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THE ROLE OF TAXES AND ZONING
IN THE EFFECTIVE CONTROL OF POLLUTION

by

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and

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45 p

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The Role of Taxes and Zoning and Effective Management of Pollution

Abstract:

Previous analysis of pollution presumes separation between pollutees and polluters. In the model developed here, all possible combinations of housing and industry locations are allowed. In the resulting world of nonconvexities and multiple local optimum, Pigouvian taxes are generally insufficient. Accordingly, our analysis considers a wide range of policy instruments extending from Pigouvian taxes, double taxes, to zoning regulations. The results demonstrate that the management of pollution requires the recognition of two separate regimes determined by the type of convexity or concavity of the pollution dispersion function. When this function is convex, the optimal solution requires no zoning of housing. When the dispersion function is concave in emissions, the optimal allocation implies zoning into industrial and residential zones and, in some circumstances, taxes equal to total damages. To achieve effective management under limited information regarding the pollution dispersion function, it is argued that zoning restrictions cab be determined by trial and error through observation of changes in land rents.

Running Head: Effective Pollution Management

THE ROLE OF TAXES AND ZONING IN THE EFFECTIVE
CONTROL OF POLLUTION

Introduction

Two major local externalities are recognized in the literature: congestion
and pollution. Congestion is defined to be the external effect a participant
in a particular land use has on other participants in the same land use and
pollution to be the external effect a particular use of land has on other
types of land use. Because economic activity always involves the use of land
(even space flights need baseland), it follows that these two types of ex-
ternal effects exhaust all possible local external effects. As congestion
effects have been treated extensively in the literature (Hochman, 1982a,
1982b), this paper is dedicated to the problem of pollution externalities,
namely, the characterization of the optimal joint location and allocation of
resources of a polluting industry and residential housing in which workers of
the industry locate.

The extent of geographical proximity between polluter and pollutee greatly
affects the level of damages caused by the pollution. This implies two impor-
tant consequences: first, that spatial considerations cannot be ignored when
analyzing a pollution problem; and, second, that, unlike the case of congestion
(in which the ones causing and the ones suffering from the external effect are
the same), spatial separation between polluters and pollutees can be utilized
as a means to control pollution effects. In spite of these obvious implica-
tions, only in 1974 do we find the first attempt to model pollution effects in
a spatial model. A model of a polluting industry and its effects on adjacent
residents was constructed at that time by Tietenberg (1974a, 1974b). He

showed that, varying with location, Pigouvian (per unit of pollution) taxes levied on producers can insure optimality. The major limitation of Titenberg's analysis is that the locations of the economic activities in his model are predetermined.

Henderson extended Tietenberg's approach and allowed industry and housing to locate optimally but in a predetermined order of zoning, i.e., he assumed a single central business district (CBD) in which the polluting industry is located adjacent to and surrounded by a single continuous residential area in which the residents are both the employees and the pollutees of the CBD industry. Henderson's main finding was that, under these assumptions, a single Pigouvian tax is not sufficient to assure both optimality of the location and level of economic activities and of discharge of effluents by the polluting CBD. A lump-sum tax on each of the polluters is needed in addition to the differential Pigovian taxes to assure optimality of all these activities.

Strotz and Wright and Hochman (1978) also considered a polluting CBD surrounded by a residential zone. Both concentate on location and the latter, also, on intensity of land use in the residential ring. Both show, among other things, the possibility of a positive rent gradient in the neighborhood of the CBD; Hochman's work shows, also, the possibility of a buffer zone between the residential area and the CBD.

In Hochman and Ofek, the residential ring is collapsed into a single point, and attention is placed on the industrial area. Their paper confirms Henderson main result, namely, the insufficiency of the Pigovian tax. However, it proves that a single type of tax on polluters can still insure optimality--contrary to Henderson's double tax scheme--but this tax is not Pigouvian. It is, rather, a differential per unit land tax equal to the

marginal damages to pollutees contributed by the pollution produced at that unit of land. It is further shown that the per unit land taxes can be replaced by regulation of land use. The amount of emission per unit of land at a given location is not to exceed a predetermined (optimal) level of pollution. When this is done, efficiency is ensured. Furthermore, optimal pollution taxes are capitalized into land rents, and the local governments can obtain this income by using nondistorting land taxes. It is also argued that optimal regulations maximize long-run land values and thus provide the local government with a trial-and-error guidance to optimal land regulations.

In all of these papers, separation between residential pollutees and producers-polluters is assumed. Also assumed is the existence of a single zone of each type. Potential relative locations of industry and residence are numerous: the two land uses can occupy the same space with different densities in different zones, and they can be arranged in consecutive separate zones of varying sizes separated by buffer zones or any combination of the two. None of these possibilities has ever been considered, let alone tested for optimality. Thus, the potential use of relative location as an instrument in controlling pollution has not been investigated in the economic literature. This is one problem that this paper attempts to resolve.

The Pigouvian tax subsidy solution to externality problems has dominated the theoretical thinking of economists for many years. Pigou's own thinking about externality problems, however, was hardly dominated by the tax approach. In considering the external effects of location choices, for example, he states:

". . . thus it is coming to be recognized as an axiom of government
that, in every town, power must be held by some authority to limit
the quantity of building permitted to a given area--for the erection
of barrack buildings may cause great overcrowding of area even though
there be no overcrowding of rooms" (p. 194).

This situation, Pigou argues, results because "the interrelations of the
various private persons affected are highly complex, the government may find
it necessary to exercise some means of authoritative control in addition to
providing a bounty" (p. 194). Marshall had a more concrete suggestion quoted
with approval by Pigou, viz., "that every person putting up a house in a
district that has got as closely populated as is good should be compelled to
contribute towards providing free playgrounds" (Pigou, pp. 192 and 193). It
is interesting that neither Marshall nor Pigou recommended simply a tax solu-
tion for this type of externality. It is, indeed, apparent that several
alternative approaches exist.

Coase's seminal article stimulated a further examination of the tax-
subsidy approach, in general, and incidentally questioned the validity of the
single tax as a means of controlling externalities. Coase suggests:

". . . if the factory owner is to be made to pay a tax equal to the
damage caused, it would clearly be desirable to institute a double
tax system, and to make residents of the district pay an amount equal
to the additional cost incurred by the factory owner (or the consumers
of his products) to avoid the damage" (p. 41).

Baumol suggests that Coase is referring to the case of multiple optima.
For the case of a single optimum, Baumol argues that:

". . . the optimal price for the enternality-generating product is equal to the (Pareto-optimal) level of its entire social marginal cost, . . . while the optimal price for any item, . . . which generates no externalities is simply its marginal private cost" (p. 311).

That is, a single tax suffices.

Nonconvexities and multiple local optima have been increasingly recognized as an important issue in externality problems. The basic difficulty is that externalities can be associated with nonconvexities in preference or production sets, and these nonconvexities can lead to multiple tax equilibria. Treatments of the connection between externalities and nonconvexities include Portes; Baumol and Bradford; Baumol and Oates; Starrett and Zeckhauser; Kohn and Aucamp; and Gould. Baumol and Oates argue that spatial separation of the polluter and pollutee (or zoning) can only limit the magnitude of a nonconvexity, not prevent it. Page and Ferejohn argue that most damage functions are convex due to diminishing returns resulting from the assimilated capacity of the environment and biological defense mechanisms. In their framework, convex damage functions lead to interior solutions with efficient pollution levels that are neither zero nor very high, while concave functions are likely to give corner solutions that require benefit-cost analysis.

In the face of nonconvexities and spatial configurations, Pigouvian taxes may not always be enough. Henderson argued that a single Pigouvian tax is not sufficient and an additional lump sum tax is needed. Hochman and Ofek found that a single optimal tax exists in this case, but it is not a Pigouvian tax.

In different settings, a number of other authors have recommended double tax schemes of one sort or another. For example, treating the number of firms in the industry as endogenous, Carlton and Loury argue that both a Pigouvian tax and an annual license fee are required to achieve an optimum. Mohring and Boyd, following Coase, suggest that taxing both the polluter and pollutee is required to induce an equilibrium in which there is an incentive to the pollutee to relocate should changing circumstances make doing so globally efficient. They argue that taxing only the polluter will not necessarily provide this incentive.

Hochman and Hochman investigate the cumulative effects of pollution in a general equilibrium model with endogenous regeneration. They find that taxing polluters alone is not enough; taxes on pollutees' household size should be levied, as well, in order to control family size and, with it, the number of future pollutees. These results resemble and corroborate those of Mohring and Boyd's intertemporal partial equilibrium model. The question of taxes-- Pigouvan single or double or any other taxes--as policy instruments versus zonning and regulation of land use is another important topic of this paper.

In the model employed, we assume away everything which is not essential to characterize a polluting industry and its effects on nearby residents; thus, we assume a linear city. The production function is of constant returns to scale so that the only reason for agglomeration of housing and/or industry is pollution with its bad effects. Actually, when we assume away the ills of pollution in the text, the resulting layout of the city is a workplace and residence located together so that no commuting occurs and the city is uniform. On the other hand, all possible combinations of relative locations of housing and industry are allowed in the model. Also considered is a wide

range of policy instruments extending from Pigouvian taxes, double taxes, and zoning regulations.

Our main results are the following. Pollution should be divided into two types. Each type requires different policy instruments and results in a fundamentally different layout of the city. The factor which determines each type is the convexity or concavity of the pollution dispersion function in the variable level of emissions. When the function is convex, the optimal solution requires no zoning of housing, and production activities should take place in the same location. Pollution emissions should be controlled, and this can be achieved by the usual differential Pigouvian taxes. When the dispersion function is concave in emissions, the optimal allocation implies zoning into industrial and residential zones--possibly with buffer zones between them. The size of the zones and their number depend, other things being equal, on the level of concavity, i.e., on the absolute value of the second derivative of the dispersion function with respect to emissions. For example, when this derivative is zero, each zone is of infinitesimal width, and their number is infinite so that the two activities are separated by an infinitesimal distance and are, therefore, practically mixed. Hence, this border case fits both the concave and the convex solutions. In a proper concave case, Pigouvian taxes do not achieve optimal location; instead, a per unit land tax equal to the marginal damages caused by the pollution produced by this unit of land in the optimum is needed.[1] This particular tax will achieve both optimal emissions by and optimal locations of the industry and will lead to the optimal zoning as well; however, to levy the correct tax, information about the dispersion function is needed. When an optimal land tax is needed and Pigouvian taxes are used, efficiency is not achieved and vice

versa. This is in addition to the fact that information on utility functions or damage functions is now known as well; therefore, we cannot ever hope to be able to calculate the correct taxes.

An alternative approach is to regulate land use directly and use zoning as policy instruments having in mind the goal of maximizing land values by trial and error. Details of this recommended approach can be found in Hochman and Ofek and in Hochman (1982a, 1982b).

Our analysis begins with the specification of the model in the second section. After deriving the optimal and decentralized solutions in the third section and characterizing the solution in the fourth section, we end with the implications of our results for effective pollution control policies. In Appendix A the proof that concave dispersion functions lead to zoning is provided, and in Appendix B additional necessary conditions for this case are derived. Appendix C provides the detailed derivation of the necessary conditions for the general case.

Model Specification

Assume a linear strip of unit width and a given length, L, beginning at the origin and stretching in the horizontal positive direction. If x designates distance from the origin, then $x = 0$ and $x = L$ are the two ends of this strip of land. Define

$a(x) f[n(x), e(x)]$ = total production per unit of land at x where $a(x)$ is the ratio of land in the zone occupied by the industry

$n(x)$ = number of workers per unit of land at x

$e(x)$ = amount of emissions of the industry per unit of land at x.

The function f is assumed to be an increasing function of its arguments but at a decreasing rate; specifically, f is a linear, homogeneous production function.

A linear, homogeneous production function will allow the identification of pollution effects under varying conditions rather easily. The layout of the city under the constant returns to scale assumption and without undesirable pollution effects is straightforward; hence, any distortion of this simple pattern when we introduce pollution is entirely due to the latter effect. This is also the reason why city size L is assumed rather than determined exogneously.

We also assume a homogeneous population with free and costless mobility in the economy and, hence, a fixed utility level everywhere, i.e.,

(1) $$U[h(x), Z(x), C(x)] = U_0$$

where

  $U(\cdot)$ = utility function

  $h(x)$ = amount of housing consumed by the household at x and produced
    there from land only

  $Z(x)$ = amount of composite good consumed by the household at x (the
    price of Z is assumed to be a unit)

  $C(x)$ = concentration of pollution at x which results from emissions
    of the industry throughout the city.

The utility function, $U(\cdot)$, is assumed to be quasi concave in h, Z, and $(-C)$.

Define the positive direction of x as north and the opposite direction as south. The level of $C(x)$ is determined by the emissions in all areas as described by the following:

(2) $C(x) = \int_0^x D^N[e(y)a(y), x - y]dy + \int_x^L D^S[e(y)a(y), y - x]dy.$

The functions $D^i(e, y)$, $i = S$ or N, fulfill, for $e, y > 0$,

(3) $\dfrac{\partial D^i(e, y)}{\partial e} = D_1^i > 0,$

$\dfrac{\partial D^i(e, y)}{\partial y} = D_2^i < 0,$

and

$\dfrac{\partial^2 D^i}{\partial y^2} < 0.$

The functions $D^i[e(x)a(x), |x - y|]$ are dispersion functions which convert pollution emitted at x, $e(x)$ to its contribution to pollution concentration at y. This type of function was first introduced into the theoretical literature by Tietenberg (1974a, 1974b). Note that we allow for the possibility of different dispersion effects between different directions (north and south) by assuming two different functions, D^N and D^S, for each direction from the emission site.

Let $T(x)$ be the number of workers traveling from home northward crossing x minus the number of workers crossing x traveling from home to work southward. Note that $T(x)$ can be positive or negative. The contribution to $T(x)$ at location x is the number of residents at x, $b(x)/h(x)$, minus the number of workers, $a(x)n(x)$, where $b(x)$ is the proportion of land at x occupied by housing. Thus,

(4a) $\dot{T}(x) = \dfrac{b(x)}{h(x)} - a(x)n(x),$

where a dot above the function indicates differentiation with respect to distance or, alternatively,

(4b)
$$T(x) = \int_0^x \left[\frac{b(y)}{h(y)} - a(y)n(y) \right] dy$$

with

(4c)
$$T(0) = 0$$

(4d)
$$T(L) = 0.$$

The last condition, (4d), implies that the total number of households in the city also equals the total number of city workers. This is so since we assume that each household contributes a single worker to the labor force. The relevant land-utilization constraints are:

(5a)
$$a(x) + b(x) - 1 < 0$$

(5b)
$$a(x) \geq 0 \qquad b(x) \geq 0.$$

When these constraints are not effective, it means that at least some land at x is vacant.

The variable $T(x)$ represents the number of commuters crossing x northbound daily. This follows since each worker will try to minimize his travel costs. Note that $T(x)$ also equals the number of households minus the number of workers south of x. Finally, let V be the cost of commuting a unit distance. We also assume the city is located along a road so that the price of the export good is independent of location. Let P be the price of the export good produced in the city.

Given the above definitions, the net city surplus (which is also the net city export) (see Hochman, 1981) is given by

$$(6) \qquad \pi = \int_0^L \left[Paf(n, e) + \frac{b}{h} (I - Z) - |T| \cdot V \right] dx$$

where I is the nonearned income of a household. Note that, for simplicity of notation, the variable x is omitted whenever there is no risk of confusion. Maximization of π subject to (1), (2), (4a), and (5) with (4c) and (4d) as terminal conditions is a necessary condition for a Pareto optimum in the economy as a whole (see Hochman, 1981)[2]. To solve this maximization problem, define the function

$$(7) \qquad \text{Sign } x = \begin{cases} +1 \text{ iff } x > 0 \\ 0 \text{ iff } x = 0 \\ -1 \text{ iff } x < 0. \end{cases}$$

Sign x is differentiable everywhere except at x = 0. If we set $[d(\text{sign } x)/dx]_{x = 0} = 0$, then, for all x, $d(\text{sign } x)/dx = 0$; and we have

$$(8) \qquad \begin{aligned} |T(x)| &= [\text{sign } T(x)] \cdot T(x) \\ \frac{d[|T(x)|]}{dT(x)} &= \text{sign } T(x). \end{aligned}$$

The Optimal and the Decentralized Solutions

The necessary conditions for the optimization problem are (see Appendix C for details of derivation)

$$(9) \qquad a(x) Pf_1[n(x), e(x)] - a(x) \psi(x) = 0$$

$$(10) \qquad \begin{aligned} a(x) Pf_2 - a(x) \Big\{ &\int_x^L \eta(y) D_1^N[a(x)e(x), y - x]dy \\ &+ \int_0^x \eta(y) D_1^S[a(x)e(x), x - y]dy \Big\} = 0, \end{aligned}$$

where $\psi(x)$ is the costate of the state variable $T(x)$; $\eta(y)$ is the shadow price of the pollution concentration constraint (2)[2].

(11)
$$Pf[n(x), e(x)] - \psi(x)n(x) - e(x) \left\{ \int_x^L \eta(y) \, D_1^N[a(x)e(x), y - x]dy \right.$$
$$\left. + \int_0^x \eta(y) \, D_1^S[a(x)e(x), x - y]dy + \gamma(x) - \rho(x) \right\} = 0$$

where $\gamma(x)$, $\mu(x)$, and $\rho(x)$ are the shadow prices of (5a) and (5b), respectively, and fulfill[3]

(12a) $\qquad \gamma(x) \geq 0 \qquad \gamma(x)a(x) = 0$

(12b) $\qquad \mu(x) \geq 0 \qquad \mu(x)b(x) = 0$

(12c) $\qquad \rho(x) \geq 0 \qquad \rho(x)[1 - a(x) - b(x)] = 0$

(13) $\qquad b(x)\left[I(x) + \psi(x) - Z(x) - \frac{U_h}{U_z} h(x) \right] = 0$

(14) $\qquad \eta(x) = -\frac{b(x)}{h(x)} \frac{U_c(x)}{U_z(x)}$

(15) $\qquad \frac{U_h}{U_z} + \mu(x) - \rho(x) = 0$

(16) $\qquad \dot{\psi}(x) = V \cdot \text{sign}[T(x)].$

From equation (9), we learn that $\psi(x)$ at locations where the industry is located [i.e., $a(x) > 0$] equals the value of the marginal productivity of labor and, therefore, represents the wage rate in the decentralized market solution. From equation (14), we learn that $\eta(x)$ is equal to the marginal damages caused to the residents occupying a unit area at x by concentration of pollution at this location.

To achieve efficiency, equation (10) suggests that, at each location where the industry operates [$a(x) > 0$], the industry must emit pollution up to the level at which the value of the marginal productivity of pollution emissions equals the marginal damages caused by this pollution and where those marginal damages are given by

(17)
$$M(x) = \int_x^0 \eta(y) \, D_1^N[a(x)e(x), y - x]dx$$
$$+ \int_0^x \eta(y) \, D_1^S[a(x)e(x), x - y]dy.$$

To fulfill equation (10) for the decentralized case, government intervention is needed. One form of this intervention is a per unit emission tax equal to $M(x)$ levied on the industry at x. $M(x)$ is the well-known Pigouvian tax. Other forms of intervention are possible; they will be developed later.

Let us define $R^I(x)$ as the bid rent function of the industry. It is generated by

(18) $$R^I(x) = Pf[n(x), e(x)] - \psi(x)n(x) - e(x)M(x),$$

where $n(x)$ and $e(x)$ are determined so that they fulfill conditions (9) and (10). Hence, $R^I(x)$ is the maximum amount the industry can pay for land at x without suffering losses [provided that $M(x)$ is imposed as an emission tax].

Let $R^h(x)$ be the households' bid rent function for land at x, i.e.,

$$(19) \qquad R^h(x) = \frac{U_h}{U_z},$$

where h and z are determined so that equations (1) and (13), with $b(x) > 1$, are satisfied. As before, $R^h(x)$ is the maximum amount the households are willing to pay per unit of housing (land).

Equations (11) and (12) imply that

$$(20) \qquad \begin{aligned} R^I(x) &\leq \rho(x) \\ R^I(x) &= \rho(x) \iff a(x) > 0. \end{aligned}$$

From equations (12) and (15), we also note that

$$(21) \qquad \begin{aligned} R^h(x) &\leq \rho(x) \\ R^h(x) &= \rho(x) \iff b(x) > 0. \end{aligned}$$

In a decentralized competitive solution, $\rho(x)$ is the land rent; equations (20) and (21) actually imply that an activity (of production or consumption) will take place at a given location if, and only if, the bid rent function equals the land rent. Furthermore, this equality also determines the value of ρ, i.e.,

$$(22) \qquad \rho(x) = \max [0, R^h(x), R^I(x)].$$

Equation (13), with $b(x) > 0$, serves as a budget constraint. Note that $\psi(x)$ now stands for the earned income the household has at the residence net of commuting costs. If the industry is operating at x, this is also the wage

rate at x. However, if the industry does not operate at x, then $T(x) \neq 0$ and, therefore, equation (16) implies that the wages actually received by the people are higher by the commuting costs to the nearest location where an industry exists. It should also be noted that wages need not be equal even at locations where industry operates since different levels of pollution and land rents require different total earnings needed to insure the predetermined level of utility.

Characterization of the Solution

The solution can be characterized for two alternative assumptions concerning the dispersion function. These assumptions describe two extreme cases. All other cases can be described as combinations of these two basic alternatives.

First assume that $D^i(e, y)$ is convex in e [i.e., $D^i(e + \Delta e, y) > D^i(e, y) + D_1^i(e, y) \Delta e$]. This implies that $D_{11} > 0$. In this case, the second-order conditions of an internal solution of the problem exist. Hence, we have

$$(23a) \qquad a(x) > 0 \qquad 0 \leq x \leq L$$

$$(23b) \qquad b(x) > 0 \qquad 0 \leq x \leq L.$$

Furthermore, we also have

$$(24) \qquad a(x) + b(x) = 1 \qquad 0 \leq x \leq L.$$

The reason for the last equality follows from both the concavity of the production function which makes it worthwhile to expand over costless space and the quasi concavity of the utility function which makes it worthwhile to expand housing over costless space.

Equations (23a), (23b), and (24) imply that, in all locations, both residents and housing will coexist. It should be noted that this case obtains when $U_C = 0$ independently of the sign of D_{22}. In this last case, $T(x) = 0$ everywhere, i.e., workers reside next to their working place and do not commute.

For the case of $U_C < 0$, assume that the effect of the pollution disappears after a certain distance, S_0, that is,

(25) $\qquad D^i(e, y) = 0$ for $y > y_0$ and all e $\qquad i = S, N.$

For this case, we restrict ourselves to the ordinary local cases of pollution and not to activities which may affect very distinct locations.

Let the Golden Path be the constant value solution of the set of equations, i.e., the solution which satisfies

(26) $$\dot{C} = \dot{T} = \dot{\psi} = 0.$$

Under these circumstances, we note from equation (16) that sign $T(x) = 0$; hence,

(27) $$T(x) = 0.$$

Thus, as in the case of no pollution ($U_C = 0$), workers do not commute but locate next to their working place. The industry continues to pay the Pigouvian pollution tolls and, therefore, limits the amount of pollution it emits. The number of workers, emissions, and concentration levels are the same everywhere along the Golden Path; if the city would have stretched indefinitely in both directions, this Golden Path would solve the set of equations.

In figure 1, this result is depicted when the terminal conditions (4c) and (4d) are taken into account and $D^N(e, y) = D^S(e, y) = D(e, y)$. We see that, near the edge of town, emissions come from only one direction; thus, the air is cleaner and housing rents are higher. For this region, households will outbid industry. Since rents are higher, housing size is smaller. The industry which does locate at the edge of town will pay lower wages and lower pollution taxes (because pollution affects fewer households). Therefore, land use will be more intensive--that is, both n and e are higher. As rents decrease, the industry occupies more land and becomes less intensive and housing occupies less land and also becomes less intensive. At a certain distance, the industry occupies more land and pollutes more than over the Golden Path.

The land-use structure described in figure 1 is not usually observed in the real world. This is presumably because economies of scale, which tend to agglomerate the industry in a single location, are disregarded in this study. The framework also disregards the fact that the dispersion function is not symmetric. If, for example, $D^S(e, y) = 0$, then industry would be highly dense in the south with housing in the north and the Golden Path in between as sketched in figure 2. This land-use pattern is much more familiar where the northern region consists mainly of housing (and clean industries) and the southern region consists mainly of hardy polluting industries.

Assume now that D^i is concave in emissions [i.e., $D^i(e + \Delta e, y) < D^i(e, y) + \Delta e D_1^i(e, y)$]. In Appendix 1, it is shown that this condition is sufficient for a zoning solution, i.e., the optimal allocation implies a division into areas, each containing exclusively a single land use--either housing or industry; between any two such zones, buffer zones may exist over
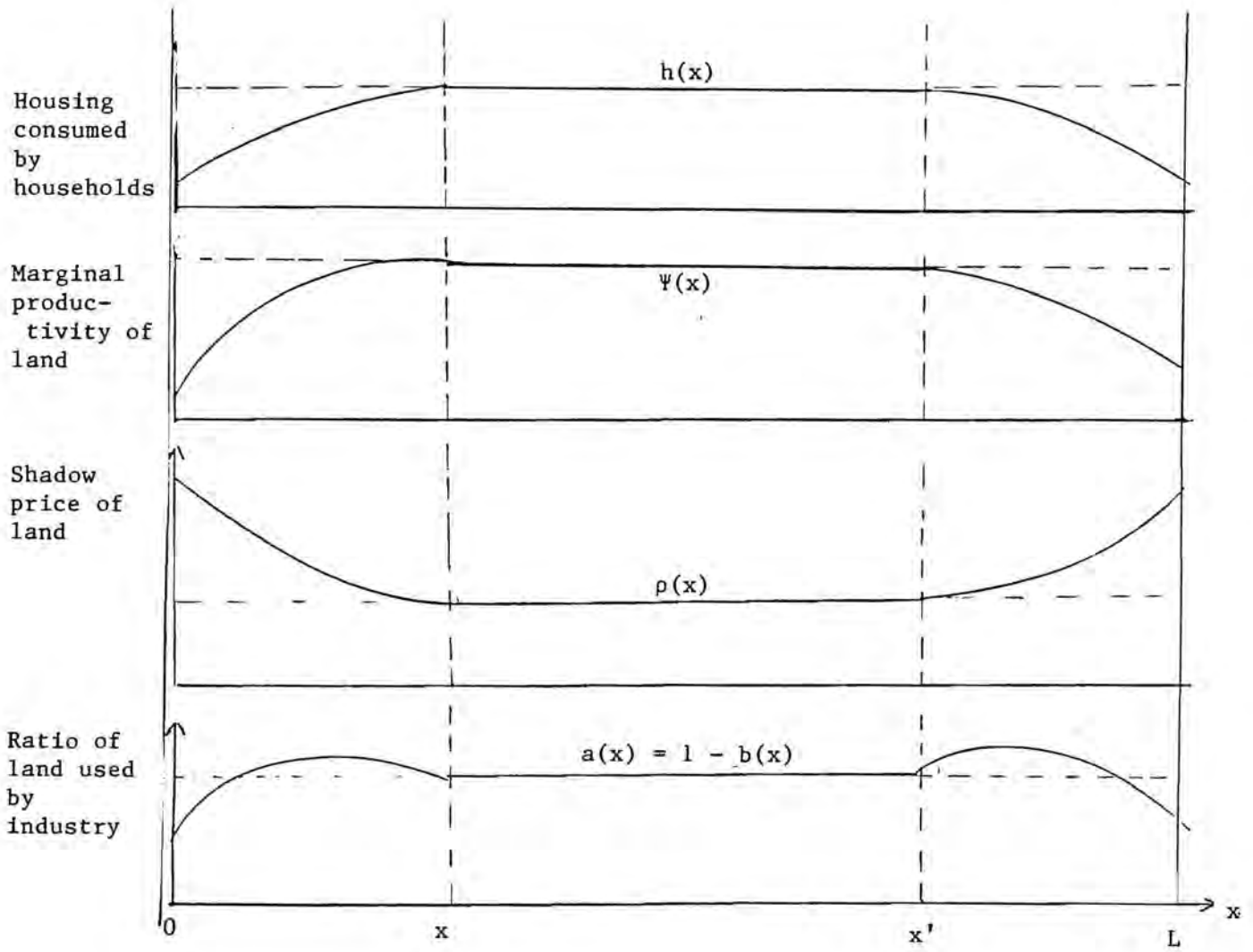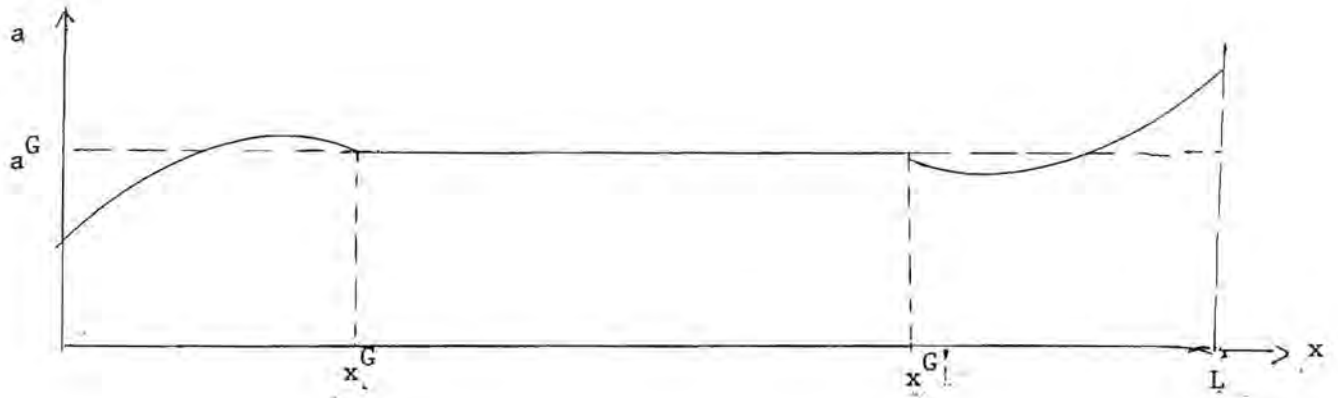
Fig. 1. Land Use Structure

Fig. 2. The Ratio of Land Used by Industry if Pollution Only Flowed North.

which no activity takes place whatsoever. Let X_{4N-i} denote the borders of these zones, $N = 1, 2, \ldots, \overline{N}$, $i = 1, 2, 3, 4$, $X_0 = 0$, and $X_{\overline{N}} = L$. Also, X_{4N-4} is the northern border of an industrial zone; X_{4N-3} is the southern border; and X_{4N-2} and X_{4N-1} are the northern and southern borders, respectively, of a residential zone. Between these zones are buffer zones (see figure 3). The specification of the problem in this case, as well as the method of obtaining the following results, is specified in Appendix B.

In the industrial areas, we have the following condition to be fulfilled on the border and, therefore, everywhere else inside the zone. Thus, for $X_{4N-4} \leq x \leq X_{4N-3}$, we have

(28) $$Pf[n(x), e(x)] - n(x)\psi(x) - Td(x) \geq 0,$$

where

$$Td(x) = \sum_{k=1}^{n-1} \int_{X_{4k-2}}^{X_{4k-3}} \eta(y) \, D^S[e(x), x - y]dy$$

$$+ \sum_{k=n}^{\overline{N}} \int_{X_{4k-2}}^{X_{4k-1}} \eta(y) \, D^N[e(x), y - x]dy \qquad X_{4N-4} \leq x \leq X_{4N-3}.$$

Note that Td, unlike M, equals the total value of the damages caused by pollution at x; equation (28) implies that, at each point inside the industrial zone, the social net gain [which is described by equation (28)] must be nonzero. If there is a buffer zone next to the industrial zone, then at this border the equality in equation (28) will hold. Throughout the buffer zone, an opposite inequality to (28) holds.
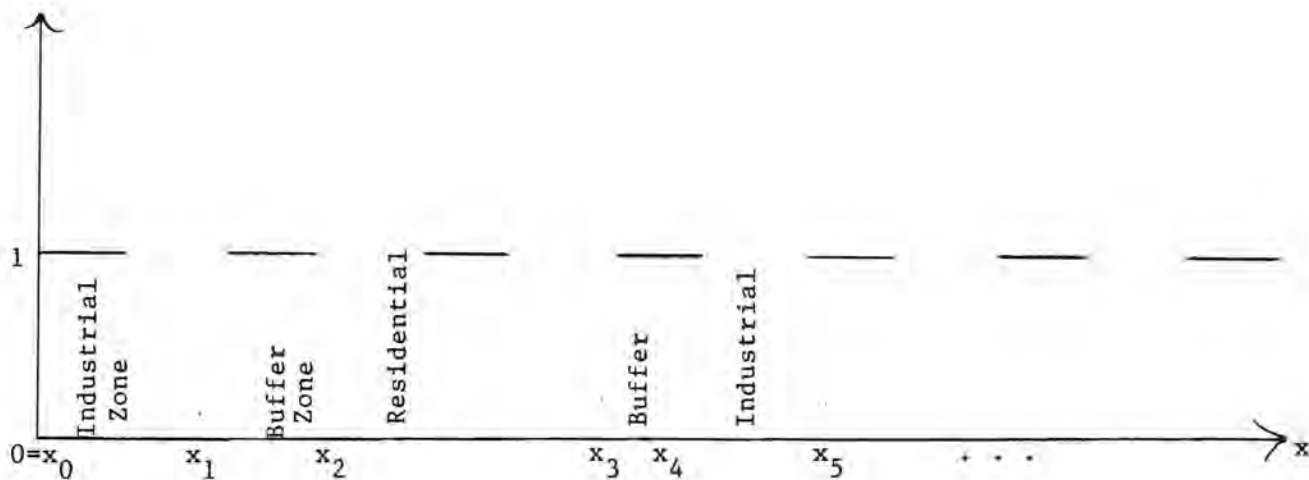
Fig. 3. Zoning Solution When D^i is Concave in Emissions.

For any point inside a residential zone (X_{4N-2}, X_{4N-1}), we must have

(29) $\qquad \dfrac{1}{h(x)} [I + \psi(x) - Z(x)] \geq 0 \qquad X_{4N-2} \leq x \leq X_{4N-1}.$

Again, if there are buffer zones, this value equals zero at the boundary and the inequality is reversed throughout the buffer zone. If, however, there is no buffer zone between an industrial and residential zone--say, $\tilde{x} = X_{4N-3} = X_{4N-2}$ at that point--we have

(30) $\qquad \dfrac{1}{h(x)} [I + \psi(\tilde{x}) - Z(\tilde{x})] = Pf[n(\tilde{x}), e(\tilde{x})] - n(\tilde{x})\psi(\tilde{x}) - Td(\tilde{x}) \geq 0.$

(For proof of these results, see Appendix B.)

To satisfy a decentralized solution for this case, the Pigouvian tax of the previous section has to be replaced by a new tax on the industry. This new tax, placed on units of land, is equal to the total damages caused by the industry at a given location.[4] Under these circumstances, the left-hand side of equation (28) describes the land rent in the industrial zone; and, as noted for the first case, the left-hand side of equation (29) describes the residential land rents. Equations (28), (29), and (30) simply state that each of the zones should extend up to the point where the land rent is zero. If the zones have a mutual border (no buffer zone), the industrial and residential rents at this border should be equal and greater than zero. Of course, these conditions are always satisfied in a competitive equilibrium given that the optimal per unit land tax is levied.

Implications of Pollution-Control Policies

To achieve efficiency, we have identified two distinct methods for pollution taxation corresponding to concave and convex dispersion functions.

Furthermore, if one method is implemented when the other method is needed, a case of undertaxation occurs; namely, the industry pays less than it should and, consequently, at each location excess land is devoted to industrial activities. This implies that exact information on the nature of the dispersion functions is needed before the proper method of taxation can be determined. However, even this information by itself is not sufficient since we may very well have a situation in which, for some range of e, D_{11} has one sign and for the other range has the opposite sign. In this case, even if we had all the necessary information available, local optima is likely to result. Hence, decisions based on information concerning marginal changes might be misleading. Matters might be even more complicated if the sign of D_{11} changed with the distance from the source of pollution.

All this brings us back to zoning regulations, namely, that the local government has to determine at each location the maximum amount of pollution which can be emitted. Of course, these restrictions can never be calculated accurately, but they can be determined by trial and error through observation of changes of land rents.[5]

In summary, we have two distinct cases--one in which $D_{11} > 0$ where a Pigouvian tax is needed and a second case in which $D_{11} < 0$, given that the industry is properly located, where a tax equal to total damages is needed. Whenever one tax is used instead of the other, an inefficient allocation will result due to insufficient taxation. This implies both excess emissions and excess use of land by the industry.

A simple rule can be devised, however; the optimal tax is always the higher of the two, i.e., optimal tax = max (M, Td). In the case where $D_{11} < 0$, optimal taxation by itself need not necessarily be sufficient. If

the land uses are mixed and industry and housing are located in the same location, we show in Appendix A that total land values will increase if zoning restrictions are implemented. However, when industry and housing change location as a result of zoning restrictions, the benefits--i.e., increase in land rents due to the decrease in concentration levels--occur throughout the city; but the costs--namely, the increase in transportation costs--place burdens on only the industry and households where these changes take place. In other words, most of the benefits are external to the individual actors involved while the costs are not. Therefore, if activities are mixed, taxation by itself may not be sufficient to produce the optimal allocation of land.

Appendix A

Zoning exists in section G if, for all $x \in G$,

(A.1a) $$a(x)b(x) = 0.$$

Effective zoning exists if (A.1b) holds in addition to (A.1a).

(A.1b) $$a(x) + b(x) > 0.$$

Note that (A.1b) implies that $a(x) + b(x) = 1$. When (A.1a) does not hold at any point $x < G$, zoning does not exist in x.

Define $\beta(x, y)$, β_G, and $\underline{\beta}_G$ as

(A.1c) $$\beta(x, y) = 1 + \frac{b(x)}{a(y)}$$

(A.1d) $$\overline{\beta}_G = \max_{x,y \in G} \beta(x, y)$$

(A.1e) $$\underline{\beta}_G = \min_{x,y \in G} \beta(x, y).$$

Thus, if effective zoning exists, $\beta(x, y) = \infty$ or 1. If zoning does not exist in a finite section G, then,

(A.1f) $$1 < \underline{\beta}_G \le \beta(x, y) \le \overline{\beta}_G < \infty.$$

Lemma 1: If $D_{11}^i(e, S) < 0$, $D^i(e = 0) = 0$, and zoning does not exist in a continuous section, $G \subset (0, L)$, then, by introducing zoning to G, pollution concentration everywhere in the city falls.[6]

<u>Proof</u>: Choose an arbitrary point x_0 and a $\Delta S > 0$ as small as is desired so that $(x_0, x_0 + \Delta S) \subset G$. Then, choose Δx_0 so that

(A.2)
$$\frac{\Delta S}{\bar{\beta}_G} \leq \Delta x_0 \leq \frac{\Delta S}{\underline{\beta}_G} .$$

Let Δx_1 be defined by (A.3).

(A.3)
$$\int_{x_0}^{x_0+\Delta x_0} \rho(x)dx = \int_{x_0+S-\Delta x_1}^{x_0+S} a(x)dx .$$

Equations (A.2) and (A.3) imply that

(A.4)
$$\underline{S} < \Delta x_0 + \Delta x_1 < S \text{ (see appendix figure A1).}$$

Now move all households in $(x_0, x_0 + \Delta x_0)$ to $(x_0 + S - \Delta x_1, x_0 + S)$ and all the industry in $(x_0 + S - \Delta x_1, x_0 + S)$ to $(x_0, x_0 + \Delta x_0)$ without changing any of the production or consumption input and output levels except, of course, the concentration level. The changes in the concentration levels at different locations in the city are:

$$\Delta C(x)\Big|_{x<x_0} = \int_{x_0}^{x_0+\Delta x_0}\Big\{D^N[a(y)e(y) + b(y)e(y) + S - \Delta x_1, y - x]$$

$$- D^N[a(y)e(y), y - x]dy - \int_{x_0+S-\Delta x_1}^{x_0+S} D^N[a(y)e(y), y - x]dy\Big\}$$

(A.5)
$$\cong \Delta x_0\Big\{D^N[a(x_0)e(x_0) + b(x_0)e(x_0 + S), x_0 - x]$$

$$- D^N[a(x_0)e(x_0), x_0 - x] - \frac{b(x_0)}{a(x_0 + S)} D^N[a(x_0 + S)e(x_0 + S), x_0 + S - x]\Big\}$$
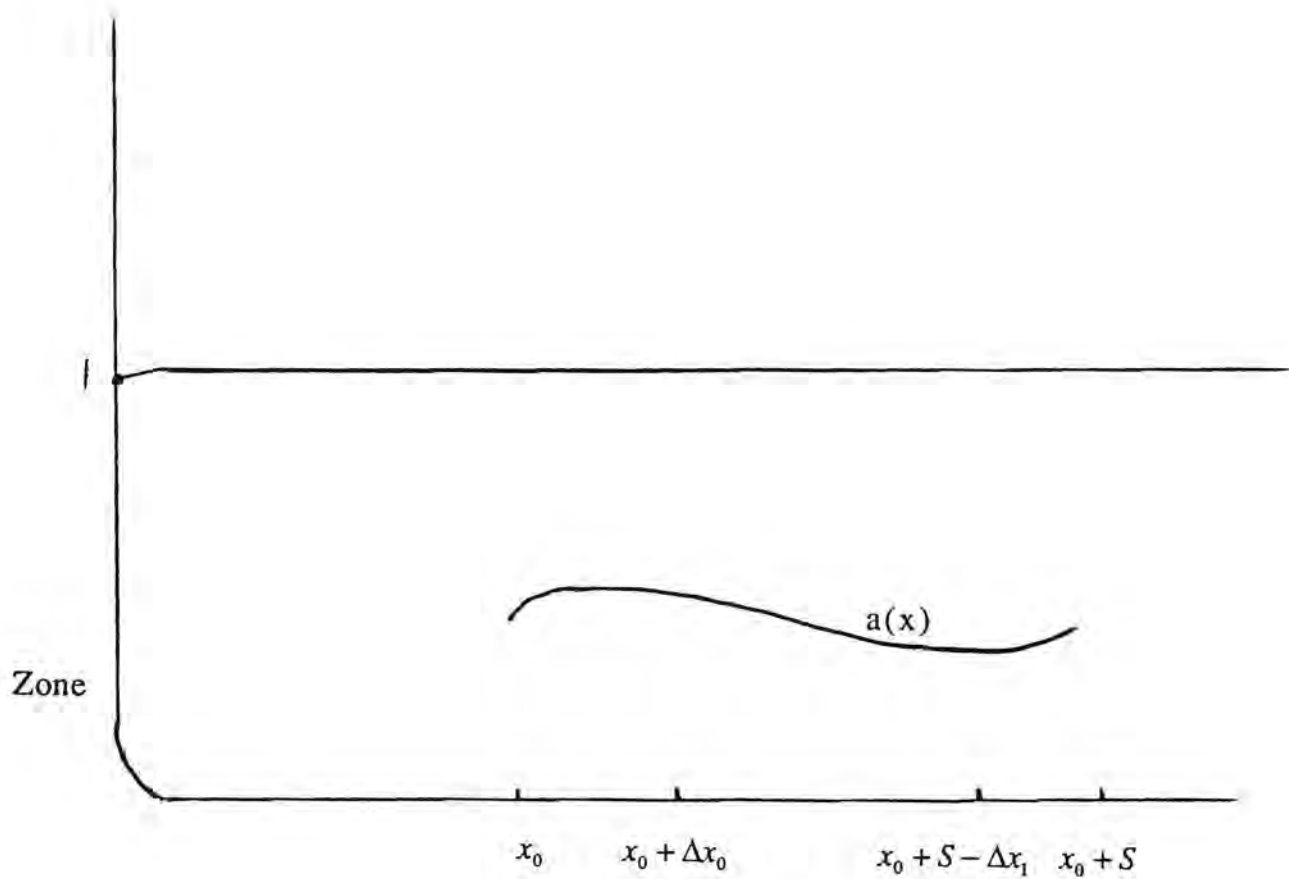
Fig. A1. Illustration of Equation (A.4).

The last equality in (A.5) is an approximation which becomes an equality for an infinitesimally small Δx_0.

$$D[a(x_0)e(x_0) + b(x_0)e(x_0 + S), x_0 - x] < D[a(x_0)e(x_0), x_0 - x]$$

(A.6)

$$+ D_1[a(x_0)e(x_0), x_0 - x] b(x_0)e(x_0 + S).$$

The inequality in (A.6) follows from the convexity of D with respect to e. Equation (A.6) holds for all S.

(A.7) $D[a(x_0 + S)e(x_0 + S), x_0 + S - x] = D[a(x_0 + S)e(x_0 + S), x_0 - x] + RE(S)$

where $RE(S) < 0$ due to convexity in S (i.e., $D_{22} < 0$) and approaches zero with S.

Substituting (A.6) and (A.7) in (A.5), we obtain

$$\Delta C\Big|_{x < x_0} < \Delta x_0 \left\{ D_1[a(x_0)e(x_0), x_0 - x] b(x_0)e(x_0 + S) \right.$$

$$- \frac{b(x_0)}{a(x_0 + S)} \left(D[a(x_0 + S)e(x_0 + S), x_0 - x] + RE(S) \right) \right\}$$

(A.8)

$$= \Delta x_0 \frac{b(x_0)}{a(x_0 + S)} \left\{ D_1[a(x_0)e(x_0), x_0 - x] a(x_0 + S)e(x_0 + S) \right.$$

$$\left. - D[a(x_0 + S)e(x_0 + S), x_0 - x] - RE(S) \right\}.$$

The term in the brackets is negative due to the assumed properties of D and, therefore, for sufficiently small S. Since RE' approaches zero with S, the term on the right-hand side of equation (A.8) is negative. In other words, we

can find a sufficiently small length of strip S so that the concentration at all x, $x \leq x_0$, is reduced provided that $D_{ee} < 0$ and $D(e = 0, S) = 0$.

In the same fashion, it can be shown that

$$\Delta C\Big|_{x > x_0 + S} \cong \Delta x_0 \Big\{ D^S[a(x_0)e(x_0) + b(x_0)e(x_0 + S), x - x_0]$$

$$- D^S[a(x_0)e(x_0), x - x_0] - \frac{b(x_0)}{a(x_0 + S)} D^S[a(x_0 + S)e(x_0 + S), x - x_0 - S]\Big\}$$

(A.9)

$$= \Delta x_0 \frac{b(x_0)}{a(x_0 + S)} \Big\{ D_1^S[a(x_0)e(x_0), x - x_0] \, a(x_0 + S)e(x_0 + S)$$

$$- D^S[a(x_0 + S)e(x_0 + S), x - x_0]\Big\} + RE(S) < 0.$$

$$\Delta C\Big|_{x_0 + S - \Delta x_1 \leq x \leq x + S_0} \cong \Delta x_0 \Big\{ D^S[a(x_0)e(x_0) + b(x_0)e(x_0 + S, S]$$

$$- D^S[a(x_0)e(x_0), S] - \frac{b(x_0)}{a(x_0 + S)} D[a(x_0 + S)e(x_0 + S), 0]\Big\}$$

(A.10)

$$< \Delta x_0 \frac{b(x_0)}{a(x_0 + S)} \Big\{ D_1^S[a(x_0)e(x_0), S] \, a(x_0 + S)e(x_0 + S)$$

$$- D[a(x_0 + S)e(x_0 + S), S]\Big\} + RE(S) < 0.$$

Equations (A.8), (A.9), and (A.10) imply that the concentration $C(x)$ at all x declines when zoning is imposed. Furthermore,

(A.11) $\lim\limits_{\Delta S \to 0} \dfrac{\Delta S}{\Delta x_0} = \beta(x_0, x_0) = 1 + \dfrac{b(x_0)}{a(x_0)} = \dfrac{1}{a(x_0)}$ [see equation (A.2)].

Hence, from (A.8), (A.9), (A.10), and (A.11),

(A.12)

$$\frac{dC(x)}{dS(x_0)} = \lim_{S \to 0} \frac{\Delta C}{\Delta S} = b(x_0) \left\{ D_1^i[a(x_0)e(x_0), |x - x_0|] \, a(x_0)e(x_0) \right.$$

$$\left. - D^i[a(x_0)e(x_0), |x - x_0|] \right\} < 0.$$

Lemma 2. A reduction ΔC in concentration at x will increase the bid rent of housing at x by

(A.13)
$$-\Delta C\left(\frac{\partial R^h}{\partial C}\right) = -\frac{b(x)}{h(x)} \frac{U_C}{U_z} \Delta C.$$

Proof: From equations (13) and (19) in the text, it follows that

(A.14)
$$R^h_{(x)} = \frac{1}{h(x)} (I + \psi - z).$$

Hence, the effect of a change of C on $R^h_{(x)}$ is due only to its effect on z (income and housing are not allowed to change). The intuition is that, since concentration is reduced, households need less compensation for pollution damages in order to maintain their utility levels. The exact number of z households willing to sacrifice per unit land is

(A.15)
$$\frac{\partial \rho(x)}{\partial C(x)} = -\frac{b(x)}{h(x)} \frac{\partial z(x)}{\partial C(x)} = \frac{b(x)}{h(x)} \frac{U_C}{U_z} < 0.$$

The last equality follows from differentiation of (1). Equation (A.13) follows immediately.

Note that (A.15) takes into account only the effects on residential land rents. But a decrease in concentration will also reduce the price of

pollution due to its effect on $\eta(y)$. We disregard this component and consider (A.15) to be the only effect. The total change in land rents due to a marginal zoning, dS, follows from (A.15) and (A.12), i.e.,

$$\frac{d(TR)}{\partial S} = -b(x_0) \int_0^L \eta(y) \left\{ D_1[a(x_0)e(x_0), |y - x_0|] \, a(x_0)e(x_0) \right.$$

(A.16)

$$\left. - D[a(x_0)e(x_0), |y - x_0|] \right\} dy > 0.$$

The total number of workers displaced is $n(x_0 + S)a(x_0 + S)\Delta x_1$. The total amount of added commuting costs, which we designate as $\Delta CC(S)$, is, therefore,

(A.17)
$$\Delta CC(S) = \Delta S V b(x) \, \Delta x_0 \left[\frac{1}{h(x_0)} + n(x_0 + \Delta S) \right].$$

Hence,

(A.18)
$$\frac{d(CC)}{dS} = \lim_{\Delta S \to 0} \frac{\Delta CC(S)}{\Delta S} = V b \Delta(x_0) \left[\frac{1}{h(x_0)} + n(x_0 + \Delta S) \right] = 0.$$

The last equality in (A.18) follows since Δx_0 disappears with ΔS, while all the other variables approach a finite limit.

Added travel costs also affect the rents through their effect on ψ. Increased travel costs reduce ψ at the locations of both labor and residency and, hence, reduce total rents. In Hochman (1981) it is shown that the net land rents exactly equal net city gains (surplus). Thus, we see that, while the benefits of zoning due to a decrease in concentration tend toward a positive limit, the costs disappear with ΔS. Hence, it is always possible to set ΔS sufficiently small so that the benefits exceed the costs. The

last arguments and lemmas can be summarized in the major result of this Appendix.

Theorem: When $D_{11} < 0$, zoning in the entire range is a necessary condition for optimization.

Appendix B

In this Appendix we show that in the case of zoning, i.e., residential and industrial activities which take place in separate areas, relations (28), (29), and (30) in the text are additional necessary conditions for optimum resource allocation. To demonstrate this outcome, let X_{4N-4}, $N = 1, 2, 3, \ldots$, designate the left border of an industrial zone and X_{4N-3} its right border; let X_{4N-2} and X_{4N-1} designate the left and right borders, respectively, of a residential zone. Between the adjacent residential and industrial zones, a buffer zone exists (see figure 3). Given zoning, the net city surplus, π, can be derived [alternatively to equation (6) in the text] by

$$
\begin{aligned}
\pi = \sum_{N-1}^{\infty} &\left\{ \int_{X_{4N-4}}^{X_{4N-3}} \left(Pf[n(x), e(x)] - V|T(x)| \right) dx \right. \\
&\left. + \int_{X_{4N-2}}^{X_{4N-1}} \left(\frac{1}{h(x)} [I - Z(x)] - V|T(x)| \right) dx \right\} \\
&- (X_{4N-2} - X_{4N-3}) \, V|T(X_{4N-2})| \\
&- (X_{4N} - X_{4N-1}) \, V|T(X_{4N-1})|.
\end{aligned}
$$

(B.1)

Similarly, instead of equation (4a) in the text, we have

(B.2)
$$
\overset{\circ}{T}(x) = \begin{cases} \dfrac{1}{h(x)} & \text{for } X_{4N-2} \leq x \leq X_{4N-1} \\[2ex] -n(x) & \text{for } X_{4N-4} \leq x \leq X_{4N-3} \\[2ex] 0 & \text{otherwise.} \end{cases}
$$

The variable $C(x)$ is now given by (B.3) instead of equation (2)

(B.3)

$$C(x) = \sum_{N=1}^{\tilde{N}} \int_{X_{4N-4}}^{X_{4N-3}} D^N[e(y), x - y]dy$$

$$+ \sum_{N=\tilde{N}+1}^{\infty} \int_{X_{4N-4}}^{X_{4N-3}} D^S[e(y), y - x]dy = 0.$$

For all, x in residential areas and $\tilde{N}$ are such that $X_{4\tilde{N}-2} \leq x \leq X_{4\tilde{N}-1}$. In addition, we have the four following constraints:

(B.4a) $\qquad\qquad X_{4N-3} - X_{4N-4} \geq 0$

(B.4b) $\qquad\qquad X_{4N-2} - X_{4N-3} \geq 0$

(B.4c) $\qquad\qquad X_{4N-1} - X_{4N-2} \geq 0$

(B.4d) $\qquad\qquad X_{4N} - X_{4N-1} \geq 0.$

The restructured optimization problem is now to maximize π as defined in (B.1) subject to (B.2), (B.3), (B.4), and equations (1), (5a), and (5b) from the text with (4c) and (4d) as terminal conditions. The variables, X_{4N-i}, are now added to the list of variables as control parameters. In addition to the necessary conditions specified in the text, the following necessary transversality conditions are now required:

$$Pf[n(X_{4N-i}), e(X_{4N-i})] - \psi(X_{4N-i})n(X_{4N-i})$$

(B.5a)

$$- \left\{ \sum_{k=1}^{N-1} \int_{X_{4k-2}}^{X_{4k-3}} n(y)D^S[e(X_{4N-i}), X_{4N-i} - y]dy \right.$$

$$\left. - \sum_{k=N}^{\tilde{N}} \int_{X_{4k-2}}^{X_{4k-1}} n(y)D^N[e(X_{4N-i}), y - X_{4N-i}]dy \right\}$$

$$+ (-i)^i (\sigma_{4N-i} - \sigma_{4N-i-1}) = 0, \qquad \text{for } i = 3, 4.$$

(B.5b) $I - z(X_{4N-i}) + \psi(X_{4N-i}) + (-i)^i(\sigma_{4N+1-i} - \sigma_{4N-i})h(X_{4N-i}) = 0$

$$\text{for } i = 1, 2$$

where σ_{4N-i}, $i = 3, 2, 1, 0$ are, respectively, the shadow prices of (B.4a) - (B.4d).

We have $\sigma_{4N-i} \geq 0$ and

(B.6a) $$\sigma_{4N-3}(X_{4N-3} - X_{4N-4}) = 0$$

(B.6b) $$\sigma_{4N-2}(X_{4N-2} - X_{4N-3}) = 0$$

(B.6c) $$\sigma_{4N-1}(X_{4N-1} - X_{4N-2}) = 0$$

(B.6d) $$\sigma_{4N}(X_{4N} - X_{4N-1}) = 0.$$

Equations (B.5a) and (B.5b), together with (B.6), lead to equations (28), (29), and (30) in the text.

Appendix C

Let

$$\mathcal{L} = \int_0^L [Paf(n, e) + \frac{b}{h} (I - z) - |T| v] \, dx$$

$$+ \int_0^L \lambda(x) [U(h, z, c) - u_0] \, dx + \int_0^L dx \eta(x) \; c(x) - \int_0^x D^N[e(y) \, a(y), x - y]$$

(C.1)

$$- \int_x^L D^S[e(y) \, a(y), y - x] \, dy + \int_0^L \zeta(x) \; T(x) - \int_0^x [\frac{b(y)}{h(y)} - a(y) \, n(y)] \, dy \; dx$$

$$+ \int_0^L [\rho(a + b - 1) + \gamma a + \mu b] \, dx.$$

The necessary conditions are as follows. (The variable of differentiation is noted on the left-hand side of each equation. Note that the differentation is pointwise and that a function with a number as a subscript indicates derivation of the function with respect to the variable of the order of the subscript.)

$n(x)$ (C.2) $a(x) \, Pf_1 + \int_x^L z(y) \, dy = 0.$

$e(x)$ (C.3) $a(x) Pf_2 - \int_x^L \eta(t) \, D_1^N[e(x) \, a(x), t - x] \, dt$

$$- \int_0^x \eta(t) \, D_1^S[e(x) \, a(x), x - t] \, dt = 0$$

$a(x)$ (C.4) $Pf - e(x) \quad \int_x^L \eta(t) \, D_1^N[e(x) \, a(x), t - x] \, dt$

$$+ \int_0^x \eta(t) \, D_1^S[e(x) \, a(x), x - t] \, dt \; + n(x) \int_x^L \zeta(t) \, dt$$

$$+ \rho(x) + \gamma(x) = 0$$

$h(x)$ (C.5) $- \frac{b(x)}{h(x)^2} [I - z(x)] + \lambda(x) \, U_h + \frac{b(x)}{h(x)^2} \int_x^L \zeta(t) \, dt = 0$

$b(x)$ (C.6) $\frac{1}{h} (I - z) - \frac{1}{h(x)} \int_x^L \zeta(t) \, dt + \rho(x) + \mu(x) = 0$

$z(x)$ (C.7) $- \frac{b}{h} + \lambda U_z = 0.$

By using equation (8) from the text, we obtain:

$T(x)$ (C.8) $- [\text{sign } T(x)] \cdot v + \zeta(x) = 0$

$c(x)$ (C.9) $\lambda(x) U_c + \eta(x) = 0.$

Define the costate of $T(x)$ to be $\psi(x)$:

$$(C.10) \qquad \psi(x) \overset{\text{def}}{=} \int_x^L \psi(t) \, dt;$$

then

$$(C.10') \qquad \dot{\psi}(x) = \zeta(x).$$

By substituting $\psi(x)$ and $\psi(x)$ from (C.10) into (C.2) minus (C.9) and eliminating from the equations $\lambda(x)$ by substituting from (C.7), we obtain the necessary conditions as specified in the text.

Footnotes

[1]Henderson suggested a lump-sum tax equal to the difference between Pigouvian taxes and our per-unit land tax at the borderline of the zone. This double tax scheme works in Henderson's model since only one industrial zone with one border exists. In our case almost all zones have more than one border, and the lump sun has to differ in the two borders which is impossible to achieve. In practice the situation is even more complicated since the number of borders of a single zone can be large.

Carlton and Loury's double tax scheme can be considered as being two Pigouvian taxes levied on two separate externalities that happen to have a joint damage function; hence, no real "double tax" scheme is identified in their cases. In conclusion, no double tax on polluters is necessary--yet the necessary tax in the case of a concave dispersion function is not a Pigouvian but a land tax.

[2]It is proved there that, if the economy is divided into disjoint areas each facing prices of imports and exports of the area as fixed (including utility level of population) and if the nonearned income of households is independent of city of residency, then maximization of the net city surplus (equal to the value of net city gains when nonearned income is considered as well) is a necessary condition of Pareto optimality in the economy. This condition is termed local efficiency. (Note that assuming city rents divided among residents, a common assumption in urban

economic literature, contradicts the assumption of nonearned income independent of city of residency.) The intuitive rationale behind the condition is that the whole city may be considered as a price-taking production unit. The net surplus is then the net city gains. As in classical economic literature, a profit-maximizing behavior (maximizing net city surplus) is efficient.

[3]A subscripted function indicates the differential of the function with respect to the variable whose order is indicated by the subscript.

[4]Henderson's double tax (the Pigouvian plus a lump-sum tax) will also lead to the efficient solution. Note that his suggestion that total damages may be smaller than Pigouvian taxes paid and, therefore, that the lump-sum tax may be a subsidy is impossible. Total damages are less than Pigouvian taxes paid only if $D_{11}^i > 0$. When that occurs, we do not have a zoning solution; and Pigouvian taxes alone are sufficient.

[5]For the details, see Hochman and Ofek.

[6]Note that any set that has both--points with effective zoning and points that have no zoning--can be divided into subsets that are either with or without zoning except for a countable number of points. The above proof can be extended to include such sets.

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