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The Smoothing Effect of Carpool Lanes on Freeway Bottlenecks

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Abstract

Real data show that reserving a lane for carpools on congested freeways induces a smoothing effect that is characterized by significantly higher bottleneck discharge flows (capacities) in adjacent lanes. The effect arises because disruptive vehicle lane changing diminishes in the presence of a carpool lane. The effect is reproducible across days and freeway sites: it was observed, without exception, in all cases tested.

Queueing analysis shows that the effect greatly reduces the times spent by people and vehicles in queues. By ignoring the smoothing effect at one of the sites we analyzed, for example, one would predict that its carpool lane increased both the people-hours and the vehicle-hours traveled by well over 300%; when in reality the carpool lane and its attendant smoothing reduced both measures. The effect is so significant, in fact, that even a severely underused carpool lane can in some instances increase a freeway bottleneck's total discharge flow. This happens for the site we analyzed when carpool demand is as low as 1200 vph. It follows that strategies designed to induce smoothing by other means also hold promise for managing congestion, both for freeways that have carpool lanes and those that do not. Possible strategies of this kind are discussed.

1. Introduction

Carpool lanes are deployed on urban freeways for the exclusive use of vehicles that carry more than a predetermined number of occupants. The usefulness of these lanes seems to be a subject of debate. On one hand, they tend to be underused, and as a consequence a number of studies report that carpool lanes unduly penalize Low Occupancy Vehicles (LOVs) by creating congestion in non-carpool lanes (e.g. Schofer and Czepiel, 2000; Chen, et al, 2005; Kwon and Varaiya, 2008). And since an underutilized carpool lane wastes a freeway's queue storage space, it extends the queue length in adjacent lanes.

On the other hand, the damage done by this queue extension effect tends to be small for most freeways (Daganzo and Cassidy, 2008). And we know, of course, that by enabling high occupancy vehicles to bypass LOV-queues, carpool lanes can reduce the time that people collectively spend commuting (e.g. Cassidy, et al 2006).

Moreover, there is limited evidence suggesting that these lanes, even when underutilized, can *diminish* freeway congestion. Menendez and Daganzo (2007) have predicted based on simulation experiments that carpool lanes diminish lane-changing maneuvers, and that this, in turn, smoothes (and increases) bottleneck flows in adjacent lanes. This conjecture is consistent with earlier work showing that disruptive lane changes cause capacity drops at bottlenecks without carpool lanes (Cassidy and Rudjanakanoknad, 2005; Laval and Daganzo, 2006). These findings are intriguing: if the smoothing effect turns out to be real, it would mean that carpool lanes can sometimes benefit all freeway commuters, and not just carpoolers; and would shed light on new ways to control freeway congestion.

The present paper uses detailed video data to demonstrate the existence of the smoothing effect, and to unveil the mechanism that causes it (sec. 2). Detector data from all suitable sites in the San Francisco Bay Area are next used to show that the effect arises consistently, significantly and reproducibly across days and sites (sec. 3). Queueing theory is then used to quantify its impacts (sec. 4). Finally, the paper discusses how to exploit the effect on freeways with and without carpool lanes (sec. 5).

2. The Effect and its Causal Mechanism

Traffic data collected from videos are used below to demonstrate (i) the existence of the smoothing effect at a freeway merge bottleneck; and (ii) the role in this played by a carpool lane that runs through the bottleneck.

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2.1 The merge bottleneck and evidence of smoothing

This section examines a day in which a queue formed in the early portion of a rush, before the carpool restriction went into force; and demonstrates that the queue discharge rate increased when the carpool restriction did take effect. The underlying causal mechanism is unveiled in Sec. 2.2.

Our study site is shown in Fig. 1. The median lane (lane 1) of that freeway is reserved for carpools on weekdays during the morning rush (5:00 to 9:00), and again in the afternoons (15:00 to 19:00). The remaining lanes are labeled 2 through 4.

Video cameras were erected on the over-crossings for pedestrians and for Tennyson Rd., and these cameras recorded traffic during part of an afternoon rush (on July 19, 2006). Vehicle arrival times at locations X_1, X_2 and X_3 were manually extracted from the videos and, as is customary, cumulative curves of vehicle count for all lanes combined were plotted on an oblique coordinate system (O-curves), as shown in Fig. 2. Note that the slopes of the O-curves are the excess flows over a background flow, which is 6800 vph in the present case; and that the curves in Fig. 2 were constructed in such ways that superimposed curves indicate free-flow traffic (flow = demand), and separated curves indicate delays: the wider the separation the longer the delays (see Cassidy and Windover, 1995; and Muñoz and Daganzo, 2002).

In Fig. 2, curves 2 and 3 are superimposed, and below curve 1. Thus, traffic was freely flowing between X_2 and X_3, but delays arose between X_1 and X_2; i.e. a bottleneck formed between the latter two locations. The curves at these two locations diverged for good at about 14:43 hrs when a disruption temporarily reduced the total flow at X_2. Less than 3 minutes later (at approximately 14:45:30) and well before the carpool restriction was activated, flow dropped further to about 6950 vph. Thus, the carpool lane did not contribute to the bottleneck formation and capacity drop. Instead, and as is typical of merge bottlenecks without carpool lanes, the queue first formed in the shoulder lane and then spread to all lanes; see Cassidy, et al (2006) for more details. The carpool lane did begin to exert influence a short time later, however; and the influence was favorable.

Figure 3 displays an O-curve for the median (carpool) lane measured at X_3. As one might expect, flow diminished both before and after 15:00 hrs, as LOVs exited the lane. Surprisingly, a comparison of Figs. 2 and 3 from 14:52 to 15:10 reveals that the total flow across all lanes (including the carpool lane) remained quite steady at rates approaching 7000 vph, even as the carpool lane was being vacated.

These patterns indicate that the diminished carpool-lane flow was compensated by increased queue discharge rates (capacity) in adjacent lanes. The effect was sustained from then on, and this is underscored by extending some of the curves in Figs. 2 and 3 beyond 15:10.
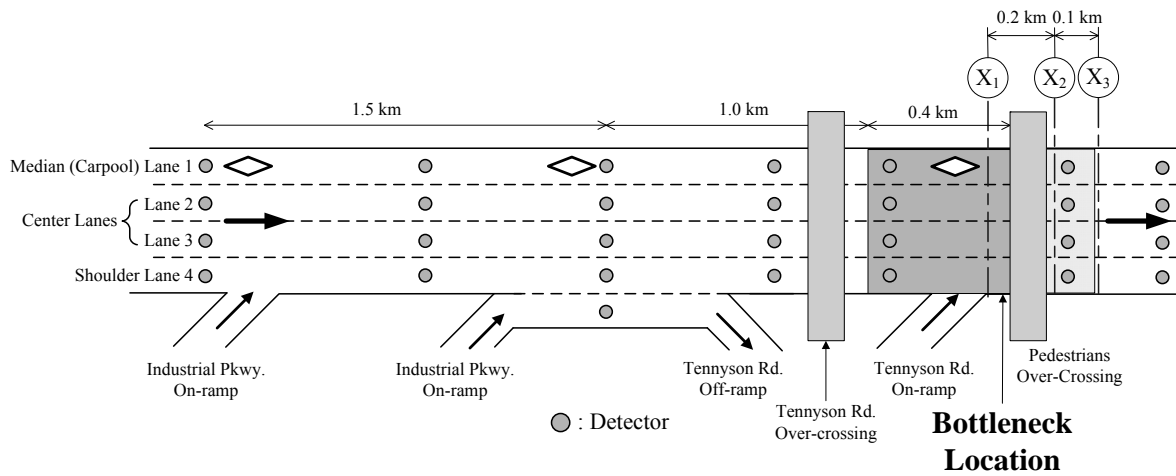
2

Figure 1 Study Site: I-880 North, Hayward, California
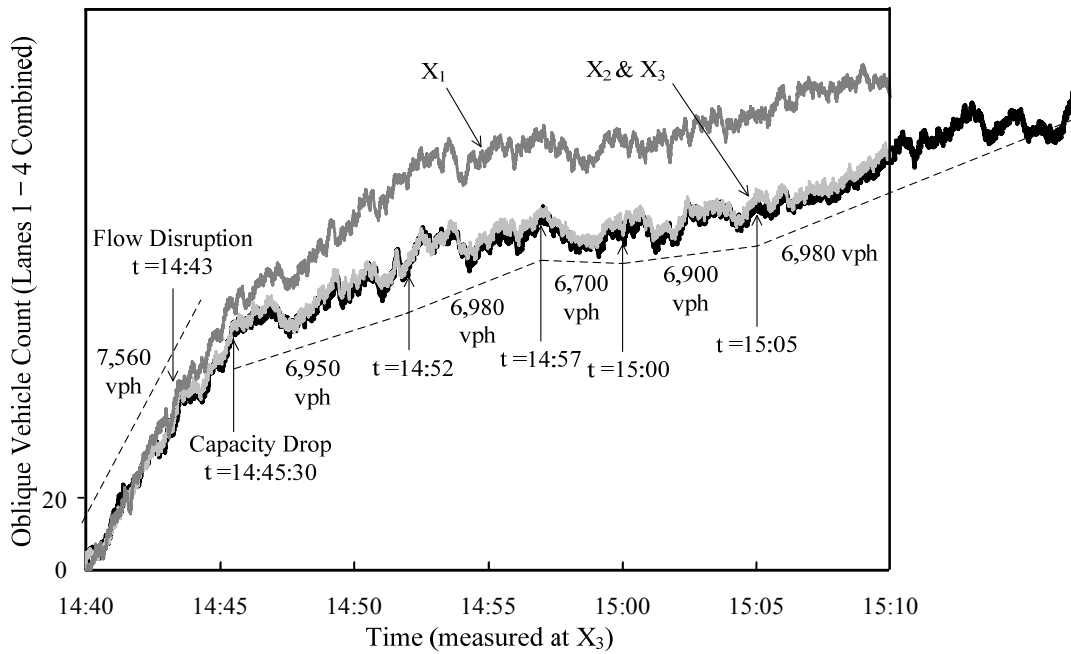Shaded segments were subject to video surveillance

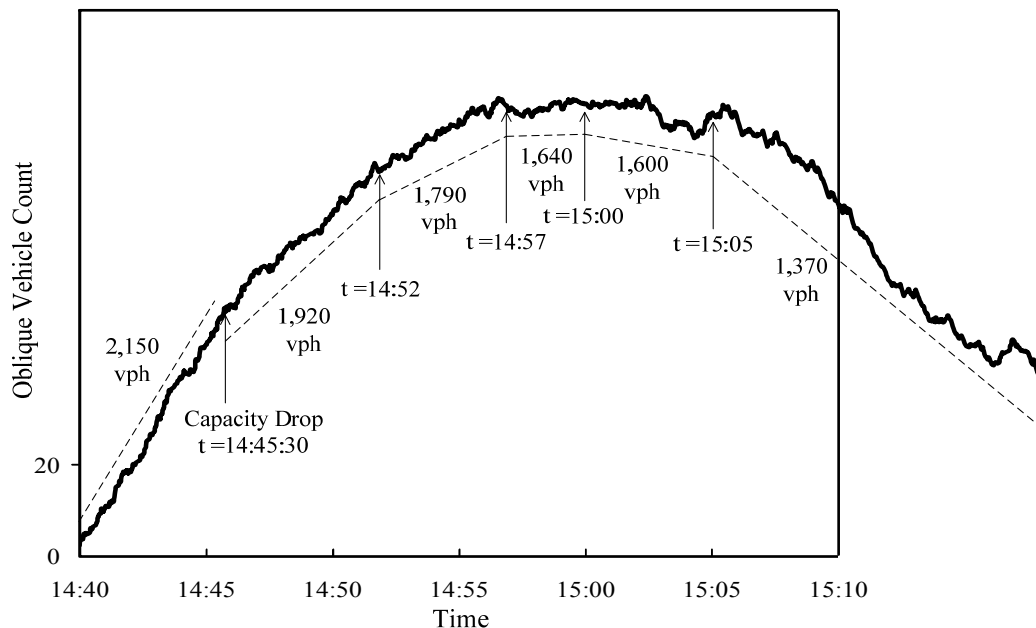Figure 2 O-Curves for Lanes 1 – 4 at X_1 through X_3 (July 19, 2006)

Figure 3 O-Curve of Median (Carpool) Lane at X_3 (July 19, 2006)

2.2 The cause of smoothing

We show here that the increase in discharge flows is due to a decline in lane changing rates caused by the carpool restriction. Lane-changing rates were extracted from videos over the 0.4-km segment between the over-crossings in Fig. 1. We consider first the connection between discharge rates and lane-changing rates, and then show that the decline in the latter can be attributed to the carpool lane.

To understand the connection between lane changing and discharge flow, we examined the lanes, one at a time, starting with lane 2. The boldfaced O-curve in Fig. 4a shows that lane-changing maneuvers in and out of that lane began to diminish minutes before 15:00 hrs, the carpool lane activation time (as highlighted by the downward-bending dashed line in the figure). An abrupt increase in the lane's discharge flow followed close on the heels of this event, as revealed by the thin O-curve of vehicle count (and highlighted by the upward-bending dashed line). Figure 4b reveals that a similar pattern was observed a few minutes earlier in lane 3: lane changing diminished and discharge flow rose very soon thereafter, beginning sometime around 14:52 hrs. The phenomenon was not observed in lane 4, however; see Fig. 4c.

Thus, we see that in each of the two lanes closest to the carpool lane, a reduction in lane-changing rate was closely followed by an increase in discharge flow. The timing of these events so close to 15:00 hrs strongly suggests that they were caused by the carpool restriction.
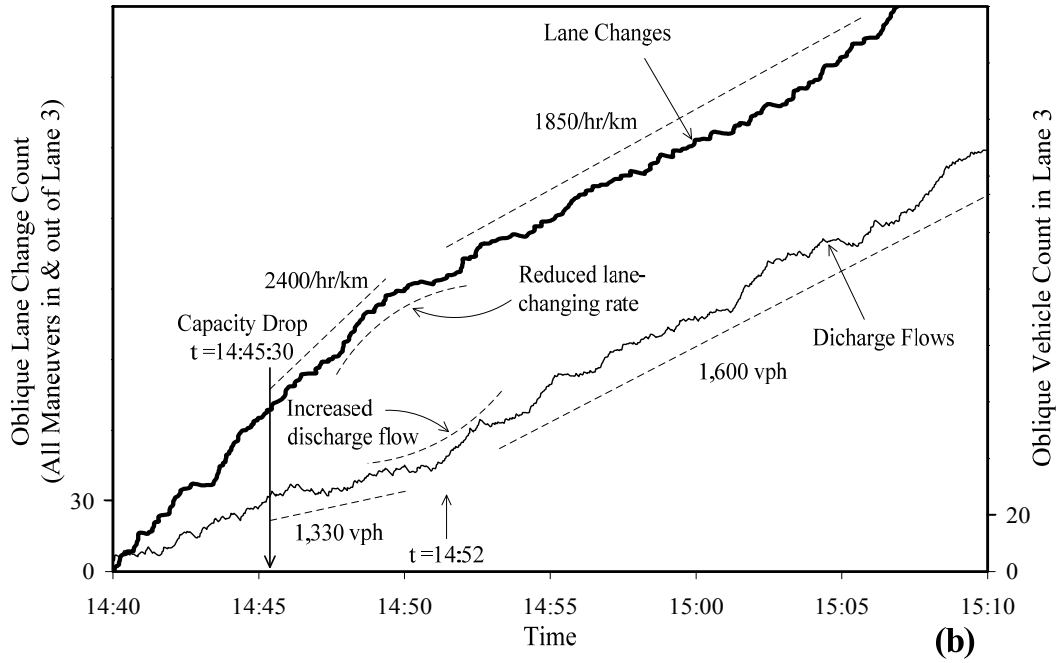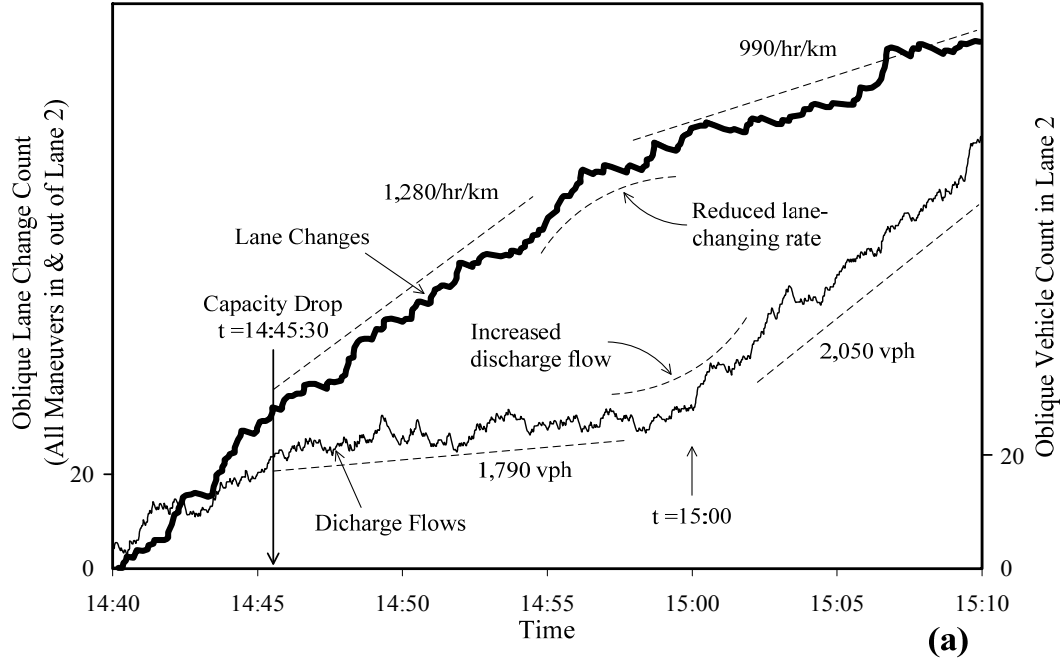
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Figure 4 Oblique Cumulative Curves of Lane-Changing and Discharge Flow (July 19, 2006);
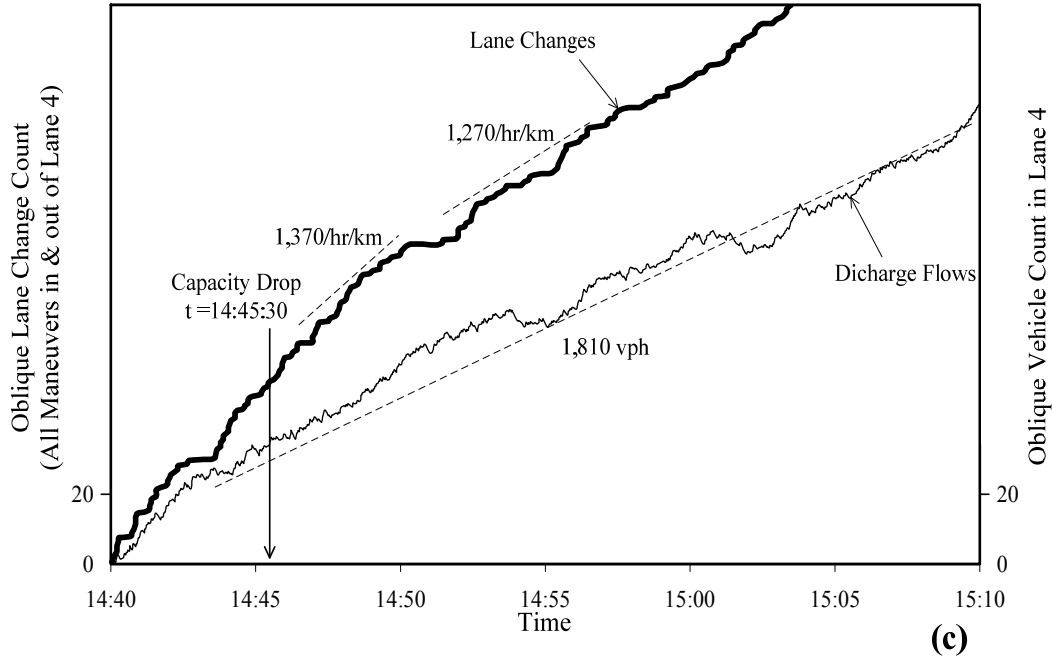(a) Lane 2; (b) Lane 3

Figure 4 (cont'd) Oblique Cumulative Curves of Lane-Changing and Discharge Flow (July 19, 2006); (c) Lane 4

Appendix A takes a more detailed look at the data and shows that the observed patterns (including the 8-min discrepancy between the pattern changes in lanes 2 and 3) were indeed caused by the migration of vehicles away from the carpool lane, further solidifying the idea that the carpool restriction is at the root of the reductions in lane changing and improvement in discharge flow.

The next section shows that the smoothing effect arises consistently and reproducibly at different sites.

3. Repeated Observations

We examined the entire network of carpool facilities in California's San Francisco Bay Area during multi-week study periods, and identified all the sites in which a bottleneck was active for at least 30 mins before and after its carpool restriction switched on or off. This filtering method is logical, since we are comparing the bottleneck's center-lane discharge flows with and without the carpool lane, while holding all else approximately constant. Although we found only two suitable sites (the site in Fig. 1 and one

additional site), multiple instances passed our filter at each site. The smoothing effect arose in every instance. We show this for our first site in sec. 3.1; and for the second site in sec. 3.2.

3.1 Reproducibility across days: Site 1

This site was examined every weekday in Aug. and Sept. 2007, and eight suitable instances turned up. These eight periods are in addition to the one used in sec 2; four came from late portions of the morning rush, and four from early portions of the afternoon rush. No other instances were found in which the bottleneck's active period overlapped both the carpool lane's active and inactive periods.

Table 1 summarizes the data. For each of the eight periods, it presents 30-min average bottleneck discharge flows in the two center lanes combined, with and without the carpool restriction. (Discharge flows from 5-min transition periods on each side of the carpool lane activation and deactivation instants are excluded.) The table shows that the smoothing effect arose without exception, and did so significantly and consistently. The resulting rise in center-lane discharge flows ranged from 9.5% to 13%, with an average of 10.5%, in the early afternoons; and from 18% to 21%, with an average of 19.5%, in the late mornings. The late morning and early afternoon differences are statistically significant, so something must be causing them. As we shall see momentarily, a similar discrepancy arises at the second site.

Table 1 Discharge Flows from Lanes 2 and 3 Combined at I-880 North Study Site

(1)	(2)	(3)	(4)	(5)
Observation Date	Flows with carpool lane (vph)	Flows without carpool lane (vph)	Increase due to smoothing (vph)	% increase
Early Afternoons	**15:05 ~ 15:35**	**14:25 ~ 14:55**		
2007-08-02	3670	3350	320	9.5
2007-08-03	3690	3370	320	9.5
2007-08-23	3740	3400	340	10
2007-09-11	3650	3240	410	13
Late Mornings	**8:25 ~ 8:55**	**9:05 ~ 9:35**		
2007-08-21	3290	2750	540	20
2007-08-22	3530	2990	540	18
2007-08-23	3370	2830	540	19
2007-09-04	3430	2840	590	21

3.2 An additional site

The second site is shown in Fig. 5. A bottleneck forms at the entrance to the curved section during the afternoon rush. The site was canvassed for suitable study instances from May through September 2007. Four instances were found: all during the afternoon rush. Two straddled the carpool lane's activation time (at 15:00 hrs) and two its deactivation time (at 19:00 hrs).

Again, the smoothing effect emerged without exception; see Table 2 which presents the discharge flows measured in the two center lanes for each of the four cases. The effect is again significant and consistent: discharge flows increased by 8% and 12% in the beginning of the afternoon rush; and by 18% and 19% at the end of the rush; and the discrepancies between early and late measurements are statistically significant.

Since for both sites, we see that the smoothing effect is less significant at the start of the afternoon rush, we look more deeply at the data in Tables 1 and 2 and see that all the discharge flows are significantly higher at this time of day than at the end of a rush. The pattern indicates that early-afternoon drivers are more aggressive, perhaps because they are trying to "beat the rush" for the remainder of their trips and are less affected by lane changes. We therefore conjecture that, during the beginning of a morning rush, center-lane discharge flows would increase by 10% due to smoothing, as they do near the start of the afternoon rush; and that 15% might be a good average to use for planning purposes.
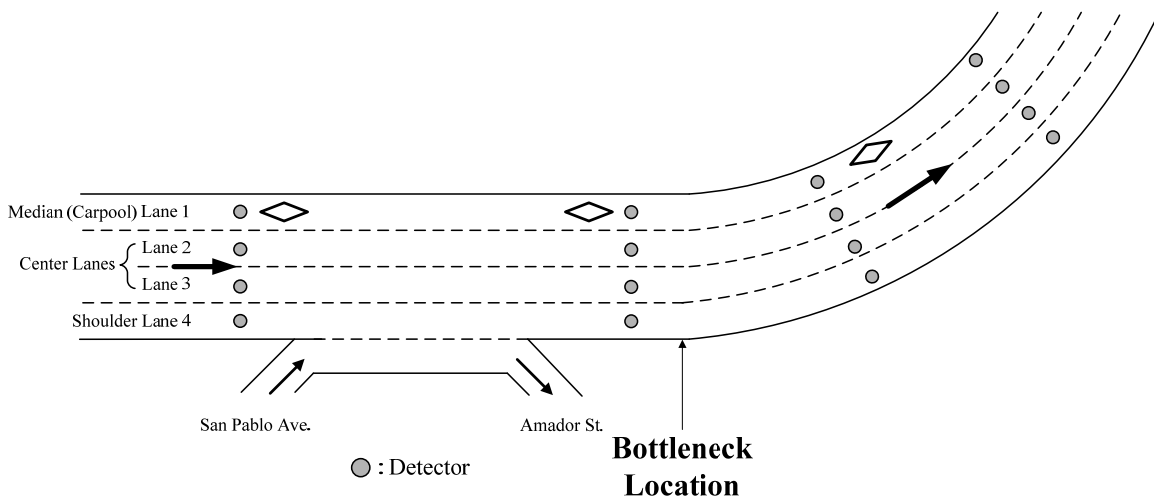

Figure 5 Second Study Site: I-80 East, Richmond, California

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Table 2 Discharge Flows from Lanes 2 and 3 Combined at I-80 East Study Site

(1)	(2)	(3)	(4)	(5)
Observation Date	Flows with carpool lane (vph)	Flows without carpool lane (vph)	Increase due to smoothing (vph)	% increase
Early Afternoons	**15:05 ~ 15:35**	**14:25 ~ 14:55**		
2007-05-30	3880	3590	290	8
2007-07-23	3870	3450	420	12
Late Afternoons	**18:25 ~ 18:55**	**19:05 ~ 19:35**		
2007-07-10	3660	3100	560	18
2007-09-11	3350	2820	530	19

4. The Real Impacts of Carpool Lanes on People and Vehicle Delay

This section explores the *real* impacts of carpool lanes; i.e. by recognizing smoothing. We compare the PHT and VHT for an afternoon at the site in Fig. 1 under three scenarios: (i) no carpool restriction; (ii) a carpool lane with realistic consideration of smoothing; and (iii) the hypothetical (and unrealistic) case of a carpool lane that does not induce smoothing. Predictions were made with the queueing/kinematic wave model of Newell (1993). Details are provided in Appendix B. Inputs to the analysis were estimated from the site's data: discharge flows were set equal to the average rates over multiple afternoons; and input flows were set equal to those measured at the upstream detector station during an afternoon when the queue did not grow beyond these detectors. This allowed us to measure upstream demand precisely, but corresponds to a day with lower than usual congestion. Thus, our results underestimate the differences that arise between our three scenarios on more typical days. Results are shown in Table 3.

Note from columns 2 through 4 which compare system performance with and without the carpool lane, the carpool lane reduces PHT by 30% compared to the case of no carpool lane. This is reassuring, since PHT-reduction is a commonly-cited reason for deploying carpool lanes in the first place (Turnbull and Capelle, 1998; Bracewell, et al. 1999; Henderson, 2003). But more remarkably, and thanks to the smoothing effect, the carpool lane reduces VHT by 15%.

Let us now see what a conventional analysis (wrongly ignoring the smoothing effect) as in Dahlgren (1998, 2002) and Kirshner (2001) would have predicted. The result is shown in column 5. By ignoring the smoothing effect, one would incorrectly attribute very large delays to the carpool lane. One

would be predicting increases well in excess of 300% both for PHT and VHT when instead the carpool lane would reduce both. This example clearly shows that one cannot assess the *real* impacts of a carpool lane without accounting for smoothing. A question of interest then is: what fraction of traffic must be carpools to justify a carpool lane?

Table 3 Predicted PHT's and VHT's

(1)	(2)	(3)	(4)	(5)	(6)
	No carpool restriction	Carpool lane with (real) Smoothing effect	% difference	Carpool lane without smoothing effect (hypothetical)	% difference with (2)
PHT (person-hrs)	2450	1900	– 30	10340	+ 322
VHT (veh-hrs)	1950	1700	– 15	9080	+ 365

Queueing analysis also shows that the carpool lane is beneficial even when demands for that lane are quite low. The boldfaced curves in Figs. 6(a) and (b) show the PHT and VHT obtained at a 4-lane site like ours, with and without a carpool lane, as a function of the percentage of freeway demand that is comprised of carpools, α. Note from Fig. 6(a) that the carpool lane reduces PHT when α is as low as 17%; and from Fig. 6(b) that only a slightly higher α (17.3%) is required to reduce the VHT. In the present case, $\alpha \approx 17\%$ corresponds to carpool-flows that are less than 1200 vph. So we see that, with smoothing, even a very underused carpool lane can reduce this freeway's PHT and VHT, with its attendant externalities, and therefore be a win-win proposition for society.
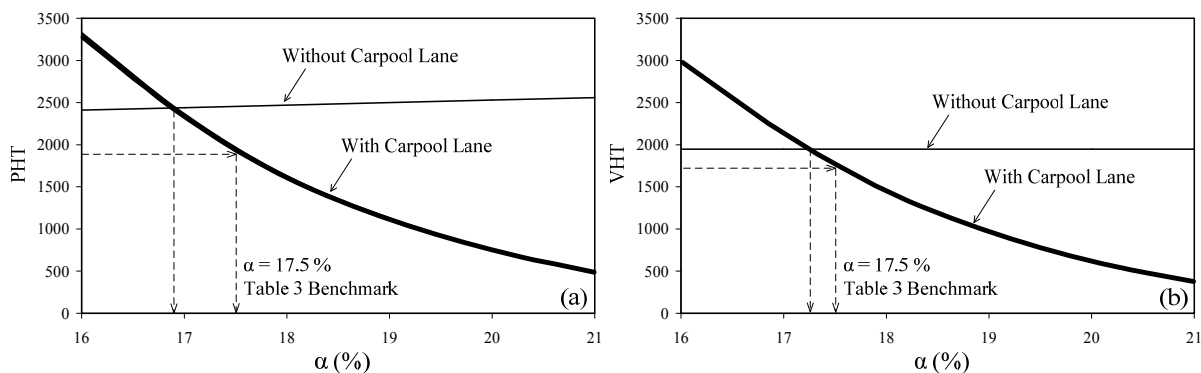

Figure 6 Predictions with and without Carpool Lane as functions of α, (a) PHT's; (b) VHT's

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5. Conclusions and Recommendations

This paper has shown that carpool lanes passing through bottlenecks significantly increase the discharge flows in lanes adjacent to the carpool lanes. The effect was consistently reproduced across days and sites. The effect is so pronounced that even an underutilized carpool lane can increase a bottleneck's total discharge rate. Queueing analysis illustrates that carpool lanes with flow as low as 1200 vph can reduce not only people delay, but even vehicle delay, and that the influence of smoothing on all this is very large. Thus, one cannot realistically assess the impacts of a carpool lane without accounting for smoothing. Given that smoothing was not observed furthest from the carpool lane in lane 4, the effect when induced by a carpool lane may be especially strong on narrow freeways with few lanes.

The findings suggest that freeway congestion could also be reduced by inducing the smoothing effect through other means. For example, roadside signing and (solid) painted lane striping might be used near certain bottlenecks to limit the disruptive impacts of lane changing. Disruptive lane changing might also be reduced in some cases by sorting drivers (and vehicle classes) across lanes according to their preferred travel speeds; or in other cases by inducing a more even distribution of flows across lanes; and these outcomes might be achieved by imposing lane-specific speed limits, based perhaps on real-time measurements of traffic. The above measures could be deployed on any freeway, whether or not it includes a carpool lane. For freeways with severely underused carpool lanes, one might even try to induce smoothing by rescinding carpool restrictions near bottlenecks at certain times only, e.g., as described in Daganzo, et al (2002). Though this latter dynamic strategy may be unconventional, simulations in Menendez and Daganzo (2006) indicate that it can significantly increase bottleneck capacity. Field experiments to test some of these ideas are now being planned.

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Appendix A: The Connection between the Carpool Restriction and Lane Changing Patterns

The evidence presented in this appendix indicates that *favorable* lane-changing patterns (i.e. patterns that ultimately induced the higher discharge flows in lanes 2 and 3) were triggered by the carpool restriction. This is explained with Fig A1. It uses arrows to illustrate the time-varying lane-changing patterns measured over the 0.4-km stretch upstream of the I-880 bottleneck (the darker shaded area in Fig. 1). Thin, solid arrows denote initial lane-changing rates; thick arrows increased rates; and dashed arrows diminished rates. Note from the line-weights of the arrows how the lane-changing rates diminished when comparing the period before 14:52 (before the carpool restriction came into play) to the period after 15:05 (when the restriction was in effect), as was mentioned in the text. Let us now examine the sequence of events in more detail.
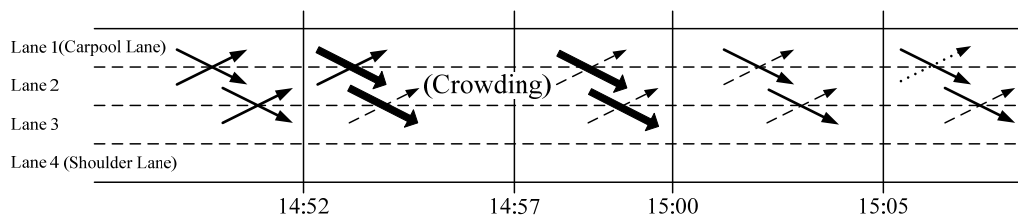

Figure A1 Lane-Changing Patterns Observed over Time

Consider first the vehicle maneuvers made out of lane 1 (the carpool lane) and into lane 2. This migration rate increased from 14:52 to 15:00 hrs, as depicted with the two boldfaced arrows that project from lane 1 to 2. We attribute this temporary increase to the impending carpool restriction since LOVs are required to vacate lane 1 by 15:00 hrs: early responses are to be expected since LOV-drivers risk fines for carpool violations.

The impending carpool restriction also discouraged drivers from maneuvering into lane 1 from lane 2. These movements started to decline at 14:57, and continued to diminish after the carpool lane activated, as depicted with the dashed arrows from lane 2 to 1.

The imbalance in lane-changing rates between lanes 1 and 2 created crowded conditions in lane 2 promptly after 14:52, and this had two effects. First, for a time the crowding pushed vehicles from lane 2 into lane 3. As shown by the thicker arrows, this push subsided at 15:00 hrs, when the heightened migration from lane 1 subsided as well. Second, the crowding discouraged maneuvers made into lane 2 from lane 3, as depicted with the dashed arrows from lane 3 to 2. Measurements quantifying the patterns of Fig. A1 are furnished in Figs. A2 – A4.

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In summary, we see that all the changes in the detailed traffic patterns between 14:52 and 15:05 hrs can be traced back to the to the carpool restriction. Since we cannot think of another plausible explanation, we conclude that the changes are indeed caused by the restriction.
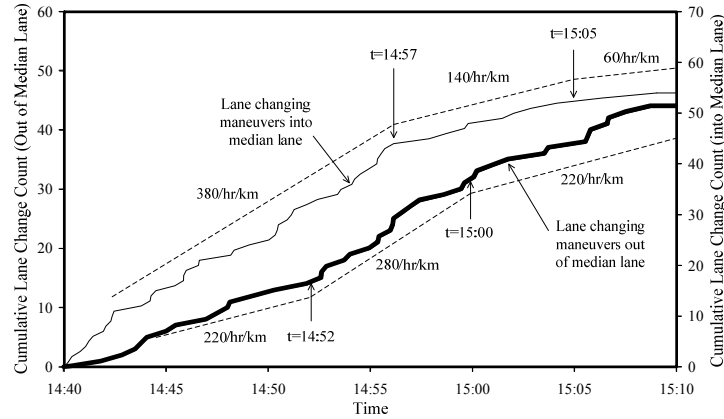

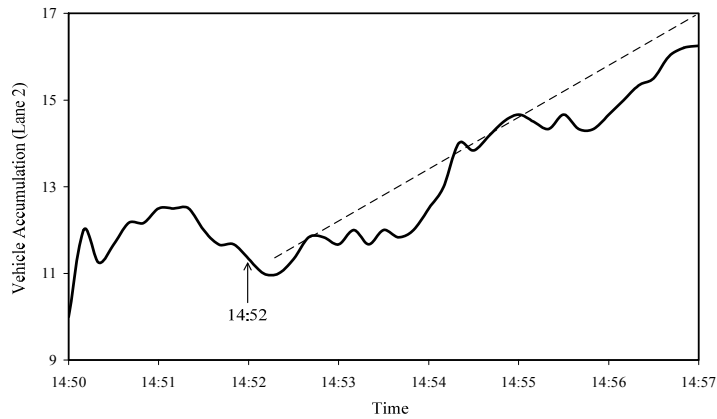Figure A2 Cumulative Curves of Lane Changes between Lanes 1 and 2 (July 19, 2006)

Figure A3 Vehicle Accumulation in Lane 2 (July 19, 2006) Measured over the 0.4-km Darker Shaded Segment in Fig. 1
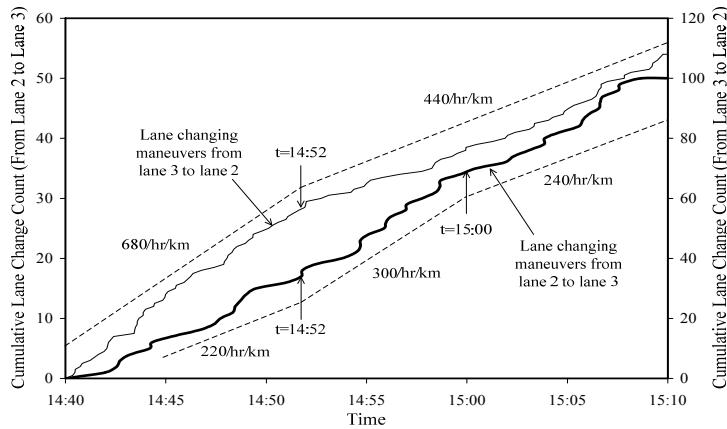

Figure A4 Cumulative Curves of Lane Changes between Lanes 2 and 3 (July 19, 2006)

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Appendix B: PHT and VHT Predictions for Site 1 (I-880)

This appendix presents our queueing analysis for site 1; see Fig. B1. Given are: (i) the demands on the freeway at X_U, denoted $q^f(t)$ (veh/hr), (ii) the fixed metered inputs from the on-ramps, denoted q_U, q_M and q_D; (iii) the discharge flows past X_D (immediately upstream of the Tennyson onramp), (iv) the fixed fraction of both LOVs and carpools exiting at X_M, β, and (v) the fixed fraction of total flow that is comprised of carpools, α. See Table B1 for these values.

Delays are calculated assuming that: (i) there is no delay beyond X_D; (ii) carpools entering from the on-ramps experience no significant delays as they access the carpool lane and do not create a more restrictive bottleneck in doing so; (iii) carpools exiting at X_M are already in the General Purpose (GP) lanes prior to arriving at X_U; and (iv) all vehicles delayed in GP lanes obey the kinematic wave theory with the parameters of Table B1.

We use the queuing representation of kinematic wave theory proposed in Newell (1993). Some care is required because the system exhibits two distinct phases: before and after carpool-lane deactivation. We consider first the scenario with a carpool lane that does not induce smoothing because it turns out to be the most complex. As a preliminary step, we construct a queuing diagram (Fig. B2) that only keeps track of those vehicles delayed in the GP lanes and destined for X_D: LOVs before 19:00 hrs (phase 1) and all vehicles after 19:00 hrs (phase 2). The delay of other vehicles will be calculated as a side product.

Note that the two phases are separated by a brief transition with curves shown by dotted lines. During this transition carpools and LOVs mix across all four lanes, and the change in discharge flow propagates upstream from X_D. This takes about 4 minutes. Since the transition is so short relative to the rush, it does not have to be modeled precisely. Thus the dotted curves are drawn linearly.

In the first phase, prior to 19:00, the V-curve displays the known cumulative number of desired departures at X_D for all LOVs, ending with vehicle N_A at point A. In the second phase, after the transition, the cumulative count $V(t') - N_A$ at some time $t' > 19:00$ hrs includes all vehicles with desired departures between 19:00 and t', including all the carpools present on the freeway at 19:00 and destined for X_D. This cumulative count is known from the data.

The D-curve is constructed in the conventional queuing way using as the service rate the discharge rate of the GP lanes minus the inflow from the Tennyson on-ramp. Note that the slope changes at 19:00 as the number of GP lanes changes from 3 to 4. The area between curves V and D is only the delay to those vehicles in the GP lanes destined for X_D.

To obtain the delay to all vehicles including those exiting via the off-ramp at X_M, we construct the departure curve at X_M which isolates the delay between X_U and X_M (darker area in the figure). This

construction is easy because the horizontal distances between the *M*- and *D*-curves in the two phases are the known vehicle delays in the segment from X_M to X_D (Lawson, et al, 1997). Knowing the *M*-curve, we can now compute the total delay in the system. Since only a fraction $1-\beta$ of the vehicles experiencing delay in the X_U to X_M segment is captured in the figure (the figure ignores the fraction β that exits at X_M), the total delay in that segment is $1/(1-\beta)$ times greater than the darker area shown. To this we must add the lighter shaded area in the figure and the delay to the carpool lane vehicles. The latter was estimated as the product of the carpools' average extra pace, *Δp*,(see Table B1) and their vehicle-miles traveled on the site.

To convert this total delay into VHT, we must add the free-flow travel hours; i.e. the product of the vehicle-kilometers traveled and the free-flow speed, both known. To obtain PHT, averages for the number of occupants per LOV and per carpool (known from earlier field observation, Caltrans, 2004) were used to convert VHT to PHT on both the upstream and downstream parts of our freeway. This concludes the analysis of the most difficult case.
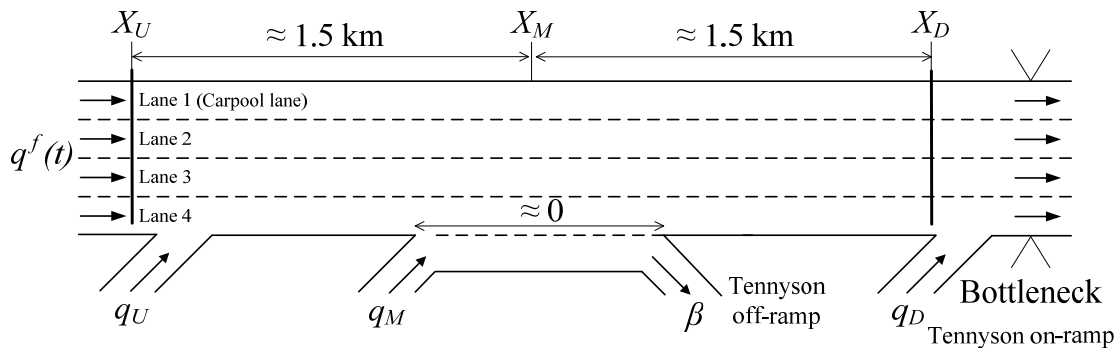

Figure B1 I-880 Study Site with Added Notation

Table B1 Estimated Inputs

Input variables (Units)	$q_f(t)$ (vph)	q_U (vph)	q_M (vph)	q_D (vph)	β (%)	α (%)	Δp (hrs/km)	LOV average person Occupancy	Carpool average person occupancy
Value(s) estimated from data	5400-6300	500	700	700	12	17.5	0.043	1	2.5
Triangular-shaped fundamental diagram (estimated from data): free flow vehicle speed = 105 km/hr, jam density = 75 vehs/km/lane, GP-lane capacity (discharge flow) = 1,900 vph/lane (with smoothing), or 1,740 vph/lane (without smoothing or without carpool lane).									

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The case of a carpool lane with smoothing, and the case of no carpool restriction were analyzed in similar, but simpler fashion. For the case of no carpool restriction, the queueing analysis was of a FIFO system with a single bottleneck capacity (commensurate with four freeway lanes and no smoothing). The case of a carpool lane with smoothing was also analyzed using a single capacity for the (3-lane) bottleneck, because the rush-hour queue predicted in this case did not persist beyond the carpool lane deactivation time.
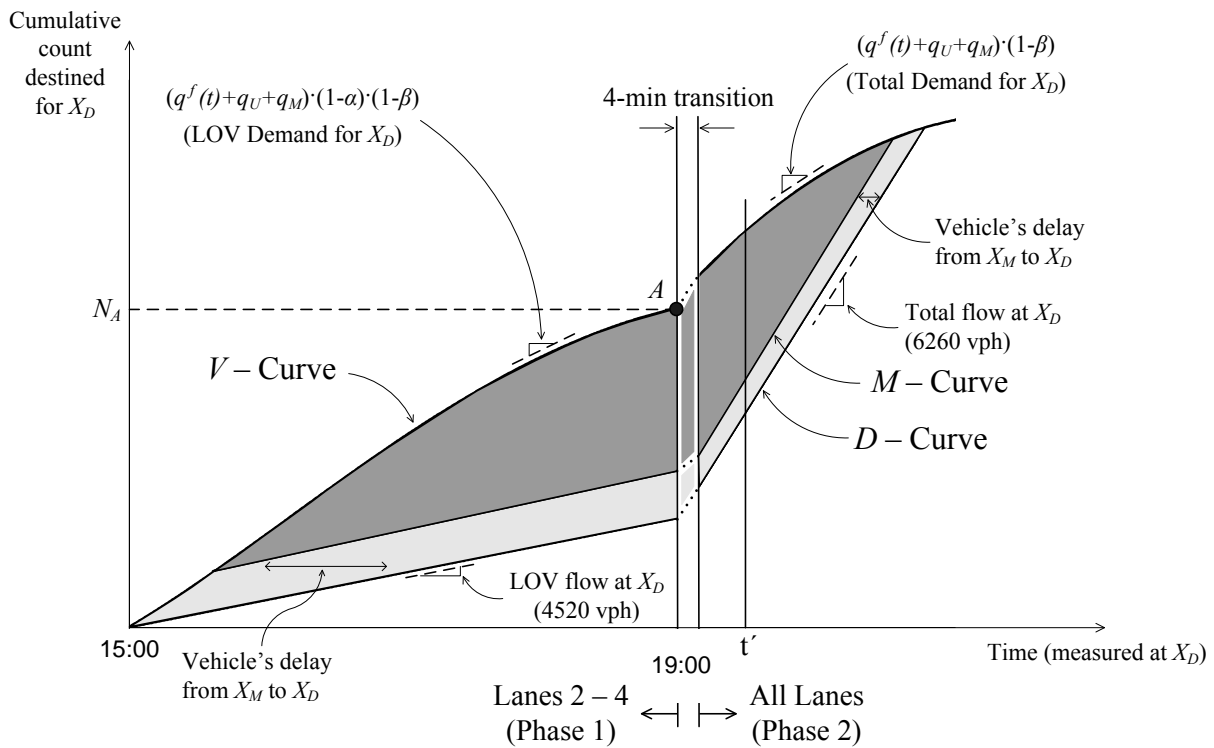
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Figure B2 Queueing Diagram for the Case of a Carpool Lane without Smoothing

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