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The 9-Intersection: Formalism and Its Use for Natural-Language Spatial Predicates (94-1)

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THE 9-INTERSECTION: FORMALISM AND ITS USE FOR NATURAL-LANGUAGE SPATIAL PREDICATES

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PREFACE AND ACKNOWLEDGEMENTS

This report contains two papers, plus supplementary material. The first paper develops and presents the formal mathematical definitions of the 9-intersection; it is under consideration by a journal at the time of this writing. The second paper reports of cognitive testing, based on the mathematical model presented in the first paper, it was submitted to a journal in February 1993, but at this writing, no decision had yet been reached. Lastly, it contains the complete set of stimuli used in Mark and Egenhofer's experimental work up to January 1994.

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Table of Contents

Categorizing Binary Topological Relations Between Regions, Lines, and Points in Geographic Databases*

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Abstract

One of the fundamental concepts necessary for the analysis of spatial data in a Geographic Information System (GIS) is a formal understanding of the geometric relationships among arbitrary spatial objects. Topological relations, a particular subset of geometric relations, are preserved under topological transformations such as translation, rotation, and scaling. A comprehensive formal categorization of such binary topological relations between regions, lines, and points has been developed that is based upon the comparison of the nine intersections between the interiors, boundaries, and exteriors of the two objects. The basic criterion for the distinction of different topological relations is whether the intersections are empty or not, thus identifying 2^9 mutually exclusive topological relations. It is derived which of these 512 binary relations actually exist in $\mathbb{R}^2$ between regions, lines, and points. An equivalent model is developed that replaces the intersections with exteriors by subset conditions of the closure so that efficient implementations of topological relations are possible in geographic information systems.

1 Introduction

Queries in spatial databases, such as Geographic Information Systems (GISs) [25, 48], image databases [7, 64], or CAD/CAM systems [63], are often based on the relationships among spatial objects. For example, in geographic applications typical spatial queries are, "Retrieve all cities that are within 5 miles of the interstate highway I-95" or, "Find all highways in the states adjacent to Maine." Current commercial database query languages, such as SQL [6] and Quel [67], do not sufficiently support such queries, because they provide only tools for comparing equality or order of such simple data types as integers or strings. The incorporation of spatial relations over geometric domains into a spatial query language has been identified as an essential extension beyond the power of traditional query languages [19, 64]. Some experimental spatial query languages support queries with one

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or the other spatial relationship (Table 1); however, their diversity, semantics, completeness, and terminology vary dramatically [16, 32].

	Spatial Relationships
[30]	left of, right of, beside, above, below, near, far, touching, between, inside, outside
ATLAS [70]	area adjacency, line adjacency, boundary relationship, containment, distance, direction
MAPQUERY [25]	on, adjacent, within
KBGIS [65]	containment, subset, neighborhood, near, far, north, south, east, west
KGIS [41]	distance, overlay, adjacent, overlap
PSQL [64]	covering, coveredBy, overlapping, disjoint, nearest, furthest, within, outside, on_perimeter
SQL extension [39]	adjacent, contains, contains_point, enclosed_by, intersect, near, self_intersect
Geo-Relational Algebra [34]	equal, not equal, inside, outside, intersect
Spatial SQL [16]	disjoint, equal, meet, overlap, concur, commonBounds

Table 1: Terms proposed or used for spatial relationships in query languages.

Spatial queries can be easily solved if *all* geometric relationships between the objects of interest are explicitly stored; however, such a scenario is unrealistic, even for relatively small data collections [12], because it would need tremendous amounts of storage space—n^2 values for each kind of spatial relationships between n objects—and imply complex maintenance procedures. For instance, a GIS that explicitly recorded the geographic directions between any two objects would require extensive update operations because, with the addition of any new object, one must also determine and subsequently store the corresponding direction values from the new object to all objects already known in the database, and vice-versa (i.e., $2n$ new entries for a database with n objects). In lieu of recording all spatial relationships, it is more common to derive them, e.g., from their geometry or spatial location. This process needs a thorough understanding of *what* possible geometric relationships are and *how* they can be determined.

The development of a coherent, mathematical theory of spatial relations to overcome shortcomings in almost all geographic applications [5] is one of the goals of current GIS research [1, 56]. A formal definition, for instance, is a prerequisite for the query execution in a compiler and for reasoning about the relationships among spatial objects. Its benefits will be threefold: (1) Such a formalism may serve as a tool to identify and derive relationships. Redundant and contradicting relationships can be avoided such that a minimal set of fundamental relationships can be defined. (2) The formal methods can be applied to determine the relationship between any two spatial objects given in a formal representation. Algorithms to determine relationships can be specified exactly, and mathematically sound models will help to define the relationships formally. (3) The fundamental relationships can be used to combine more complex relationships.

The exploration of spatial relationships is a multi-disciplinary effort. Cognitive scientists, psychologists, and linguists are interested in how humans perceive the inter-relationships between spatial objects and their studies focus on the use of spatial predicates and relations in natural language [40, 49, 69]. Cartographers and geographers collected terms and prototypes of spatial relations. An early compilation of primitive spatial relations [30] lacks a formal underpinning, but is close to a list [42] that is based on a cognitive linguistics approach [47].

The scope of this paper is to use formal methods for the identification of different topological relations, a particular subset of geometric relations. Their characteristic is that they are preserved under topological transformations such as translation, rotation, and scaling. Topological information is a purely qualitative property and excludes any consideration of quantitative measures. For example, two parcels are neighbors if they share a common boundary and the neighborhood relationship is independent of the length of the boundary or the number of common boundary segments. It is important to keep in mind that topological equivalence does not preserve distances and directions, which are spatial relations that are part of other investigations [8, 26, 36, 59]; therefore, the subsequent investigations are based upon continuity, which is described in terms of *coincidence* and *neighborhood*, and no reference to the notions of distance and direction will be made. Other spatial relations, excluded from the investigations in this paper, are approximate relations, such as *close* [62] and *about five miles north-easterly of* [13], or relations that are expressions about the motion of one or several objects such as *through* and *into* [69].

We concentrate on the geometry of the objects—regions, lines, and points—irrespective of their particular meanings. While certain spatial terms may be specific to particular applications, in general all spatial relations are based upon fundamental geometric principles and models. A consistent and least redundant approach requires that the common concepts are identified at the geometry level in the form of a fundamental set of spatial relations. These generic relationships can then be applied for the definition of application-specific relationships. Linguists' observations about the use of natural language terms for the description of spatial relations support this approach [40, 69]. In the English language, spatial relations and prepositions are independently used of the size and material of the reference objects, yet context in which a specific relationship occurs is essential for the selection of the correct terms.

The remainder of this paper is organized as follows: Section 2 summarizes the spatial data model, for which the topological relations will be investigated. Section 3 introduces the 9-intersection as our model to formalize binary topological relations. Their existence for regions, lines, and points in $\mathbf{R}^2$ is investigated in Section 4. In Section 5 our model is compared with other formalisms for spatial relations and the conclusions in Section 6 describe an implementation and discuss future research activities based on these results.

2 Spatial Data Model

In order to describe the kinds of spatial objects one deals with and to determine what their particular properties are, it is necessary to introduce a spatial data model. A spatial data model is a formalization of the spatial concepts that humans employ when they organize and structure their perception of space [24, 27]. These concepts differ depending on the observers' experiences and the context in which a person views some situation. Formalizations of spatial concepts are necessary, because computer systems are essentially formal systems that manipulate symbols according to formal rules. The role of a spatial data model is similar to the conceptual schema in the 3-schema view: concepts get separated from the actual implementations, thus implementations of certain parts of the large GIS software system become more independent and may be updated without affecting the remaining software parts.

Here, the formalism will primarily serve as a means to verify that the readers' assumptions and expectations about spatial concepts concur. Without such a formal framework it would be impossible to investigate and discuss the formalization of topological relations, because it may vary considerably depending on the data model selected.

2.1 Cells and Cell Complexes

The spatial data model, upon which the definition of topological relations is based, uses *algebraic topology* [3, 66], a branch of geometry deals with the algebraic manipulation of symbols that represent geometric configurations and their relationships to one another. The application of algebraic topology has been the subject of extensive research in geographic information systems [11, 73] and led to today's most common spatial data model in GISs for modeling discrete spatial data [24, 27], e.g., in Arc/Info [53] and TIGRIS [37], and a cartographic data transfer standard [57].

The algebraic-topology spatial data model is based on primitive geometric objects, called *cells*, which are defined for different spatial dimensions[1]: A 0-cell is a node (the minimal 0-dimensional object); a 1-cell is the link between two distinct 0-cells; and a 2-cell is the area described by a closed sequences of three non-intersecting 1-cells. A *face* of an n-cell A is any $(0 \ldots n)$-cell that is contained in A.

This spatial data model differs from the simplicial data model [20, 28] primarily in one property: simplices are convex hulls, while cells may have arbitrarily shaped interiors.

The topological primitives relevant for the forthcoming investigations are the closure, interior, boundary, and exterior of a cell.

Definition 1 *The closure of an n-cell A, denoted by $\overline{A}$, is the set of all faces r-f of A, where $0 \leq r \leq n$, i.e.,*

$$\overline{A} = \bigcup_{r=0}^{n} r\text{-}f \in A$$

Definition 2 *The set-theoretic boundary of an n-cell A, denoted by ∂A, is the union of all r-faces r-f, where $0 \leq r \leq (n-1)$, that are contained in A:*

$$\partial A = \bigcup_{r=0}^{n-1} r\text{-}f \in A$$

Definition 3 *The interior of a cell A, denoted by A°, is the set difference between A's closure and A's boundary:*

$$A^\circ = \overline{A} - \partial A$$

Definition 4 *The exterior of a cell A, denoted by A^-, is the set of all cells in the universe $\mathcal{U}$ that are not elements of the closure:*

$$A^- = \mathcal{U} - \overline{A}$$

From the elementary geometric objects, more complex ones can be formed as their aggregates, called *cell complexes*. The operations on cell complexes are defined in terms of the operations on cells. Let x be the number of cells $(A_1 \ldots A_x)$ that constitute a complex C.

[1]The definition of the topological dimension of a space is based on the concept of a refinement [55]. Examples of one-dimensional spaces are a line and the border of a circle; common two-dimensional spaces are the open and the closed disks, and their topological images. An n-cell has the same dimension n as its embedding space if the cell exists in that space, but there is no homeomorphic mapping for the cell into an $(n-1)$-space.

Definition 5 *The boundary of C is the set of all boundaries of the x n-cells A_i that constitute C and are part of a single A in C, i.e.,*

$$\partial C = (\bigcup_{i=1}^{x} \partial A_i) - (\bigcup_{i=1}^{x} \bigcup_{j=i+1}^{x} (\partial A_i \cap \partial A_j))$$

Definition 6 *The interior of an n-complex C, denoted by C°, is the set of all $(0\ldots n)$-cells in the closure of $A_i \in C$ that are not elements of C's boundary, i.e.,*

$$C^\circ = (\bigcup_{i=1}^{x} \overline{A}_i) - \partial C$$

Definition 7 *The exterior of a complex, denoted by C^-, is the intersection of the exteriors of all cells A_i that are part of the complex, i.e.,*

$$C^- = \bigcap_{i=1}^{x} A_i^-$$

From these definitions, it follows that (1) interior, boundary, and exterior of a cell (or a cell complex) are mutually exclusive and (2) their union coincides with the universe.

Subsequently, the term *cell* will be used as a synonym for *complexes*. For the sake of clarity, some of the interior faces will be omitted in the figures.

2.2 Integrated Topology

In order to compare cells for coincidence, it is necessary to embed all cells into the same universe. This integration allows for the solution of topological operations on a purely symbolic level, without any consideration of metric. This fundamental topological structure has to fulfill two completeness axioms [28]:

- Completeness of incidence: The intersection of two cells is either empty or a face of both cells. Hence, no two geometric objects must exist at the same location. For example, though a 1-cell may represent both a part of a state boundary and a part of the border of a nation, the geometry of the 1-cell will be recorded only once.

- Completeness of inclusion: Every n-cell is a face of a $(n + 1)$-cell. Hence, in a 2-dimensional space every 0-cell is either start- or end-node of a 1-cell, and every 1-cell is in the boundary of a 2-cell.

It is further assumed that the closure of each cell is strictly inside the universe ($A \subset R^2$), i.e., no cell is outside of or on the border of the universe.

The embedding of the cells into a universe gives rise to the definition of the codimension. The *codimension* defines the difference between the dimension of the embedding space and the dimension of a cell. For example, codimension 1 for a 2-cell describes that it is located in a 3-space. The codimension can be never less than zero and it is zero if and only if the cell and the space are of the same dimension.

2.3 Cells for Regions, Lines, and Points

Within the context of this paper, we are interested in a subset of cell complexes that are most commonly used in geographic and cartographic applications. The complexes are "homogeneously n-dimensional" and not partitioned into non-empty, disjoint parts. The commonly used geographic features of points, lines, and regions are then defined as follows:

- A *region* is a 2-complex in $\mathbf{R}^2$ with a non-empty, connected interior.
  - A *region without holes* is a region with a connected exterior and a connected boundary (thus also called a *region with connected boundaries*) (Figure 1a).
  - A *region with holes* is a region with a disconnected exterior and a disconnected boundary (Figure 1b).
- A *line* is a sequence of connected 1-complexes in $\mathbf{R}^2$ such that they neither cross each other nor form closed loops.
  - A *simple line* is a line with two disconnected boundaries (Figure 1c).
  - A *complex line* is a line with more than two disconnected boundaries (Figure 1d).
- A *point* is a single 0-cell in $\mathbf{R}^2$.

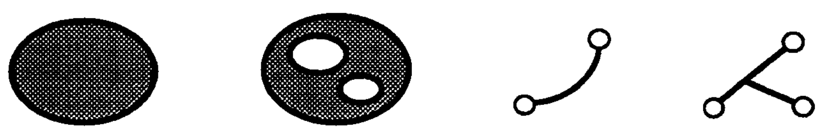

Figure 1: A region with (a) connected and (b) disconnected boundary; and a (c) simple and (d) complex line.

3 9-Intersection as a Model for Topological Relations

The binary topological relation R between two cells, A and B, is based upon the comparison of A's interior (A°), boundary (∂A), and exterior (A^-) with B's interior (B°), boundary (∂B), and exterior (B^-). These six *object parts* can be combined such that they form nine fundamental descriptions of a topological relation between two n-cells. These are:

- the intersection of A's *interior* with B's *interior* (and spelled "boundary-boundary intersection"), denoted by ($A^\circ \cap B^\circ$),
- the intersection of A's interior with B's boundary ($A^\circ \cap \partial B$),
- A's interior with B's exterior ($A^\circ \cap B^-$),
- the boundary-boundary intersection $\partial A \cap \partial B$,
- A's boundary with B's interior ($\partial A \cap B^\circ$),
- A's boundary with B's exterior ($\partial A \cap B^-$),

- the intersection of the two exteriors ($A^- \cap B^-$),
- A's exterior with B's boundary ($A^- \cap \partial B$), and
- A's exterior with B's interior ($A^- \cap B^\circ$).

Sometimes, we will also refer to more general terms like, "A's interior intersections," which encompasses the three intersections $A^\circ \cap \partial B$, $A^\circ \cap B^\circ$, and $A^\circ \cap B^-$, or "B's boundary intersections," which are $\partial A \cap \partial B$, $A^\circ \cap \partial B$, and $A^- \cap \partial B$.

The framework for the description of the topological relation between two cells, A and B, is the ordered set of these nine intersections, called the *9-intersection*, which is concisely represented as a 3×3-matrix.

$$R(A,B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

Every different set of 9-intersections describes a different topological relation, and relations with the same specifications will be considered to be *topologically equivalent*; therefore, the 9-intersection can be employed to analyze whether or not two different configurations have the same topological relation [23]. Topological relations are characterized by the *topological invariants* of these nine intersections, i.e., properties that are preserved under topological transformations [55]. Examples of topological invariants applicable to the 9-intersection are the content (i.e., emptiness or non-emptiness) of a set, the dimension, the number of separations, and the sequence of disconnected intersections of different dimensions along the boundary [21, 38].

For the 9-intersection mode, the *content* of the nine intersections was identified as a simple and most general topological invariant [21]. It characterizes each of the nine intersections by a value *empty* ($\emptyset$) or *non-empty* ($\neg\emptyset$). For example, the 9-intersections based on empty/non-empty intersections for a configuration in which region A covers region B is:

$$R(A,B) = \begin{pmatrix} A^\circ \cap B^\circ = \neg\emptyset & A^\circ \cap \partial B = \emptyset & A^\circ \cap B^- = \emptyset \\ \partial A \cap B^\circ = \neg\emptyset & \partial A \cap \partial B = \neg\emptyset & \partial A \cap B^- = \emptyset \\ A^- \cap B^\circ = \neg\emptyset & A^- \cap \partial B = \neg\emptyset & A^- \cap B^- = \neg\emptyset \end{pmatrix}$$

or briefly:

$$R(A,B) = \begin{pmatrix} \neg\emptyset & \emptyset & \emptyset \\ \neg\emptyset & \neg\emptyset & \emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{pmatrix}$$

Subsequently, the latter notation will be used as a shortcut. The sequence of the nine intersections, from left to right and from top to bottom, will always be (1) interior, (2) boundary, and (3) exterior.

The nine empty/non-empty intersections describe a set of relations that provides a complete coverage—any set is either empty or not empty and *tertium non datur*. Furthermore, they are mutually exclusive so that the union (OR) of all specifications is identically true, i.e., one of the specified relations holds true for any possible configuration, and the intersection (AND) of any two specified relations is identically false, i.e., only a single one exists between two cells.

For the goal of this paper—the formal identification of existing topological relations—it is extremely useful that the 9-intersection can concisely describe topological properties and constraints of both existing and non-existing relations.

3.1 Topological Properties

A variety of topological properties between two cells, A and B, can be expressed in terms of the 9-intersection [18]. Those intersections that do not matter and, therefore, can take an arbitrary value will be marked by a "wild card" (_).

Let a_i and b_j be arbitrary non-empty parts of A and B, respectively.

- If a_i is *disjoint* from b_j then the intersection between these two parts must be empty, while the other eight intersections can take any arbitrary value. For example, if A's boundary is disjoint from B's interior then the 9-intersection between A and B must match the following pattern:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) = \begin{pmatrix} - & - & - \\ \emptyset & - & - \\ - & - & - \end{pmatrix}$$

- If a_i *intersects* with b_j then the intersection between these two parts must be non-empty. For example, if A's interior intersects with B's boundary then the 9-intersection between A and B must match the following pattern:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) = \begin{pmatrix} - & \neg\emptyset & - \\ - & - & - \\ - & - & - \end{pmatrix}$$

- If a_i is a *subset* ($\subseteq$) of b_j then the intersection between these two parts must be non-empty. Furthermore, the two intersections between a_i and the other two parts of B, b_k and b_l, must be empty, because the parts are pairwise disjoint, otherwise, there would be some part of a_i outside of b_j, which would contradict the subset relation. For example, if A's boundary is a subset of B's interior (Figure 2a), then the 9-intersection between A and B must match the following pattern:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) = \begin{pmatrix} - & - & - \\ \neg\emptyset & \emptyset & \emptyset \\ - & - & - \end{pmatrix}$$

- Likewise, if a_i is a *subset* of two parts, b_j and b_k ($j \neq k$), such that $a_i \not\subseteq b_j$ and $a_i \not\subseteq b_k$, then the intersections with these two parts must be non-empty, while the intersection between a_i and the third part of B must be empty. For example, if $\partial A \subseteq (\partial B \cup B^\circ)$ such that $\partial A \not\subseteq \partial B$ and $\partial A \not\subseteq B^\circ$ (Figure 2b), then the 9-intersection between A and B must match the following pattern:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) = \begin{pmatrix} - & - & - \\ \neg\emptyset & \neg\emptyset & \emptyset \\ - & - & - \end{pmatrix}$$

- A consequence of the first subset rule is that if two object parts, a_i and b_j, *coincide*, then the intersection between a_i and b_j must be non-empty, while the other four intersections, having either a_i or b_j as an argument, must be empty. This follows from $a_i = b_j$ if $a_i \subseteq b_j$ and $b_j \subseteq a_i$. For example, if the two boundaries of A and B coincide then the 9-intersection between A and B must match the following pattern:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) = \begin{pmatrix} - & \emptyset & - \\ \emptyset & \neg\emptyset & \emptyset \\ - & \emptyset & - \end{pmatrix}$$

Figure 2: (a) A's boundary being a subset of B's interior and (b) A's boundary being a subset of B's interior and boundary.

3.2 Constraints for Non-Existing Relations

In a similar way, the 9-intersection can be used to describe "negative" topological constraints, i.e., configurations that cannot exist. Non-existing configurations may be due to particular properties of the objects (e.g., regions or lines), the embedding space (e.g., 2-D plane or surface of a 3-D object), the relation between the objects and the embedding space (i.e., the codimension), or the spatial data model (e.g., discrete or continuous). The following example is to illustrate the idea of representing non-existing relations in terms of the 9-intersection. Between two non-empty cells in $\mathbb{R}^2$, there must be at least one non-empty intersection, otherwise, no geometric interpretation can be found. In terms of the 9-intersection, it is impossible that all nine intersections are empty; therefore, the following condition holds:

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{pmatrix}$$

Multiple conditions for non-existing relations may be correlated such that the same non-existing relation, described by two patterns of 9-intersections, is a member of different conditions. For example, if one condition C_i is more specific than another condition C_j then all of C_i's non-existing intersections are included in the set of C_j's non-existing intersections. Using the 9-intersection, such dependencies can be easily detected by comparing the corresponding values of the two 9-intersections. For example, the condition C_1 that "all nine intersections must not be empty" is implied by condition C_2 that "the exteriors of two cells must always be non-empty," because $C_1 \subset C_2$:

$$C_1 = \begin{pmatrix} \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset \end{pmatrix} \quad C_2 = \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & \emptyset \end{pmatrix} \quad \Rightarrow \quad C_1 \subset C_2$$

4 Existing 9-Intersections in $\mathbb{R}^2$

This section focuses on the binary relations in $\mathbb{R}^2$ between an m-cell and an n-cell, where $0 \leq m, n \leq 2$. Based upon the empty/non-empty 9-intersections, 2^9 topological relations are possible between two cells; however, only a smaller number of them can be realized in a particular space. Some of them depend on the dimensions and codimensions of the cells. The goal of this section is to identify which topological relations may be realized and which ones may not.

The approach taken is a three-step process:

- the formalization of topological conditions for *non-existing* relations in terms of the empty/non-empty 9-intersections, which are translated into specification patterns for non-existing topological relations;
- the calculation of the set of 9-intersections that exist between two cells as the set of all 512 possible relations, reduced by the union of all non-existing relations; and
- the verification of the existence of the remaining relations by realizing prototypical geometric configurations in $\mathbf{R}^2$.

Since different topological conditions apply depending on the codimensions of the objects involved, the investigations will be separated into relations between two regions in 2-D (Section 4.1); two lines in 2-D (Section 4.2); a region and a line in 2-D (Section 4.3); and the trivial relations with points in 2-D (Section 4.4). Subsequently, we present one combination of conditions that leads to the set of existing binary topological relations between any combination of regions, lines, and points. Numerous other combinations of conditions are possible. Though our set of conditions is not necessarily minimal, it is such that (1) no condition is part of another condition and (2) no condition is covered by any combination of other conditions. The first property can be easily checked by comparing the 9-intersections of all conditions (Section 3.2). To evaluate the second property a test program was used to compare those 9-intersections that fulfilled all n conditions with the 9-intersections that fulfilled only n-1 conditions. If the latter set was equal to the first set, then the condition left out was implied by the combination of the other relations and, therefore, redundant.

4.1 Relations between Two Regions with Codimension 0

4.1.1 Conditions for Regions.

The intersection between two exteriors is only empty if at least one of the two regions coincides with $\mathbf{R}^2$, or if the union of the two cells is the universe. This follows immediately from $\overline{A} \cup A^- = \mathbf{R}^2$ and $\overline{B} \cup B^- = \mathbf{R}^2$: $A^- \cap B^-$ is only empty if either $\overline{A} = \mathbf{R}^2$, or $\overline{B} = \mathbf{R}^2$, or $\overline{A} \cup \overline{B} = \mathbf{R}^2$. All three scenarios are impossible for the cell data model, because $A \subset \mathbf{R}^2$ and $B \subset \mathbf{R}^2$. Thus also $(A \cup B) \subset \mathbf{R}^2$; therefore, the following condition holds:

Condition 1 *The exteriors of two cells intersect with each other, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \;\neq\; \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & \emptyset \end{pmatrix} \tag{1}$$

The following three conditions are based upon a particular property of this spatial data model, namely the fact that if the boundaries of two regions do not coincide then there is either some interior or exterior between them. This implies that if A's interior does not intersect with B's exterior then the interiors must intersect (Condition 2), A's boundary must not intersect with B's exterior (Condition 3), and A's interior must not intersect with B's boundary (Condition 4).

Condition 2 *If both interiors are disjoint then A's interior intersects with B's exterior, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \;\neq\; \begin{pmatrix} \emptyset & - & \emptyset \\ - & - & - \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} \emptyset & - & - \\ - & - & - \\ \emptyset & - & - \end{pmatrix} \tag{2}$$

Condition 3 *If A's interior is a subset of the B's closure then A's boundary must be a subset of B's closure as well, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} - & - & \emptyset \\ - & - & \neg\emptyset \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ - & - & - \\ \emptyset & \neg\emptyset & - \end{pmatrix} \qquad \text{(3a \& b)}$$

Condition 4 *If A's interior intersects with B's boundary then it must also intersect with B's exterior, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} - & \neg\emptyset & \emptyset \\ - & - & - \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ \neg\emptyset & - & - \\ \emptyset & - & - \end{pmatrix} \qquad (4)$$

A cell with a non-empty boundary cannot have all three boundary intersections empty. $\partial A = \neg\emptyset$ implies $\partial A \cap \mathbf{R}^2 = \neg\emptyset$. Since $\partial B \cup B^\circ \cup B^- = \mathbf{R}^2$ it follows that $\partial A \cap (\partial B \cup B^\circ \cup B^-) = \neg\emptyset$, which is only true if at least one part of B intersects with A's boundary.

Condition 5 *A's boundary intersects with at least one part of B, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} - & - & - \\ \emptyset & \emptyset & \emptyset \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & \emptyset & - \\ - & \emptyset & - \\ - & \emptyset & - \end{pmatrix} \qquad \text{(5a \& b)}$$

Since the boundary of a region separates its interior from the exterior, every path from the exterior to the interior crosses the boundary (Jordan-Curve-Theorem) [66]. This gives rise to the following four conditions:

Condition 6 *If both interiors are disjoint then A's boundary cannot intersect with B's interior, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} \emptyset & \neg\emptyset & - \\ - & - & - \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} \emptyset & - & - \\ \neg\emptyset & - & - \\ - & - & - \end{pmatrix} \qquad \text{(6a \& b)}$$

Every connected object part that intersects with both the interior and exterior of another object must also intersect with that object's boundary. For arbitrary regions, only the interior is connected.

Condition 7 *If A's interior intersects with B's interior and exterior, then it must also intersect with B's boundary, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} \neg\emptyset & - & - \\ \emptyset & - & - \\ \neg\emptyset & - & - \end{pmatrix} \vee \begin{pmatrix} \neg\emptyset & \emptyset & \neg\emptyset \\ - & - & - \\ - & - & - \end{pmatrix} \qquad \text{(7a \& b)}$$

Unless the boundaries of two regions coincide, at least one boundary must intersect with the other region's exterior.

Condition 8 *If both boundaries do not intersect with each other then at least one boundary must intersect with its opposite exterior, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \quad \neq \quad \begin{pmatrix} - & - & - \\ - & \emptyset & \emptyset \\ - & \emptyset & - \end{pmatrix} \qquad (8)$$

Likewise, if the interiors of two regions are separated then at least one boundary must intersect with the opposite exterior.

Condition 9 *If both interiors do not intersect with each other then at least one boundary must intersect with its opposite exterior, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} \emptyset & - & - \\ - & - & \emptyset \\ - & \emptyset & - \end{pmatrix} \tag{9}$$

4.1.2 Conditions for Regions without Holes.

Conditions (1)–(9) apply to regions—independent of whether they have holes or not. Regions without holes are a more restricted class of spatial objects than regions and, therefore, their topological relations have further constraints. The crucial property of a region without holes is that its boundary is connected. This fact, in combination with the Jordan-Curve-Theorem, gives rise to the definition of the following three conditions:

Condition 10 *If both boundaries intersect with the opposite interiors then the boundaries must also intersect with each other, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} - & \neg\emptyset & - \\ \neg\emptyset & \emptyset & - \\ - & - & - \end{pmatrix} \tag{10}$$

Condition 11 *If A's interior intersects with B's exterior then A's boundary must also intersect with B's exterior, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} - & - & \neg\emptyset \\ - & - & \emptyset \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ - & - & - \\ \neg\emptyset & \emptyset & - \end{pmatrix} \tag{11}$$

Condition 12 *If the interiors do not intersect with each other then A's boundary must intersect with B's exterior, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} \emptyset & - & - \\ - & - & - \\ - & \emptyset & - \end{pmatrix} \vee \begin{pmatrix} \emptyset & - & - \\ - & - & \emptyset \\ - & - & - \end{pmatrix} \tag{12}$$

4.1.3 Realization of Region Relations.

The 9-intersections of the existing relations between two regions can be determined by successively applying these conditions and canceling the corresponding non-existing 9-intersections from the set of all 512 relations. Eighteen relations exist in $\mathbb{R}^2$ if the region boundaries are connected or disconnected, eight of which can be realized only for regions with connected boundaries. The existence of the topological relations corresponding to the 9-intersections has been verified by finding their geometric interpretations. Figure 3 shows prototypes of the eight relations between arbitrary regions (R_0–R_7) and the ten particular relations between regions with disconnected boundaries (R_8–R_{17}), respectively.

Some of the conditions for regions are generic so that they apply also for other cells:

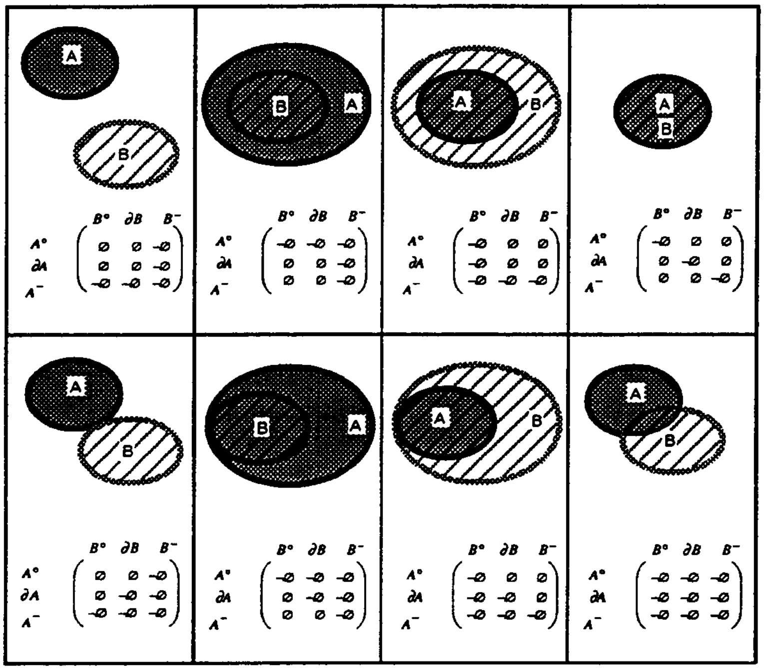

Figure 3: A geometric interpretation of the 8 relations between two regions with connected boundaries

- Condition (1) holds for any two non-empty cells.
- Conditions (2)–(4) hold for any two non-empty cells, A and B, of the same dimension. If the dimension of A is greater than the dimension of B then only the first part of each condition applies.
- Condition (5) holds for any two cells with non-empty boundaries.
- Conditions (6)–(12) apply only to regions with codimension 0.

4.2 Relations between two Lines with Codimension > 0

4.2.1 Line Conditions.

Lines are non-empty cells with non-empty boundaries, therefore, Conditions (1)–(5) apply. Additional constraints must hold for two lines due to the property of the spatial data model that another point exists between any two distinct points; therefore, if the exterior of one line intersects with the boundary of another line, the exterior must also intersect with the interior of the other line. This implies:

Condition 13 *If A's closure is a subset of B's interior then either A's exterior intersects with both B's boundary and B's interior, or not at all, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} - & - & \emptyset \\ - & - & - \\ \emptyset & \neg\emptyset & - \end{pmatrix} \vee \begin{pmatrix} - & - & \emptyset \\ - & - & \neg\emptyset \\ \emptyset & - & - \end{pmatrix} \vee \tag{13a}$$

$$\begin{pmatrix} - & - & \emptyset \\ - & - & - \\ \neg\emptyset & \emptyset & - \end{pmatrix} \vee \begin{pmatrix} - & - & \neg\emptyset \\ - & - & \emptyset \\ \emptyset & - & - \end{pmatrix} \tag{13b}$$

4.2.2 Simple Line Conditions.

If the two lines are simple then both boundaries consist of two points, each of which has no extend and, therefore, can only intersect with one part of another object. This particular property of simple lines leads to the following condition:

Condition 14 *Each boundary can intersect with at most two opposite parts, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} - & \neg\emptyset & - \\ - & \neg\emptyset & - \\ - & \neg\emptyset & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \\ - & - & - \end{pmatrix} \tag{14a \& b}$$

Likewise, the fact that the boundary of a simple line A is a subset of the boundary of another simple line B implies that no part of the boundary can be outside of A's boundary. If there were some part of B's boundary outside of A's boundary, this would mean that B's boundary has more than two disconnected boundaries, and then the line would not be simple anymore.

Condition 15 *If A's boundary is a subset of B's boundary, then the two boundaries coincide, and vice-versa, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \neq \begin{pmatrix} - & \neg\emptyset & - \\ \emptyset & \neg\emptyset & \emptyset \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ \emptyset & \neg\emptyset & \emptyset \\ - & \neg\emptyset & - \end{pmatrix} \vee \tag{15a}$$

$$\begin{pmatrix} - & \emptyset & - \\ \neg\emptyset & \neg\emptyset & - \\ - & \emptyset & - \end{pmatrix} \vee \begin{pmatrix} - & \emptyset & - \\ - & \neg\emptyset & \neg\emptyset \\ - & \emptyset & - \end{pmatrix} \tag{15b}$$

4.2.3 Realization of Line Relations.

There are 57 relations between two lines, 33 of them can be also realized between simple lines. Figures 4 and 5 show the 9-intersections and corresponding geometric interpretations of the 33 relations between two simple lines and of the 24 relations that exist only for complex lines, respectively.

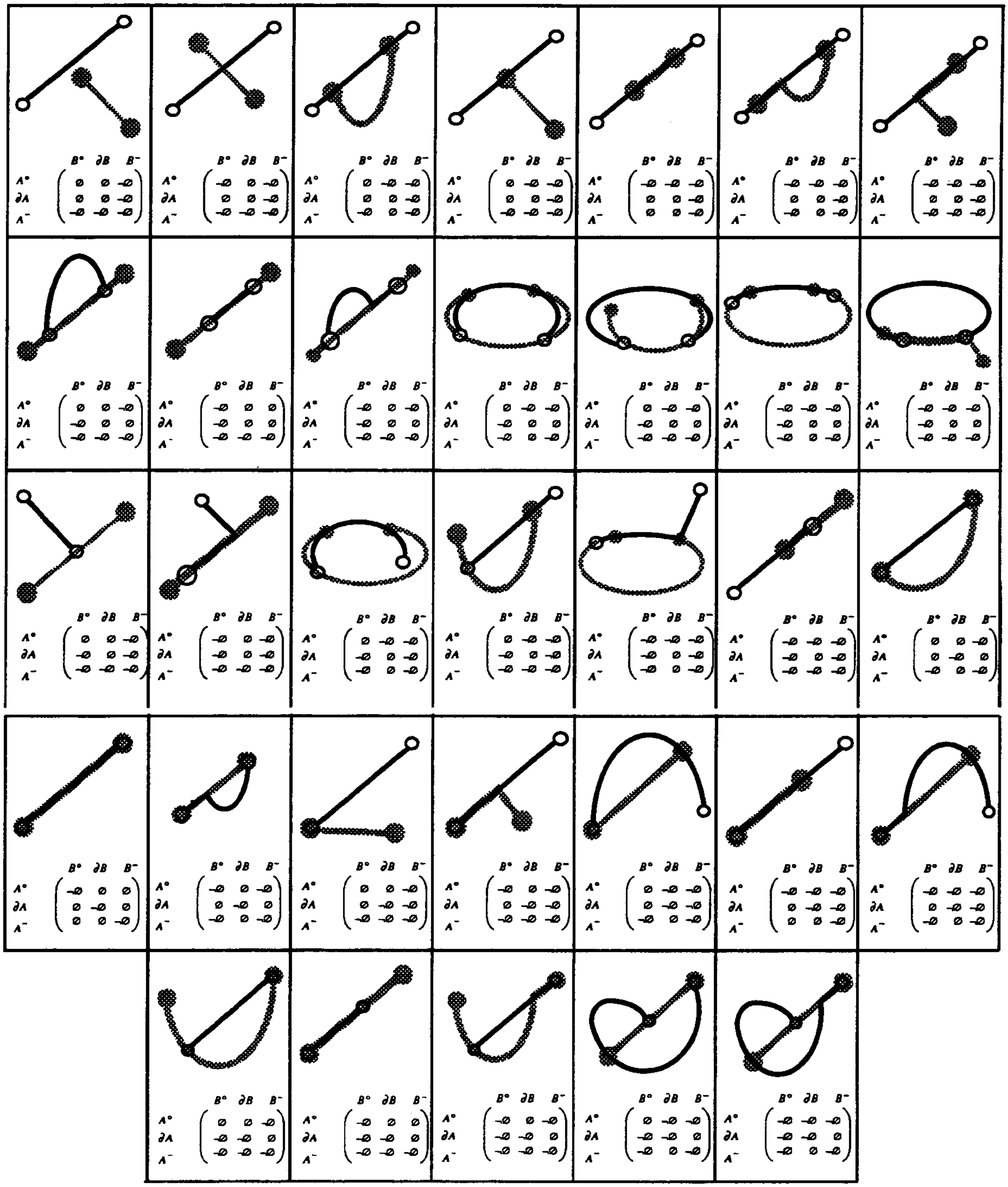

Figure 4: A geometric interpretation of the 33 relations that can be realized between two simple lines.

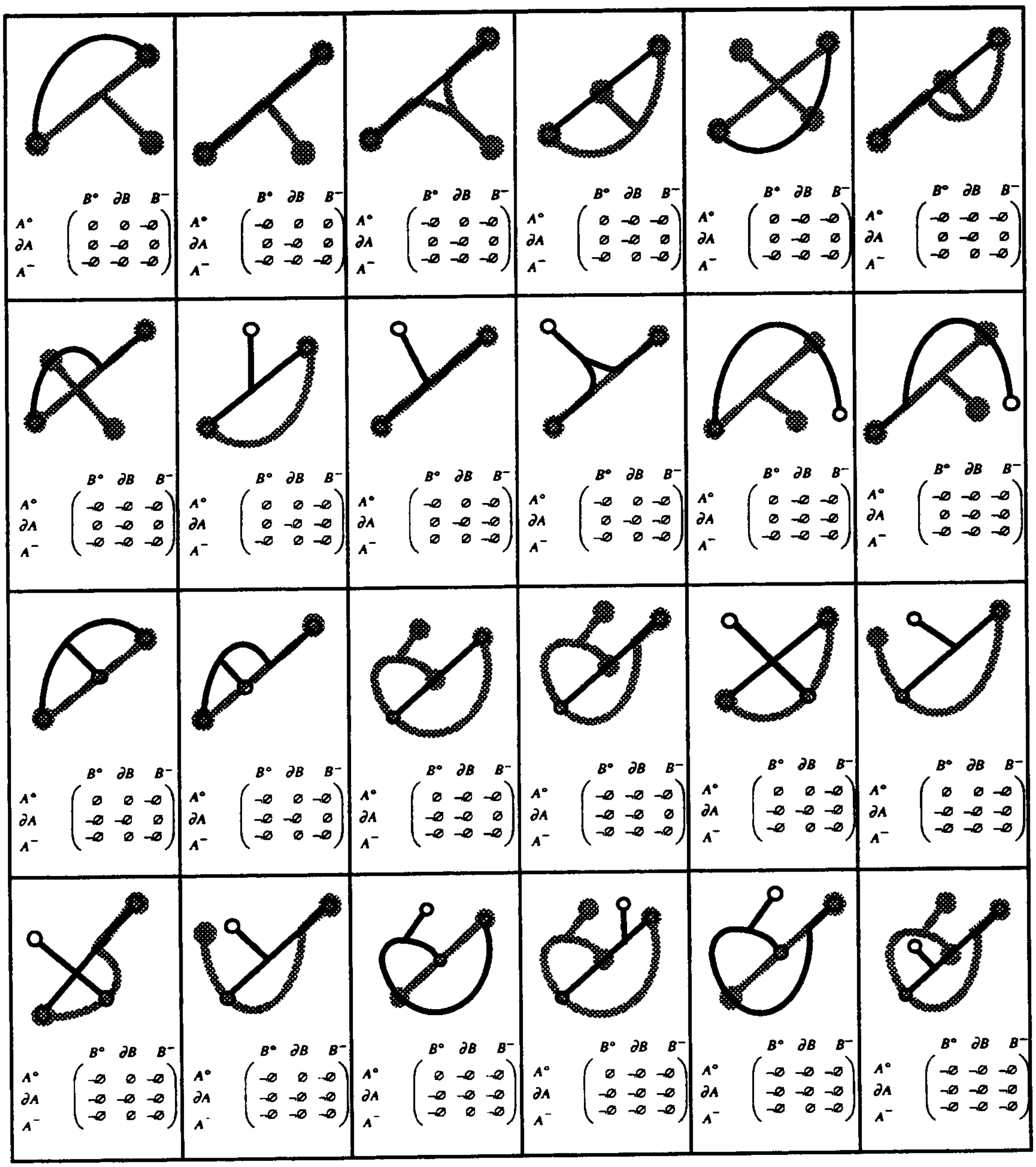

Figure 5: A geometric interpretation of the 24 additional relations between two non-simple lines.

4.3 Relations between a Region and a Line

The relations between a region and a line involve two objects of different dimensions, therefore, conditions that hold between a region and a line do not necessarily hold between a line and a region.

From the previous definitions for regions, the symmetric Condition (1), and the asymmetric parts of Conditions (3a), (5a), (6a), and (7a) apply also for the relations between a region and a line. Further constraints are due to the fact that the regions and lines have different dimensions. The dimension of the interior of a region A is always greater than the dimension of the closure of a line B, therefore, $A^\circ \supset \overline{B}$:

Condition 16 *The interior of a region A always intersects with the exterior of a line B, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\simeq \begin{pmatrix} - & - & \emptyset \\ - & - & - \\ - & - & - \end{pmatrix} \tag{16}$$

By definition, a line has a non-empty boundary and contains no loops. A region's boundary, on the other hand, is a closed 1-cell. This implies that the closure of a line is at most a true subset ($\subset$) of the region's boundary:

Condition 17 *The boundary of a region A always intersects with the exterior of a line B, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\simeq \begin{pmatrix} - & - & - \\ - & - & \emptyset \\ - & - & - \end{pmatrix} \tag{17}$$

The interior of a line is always non-empty, which implies the following condition:

Condition 18 *The interior of a line B must intersect with at least one of the three parts of a region A, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\simeq \begin{pmatrix} \emptyset & - & - \\ \emptyset & - & - \\ \emptyset & - & - \end{pmatrix} \tag{18}$$

Twenty 9-intersections fulfill these conditions for the topological relations between two lines. One of them can be realized only if the line consists of more than one segment, i.e., if it is a non-simple line. If the line has only a single segment then the intersections must also fulfill the conditions for simple lines, (14a) and (15a). The 9-intersections and their geometric representations for a region and a line are shown in Figure 6.

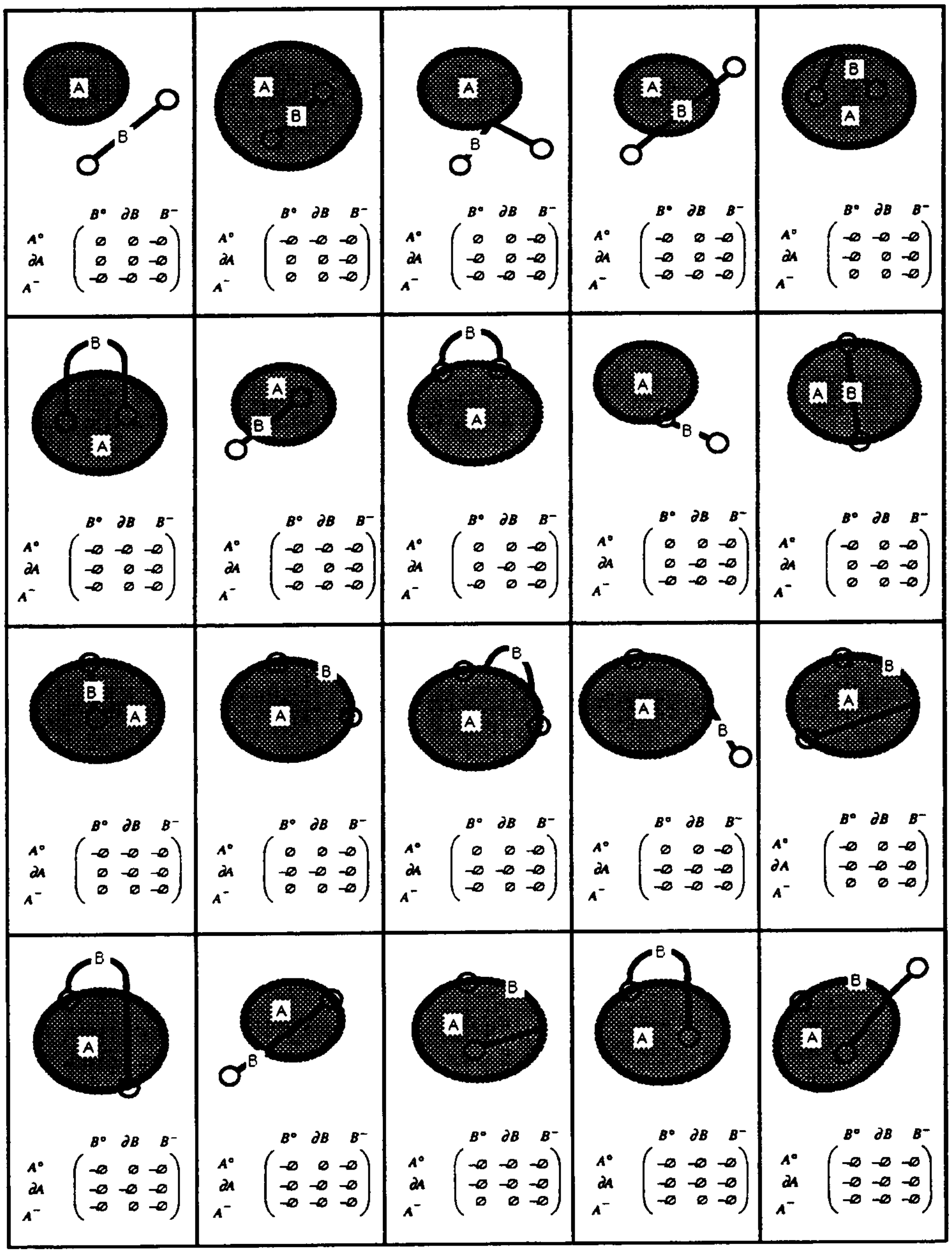

Figure 6: A geometric interpretation of the 20 relations between a region and a line (one of them can be only realized if the line is non-simple).

4.4 Relations with Points

Since the boundary of a point is empty, it is irrelevant to analyze its three boundary intersections. This leaves six significant intersections for describing the topological relations between a non-point (region or line) and a point and gives rise to 2^6 possible relations. The conditions for non-existing intersections are based on the fact that a point is always a true subset ($\subset$) of one of the three parts—interior, boundary, and exterior—of a non-point object.

Condition 19 *Interior, boundary, and exterior of any non-point object A intersect with the exterior of a point B, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\Rightarrow \begin{pmatrix} - & - \\ - & \emptyset \\ - & - \end{pmatrix} \vee \begin{pmatrix} - & \emptyset \\ - & - \\ - & - \end{pmatrix} \vee \begin{pmatrix} - & - \\ - & - \\ - & \emptyset \end{pmatrix} \tag{19}$$

Condition 20 *The interior of a point can only intersect with a single part of another object, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\Rightarrow \begin{pmatrix} \neg\emptyset & - \\ \neg\emptyset & - \\ - & - \end{pmatrix} \vee \begin{pmatrix} - & - \\ \neg\emptyset & - \\ \neg\emptyset & - \end{pmatrix} \vee \begin{pmatrix} \neg\emptyset & - \\ - & - \\ \neg\emptyset & - \end{pmatrix} \tag{20}$$

Condition 21 *The interior of a point must be a subset of one of the three parts of another object, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\Rightarrow \begin{pmatrix} \emptyset & - \\ \emptyset & - \\ \emptyset & - \end{pmatrix} \tag{21}$$

This leaves three combinations of intersections between a point and a non-point that can be realized for point-region and point-line configurations).

Finally, for the sake of completeness, the trivial case between two points. Since both boundaries are empty, there are only four relevant intersections, that is, the intersections between interiors and exteriors. Condition (1) for the 9-intersection can be immediately applied to these four intersections:

Condition 22 *Both exteriors must intersect, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\Rightarrow \begin{pmatrix} - & - \\ - & \emptyset \end{pmatrix} \tag{22}$$

Since a point is "atomic," it cannot intersect with more than one part of another cell. On the other hand, points are non-empty and therefore, they must intersect with at least one part of another cell.

Condition 23 *The interior of a point intersects with exactly one opposite object part, i.e.,*

$$R_{\{\emptyset,\neg\emptyset\}}(A,B) \not\Rightarrow \begin{pmatrix} \emptyset & \emptyset \\ - & - \end{pmatrix} \vee \begin{pmatrix} \emptyset & - \\ \emptyset & - \end{pmatrix} \vee \begin{pmatrix} \neg\emptyset & \neg\emptyset \\ - & - \end{pmatrix} \vee \begin{pmatrix} \neg\emptyset & - \\ \neg\emptyset & - \end{pmatrix} \tag{23}$$

This leaves two combinations of intersections for which the corresponding topological relations, disjoint and equal, can be realized between two points.

5 Related Work

A common thread in most spatial reasoning systems is the attempt to formalize spatial reasoning tasks by translating the problem into Cartesian coordinate space and to use common Euclidean geometry to find the solution [14, 51, 54, 72]. The field of *geometric reasoning* is based on this premise [46]. By using a propositional representation, such as predicates for the relations between objects, it is possible to describe qualitative spatial concepts without the need to bring them into a quantitative environment [9].

Computational approaches focusing on mathematical models to formalize relations among symbolic representations of conceptually modeled objects have been mainly investigated in artificial intelligence and engineering. Various models for *cardinal directions*, such as north, east, and northeast, have been discussed [59] and formalized for point objects [26], and their properties have been analyzed and compared with desirable properties of models for cardinal directions. It has also been proposed to derive topology from metric by using the primitives of *distance* and *direction* in combination with the logical connectors *AND*, *OR*, and *NOT* [58], which is only described for precise metric positions and leads to serious implementation problems in computers [31, 52] due to the finiteness of the underlying number system [20, 29].

5.1 Symbolic Projections

The most extensively investigated formalism for spatial relations is based on a segmentation of the plane, called *symbolic projections* [9]. Symbolic projections translate exact metric information into a qualitative form and allow for reasoning about the spatial relations among objects in a 2-D plane [8]. The order in which objects appear, projected vertically and horizontally, is encoded into two strings, called *2D string*, upon which spatial queries are executed as fast substring searches [9]. Initially, this approach has been proposed only for non-overlapping objects (using the two operators "less" and "equal"). An extension of this algebra with the operator "edge-to-edge" [43] allows for overlapping objects. By including the "empty space" into the 2D strings ambiguities that may exist for certain configurations can be resolved [44].

It was shown that symbolic projections and the 9-intersection are both suitable for powerful spatial reasoning [8, 18]. The major differences are:

- Symbolic projections and their derivatives subdivide the space, while the 9-intersection considers the objects and how they are embedded into space.
- Symbolic projections are primarily based on the relation "less" along to perpendicular axes, therefore, modeling directions such as north, south, east, and west, from which topological relations are derived. The 9-intersection, on the other hand, is only concerned with topological relations.
- Unlike the 9-intersection, which is invariant under topological transformations, symbolic projections depend on the orientation of the objects and, therefore, they are not invariant under rotation.
- The shapes of the objects (convex/concave) matter for the relations modeled by the symbolic projections, while the 9-intersection is independent of the shape of the objects.

5.2 Derivatives of Allen's Interval Relations

Another popular framework are the relations between one-dimensional intervals, initially proposed for modeling time [4]. They have been frequently extended to describe spatial relations in 2- and

3-dimensional space [33, 36, 61]. Some of the extension from 1-dimensional intervals, initially designed to model time, carry over the ordering (start/end) of the interval boundaries to the higher dimension. Most of these approaches assume that spatial objects are described by their bounding rectangles, to which Allen's approach can be easily generalized; however, rectangles are sometimes only crude approximations of the actual shapes of the objects and, therefore, they represent only a simplified model of spatial data. Variations for imprecise boundaries, using fuzzy logic [74], have been also studied [15].

The 9-intersection can be also considered a derivative of Allen's approach. Initially it was proposed to use only the four intersections of the two interiors and boundaries [17, 23], which was shown to be sufficient for codimension 0 [21]. Pigot's extension for triangles in $\mathbb{R}^3$ uses the five intersections of A's boundary with B's interior, boundary, "exterior," "above," and "below" [60]. Actually, this "exterior" is the exterior of B projected into $\mathbb{R}^{n-1}$, and "above" and "below" are then the two sets in $\mathbb{R}^n$ that are separated by the union of B's interior, boundary, and "exterior." Based on this classification schema, a total of fourteen topological relations are distinguished between two triangles in $\mathbb{R}^3$.

5.3 4-Intersection

The initial model for binary topological relations was developed for regions embedded in $\mathbb{R}^2$ [21]. This model, called the *4-intersection*, considers the two objects' interiors and boundaries and analyzes the intersections of these four object parts for their content (i.e., emptiness and non-emptiness). Several researchers have tried to model line-region and line-line relations in $\mathbb{R}^2$ just with the 4-intersection [10, 35, 68]. It is obvious that the 4-intersection is a subset of the 9-intersection, so that the 9-intersection would be able to distinguish more details than the 4-intersection. For region-region configurations in $\mathbb{R}^2$, the 4-intersection and the 9-intersection provide the same eight relations; however, for line-line and region-line relations, the 4-intersection distinguishes only 16 and 11 relations, respectively. The major difference for line-line relations is that the 4-intersection does not suffice to establish an equivalence relation [22], because several different line-line configurations have the same empty/non-empty 4-intersection. Similarly for region-line relations, the 4-intersection does not distniguish between certain topologically distinct configurations that may be critical for defining natural-language spatial predicates to be used in spatial query languages [50]. With the 9-intersection, these problems are overcome.

6 Conclusions

6.1 Summary

A formalism for the definition of binary topological relations has been presented that is based upon purely topological properties and, therefore, independent from the existence of such non-topological concepts as distance or direction. Binary topological relations are described by putting the three topologically distinct parts of one object—its interior, boundary, and exterior—into relation with the parts of the other object. Formally, this has been described as the 9-intersection, i.e., all possible set intersections of the parts. The criterion for distinguishing different topological relations is the content of the 9-intersections, i.e., whether the intersections are empty or non-empty.

The search for a method that provides also an efficient implementation led to the separation of the 9-intersection into the primary criterion or 4-intersection—empty or non-empty intersections between interiors and boundaries—and the secondary criterion, whether or not boundaries and interiors are subsets of the other objects closure. The 4-intersection representation proved to be sufficient for modeling the topological relations between two n-cells if their boundaries are connected and their codimensions are zero; however, the 4-intersection is insufficient if the objects are embedded into a higher dimensional space and the secondary criterion has also to be examined to resolve ambiguities; however, the 4-intersection is insufficient if the objects are embedded into a higher dimensional space and the secondary criterion has also to be examined to resolve ambiguities.

6.2 Implementation

A variation of the 9-intersection has been implemented in MGE-Dynamo [38]. Since each part of a cell is an aggregate of primitives with a unique identifier, the relevant operations, such as interior and boundary, can be implemented as symbolic, rather than arithmetic, operations. The implementation needs four fundamental operations:

- testing whether an intersection of two parts is empty;
- testing whether an intersection of two parts is non-empty;
- testing whether a part is included in another part; and
- testing whether a part is not included in another part.

These are standard operations, for which most efficient implementations have been proposed, for instance in language compilers [2].

The particular benefit of this approach for the implementation of a GIS is that it provides a complete coverage of binary topological relations. Users can build from them customized topological relations, accessible in their spatial query language [39]. For example, some applications may disregard the topological difference between *inside* and *coveredBy* and integrate the two into a single relation, say *within*, such that *within* has a non-empty interior intersection, an empty and a non-empty boundary-intersection, while the value of the boundary intersection does not matter.

This framework may also serve as an internal representation for a graphical spatial query language in which users sketch the spatial constraints graphically. In order to process such queries in a geographic database, the topological constraints contained in the sketch must be parsed and translated into a symbolic representation such a the 9-intersection.

6.3 Discussion and Future Work

The results of this paper represent a significant advancement in the investigations of formalisms for topological relations. Compared to our previous results [21], the novel findings are:

- The application of the framework of empty and non-empty intersections to objects with codimension greater than zero. This was achieved by introducing the 9-intersection.
- The inclusion of objects with connected *or* disconnected boundaries, giving rise to treat lines and n-dimensional objects ($n > 1$) with holes.
- With the 9-intersection we have found a model within which topological constraints can be formalized and compared.

Issues still to be investigated include:

- Topological relations between complex objects, i.e., objects that are made up of simpler ones—either of the same dimension or mixed dimensions, such as a line ending at a region and both together form a single object.

- Optimization strategies of queries with multiple topological constraints are necessary to improve the processing of complex spatial queries. For a small subset—the eight relations between two regions without holes—we have derived the composition table [18] upon which a *relation algebra* [71] can be based.

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Modeling Spatial Relations Between Lines and Regions: Combining Formal Mathematical Models and Human Subjects Testing[1]

David M. Mark and Max J. Egenhofer

ABSTRACT. This paper describes the results of a series of human-subjects experiments to test how people think about spatial relations between lines and regions. The experiments are centered on a formal model of topological spatial relations, called the 9-intersection. For unbranched lines and simplyconnected regions, this model identifies 19 different spatial relations. Subjects were presented with two or three geometrically-distinct drawings of each spatial relation (40 drawings in all), with the line and region said to be a road and a park, respectively. In the first experiment, the task was to group the drawings so that the same phrase or sentence to describe every situation in each group. A few subjects differentiated all 19 relations, but most identified 9 to 13 groups. Although there was a great deal of variation across subjects in the groups that were identified, the results confirm that the relations grouped by the 9-intersection model are the ones most often grouped-by the subjects. No consistent language-related differences were identified among 12 Englishspeaking subjects, 12 Chinese-speaking subjects, and 4 other subjects tested in their own native languages. A second experiment presented the subjects with a short sentence describing a spatial relation between a road and a park, and the same 40 diagrams. Each subject was asked to rate the strength of their agreement or disagreement that the sentence described each relation. For each of the two different predicates tested-"the road crosses the park" and "the road goes into the park"-there was a great deal of consensus across the subjects. The results of these experiments suggest that the 9-intersection model forms a sound basis for characterizing line-region relations, and that many spatial relations can be well-represented by particular subsets of the primitives differentiated by the 9-intersection.

KEYWORDS: spatial relations, GIS, cognition, natural language

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1. Introduction

Over the last several decades, research on fundamental theories of spatial relations has been driven by at least three independent motivations. Mathematicians and some mathematical geographers have searched for situations that can be distinguished in a formal sense (Peuquet 1988; Herring 1991). Largely independently, cognitive scientists have described informally how spatial relations are expressed and manipulated in natural language and thought (Talmy 1983; Herskovits 1986; Retz-Schmidt 1986). And, during the same period, designers of software for geographic information systems (GISs) have developed solutions that would allow them to implement those spatial relations and concepts that are needed for the operation of actual working GISs. The last of these approaches often has produced ad hoc results that are difficult or impossible to generalize from or to extend. While the approach based on mathematics will generate sound definitions as the basis for query algebras, it is not clear how closely such "artificial" models represent human thinking. It seems obvious that research leading to fundamental theories of spatial relations must take human spatial cognition into account, but up to now, studies of locative expressions in cognitive science have usually dealt either with very general principles or with narrowly defined situations often involving non-geographic spaces.

What is missing in almost all of this research is the human factor. Even the cognitive scientists have typically studied published grammars, or used their own intuitions about language as a basis for formalization, and rarely have tested the concepts they develop with human subjects. Several research questions arise: What aspects of spatial relations do people pay attention to during spatial reasoning and decision-making? Of the unlimited number of possible differences that could be distinguished mathematically, what distinctions do people actually make, and what detailed situations do they group when they reason about spatial relations or describe them in natural language? How is this differentiation of spatial concepts influenced by the task that the person is trying to perform, by the native language of the person, by their culture, or by individual differences? To be general, a model of the distinctions that people make in the context of geographic problem solving, or in simply talking about geographic space and spatial relations, must include all of the required distinctions needed for spatial reasoning, for any people and for any problem domains.

A basic thesis of this paper is that human-subjects experiments can guide mathematicians and software engineers as to which distinctions are worth making, and which are not. It describes such experimental work, which we believe demonstrates that the

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interplay between formal mathematics and human subjects testing is of eminent value in the search for fundamental theories of spatial relations.

2. Spatial Relations

Calls for general theories of spatial relations have been issued prominently in the GIS literature for a decade or so (Boyle et al. 1983; Abler 1987; Frank 1987; Peuquet 1988, NCGIA 1989). Indeed, Boyle et al. listed the lack of such a general theory as a major problem for the development of GIS. Thus, it is not surprising that one of the five high-priority topics for research by the proposed U. S. National Center for Geographic Information and Analysis (NCGIA) was defined to be a search for "a general theory of spatial relationships" (Abler 1987, 304). Abler went on to elaborate that the goal is "a coherent, mathematical theory of spatial relationships" (Abler 1987, 306). On the same page, he also stated:

> "Fundamental spatial concepts have not been formalized mathematically and elegantly. Cardinal directions are relative concepts, as are ideas basic to geography such as near, far, touching, adjacent, left of, right of, inside, outside, above, below, upon, and beneath."

The successful proposal for the NCGIA featured this topic prominently in its proposed research program, and stated that "the search for 'fundamental spatial concepts' must be conducted in the cognitive sciences in parallel with searches in mathematics" (NCGIA 1989). Cognitive science can be characterized as follows:

> "Cognitive science is a new field that brings together what is known about the mind from many academic disciplines: psychology, linguistics, anthropology, philosophy, and computer science. It seeks answers to such questions as: What is reason? How do we make sense of our experiences? What is a conceptual system and how is it organized? Do all people use the same conceptual system? If so, what is that system? If not, exactly what is there that is common to the way all human beings think? The questions aren't new, but some recent answers are." (Lakoff 1987, xi)

Research at the NCGIA during 1989 and 1990 made considerable progress on the formal side (Egenhofer 1989a, 1989b; Egenhofer and Herring 1990; Egenhofer and Franzosa 1991), but that work was not linked to cognitive principles. NCGIA researchers also conducted research on spatial cognition, but concentrated primarily on wayfinding (Freundschuh 1989, 1991; Gould 1989; Mark 1989; Freundschuh et al. 1990; Gopal and Smith 1990; Mark and Gould 1992), and did not directly address the sorts of fundamental spatial relations needed for GIS from a cognitive perspective.

Recently, there has been a burst of publications that attempt to extend the formal work noted above by formalizing fairly fine distinctions among spatial relations (Pigot 1990; Egenhofer and Herring 1991; Svensson and Zhexue 1991; Clementini et al. 1992; Hadzilacos and Tryfona 1992; Hazelton et al. 1992). Although the distinctions made in these papers may be valid, and perhaps exhaustive within specific domains, mathematical methods alone cannot establish whether these are the most appropriate distinctions to make for human spatial reasoning and problem-solving. These and other formal developments must be evaluated and refined through human-subjects experiments, and we have begun a series of such experiments to attempt to do such evaluations, the first of which are reported in this paper.

2.1 Previous Published Definitions of Spatial Relations Between Lines and Regions

Most categorizations of spatial relations distinguish between topological relations, such as inclusion or overlap, and metrical relations, such as distance and directions (Pullar and Egenhofer 1988; Worboys; and Deen 1991). In this paper, we concentrate on topological relations, although some of the relations we examine may have other geometric constraints. Peuquet (1988), Frank (1991), and Freksa (1992) provide contributions to some important aspects of metrical spatial relations.

Spatial query languages contain many spatial predicates (Frank 1982; Roussopoulos et al. 1988; Herring et al. 1988; Egenhofer 1991; Raper and Bundock 1991); however most of these predicates lack formal definitions, at least in the publications describing them. While a great deal of attention has been paid in the GIS literature to spatial relations between two regions (Freeman 1975; Claire and Guptill 1982; Peuquet and Zhan 1987; Egenhofer and Franzosa 1991; Hernandez 1992), and to spatial relations between points and regions (the classic "point-in-polygon" problem, for example), there has been relatively little published work on relations between lines and regions.

In a relatively early paper on spatial abstract data types, Cox et al. (1980) mentioned just three pairs of Boolean relations, stating that their arguments could be points, lines, or regions. They called these relations "equality," "sharing," "exclusivity," and their negations, but did not give definitions, other than to note that equality and sharing are symmetric whereas exclusivity is not. They give point-in-polygon (region) as a special case of "sharing," which implies that "sharing" is true if the objects have one or more points in common. "Equal" would appear to be a subset of "sharing," but this cannot be confirmed from the information included in the published paper.

In the late 1980s, several other papers appeared that defined spatial relations. Giiting (1988) listed three Boolean spatial relations between a line and a region: "inside," "outside," and "intersect." Roussopoulos et al. (1988) listed three pairs of Boolean spatial relation operators between a line and a region: "intersect"/ "not-intersect; "within"/ "not-within;” and "cross"/ "not-cross." Once again, however, no details are given in the paper for the exact definition of these predicates. Menon and Smith (1989) included metric spatial relations between points and lines (distance, direction), but no Boolean predicates. Bennis et al. (1991) added the idea of asymmetric spatial relations between line objects and region objects, that is, some spatial relations apply between a line and a region, but not between a region and a line. For example, a region can be left-of or right-of a directed line that is coincident with part of its boundary, but without an external reference frame, a line cannot be left-of (or right-of) any region. The other Boolean spatial relations Bennis et al. presented were "overlap" and "inclusion," and to these they added the metrical relations "distance" and "direction."

2.2 The "9-Intersection" Definition of Topological Spatial Relations

Recently, Egenhofer and Herring (1991) extended a previously-published formal categorization of topological spatial relations between two spatial regions (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991) to account for binary relations in two-dimensional space IR^2 between objects other than regions, such as between two fines, or between a fine and a region. For line-region relations the following definitions are relevant

- A *line* is a sequence of 1 ... *n* connected 1-cells--connection between two geometrically independent 0-cells (nodes)-such that they neither cross themselves nor form cycles. A line defined in this way is equivalent to a non-directed "Chain" in the U.S. Spatial Data Transfer Standard (SDTS) (Fegeas et al. 1992). Nodes at which exactly one 1-cell ends will be referred to as the *boundary* of the line, whereas nodes that are an end point of more than one 1-cell are interior nodes. The *interior* of a line is the union of all interior nodes and all connections between those nodes. Finally, the exterior is the difference between the embedding space IR^2 and the union of the interior and boundary (Figure 1). In this paper, we focus on simple lines, which have exactly two boundary nodes.

- A *region* is defined as a connected, homogeneously 2-dimensional 2-cell; this is termed a "GT-Polygon" in SDTS- Its *boundary* forms a Jordan curve separating the regions *exterior* from its *interior* (Figure 2).

The 9-intersection describes binary topological relations in terms of the intersections of the interiors, boundaries, and exteriors of the two spatial objects. The nine possible intersections among the six object parts (each of the line's interior, boundary, and exterior with each of the region's interior, boundary, and exterior) are preserved under topological transformations and provide a framework for the formal description of their topological relationship.

A variety of topological invariants can be applied to analyze the intersections. The most general topological invariant is the distinction of the content (emptiness or non-emptiness) of the intersections. This can be concisely represented as a 3x3 "bitmap" (Figure 3). With each of these nine intersections being empty or non-empty, the model has 512 (29) possible relations between the objects. Most of these combinations of the nine intersections are, however, impossible for connected objects in the 2-dimensional Cartesian plane. In fact, between an unbranched line and a region, just 19 distinct topological relations are possible (Figure 4). More detailed distinctions would be possible if further criteria were employed to describe the non-empty intersections, such as the dimensions of the intersections (0- or 1-dimensional) or the number of separate components per intersection (Egenhofer and Herring 1990; Clementini *et al*. 1992). Such additional distinctions are, however, ignored in this paper.

The 19 situations can be presented in a diagram that links cases where exactly one of the nine intersections is different but the others are the same; Egenhofer and Al-Taha (1992) presented such a diagram for spatial relations between two regions. This diagram (Figure 5) has a particular "symmetry” such that situations in the equivalent positions on the left and right sides differ only in the fact that the "interior" and "exterior" of the region have been interchanged. In most cases, it also is possible to transform between neighboring situations through smooth geometric transformations.

Whereas the 9-intersection can be shown to be a correct and complete characterization of a system of spatial relations, mathematics alone cannot indicate whether the distinctions made are relevant, or whether relevant distinctions have been omitted. As noted above, additional distinctions can be made, and the number of such possible distinctions may be essentially unlimited. Thus any particular level of detail in topological distinctions may be viewed as a level of abstraction, and the question becomes: is the level of abstraction for spatial relations represented by the 9-intersection an appropriate level for GIS or for cognitive science? In particular, the 9-intersection model distinguishes 19 different line-region relationships. However, the intuition of at least some researchers in the field seems to indicate that most people would not distinguish that many different kinds of topological relations between a line and a region. To evaluate this intuition, we developed experiments to examine how people categorize spatial relations between lines and regions in a geographic context.

3. The First Experiment

3.1 Experimental Design

For the first experiment, we produced 40 drawings of a line and a region. The region was identical in each drawing, and was bounded by a thin solid line and filled with a gray tone. The line was drawn with a line weight twice that of the region boundary. The position of the line relative to the region was different in each case, and the lines were positioned so as to provide two (or, in two cases, three) geometrically distinct examples of each of the 19 topologically-distinct cases of line-region relations. Whenever possible, the line was straight in one example of each 9-intersection situation, and curved in the other(s). Subjects were told that the region was a "park" and that the line was a "road," although the representations of those features did not follow standard cartographic symbology. Some examples are shown in Figure 6, and others appear in subsequent diagrams in this paper.

The 40 drawings were then printed on individual cards 7.0 by 10.8 cm (about the size of standard playing cards), and were shown to 12 native speakers of English, 12 native speakers of Chinese, 3 native speakers of German, and one native speaker of Hindi. With one exception[2], the instructions were given and responses were recorded in the native language of the subject. In English, the instructions were:

> "Here are 40 different sketches of a road and a state park. Please arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group."

When the subject completed the task, the experimenter recorded the groups, and elicited the descriptive phrase or sentence for the spatial relation for each group. Lastly, most of the subjects were asked to select the "best example" from each group, as a prototype.

3.2 Results

3.2.1 Number of Groups

The numbers of groups identified by the subjects varied widely, from 4 to 20 (Figure 7). The median number of groups was somewhat higher for the Chinese subjects, but there is sufficient variation within each group that it is clear that there is no systematic difference in group sizes across the languages tested. Attention thus was turned to the actual groupings.

3.2.2 Groupings

Groupings by individual subjects can be examined visually after plotting them on the diagram introduced in Figure 5, above. We found that the groups normally appeared as connected subgraphs of this diagram. However, there was a great variation in the groupings by individual subjects (for examples, see Figures 8, 9, and 10). Even for individual subjects, the groups seemed at times to overlap. However, because we required each stimulus to be put into exactly one group, such overlaps could not be detected from the data.

3.2.3 Validity of the 9-Intersection

With 40 stimuli, there are 760 distinct pairs of stimuli (n(n-l)/2). Each of these stimulus pairs was grouped together by between all (28) and none (0) of the 28 subjects. Since we did not observe any particular differences across languages, we considered all 28 subjects together, counting how often each pair of stimuli was aggregated and ranking the 760 possible pairs by their grouping frequencies. If we consider only the basic set of 38 stimuli (two for each situation), **the 19 most frequently grouped pairs were exactly the 19 that were within the 9-intersection classes.** This appears to be a strong confirmation of the fundamental nature of the 9-intersection model. Seven of these within-situation pairs were grouped by all 28 subjects, and 4 others were grouped by 27 subjects. Interestingly, all of these categories were around the margin of the diagram. The remaining eight 9-intersection situations, toward the middle of the diagram, were grouped by between 23 and 25 subjects, still more frequently than any between-situation pair. In fact, the most frequently grouped between-situation pair was grouped by 22 of 28 subjects, and only 5 such pairs were grouped by 20 or more subjects.

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[2] The Hindi-speaking subject, was asked the question in English, and asked to use Englishlanguage phrases to describe the categories.

3.2.4 Prototype Effects

Part of the experiment involved asking each subject to designate one stimulus from each group as the best example of that group, to serve as a prototype. Figure 11 shows the frequencies of prototype choice for 23 of the subjects, the 12 Chinese speakers plus 11 of the 12 English-language subjects. Many of the prototypes for the line-to-region relation categories were at the ends of the categories, rather than in the centers. Initially, this was surprising, because for other prototype studies (for example, Berlin and Kay's 1969 study of color categories), prototypes are usually near the centers of categories. However, most of the prototypes for line-region relations were cases in which the body (interior) of the line fell entirely into one of the three parts (interior, boundary, exterior) of the region. To put this another way, the spatial relations in the prototypes seem more "simple." The more complicated cases that fall toward the central part of the diagram (Figure 11) were seldom considered to be "best" examples of relationships, probably because they combined elements of several different relationships. When the prototypes for all categories for the 23 subjects are aggregated, situations around the edge of the diagram are selected far more often than those toward the middle.

For several cases, a situation on the right side of the diagram was selected as a prototype considerably more often than the equivalent diagram on the left. One example is the case where the road is entirely outside the park, except for touching it at one end. That situation was a group prototype for 16 of the 23 subjects for whom prototype information was tabulated, whereas its inside-out equivalent was a prototype by only 7 subjects. In fact, the two stimuli in which the road ended at the park boundary from outside were isolated as a group of two by 13 subjects (making them automatically their own prototype); 2 additional subjects added just one other stimulus to the pair, and only 1 subject used this pair as the prototype for a larger group (10 stimuli). Of the 7 subjects who marked the situation where the road touched the boundary at one end but otherwise was inside as a prototype, 4 had the two stimuli as an isolated pair, and the other 3 had a member of that pair as a prototype of a larger group, or 4, 6, or 8 stimuli. The right-left asymmetry in prototype selection was somewhat more common among the English-language subjects (right-side prototypes outnumber left-side prototypes by 48-34) than among the Chinese-language subjects (right-side prototypes outnumber left-side prototypes by 58-49), but the cross-language difference is not strong.

3.2.5 Similarity Among Subjects

In order to examine similarities and differences among the subjects, an index of similarity between each pair of subjects was computed. First, for each subject, a 40 by 40 binary symmetric matrix was determined, in which a "1" in any position indicates that the subject placed the pair of stimuli denoted by the row and column in the same group, and a "0" otherwise. The fewer groups a subject made, the more within-group pairs there are, and the more -1's there are in the binary matrix for that subject. Then, for each pair of subjects, we counted the number of places in their binary matrices that were identical (that is, the two subjects treated that pair of stimuli identically, either grouping them or not), and divided this count by 1,600 to get a similarity index that would be 1.0 if the two subjects came up with identical groupings. Two subjects did indeed have identical responses, so the maximum value of the similarity index was 1.0, and the minimum observed value for any pair of subjects was 0.736.

In addition to the human-subjects data, we created 4 "synthetic" subjects, one being the exact groupings of the 9-intersection model, and the other 3 from topological models that would result if certain distinctions made by the 9-intersection model were ignored. In two models, the boundary of the region is either merged with its interior ("region-closure model") or the exterior of the region ("open-region model"). A final model lumped the interior and boundary of both the line and the region ("line- and region-closure model"), thus letting the model degenerate to just the "contains," "overlaps," and "disjoint" relations that some previouslye-published categorizations of spatial relations had recognized. Some of these classifications had much lower similarities to the data from the human subjects, with indices as low as about 0.55.

In the analysis we employed multidimensional scaling (MDS), a technique for determining configurations of points given only a matrix of inter-point distances or similarities. Usually, no configuration would replicate the interpoint similarities exactly, and so MDS finds the configuration that best fits the data according to some goodness-of-fit criterion. The solution does not determine such factors as scale, rotation, or reflection, and so the axes of the output configuration are arbitrary. The similarity indices among all pairs of subjects and models, as discussed in the preceding paragraph, were entered into SPSS-Xs multidimensional scaling procedure, and this produced a 2-dimensional configuration of points (Figure 12). Except for the fact that there are very few Chinese subjects appearing as outliers on the diagram, the subjects do not seem to cluster by language. This seems to confirm impressions gained from visual inspection of the subjects' groupings, that individual differences within languages are greater than distances between languages. Note that the "region-closure," "open-region," and "line- and region-closure" models fall outside the convex hull of the data for the 28 subjects, although data for several subjects plotted closer to the "regionclosure" model than to the 9-intersection model itself.

3.2.6 A Rare Example Of Discrimination By Geometry

Whereas almost all subjects appeared to emphasize topological factors in their responses to the grouping task, one of the English-language subjects apparently used a geometric criterion to classify some of the stimuli. Figure 13 shows the stimuli involved in this exception. The particular geometry of the four cases on the upper right-hand side of Figure 13 caused them to be grouped together by this subject, who noticed that these had a straight segment in exactly the same position relative to the concavity on the lower ("southern") side of the park. However, none of the other 27 subjects grouped these 8 stimuli in this way, and the exception does not contradict the general tendency among subjects to classify the stimuli primarily according to topological criteria and to generally ignore geometric characteristics.

3.3 Summary of the First Experiment

This first experiment has shown that there is a great deal of variation in the ways in which people classify spatial situations that involve roads (lines) and parks (regions). There are, however, underlying patterns. One of the strongest of these is the 9-intersection model. Whereas it is possible that the experiment contains biases that promote the recognition of the 9-intersection distinctions by the subjects[3], that model definitely emerged as an underlying structure. The 19 pairs of stimuli most often grouped by the subjects were exactly the 19 cases that the 9-intersection model does not distinguish. Also, since the differences between the two members of each topologically-similar pair of road-park examples were geometric--different orientation, shape, and length of the line, etc.-the outcome suggests that people often ignore such quantitative differences and are primarily concerned with qualitative (topological) differences. The results of the experiment suggest that many of the qualitative differences that people make regarding spatial relations are captured by the 9-intersection model.

The diagram introduced in Figure 5, constructed analytically on the basis of "least distinguishable differences" in the 9-intersection, reflected subjects' judgments very well: the 37 most-frequent pairs, and 53 of the 58 most frequent pairs, were either within 9-intersection classes, or were between adjacent classes on that diagram. Figure 14 shows a consensus diagram, which groups all pairs that were combined by 14 or more of the 28 subjects. The groups are somewhat larger on the left side of the diagram than on the right, which has a larger number of isolated 9-intersection classes.

Although there are a few intriguing suggestions of language-based differences in the results, some of which have been reported above, the experiment described provides no solid evidence of differences in judgments about lineregion spatial relations between speakers of the languages tested. It is quite possible that no such differences exist for roads and parks, or even more generally regarding lines and regions. If such differences exist between English and Chinese, they are probably subtle, at least with respect to this experimental design, and thus larger samples or more focused experiments (or both) will be needed to establish any differences that may exist. It is also possible that, by coincidence, Chinese and English happen to be very similar for the spatial situations included in the experiment; speakers of other languages will have to be tested before any generalizations about crosslinguistic universal principles can even be proposed, let alone substantiated.

The high individual differences across subjects, both in groupings and in the language used to describe those groups, make it difficult to examine the possible meanings of various locative phrases, or to relate the results to more practical issues of queries in a GIS context. Therefore, a second, more specific experiment was designed.

4. The Second Experiment

4.1 Experimental Design

To further evaluate the model described above, we designed a more specific test, in which subjects were presented with sentences in English that described a spatial relation between a "road" and a "park." Potential spatial predicates to be tested were drawn from the subjects' responses in the first experiment. From these, we selected "the road crosses the park" and "the road goes into the park" for testing. The test instrument consisted of six pages. The first page (for the test of "cross") presented the following instructions:

"Each of the accompanying 40 diagrams represents a State park and a road. Please examine each map, decide how strongly you agree or disagree with the statement that in that case, 'the road crosses the park,' and mark your response on the scale from 1 to 5 under each diagram."

This instruction page was followed by five pages, each with 8 road-park diagrams. The top half of the first page, containing stimuli 1, 2, 5, and 6, is shown in Figure 15.

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[3] One of the English-language subjects, who was making rather fine distinctions, noticed that identical topological relations usually came in pairs, and then used this as part of his classification, trying to find a "twin" for any apparently-isolated stimulus, and also looking for ways to divide any groups of three he had formed. In fact, the two "extra" examples that were added to the stimulus set created a problem for him at the end, as he worked on the two groups of 3 stimuli for quite a while before finally leaving one of them, and breaking the other into a group of 2 and a singleton. It is possible that other subjects used similar reasoning.

Each subject was asked to compare the sentence to each of the 40 diagrams that were used in the first experiment, and to evaluate on a scale of 1 to 5 the strength with which they disagreed (1) or agreed (5) that the sentence described the situation portrayed in that diagram. Then the average rating for all subjects was obtained for each diagram, and this average was rescaled so that 0.0 would represent "strongly disagree" and 1.0 would indicate "strongly agree.- Since these ratings were quite similar for the 2 or 3 examples of each of the 19 relations distinguished by the 9-intersection model, we further averaged the results across the stimuli for each of those 19 relations[4]. These summary ratings are empirical estimates of the probability that a subject would consider that a drawing illustrating that topological relation represents the concept to which the sentence refers.

4.2 The Road "Crosses" the Park

The first spatial relation that we tested was the concept of a line "crossing" a region. Some *a priori* analysis suggests that in order for a line to "cross" a region, it should satisfy two topological constraints: the line must have some intersection with the interior of the region, but also should not terminate within the region. As linguist Leonard Talmy has discussed (Talmy 1983), the prototypical meaning of "cross" involves completion of a side-to-side traversal of a two-dimensional entity, and thus there is a possibility that metrical properties will be important. Figure 16 illustrates the two subsets of the 9-intersection diagram that are excluded by the restrictions noted above, and the five remaining relations, which should correspond to "cross."

We collected agreement ratings for "the road crosses the park" from 13 native English-speakers and for three other subjects[5]. The results are illustrated in Figure 17, and confirm the conceptual model outlined above. The stimuli fall into 3 groups. The 5 spatial relations that were predicted to "cross" the park had the highest mean ratings, 0.68 or above. It is interesting to note that the highest agreement ratings are for the two situations at the ends of the “crosses" class; this supports the generalization presented in section 3.2.4 above, that best examples (prototypes) tend to be at the ends of categories. The 7 relations for which the road does not enter the park's interior at all had the lowest ratings, 0.14 or lower. The 7 cases in which the road enters the park but ends inside it had intermediate mean ratings, between 0.21 and 0.36. Evidently, ending inside the park does not exclude a road from "crossing" a park as strongly as not entering the park at all.

Talmy's (1983) emphasis on a side-to-side traversal suggests a further restriction might exist regarding geometry. Talmy presented the following example (in Mark *et al*. 1989a): if a person walks from one end of a pier to the other, straight down the middle, it would not be appropriate to say in English that the person crossed the pier, even though the walk had completed a traverse from one part of the pier's boundary to another. Thus, some situations that meet the topological restrictions noted above might still be excluded from the class of "roads crossing the park" by geometric properties. In fact, our stimulus #39, in which the road comes in one end, curves, and goes out through that same end, had a "cross" rating of 0.45, whereas the two topologically-identical stimuli in which the road traversed the park between two opposing sides had mean ratings of 0.84 and 1.00. Influences of geometry will be a focus of some testing in our further research.

4.3 The Road "Goes Into" the Park

The same set of stimuli and instructions were run with the phrase "the road goes into the park," which was a spatial-relation category that several of the English-language subjects in the classification experiment (first experiment) came up with, and which usually had as its prototype the situation with one end of the road outside the park, and the other end inside (the situation at the top of the 9-intersection diagram; see Figure 18).

Data on agreement with the phrase "the road goes into the park" were collected for 7 subjects. Again, there was considerable consensus within 9-intersection relations, across both subjects and stimuli. The results, presented in Figure 18, show high agreement for all cases in which the body of the road intersects the interior of the park, as long as at least one end of the road is outside or on the boundary. Furthermore, the relation at the top of the diagram, which as just noted was often the category prototype in the classification experiment, had a rating of 0.95, second highest of all the stimuli.

4.4 Summary of the Second Experiment

Comparing the results of this experiment for the two sentences tested, we find that some situations were strongly confirmed as belonging both to "the road crosses the park" and "the road goes into the park." Other situations belong to one concept and not the

[4] The first 38 stimuli (two for each 9-intersection class) were drawn with the road in an "'ordinary' relation to the park. Stimuli #39 and #40, however, had specific geometries designed to examine specific aspects of road-park relations. These two special stimuli were excluded from the averages calculated for the 9-intersection relations, and results for #39 are reported separately below.

[5] These three additional subjects, two who were native speakers of Chinese and one of Hindi, were all fluent in English and were tested in English.

other, and still others fit neither of these descriptions. This supports the idea that no single set of mutually-exclusive and collectively-exhaustive spatial predicates could satisfy all queries or natural language descriptions. On the other hand, the results give us further confidence that the 19 line-region relations distinguished by the 9-intersection model have promise as a set of primitives, to be used as building blocks in developing a potentially large number of higher-level spatial concepts.

5. Future Research

Future research is indicated in many directions. Clearly, the experiments described in this paper should be repeated with larger samples. The cross-linguistic dimension of the problem also is worth pursuing, because of the implications for GIS user interfaces, query languages, and cross-linguistic technology transfer (Mark et al., 1989b; Frank and Mark, 1991; Gould et al., 1991). There also is potential to contribute to our understanding of the differences by which different languages express spatial concepts (Talmy, 1983), and thus subjects should be tested in other languages. The possible influence of the hypothetical phenomena in the drawings also is worth investigating. Would the results be significantly different if the test drawings for the line-region relation were described as a storm track and an island or peninsula? Or a road and a gas cloud? And does scale (scope) matter, that is, would the categorization be different if the line and region were things on a table-top, or were at continental scales?

It also would be interesting and potentially valuable to perform human subjects experiments regarding the acceptability of hypothetical GIS responses to hypothetical quasi-natural-language queries regarding spatial relations between line features and region features. The second experiment was designed to test this aspect of the problem, but probably would be more clearly applicable to GIS if the queries were from a GIS context and if the test were performed on a computer rather than on paper. Also, we feel that the model described herein would provide a good basis for analyzing line-region queries provided in GIS software, or in testing spatial relations defined in the literature.

In addition to the specific results obtained, we feel that the studies reported in this paper demonstrate the value of human subjects testing and empirical evidence in the development and evaluation of formal models for spatial relations. The 9-intersection model for lines and regions can be understood more fully in light of data from human subjects. We hope that more researchers from the GIS and cartographic communities will combine experimentation and mathematical rigor to determine the strengths and limitations of the infinity of possible spatial relations that could be formally defined.

6. Acknowledgments

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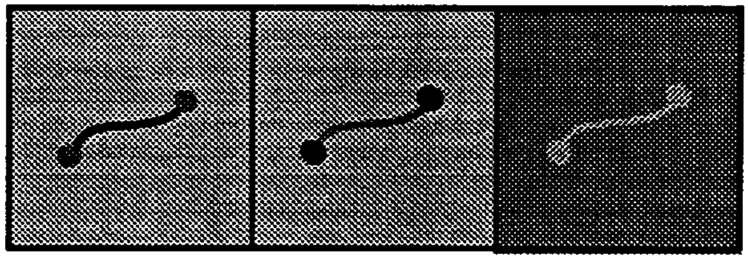


Figure 1. Interior, boundary, and exterior of a line in $\mathbb{R}^2$.

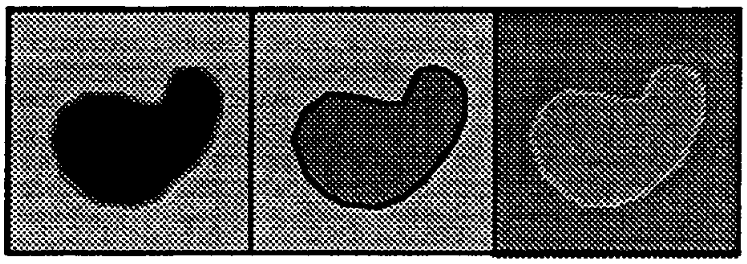


Figure 2. Interior, boundary, and exterior of a region in $\mathbb{R}^2$.

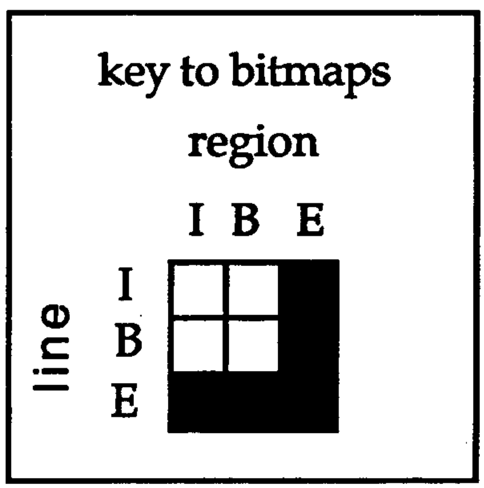


Figure 3. The 3x3 "bitmap" of a 9-intersection. White pixels represent empty intersections and black pixels stand for non-empty pixels. The left column of each bitmap has a black square for each part of the line (from the top, the Interior, Boundary, and Exterior) that intersects the Interior of the line; the column in the middle indicates the same for the Boundary of the region; and the right column represents the three intersections with the region's Exterior.

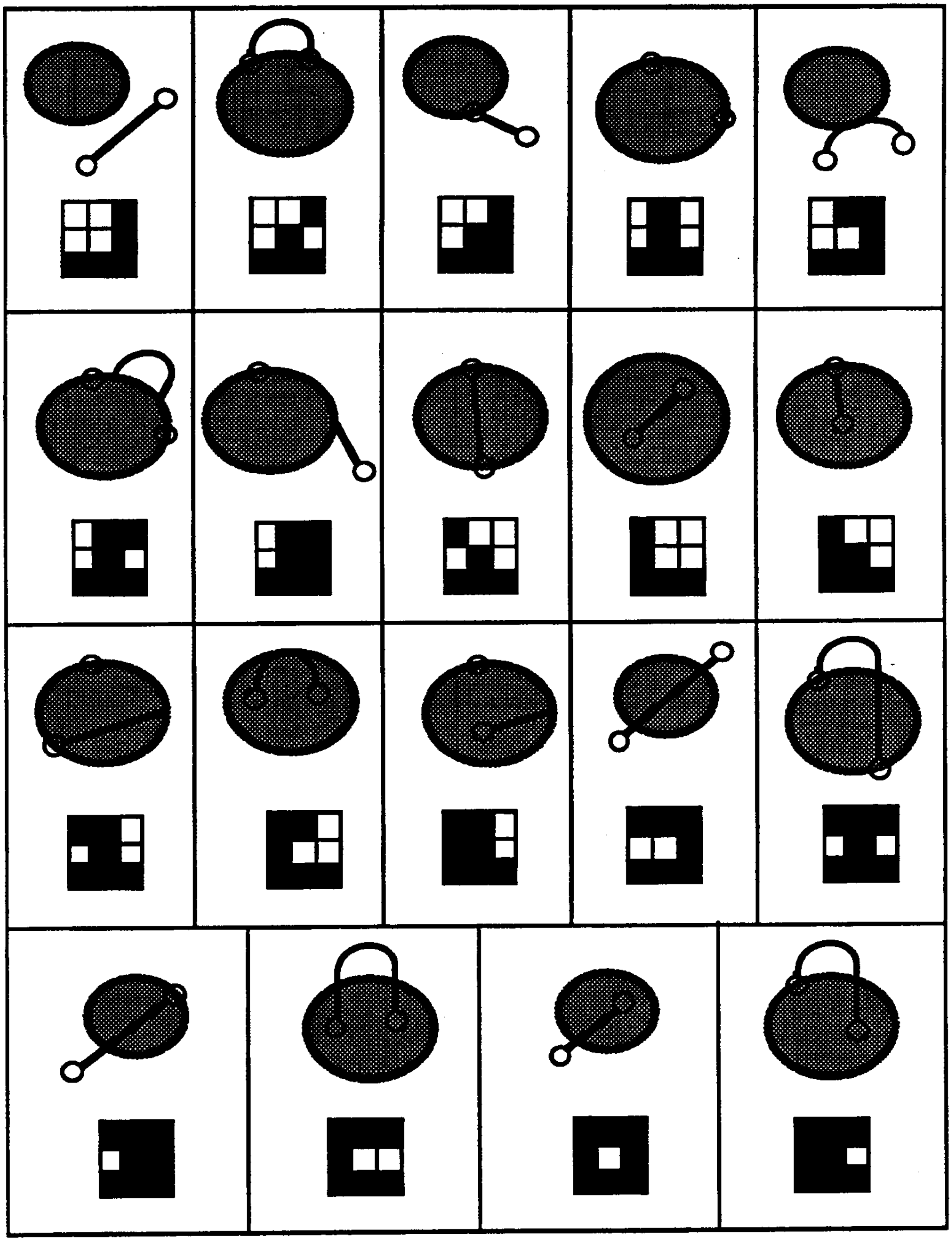


Figure 4. The 19 topological relations distinguished by the 9-intersection, together with their empty/non-empty bitmaps.

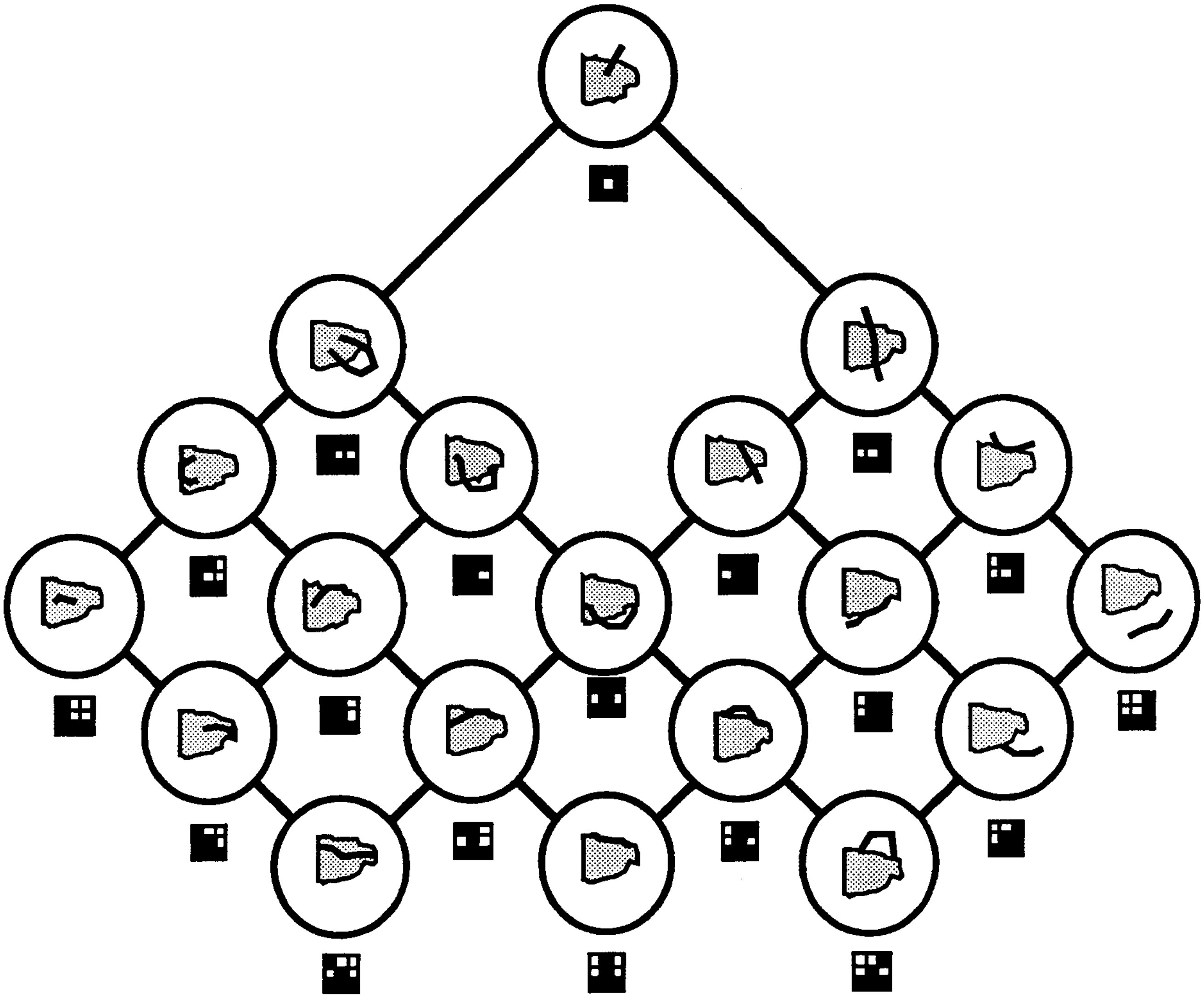


Figure 5. Diagram illustrating relationships among the 19 topologically-distinct spatial relations between a line feature and an region feature according to Egenhofer and Herring's "9-intersection" model. Situations are connected in the diagram if they differ for exactly one of the nine "intersections."

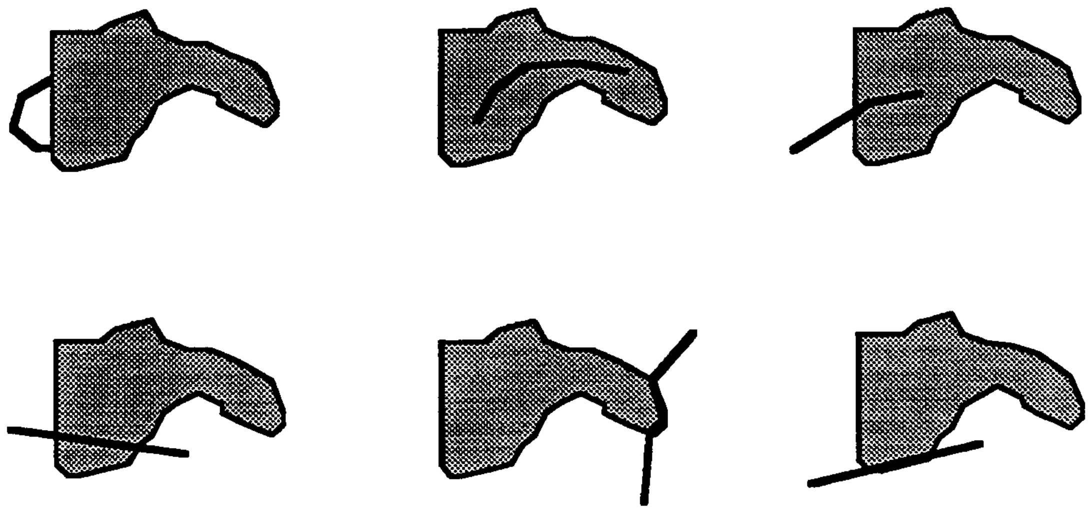


Figure 6. Some examples of the stimuli used in this research. The right and middle examples in the lower row are topologically identical but geometrically-distinct.

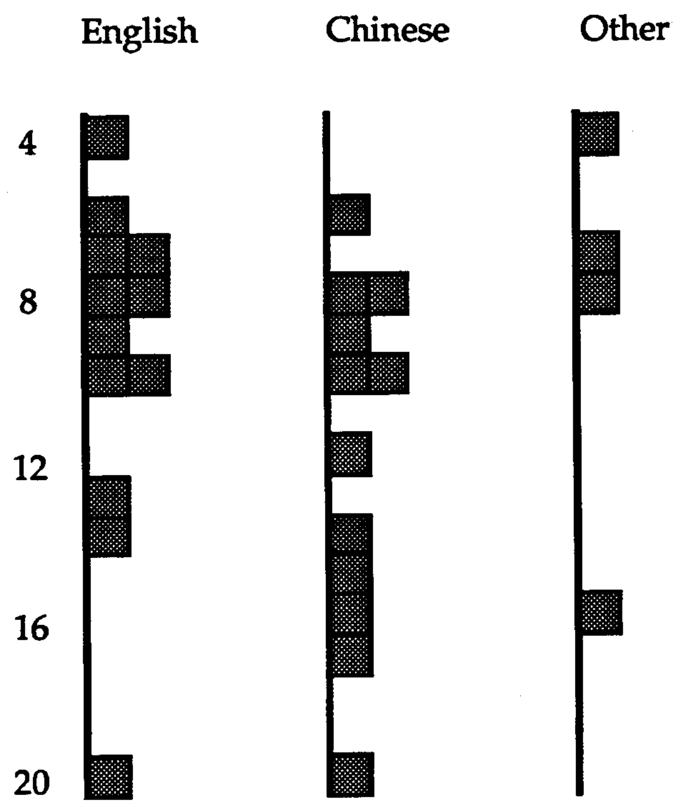


Figure 7. Histogram showing the number of groups defined by the subjects.

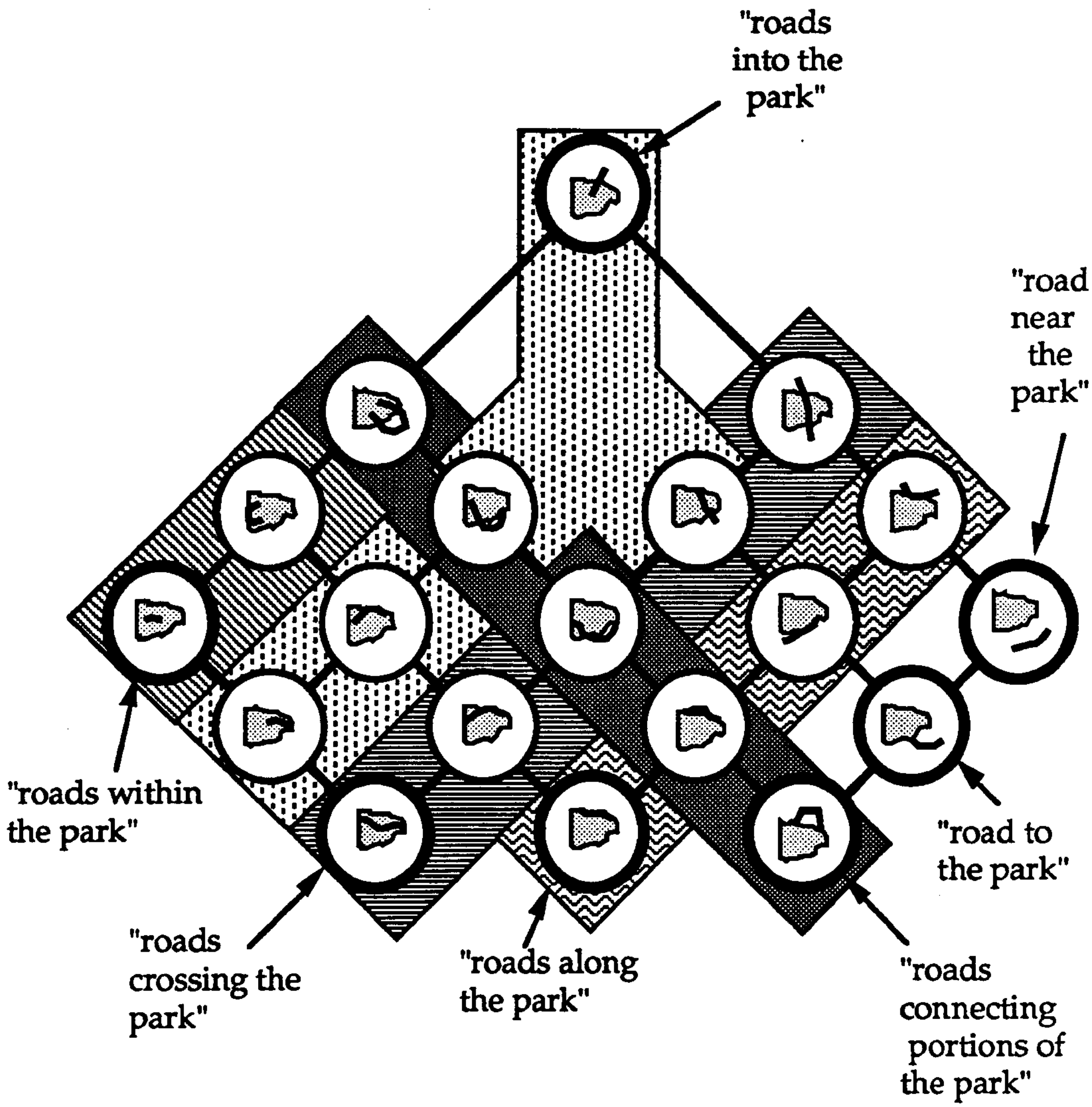


Figure 8. The responses of one of the English-language subjects, plotted on the diagram introduced in Figure 5. Each shaded polygon represents a group; the shadings are arbitrary and simply are intended to discriminate the groups within one diagram. If the group polygon covers just half of a circle representing one of the 9-intersection situations, that means that only one of the two diagrams representing that situation was placed in the group. A heavy circle boundary designates the group prototype; a heavy circle on the white general background indicates that the subject isolated the two or three diagrams representing that 9-intersection situation as a small group.

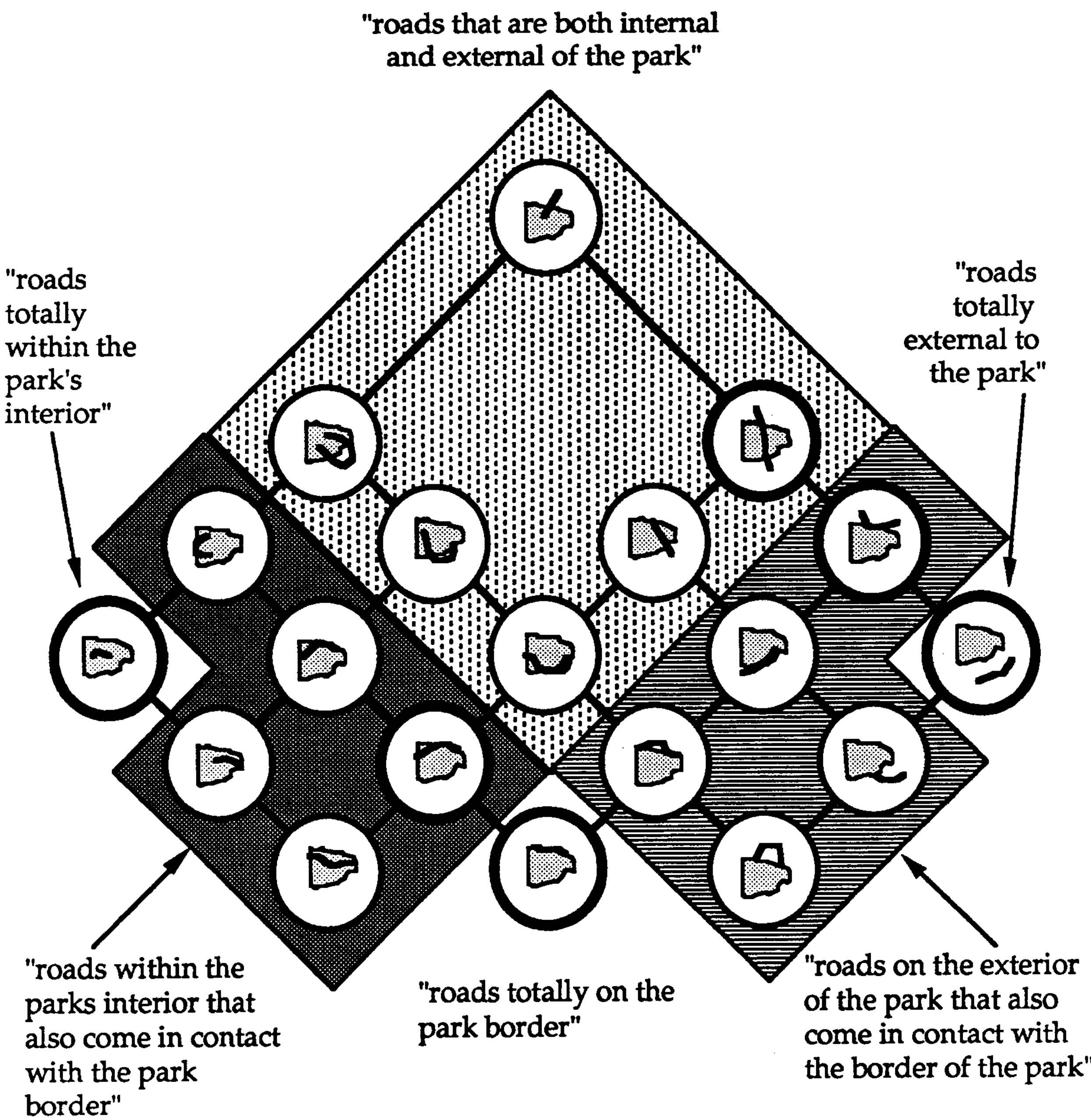


Figure 9. Diagram for another English-language subject. Note that these groups are much more compact than those produced by the previous subject (Figure 8), and that the diagram is perfectly symmetric.

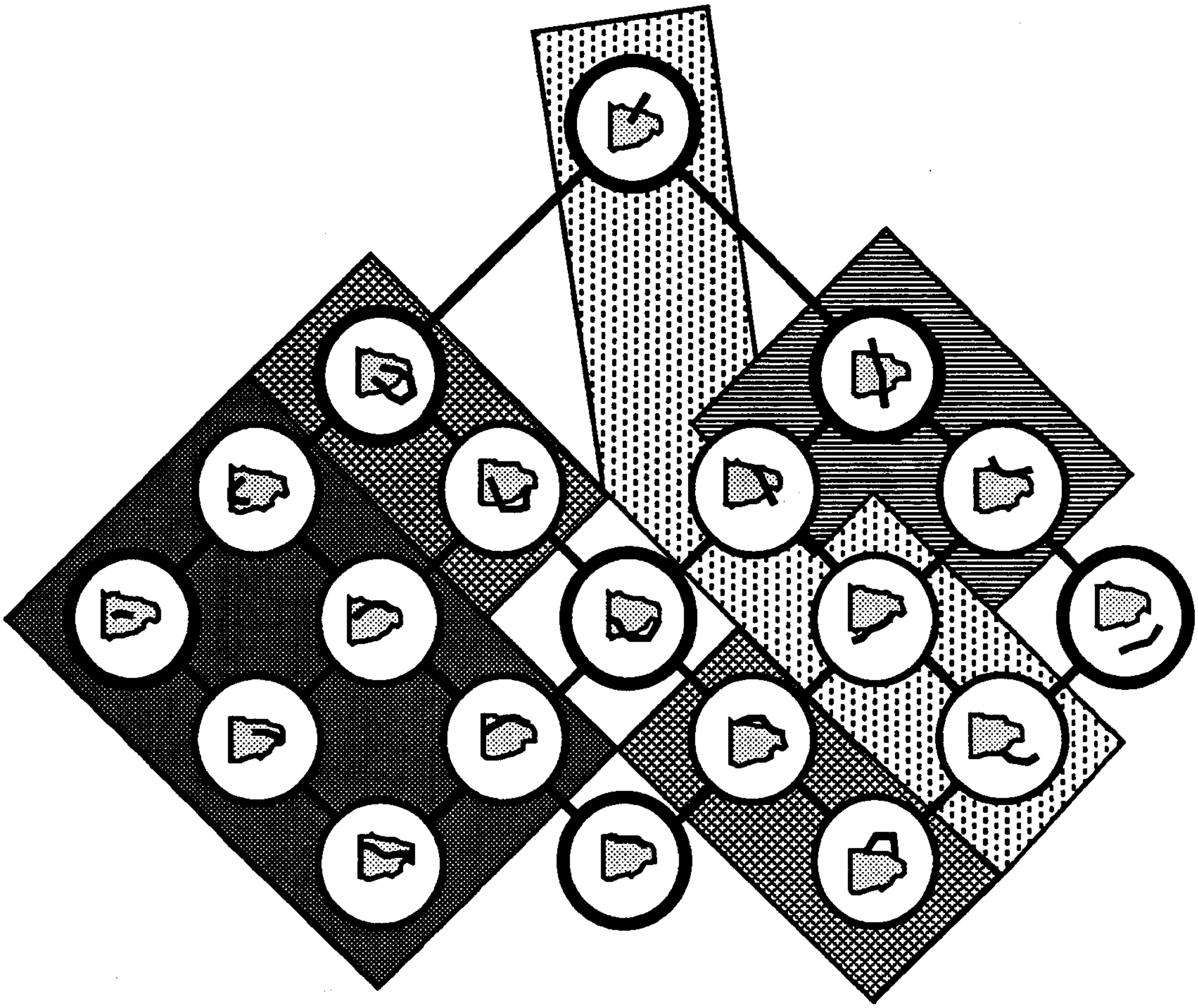


Figure 10. Diagram for one of the German-language subjects. This subject provides an example of the tendency of several subjects to aggregate much more on the left side of the diagram (where the road in within the park), but to differentiate more finely on the right side.

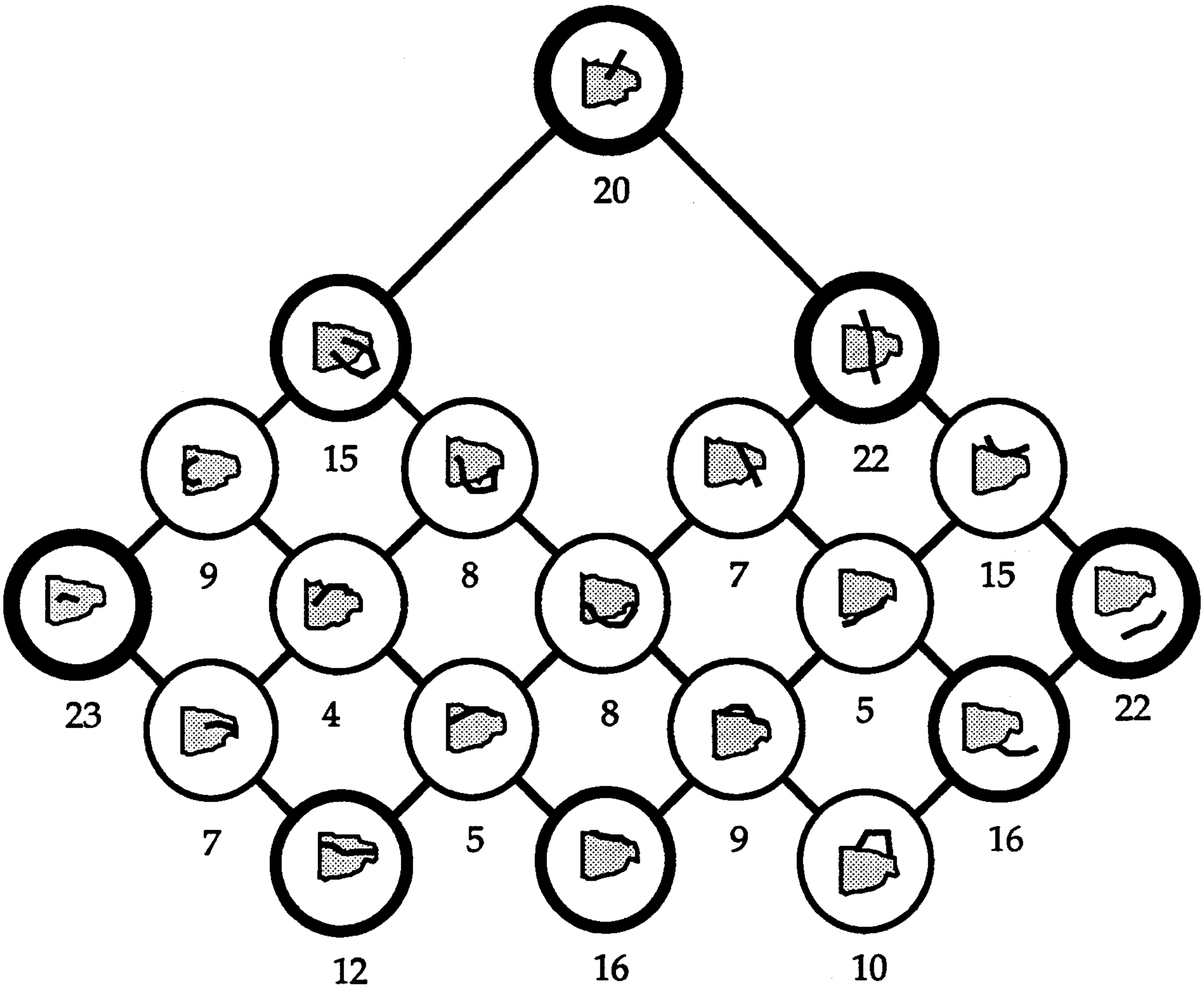


Figure 11. Frequency with which each situation was chosen as a group prototype by 23 subjects.

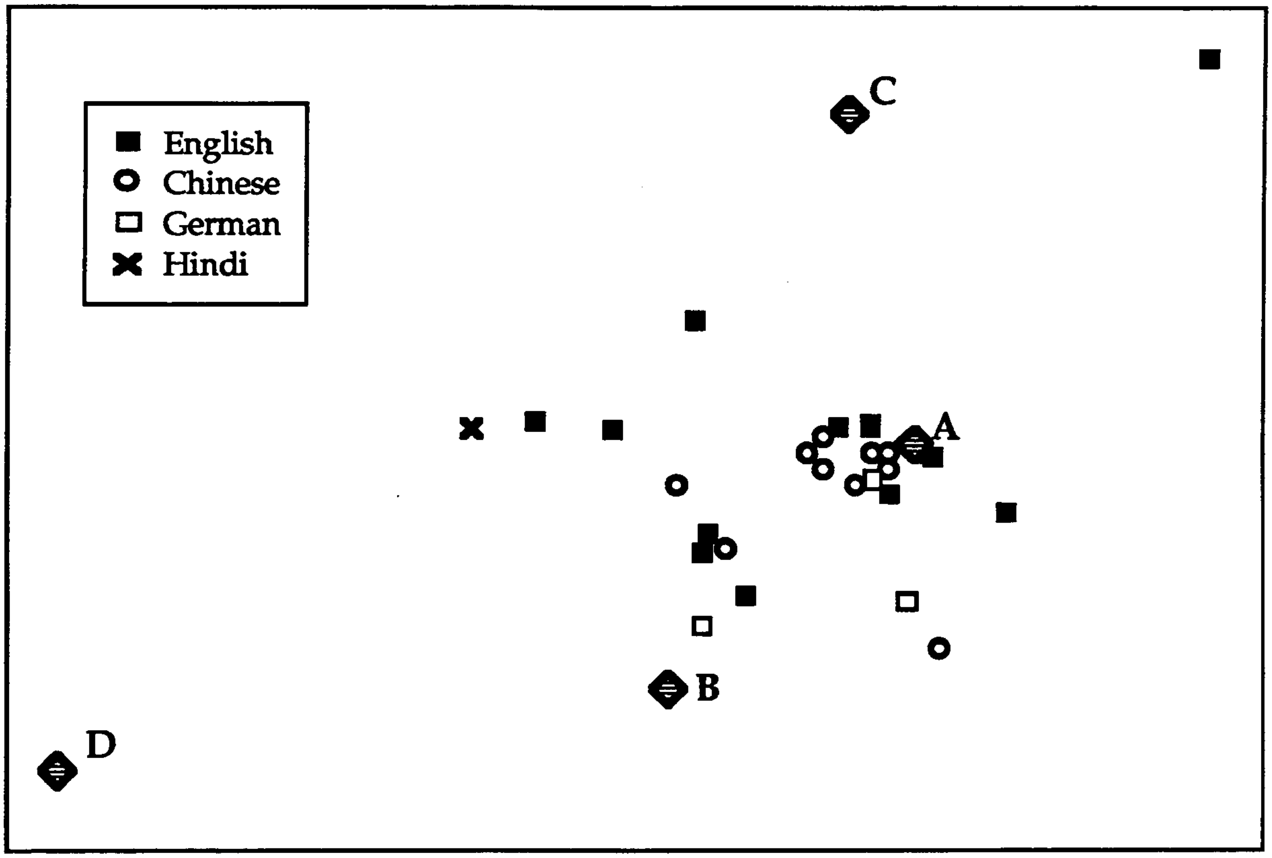


Figure 12. Configuration of the 28 subjects and 4 models of spatial relations, produced by multidimensional scaling. The models are indicated by shaded diamond shapes: A, the 9-intersection model with 19 groups; B, the region-closure model (combining the interior and boundary of the region); C, the open-region model (combining the boundary and exterior of the region); and D, the line- and region-closure model (boundaries of both the line and the region are merged with their interiors). The axes of the diagram are arbitrary dimensions.

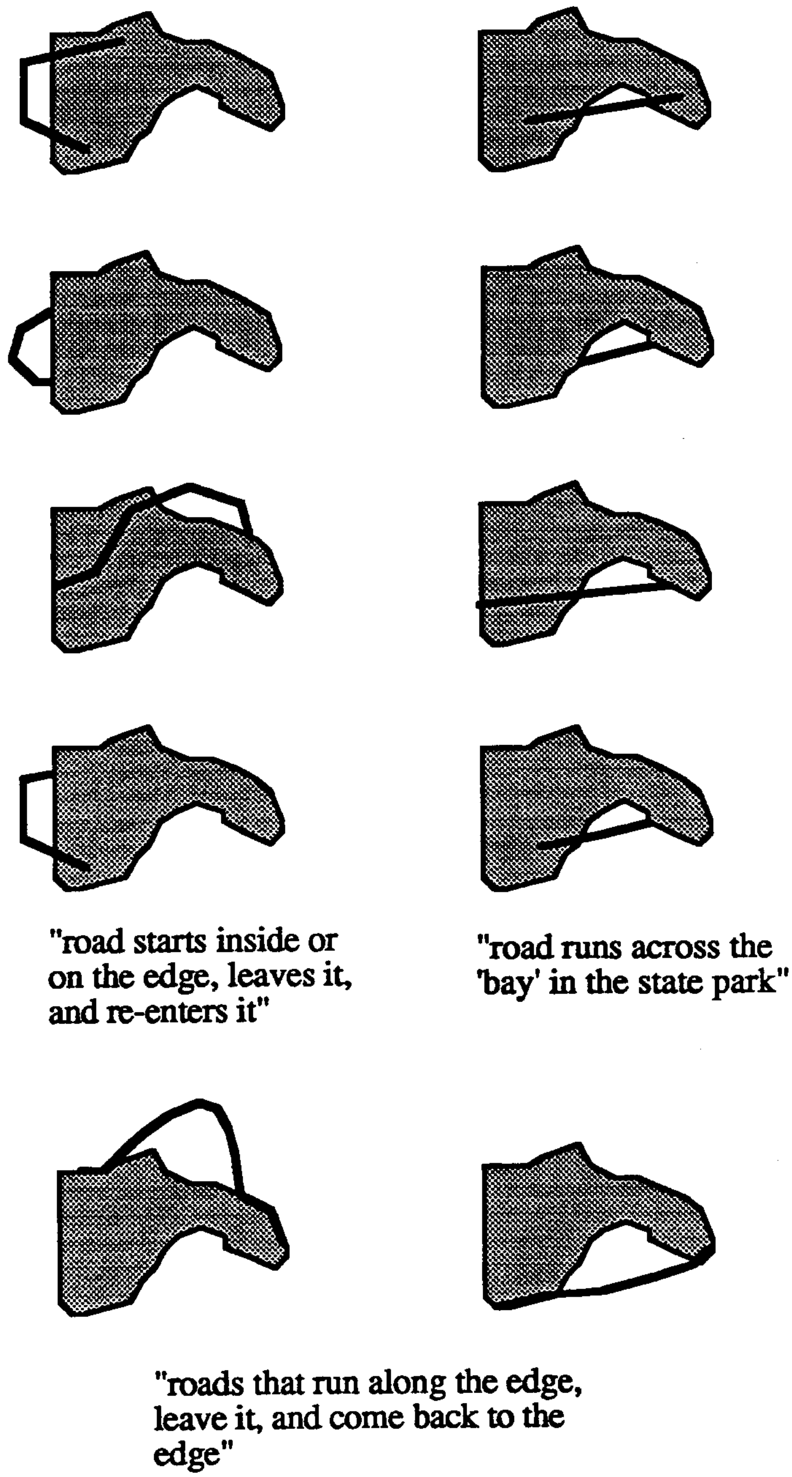


Figure 13. Three groups of stimuli according to one of the English-language subjects. The four stimuli in the upper left part of the diagram were described by the sentence "starts inside or on the edge, leaves it, and re-enters it," whereas the four in the upper right were grouped under "road runs across the "bay" in the state park." The lower two examples were grouped together and described by the phrase "roads that run along the edge, leave it, and come back to the edge." Each of the left-right pairs of park drawings above represents two realizations of the same 9-intersection relation.

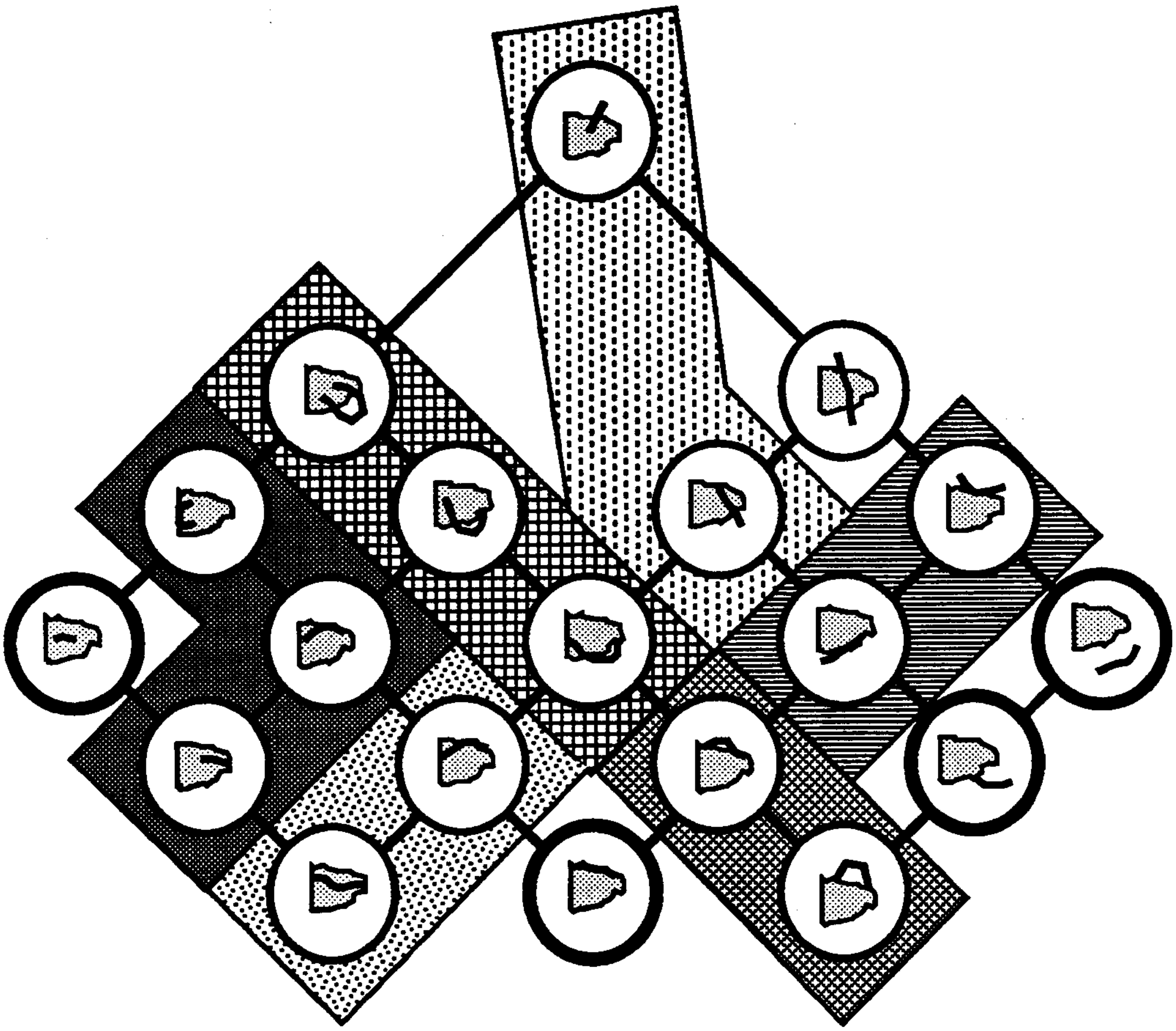


Figure 14. Consensus diagram for all 28 subjects. For the 19 9-intersection situations on the diagram, all within-situation stimulus pairs were grouped together by well over half of the subjects. Heavy circles surround those 9-intersection situations that were not grouped with any other stimuli by 14 or more subjects. The shaded zones surrounding the remaining situations indicate groups for which every within-group pair was combined by at least 14 subjects.

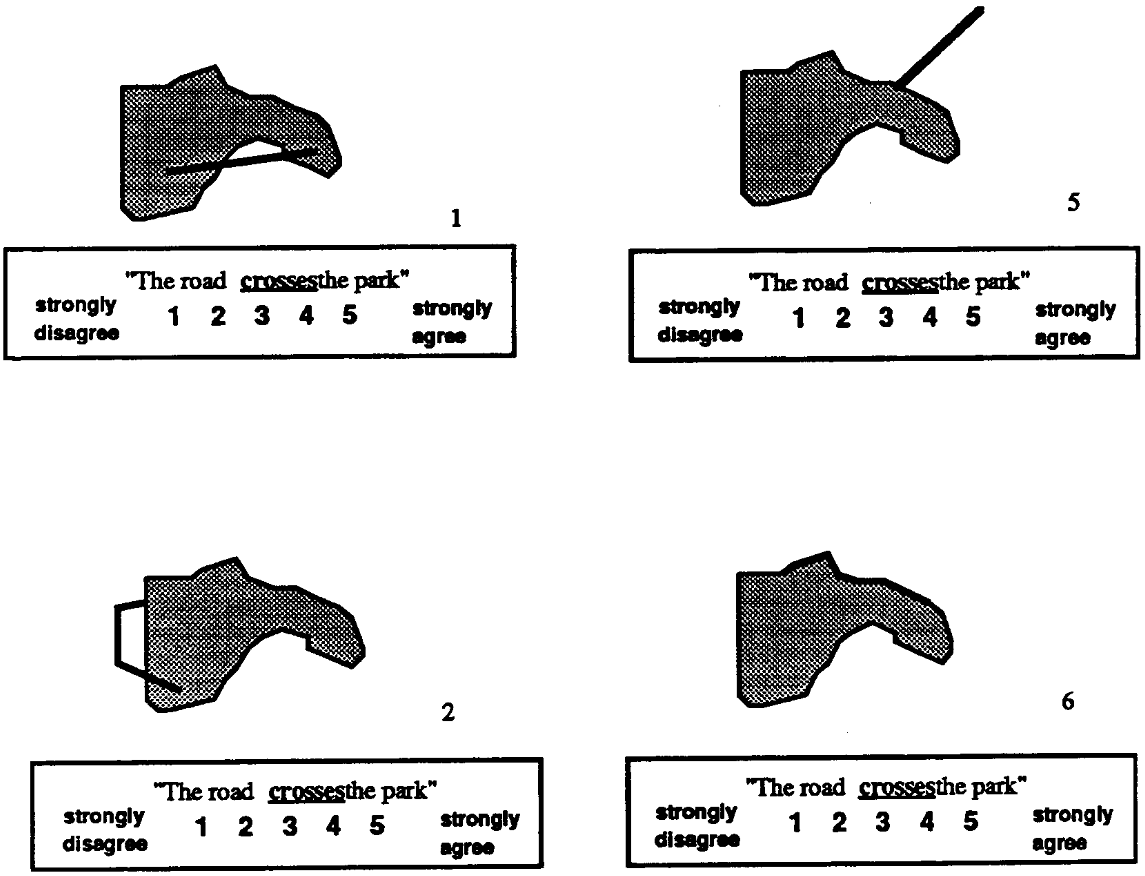


Figure 15. Four of the stimuli (the top half of the first page of the test instrument) used in the experiment designed to evaluate the concept of a linear feature "crossing" an area feature.

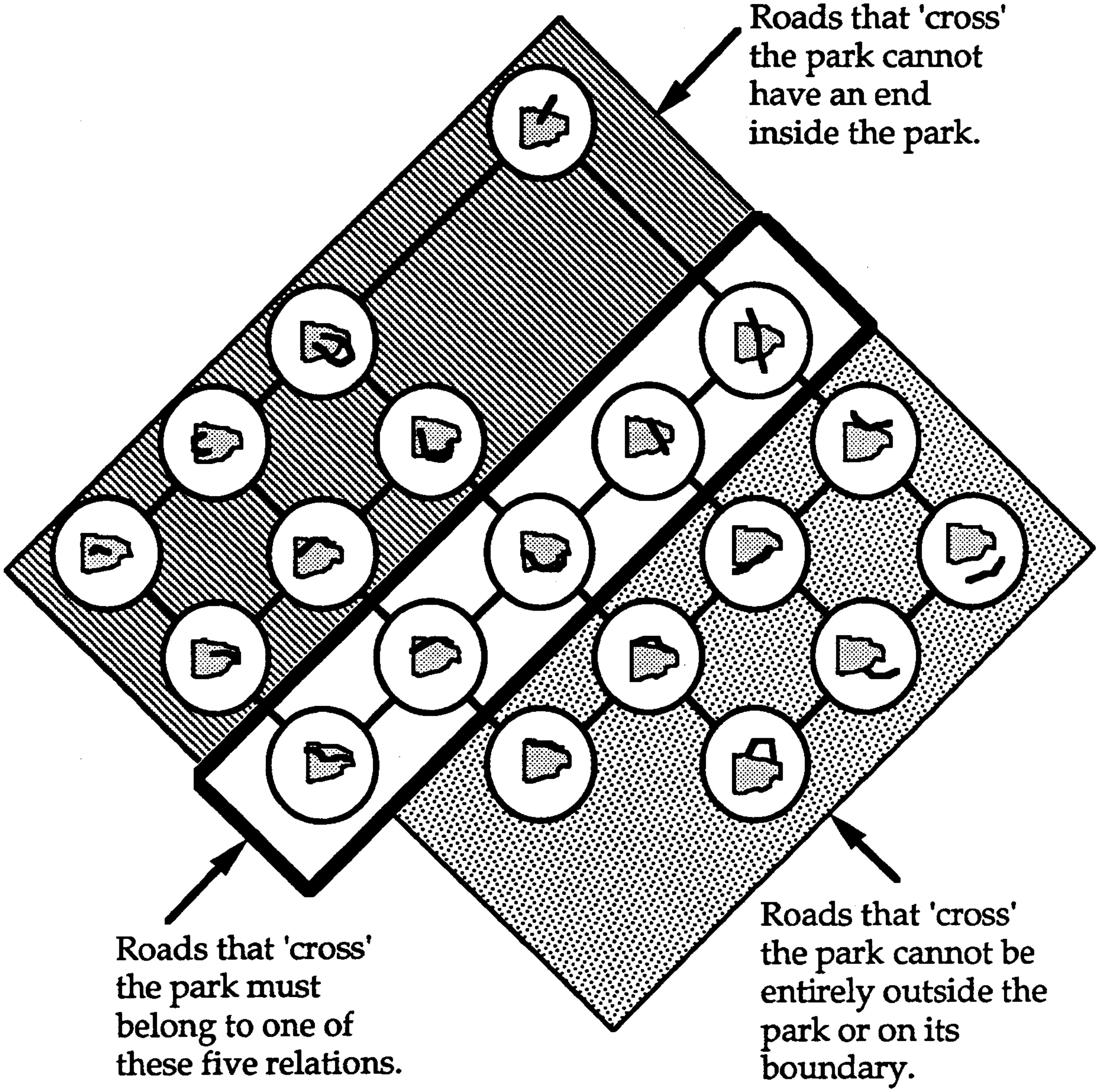


Figure 16. *A priori* analysis of the probable constraints on the concept of a linear feature "crossing" a region feature.

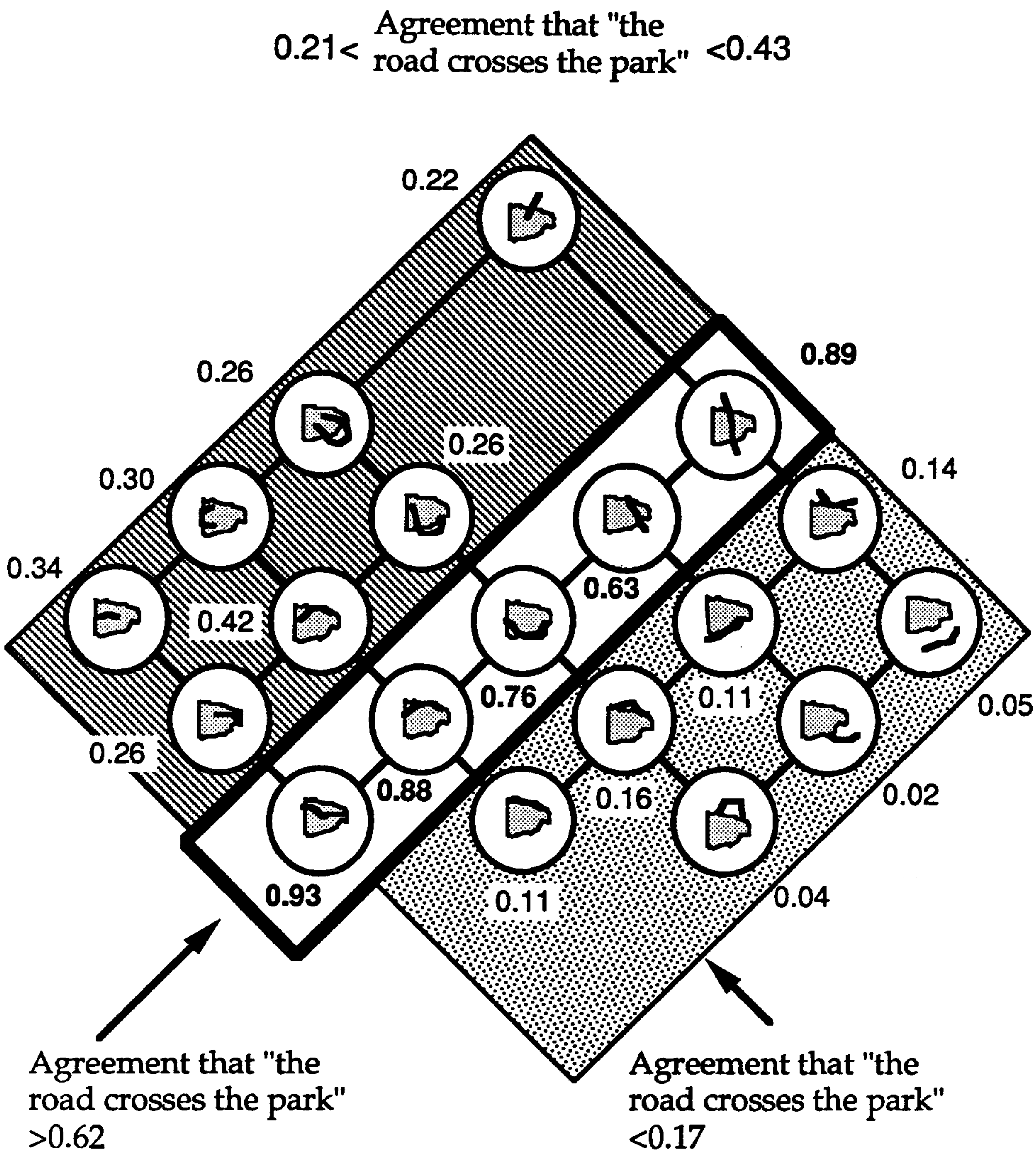


Figure 17. Strength of agreement that "the road crosses the park" by 9-intersection relation, averaged across 16 subjects and 2 or 3 cases per relation.

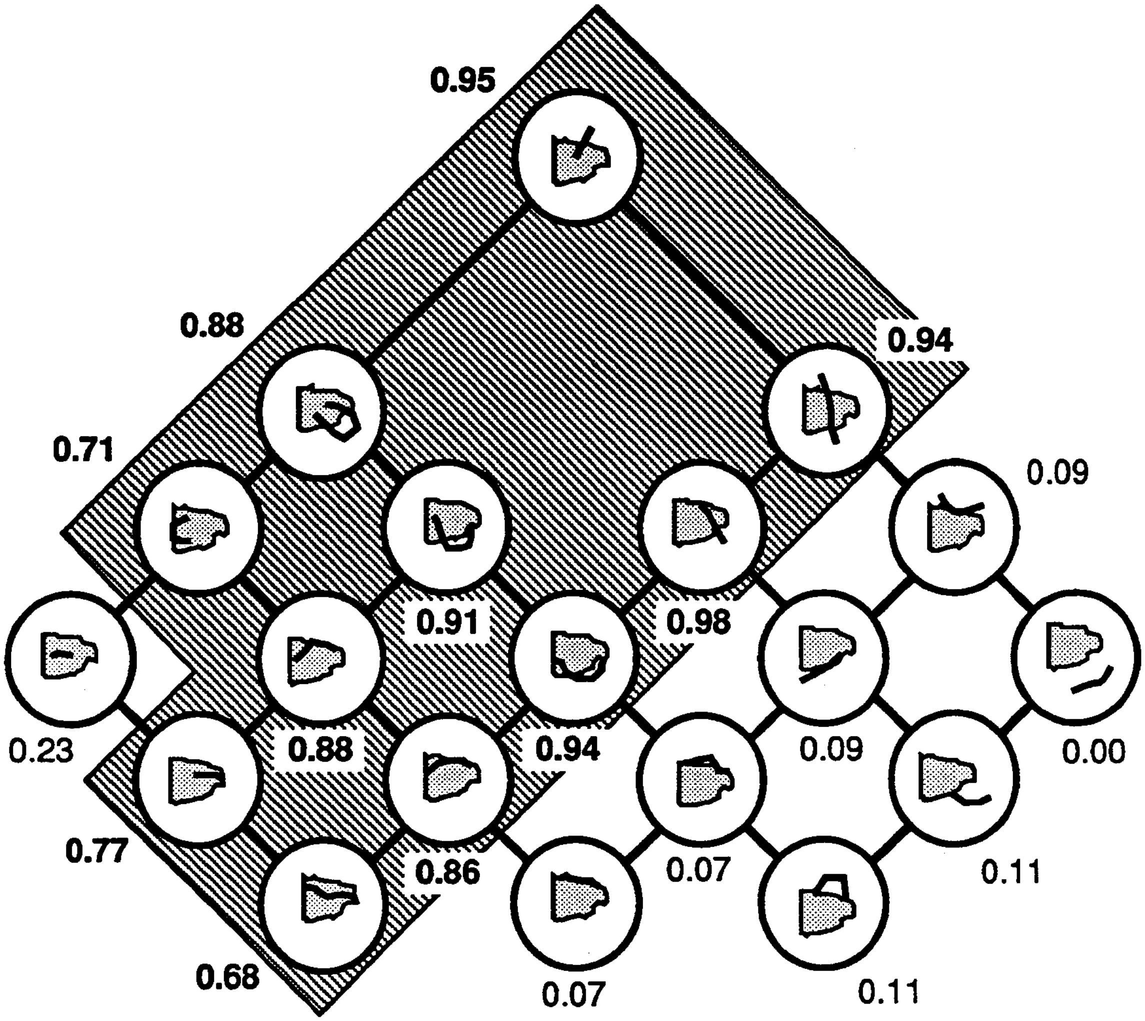
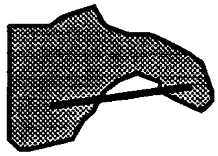


Figure 18. Strength of agreement that "the road goes into the park" by 9-intersection relation, averaged across 7 English-speaking subjects and 2 or 3 cases per relation. Situations with values above 0.5 are surrounded by shaded box.

Stimuli and Instructions Used in Mark and Egenhofer's Experiment 2

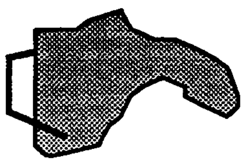
Each of the accompanying 40 diagrams presents a State park and a road. Please examine each map, decide how strongly you agree or disagree with the statement that in that case, "the road crosses the park", and mark your response on the scale from 1 to 5 under each diagram.



1

"The road <u>crosses</u> the park"

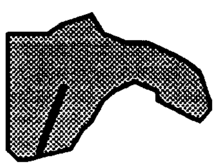
strongly disagree 1 2 3 4 5 strongly agree



2

"The road <u>crosses</u> the park"

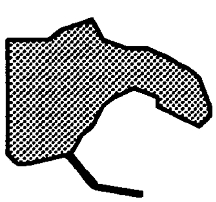
strongly disagree 1 2 3 4 5 strongly agree



3

"The road <u>crosses</u> the park"

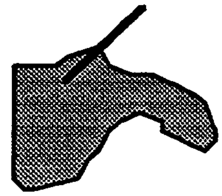
strongly disagree 1 2 3 4 5 strongly agree



4

"The road <u>crosses</u> the park"

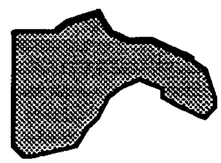
strongly disagree 1 2 3 4 5 strongly agree



5

"The road <u>crosses</u> the park"

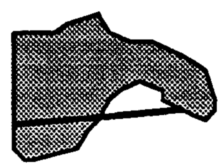
strongly disagree 1 2 3 4 5 strongly agree



6

"The road <u>crosses</u> the park"

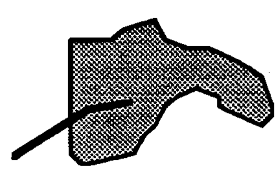
strongly disagree 1 2 3 4 5 strongly agree



7

"The road <u>crosses</u> the park"

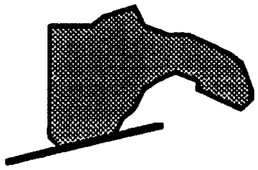
strongly disagree 1 2 3 4 5 strongly agree



8

"The road <u>crosses</u> the park"

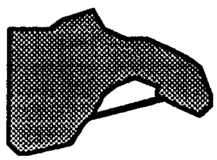
strongly disagree 1 2 3 4 5 strongly agree



9

"The road crosses the park"

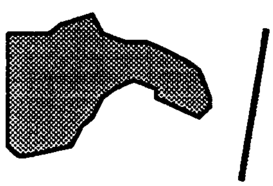
strongly disagree 1 2 3 4 5 strongly agree



10

"The road crosses the park"

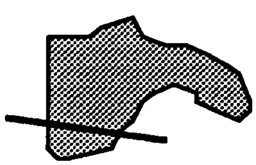
strongly disagree 1 2 3 4 5 strongly agree



11

"The road crosses the park"

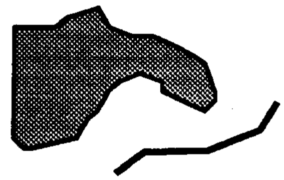
strongly disagree 1 2 3 4 5 strongly agree



12

"The road crosses the park"

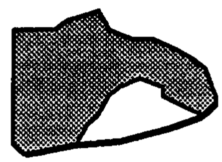
strongly disagree 1 2 3 4 5 strongly agree



13

"The road crosses the park"

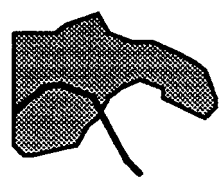
strongly disagree 1 2 3 4 5 strongly agree



14

"The road crosses the park"

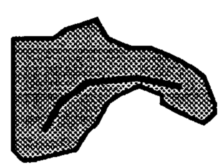
strongly disagree 1 2 3 4 5 strongly agree



15

"The road crosses the park"

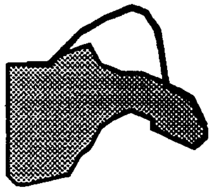
strongly disagree 1 2 3 4 5 strongly agree



16

"The road crosses the park"

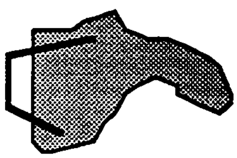
strongly disagree 1 2 3 4 5 strongly agree



17

"The road crosses the park"

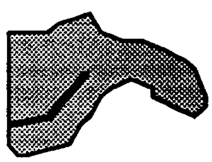
strongly disagree 1 2 3 4 5 strongly agree



18

"The road crosses the park"

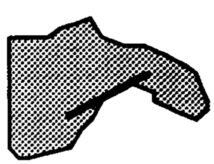
strongly disagree 1 2 3 4 5 strongly agree



19

"The road crosses the park"

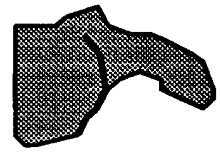
strongly disagree 1 2 3 4 5 strongly agree



20

"The road crosses the park"

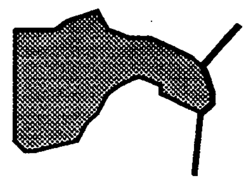
strongly disagree 1 2 3 4 5 strongly agree



21

"The road crosses the park"

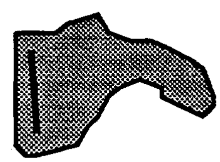
strongly disagree 1 2 3 4 5 strongly agree



22

"The road crosses the park"

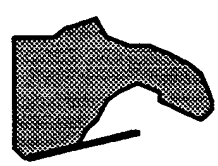
strongly disagree 1 2 3 4 5 strongly agree



23

"The road crosses the park"

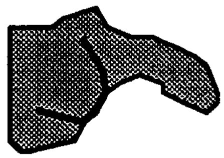
strongly disagree 1 2 3 4 5 strongly agree



24

"The road crosses the park"

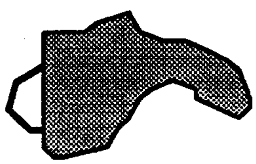
strongly disagree 1 2 3 4 5 strongly agree



25

"The road crosses the park"

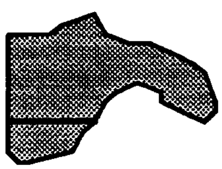
strongly disagree 1 2 3 4 5 strongly agree



26

"The road crosses the park"

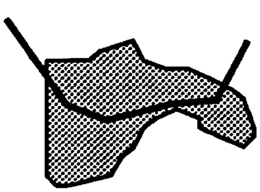
strongly disagree 1 2 3 4 5 strongly agree



27

"The road crosses the park"

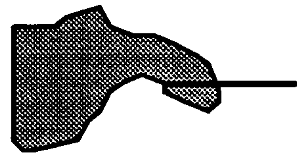
strongly disagree 1 2 3 4 5 strongly agree



28

"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree



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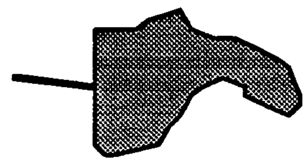
"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree



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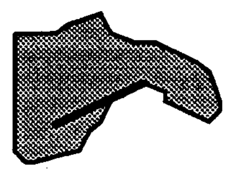
"The road crosses the park"

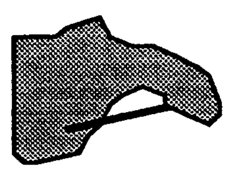
strongly disagree 1 2 3 4 5 strongly agree



31

"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree



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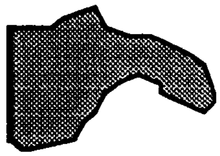
"The road crosses the park"

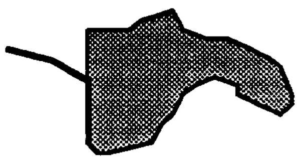
strongly disagree 1 2 3 4 5 strongly agree



33

"The road crosses the park"

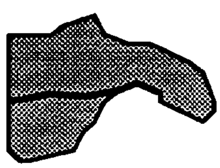
strongly disagree 1 2 3 4 5 strongly agree



34

"The road crosses the park"

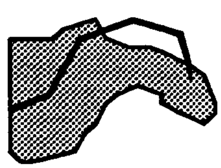
strongly disagree 1 2 3 4 5 strongly agree



35

"The road crosses the park"

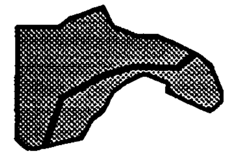
strongly disagree 1 2 3 4 5 strongly agree



36

"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree



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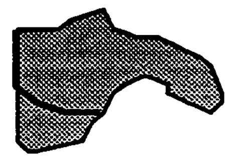
"The road crosses the park"

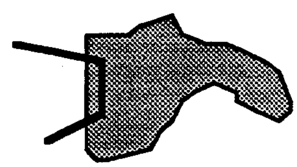
strongly disagree 1 2 3 4 5 strongly agree



38

"The road crosses the park"

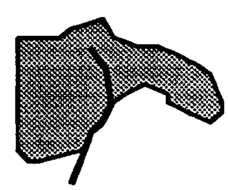
strongly disagree 1 2 3 4 5 strongly agree



39

"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree



40

"The road crosses the park"

strongly disagree 1 2 3 4 5 strongly agree

Stimuli and Instructions Used in some Subsequent Experiments by Mark and Egenhofer

Spatial Relations Survey: Instructions

This survey is part of a research project on spatial relations, being conducted by Dr. David Mark of UB's Geography Department. **<u>Your participation in this survey is completely voluntary</u>.** If you do not wish to participate, simply return the test booklet and answer form unmarked, either immediately, or when the forms are being collected. You may withdraw from the survey at any time without penalty. If you do decide to complete the survey, please take it seriously.

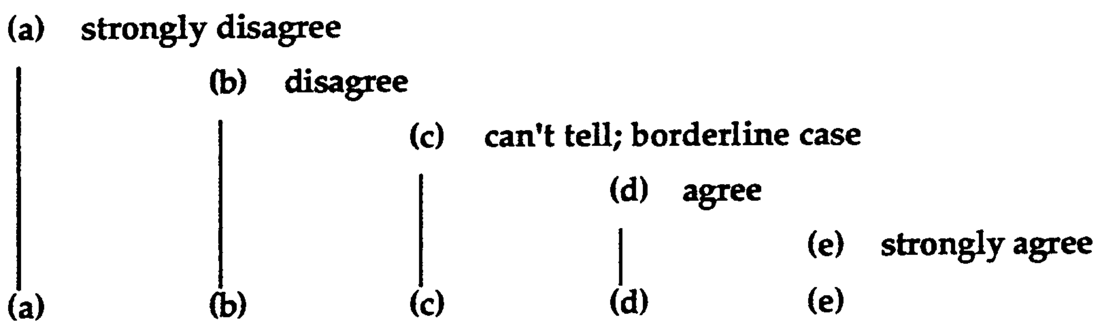
The surveys are anonymous; we only want your responses to the 60 diagrams in this booklet, plus some minimal background information. Please make all your responses on the computer-readable answer form provided; **<u>please do not mark the test booklet</u>.** The answer form should be filled out using a Number 2 pencil.

Before you begin the main part of the survey, please indicate the following items on side 2 of the answer form:

1. Please print your native ('first') language in the space marked 'name', and fill out the 'bubbles' under the letters accordingly.

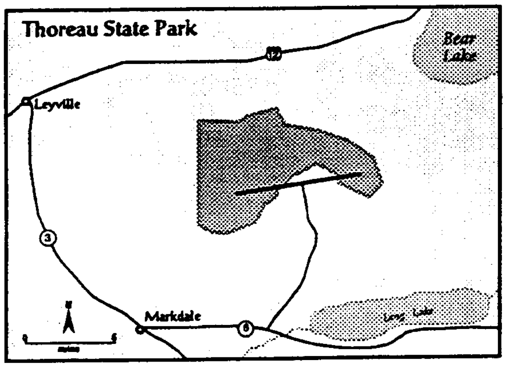
2. Please indicate your sex (male, female) and your month and year of birth, and fill out the 'bubbles' accordingly.

After filling out this background information, please begin the main survey. Examine each of the 60 maps, and determine how well you think the sentence printed at the top of side 1 of your answer form fits the spatial (geographic) relationship between the thicker dark road and the park. Your judgment should be on the scale of (a) "strongly disagree" to (e) "strongly agree":

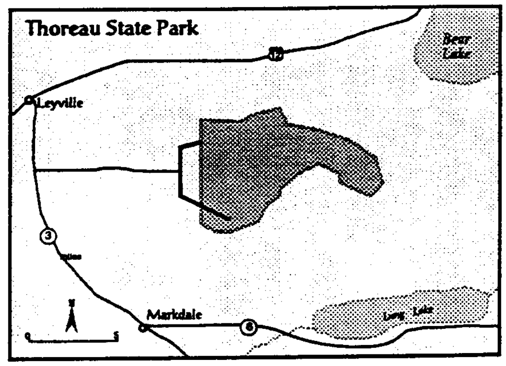


Please note that there are no 'correct' or 'wrong' answers, and that not everyone in the class has the same test sentence that you have.

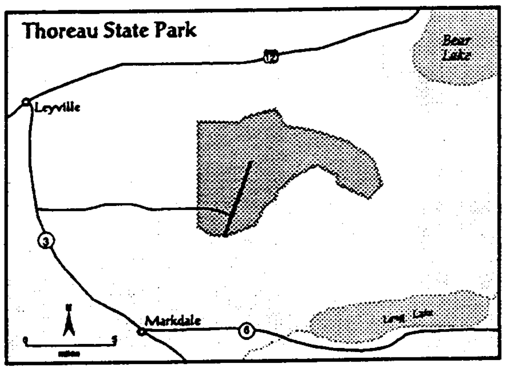
If you want to receive a copy of the results of this survey, please put your name and address on a piece of paper, and hand it in either when you hand in the survey sheet, or later.


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

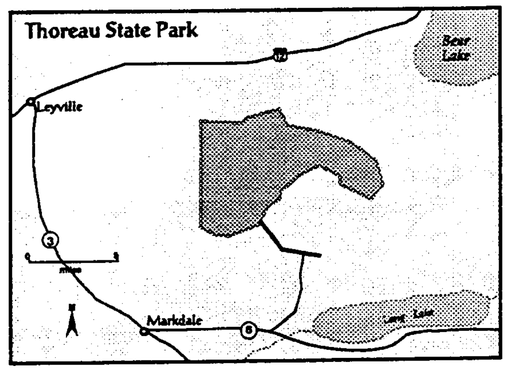
1


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

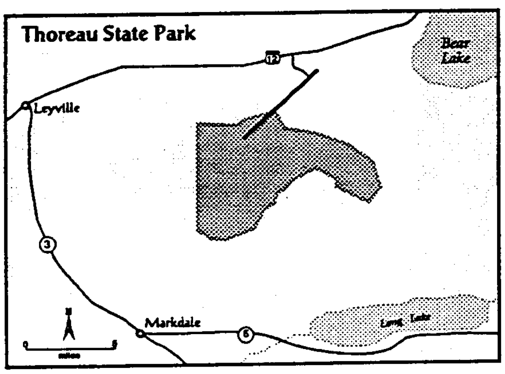
2


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

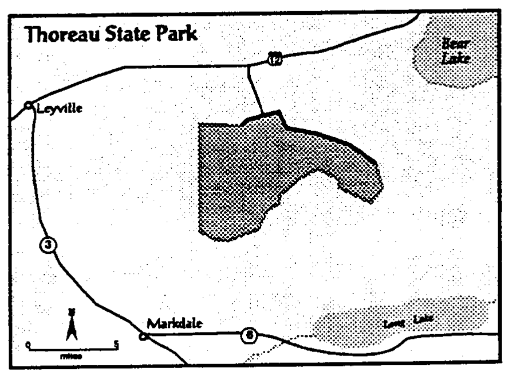
3


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

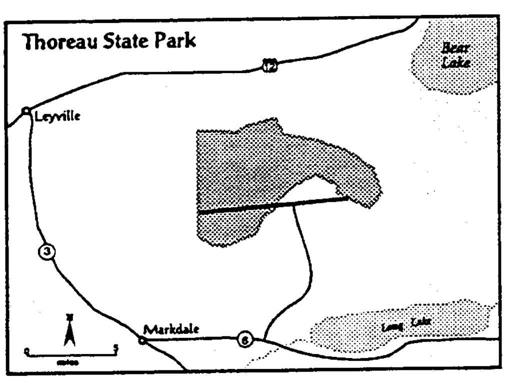
4


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

5


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
6

6


Thoreau State Park
Bear Lake
Leyville
12
3
Markdale
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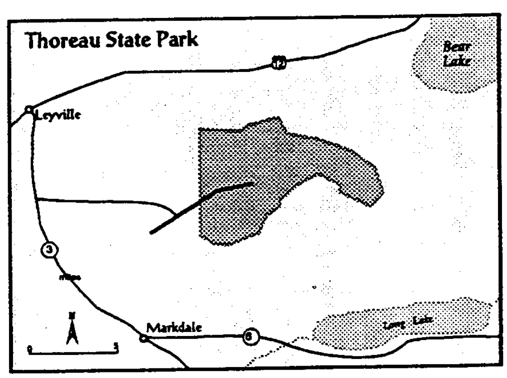
7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Leyville
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Markdale
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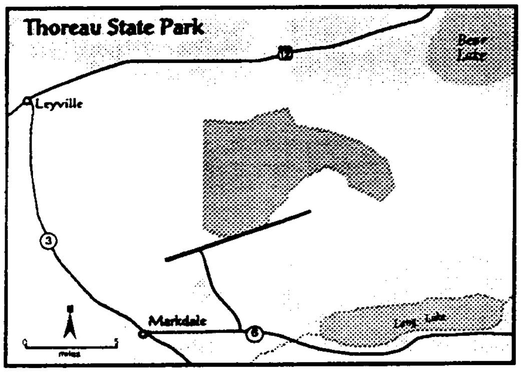

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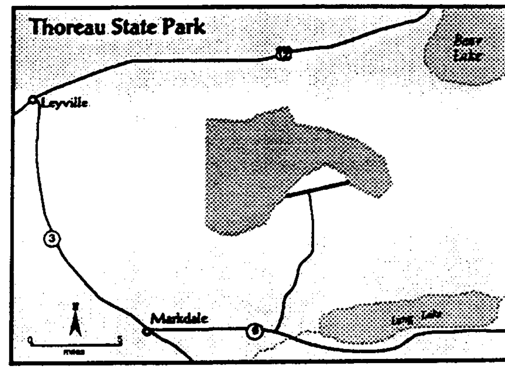

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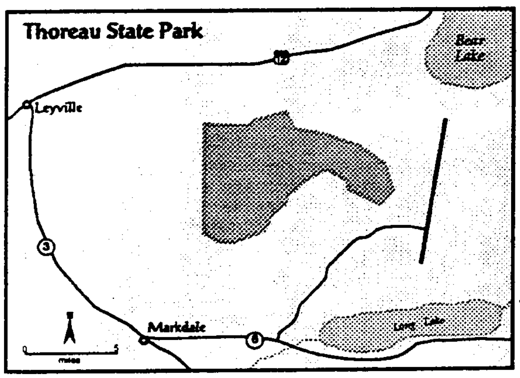

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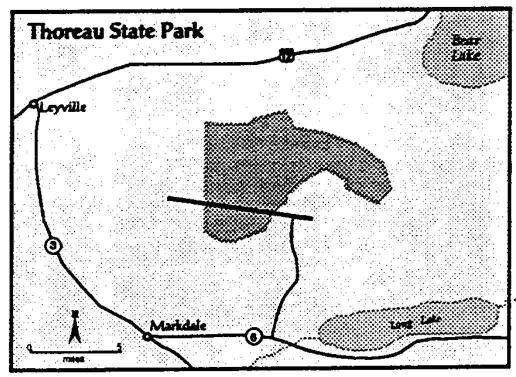

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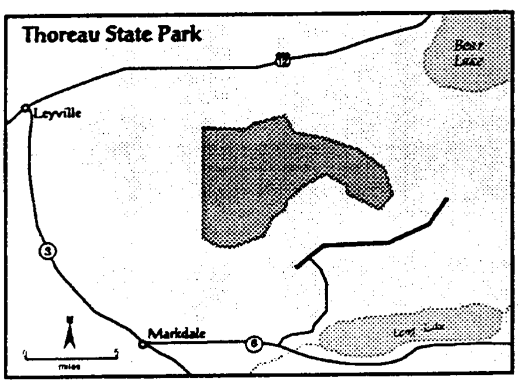

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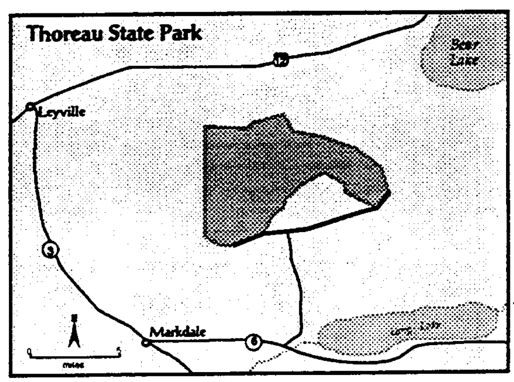

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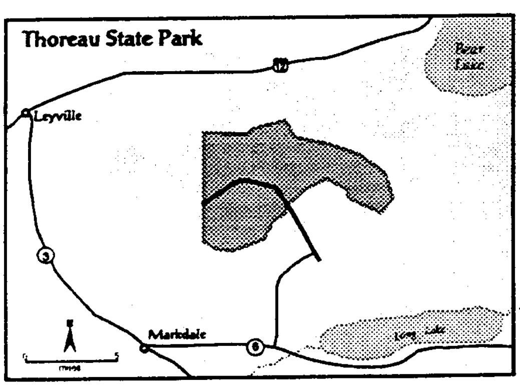

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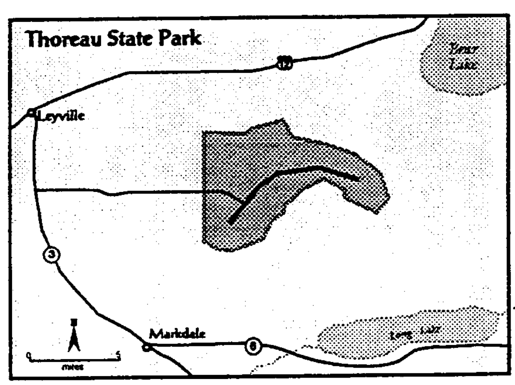

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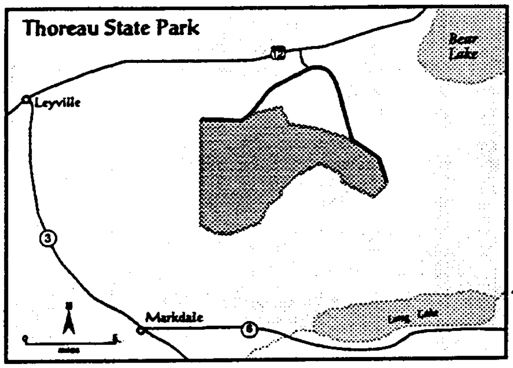

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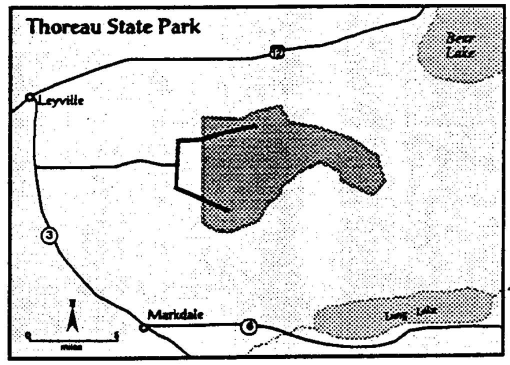

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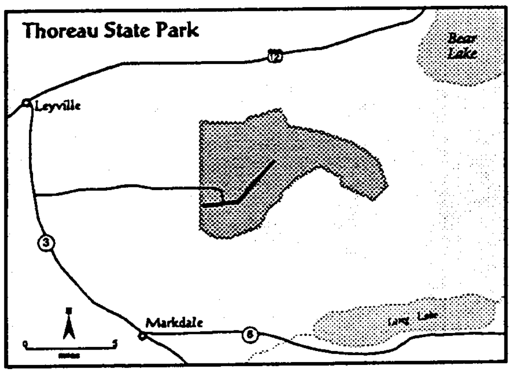

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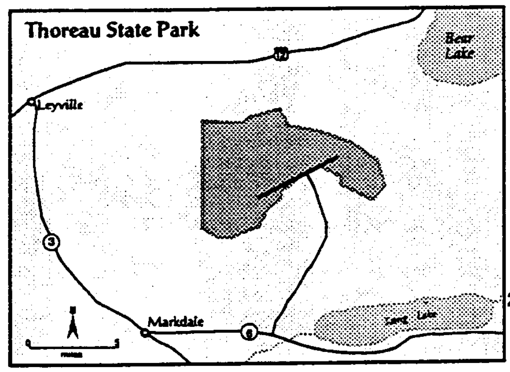

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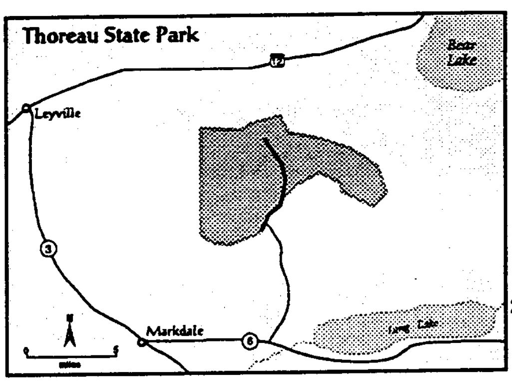

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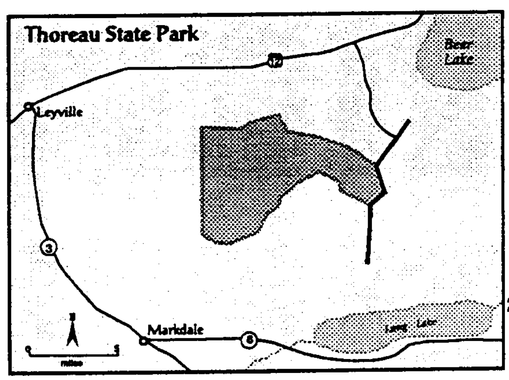

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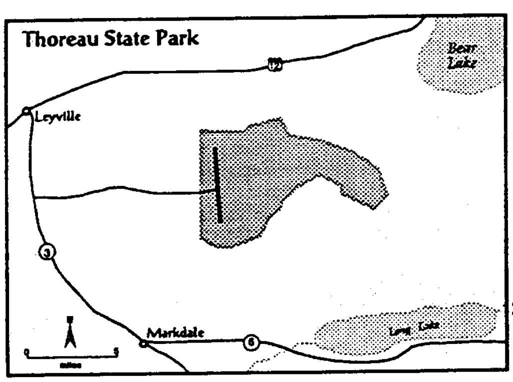

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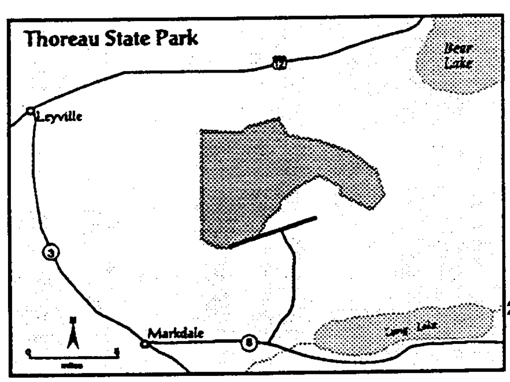

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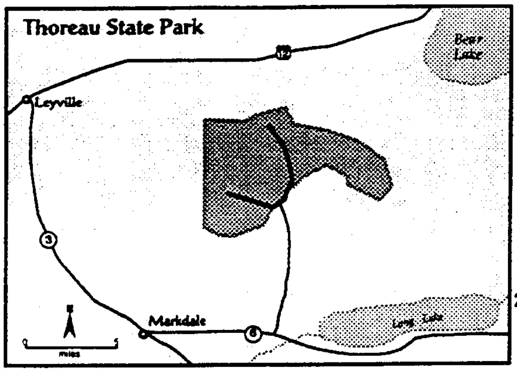

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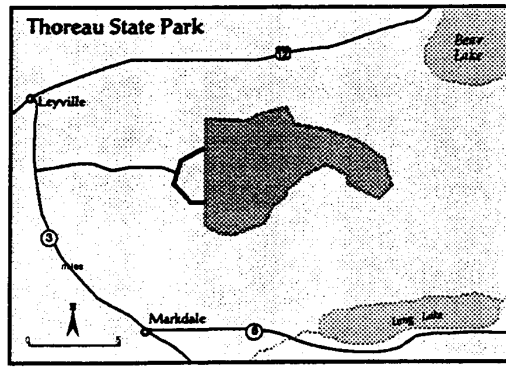

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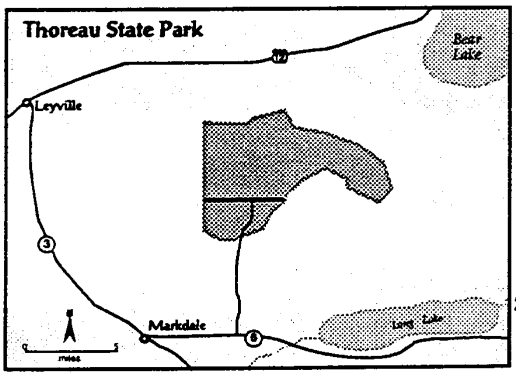

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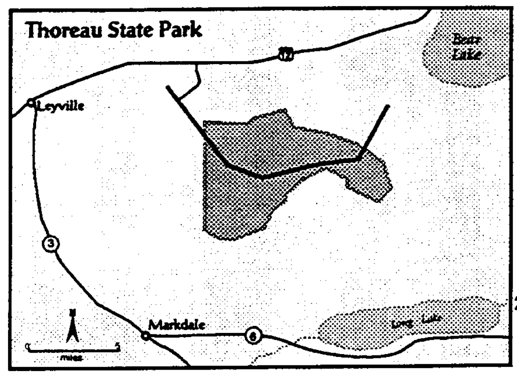

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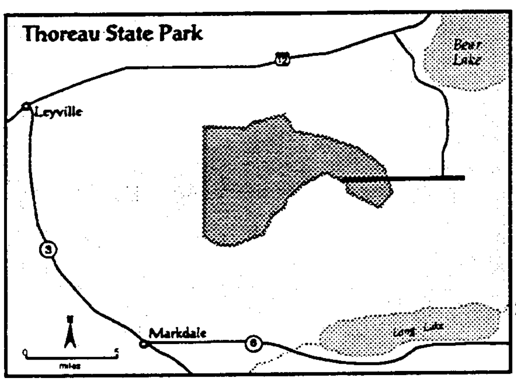

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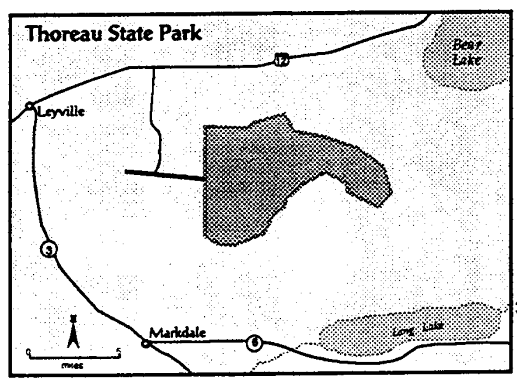

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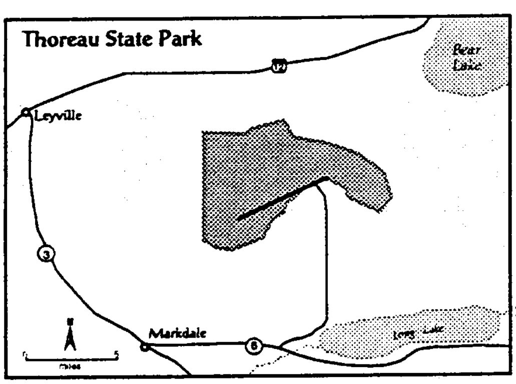

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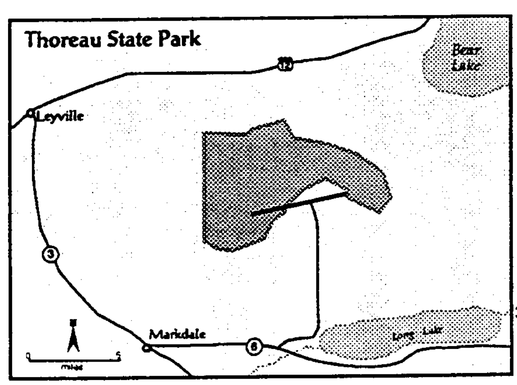

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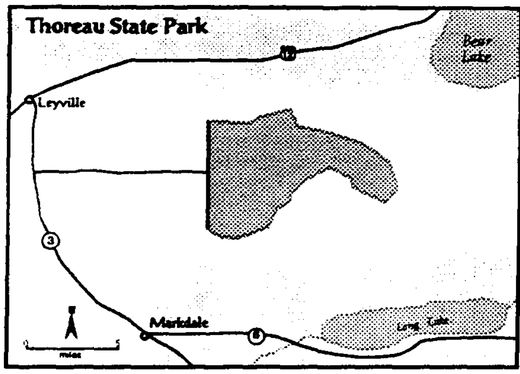

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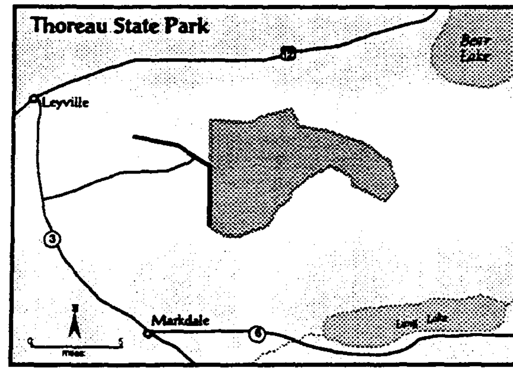

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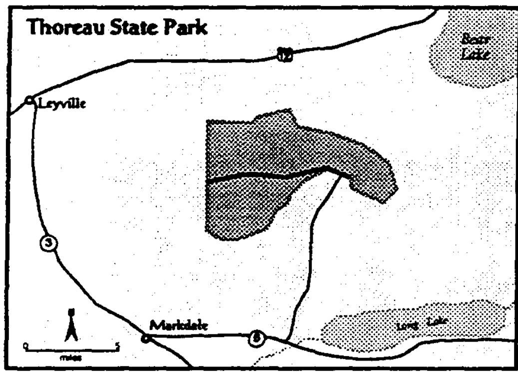

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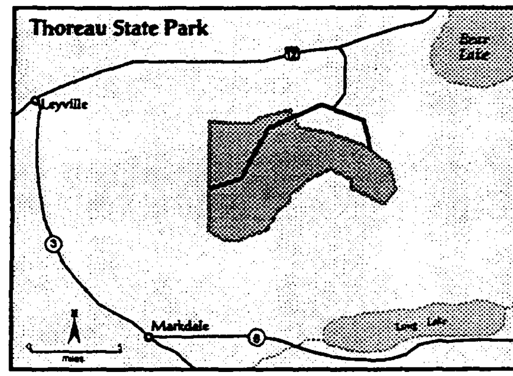

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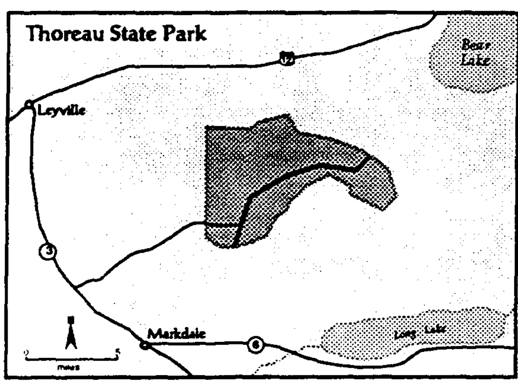

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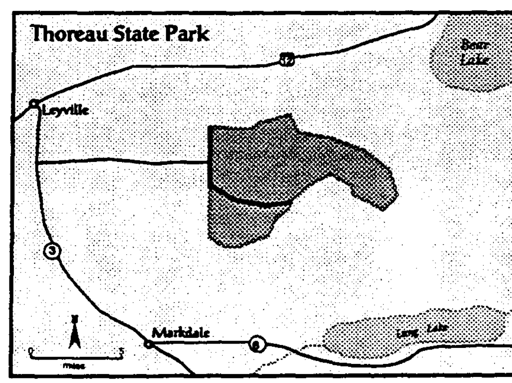

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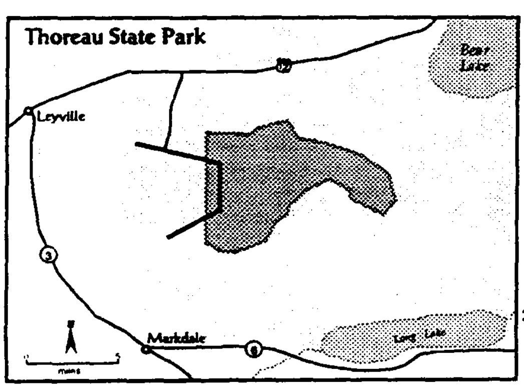

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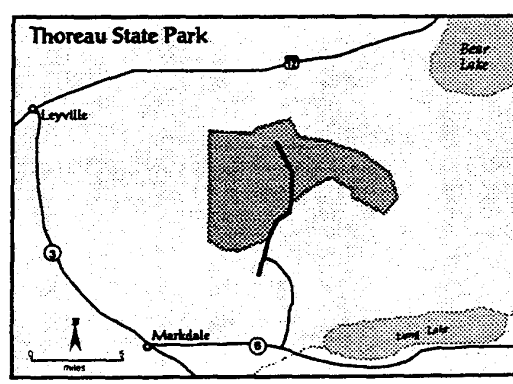

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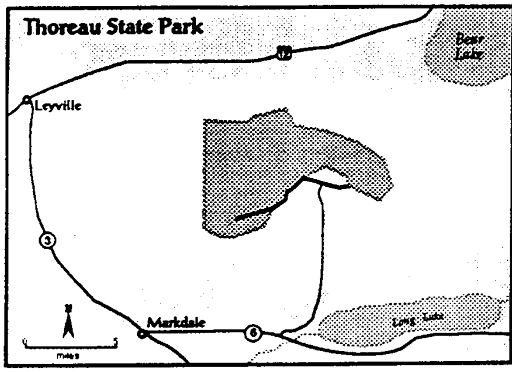

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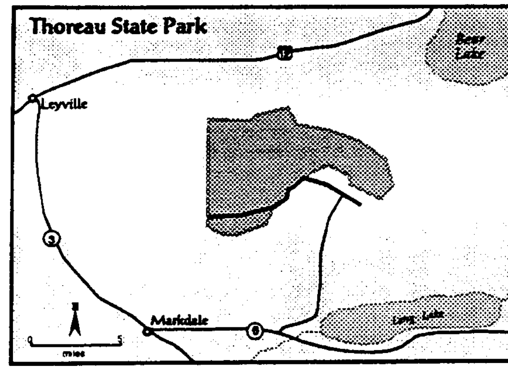

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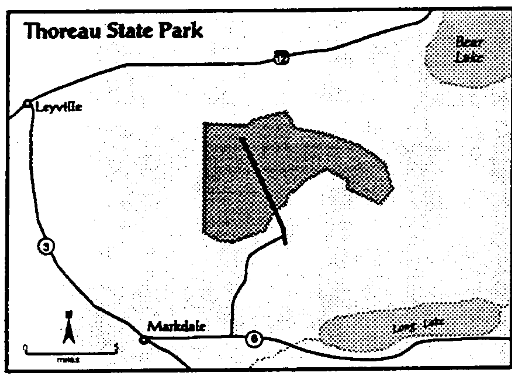

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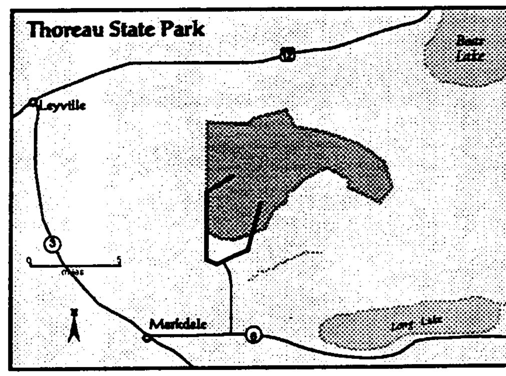

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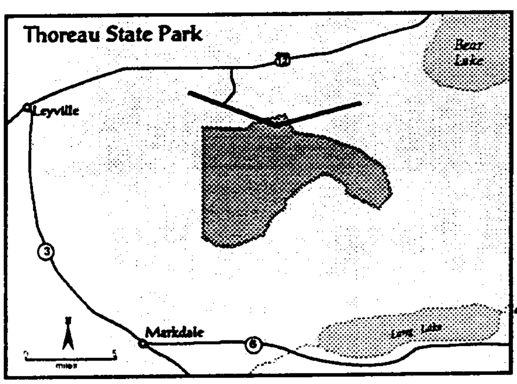

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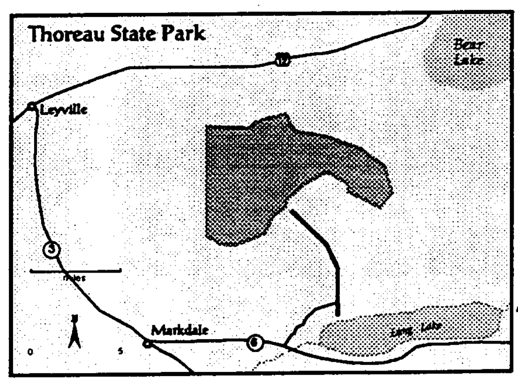

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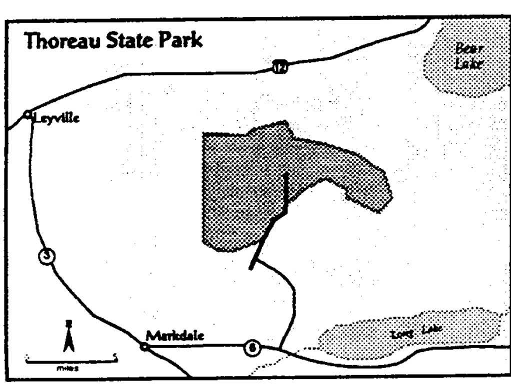

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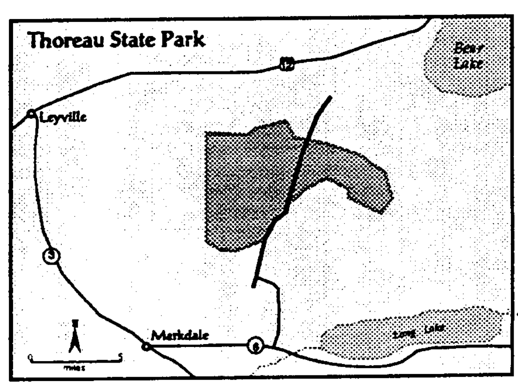

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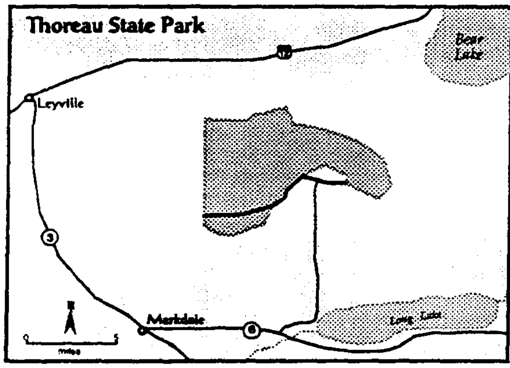

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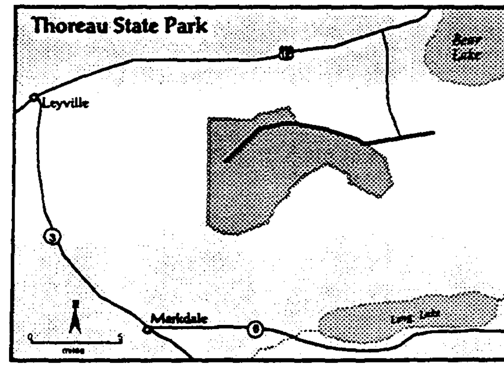

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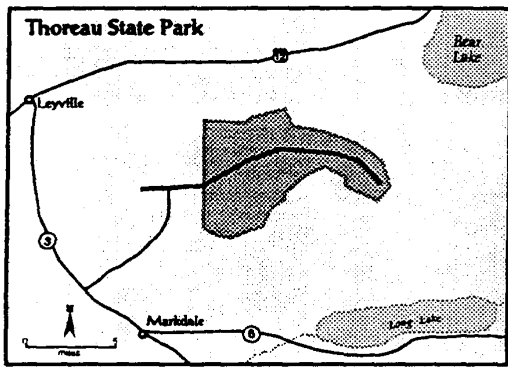

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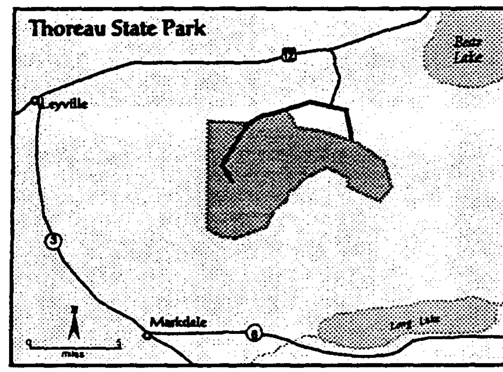

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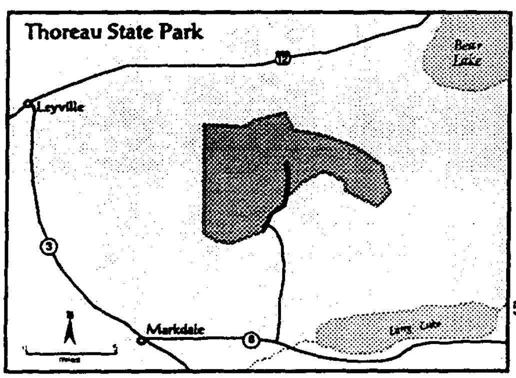

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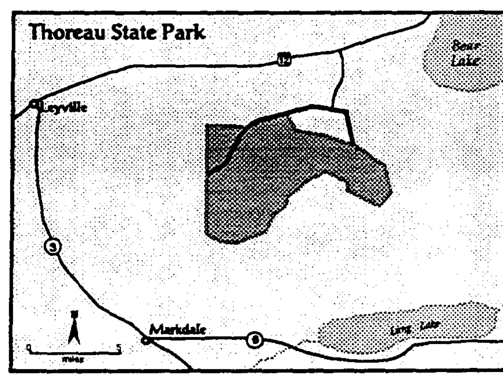

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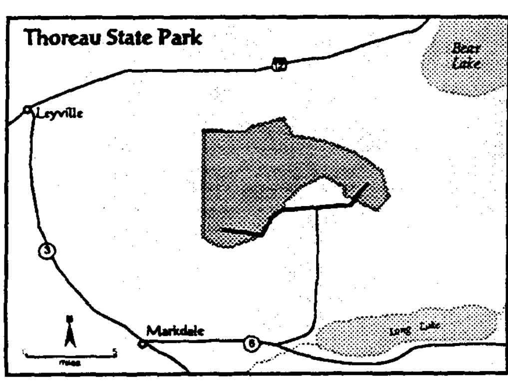

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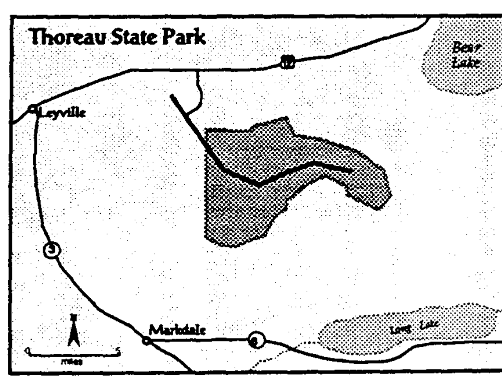

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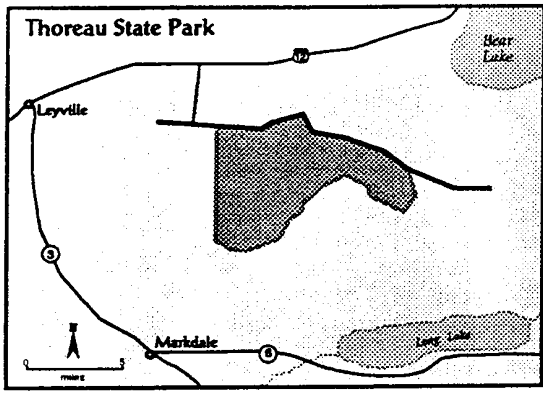
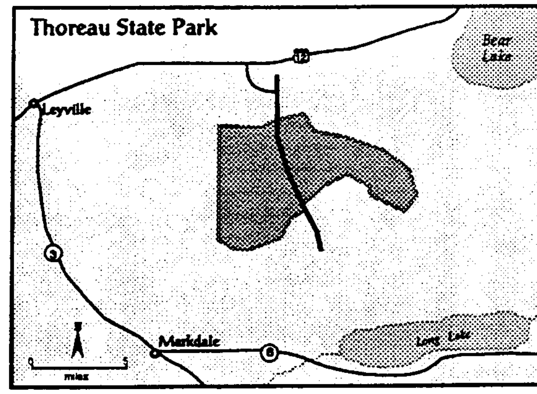

Thoreau State Park
Bear Lake
Leyville
Markdale
Long Lake

57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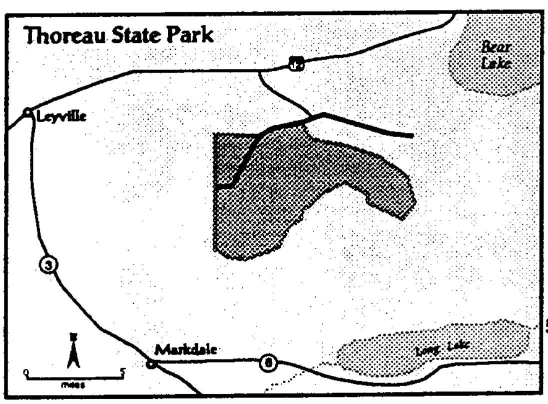
Markdale
Long Lake

58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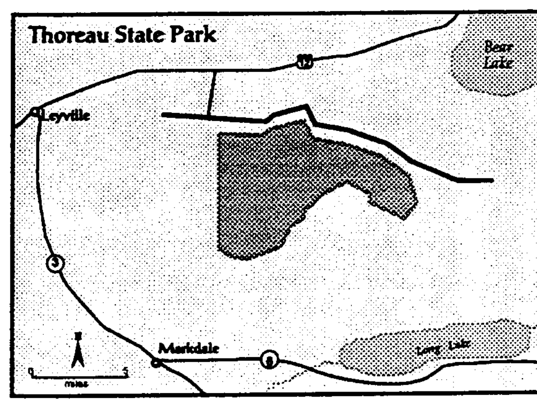
59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60