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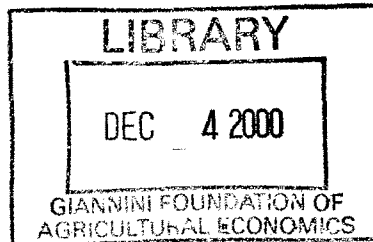
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FITTING THE GLASS SLIPPER:
OPTIMAL CAPITAL STRUCTURE IN THE FACE OF LIABILITY

by

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**FITTING THE GLASS SLIPPER:
OPTIMAL CAPITAL STRUCTURE IN THE FACE OF LIABILITY***

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October 2000

Abstract

The model presented in this paper juxtaposes two theories for why a firm might offer creditors a security interest to back up a loan. One theory holds that issuing secured debt allows the firm's owners to reduce expected payments in the event of bankruptcy to so-called "non-adjusting" creditors, who cannot or do not adjust the size of their claims in response. An important class of such non-adjusting claims are liability claims on the firm. The other theory holds that issuing secured debt solves an underinvestment problem: the firm may only be able to finance a growth opportunity if it offers new investors a security interest.

Recognizing that most real-world firms face both non-adjusting claims *and* growth opportunities, we combine the two theories in a single model. We find that firms generally choose an interior secured-debt ratio, and all firms smaller than a critical size choose a strictly higher secured-debt ratio than firms larger than the critical size. Moreover, the relationship between the optimal secured-debt ratio and firm size is highly nonlinear in ways consistent with the empirical evidence: the optimal ratio may or may not initially increase in firm size, then tends to decrease, and then becomes constant.

JEL: K13, K32, Q28, G33, L11

Keywords: secured debt, tort liability, bankruptcy, judgment proof problem, firm size

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1. INTRODUCTION

The model presented in this paper juxtaposes two theories for why a firm might offer creditors a security interest to back up a loan. One theory, first analyzed formally by Scott (1977), is that issuing secured debt allows the firm's owners to reduce expected payments in the event of bankruptcy to so-called "non-adjusting" creditors, who cannot or do not adjust the size of their claims in response. Such non-adjusting claims (*NACs*) include (1) current or future liability claims on the firm, which are generally determined by the courts without taking into account possible subordination of the claims in bankruptcy; (2) tax and other government claims, which are generally fixed by law; and (3) claims by various creditors that are simply too small to make adjustments worthwhile, given the transaction costs involved. Hereafter, we refer to this as the "*NAC*-minimization theory." The other theory, first analyzed formally by Stulz and Johnson (1985), is that issuing secured debt solves an underinvestment problem: the firm may only be able to finance a new project with positive net present value (*NPV*) if it offers new investors a security interest. Hereafter, we refer to this as the "*NPV*-maximization theory."

Both theories feature prominently in the ongoing legal debate on reform of the "adequate protection" provisions of the U.S. Bankruptcy code, which stipulate that secured creditors are entitled to the full value of their bankruptcy claim before any unsecured claims are paid. For example, in an important contribution to this debate, Bebchuk and Fried (1996) argue that the case for full priority of secured claims in bankruptcy is an "uneasy" one, because of inefficiencies induced by *NAC*-minimization. In particular, they argue that the firm's ability to divert value from non-adjusting creditors through issuing secured debt distorts its incentives to invest or spend on safety measures to prevent accidents. In the same vein, LoPucki (1994) argues that "[w]hen all assets, including the future income stream of the tortfeasor, are encumbered to their full value, the company's real exposure to tort liability can be almost eliminated," and suggests that only reputational concerns and transaction costs currently keep many firms from fully exploiting this ability to externalize tort claims. As transaction costs decline, however, because of technological advances and changes to the Bankruptcy Code that facilitate the creation of security interests, he foresees that firms will move increasingly to an all-secured-debt capital structure in which tort liability will be almost entirely externalized.² Other contributions critical of secured debt include Buckley (1986), Leebron (1991), Ingberman (1994), and Hudson (1995).

On the other side of the debate, authors such as Triantis (1992), Carlson (1994), Harris and Mooney (1994), Schwartz (1997) and in particular Schwarcz (1997) emphasize the efficiency gains implied by the *NPV*-maximization theory. In a direct rejoinder to Bebchuk and Fried (1996), Schwarcz argues, for example, that such efficiency gains make the case for full priority an "easy" one. He suggests that in reality firms hold off on encumbering their assets, in order to maximize the collateral they have available to pledge in case of a potential future economic downturn. Citing Stulz and Johnson (1985), he notes that in situations of financial distress, issuing secured new

² A front-page article in the *New York Times* of May 24, 1998 provides some anecdotal support for this viewpoint. Under the headline "Taxi Owners Deftly Dodge Claims of Accident Victims," it describes how owners of New York taxi medallions, which are worth an average of \$275,000 each, routinely use these medallions as security for loans, in order to prevent accident victims from seizing them in court.

loans may be the only way that firms can finance positive-*NPV* projects. Because such projects may then reduce the probability of default, Schwarcz argues that full priority of secured claims in fact *increases* the expected value of unsecured claims—including non-adjusting ones.

Clearly, when considered in isolation, each theory implies radically different policy recommendations. If it is true that firms offer security interests only to divert value from non-adjusting creditors, then policy measures are called for that reduce the effective priority of secured debt, for example by treating some fraction of secured claims as unsecured³ or by giving involuntary claims “super-priority” status.⁴ If, on the other hand, firms offer security interests only to avoid inefficient bankruptcies, then a case can be made for policy measures that enhance the availability of secured debt, for example by reducing the transaction costs of securing loans and by broadening the range of assets that can be pledged as collateral.

Each theory has radically different empirical implications as well. If the *NAC*-minimization theory is correct, one would expect to see firms encumber all, or substantially all, of their assets. If, on the other hand, the *NPV*-maximization theory is correct, one would expect only distressed firms to have any encumbered assets at all. The statistical evidence on secured lending, which indicates a wide divergence of secured-debt ratios, appears to be inconsistent with either theory. For example, using 36,845 firm-year observations from the COMPUSTAT annual industrial file from 1981 through 1992, Barclay and Smith (1995) find that the ratio of secured debt to total firm value averages 10%, with a standard deviation of 15%, and that the ratio of secured debt to total fixed claims (capitalized leases, debt, and preferred stock) averages 36%, with a standard deviation of 39%. Strikingly, 37% of all firms secure none of their debt, while 11% secure all of it. When they run Tobit regressions of both secured-debt ratios on various firm characteristics, two clear patterns emerge. First, there is a strong negative relationship between both secured-debt ratios and firm size, measured by the log of firm value. This confirms casual observations that large firms often have no secured debt at all, while small firms often have high proportions of secured debt. Second, there is also a negative relationship between both secured-debt ratios and the market-to-book ratio. If the latter ratio is interpreted as a proxy for firms’ growth opportunities and thereby for the potential of future underinvestment problems, then this result at least appears consistent with the *NPV*-maximization theory. Nevertheless, this theory leaves unexplained why a significant fraction of firms should choose all-secured-debt structures, and also why those firms tend to be small.

Complicating the empirical picture even further, Mann (1997) suggests, based on extensive interviews with CFOs of large U.S. banks, that in recent years there has been a marked shift away from secured debt in the important market for small-business loans of less than \$100,000. When we replicate Barclay and Smith’s (1995) regressions on 43,720 COMPUSTAT firm-year observations from 1980 through 1999 but add a quadratic term for firm size, we indeed find that this term is highly significant and enters in a manner consistent with Mann’s suggestion. Figure 1 shows a box

³ Bebchuk and Fried (1996) note that such a partial-priority rule was proposed in 1985 by the German Commission on Bankruptcy Law.

⁴ Under environmental “superlien” laws currently in force in nine U.S. states, a state environmental agency that cleans up a hazardous-waste site can seek reimbursement for its expenses by filing a lien on the assets of the site’s owner, and this lien takes priority over even pre-existing secured claims.

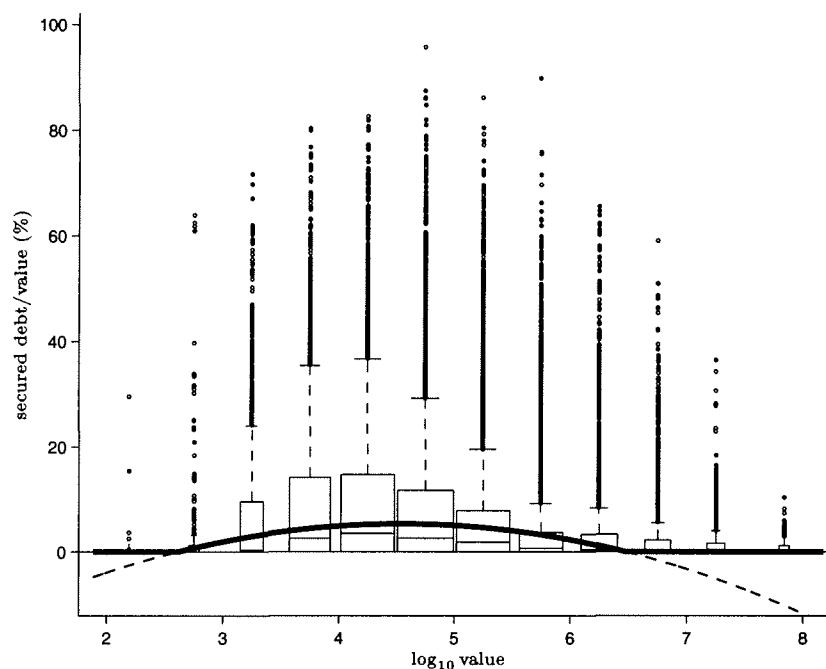


FIGURE 1. Box plot and predicted profile of the ratio of secured debt to firm value, plotted against the \log_{10} of firm value.

plot of the observed ratios of secured debt to total firm value plotted against the \log_{10} of firm value in \$mn. Also shown is the predicted relationship of the two variables when the remaining regressors are held constant at their sample means. A clear inversely U-shaped profile emerges for firms with values below \$3 million, which comprise 93% of the sample. The 7% of firms above this value tend to have secured-debt ratios close to or at zero.⁵

Our paper is the first, to our knowledge, to model the firm's choice of secured-debt ratio when—as is surely true in reality—it faces both non-adjusting claims *and* growth opportunities.⁶ The key insight of the paper is that the firm, when faced with both a *NAC*-minimization incentive to choose a high secured-debt ratio *and* a *NPV*-maximization incentive to keep the ratio low, will need to balance two conceptually very different kinds of risk. The *NPV*-maximization incentive involves management of what we shall call “market shocks”: increases in costs or reductions in demand

⁵ The estimated regression is

$$\text{SECVAL} = -20.3 - 3.70 \text{ MKTBK} + 4.79 \text{ REGUL} + 0.02 \text{ ABNORM} + 4.33 \text{ TAXLCF} + 13.0 \text{ LVAL} - 1.43 \text{ LVALSQ},$$
with p -values < 0.001 for all coefficients except *ABNORM* (which is insignificant, at $p = 0.90$). We refer the reader to Barclay and Smith's (1995) original paper for an explanation of the regressors and of the procedures used to remove outliers. Qualitatively, the inversely U-shaped profile is robust when we use the ratio of secured debt to total fixed claims as the dependent variable; regress either ratio on alternative measures of firm size such as total assets, sales, or number of employees; or run the regression as a cross-section, averaging the time-series observations for each firm. The profile remains present also in a majority of industries when we disaggregate the data by 2-, 3-, or 4-digit SIC codes, provided the industry sample sizes are large enough to obtain some resolution.

⁶ Ingberman (1994), whose paper is perhaps closest in spirit to ours, also analyses a tradeoff between “underinvestment” and risk externalization, but uses the former term to refer to a sub-optimal (i.e., not cost-minimizing) initial choice of capital intensity of production by the firm. He finds that if secured debt has full priority over tort claims, the firm will encumber all its capital. A paper by Berkovitch and Kim (1990) is closest to ours in terms of modeling assumptions, but investigates a different set of issues.

that reduce the firm's profits and may drive it into a position of financial distress. In contrast, the *NAC*-minimization incentive involves management of what we shall call "event shocks": explosions, spills, incidents of misbehavior by employees, product failures, etc., that give rise to liability or compensation claims. Because these risks vary in different ways with firm size, we find that the secured-debt ratio that solves the firm's risk-balancing problem varies with firm size as well. We identify conditions under which this ratio varies in a way that is consistent with the empirical evidence discussed above.

To understand how the two kinds of risk depend on firm size, consider for concreteness the situation facing a shipper operating several oil tankers. A market shock to this shipper may come in the form of a sharp decline in the demand for oil-tanker services. It is clearly reasonable to presume that demands for the services of any two tankers are highly correlated with each other; we will make the extreme assumption of perfect positive correlation. An event shock to the shipper may come in the form of an oil spill caused by one of its tankers, resulting in tort liability. A clearly reasonable presumption is that the probabilities of any two tankers being involved in such incidents are more or less independent of each other; we will make the extreme assumption of perfect independence. Moreover, if the probability of a single incident per period is sufficiently small, the probability of multiple incidents per period will be of second-order significance. Under these conditions, we can reasonably assume that probability of a shipper being held responsible for an oil spill in a given period will be proportional to the number of tankers that the shipper operates. Note that under these assumptions, the *expected* magnitude of both market and event shocks is directly proportional to firm size. Beyond this, however, the relationship between the two kinds of shocks and firm size is quite different: for market shocks, the *probability* of a shock is invariant with respect to firm size, while for event shocks it is proportional; for market shocks, the *magnitude* of any given shock is proportional to firm size, while for event shocks it is invariant. These differences will drive the results presented below.

The paper has three main results. First, when both the *NAC*-minimization incentive and the *NPV*-maximization incentive are operative, firms generally choose an interior secured-debt ratio. Second, under quite general assumptions about growth opportunities, all firms smaller than a critical size choose a strictly higher secured-debt ratio than firms larger than the critical size. Third, under slightly more restrictive assumptions, the specific relationship between the optimal secured-debt ratio and firm size is highly nonlinear in ways consistent with the empirical evidence: the optimal ratio may or may not initially increase in firm size, then tends to decrease, and then becomes constant.

The structure of the paper is as follows. Section 2 builds intuition for our results by first considering Myers's (1977) model of a firm with no assets in place and only a single growth opportunity. Whereas Myers shows that the firm will optimally issue no debt, we show that if the firm in addition faces the probability of an event shock, resulting in a non-adjusting claim, it will optimally choose a positive debt ratio. Section 3 presents our own model, in which we further generalize Myers' model in two ways: we introduce an asset in place and we consider firms of different sizes. Section 4 solves the model, working backwards through time: first, we determine how the firm will optimally finance the growth opportunity *if* it is undertaken; then we analyze the firm's

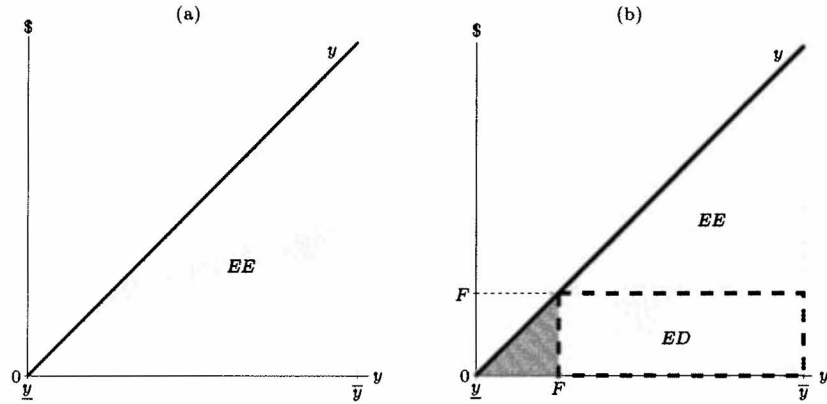


FIGURE 2. The Myers model with and without part-debt financing.

decision whether or not to indeed undertake the growth opportunity; and finally we determine how the firm will optimally finance the asset in place. Section 6 concludes.

2. THE MYERS (1977) MODEL WITH A NON-ADJUSTING CLAIM

Following Myers (1977), consider a firm which at time 0 has no assets in place but purchases at cost I an option on a future growth opportunity. Specifically, the option entitles the firm to costlessly acquire an asset Y at time 1 that will yield a stochastic payout y . Assume for simplicity that Y is distributed uniformly on $[\underline{y}_H, \overline{y}_H]$ and assume zero discounting. Also, suppose initially that the option purchase is all-equity financed. Clearly, the firm will then exercise the option at time 1 if and only if the realization of y is positive, i.e., if $y > 0$. In Figure 2(a), the shaded triangle labeled EE represents the expected payouts to equityholders from all such projects. The market value of the firm at time 0 is just equal to these payouts minus the up-front cost I of the growth option:

$$EV = EE - I.$$

Next, suppose that at time 0 the firm finances a fraction σ of the cost of the growth option by issuing debt with promised payment F , and that this debt matures *after* the firm's investment option expires. As Myers demonstrates using a diagram analogous to Figure 2(b), an underinvestment problem then arises.⁷ From the point of view of equityholders at time 1, the growth option is now only worth exercising if $y > F$. This is because if $y \leq F$, the entire payout of the Y -project will be absorbed by the initial creditors, leaving equityholders with nothing. As a result, the firm will "underinvest" at time 1, in the sense that there are projects that its equityholders will forgo even though the projects have strictly positive *total* payouts y .

In Figure 2(b), the expected payout to creditors over all projects that *are* undertaken corresponds to the shaded rectangle labeled ED . Equityholders receive the residual payout $y - F$ from each project, so their expected payouts correspond to the shaded triangle labeled EE . To find the market value of the firm at time 0 we must now subtract from the expected payouts EE to

⁷ Figure 2(b) differs from Myers' Figure 2 in that Myers assumes that acquiring the asset requires an additional outlay at time 1. To simplify the exposition, we set this outlay to zero.

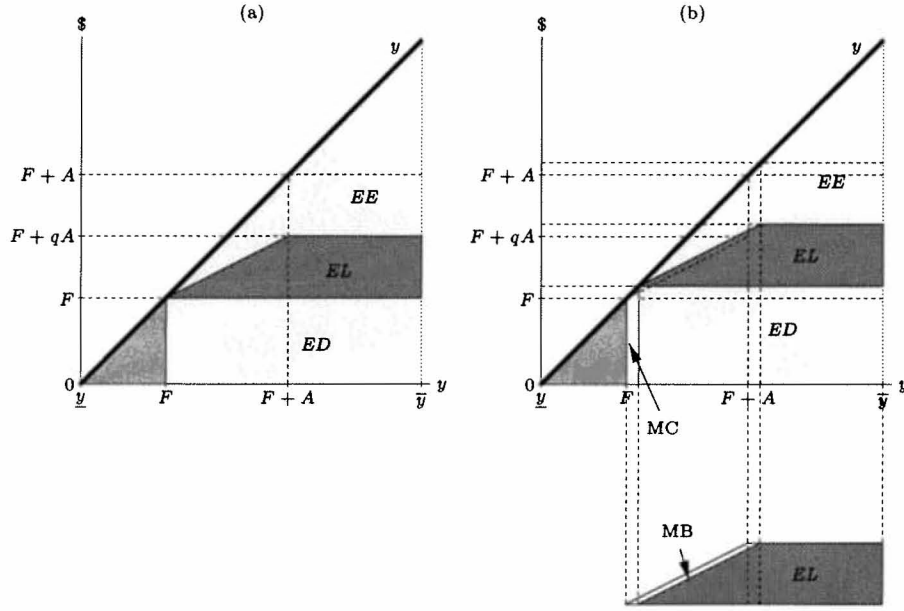


FIGURE 3. The Myers model augmented with a non-adjusting claim.

equityholders their share $1 - \sigma$ of the up-front investment I :

$$EV = EE - (1 - \sigma)I. \quad (1)$$

However, given perfect capital markets, creditors must in equilibrium just break even in expectation when they correctly anticipate that the firm's equityholders will at time 1 only undertake projects with payout y greater than F . The equilibrium face value F will therefore be such that the expected payouts ED to creditors will just equal their up-front investment of σI . Substituting this market equilibrium condition $ED = \sigma I$ into expression (1) then yields

$$EV = ED + EE - I. \quad (2)$$

For later reference, note that the market value EV is also equal to the expected net present value $ENPV$ of the projects undertaken by the firm. Since all payouts to those projects are divided up between creditors and equityholders, $ENPV$ is just equal to the sum of the payouts, $ED + EE$, less the up-front cost I .

Comparing the two panels of Figure 2 shows, however, that firm's expected net present value, and thereby its market value, is unambiguously lower if it finances any part of the up-front investment with debt. The difference, represented by the hatched triangle in Figure 2(b), corresponds exactly to the expected value of all projects that have positive total payouts y , but which the firm will forgo in the presence of pre-existing debt. As Myers discusses at length, this underinvestment problem arises because equityholders cannot credibly commit to undertake all future projects with positive net present value, and because the cost of renegotiating existing debt contracts is generally prohibitive. In order to avoid underinvestment at time 1, the firm's optimal policy at time 0 is therefore to use only equity to finance the cost I of the option.

The firm's optimal policy changes, however, when we make just one modification to Myers' framework: we assume that *after* committing to the growth opportunity but *before* its debt matures, the firm will with some probability q face a non-adjusting claim of size A —a liability claim for an accident, for example.

Provided q is less than unity, limited liability then implies that equityholders at time 1 will still undertake the project whenever the payout y exceeds F , because for all such realizations of y the equityholders' expected payout from the project will still be positive. Specifically, for realizations of y between F and $F + A$, their expected payout is $(1 - q)(y - F)$, because if an accident occurs, with probability q , the firm will not be able to pay both the creditors' claim F and the liability claim A ; as a result, the firm will go bankrupt, leaving equityholders with no payout. It is only when no accident occurs, with probability $1 - q$, that the equityholders receive the residual payout $y - F$. In contrast, for high realizations of y , greater than $F + A$, the firm will be able to pay all claims even if an accident occurs; the equityholders' expected payout for such realizations is therefore $y - F - qA$. In Figure 3(a), the expected payouts to equityholders from all projects undertaken correspond to the shaded area labeled EE . Assume for simplicity that the liability claim A ranks behind the creditors' claim F in bankruptcy—as would be the case if the debt were secured against the future returns to the project.⁸ The expected payouts to liability claimants then correspond to the cross-hatched area labeled EL in the figure, and those to creditors again to the rectangle labeled ED .

The market value of the firm at time 0 is still given by (2), but is now no longer equal to the expected net present value of the projects undertaken by the firm. The expected total payouts to all projects undertaken are now divided up between creditors, liability claimants, and equityholders, so we have $ENPV = ED + EL + ED - I$. After substituting this into (2), we obtain

$$EV = ENPV - EL. \quad (3)$$

Note that this expression captures exactly the NPV -maximization incentive and NAC -minimization incentive discussed in the introduction. It shows that in the presence of a non-adjusting claim, the firm can maximize its market value in two ways, namely (1) by maximizing the net present value $ENPV$ of all projects it undertakes, and (2) by minimizing the expected payments EL to its liability claimants.

To see how these two incentives play out in the firm's financing decision at time 0, suppose that the firm increases the fraction of I that is debt- rather than equity financed, thereby increasing the face value of the debt it holds at time 1 by an amount dF . As shown in Figure 3(b), doing so has an associated marginal cost and a marginal benefit. The marginal cost arises, as in Myers' model, because the firm will now forgo projects with payouts y in the interval $(F, F + dF]$. As a result, the loss triangle from underinvestment increases by the area labeled MC and the market value of the firm falls accordingly. This marginal cost represents the firm's NPV -maximization incentive to keep its debt ratio low: the lower its debt ratio, the higher the expected net present value $ENPV$ of the projects it will undertake. The marginal benefit arises because for all realizations of y between $F + dF$ and $F + dF + A$ there is a reduction in the payout that liability claimants receive in

⁸ See the next footnote for why this simplification is harmless.

bankruptcy, from $y - F$ to $y - (F + dF)$. In effect, the creditors' now larger claim $F + dF$ "crowds out" part of the liability claim.⁹ This marginal benefit, which corresponds to the area labeled MB in Figure 3(b), represents the firm's NAC -minimization incentive to raise its debt ratio: the higher its debt ratio, the lower the expected payouts EL to liability claimants.

With the introduction of a non-adjusting claim, the optimal debt ratio therefore need not be zero, as in Myers' model, but will be some positive ratio at which the marginal cost of any further increase just offsets the marginal benefit, i.e., where the NPV -maximization incentive just balances the NAC -minimization incentive. The model presented in the next section will be used to analyze how this optimal debt ratio varies with firm size. This will be shown to depend on (1) how firm size correlates with the expected size A of the liability claim, and thereby with the range of y values such that the firm is bankrupted if the liability claim arises, and (2) the probability weights $g(y)$ that investors assign to realizations in this range.

3. ASSUMPTIONS OF THE MODEL

The model we analyze in this paper generalizes that of the previous section in various ways. Most importantly, we introduce a notion of firm size, as well as a project X which the firm undertakes at time 0.

Investment opportunities: Consider, then, a firm of size $k = 1$ established at time 0 through an investment outlay of I in a risky project X . It is known at time 0 that at time 1 a new investment opportunity will come along in the form of a risky project Y requiring an investment outlay of i . Having the asset underlying X in place is a precondition for the growth opportunity Y to arise, so that Y cannot be undertaken independently. (For example, X may be a manufacturing project and Y an advertising campaign or a project to maintain or upgrade the manufacturing facilities underlying X .) At time 2, the firm is liquidated and its gross returns are divided among equityholders and other claimants.

There are two states of the world at time 2. In what we shall call the "distress" state, which occurs with probability $1 - p$, the return x to project X is equal to x_L , where $0 < x_L < I$; in what we shall call the "no-distress" state, which occurs with probability p , the return is x_H , where $x_H > I$. We assume that the project has positive net present value $NPV \equiv (1 - p)x_L + px_H - I > 0$. The return y to project Y is variable also: in the distress state of the world it is equal to y_H , where $y_H > i$; in the no-distress state it is y_L , where $0 < y_L < i$. The assumption of perfect negative correlation is convenient but not required for any of our results: all that we need is that x and y are not too strongly *positively* correlated (see section 5).

As in Myers' model, we assume that at time 0 there is some initial uncertainty about Y . Specifically, investors know the parameters i and y_L , but know only the *distribution* $G(y_H)$ of y_H . The support of $G(y_H)$ is $[y_H, \overline{y_H}]$, where $\overline{y_H} \leq \infty$ and y_H is such that $npv(y_H) \equiv (1 - p)y_H + py_L - i =$

⁹ Such crowding out would still occur if the debt were unsecured. The debt claim would then rank equally with the liability claim and both classes of claimants would be paid *pro rata*, i.e., receive a share of y in proportion to the size of their claim in bankruptcy. Raising F by dF would then reduce the fraction of y paid to liability claimants from $A/(F + A)$ to $A/(F + dF + A)$. Qualitatively, the conclusion that the firm would optimally issue a positive amount of debt remains the same.

0, so that all projects with $y_H > \underline{y}_H$ have strictly positive net present value. Given that $y_L < i$, it then follows that $y_H > i$ for all y_H . We assume also that at time 1, before equityholders have to decide whether to undertake Y , the uncertainty about y_H is completely resolved, in the sense that investors learn exactly what return y_H the Y -project will yield in the no-distress state. They do not, however, learn whether that state will in fact obtain at time 2.

Financial instruments: The firm's owners can finance their activities either with equity capital or with secured, zero-coupon debt. This clearly restrictive assumption allows us to isolate what we believe are the key priority relationships between various claims on x and y , namely

1. Initial creditors, who lend the firm some fraction σ of I , have first claim on all returns x from the X -project, up to the face value F of their loan.
2. New creditors, who lend the firm some fraction τ of i if it undertakes the Y -project, have first claim on all returns y up to the face value f of their loan.
3. Any residual claims $F - x$ or $f - y$ by initial or new creditors rank ahead of the equityholders' claim on either x or y .

For simplicity, we assume that all investors are risk neutral and that the risk-free interest rate is zero.

Timeline of decisions and information: The firm's owners (we use this term interchangeably with equityholders) face three decisions. First, at time 0, they choose what fraction σ of I to finance with debt. At that point, they (and all other investors) know only the distribution $G(y_H)$ of y_H . Second, at time 1, they (and all other investors) observe the realization of y_H and decide whether or not to undertake project Y . Third, if they decide to undertake the Y -project, they choose at time 1 also what fraction τ of i to finance with debt. All of the above is known to both equityholders and creditors at times 0 and 1. The state of the world—and thereby the realizations of x and y —is not revealed, however, until time 2.

Firm size: A key variable in our model is firm “size,” denoted by k . For reasons that will later become clear, it will be useful to interpret k as measuring some more or less physical quantity associated with the firm, such as the size of its workforce or the number of units of productive capital it employs (the number of oil tankers operated by a shipper, say, or the number of product lines of a manufacturer). Of course, quantities of this kind correlate strongly with financial measures of size often used in empirical work, such as sales or market value.

When comparing firms of different sizes, we make the following assumptions about how investment opportunities vary with k . First, we assume that the probability $1 - p$ of the distress state occurring—which is the “market shock” discussed in the introduction—is independent of k , for example because firms of all sizes are subject to the same, industry-wide demand or supply shocks. Second, we assume that projects X and Y “scale up” perfectly, so that a firm of size k requires an investment of kI in project X , which then generates returns of kx_L or kx_H , and similarly for project Y . If we then redefine F and f to represent the face values of debt per size unit of size, so that the actual face values for a size- k firm are kF and kf , then all of the above assumptions for a firm of size $k = 1$ apply *per unit of size* also to firms of size $k > 1$.

Non-adjusting claim: With positive probability, the firm will incur liability for damages resulting from some “event shock” that occurs between times 1 and 2. This liability is a non-adjusting claim. As discussed in the introduction, event shocks can take many forms, ranging from accidents such as explosions or environmental spills to the discovery of product defects, or lawsuits by disgruntled employees. For expositional simplicity, however, we shall refer to the event shock as an “accident” and to the liability claimants as “accident victims.”

Of crucial importance to our analysis is how the non-adjusting claim varies with firm size. To motivate the assumptions we make, consider again a shipper operating a single oil tanker. The shipper faces a probability $1 - p$ of experiencing a 10% drop in its operating profits x , because of a market-wide slump in demand, say. At the same time, the shipper also faces a probability q of having to pay oil spill damages A . Now consider a larger, but otherwise identical shipper operating k tankers. This larger shipper may well face the same probability $1 - p$ of a 10% reduction in its k times larger operating profits kx , consistent with our assumptions above. It does not seem plausible, however, to assume that the larger shipper also faces the same probability q of having to pay k times larger oil spill damages kA . Rather, if we assume that q is very small, so that the probability of two or more oil spills occurring per period is negligible, exactly the opposite would be true: the shipper operating k tankers would face a k times larger probability kq of having to pay for a similar-size oil spill with damages A . This opposite assumption is in fact the one we shall make. Note that, because our concern is with accident claims that are large enough to potentially bankrupt smaller firms, it is reasonable to assume that q is indeed very small.

In order to focus the analysis on what is new in our model, we abstract from the firm’s decision of how much to invest in precautionary measures, by assuming that both the probability of an accident q and the size of the damages A are exogenous.¹⁰ We abstract also from the firm’s insurance decision, implicitly assuming that all firms carry the same coverage (possibly zero) per size unit k . The variable A can then be interpreted as the uninsured portion of per-unit accident damages. Alternatively, q can be interpreted as the per-unit probability of a claim arising that falls outside the contract terms of any firm’s insurance.

As for the priority ranking of tort claims in bankruptcy, the institutional reality is that tort claims are treated as unsecured, and therefore rank equal in priority with the residual claims of secured creditors. We simplify, however, by assuming that tort claims rank behind any creditors’ claims, even residual ones, though still ahead of equity.

Payouts in the no-distress state: A final assumption, made purely for expositional convenience, is that payouts in the no-distress state are always sufficient to make all fixed claimants whole, including accident victims. Specifically, we assume that

$$\begin{aligned} x_H &> I/p + A. \\ x_H + y_L &> (I + i)/p + A. \end{aligned}$$

Under these assumptions, even if the X -project is financed purely by debt, F cannot exceed I/p and $F + f$ cannot exceed $(I + i)/p$. Since $k \geq 1$, it follows that if only the X -project is undertaken,

¹⁰ It would not be hard to endogenize these quantities and replicate existing results in the literature that the firm’s investment in precaution will be sub-optimal whenever it externalizes accident liability.

then for all firm sizes k , on a per-unit basis,

$$x_H > F + A/k.$$

If the Y -project is undertaken as well, then

$$x_H + y_L > F + f + A/k.$$

4. SOLUTION OF THE MODEL

As usual, we solve the model backwards in time. First, we show that if any given project Y is undertaken at time 1, equityholders in firms of all sizes optimally finance the Y -project entirely with secured debt, i.e., they choose $\tau^* = 1$. Next, we show that, given that $\tau^* = 1$, equityholders will undertake the Y -project at time 1 for any given realization of y_H if and only if doing so raises expected payouts to initial creditors and accident victims by less than the project's net present value $npv(y_H)$. Finally, we show how this condition feeds back into the equityholders choice of σ^* at time 0.

4.1. Optimal financing of the Y -project

We begin the analysis by determining the equityholders' optimal choice of τ , denoted τ^* , if they decide to undertake a given Y -project with given payout y_H in the distress state. Let $E^Y(y_H)$ denote the expected payouts at time 2 to equityholders given that the Y -project is undertaken, where this expectation is evaluated at time 1. Similarly, let $D^Y(y_H)$, $d^Y(y_H)$, and $L^Y(y_H)$ denote the expected payouts at time 2 to initial creditors, new creditors, and liability claimants. At time 1, any initial investment in the X -project by equityholders is already sunk. As a result, the market value $V^Y(y_H)$ of the firm at time 1 is just equal to the expected payout $E^Y(y_H)$ to equityholders net of their initial investment of $(1 - \tau)i$ in the Y -project:

$$V^Y(y_H) = E^Y(y_H) - (1 - \tau)i.$$

Equityholders choose τ and f so as to maximize $V^Y(y_H)$, subject to the condition that in capital market equilibrium the expected payouts $d^Y(y_H)$ to new creditors must just equal their up-front investment of τi . Their optimization problem can therefore be written as

$$\max_{\tau, f} V^Y(y_H) = E^Y(y_H) - (1 - \tau)i \quad \text{s.t.} \quad d^Y(y_H) = \tau i.$$

Substituting the constraint into the objective function and eliminating τ transforms the problem into an unconstrained one of finding the optimal face value f^* that maximizes the combined payoffs to equityholders and new debtholders:

$$\max_f V^Y(y_H) = E^Y(y_H) + d^Y(y_H) - i. \quad (4)$$

This problem can be simplified further by making use of the accounting identity

$$NPV + npv(y_H) + I + i = D^Y(y_H) + d^Y(y_H) + L^Y(y_H) + E^Y(y_H), \quad (5)$$

which follows from the fact that total expected payouts at time 2, i.e., $(1-p)(x_L+y_H)+p(x_H+y_L) = NPV + npv(y_H) + I + i$, are divided up between the four classes of claimants. After substituting this identity, the equityholders' problem becomes

$$\max_f V^Y(y_H) = NPV + npv(y_H) + I - D^Y(y_H) - L^Y(y_H). \quad (6)$$

Because the first three terms of the objective function are constants (recall that y_H is known at time 2), the only effect on the market value of raising f must be via its effect on expected payouts $D^Y(y_H)$ and $L^Y(y_H)$ to initial creditors and liability claimants. Even without writing out expressions for $D^Y(y_H)$ and $L^Y(y_H)$ it is clear, however, that if the new creditors' claim f on the total payouts is increased, the payouts $D^Y(y_H)$ and $L^Y(y_H)$ to initial creditors and liability claimants can only fall or stay constant. Consequently, raising f can only increase the firm's market value or at worst leave it unchanged. For expositional simplicity, we can therefore assume that equityholders will always raise f as much as possible by setting $\tau^* = 1$, i.e., by financing the entire cost i of the Y -project with debt.

The intuition for this result can be grasped most easily from writing the problem as in (4). Because the new debt issued to finance i is sold on terms such that new creditors just break even in expectation, there is no loss of generality in imagining that the equityholders in fact write *themselves* a new secured loan for τi , while putting up the remainder $(1-\tau)i$ in the form of equity. As secured new creditors, however, their claim on Y -payouts is senior to any liability claim and any residual claim by initial creditors, whereas as equity-holders, their claim is junior to all other claims. It follows that they will maximize their *overall* net payout $E^Y(y_H) + d^Y(y_H) - i$ by setting $\tau^* = 1$, because this maximizes the priority given to payouts they receive at time 2 in return for their overall investment of i .

4.2. The decision whether or not to undertake the Y -project

Moving back through the equityholders' decision tree, consider next their decision at time 1 whether or not to indeed undertake Y . Given that they will set $\tau^* = 1$ if they do undertake Y , so there is no up-front equity investment of $(1-\tau)i$, the decision reduces to a simple comparison of $E^Y(y_H)$ and E^X , where the latter denotes the expected payouts to equityholders at time 2 if only project X is undertaken.

An expression for the difference between $E^Y(y_H)$ and E^X can be derived by rearranging some accounting identities. First, we can rearrange the accounting identity $NPV = D^X + L^X + E^X - I$ to write E^X as

$$E^X = NPV + I - D^X - L^X. \quad (7)$$

Next, using our result above that $\tau^* = 1$, so that in market equilibrium $d^Y(y_H) = i$, we can simplify and rearrange accounting identity (5) to

$$E^Y(y_H) = NPV + npv(y_H) + I - D^Y(y_H) - L^Y(y_H). \quad (8)$$

represents payout pairs (x, y) at which, if an accident occurs, the liability claim can be satisfied as well.

Consider now first the no-distress state, represented by the black dot at payoff pair (x_H, y_L) . If, in this state, the Y -project is forgone and no accident occurs, the residual payout to equityholders will be $x_H - F$, after paying F to the initial creditors. In the figure, this residual payout corresponds to the horizontal distance between point (x_H, y_L) and the solid vertical line at F . If, on the other hand, the Y -project is undertaken and again no accident occurs, the residual payout will be *smaller*, namely $x_H + y_L - F - f^*$, after paying F and f^* to initial and new creditors. In the figure, this smaller residual payout corresponds to the horizontal distance between (x_H, y_L) and the solid diagonal line. The reason why the residual payout is lower with the Y -project is that the new creditors' claim f^* cannot be satisfied out of the Y -payout y_L , which leaves them with a residual claim $f^* - y_L$ on the X -payout. Because this claim ranks ahead of the equityholders' claim on $x_H - F$, the payout to equityholders is correspondingly reduced. Almost the same happens also if an accident does occur: the equityholders' residual payout is then $x_H - F - A/k$ if the Y -project is forgone, but only $x_H + y_L - F - f^* - A/k$ if it is undertaken: the difference is again $f^* - y_L$. Regardless of whether an accident occurs, therefore, equityholders incur a *loss* of $f^* - y_L$ in the no-distress state when they undertake the Y -project. In the figure, this loss is represented by the black double arrow.

An immediate implication is that the marginal Y -project must offer equityholders an expected *gain* in the distress state that just offsets their expected loss in the no-distress state, thereby leaving equityholders just indifferent about undertaking the project. In Figure 4, this is the case (for suitably chosen values of k , q , and p) at the payout combination (x_L, \widehat{y}_H) in the distress state. Note that in the case drawn, F is greater than x_L . Equityholders therefore receive nothing in the distress state if the Y -project is forgone, because the firm goes bankrupt and the entire X -payout x_L accrues to the initial creditors. Also, the per-unit liability claim A/k is drawn so large (i.e., the firm's size k is so small) that at the marginal Y -project the firm is bankrupted in the distress state if an accident occurs. Equityholders then again receive nothing. It follows that the equityholders' gain from undertaking the Y -project is just their residual payout $x_L + \widehat{y}_H - F - f^*$ if an accident does *not* occur. In the figure, this gain is represented by the white double arrow.

In addition to the critical Y -payout \widehat{y}_H which defines the marginal project, Figure 4 also shows a second critical Y -payout which will turn out to be important when we examine the firm's optimal initial secured-debt ratio σ^* in the next subsection. This payout, labeled \widetilde{y}_H , represents the minimum value of y_H such that the per-unit liability claim A/k can be fully satisfied in the distress state after the claims of initial and new creditors have been satisfied. Mathematically, it is therefore defined by the identity

$$x_L + \widetilde{y}_H = F + f^* + A/k. \quad (12)$$

The key point to note is that if a firm is sufficiently small, and its per-unit liability claim A/k correspondingly large, then the value of \widetilde{y}_H will exceed the value of \widehat{y}_H , as is the case in Figure 4. For a firm that is "small" in this sense, there will be a range of Y -projects with payouts in the distress state between \widehat{y}_H and \widetilde{y}_H which equityholders find profitable enough in expectation to undertake (because $y_H > \widehat{y}_H$) but which nevertheless leave the firm bankrupt in case of an accident

4.3. Optimal financing of the X -project

Turning now to the equityholders' optimization problem at time 0, recall that once the value of y_H for a given Y -project is revealed at time 1, it will be undertaken if and only if y_H exceeds \widehat{y}_H . Anticipating this optimal decision rule of equityholders at time 1, equityholders at time 0 will expect to receive payout E^Y for any value of y_H greater than \widehat{y}_H , and E^X otherwise. Aggregating over all possible future values of y_H and using that $E^Y = E^X + \Delta E$, we can therefore write the payouts to equityholders as expected at time 0 as

$$EE \equiv E^X + \int_{\widehat{y}_H}^{\overline{y}_H} \Delta E(y_H) g(y_H) dy_H,$$

and similarly for the expected payouts ED to initial creditors and EL to liability claimants. As for new creditors, these of course only come into play if y_H exceeds \widehat{y}_H , so the payouts to them as expected at time 0 are just

$$Ed \equiv \int_{\widehat{y}_H}^{\overline{y}_H} d^Y(y_H) g(y_H) dy_H.$$

It is useful to also define the expected net present value of the firm as

$$ENPV \equiv NPV + \int_{\widehat{y}_H}^{\overline{y}_H} npv(y_H) dG(y_H).$$

In equilibrium, F will be such that $ED = \sigma I$, and f such that $d^Y = i$, or equivalently $Ed = (1 - G(\widehat{y}_H))i$. Using these identities and the accounting identity $ENPV - I - (1 - G(\widehat{y}_H))i = ED + Ed + EE + EL$, we can rewrite the equityholders' constrained optimization problem at time 0, namely

$$\max_{\sigma, F, f} EV = EE - (1 - \sigma)I \quad \text{s.t.} \quad ED = \sigma I, \quad Ed = (1 - G(\widehat{y}_H))i, \quad (13)$$

as the unconstrained problem¹²

$$\max_F EV = ENPV - EL. \quad (14)$$

Note that this expression captures exactly the NPV -maximization incentive and NAC -minimization incentive discussed in the introduction. It shows that in the presence of a non-adjusting claim, the firm can maximize its market value in two ways, namely (1) by maximizing the net present value $ENPV$ of all projects it undertakes, and (2) by minimizing the expected payments EL to its liability claimants.

Differentiating with respect to F yields

$$\begin{aligned} \frac{\partial EV}{\partial F} &= \frac{\partial ENPV}{\partial F} - \frac{\partial EL}{\partial F} \\ &= -npv(\widehat{y}_H)g(\widehat{y}_H)\frac{\partial \widehat{y}_H}{\partial F} + \Delta L(\widehat{y}_H)g(\widehat{y}_H)\frac{\partial \widehat{y}_H}{\partial F} - \left[\frac{\partial L^X}{\partial F} G(\widehat{y}_H) + \int_{\widehat{y}_H}^{\overline{y}_H} \frac{\partial L^Y(y_H)}{\partial F} dG(y_H) \right]. \end{aligned}$$

¹² To simplify the exposition, we implicitly assume that the constraints $\sigma \geq 0$ and $\sigma \leq 1$ do not bind at the solution to (13). The first of these constraints can be shown to never bind. The second may bind if I is sufficiently low, and the optimal face value F is then obviously that at which $ED = I$.

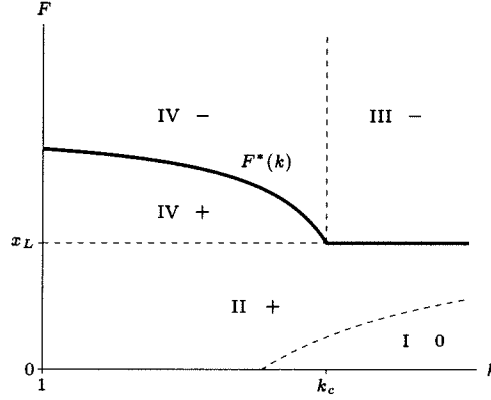


FIGURE 6. Regions in (k, F) -space discussed in the text, corresponding sign of $\partial EV/\partial F$, and optimal face value $F^*(k)$.

This expression can be simplified by making use of the identity

$$npv(\widehat{y}_H) = \Delta D(\widehat{y}_H) + \Delta L(\widehat{y}_H), \quad (15)$$

which, by the reasoning that led to equation (9), must hold at the “marginal project” with $y_H = \widehat{y}_H$. Using this identity, the expression becomes

$$\frac{\partial EV}{\partial F} = - \left[\Delta D(\widehat{y}_H) g(\widehat{y}_H) \frac{\partial \widehat{y}_H}{\partial F} \right] + \left[- \frac{\partial L^X}{\partial F} G(\widehat{y}_H) - \int_{\widehat{y}_H}^{\overline{y}_H} \frac{\partial L^Y(y_H)}{\partial F} dG(y_H) \right]. \quad (16)$$

The first bracketed term on the right-hand side can be interpreted as the marginal cost to equityholders from raising F , because if $\partial \widehat{y}_H/\partial F > 0$, raising F by dF induces equityholders at time 1 to forgo a range of marginal projects with probability weight $g(\widehat{y}_H) \partial \widehat{y}_H/\partial F dF$. From the point of view of equityholders at time 0, the resulting loss is equal to the net present value $npv(\widehat{y}_H)$ of each forgone project, offset by the share of $npv(\widehat{y}_H)$ that would have been absorbed by accident victims anyway, namely $\Delta L(\widehat{y}_H)$. By identity (15), the difference is exactly equal to the share of $npv(\widehat{y}_H)$ that would have been absorbed by initial creditors, namely $\Delta D(\widehat{y}_H)$. Intuitively, since initial creditors always break even in expectation, any reduction in their payouts—such as the loss of $\Delta D(\widehat{y}_H)$ from a marginal project forgone—is ultimately passed on to equityholders in the form of a reduction in the loan σI that equityholders can obtain at any given face value F .

The second bracketed term in (16) can be interpreted as the marginal benefit to equityholders from raising F because of the induced marginal reduction in payments to accident victims in states of the world other than that in which $y_H = \widehat{y}_H$. If $y_H < \widehat{y}_H$, so that the Y -project is forgone, this reduction is equal to $\partial L^X/\partial F$; if $y_H > \widehat{y}_H$, so that the Y -project is undertaken, it is equal to $\partial L^Y/\partial F$.

In order to determine the optimal face value F^* , and investigate how it varies with firm size k , it is useful to distinguish the four regions in (k, F) -space labeled I–IV in Figure 6. The essential distinction between these regions is that different combinations of the marginal cost and benefit terms in expression (16) evaluate to zero, resulting in the signs of $\partial EV/\partial F$ indicated in the figure. Equivalently, in each of the regions, different combinations of the NPV -maximization and NAC -minimization incentives are operative.

Region I: (k, F) such that $x_L \geq F + A/k$.

The inequality that defines this region implies that firm size k is so large, and the face value F so small, that both initial creditors and accident victims can be made whole out of just the X -payouts, even in the distress state. As a result, we have $D^X = D^Y = F$, so $\Delta D = 0$, and we have $L^X = L^Y = qA$, so $\partial L^X/\partial F = \partial L^Y/\partial F = 0$. Substituting these results into (16) then shows that the marginal cost and marginal benefit terms are both zero, so $\partial EV/\partial F = 0$ also.

Intuitively, because in this region of (k, F) -space neither initial creditors nor accident victims absorb any Y -payouts, there is no underinvestment problem and therefore no NPV -maximization incentive to reduce F . At the same time, F is so low and A/k so small that raising F never crowds out any accident payments, so there is NAC -minimization incentive to raise F either.

Region II: (k, F) such that $F \leq x_L < F + A/k$.

In this region, F is still low enough for initial creditors to be made whole out of just the X -payouts, even in the distress state. We therefore still have $\Delta D = 0$, and so the marginal cost term in (16) is still zero. The marginal project in this region is characterized by the identity $npv(\widehat{y}_H) = \Delta L(\widehat{y}_H)$, so that equityholders do not care about losing it—its entire net present value accrues to liability claimants anyway. However, because the sum of the initial creditors' claim F and the liability claim A/k exceeds x_L , liability claimants do gain in the distress state if a Y -project is undertaken. As a result, there is a net marginal benefit to raising F , because doing so crowds out expected payouts EL to liability claimants at values of y_H other than \widehat{y}_H . For example, if y_H turns out to be less than \widehat{y}_H , so that the Y -project is forgone, raising F reduces the liability claimants' residual payout $x_L - F$ in the distress state. Also, if the firm is "small" and y_H turns out to lie between \widehat{y}_H and \widetilde{y}_H , raising F reduces the liability claimants' residual payout $x_L + y_H - F - f^*$. Aggregating over all realizations of y_H , the marginal benefit term in (16) is unambiguously positive, and the same is therefore true of $\partial EV/\partial F$. It follows, then, that in Region II the NAC -minimization incentive is operative, but the NPV -maximization incentive is not.

Region III: (k, F) such that $F > x_L$ and $k \geq k_c$.

In this region, the face value F of the initial debt is greater than x_L and firms are "large" in the sense that $\widehat{y}_H \geq \widetilde{y}_H$, or equivalently, $k \geq k_c$, where k_c denotes the critical firm size at which \widetilde{y}_H becomes equal to \widehat{y}_H for face values $F > x_L$.¹³ Because F exceeds x_L , ΔD is positive in Region III for all y_H , and as a result there is a net marginal cost to raising F from losing the marginal Y -project. There is, however, no net marginal benefit from raising F , because doing so does not crowd out any payouts to liability claimants. This follows because if y_H turns out to be less than \widehat{y}_H , so that the Y -project is forgone, initial creditors absorb the entire X -payout x_L in the distress state anyway, leaving nothing for liability claimants; if, on the other hand, y_H turns out to be greater than \widehat{y}_H , so that the Y -project is undertaken, then because firms are large in Region III and the per-unit liability claim A/k is correspondingly small, liability claimants receive their entire claim A/k even in the distress state.¹⁴ In either case, raising F has no effect on the liability claimants' payout. It follows that $\partial EV/\partial F$ is unambiguously negative in Region III and exactly the opposite

¹³ Recall from subsection 4.2 that $\partial \widehat{y}_H/\partial F = \partial \widetilde{y}_H/\partial F$ when $F > x_L$, so k_c does not depend on F .

¹⁴ Recall the discussion of Figure 5.

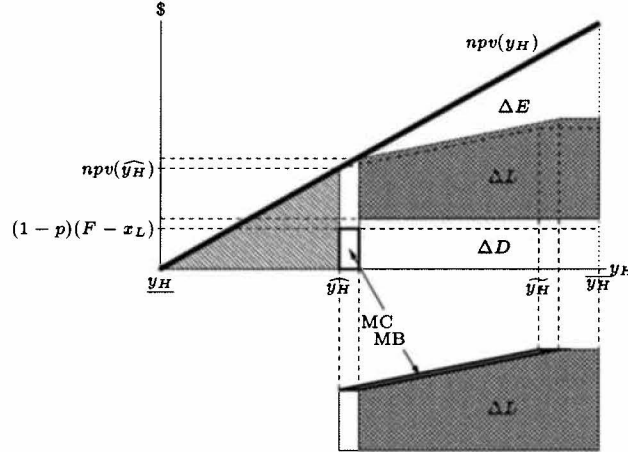


FIGURE 7. The marginal cost and benefit of raising F in Region IV.

situation obtains from that in Region II: the NPV -maximization incentive is operative, but the NAC -minimization incentive is not.

Region IV: (k, F) such that $F \geq x_L$ and $k < k_c$.

In this region, the face value F of the initial debt is again greater than x_L , and firms are “small” in the sense that $\widehat{y}_H < \widetilde{y}_H$ or $k < k_c$. It turns out that it is only in Region IV that both the NPV -maximization incentive and the NAC -minimization incentive are operative, in the sense that there is both a marginal cost and a marginal benefit of raising F , making the sign of $\partial EV / \partial F$ ambiguous.

Graphically, we can represent these marginal effects in much the same way as we did in Figure 3 for the simple model of Section 2. Where Figure 3 showed, for each value of y , the breakdown of the Y -project’s total payout y into the various stakeholders’ payouts $D(y)$, $L(y)$, and $E(y)$, Figure 7 shows, for each value of y_H , the breakdown of the Y -project’s net present value $npv(y_H)$ into the various stakeholders’ payout *increases* $\Delta D(y_H)$, $\Delta L(y_H)$, and $\Delta E(y_H)$ if the Y -project is undertaken. Note that at the marginal project, with payout \widehat{y}_H in the distress state, identity (11) holds: the payout increases $\Delta D(\widehat{y}_H)$ and $\Delta L(\widehat{y}_H)$ to initial creditors and liability claimants absorb the entire net present value $npv(\widehat{y}_H)$. This then leaves equityholders just indifferent about undertaking the project, since $\Delta E(\widehat{y}_H) = 0$.

The areas labeled MC and MB in the figure represent the marginal cost and benefit of raising F . As discussed under expression (16), the marginal cost is equal to the initial creditors’ loss of $\Delta D(\widehat{y}_H)$ from marginal projects forgone, multiplied by the probability weight $g(\widehat{y}_H) \partial \widehat{y}_H / \partial F dF$ on such projects. Equivalently, it is the loss of net present value $npv(\widehat{y}_H)$ from the projects less the share $\Delta L(\widehat{y}_H)$ of that value that would have been absorbed by liability claimants. The marginal benefit is the reduction in expected payouts to liability claimants for all projects with y_H -values between \widehat{y}_H and \widetilde{y}_H . Recall that for all such projects, an accident in the distress state will bankrupt the firm, leaving liability claimants with only the residual payout $x_L + y_H - F - f^*$ after initial and new creditors have satisfied their higher-ranking claims. Clearly, any increase in the initial creditors’ claim F will reduce—or “crowd out”—this residual payout.

To find exact expressions for both the marginal cost and benefit, we need exact expressions for the payouts to initial creditors and liability claimants given different realizations of y_H . First, if $y_H \in [\underline{y}_H, \widehat{y}_H]$, the Y -project is forgone, so initial creditors receive only x_L in the distress state, leaving nothing for liability claimants:

$$\begin{aligned} D^X &= (1-p)x_L + pF, \\ L^X &= kq[(1-p) \cdot 0 + pA/k]. \end{aligned}$$

If $y_H \in [\widehat{y}_H, \widetilde{y}_H]$, the Y -project is undertaken and initial creditors are made whole in both states, while liability claimants receive the residual $x_H + y_H - F - i$ in the distress state:

$$\begin{aligned} D^Y(y_H) &= (1-p)F + pF, \\ L^Y(y_H) &= kq[(1-p)(x_L + y_H - F - i) + pA/k]. \end{aligned}$$

Lastly, if $y_H \in [\widetilde{y}_H, \overline{y}_H]$, initial creditors receive the same payouts, but liability claimants are made whole in both states:

$$L^Y(y_H) = kq[(1-p)A/k + pA/k].$$

It follows that $\partial L^X/\partial F = 0$, $\partial L^Y/\partial F = -kq(1-p)$ for $y_H \in [\widehat{y}_H, \widetilde{y}_H]$, and $\partial L^Y/\partial F = 0$ for $y_H \in [\widetilde{y}_H, \overline{y}_H]$. Also, at $y_H = \widehat{y}_H$,

$$\begin{aligned} \Delta D(\widehat{y}_H) &= (1-p)(F - x_L), \\ \Delta L(\widehat{y}_H) &= kq(1-p)(x_L + \widehat{y}_H - F - i). \end{aligned}$$

Substituting these expressions into (15) and solving for \widehat{y}_H then yields

$$\widehat{y}_H = \frac{p(i - y_L)}{(1 - kq)(1 - p)} + F - x_L + i, \quad (17)$$

so that $\partial \widehat{y}_H/\partial F = 1$. Substituting all these results into equation (16) then yields the first-order condition for an interior optimum F^* in Region IV,

$$\frac{\partial EV}{\partial F} = \frac{\partial ENPV}{\partial F} - \frac{\partial EL}{\partial F} = -(1-p)(F - x_L)g(\widehat{y}_H) + kq(1-p)[G(\widetilde{y}_H) - G(\widehat{y}_H)] = 0. \quad (18)$$

Although we cannot solve the condition explicitly for F^* without assuming a specific functional form for $G(y_H)$, a number of general observations can be made. First, it is immediate that $\partial EV/\partial F$ is strictly positive when $F = x_L$. Combined with our result that $\partial EV/\partial F > 0$ for Region II, this implies that all firms of size $k < k_c$ will choose an optimal F^* strictly greater than x_L , and therefore an optimal σ^* strictly greater than x_L/I . In contrast, combining our results for Regions I–III shows that firms of size $k \geq k_c$ will optimally choose $F^* = x_L$ and therefore $\sigma^* = x_L/I$.

Note also that, because the cost I of project X enters nowhere into (18) and can take on any value greater than x_L , the optimal face values F^* for firms of all sizes are consistent with arbitrary interior values of σ^* . Put differently, specific bounds on either x_L or I must be assumed to make σ^* equal to either zero or one.

We summarize these observations in the following propositions:

Proposition 1. *When faced with both a non-adjusting claim and investment opportunities, firms generally choose an interior secured-debt ratio.*

Proposition 2. *All firms of size greater than or equal to some critical size k_c choose the same secured-debt ratio, whereas all firms of size strictly less than k_c choose a strictly higher ratio.*

Whereas both these results hold for arbitrary investor expectations $G(y_H)$, our third result, that F^* declines with k everywhere *within* the range $k < k_c$, turns out to require a substantive restriction on these expectations. Totally differentiating the first-order condition yields

$$\frac{dF^*}{dk} = -\frac{\partial^2 EV/\partial F\partial k}{\partial^2 EV/\partial F^2}. \quad (19)$$

Because the denominator on the right-hand side is negative by the second-order condition, the sign of dF^*/dk is equal to that of the numerator, i.e., to the sign of

$$\frac{\partial}{\partial k} \{-(1-p)(F - x_L)g(\widehat{y}_H) + kq(1-p)[G(\widetilde{y}_H) - G(\widehat{y}_H)]\}. \quad (20)$$

We relegate to appendix A a formal proof that this expression is strictly negative as long as the density $g(y_H)$ does not decline “too rapidly” in y_H on the interval $[\widehat{y}_H, \widetilde{y}_H]$, in a sense made precise. Here, we demonstrate strict negativity for the simplest case of a uniform distribution.

Writing $g(y_H) = 1/(\overline{y}_H - \underline{y}_H) \equiv g$ for all y_H and $G(y_H) = (y_H - \underline{y}_H)g$, the first-order condition for the uniform case reduces to

$$-(1-p)(F^* - x_L)g + kq(1-p)(\widetilde{y}_H - \widehat{y}_H)g = 0.$$

Solving for F^* and substituting from equations (12) and (17) yields

$$\begin{aligned} F^*(k) &= x_L + kq(\widetilde{y}_H - \widehat{y}_H) \\ &= x_L + kq \left[\frac{A}{k} - \frac{p(i - y_L)}{(1 - kq)(1 - p)} \right] \\ &= x_L + qA - \frac{kq}{1 - kq} \frac{p(i - y_L)}{1 - p}. \end{aligned}$$

It follows that for $k < k_c$, the optimal F^* is a strictly declining function of k . At $k = k_c$, we have $\widetilde{y}_H = \widehat{y}_H$, so $F^*(k_c) = x_L$. As noted above, $F^*(k) = x_L$ for $k > k_c$ also.

Some intuition for this result is provided by Figure 8. Recall that the net marginal benefit arises in Region IV only if two events occur: (1) an accident occurs in the distress state, with probability $kq(1-p)$, and (2) the y_H -value of the Y -project lies between \widehat{y}_H and \widetilde{y}_H , with probability $G(\widetilde{y}_H) - G(\widehat{y}_H)$. Only when both events occur simultaneously will an increase in F result in a reduction of the residual payout $x_L + y_H - F - f^*$ to liability claimants. Clearly, the probability of event (1) increases with firm size, and in fact does so proportionally. On the other hand, because the range $[\widehat{y}_H, \widetilde{y}_H]$ shrinks and eventually vanishes as k approaches the critical size k_c , the probability of event (2) decreases with firm size.¹⁵

¹⁵ In fact, the range $[\widehat{y}_H, \widetilde{y}_H]$ shrinks not only because \widetilde{y}_H falls with firm size, as explained in the discussion of Figures 4 and 5, but also because \widehat{y}_H increases with firm size. This follows because equityholders lose $f^* - y_L$ in the no-distress state from undertaking the marginal Y -project regardless of whether an accident occurs, but gain $x_L + \widehat{y}_H - F - f^*$ in the distress state only if no accident occurs, with probability $1 - kq$. The larger the firm, therefore, the smaller the probability that they will gain. To compensate, \widehat{y}_H , and thereby the gain $x_L + \widehat{y}_H - F - f^*$ itself, must be larger.

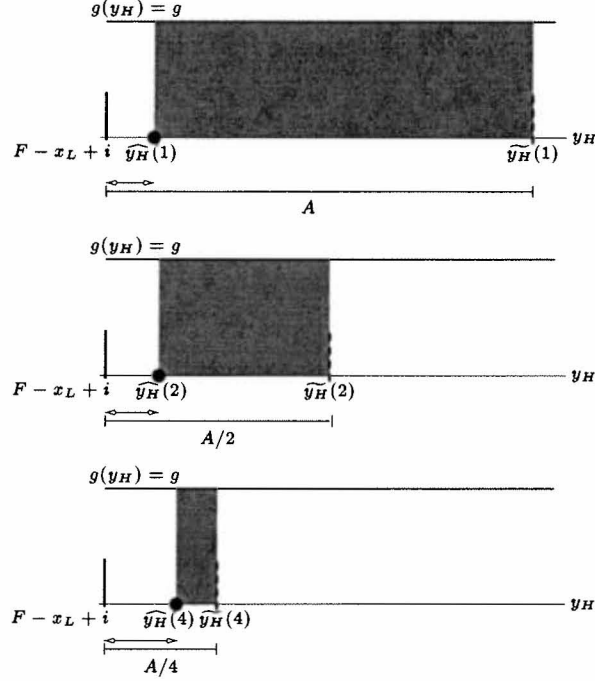


FIGURE 8. Total probability weight on y_H lying between \widehat{y}_H and \widetilde{y}_H , with constant local probability weight $g(y_H)$.

Because of these competing size effects, the way in which the expected marginal benefit varies with firm size will depend on the *rate* at which the range $[\widehat{y}_H, \widetilde{y}_H]$ shrinks, as well as on the shape of the density $g(y_H)$.

Figure 8 illustrates this size dependence for the case of a uniform density $g(y_H) = g$ and firm sizes $k = 1, 2$, and 4 . It is visually obvious that for every doubling of firm size, and thereby a doubling of the probability of event (1), the probability of event (2) shrinks by more than one-half. As a result, the product $kq(1-p)[G(\widetilde{y}_H) - G(\widehat{y}_H)]$ of these two probabilities, and thereby the marginal benefit of raising F , unambiguously falls with firm size, while the marginal cost $(1-p)(F - x_L)g$ remains constant. As a result, larger firms within Region IV will choose unambiguously lower values of F^* , in order to equate the constant marginal cost of raising F to the declining marginal benefit. Combined with our results for Regions I–III, this then yields the relationship between F^* and firm size illustrated in Figure 6.

Clearly, however, the assumption of a uniform density $g(y_H)$ is rather implausible. More likely, the density will decline in y_H , reflecting that highly profitable investment opportunities are less likely to come along than moderately profitable ones.

Figure 9 illustrates the typical size dependence in this case. In the two top panels, representing firms of sizes $k = 1$ and 2 , $G(\widetilde{y}_H) - G(\widehat{y}_H)$ shrinks by *less* than one-half as firm size doubles. In the two bottom panels, however, representing firms of sizes $k = 2$ and 4 , $G(\widetilde{y}_H) - G(\widehat{y}_H)$ shrinks by *more* than one-half as firm size doubles again. The more plausible assumption of declining probability weights therefore tends to generate the complicated relationship between F^* and firm size illustrated in Figure 10: initially, for very small firms, F^* increases in firm size, but eventually,

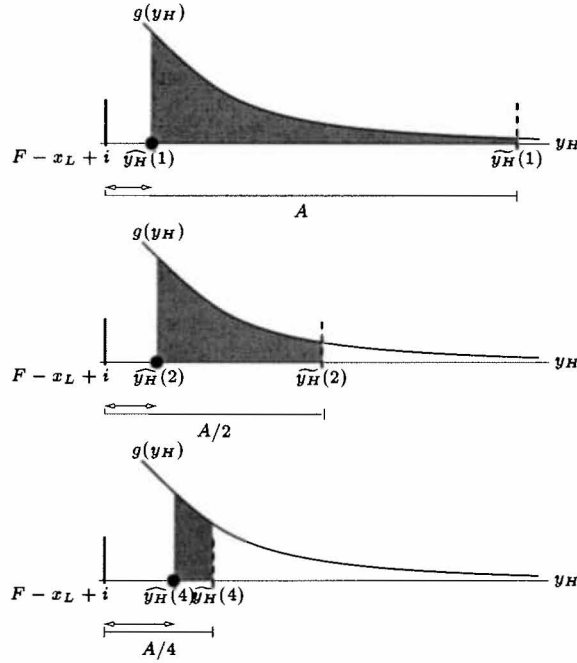


FIGURE 9. Total probability weight on y_H lying between \widehat{y}_H and \widetilde{y}_H , with declining local probability weight $g(y_H)$.

as k approaches k_c and the range $[\widehat{y}_H, \widetilde{y}_H]$ becomes small, F^* declines, until at the critical size k_c it becomes constant and equal to x_L .

We summarize these observations in the following proposition:

Proposition 3. *Subject to a restriction on investors' expectations about future investment opportunities—namely that the probability weight $g(y_H)$ must not decline “too rapidly” in y_H on the interval $[\widehat{y}_H, \widetilde{y}_H]$ —the secured-debt ratio chosen by all firms of size less than k_c strictly decreases in firm size. If this restriction fails, then the secured-debt ratio chosen by firms of size less than k_c will be an inversely U-shaped function of firm size.*

Note that this result is consistent with Mann's (1997) suggestion that loans to the very smallest firms are often unsecured, so that it is medium-sized firms that take on most secured debt. It is also consistent with the empirical evidence presented in the introduction in support of Mann's suggestion.

5. ROBUSTNESS OF THE RESULTS

Several of the model's simplifying assumptions can easily be relaxed without qualitatively affecting the results.

This is true, for example, of the assumption that residual claims of secured creditors rank ahead of liability claims in bankruptcy. If instead residual claims are given equal rank, as is true in reality, it is easy to see that Region I in Figures 6 and 10 is unchanged, as is the intuition for why $\partial EV/\partial F = 0$ in this region. Similarly, although the value of k_c changes, and thereby the boundary

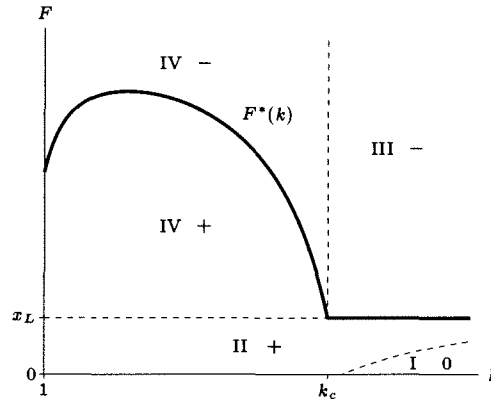


FIGURE 10. $F^*(k)$ for “sufficiently” declining $g(y_H)$.

between Regions III and IV, these regions are still identifiable and the intuitions for the sign of $\partial EV/\partial F$ in each of them still apply. The expressions that define \widehat{y}_H in each of the regions become much more convoluted, however, which significantly complicates the algebra.

Using Figure 11, we can also provide a heuristic argument for why our assumption of perfect negative correlation between the X - and Y -payouts can be substantially weakened without qualitatively affecting our results. The figure is identical to Figure 4, except that two additional states of the world have been introduced, with payouts (x_L, y_L) and (x_H, y_H) . Clearly, by raising the probability weight on these states, we could make the correlation between the X - and Y -payouts arbitrarily close to perfectly positive. However, in the (x_L, y_L) state, regardless of whether the Y -project were undertaken, initial creditors would receive x_L and accident victims would receive nothing. Similarly, in the (x_H, y_H) state, again regardless of whether the Y -project were undertaken, both initial creditors and accident victims would be made whole. Introducing these states would therefore *qualitatively* affect neither of the terms ΔD or ΔL in condition (10), which determines whether a Y -project is in fact undertaken. Any effect would be merely quantitative, via changes in the probability weights entering the terms.

The same point can also be made in terms of the equityholders’ gains and losses from undertaking a given Y -project. Complicating the model by introducing an (x_L, y_L) state adds neither a gain nor a loss to equityholders, because they receive nothing in this state regardless of whether Y is undertaken. Introducing an (x_H, y_H) state adds a gain $y_H - f$, equal to the difference between payout $x_H + y_H - F - f$ if the Y -project were undertaken and $x_H - F$ if it were not. This additional gain is represented by the double white arrow in the top-right quadrant of Figure 11. This gain, however, is independent of the face value F of the initial debt, and therefore does not qualitatively affect the equityholders’ choice of F at time 0. Clearly then, it is crucial to our analysis that the (x_L, y_H) and (x_H, y_L) states of the world have *some* positive probability of arising. Without the (x_H, y_L) state, there would be no loss to offset any gains to equityholders in the remaining states, so there would be no underinvestment problem and thereby no tradeoff between the NPV -maximization incentive and the NAC -minimization incentive. Without the (x_L, y_H) state, the equityholders’ choice of F at time 0 would have no effect on their decision at time 1 whether or not to undertake Y . Our results therefore do require less-than-perfect positive correlation between the X - and Y -payouts,

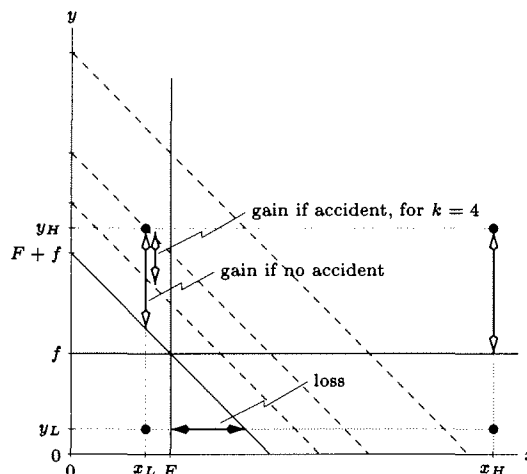


FIGURE 11. Equityholder gains and losses in different states of the world from undertaking the Y -project.

but do not depend on our carrying this assumption to the extreme of perfect *negative* correlation. This is done merely for expositional convenience.

Of course, the preceding argument is predicated on the inequality $F \geq x_L$ holding, as in Figure 11. It is easy to show, however, that even when states (x_L, y_L) and (x_H, y_H) are given positive weight, equityholders will choose σ at time 0 such that this inequality indeed holds.

6. CONCLUSION

To our knowledge, our model is the first to potentially survive what LoPucki (1994) has humorously called the “glass slipper” test—a repeated challenge by Schwartz (1981, 1984, 1989), in a series of seminal contributions to the legal debate on secured-debt reform, to produce a theory of secured debt that is consistent with the empirical evidence.

Our results imply sharper empirical predictions than have thus far been tested, however. In particular, our model predicts not just the negative relationship between firm size and secured debt as a ratio of firm value found by Barclay and Smith (1995), but predicts also that this relationship should be non-linear, leveling off for very large firms. Moreover, if our model is correct in explaining this relationship from the different rates at which non-adjusting claims and investment opportunities scale up with firm size, then one should expect the relationship to be less pronounced in industries where large liability claims are rare.

Among possible theoretical extensions to our model is the introduction of unsecured debt as a third financing option available to the firm’s owners. For consistency with Barclay and Smith’s empirical findings, such a model would have to imply a negative relationship between firm size and not only the optimal ratio of secured debt to firm value, but also the optimal ratio of secured to total debt.

APPENDIX A.

In this appendix, we derive an elasticity condition on $g(y_H)$ which is sufficient for dF^*/dk to be strictly negative for all $k < k_c$ in Region IV. We also show that if the condition fails, then dF^*/dk may be positive for the smallest firms, but will still be negative for firms of size k sufficiently close to k_c .

First, note from dividing (18) through by $q(1-p)g(\widehat{y}_H)$ that the sign of dF^*/dk is equal to that of

$$\frac{\partial}{\partial k} \left\{ \frac{k[G(\widehat{y}_H) - G(\widehat{y}_H)]}{g(\widehat{y}_H)} \right\}. \quad (\text{A1})$$

Suppose now that $g(y_H)$ can be approximated on the interval $[\widehat{y}_H, \widetilde{y}_H]$ by a function of the form $\alpha e^{-\beta y_H + \gamma}$, where α , β and γ are parameters. The expression in braces in (A1) then reduces to

$$\frac{k}{\beta} \left[1 - e^{-\beta(\widetilde{y}_H - \widehat{y}_H)} \right],$$

with derivative w.r.t. k

$$\frac{1}{\beta} \left[1 - e^{-\beta(\widetilde{y}_H - \widehat{y}_H)} \right] + k e^{-\beta(\widetilde{y}_H - \widehat{y}_H)} \left[\frac{\partial \widetilde{y}_H}{\partial k} - \frac{\partial \widehat{y}_H}{\partial k} \right].$$

Substituting from (12) and (17), using that

$$\begin{aligned} \frac{\partial \widetilde{y}_H}{\partial k} &= -\frac{A}{k^2} \\ \frac{\partial \widehat{y}_H}{\partial k} &= \frac{qp(i - y_L)}{(1 - kq)^2(1 - p)} \end{aligned}$$

and defining $P \equiv p(i - y_L)/((1 - kq)(1 - p))$, the derivative becomes

$$\frac{1}{\beta} \left[1 - e^{-\beta(\frac{A}{k} - P)} \right] - e^{-\beta(\frac{A}{k} - P)} \left[\frac{A}{k} - P + \frac{1}{1 - kq} P \right], \quad (\text{A2})$$

where $A/k - P = \widetilde{y}_H - \widehat{y}_H > 0$ everywhere in Region IV.

If β is positive, implying that $g(y_H)$ decreases in y_H on the interval $[\widehat{y}_H, \widetilde{y}_H]$, the condition that the derivative (A2) be negative can be written as

$$e^{\beta(\frac{A}{k} - P)} - 1 - \beta \left(\frac{A}{k} - P \right) - \beta \frac{1}{1 - kq} P < 0 \quad (\text{A3})$$

after multiplying through by $\beta e^{\beta(A/k - P)}$. Treated as a function of β , the left-hand side of (A3) is convex with two roots, one zero and the other strictly positive. The required inequality therefore holds everywhere between these two roots, i.e., for all β that are not “too large.” If β is negative, the multiplication by $\beta e^{\beta(A/k - P)}$ reverses the required inequality, which then holds for all such β .

Combining these results for positive and negative β with those for the uniform case derived in section 4 shows that dF^*/dk is strictly negative in Region IV as long as $g(y_H)$ does not decline “too rapidly” on the interval $[\widehat{y}_H, \widetilde{y}_H]$.

Differentiating (A2) with respect to k yields

$$\frac{1}{k} e^{-\beta(\frac{A}{k}-P)} \left\{ -\beta \left[\frac{A}{k} + \frac{kq}{1-kq} P \right]^2 - \frac{2kq}{(1-kq)^2} P \right\},$$

which is clearly negative for all $\beta > 0$. This shows that if $g(y_H)$ declines on $[\widehat{y}_H, \widetilde{y}_H]$, $F^*(k)$ is concave everywhere in Region IV. Also, in the limit as k approaches k_c from below, $A/k - P = \widetilde{y}_H - \widehat{y}_H$ goes to zero, so (A2) goes to $-P/(1-kq) < 0$. Combined with the concavity result, this implies that if $g(y_H)$ does decline “too rapidly” on $[\widehat{y}_H, \widetilde{y}_H]$ (i.e., if β is too large), then dF^*/dk will be positive for only for the smallest firms; it will still be negative for firms of size k close enough to k_c .

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