then

$$\int_0^T r^2 g'(r, x^*)dr \leq \int_0^T \mathcal{R}^2(x^*)g'(r, x^*)dr.$$

By the fundamental theorem of calculus, the latter is in fact $\mathcal{R}^2(x^*)[g(T, x^*) - g(0, x^*)] = 0$ and so assumption (2) is satisfied. Of course, our relatively weak assumptions on endowments are possible because we have added heterogeneity assumptions: they are not present in Hildenbrand's model.

The assumption that endowments and preferences are independently distributed may be weakened in the following way: Let

$$\mathcal{E}_n^s = (U^* \times E, \mathcal{F} \times \mathcal{B}, \nu_n^s \times \mu^s),$$

$s = 1, 2, ..., M$ be M sequences of economies with the property that the mean endowments of the sequences of economies are collinear, i.e., $\int_E x d\mu^s = \theta^s \bar{x}$, where $\theta^s > 0$ for $s = 1, 2, ..., M$. Then we may construct the sequence of economies

$$\mathcal{E}_n^* = (U^* \times E, \mathcal{F} \times \mathcal{B}, \lambda_n)$$

where the probability measure λ_n is defined by

$$\lambda_n(A) = \psi^1(\nu_n^1 \times \mu^1)(A) + \psi^2(\nu_n^2 \times \mu^2)(A) + ...\psi^M(\nu_n^M \times \mu^M)(A),$$

where A is any measurable subset of $U^* \times E$ and ψ^s are positive and constant for all n with $\sum_{s=1}^M \psi^s = 1$. The economies $\mathcal{E}_n^*$ will have the mean endowment $(\sum_{s=1}^M \psi^s \lambda^s)\bar{x}$. For this new sequence of economies, preferences and endowments need not be independently distributed. Instead, we have assumed that each economy in the new sequence can be divided into M "blocks"; within each block preferences and endowments are independent. If each of the sequences $\mathcal{E}_n^s$, $s = 1, 2, ..., M$, satisfies the assumptions of Theorem 5.7 (respectively Theorem 5.10) then the conclusion of the theorem will be valid for the sequence $\mathcal{E}_n^*$. With some reflection, this should be clear. Similarly, Theorems 2.4 and 3.2 remain valid for economies where the independence assumption is relaxed in the manner just described.

Finally, we add that for Theorems 5.7 and 5.10, the assumption that E is bounded away from zero may be dropped, provided we assume that at the neighbourhood of any price $\frac{df_i}{dp_j}(u, p, w)$ is bounded uniformly for all $w \in (0, W]$, W a finite number. In that way, Lemma 5.1 will still be true; while the proofs of Propositions 5.5 and 5.9 will involve an additional layer of approximations, the propositions will also remain valid (see Appendix). Proposition 5.6 is unaffected by the removal of the assumption.

Appendix

We will first indicate how we may modify the proof of Propostion 5.5 when E is not assumed to be bounded away from zero:

As in the original proof we choose $w_U = \sup\{w : w = p.x, p \in P, x \in E\}$; w_U is finite since E is compact. But we cannot choose $w_L = \inf\{w : w = p.x, p \in P, x \in E\}$ as the infimum may now be zero. Instead, we first choose w_0, with $0 < w_0 < w_U$. Then we may split the matrix $B_n(p)$ into three matrices by

$$B_n(p)_{ij} = \int_{E \setminus N(0)} x_j \int_R [\frac{df_i}{dw}(p, (p.x)e^{-t}) - \frac{df_i}{dw}(p, w_0 e^{-t})] h_n(t) dt d\mu$$

$$+ \int_{E \setminus N(0)} x_j \int_R \frac{df_i}{dw}(p, w_0 e^{-t}) h_n(t) dt d\mu.$$

$$+ \int_{N(0)} x_j \int_R \frac{df_i}{dw}(p, (p.x)e^{-t}) h_n(t) dt d\mu$$

where $N(0)$ is a neighbourhood of zero in R_+^l. We name the matrices B_n^1, B_n^2 and B_n^3. When $N(0)$ is small enough, we can guarantee that both $|v.B_n^3(p)v|$ and $|v.B_n^2(p)v|$ will be less that ϵ for all (p, v) in Q and for all n: the former because each entry in the matrix can be made arbitrarily small, the latter because $v.\bar{x} = 0$, as in the original proof. So we only have to deal with $|v.B_n^1(p)v|$. For n sufficiently large, this too will be arbitrarily small. The arguments in the original proof can be re-applied, with w_L chosen to satisfy

$$0 < w_L < \min\{\inf_{p \in P} p.(E \setminus N(0)), w_0\}.$$

The proof of Proposition 5.9 may be modified in the following way: Keep to the original proof until we focus on the term

$$\int_S \frac{1}{2(p.x^*)^2}[\int_R \int_0^{Tp.x^*} (f(u_t, p, w).v)^2 g'(\frac{w}{p.x^*}, x^*) dw d\mu_n] d\nu.$$

Since E need not be bounded away from zero, the $\tilde{r}$ and Y we postulated in the original proof need not exist. Instead, we choose a positive number Y' so small that

$$Y' < \min\{Tp.x^* : p \in P, x^* \in S\},$$

$$|\int_S \frac{1}{2(p.x^*)^2}[\int_R \int_0^{Y'} (f(u_t, p, w).v)^2 g'(\frac{w}{p.x^*}, x^*) dw d\mu_n] d\nu| < \epsilon,$$

and, lastly, that

$$\int_{\frac{Y'}{p.x^*}}^{T} r^2 g'(r, x^*) dr < \epsilon$$

for all $x^* \in S$, $p \in P$. Remember that we have assumed g' to be uniformly bounded, so the last inequality is possible. After Y' had been chosen, the arguments in the original proof can be repeated with Y' in place of Y.

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