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Longitudinal Control of a Platoon of Vehicles; II: First and Second Order Time Derivatives of Distance Deviations

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1 Introduction

This report is an addendum to PATH Research Report UCB-ITS-PRR-89-3. In that report the linear control law used $\dot{\Delta}$ and $\ddot{\Delta}$ for the longitudinal control law of a platoon of vehicles. The purpose of this addendum is to establish the benefit resulting from having both 6 and $\ddot{\Delta}$ available. For convenience we shall refer to equations of [She.11 by their own equation numbers in that report.

In the spirit of [She.l] we shall examine linear control laws for the longitudinal control of a platoon of vehicles which use:

1. No $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ (i = 1,2,. . .) terms in the linear control law [She.l;(-L.l)-(4.211.
2. No $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ (i $= 2, 3, \ldots$) terms in the linear control law [She.l;(4.l)-(4.2)].
3. Full feedback of $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ (i $= 1, 2, \ldots$) in the linear control law [She.1;(4.1)-(4.2)].

2 Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i = 1,2,...

Setting $c_{vi} = 0$ and $c_{ai} = 0$ for $i = 1, 2, \ldots$ in [She.l;(4.1)] and [She.l;(42)] we obtain

$$c_1 := c_{p1}\Delta_1(t) + k_{v1}v_l(t) + k_{a1}a_l(t) \tag{2.1}$$

and for i $= 2, 3, \ldots$

$$c_i := c_{pi}\Delta_i(t) + k_{vi}\left[v_l(t) - v_i(t)\right] + k_{ai}\left[a_l(t) - a_i(t)\right] \qquad (2.2)$$

Setting $c_{vi} = 0$, $c_{ai} = 0$ (i $= 1, 2, \ldots$), and noting [She.l;(5.2),(5.4),(5.6)] results in the following transfer functions for the platoon of identical vehicles:

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{\tau s^2 + (1 + \tau d_1 - k_{a1n})s + (d_1 - k_{v1n})}{\tau s^3 + (1 + \tau d_1)s^2 + d_1 s + c_{p1n}} \qquad (2.3)$$

$$\hat{h}_{\Delta_2\Delta_1}(s) = \frac{-k_{an}s^2 - k_{vn}s + c_{p1n}}{\tau s^3 + (1 + \tau d_1 + k_{an})s^2 + (d_1 + k_{vn})s + c_{pn}} \qquad (2.4)$$

$$\hat{g}(s) = \frac{c_{pn}}{\tau s^3 + (1 + \tau d_1 + k_{an})s^2 + (d_1 + k_{vn})s + c_{pn}} \qquad (2.5)$$

From (2.3) we note the following:

- We do not have complete freedom in choosing the poles of $\hat{h}_{\Delta_1 w_l}$,because we have only one design parameter at our disposal, namely c_{p1n}.
- We need to choose c_{p1n} so that the roots of the denominator polynomial in (2.3) all lie in the open left-half plane. Choosing τ = 0.2 $sec, d_1 =$ 0.03, and $c_{p1n} =$ 0.0002 gives 3 negative real poles for the denominator polynomial of $\hat{h}_{\Delta_1 w_l}$.
- Any other choice of c_{p1n}, which gives 3 negative real poles for the denominator of $\hat{h}_{\Delta_1 w_l}$ with the above values of τ and d_1, will be close to 0.0002. This is clearly an undesirable time constant.

3 Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for $i =$ 2,3,. . .

Suppose now that the first vehicle has $\dot{\Delta}_1$ and $\ddot{\Delta}_1$ available, hence uses the control law [She.l;(4.1)]:

$$c_1 := c_{p1}\Delta_1(t) + c_{v1}\dot{\Delta}_1(t) + c_{a1}\ddot{\Delta}_1(t) + k_{v1}v_l(t) + k_{a1}a_l(t) \qquad (3.1)$$

Setting $c_{vi} = 0$ and $c_{ai} = 0$ for i $= 2, 3, \ldots$ in [She.1;(4.2)] we obtain for i $=$ 2,3,...

$$c_i := c_{pi}\Delta_i(t) + k_{vi}\left[v_l(t) - v_i(t)\right] + k_{ai}\left[a_l(t) - a_i(t)\right] \qquad (3.2)$$

Setting $c_{vi} = 0$, $c_{ai} = 0$ (i = 2,3,. . .), and noting [She.l;(5.2),(5.4),(5.6)] results in the following transfer functions for the platoon of identical vehicles:

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{\tau s^2 + (1 + \tau d_1 - k_{a1n})s + (d_1 - k_{v1n})}{\tau s^3 + (1 + \tau d_1 + c_{a1n})s^2 + (d_1 + c_{v1n})s + c_{p1n}} \qquad (3.3)$$

$$\hat{h}_{\Delta_2 \Delta_1}(s) = \frac{(c_{a1n} - k_{an})s^2 + (c_{v1n} - k_{vn})s + c_{p1n}}{\tau s^3 + (1 + \tau d_1 + k_{an})s^2 + (d_1 + k_{vn})s + c_{pn}} \qquad (3.4)$$

$$\hat{g}(s) = \frac{c_{pn}}{\tau s^3 + (1 + \tau d_1 + k_{an})s^2 + (d_1 + k_{vn})s + c_{pn}} \qquad (3.3)$$

From (3.3) we note' the following:

- Since $\dot{\Delta}_1$ and $\ddot{\Delta}_1$ are available at the first vehicle, we have complete freedom in choosing the poles and the zeros of $\hat{h}_{\Delta_1 w_l}$.
- Given the freedom in selecting the poles of $\hat{h}_{\Delta_1 w_l}$, we can select the poles such that c_{p1n} will be much greater than $d_1 - k_{v1n}$ in (3.3)(i.e., the steady state error $\hat{h}_{\Delta_1 w_l}$ (0) is very small). Hence, with the above linear control law, the platoon tracks the lead vehicle to within spacings of two to three feet.(see fig. 3)

From (3.5) we note the following:

- We have complete freedom in choosing the poles of $\hat{g}(s)$.
- Since the numerator polynomial of $\hat{g}(s)$ is a constant, $|\hat{g}(j\omega)|$ behaves proportional to $\frac{1}{\omega^3}$ for large frequencies w.

4 Linear control law with $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i = 1,2,. . .

Noting [She.l;(5.2),(5.4),(5.6)],and choosing identical characteristic polynomials for $\hat{h}_{\Delta_1 w_l}$ and $\hat{g}$ results in the following transfer functions for the platoon of identical vehicles:

$$h_{\Delta_1 w_l}(s) = \frac{\tau s^2 + (1 + \tau d_1 - k_{a1n})s + (d_1 - k_{v1n})}{\tau s^3 + (1 + \tau d_1 + c_{a1n})s^2 + (d_1 + c_{v1n})s + c_{p1n}} \qquad (4.1)$$

$$\hat{g}(s) = \hat{h}_{\Delta_2\Delta_1}(s) = \frac{c_{an}s^2 + c_{vn}s + c_{pn}}{\tau s^3 + (1 + \tau d_1 + co,, + k_{an})s^2 + (d_1 + c_{vn} + k_{vn})s + c_{pn}} \quad (4.2)$$

From (4.1) we note the following:

- We have complete freedom in choosing the poles and the zeros of $\hat{h}_{\Delta_1 w_l}$.
- Given the freedom in selecting the poles of $\hat{h}_{\Delta_1 w_l}$, we can select the poles such that c_{p1n} will be much greater than $d_1 - k_{v1n}$ in (3.3). Hence, with the above ***linear*** control law, the platoon tracks the lead vehicle to within spacings of two to three feet.(see fig. 4)

From (4.2) we note the following:

- We have complete freedom in choosing the poles of $\hat{g}(s)$.
- Since the ***numerator*** polynomial of $\hat{g}(s)$ is a polynomial of degree 2, $|\hat{g}(j\omega)|$ behaves proportional to $\frac{1}{\omega}$ for large frequencies w. Hence, if the characteristic polynomials of $\hat{g}(s)$ in (3.5) and (4.2) are identical, the frequency response of $\hat{g}($ jw) corresponding to (4.2) will be broader than the respective frequency response corresponding to (3.5).
- Since broader frequency response corresponds to narrower impulse response in the time domain, the impulse response of $\hat{g}(s)$ (i.e., g(t)) corresponding to (4.2) is narrower than the respective impulse response related to (3.5). As a result, deviations in vehicles' spacings due to a change in the lead vehicle's velocity,w_l, will decrease more quickly to their steady-state values if $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ are used in the i-th vehicle's control law.

5 Simulation Results

To examine the behavior of a platoon of identical vehicles under the above control laws, simulations for a platoon of 16 vehicles were run using the System Build software package within MATRIXx. In all the simuiations conducted, all the vehicles were assumed to be initially traveling at the steady-state velocity of $v_0 =$ 1T.9 $m.sec^{-1}$ (i.e.. 40 m.p.h.). Beginning at time $t = 0$ sec, the lead vehicle's velocity was increased from its steady-state value of 17.9 $m.sec^{-1}$ until it reached its final value of 29.0 $m.sec^{-1}$ (i.e., 65 m.p.h.).

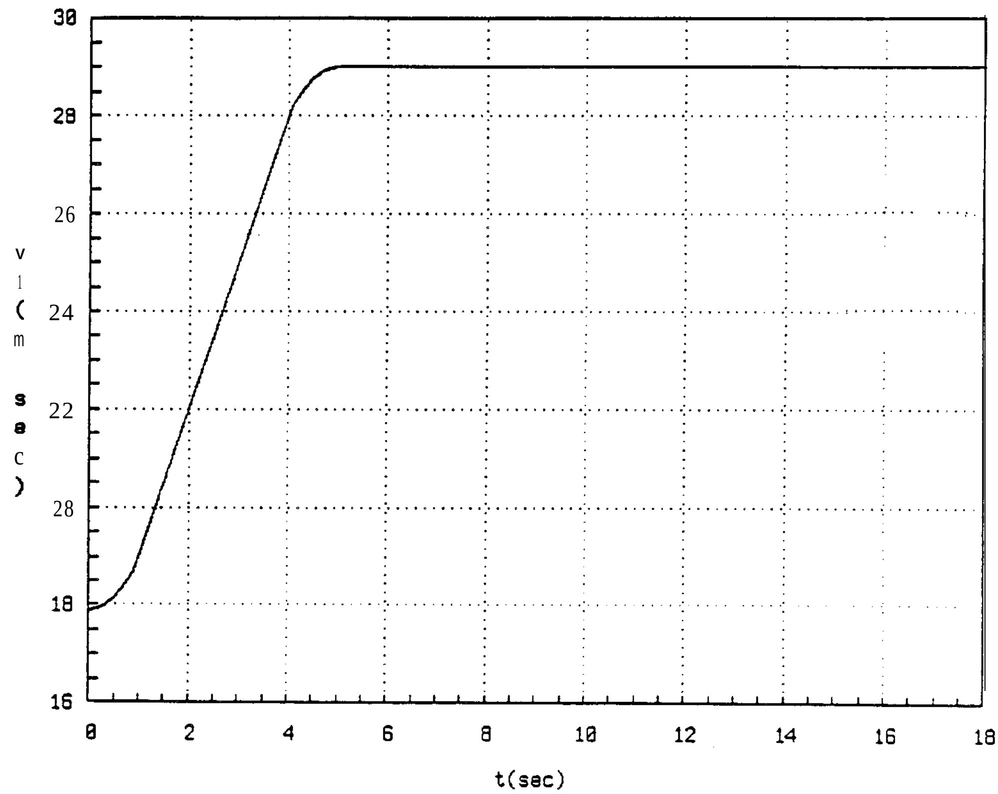

Figure 1: lead vehicle's velocity profile (v_l)

Figure 1 shows the lead vehicle's velocity profile as a function of time(t): the curve $v_l(t)$ corresponds to a maximum jerk of 2.0 $m.sec^{-3}$ and peak acceleration of 3.0 $m.sec^{-2}$ (i.e., 0.39).

The following values were chosen for the relevent parameters in the simulation:

$\tau = 0.2$ sec

$d_1 = 0.03$

- Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i $= 1, 2, \ldots$

  $c_{a1n} = 0, c_{v1n} = 0, c_{p1n} = 0.0002, k_{a1n} = 0.4, k_{v1n} = 0.02$

  $c_{an} = 0, c_{vn} = 0, c_{pn} = 24, k_{an} = 1.994, k_{vn} = 14.77$

- Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i $= 2, 3, \ldots$

  $c_{a1n} = 1.994,\ c_{v1n} = 14.77, c_{p1n} = 24, k_{a1n} = 0.4,\ k_{v1n} = 0.02$

  $c_{an} = 0, c_{vn} = 0, c_{pn} = 24, k_{an} = 1.994, k_{vn} = 14.77$

- Linear control law with $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i $= 1, 2, \ldots$

  $c_{a1n} = 1.994, c_{v1n} = 14.77, c_{p1n} = 24, k_{a1n} = 0.4,\ k_{v1n} = 0.02$

  $c_{an} = 1, c_{vn} = 9.77, c_{pn} = 24, k_{an} = 0.994,\ k_{vn} = 5$

Using the above values for the parameters, we obtain:

- Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i $= 1, 2, \ldots$

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{0.2(s + 3.02)(s + 0.017)}{0.2(s + 5)(s + 0.01)(s + 0.02)} \tag{5.1}$$

$$\hat{g}(s) = \frac{24}{0.2(s+4)(s+5)(s+6)} \tag{5.2}$$

- Linear control law with no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ for i $= 2, 3, \ldots$

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{0.2(s + 3.02)(s + 0.017)}{0.2(s+4)(s+5)(s+6)} \tag{5.3}$$

$$\hat{g}(s) = \frac{24}{0.2(s+4)(s+5)(s+6)} \tag{5.4}$$

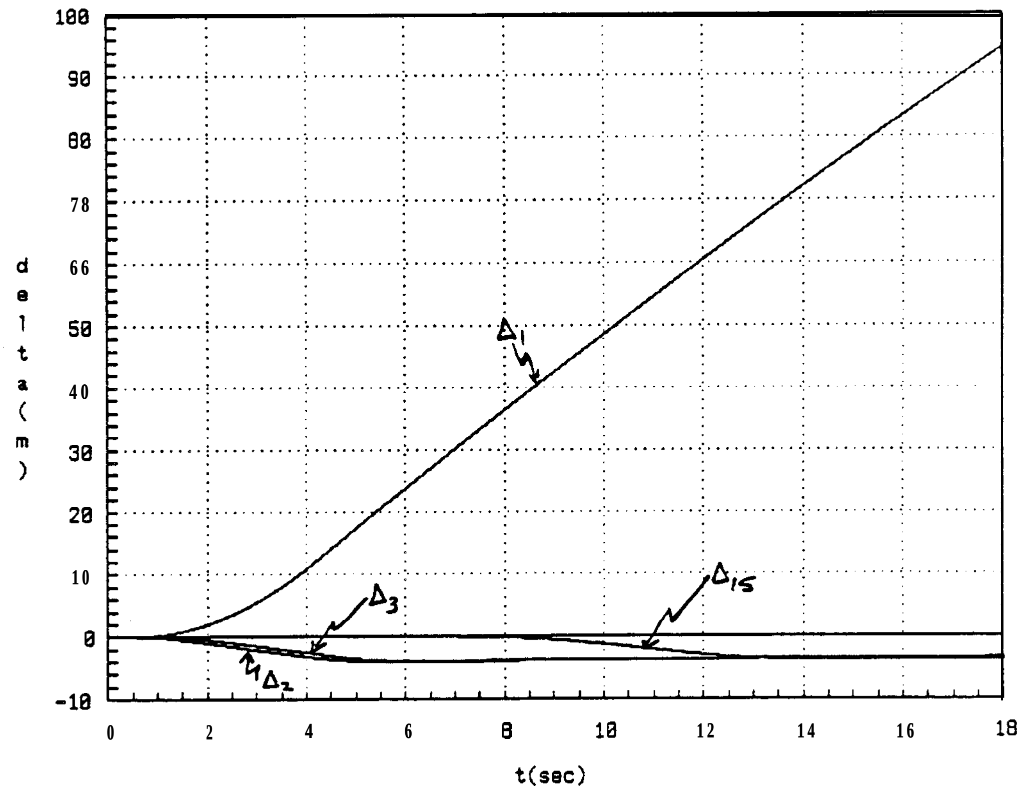

Figure 2: Δ_1,Δ_2,Δ_3,Δ_{15} vs. t- no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ terms in the control law for the i-th vehicle (i $= 1, 2, \ldots$)

- Linear control law with A, and $\ddot{\Delta}_i$ for i = $1, 2, \ldots$-

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{0.2(s + 3.02)(s + 0.017)}{0.2(s + 4)(s + 5)(s + 6)} \tag{5.5}$$

$$\hat{g}(s) = \frac{(s + 4.9)^2}{0.2(s + 4)(s + 5)(s + 6)} \tag{5.6}$$

Figure 2 shows the resulting Δ_1,Δ_2,Δ_3, and Δ_{15} with the above choices of parameters for the linear control law with no A, and 6, (i = $1, 2, \ldots$).

From (5.1) we note that $\hat{h}_{\Delta_1 w_l}(0)$ = 50. Thus, the first vehicle cannot track the lead vehicle to within close spacings due to a change in the lead vehicle's velocity. The asymptotic spacing corresponding to $\hat{h}_{\Delta_1 w_l}(0) = 50$

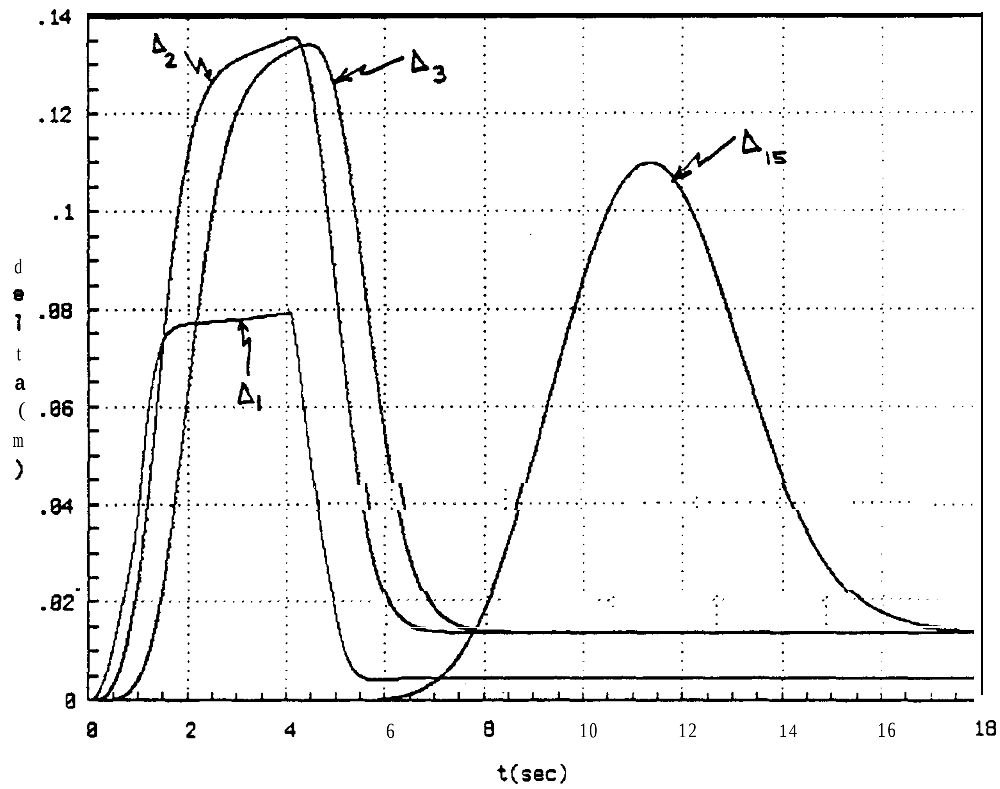

Figure 3: Δ_1,Δ_2,Δ_3,Δ_{15} vs. t- no $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ terms in the control law for the i-th vehicle $(i = 2, 3, \ldots)$

can be reduced to ***zero*** asymptotically, (as $t \rightarrow$ co), but it ***can*** be done only with a large time constant.

Figure 3 shows the resulting Δ_1,Δ_2,Δ_3, and Δ_{15} with the above choices of parameters for the linear control law with no $\dot{A}$; and $\ddot{\Delta}_i (i = 2,3,\ldots)$.

From (5.3) we note that $\hat{h}_{\Delta_1 w_l}(0)$ = 0.0004. Thus, the first vehicle tracks the lead vehicle to within close spacings due to a change in the lead vehicle's velocity.

From (5.4) we note that $\hat{g}(j\omega)$ behaves proportional to $\frac{1}{\omega^3}$ for sufficiently large values of w. Hence, the impuIse response of $\hat{g}$ (i.e., g(t)) in (5.4) is broader than the corresponding impulse response of $\hat{g}$ in (5.6) and causes increased delays (compare fig. 3 with fig. 4).

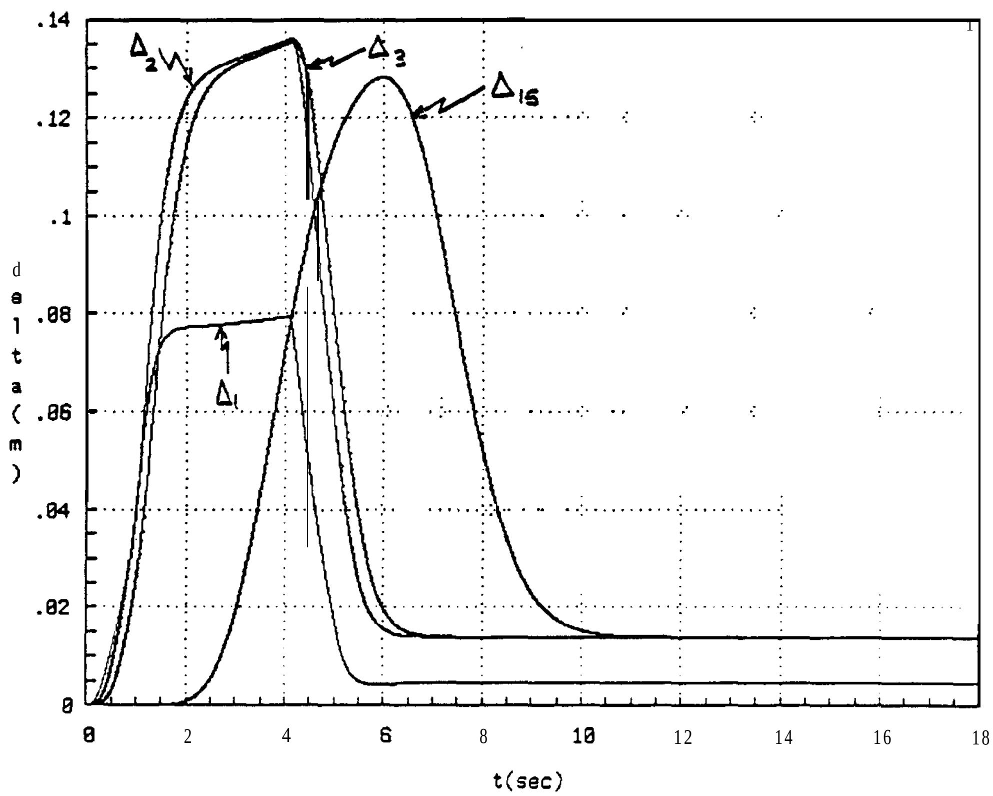

Figure 4: $\Delta_1, \Delta_2, \Delta_3, \Delta_{15}$ vs. t- full feedback of $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ terms in the control law for the i-th vehicle $(i = 1,2,\ldots)$

Figure 4 shows the resulting Δ_1,Δ_2,Δ_3, and Δ_{15} with the above choices of parameters for the linear control law with $\dot{\Delta}_i$ and $\ddot{\Delta}_i (i = 1, 2, \ldots)$.

From (5.5) we note that $\hat{h}_{\Delta_1 w_l}(0) = 0.0004$. Thus, the first vehicle tracks the lead vehicle to within close spacings due to a change in the lead vehicle's velocity.

From (5.6) we note that $\hat{g}(j\omega)$ behaves proportional to $\frac{1}{\omega}$ for sufficiently large values of w. Hence, the impulse response of $\hat{g}$ (i.e., g(t)) in (5.6) is narrower than the corresponding impulse response of $\hat{g}$ in (5.4). As a result, deviations of the vehicles from their preassigned positions due to a change in the lead vehicle's velocity will approach their steady-state values more quickly when $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ are used in every vehicle's control law.

6 Conclusion

We have shown that using A and $\ddot{\Delta}$ in the linear control laws for the longitudinal control of a platoon of vehicles benefits us as follows:

- Using $\dot{\Delta}_1$ and $\ddot{\Delta}_1$ in the linear control law enables the platoon to track the lead vehicle to within spacings of two to three feet.(see fig. 4)
- Deviations in vehicles' spacings due to a change in the lead vehicle's velocity,w_l, will decrease more quickly to their steady-state values if $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ are used in the i-th vehicle's control law.($i = 2, 3, \ldots$)(see fig. 4)

7 References

[She.11 Sheikholeslam,S.,Desoer,C.A. "Longitudinal Control of a Platoon of Vehicles 1:Linear Model,"PATH Research Report UCB-ITS-PRR-89-3,August 18,1989.