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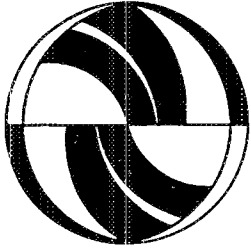**Permalink**
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HANDLING STRATEGIES FOR IMPORT CONTAINERS AT MARINE TERMINALS

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Abstract—Many types of storage systems require goods to be stacked in a storage area. The amount of handling effort required to retrieve individual items from the stacks depends on stack heights and on the adopted storage strategy. The paper focuses on container import operations at marine terminals. It presents methods for measuring the amount of handling effort required when two basic strategies are adopted, one that tries to keep all stacks the same size and another than segregates containers according to arrival time. The strategies are compared in an idealized situation. The methods should be easy to modify for the analysis of similar systems.

1. INTRODUCTION

Container import operations typically involve transferring batches of boxes from an incoming vessel to a temporary storage area where the containers wait to be claimed by trucks. To reduce the area required for temporary storage, containers are usually stacked, and special handling equipment (such as rail-mounted or rubber-tired cranes) is used to move them to and from the stacks.

It is apparent that higher stacks reduce the area required for the yard, but they require additional handling to retrieve boxes near the ground. To define a good configuration for the storage area, methods are needed for estimating the number of moves required to retrieve a box as a function of stack height and operating strategy. The extra handling effort required for higher stacks can then be traded off against space requirements and the best operating strategy can be selected for the chosen yard configuration.

This type of problem is not specific to container yards. It arises in any system in which goods are stored in stacks for a random amount of time before retrieval. The results about to be presented should also apply to such systems.

Figure 1 depicts how a typical storage yard is arranged. Stacks are aligned side by side, forming "bays" that are straddled by cranes. When a specific container needs to be retrieved, the crane must first transfer any boxes that might be on top of it to adjacent stacks, then take the container to an access lane where a truck or straddle carrier receives it. The moves required to expose the target box are called "shuffles."

In this paper, we start by developing a method for estimating the expected number of moves required to remove a single container from a bay (Section 2). In Section 3, simple formulas are derived to estimate the expected number of moves needed to retrieve several containers from a group of stacks, given a realistic operating strategy. It is assumed that no new containers are deposited in the group, so that its average stack height declines. Arrivals to the storage area are considered in the following sections. Section 4 presents an ideal situation with uniform ship arrivals and suggests two basic operating strategies. Section 5 examines the first strategy, which allows containers from different ships to be mixed in the storage area. Section 6 examines a different strategy, in which containers are segregated according to arrival time. Section 7 refines and generalizes some of the results obtained previously. Section 8 compares both strategies, and Section 9 summarizes and discusses the results.

2. RETRIEVING A SINGLE CONTAINER

To determine the expected number of handling moves required to retrieve a single container from a bay, let us start by considering a single stack with B boxes, each equally likely to be selected. In this case, the expected number of moves is clearly
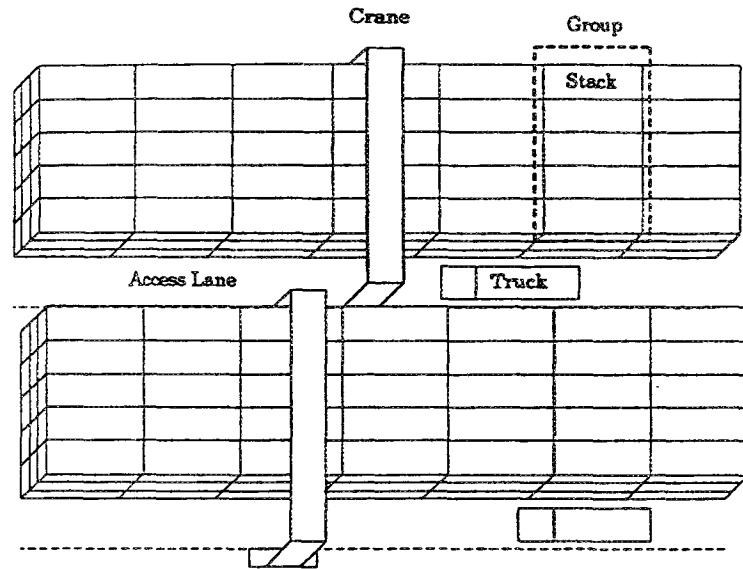
Fig. 1. Typical storage area layout.

$$E[M] = 1/2 \cdot (B + 1)$$

For a group of several stacks with different heights, the expected number of moves can be expressed as

$$E[M] = 1/2 \cdot \sum_{i=1}^{s} p_i(B_i + 1)$$

where s is the total number of stacks, B_i is the current height of stack i, and p_i is the probability that the container to be picked is in stack i.

If all containers are equally likely to be selected, p_i is proportional to B_i, and the above expression becomes

$$E[M] = 1/2 \cdot (E[B^2] + E[B])$$

or, more conveniently

$$E[M] = 1/2 \cdot \left(E[B] + \frac{Var[B]}{E[B]} + 1\right) \qquad (1)$$

Equation (1) clearly shows that the expected number of moves per container, $E[M]$, is minimal when all stacks have the same number of containers, because in that case the variance-to-mean ratio of the stack heights is zero. The result, a classical example of length-biased sampling, should not be surprising. It is analogous to the expected delay formula for passengers waiting for a bus (Cox, 1962). Because more passengers arrive during long headways (with longer delays), the average passenger delay at the bus stop is greater than half a headway. Similarly now, taller stacks, which require more moves, are more likely to contain the next box to be picked.

3. RETRIEVING SEVERAL CONTAINERS

It follows from eqn (1) that if several containers are to be extracted from a bay, requiring some shuffling moves, shuffles should aim to keep the stacks equalized. This

would reduce future handling effort. In this spirit, we consider a strategy that extracts the maximum benefit from each shuffling move by always shifting containers to the emptiest stack within a reasonable distance from the crane.

We will call a set of stacks among which shuffles can be performed a "group." A very uneven group would be likely to require shuffles, which would tend to restore its stacks to the average height rapidly. Unfortunately, perfectly even stacks cannot be achieved because after each retrieval the group will typically have one short stack (from which a container was recently removed), even if all the others should be fairly balanced.

A computer simulation was used to evaluate the variance-to-mean ratio of the stack heights in a group as it was emptied with the described strategy. The simulation program also tracked the number of shuffling moves.

Remarkably, the simulation showed that, as long as the number of stacks per group is at least equal to the initial average stack height ($s \geq E[B]$), the variance of the stack heights at any one time is approximately one-half of the mean stack height at the time, $Var[B(t)] \sim 0.5 E[B(t)]$. Thus, the expected number of shuffling moves to retrieve a container can be expressed as a function of the current average stack height, $E[B(t)]$, only

$$E[M(t)] = 1/2 \cdot (E[B(t)] + 1.5) \qquad (2)$$

This is slightly higher (0.25 moves) than it would be if the stacks were always perfectly equalized.

Figure 2 shows the expected number of moves required to remove a single container from groups of stacks of different average heights. The symbols represent values obtained from the simulation for groups of different dimensions. The solid line was obtained by using eqn (2). To illustrate the contribution of the stack height variance-to-mean ratio, the figure also shows the results obtained if $Var[B]/E[B]$ were ignored.

Equation (2) can be used to evaluate the number of moves required to retrieve several containers from a group. Let s and p denote the number of stacks and the number of boxes initially in the group. Then the expected number of movies to retrieve c containers from the group is

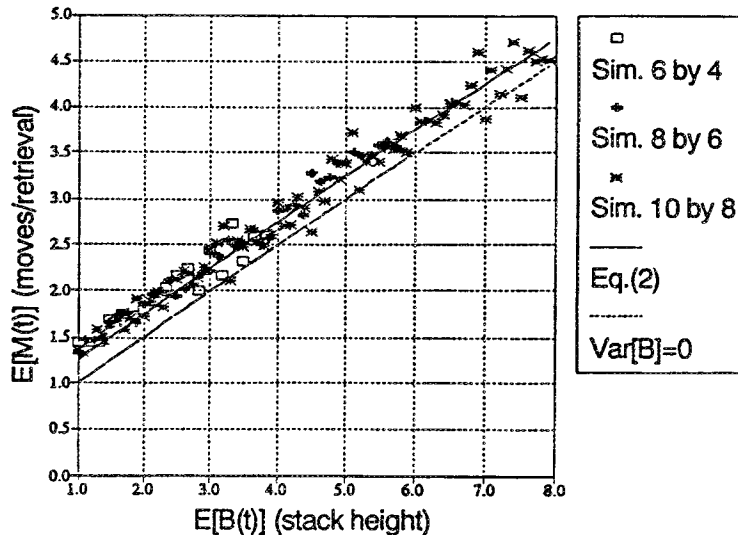$$1/2 \cdot \sum_{i=0}^{c-1} \left(\frac{p-i}{s} + 1.5 \right)$$

Fig. 2. Moves required to retrieve a single container.

where $(p - i)/s$ is the average stack height after i containers have been removed. Dividing the above expression by c, we obtain the average number of moves per box during the whole operation, $E[M]$:

$$E[M] = 1/2 \cdot \sum_{i=0}^{c-1} \left(\frac{p - (c - i)/2}{s} + 1.5 \right)$$

Letting $E[B_i]$ and $E[B_f]$ denote the initial and final average stack heights, respectively, and $E[B]$ the average stack height during the whole operation, we can rewrite this expression in a form similar to that of eqn (2):

$$E[M] = 1/2 \cdot \left(\frac{E[B_i] + E[B_f]}{2} + 1.5 \right) = 1/2 \cdot (E[B] + 1.5) \qquad (3)$$

4. AN IDEAL CASE: CONSTANT HEADWAYS AND SHIP SIZES

In the previous sections, we derived expressions for the number of moves required to retrieve one or more containers from the storage stacks. The remainder of this paper seeks to illustrate the effects of two handling strategies over long periods of time when many ships and their cargoes pass through the port. In order to keep the number of parameters down, the comparisons will be made for a situation with idealized ship traffic that can be described by only two constants. Although general (nonideal) cases can also be handled, they will not be the focus of this presentation.

We assume from now on that container dwell times can be represented as outcomes of independent identically distributed (i.i.d.) random variables that are revealed when they depart; port management has no information. If (somewhat unrealistically) we temporarily assume that all the containers in the yard are equally likely to depart at any time (as if dwell times were exponentially distributed) and we assume that an effort is made to keep the stacks even, the number of container moves can be predicted as a function of their accumulation: note that eqn (2) should hold for any container at any time with $E[B(t)] = E[P(t)]/S$, where $P(t)$ is the number of containers in the yard at time t and S is the total number of stacks.

As explained in Taleb-Ibrahimi (1989), for any given arrival pattern of containers, it is easy to construct curves depicting the cumulative number of containers to have arrived, $A(t)$, and departed, $D(t)$, from the yard, on average. For large traffic volumes the actual curves should be close to the averages so that $P(t) \sim A(t) - D(t)$ and, for large $(t_1 - t_0)$,

$$E[M] = \frac{1/2 \cdot \int_{t_0}^{t_1} ([A(t) - D(t)]/S + 1/2)dD(t)}{D(t_1) - D(t_0)}$$

This calculation can be done easily with a spreadsheet.

For our comparisons, we shall assume that ships arrive with constant headways and always unload the same number of containers, A. we also assume that the containers of a ship are removed from storage at an approximately constant rate during a period of time equal to n headways; the retrieval rate for a given ship is A/n containers per headway. The situation is depicted in Fig. 3.

Note that with this arrival/retrieval process, the storage yard always holds containers from exactly n ships and the overall retrieval rate is A (containers per headway). Therefore, the parameter n can also be viewed as the number of ships represented by at least one container in the storage stacks at any point in time.

We can take advantage of the symmetry of the problem and restrict our analysis to a period of one headway. Immediately after a ship call, all stacks are filled to a height $B = P/S$. The total number of containers in storage at this point is $P = A(n + 1)/2$. During
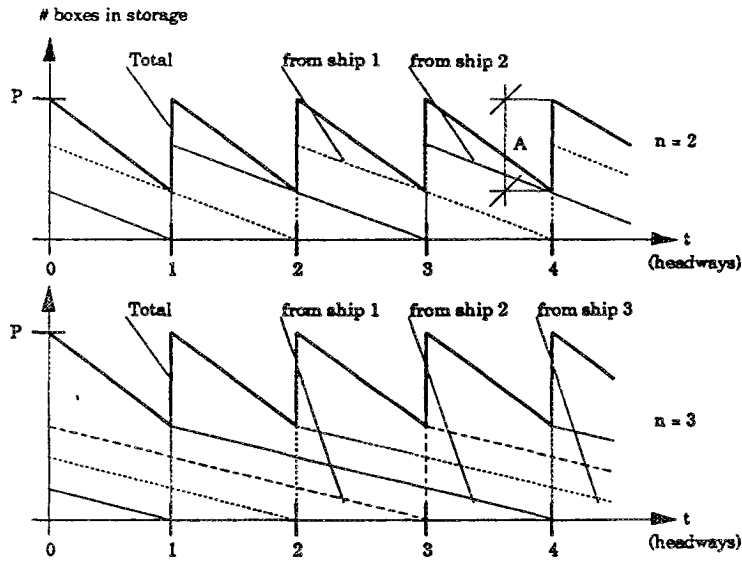
boxes in storage

Fig. 3. Ideal case.

the next headway, A containers are removed, leaving $A(n - 1)/2$ boxes in storage at the end of the period. The average number of boxes in storage during the period is $An/2$, and the average stack height is therefore

$$E[B] = \frac{An}{2S}$$

a quantity that is independent of the adopted strategy.

Little's formula of queueing theory ensures that this expression also holds for other dwell times, with mean equal to $n/2$ headways. The expression can be used with eqn (3) to estimate the total number of moves required to retrieve each container as a function of A, S and n:

$$E[M] = 1/2 \cdot \frac{An}{2S} + 0.75 \tag{4}$$

For example, if $n = 3$ and $A/S = 3$, then $E[M] = 3$.

Note that eqn (4) is valid only if at any point in time each container present in storage has an equal probability of being the next one to be retrieved. In other words, each container's departure time from the stacks should be independent of its arrival time. Such a situation could exist, but it does not seem very probable. In reality, a container that has been in storage for a long time would tend to leave before one that just arrived. The expression could also hold if containers with different probabilities of being picked were perfectly mixed in the stacks, but this also seems unlikely.

If containers have an increasing chance of being picked as their time in storage increases, as happens with uniform distributions, and nothing is done to segregate them by arrival time, the most likely boxes to be picked (hot) will tend to be "buried" under recently arrived ones (cold). This would naturally increase the expected number of moves per retrieval, $E[M]$. Section 5 examines the hot/cold penalty. Section 6 considers a strategy that avoids the penalty by storing the containers from each ship separately (segregating). The choice between the two strategies is not obvious: segregating containers introduces a "consolidation" phase that requires extra moves and will be effective only if this effort is smaller than the increase in $E[M]$ due to burying hot boxes under cold ones.

5. NONSEGREGATING STRATEGY

If new containers are placed on top of existing ones (equalizing stack heights), after each ship call the stacks would consist mostly of cold containers on top of hot ones. Then, as containers were removed, the stacks would become more mixed and the proportion of hot boxes would gradually decrease.

Because of the random nature of the shuffling moves, the evolution of stack composition is hard to model analytically. An exception occurs whenever shuffled containers are always returned to their original stacks, in the original order. In that case, containers remain stacked in the most unfavorable way possible, as illustrated in Fig. 4. The resulting formulas, derived below, are upper bounds because shuffles will tend to improve on this order.

In a general case, with an arbitrary but known ship schedule and a known cumulative distribution function (c.d.f.) for the container dwell times, $T(t)$, it is easy to determine at any time the number of containers left from each prior ship. If ship j arrived at time $H_j (H_j < t)$, then it left behind $P_j(t) = A_j[1 - T(t - H_j)]$ (or $P_j(t) = A_j T_c(t - H_j)$, where $T_c(.)$ is the complementary c.d.f.) containers at time t and these depart at a rate $d_j(t) = A_j(t)T(t - H_j)$. The expected number of moves for a container of this type is

$$E[M_j(t)] = 1/S \cdot \{1/2 \cdot P_j(t) + 0.75 + P_{j-1}(t) + \ldots \}$$

$$= A/S \cdot \{1/2 \cdot T_c(t - H_j) + 0.75 + T_c(t - H_{j-1}) + \ldots \}$$

This formula assumes that all the stacks contain the same mixture of boxes and that to get to a container of type j, all the boxes from earlier ships must be shuffled $(j - 1, j - 2, \ldots)$; in addition, a number of moves given by eqn (2) is expected for the type j container.

As a result, the expected number of moves during our period of interest is given by

$$\int_{t_0}^{t_1} \Sigma_j E[M(t)] \cdot d_j(t) dt$$

For uniformly distributed dwell times, these calculations result in simple analytical formulas.

Figure 4 illustrates how the composition of a typical stack changes over time: with
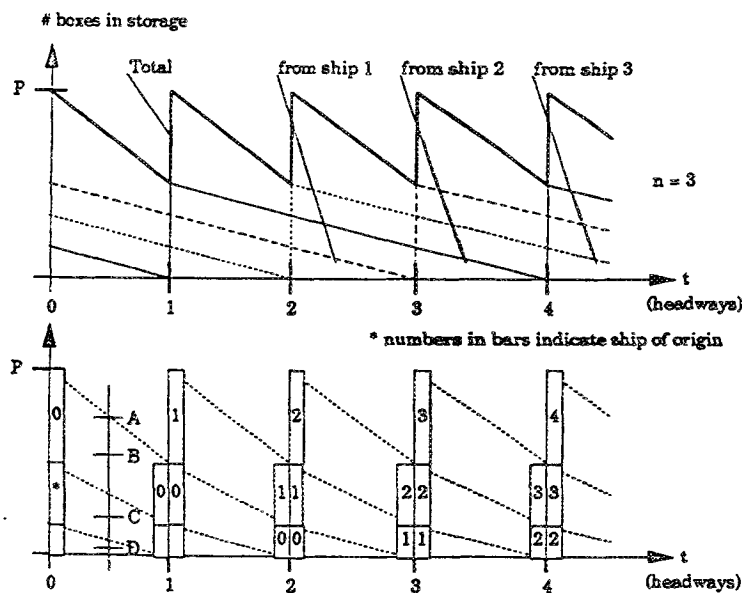

Fig. 4. Nonsegregating strategy.

our pessimistic model, the top n segments would contain the coldest containers; the middle segments of decreasing size would contain increasingly hot containers all the way to the bottom, where a single segment would contain all the hottest boxes.

The resulting formula for the expected number of moves is

$$E[M] = \left(\frac{2}{3} - \frac{1}{6n^2}\right) \cdot \frac{An}{2S} + 0.75 \qquad (5)$$

Except for the term in braces

$$f_n = \frac{2}{3} - \frac{1}{6n^2} \qquad (6)$$

this formula is identical to eqn (4). The quantity f, which corresponds to the term $1/2$ in eqn (4), can be interpreted as the expected position of a randomly selected container as a fraction of the stack height. It is apparent that if each box were equally likely to be selected, f would be equal to $1/2$ [the coefficient we used in eqns (1) through (4)]. In the worst-case scenario described above, however, f could be substantially larger: f is 0.5 for $n = 1$ and approaches $2/3$ as n tends to infinity. Therefore, even when there are many types of boxes, we can expect the additional handling effort due to burying hot boxes to be less than about $(2/3)/(1/2)$, or 33% more than that required if all containers were equal.

To see how eqns (5) and (6) arise, without algebraic manipulations, consider the example depicted in Fig. 4. Because the next container to be picked has an equal probability of being in each set of homogeneous boxes (A/n will be removed from each one within the next headway) and the departure rates are constant through the headway, we can evaluate $E[M]$ from the stack heights in the middle of a headway. The result is

$$E[M] = \frac{(0.5/2 + 1.5 + 2.5) + (1.5/2 + 2.5) + (2.3/2)}{3} \cdot \frac{1}{4.5} \cdot \frac{3A}{2S} + 0.75$$

The quantity in braces is the average of distances AB, AC and AD in Fig. 4, 2.92. This quantity is divided by the height of point A to give the coefficient f, 0.65. The factor $3A/2S$ represents the average stack height, as described in the previous section. If $A/S = 3$, $E[M] = 3.67$.

This represents a 22% increase from our previous result ($E[M] = 3$), the lower bound calculated assuming all containers were equally likely to be retrieved at any time. Although we have no evidence to support it, we would expect the actual number to be closer to the upper than to the lower bound.

6. SEGREGATING STRATEGY

Section 5 illustrated that when arriving containers are piled on top of waiting containers, the number of extra moves per container is higher than it would be if the containers had been randomly mixed. This occurred because the containers most likely to be retrieved at any given time (hot), that is, those having spent the longest time in the yard, will tend to be buried under colder containers. In this section we investigate whether handling could be reduced by segregating containers by age in the yard.

Although this type of segregation would require some extra moves before each ship arrival, enough to clear the space it would need for its containers, it may save enough moves during the actual retrieval process to be of benefit. Our objective is to identify conditions under which such segregation would be advantageous.

As in Section 5, we will calculate the number of container moves per container handled at the yard. Because ship cargoes are segregated, our unit of analysis will be one ship. The expected number of clearing and retrieval moves will be calculated for the whole ship, ending when the last of the ship's containers is retrieved. The total number of

moves will be prorated to each one of the ship's containers to obtain the expected moves per container.

Although formulas are developed in Section 7 for a general case with an arbitrary ship schedule, varying cargo sizes and any container time distribution, the comparisons with the nonsegregating strategy will be made for the ideal case examined in Section 4. The qualitative conclusions from these comparisons should extend to more general cases, which can then be evaluated more precisely with our expressions or with simulations.

In order to keep ship cargoes segregated, it is necessary to make room for each arriving ship before its arrival. Because there is no need for all the containers of one ship to be together in the yard, room is assumed to be made as follows. We assume that the stacks allocated to each ship are divided into groups (defined earlier as sets of adjoining stacks among which containers may be shuffled). The groups for a ship can be spread over the whole yard in any pattern. The number of container stacks per group, s, is assumed to be constant and equal for all ships, so that the total number of groups in the yard can be written as S/s. When empty groups are needed to receive a new shipload, they are released from partially filled groups of existing ships. Groups with the fewest containers are then emptied and their contents transferred to the remaining groups for the same ship.

With this strategy, the number of groups allocated to a ship decreases with time (in discrete amounts with each ship arrival), and the average number of containers per stack varies with time in a sawtooth pattern with discrete increases when a ship arrives and gradual decreases in between. For our ideal case with constant ship interarrival times (headways), constant ship loads A and uniform container dwell times, we assume that groups are released so as to maintain stacks of roughly equal height immediately after each ship arrival. Because the number of containers in the yard at those times is (on average) $A(n + 1)/2$, the average stack height at those times should be $B = A(n + 1)/(2S)$. To be consistent with this height, each new ship must receive A/B empty stacks, or $A/(Bs) = 2S/[s(n + 1)]$ groups when it arrives. The ship will then release one-nth of the groups with each subsequent headway: $2(S/s)/[n(n + 1)]$.

Figure 5 depicts the average number of containers per stack as a function of time $B(t)$, for $n = 3$ and $A/S = 3$. During the first headway, stack height decreases by one-third (because one-third of the containers will have departed). It reaches the maximum again after one-third of the blocks have been released for the next arriving ship (the other two-thirds is contributed by the other two ship loads still in the yard). During the second headway, stack height decreases more rapidly because the same number of containers is removed from fewer stacks. All remaining containers are cleared during the last headway.

The model introduced in Section 3 can be used to estimate the number of container moves during the times when the average stack height decreases. The moves per container
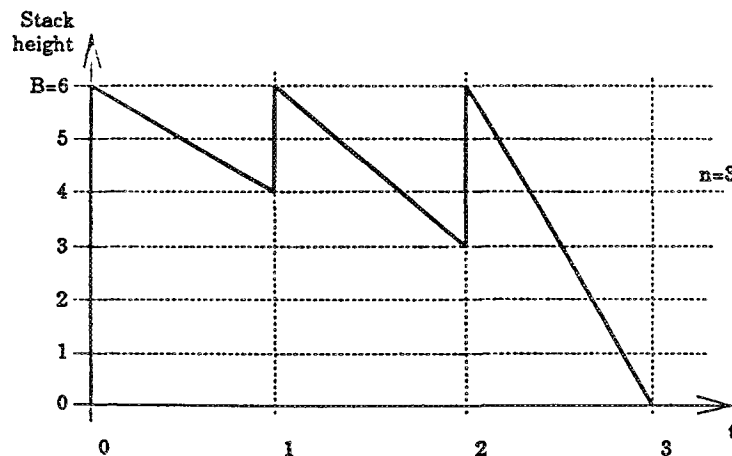

Fig. 5. Segregating strategy.

can be calculated for each headway with the aid of Fig. 5, because during each headway all the containers from each ship are equally likely to be picked.

For the example depicted in Fig. 5, average stack height decreases from the initial B = 6 to $6(2/3)$ = 4 in the first headway. Using eqn (3), we obtain $E[M]$ = $1/2(5 + 1.5)$ = 3.25 moves per container. The blocks are then consolidated, and the height of each stack is restored to 6. During the second headway, stack heights decrease from B = 6 to $6(1/2)$ = 3, yielding $E[M]$ = $1/2(4.5 + 1.5)$ = 3.00. The blocks are consolidated for the last time, and during the last headway all containers are removed $(E[M]$ = $1/2(3 + 1.5)$ = 2.25). Because the same number of containers is removed during each headway, the overall $E[M]$ is simply the average of all $E[M]$s: $1/3(3.25 + 3.00 + 2.25)$ = 2.83.

Using the same logic for an arbitrary n, we find the general formula for the expected number of retrieval moves per container, $E[M_r]$, in our ideal case (from now on we use the subscripts r and c to distinguish retrieval and clearing moves):

$$E[M_r] = \frac{B}{2} \cdot \left(1 - \frac{1}{2n} \cdot \sum_{j=1}^{n} \frac{1}{j}\right) + 0.75 = f_s \cdot \frac{An}{2S} + 0.75 \qquad (7a)$$

where f_s is the following function of n:

$$f_s = \frac{n+1}{2n} \cdot \left(1 - \frac{1}{2n} \cdot \sum_{j=1}^{n} \frac{1}{j}\right) \qquad (7b)$$

The result has the same form as eqn (5), except now the factor f, representing the expected position of a container in an average stack, is below $1/2$, significantly lower than the one found in Section 5 [eqn (6)]. It is even lower than the expected number of moves that would result if boxes were randomly mixed at all times with the same average stack height [eqn (4)]. This happens because the segregating strategy results in a few stacks of hot containers, which are emptied relatively quickly, and many with cold ones. During most of the time, therefore, the stacks most likely to be selected (the hot ones) have fewer boxes and therefore require fewer moves. Table 1 compares the factors f_s and f (defined in Section 5) for n ranging from 1 to 10:

The benefits of segregation must be traded off against the need for clearing moves. If all the blocks were cleared at the same rate, the number of clearing moves could be calculated easily. (Refer again to Fig. 5.) At the end of the first headway, all the ship stacks are 67% full, and one-third of them have to be cleared. This means that $(1/3)(2/3)$ of the containers carried by the ship must be moved. During the second headway, the same number of blocks need to be cleared, but now they are only 50% full, so $(1/3)(1/2)$ of the containers need to be moved. During the last headway there are no moves, so the number of clearing moves per container is $(1/3)[(2/3) + (1/2)]$ = 7/18, or 0.39 clearing moves per box. For an ideal case with arbitrary n, the number of clearing moves per container is

$$E[M_c] = \frac{1}{n} \cdot \left(\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n-1}{n}\right) = 1 - \frac{1}{n} \cdot \sum_{j=1}^{n} \frac{1}{j} \qquad (8)$$

If desired, the summation in eqns (7) and (8) can be approximated to within better than 1% accuracy by

Table 1. Expected position of a container in an average stack for the segregating and nonsegregating strategies

n	1	2	3	4	5	6	7	8	9	10
f_s	0.500	0.468	0.463	0.462	0.463	0.464	0.466	0.467	0.468	0.470
f	0.500	0.625	0.648	0.656	0.660	0.662	0.663	0.664	0.665	0.665

$$\sum_{j=1}^{n} \frac{1}{j} \approx \ln(m) + \frac{1}{6m^2} - 0.177$$

where $m = 2n \div 1$. Notice that this expression is only a function of n; it is independent of stack height, block size, and so forth.

Clearing moves may take more time than retrieval moves, because cleared containers have to be moved horizontally for longer distances. If blocks are randomly scattered about the yard, an upper bound to the horizontal distance per clearing move is the mean distance between two random points in the yard. In actuality the distance should be considerably less, as each cleared container can be carried to a block of our choice, subject to room availability. In fact, the selection of container destinations to minimize distance can be formulated as the Hitchcock transportation problem of linear programming, from which much lower average distances would result. The reduction is further accentuated because if cranes can find efficient back-hauls, only one-way distances need to be traveled for the most part.

Because the time spent overcoming horizontal distance is only a fraction of the total time for any crane move, it is not necessary here to model horizontal distances accurately. For comparison purposes, it should suffice to recognize that each clearing move may be equivalent to x retrieval moves, where x is a quantity somewhat >1, which can be determined by a combination of observation and analysis. In the comparisons presented in Section 8, x will be taken to be 1.2. Equation (8) is based on the assumption that all the groups are reduced at the same rate. In actuality, some groups will be reduced more rapidly than others, and by choosing to clear the groups with the fewest containers, the number of clearing moves can be lowered.

For our ideal case, this correction (in number of saved clearing moves) is of the form

$$\frac{1}{\sqrt{N}} \cdot G(n) \approx \frac{1}{\sqrt{N}} \cdot \frac{n-1}{2n}$$

where N is the number of containers per group at the beginning of a headway. (This expression is derived in the next section, which also develops expressions for $E[M_r]$ and $E[M_c]$ for a general case with arbitrary ship schedules, cargo sizes and container dwell times.)

Because $N = Bs$, the correction is maximized if the number of stacks per group is chosen as small as possible. We indicated earlier that s had to be at least equal to B in order for the retrieval process to work smoothly. Thus, we would choose $s = B$. Then

$$\sqrt{N} \approx B = \frac{A(n + 1)}{2S}$$

and the correction becomes

$$\frac{S}{A} \cdot \frac{n-1}{n(n+1)}$$

7. THE EXPECTED NUMBER OF CLEARING MOVES

This section develops a simple expression for the expected number of clearing moves per container handled (clearing moves are needed to make room for incoming ships under the segregating strategy).[1] The expression is more realistic than eqn (8) because it takes into account the fact that the operator may choose to clear the groups with the fewest containers.

We assume that shiploads of different sizes are unloaded at the port at irregular intervals, that each shipload is stored in a devoted set of groups (of stacks) and that

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[1] Although the expression is simple, its derivation is not. This section is largely independent of the remainder of the paper, however, and may therefore be skipped on a first reading.

immediately before each ship arrival the containers in some groups are cleared to make room for the incoming ship. We also assume that the number of stacks per group is a constant, s.

Containers cleared from one group are taken to groups assigned to the same ship. The number of groups and the number of containers cleared are chosen to ensure that once the newly arrived ship is unloaded, all the groups are stacked to (roughly) the same height.

We track the number of clearing moves for a ship arriving at time $t = 0$, assuming that its A containers are stacked in M_0 groups containing N_0 containers per group. (Because the number of stacks per group is a constant, the stack height at time 0 is defined by N_0—a more convenient variable for our manipulations.)

The following information is known:

A_k: cumulative distribution function of the container dwell times, assumed to be i.i.d.

H_k: time of the kth ship arrival after time $t = 0$ $(H_0 = 0)$

N_k: target number of containers per group (i.e. stack height) after the kth ship arrival

In practice, one would have to decide the N_k from the H_k and the incoming ship loads (A_k). In view of the results in Section 4, it seems that keeping the stack height (i.e. N_k) as even as possible across the yard, the same for all ships, would minimize the number of retrieval moves. If this is done, N_k is simply given by the ratio of the accumulation of containers in the yard, which can be approximated from $T(t)$ and the A_k's as shown in Taleb-Ibrahimi (1989), and the (fixed) total number of groups, S. The N_k used for the ideal case reported in Section 4 satisfies this property as it results in a constant stack height (B) after each ship arrival. The formulas developed, however, apply to any set of N_k's.

From the above information we define the following auxiliary variables:

$q_k = T(H_{k+1}) - T(H_k)$ = probability that a container is picked in the interarrival time immediately after the kth ship, that is, in the $(k + 1)$th headway

$P_k = 1 - T(H_k)$ = probability that a container is in the yard when the kth ship arrives, that is, its dwell time exceeds H_k

$p_k = q_k/P_k$ = probability that a container is picked in the interarrival time immediately after the kth ship, that is, in the $(k + 1)$th headway, given that it is in the yard at the beginning of the headway, when the kth ship arrives

$M_k = M_0 N_0 P_k/N_k$ = number of groups used by our ship after the arrival of the kth ship, that is, the beginning of headway $k + 1$ (defined as the ratio of the expected number of containers remaining, $M_0 N_0 P_k$, and the target group size, N_k)

Because M_k is assumed to be fixed, but the container departures are random, the actual group size at the beginning of headway $k + 1$ may not coincide with the target, N_k. For the purposes of calculating the expected number of clearing moves in headway $k + 1$, however, we assume that the actual group size at the beginning of the headway is N_k. The impact of this approximation should be negligible because, as we shall see, the dependence of the expected total number of moves on N_k is practically linear over the range of values over which N_k is likely to vary—the variance of N_k is proportional to the reciprocal of ship size, which is usually a large number.

Conditional on N_k, the number of containers departing from a group in the $k + 1$ headway is a binomial random variable with N_k trials and probability of success p_k, $B(N_k, p_k)$. Except for the scale on the ordinate axis, the cumulative number of groups with less than or equal to z containers at the end of headway $k + 1$ would thus resemble an empirical c.d.f. from the random variable N_k, $B(N_k, p_k)$. If N_k is large (comparable to 10, say), the cumulative curve of Fig. 6 will also resemble the c.d.f. of a normal random variable with mean $N_k(1 - p_k)$ and variance $N_k(1 - p_k)p_k$, labeled $F(z)$ (see Fig. 6).

Figure 6 depicts a case with $M_{k+1} < M_k$ and $N_{k+1} < N_k$. Because $M_{k+1} < M_k$ (as should be usual), $M_{k+1} - M_k$ groups need to be cleared for the next ship; the area labeled Y_k in the figure equals the resulting number of clearing moves. If M_{k+1} were $> M_k$, then there would be no need for any clearing moves, as not groups would have to be released for the next ship.

The area labeled Z_k in the figure represents the number of "trimming" moves that
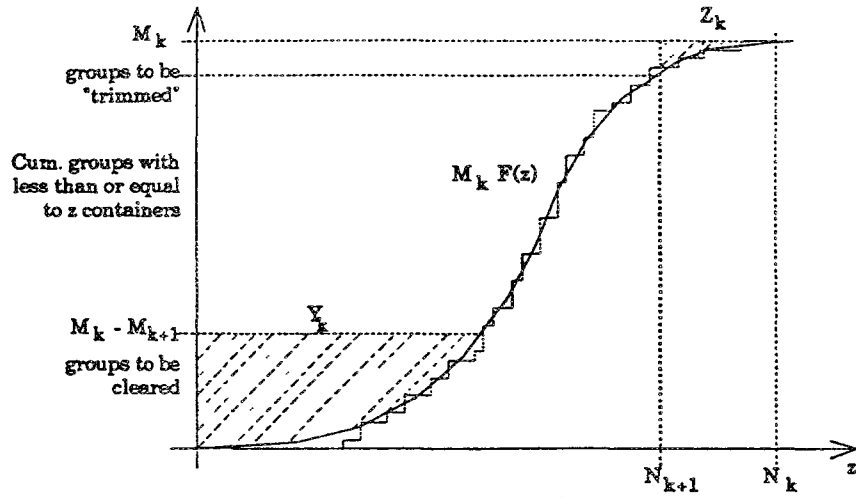
Fig. 6. Distribution of group populations.

would be needed to reduce the height of any groups with too many containers. They will be ignored here because, most likely, one would decide never to do a trimming move and instead choose to live with temporary (slight) unevenness in stack heights as a result—the increase in retrieval moves would be very small. Even if we insisted on making trimming moves, they wouldn't have to be made immediately; they could be made early in the following headway in conjunction with extra retrieval moves. As such, each trimming move would contribute only to the count of equivalent retrieval moves of $x - 1$ units. (See Section 6 for the definition of x.) Furthermore, Z_k is likely to be relatively small because only in rare occasions will N_{k+1} be substantially small than N_k to make a difference.

The area Y_k is the integral of the inverse function of Fig. 6 from zero to $M_k - M_{k+1}$. On average, it will be close to the area corresponding to the inverse of the normal approximation, $M_k F(z)$. Thus, we can write

$$E[Y_k] \approx M_k \cdot \int_0^{1-M_{k-1}/M_k} F^{-1}(y)\,dy$$

and letting $\psi(.)$ denote the standard normal c.d.f., this becomes

$$E[Y_k] \approx M_k \cdot \int_0^{1-M_{k+1}/M_k} \left\{ N_k \cdot (1 - p_k) + \sqrt{N_k \cdot (1 - p_k) \cdot p_k} \cdot \phi^{-1}(y) \right\} dy$$

After a few simple manipulations we find

$$E[Y_k] \approx M_0 N_0 \cdot \left\{ P_{k+1} \cdot (1 - M_{k+1}/M_k) + \left(\frac{P_k M_k \cdot (1 - p_k) \cdot p_k}{N_0 M_0} \right)^{1/2} \right.$$

$$\left. \cdot h(1 - M_{k+1}/M_k) \right\} \tag{9}$$

where $h(.)$ is a negative and convex function defined by

$$H(w) = \int_0^w \psi^{-1}(y)dy \tag{10}$$

If cases in which $M_{k+1} > M_k$ are allowed, this function should be defined to be zero if $w < 0$.

Because $M_0 N_0$ is the number of containers carried by the ship, requiring a total of kY_k clearing moves, it is easy to see that the expected number of moves per container is the sum over k (starting with $k = 0$) of the quantity in braces in eqn (9). The sum of the second term in eqn (9) vanishes if the N_k tends to infinity. In that case, the c.d.f. of Fig. 6 would be a step function, and all the groups would be cleared at the same rate. As such, the (negative) second term represents the moves saved because we can choose the groups having the fewest containers. The expected number of clearing moves is simplified for the ideal case in which ship loads and headways are constant and the container dwell times are uniformly distributed with a maximum dwell time equal to n ship headways. In that case, assuming that the group size also stays constant, $N_k = N$, then

$$P_k = \frac{n - k}{n}$$

and

$$p_k = 1 - M_{k+1}/M_k = \frac{1}{n - k}$$

Therefore, the quantity in braces in eqn (9) reduces to

$$\frac{1}{n} \cdot \left\{ (1 - p_k) + \frac{1}{\sqrt{N}} \cdot g(p_k) \right\} \tag{11}$$

where $g(.)$ is the following function:

$$g(w) = \sqrt{(1 - w)/w} \cdot h(w)$$

Adding eqn (11) from $k = 0$ to $k = n - 1$, we obtain

$$\frac{1}{n} \cdot (n - 1 - 1/2 - \ldots - 1/n) - \frac{1}{\sqrt{N}} \cdot G(n)$$

where $G(n)$, a positive quantity, is the negative of the average of the $g(p_k)$ for the given n. The first term of this expression coincides with the expression used in Section 6 leading to eqn (8). ($G(n)$ is given in Table 2.) The relative error in the approximation given for $n < 25$, also presented in Section 6, is < 0.05.

8. COMPARISONS

We compare both strategies for different values of n, holding the average stack height, $An/2S$, constant. Figure 7 plots the number of container moves for the ideal case

Table 2. Correction to the number of clearing moves when using the segregating strategy

n	2	3	4	5	n < 25
G(n)	0.20	0.30	0.36	0.40	~ (n − 1)/(2n)

described in Section 4. The figure shows $E[M]$ for both strategies as a function of n, for three different average stack heights ($B = 2$, 3 and 4). Note that the curves for the nonsegregating strategy represent an upper bound for $E[M]$; actual values are likely to be a little lower. The curves for the segregating strategy assume $x = 1.2$ (i.e. clearing moves require 20% more effort than retrieval moves).

The equivalent number of retrieval moves per container with the segregation strategy is $E[M] = E[M_r] + xE[M_c]$. Recall that $xE[M_c]$ is independent of stack height, unlike the expressions for the number of retrieval moves (with both the segregation and the nosegregation strategies), which increased with B. Clearly then, the segregation strategy will be relatively more appealing when land is scarce and containers have to be stacked high, as then the impact of clearing moves should be relatively smaller. Figure 7 confirms this observation. Increasing values of B have less of an effect on $E[M]$ when the segregating strategy is adopted.

Although the number of retrieval moves per container is not very sensitive to n for a given B (for $n > 2$) with either strategy, the number of clearing moves does increase with n noticeably. This should be intuitive, because each ship has to be cleared n times before all its containers have departed. Thus, one would expect situations with small n (few big ships) to favor the segregation scheme. Figure 7 also confirms this observation. The slope of the $E[M]$ curves decreases more rapidly for the nonsegregating strategy.

In general, the segregating strategy seems to perform better when n is small (≤ 3). For large values of n (≥ 10), the nonsegregating strategy reduces handling effort. For intermediate values of n (between 3 and 10 or 12), the best strategy depends on the average stack height. Shorter stacks favor the nonsegregating strategy, and segregating is better when stacks are high. For $n = 6$, for example, Fig. 7 shows that the segregating strategy is better only if the average stack height is > 3.

9. DISCUSSION

In this paper, we have developed general expressions for the expected number of moves required to retrieve a container from storage stacks under two different storage strategies. An idealized case was examined in detail in an attempt to identify the conditions favoring each strategy.

The simplicity of the expressions for this ideal case makes them easy to understand and useful as tools in the preliminary design stages of a terminal, when ship schedules and container dwell times are not known in detail. They allow terminal planners to achieve a good trade-off between stack heights (and associated land requirements) and handling effort.

For cases when the ideal scenario does not apply, more parameters are needed to analyze the problem. Although the more detailed analysis described in Sections 4 and 7 could be used, this would require much more information.

We did not attempt to optimize the frequency of consolidation moves. It is likely that with large n (> 3 or 4), consolidation would not be done n times, but a submultiple of n for each ship. Such an optimization would enhance the performance of the segregating strategy.

The comparisons were made for a uniform dwell time distribution ranging from zero to n; then the C.V. of the dwell time is in all cases $3^{-1/2} \sim 0.58$. We have seen that segregation would offer no improvement for exponentially distributed dwell times (C.V. ~ 1), which suggests that low variability in dwell times will favor segregation. This can easily be quantified with the formulas presented for uniform dwell times with a range from n' to n ($n' = 1, 2, \ldots, n$), but this is not done here for brevity. The case with $n' = n$ is a particularly obvious situation in which segregation is a must; containers then leave in the order of arrival.
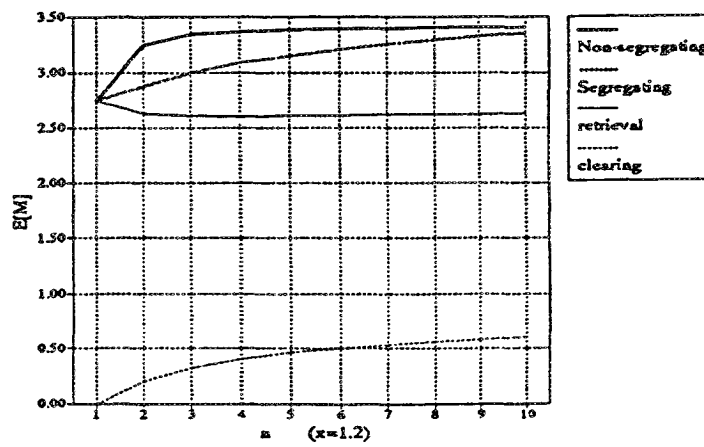
The described operating strategies were designed to compare the consequences of two fundamentally different approaches (segregating vs nonsegregating) and to keep the problem tractable. We did not attempt to identify an optimal strategy. In practice, more efficient (and complex) strategies could be developed and evaluated with simulations.
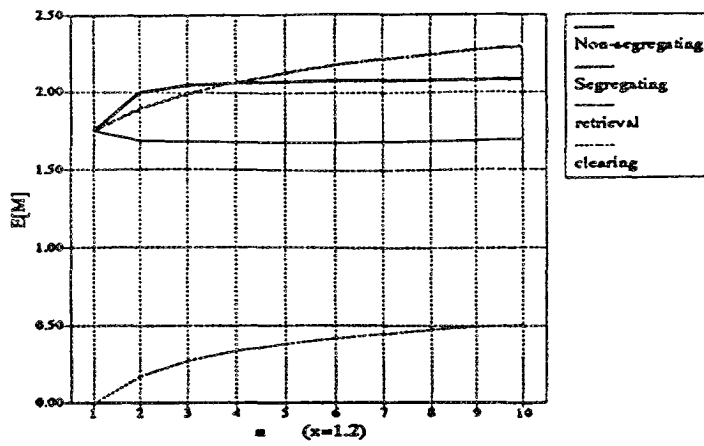
(a) E[B] = 2
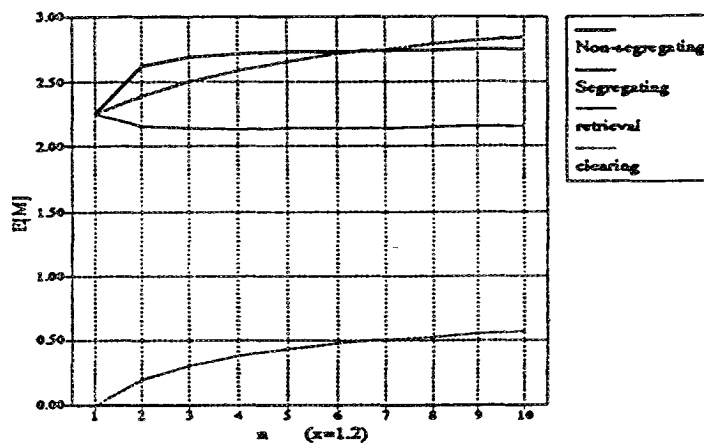
(b) E[B] = 3

(c) E[B] = 4

Fig. 7. Handling effort for each strategy.

These strategies could, for example, try to utilize shuffling moves more intelligently, perhaps by storing hot boxes on top of cold ones. It may also be advantageous to develop strategies for dynamically allocating stacks to import or export containers.

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APPENDIX

List of Symbols

A: number of import containers unloaded by a ship

B: average stack height in the yard immediately after a ship call

$B(t)$:   average stack height in the yard at time t

$E[B]$:   average stack height in the yard

f: expected fraction of a stack that must be shuffled to retrieve a container when using the nonsegregating strategy

f_s: expected fraction of a stack that must be shuffled to retrieve a container when using the segregating strategy

G: function used to estimate a correction of the expected number of clearing moves required when using the segregating strategy

H_j: arrival time of ship j

M: number of moves required to retrieve a single container from the storage stacks

M_r: number of moves required to retrieve a single container from the storage stacks when using the segregating strategy (does not include clearing moves)

M_c: number of clearing moves required per retrieval when using the segregating strategy

n: number of headways required to remove A containers from the storage area, or, equivalently, the number of ships represented by at least one container in the storage stacks at any point in time

$P_j(t)$: number of containers brought by ship j in the yard at time t

S: total number of stacks in the yard

s: number of stacks in a group

T: cumulative distribution function (c.d.f.) for container dwell times

T_c: complementary c.d.f. for container dwell times