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Inequality, Market Imperfections, and Collective Action Problems.

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Abstract

In this paper we analyze the effects of wealth inequality on the provision of public goods and management of common-property resources (CPR) when there are market imperfections in inputs that are complementary in production to the collective good. We show that for public goods inequality impedes efficiency, while for use of CPRs there is an inverse U-shaped relationship between inequality and efficiency. We discuss the implications of these theoretical results for redistributive policies such as land reform.

Keywords : Public goods, common property resources, and inequality.

J.E.L. classification numbers : D63, D71, H41, O12, O13.

1   Introduction

In social, political and economic life collective action problems are pervasive (see Olson, 1965 for an influential treatment). In this paper we examine how inequality in asset ownership among members of a group affects various types of collective action problems. Research on the interrelationship between economic inequality and collective action goes back to Olson's (1965) argument that if inequality leads to the emergence of one dominant player, then that could alleviate collective action problems through internalization of the externalities. Subsequent research in the public economics literature came up with what Cornes and Sandler (1996) call a distribution-neutrality result for pure public goods: if individual contributions are perfect substitutes in the production of the public good, then in a Cournot-Nash

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equilibrium wealth distribution (within the set of contributors) does not matter for the amount of public goods provision.[1] The neutrality result breaks down when individual contributions are complements in the production of the public good (Cornes and Sandler, 1996, pp. 184-190) or, for the impure public good model with private and pure public joint products (Cornes and Sandler, 1994).

In this paper we extend this literature to examine the effect of distribution of wealth among members of a given community on allocative efficiency in various types of collective action problems (involving public goods as well as common property resources or CPR) in the presence of missing and imperfect markets. This is particularly important in less developed countries where the life and livelihood of the vast masses of the poor crucially depend on the provision of public goods (roads, canal irrigation, public health and sanitation) and the local CPR (forestry, fishery, grazing lands), and where markets for land and credit are often highly imperfect or non-existent. Under the circumstances the role of inequality in wealth (land, capital), and of redistributive policies (land reform, credit market reform) in the provision of such collective goods can be rather complex.

Recent research in the literature on economic growth has provided some evidence that initial inequality in the distribution of income and assets tend to have a negative effect on levels and growth rates of per capita income in cross country studies (See Benabou, R., 1996). The two main reasons that are advanced for this connection are, first, inequality increases agency costs in the labor and credit markets (e.g., Banerjee and Newman 1993) and second, inequality encourages redistributive policies that discourage capital accumulation (e.g., Alesina and Rodrik, 1994). Our paper suggests a possible link that has been largely neglected in the literature : inequality may accentuate certain types of collective action problems. That this sort of a link in explaining differential growth performances across countries is not without empirical basis is suggested by recent work by Knack and Keefer (1997) who find that measures of social cohesion (measured by among other things the propensity of people to join voluntary organizations), which by their very nature are outcomes of a various types of collective action problems, positively affect per capita income growth rates across countries. They also found that the level of social cohesion is strongly and negatively associated with economic inequality.[2]

We consider two simplified settings. First we look at an economic environment where the players use as inputs one private good (say, land) and one public good (say, irrigation water) to produce a private good. The quantity of the public good is

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[1] Some of the contributions to the theoretical literature related to this result are Warr (1983), Cornes and Sandler (1984), Bergstrom, Blume and Varian (1986), Bernheim (1986) and Itaya, de Meza and Myles (1997).

[2] See also Temple and Johnson (1998). Putnam's (1993) well known study of regional disparities in Italy has also emphasized the importance of social capital and how "horizontal" social networks (i.e., those involving people of similar status and power) are more effective in generating trust and norms of reciprocity than "vertical" ones.

determined by contributions from the players, but these are not collectively decided to maximize joint surplus. Rather, it is determined by each player deciding on her individual contribution taking the contributions of other players as given. This leads to the standard problem of underprovision. What we show is that if the market for the private input is imperfect so that its marginal product is not equated across all production units, then redistributing it among players will improve efficiency by bringing the marginal products of the private inputs across production units closer to each other, and, by raising the equilibrium amount of provision of the public input. The reason forced redistribution raises joint profits in this case but are not achieved through voluntary trade are due to the presence of two market frictions : first, the non tradeability of the private input due to informational problems or transaction costs, and second, the fact that individuals do not determine the amount of the public input cooperatively.

Second, we look at a very different economic environment where players use as inputs one private good (say, boats) and one CPR (say, fishery). As in the standard textbook case, utilization of the CPR is subject to congestion: the more is the total use, the less productive it is for everybody. Here we show that the relationship between inequality and efficiency is inverse U-shaped.[3] An analogous argument shows why the distribution of wealth will be less unequal than optimal for the case of the CPR. These results are also relevant to a part of the macro political economy literature where the public fisc is sometimes taken as the commons where different interest groups lobby and compete for transfers, and the congestion externalities have implications for economic growth.[4]

The theoretical results of our paper have also some bearings for the small but growing empirical literature on inequality and collective action. Chan, Mestelman, Moir and Muller (1996) test the public goods model of Bergstrom, Blume and Varian (1986) in the laboratory and find that individuals with low incomes overcontribute (relative to the prediction of the model) to the public good, and high-income individuals undercontribute. Tang (1991) found that low variance of the average income among irrigators leads to better maintenance from a study of 23 community irrigation systems in different countries. Easter and Palanisami (1986) show that the smaller the variance of farm size, the more likely they are to form water organizations.[5] In a

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[3]In a two-period model of conservation on the local commons by players with differential assets, Dayton-Johnson and Bardhan (1999) also get a U-shaped relationship between asset inequality and conservation, but their general model is different, using linear production technology and concentrating on intertemporal conservation. See also Baland and Platteau (1997) for alternative effects of inequality in resource-use entitlements, depending on the nature of the resource-use technology.

[4]This is the implicit model of the political-economic gridlock in Indian development described in Bardhan(1984); for more explicit theoretical treatments, see Tornell and Velasco (1992), and Benhabib and Rustichini (1996).

[5]See Bardhan (1993) for a review of the case-study literature regarding the relationship between inequality and farmer cooperation in locally-managed irrigation systems in some developing

recent econometric study of 48 irrigation communities in south India Bardhan (1999) finds that the Gini coefficient for inequality of landholding among the irrigators has a significant negative effect on cooperation on water allocation and field channel maintenance (and there is some weak evidence for a U-shaped relationship). Similar results have been reported by Dayton-Johnson (1998) from his econometric analysis of 54 farmer-managed surface irrigation systems in central Mexico. In a recent paper La Ferrara (1998) has shown that the probability of formation of various types of groups such as cooperatives and rotating saving and credit associations is negatively related to inequality of asset ownership at the village level controlling for other types of heterogeneity (e.g., in education, types of economic activity) using data from rural Tanzania.

In section 2 we analyze the public goods case and in section 3 the case of the commons. In section 4 we discuss how distribution of wealth can affect the size and composition of the pool of participants (assumed as given in the earlier two sections) in these collective action problems. We end with some general remarks in section 5.

2   The Public Good Problem

Consider an economy where there are n agents. Each agent produces output using a private good (e.g., capital or land) and a public good (e.g., infrastructure). Agent i is endowed with $w_i \geq 0$ units of the private good (henceforth, wealth). The payoff of player i is:

$$\pi(k_i, X) = k_i^\alpha X^\beta - x_i - (k_i - w_i)r \tag{1}$$

where k_i denotes the amount of the private input, X denotes the amount of the public good, x_i denotes the contribution of player i towards the public good, and r is the rate of interest.[6] The public good is produced from voluntary contributions from all players, i.e., $X = g(x_1, x_2, .., x_n)$ where x_i is the contribution of agent i. We assume that the production function is subject to decreasing returns with respect to the private and the public input :

$$\alpha + \beta < 1.$$

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countries.

[6]There is some connection between our production function and those used in the literature on endogenous growth such as Romer (1990). The main difference is that in our model people partly (i.e., in a Cournot-Nash fashion) internalize the effect of their choices on the payoff of others.

2.1 Individual Contributions as Substitutes

First consider the case of a pure public good, namely, where individual contributions are perfect substitutes in the production of the public good :

$$X = \sum_{i=1}^{n} x_i.$$

The decision problem of player i is to maximize (1). Let X_{-i} denote the contributions of other players which player i takes as given. Then her decision problems can be written as

$$\max_{x_i \leq w_i} k_i^{\alpha} \left(x_i + X_{-i} \right)^{\beta} - x_i - (k_i - w_i)r$$

and the first-order conditions are

$$\alpha k_i^{\alpha-1} \left(x_i + X_{-i} \right)^{\beta} = r$$
$$\beta k_i^{\alpha} \left(x_i + X_{-i} \right)^{\beta-1} = 1$$

These two conditions yield, $k_i = \frac{\alpha \; X}{\beta r}$. Substituting back in any one of the first-order conditions, we get

$$x_i = \left(\alpha^{\alpha} \frac{1}{\beta^{\alpha-1}} \frac{1}{r^{\alpha}} \right)^{\frac{1}{1-(\alpha+\beta)}} - X_{-i}$$

Adding both sides of this condition across all individuals we get, $X = n \left(\alpha^{\alpha} \frac{1}{\beta^{\alpha-1}} \frac{1}{r^{\alpha}} \right)^{\frac{1}{1-(\alpha+\beta)}} - (n-1)X$, or,

$$X = \left(\alpha^{\alpha} \frac{1}{\beta^{\alpha-1}} \frac{1}{r^{\alpha}} \right)^{\frac{1}{1-(\alpha+\beta)}} \qquad (2)$$

The market clearing interest rate is obtained from the condition

$$\sum_{i=1}^{n} k_i = W$$

where W is the total wealth level. Hence

$$r = \frac{n\alpha \; X}{\beta W} = \frac{\alpha \; X}{\beta \bar{w}}$$

where $\bar{w} \equiv \frac{1}{n} \sum_{i=1}^{n} w_i$ is the mean value of w. Substituting in (2) we get

$$X = \beta^{\frac{1}{1-\beta}} \bar{w}^{\frac{\alpha}{1-\beta}}. \qquad (3)$$

Hence in the symmetric Nash equilibrium wealth distribution does not matter. This turns out to be a general result with identical preferences that does not depend on homotheticity of the payoff function (Bernheim, 1986).

5

We show that this result depends crucially on the absence of any form of market frictions other than the public good problem. Otherwise, even with the pure public good where individual contributions are perfect substitutes, one can have interesting wealth effects. Suppose that the input w is not tradable and as before, the public good is complementary with this input in the production process. Then the profit function of player i is

$$\pi(x_i, X_{-i}; w_i) = w_i^\alpha X^\beta - x_i.$$

The assumption that the market for w does not exist at all, while stark, is not crucial. All that is needed is that the amount of this input that a person can buy or lease in depends positively on how wealthy she is. Various models of market imperfections based on adverse selection, moral hazard, costly state verification or imperfect enforcement has been shown to lead to this property in the case of credit markets (e.g., Stiglitz and Weiss, 1981, Townsend, 1979).

Differentiating with respect to x_i we get

$$\beta w_i^\alpha \ \left(x_i^* + X_{-i}^*\right)^{\beta-1} = 1$$

This yields

$$x_i^* = (\beta w_i^\alpha)^{\frac{1}{1-\beta}} - X_{-i}^*.$$

Adding up the first-order conditions for all agents, we get

$$X^* = \sum_{i=1}^n (\beta w_i^\alpha)^{\frac{1}{1-\beta}} - (n-1)X^*.$$

This simplifies to

$$X^* = \beta^{\frac{1}{1-\beta}} \left(\frac{1}{n}\sum_{i=1}^n w_i^{\frac{\alpha}{1-\beta}}\right). \tag{4}$$

Let $w_i^{\frac{\alpha}{1-\beta}} \equiv f(w_i)$. Since $\alpha+\beta < 1$, $\frac{\alpha}{1-\beta} < 1$ and $f(.)$ is strictly concave. As a result *any* mean-preserving spread in the distribution of w will reduce X^* and conversely, *any* mean-preserving redistribution of w from the above average to below average (e.g., transferring $\varepsilon > 0$ from player 2 with $w_2 > \bar{w}$ to player 1 with $w_1 < \bar{w}$) will raise X^*. In particular, a direct comparison of (3) and (4) shows that the provision of public good is lower when the market for the private input is imperfect.

Consider the effect of redistribution from 2 to 1 (where $w_2 > w_1$) on joint profits $\Pi = \left(\sum w_i^\alpha\right) X^\beta - X$. We consider redistributions that does not completely expropriate any player. Then

$$\frac{d\Pi}{d\varepsilon} = \alpha \left(\frac{1}{w_1^{1-\alpha}} - \frac{1}{w_2^{1-\alpha}}\right) X^\beta + \left[\beta \left(\sum w_i^\alpha\right) X^{\beta-1} - 1\right] \frac{dX}{d\varepsilon}. \tag{5}$$

Since $w_1 < w_2$, for a given level of X redistribution raises joint surplus by bringing the allocation of wealth closer to that under perfect credit markets, i.e., the first term is positive. Also, we proved above that it will also raise the total amount of the public good, i.e., $\frac{dX}{d\varepsilon} > 0$. If X was chosen cooperatively to maximize joint profits, then the amount chosen will be $\beta^{\frac{1}{1-\beta}} \left(\sum_{i=1}^{n} w_i^{\alpha}\right)^{\frac{1}{1-\beta}}$. By Minkowski's inequality $\left(\sum_{i=1}^{n} w_i^{\alpha}\right)^{\frac{1}{1-\beta}} > \sum_{i=1}^{n} (w_i^{\alpha})^{\frac{1}{1-\beta}}$.[7] Hence, $\beta^{\frac{1}{1-\beta}} \left(\sum_{i=1}^{n} w_i^{\alpha}\right)^{\frac{1}{1-\beta}} > \beta^{\frac{1}{1-\beta}} \left(\frac{1}{n} \sum_{i=1}^{n} w_i^{\frac{\alpha}{1-\beta}}\right) = X^*$. This means evaluated at $X = X^*$, $\beta \left(\sum w_i^{\alpha}\right) X^{\beta-1} - 1 > 0$, i.e., we have underprovision of X with respect to the situation where X was chosen cooperatively to maximize joint profits. Suppose there are two players only and we completely equalize their wealth levels. Since $\frac{dX}{d\varepsilon} = 0$ evaluated at $w_1 = w_2 = w$ this proves perfect equality will maximize joint surplus keeping the pool of players constant.

But suppose one considers redistributions that change the number of contributors, such as by completely expropriating one player. For simplicity, let us consider a two player game. If as a result of such redistribution, one player becomes the sole decision-maker, she will choose $X = \beta^{\frac{1}{1-\beta}} (w_1 + w_2)^{\frac{\alpha}{1-\beta}}$. It is readily seen that $(w_1 + w_2)^{\frac{\alpha}{1-\beta}} > \left(\frac{w_1+w_2}{2}\right)^{\frac{\alpha}{1-\beta}}$. Also from the above argument, $\left(\frac{w_1+w_2}{2}\right)^{\frac{\alpha}{1-\beta}} > \frac{1}{2}\left(w_1^{\frac{\alpha}{1-\beta}} + w_2^{\frac{\alpha}{1-\beta}}\right)$ unless $w_1 = w_2$ in which case they are equal. This means that redistributions that effectively converts the game into a single player game gets rid of the public good problem and achieves the maximum amount of the public good. Olson (1965), as mentioned before, had highlighted this role of inequality in alleviating inefficiency in public good problems. But if one looks at joint profits, changes in the distribution of wealth $(w_1 + w_2 - \varepsilon, \varepsilon)$ as $\varepsilon \to 0$ the inefficiency due to the misallocation of the private input (the first term in (5)) becomes arbitrarily large. This means the overall surplus maximizing strategy is to achieve a completely egalitarian distribution of wealth. But the relationship between the amount of the public good and wealth inequality is non-monotonic : the former is decreasing with inequality with a local maximum at the point of perfect equality, but at the point of extreme inequality the amount of public good provision reaches a global maximum. This conclusion generalizes easily for $n > 2$. Complete expropriation of a player reduces n and hence partly alleviates the inefficiency due to the public good aspect of the problem. If this wealth is distributed among the remaining players in the pool to make the wealth distribution of participating players more equal, then that too will alleviate inefficiency. But the misallocation due to the fact that the private input is infinitely productive in the expropriated player's production unit overwhelms these gains.

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[7]See Hardy, Littlewood, and Polya (1952), p. 31-32.

The conclusion that emerges from this analysis is that :

Proposition 1 *Suppose production depends on a private input that is complementary with a public input where individual contributions are perfect substitutes and due to some form of transaction costs the marginal product of the private input is not equalized across production units. Then greater equality in the distribution of the private input will raise joint profits if there is joint diminishing returns to the private and the public input. Perfect equality maximizes joint profits.*

Admittedly, our result crucially depends on the assumption that as $w \to 0$, the marginal product of w goes to ∞ (i.e., the Inada conditions are satisfied). If this is not the case (namely the marginal product of w approaches some finite number as $w \to 0$) then there is a trade off and it is possible that extreme inequality can dominate perfect equality.

Another key assumption in the above analysis is joint diminishing returns to the private and the public input. Suppose $\alpha + \beta > 1$ even though $\alpha < 1$ and $\beta < 1$. Then our result will be reversed. In particular, the expression for X^* (in (4)) is the simple average of a convex function $w_i^{\frac{\alpha}{1-\beta}}$ and hence any mean preserving spread in the distribution of w will raise X^*. Since in this case any mean preserving spread in the distribution of w will also raise the efficiency of the use of the private input across production units, greater inequality must raise joint profits.

There is a large literature showing that small farms are more efficient than large farms in agricultural sector of developing countries. Together with the evidence of agency costs associated with sharecropping tenancy this typically provides the efficiency argument for land reform. It has been suggested that if ownership of land provides better access to credit then the sum total of productivity gains from land reform may be even greater.[8] Indeed, the limited evidence that is available on the effect of land tenure reform suggests that the productivity gains can be very large (Banerjee, Gertler and Ghatak, 1998). Proposition 1 suggests an alternative explanation for why land reform can lead to large improvements in productivity - namely, by improving efficiency of provision of public goods.

2.2   Individual Contributions as Complements

Now let us consider relaxing the assumption of individual contributions are perfect substitutes in the production of the public good. Our results are strengthened if public good production displays the weak link property. Let us take a two player example following Cornes and Sandler (1996). The only difference with their framework is

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[8]See the recent textbook by Ray, 1998 for a lucid discussion of the literature on this issue.

the presence of a non-tradable input which is complementary with the public good in production.

$$\pi(x_i, w_i) = w_i^\beta \left(x_1^{\frac{\alpha}{2}} x_2^{\frac{\alpha}{2}} \right) - x_i.$$

The first order condition for agent i $(i = 1, 2)$ is

$$\frac{\alpha}{2} \; w_i^\beta \left(x_i^{\frac{\alpha}{2}-1} x_j^{\frac{\alpha}{2}} \right) - 1 = 0.$$

If both agents choose their actions non-cooperatively then in the resulting Nash equilibrium the levels of x_i is :

$$x_i = \left[\frac{\alpha}{2} \; \left(w_i^{1-\frac{\alpha}{2}} w_j^{\frac{\alpha}{2}} \right)^\beta \right]^{\frac{1}{1-\alpha}}.$$

The equilibrium level of X is

$$\left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\alpha}} (w_1 w_2)^{\frac{\alpha\beta}{2(1-\alpha)}}$$

which is obviously concave with respect to w_1 and w_2 as $\frac{\alpha\beta}{2(1-\alpha)} < 1$ ($\frac{\beta}{1-\alpha} < 1$ as $\alpha + \beta < 1$, and $\frac{\alpha}{2} < 1$). The corresponding level of aggregate output is

$$\left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(w_1^\beta + w_2^\beta \right) (w_1 w_2)^{\frac{\alpha\beta}{2(1-\alpha)}} \right]$$

Note that this can be written as

$$\left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\alpha}} \left(w_1^{\beta \frac{2-\alpha}{2(1-\alpha)}} \; w_2^{\frac{\alpha\beta}{2(1-\alpha)}} + w_2^{\beta \frac{2-\alpha}{2(1-\alpha)}} \; w_1^{\frac{\alpha\beta}{2(1-\alpha)}} \right).$$

Since $\beta \frac{2-\alpha}{2(1-\alpha)} + \frac{\alpha\beta}{2(1-\alpha)} = \frac{\beta}{1-\alpha} < 1$, this expression is a sum of two concave functions of w_1 and w_2, hence a concave function itself. Hence this too displays inequality aversion. Ex ante aggregate surplus, which aggregate output less the cost of inputs too displays the same property :

$$\left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\alpha}} (1 - \frac{\alpha}{2}) \left(w_1^{\beta \frac{2-\alpha}{2(1-\alpha)}} \; w_2^{\frac{\alpha\beta}{2(1-\alpha)}} + w_2^{\beta \frac{2-\alpha}{2(1-\alpha)}} \; w_1^{\frac{\alpha\beta}{2(1-\alpha)}} \right)$$

From this discussion, we conclude :

Remark 1 *When public goods production displays the weak link property, and there is joint diminishing returns with respect to the private and the public inputs, then greater equality in the distribution of the private input will raise efficiency.*

3   The Commons Problem

Now we turn to the commons problem. It has the feature that an increase in the action level of another party reduces an agent's payoff. Like in the previous section, we assume that the degree to which the common property resource (CPR) is exploited by an individual, x_i, is complementary with a private input w_i in the production process and there is joint diminishing returns with respect to these two inputs.

Let the total amount of use of the CPR is X. Again, for analytical simplicity assume that the production function takes a Cobb-Douglas form :

$$
\begin{aligned}
\pi^i(x_i, X) &= w_i^\alpha \left[\left(\frac{x_i}{X} \right)^\theta X^\gamma \right]^\beta - x_i \\
&= w_i^\alpha x_i^{\beta\theta} X^{\beta(\gamma-\theta)} - x_i
\end{aligned}
$$

where α, β, θ and γ are all strictly positive and less than 1, $\theta \geq \gamma$, and in addition, the production function displays decreasing returns in the direct effects of the private inputs w_i and x_i, i.e. $\alpha + \beta\theta < 1$. We have a standard two input neoclassical production function with one difference. The second input, $G_i = \left(\frac{x_i}{X} \right)^\theta X^\gamma$ is the i-th agent's share of the common resource which is subject to congestion effects. If $\theta = \gamma$ then we have a pure private good which is not subject to any congestion effects. On the other extreme, if $\theta = 1$ then the players share the benefits of a public good such that the shares add up to 1. The shares are proportional to contributions. This is the classic example of a CPR: the productivity of the resource depends on the total use of all players, and each player receives a share which is proportional to her use.

The first-order condition for individual i is:

$$
\beta w_i^\alpha x_i^{\beta\theta-1} X^{\beta(\gamma-\theta)-1} \left[\theta X + (\gamma - \theta)x_i \right] - 1 = 0 \tag{6}
$$

In the appendix we verify that the second-order condition is satisfied. Let us write this condition in the following form:

$$
f(w_i)h(x_i, X) = 1
$$

where

$$
h(x_i, X) \equiv \beta \, x_i^{\beta\theta-1} X^{\beta(\gamma-\theta)-1} \left[\theta X + (\gamma - \theta)x_i \right].
$$

and

$$
f(w_i) = w_i^\alpha.
$$

We prove :

Proposition 2 *Suppose production depends on a private input that is complementary with a CPR and due to some form of transaction costs the marginal product of the private input is not equalized across production units. If there is joint diminishing returns to the private and CPR then the relationship between inequality and joint profits is inverse U shaped. There is a unique optimal degree of inequality that maximizes joint profits.*

Proof: First we establish some comparative static properties of x_i with respect to w_i holding X constant. Let us rewrite the first-order condition as

$$x_i = g(w_i, X).$$

Hence

$$\frac{\partial x_i}{\partial w_i}\Big|_{X \text{ constant}} = -\frac{f'(w_i)}{f(w_i)} \frac{h(x_i, X)}{h_1(x_i, X)} > 0$$

and

$$\frac{\partial^2 x_i}{\partial w_i^2}\Big|_{X \text{ constant}} = -\frac{h(x_i, X)}{h_1(x_i, X)} \frac{1}{\{f(w_i)\}^2} \left[f''(w_i) f(w_i) - \{f'(w_i)\}^2 \left(2 - \frac{h(x_i, X) h_{11}(x_i, X)}{h_1^2(x_i, X)} \right) \right].$$

Now $f''(w_i) f(w_i) = -\alpha(1-\alpha) w_i^{2\alpha-2}$, $\{f'(w_i)\}^2 = \alpha^2 w_i^{2\alpha-2}$ and

$$\begin{aligned} h_{11} &= (2 - \beta\theta)\beta\theta(1 - \beta\theta)x_i^{\beta\theta-3} X^{\beta(\gamma-\theta)} + (1-\beta\theta)\beta^2\theta(\theta-\gamma)x_i^{\beta\theta-2} X^{\beta(\gamma-\theta)-1} \\ &= \beta\theta(1-\beta\theta) x_i^{\beta\theta-3} X^{\beta(\gamma-\theta)-1}\{(2-\beta\theta) X + \beta(\theta-\gamma)x_i\} \end{aligned}$$

The expression for $\frac{h_{11}(x,X))h(x_i,X)}{(h_1(x_i,X))^2}$ is

$$\frac{(1-\beta\theta)}{\theta} \frac{\theta(2-\beta\theta) - 2(\theta-\gamma)(1-\beta\theta)z_i - \beta(\theta-\gamma)^2 z_i^2}{[(1-\beta\theta) + \beta(\theta-\gamma)z_i]^2}.$$

Since $\theta > \gamma$, we can see that the expression $\frac{h(x_i,X)h_{11}(x_i,X)}{h_1^2(x_i,X)}$ is decreasing in $\frac{x_i}{X}$. So the maximum possible value it can take is when $\frac{x_i}{X} = 0$, namely, $\frac{2-\beta\theta}{1-\beta\theta}$. Hence in this case, the sign of $\frac{\partial^2 x_i}{\partial w_i^2}\Big|_{X \text{ constant}}$ is the opposite of that of $(1-\alpha) - \alpha\frac{\beta\theta}{1-\beta\theta} = \frac{1-\alpha-\beta\theta}{1-\beta\theta} > 0$. Hence $\frac{\partial^2 x_i}{\partial w_i^2}\Big|_{X \text{ constant}}$ is always negative.

Suppose that there are two players. Adding the two first order conditions we get

$$X = g(w_1, X) + g(w_2, X)$$

Assume $w_1 < w_2$. Then consider the effect of a mean preserving spread:

$$X = g(w_1 + \varepsilon, X) + g(w_2 - \varepsilon, X)$$

Totally differentiating with respect to ε we get

$$
\begin{aligned}
dX &= \{g_1(w_1 + \varepsilon, X) - g_2(w_2 - \varepsilon, X)\}d\varepsilon + \{g_2(w_1 + \varepsilon, X) + g_2(w_2 - \varepsilon, X)\}dX \\
\frac{dX}{d\varepsilon} &= \frac{g_1(w_1 + \varepsilon, X) - g_2(w_2 - \varepsilon, X)}{1 - \{g_2(w_1 + \varepsilon, X) + g_2(w_2 - \varepsilon, X)\}}.
\end{aligned}
$$

As x_i is concave with respect to w_i, $g_1(w_1 + \varepsilon, X) > g_2(w_2 - \varepsilon, X)$ (so long as $w_1 + \varepsilon < w_2 - \varepsilon$ or $\varepsilon < \frac{w_2 - w_1}{2}$). We need to show that

$$1 - \{g_2(w_1 + \varepsilon, X) + g_2(w_2 - \varepsilon, X)\} > 0. \tag{7}$$

Totally differentiating player i's first-order condition, we get

$$g_2(w_i, X) = -\frac{h_2(x_i, X)}{h_1(x_i, X)}$$

Hence this condition can be rewritten as

$$1 + \frac{h_2(x_1, X)}{h_1(x_1, X)} + \frac{h_2(x_2, X)}{h_1(x_2, X)} > 0$$

or,

$$h_1(x_1, X)h_1(x_2, X) + h_2(x_1, X)h_1(x_2, X) + h_2(x_2, X)h_1(x_1, X) > 0. \tag{8}$$

This turns out to be the same as the strategic stability condition in the Cournot-Nash model :

$$\frac{\partial^2 \pi^1}{\partial x_1^2}\frac{\partial^2 \pi^2}{\partial x_2^2} - \frac{\partial^2 \pi^1}{\partial x_1 \partial x_2}\frac{\partial^2 \pi^2}{\partial x_1 \partial x_2} > 0$$

as the left-hand side of this expression is

$$
\begin{aligned}
&\{h_1(x_1, X) + h_2(x_1, X)\}\{h_1(x_2, X) + h_2(x_2, X)\} - h_2(x_1, X)h_2(x_2, X) \\
&= h_1(x_1, X)h_1(x_2, X) + h_2(x_1, X)h_1(x_2, X) + h_2(x_2, X)h_1(x_1, X).
\end{aligned}
$$

In the appendix we show that the strategic stability condition is indeed satisfied.

Now we consider the effect of redistribution from 2 to 1 (where $w_2 > w_1$) on joint profits $\Pi = X^{\beta(\gamma - \theta)}\left(w_1^\alpha x_1^{\beta\theta} + w_2^\alpha x_2^{\beta\theta}\right) - X$. This is, using the envelope theorem,

$$\frac{d\Pi}{d\varepsilon} = \alpha X^{\beta(\gamma - \theta)}\left(\frac{x_1^{\beta\theta}}{w_1^{1-\alpha}} - \frac{x_2^{\beta\theta}}{w_2^{1-\alpha}}\right) + \beta(\theta - \gamma)X^{\beta(\gamma - \theta) - 1}\left(w_1^\alpha x_1^{\beta\theta}\left(-\frac{dx_2}{d\varepsilon}\right) - w_2^\alpha x_2^{\beta\theta}\frac{dx_1}{d\varepsilon}\right).$$

Since we have already established that $\frac{dX}{d\varepsilon} > 0$, it must be that $\frac{dx_1}{d\varepsilon} > \left(-\frac{dx_2}{d\varepsilon}\right)$. This means the second expression is negative (since we know that x_i is increasing in $w_i, w_2^\alpha x_2^{\beta\theta} > w_1^\alpha x_1^{\beta\theta}$). We show that the first expression is positive. Rearranging the first-order condition of player i we have:

$$w_i^\alpha = \frac{x_i^{1-\beta\theta} X^{\beta(\theta-\gamma)}}{\beta\{\theta - (\theta-\gamma)\frac{x_i}{X}\}}$$

Hence

$$\frac{x_i^{\beta\theta}}{w_i^{1-\alpha}} = \frac{x_i^{-\frac{1-\alpha-\beta\theta}{\alpha}} \beta^{\frac{1-\alpha}{\alpha}} \{\theta - (\theta-\gamma)\frac{x_i}{X}\}^{\frac{1-\alpha}{\alpha}}}{X^{\frac{\beta(\theta-\gamma)(1-\alpha)}{\alpha}}}.$$

Since this expression is decreasing in x_i, $\frac{x_1^{\beta\theta}}{w_1^{1-\alpha}} > \frac{x_2^{\beta\theta}}{w_2^{1-\alpha}}$. Notice that for $w_1 = w_2$, both terms are zero. Since the second term is always negative for $w_1 < w_2$ while the first term is always positive, this means the former achieves a minimum for $\varepsilon = \frac{w_2-w_1}{2}$ while the latter achieves a maximum. On the other hand, for $\varepsilon = -w_1$ (i.e., complete expropriation of the poorer of the two players) will maximize efficiency of the CPR input. But efficiency losses due to the misallocation of the private input will go to infinity as $\varepsilon \to -w_1$ (as $w_1 \to 0$, $\frac{x_1^{\beta\theta}}{w_1^{1-\alpha}} \to \infty$). Hence there is an optimal degree of inequality to maximize surplus. ∎

Given diminishing returns with respect to the private input, and the fact that it is complementary with the CPR in production, less inequality means greater total use of the CPR. We show that with decreasing returns to the private input and the CPR jointly, the strategic stability condition is satisfied. That is, the direct effect of redistribution dominates the indirect effect via changes in the behavior of other players on the total amount of use of the CPR. Greater equality leads to greater use of the collective good in this case which lowers efficiency due to congestion. However, greater equality brings the allocation of the private input across production units closer to the first best. It turns out that with perfect equality joint profits are maximized with respect to the second effect, but are minimized with respect to the first effect. On the other hand with perfect inequality, joint profits are maximized with respect to the first effect but the marginal losses from the second effect becomes arbitrarily large (due to the fact that the marginal product of the private input goes to infinity as its amount goes to zero). This implies too much equality or too much inequality is bad for joint profits - there is an optimal degree of inequality. This is in contrast with our result in section 2 where we showed perfect equality maximizes joint surplus.

What will happen if w_i is a tradeable input which has a market price of r? Then individuals will solve

$$\pi(x_i, k_i) = k_i^\alpha x_i^{\beta\theta} X^{-\beta(\theta-\gamma)} - x_i - r(k_i - w_i).$$

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This results in the following two first-order conditions:

$$f(k_i)\frac{\partial\left(x_i^{\beta\theta}X^{-\beta(\theta-\gamma)}\right)}{\partial x_i} = 1$$
$$f'(k_i)x_i^{\beta\theta}X^{-\beta(\theta-\gamma)} = r.$$

From these two conditions we get

$$\frac{\beta}{\alpha}k_i\{\frac{\theta}{x_i} - \frac{\theta-\gamma}{X}\} = \frac{1}{r}.$$

Given that the game is symmetric, let us focus on the symmetric equilibrium with $k_i = k^*$ and $x_i = x^*$ for all i which always exists.[9] By the above analysis, the efficiency of this outcome will be less than that with the optimal degree of wealth inequality and credit market imperfections. This is, therefore, a second best result - the congestion externality relating to the CPR and the imperfection in the market for the private input partly offset each other.

4 The Pool Effect

So far we have looked at a given set of individuals and examined how the distribution of wealth among them affect the level of efficiency in two types of collective action problems : the public good problem and the commons (or, congestion) problem. In this section we endogenize the decision to enter the pool and examine the role of the distribution of wealth in determining the size and composition of the pool of individuals who are participating in these two types of collective action problems.

This is often quite important in development policy. For example, in villages in many poor countries new access to private pumpsets for extracting groundwater (which only the better-off farmers can buy because of severe credit market imperfections) has often led to the exit of these farmers from previous cooperative arrangements in common surface water management. Similarly, on the entry side, advocates of land reform have suggested that redistribution of land to the landless may improve the participation and contribution in common water management schemes by a hitherto excluded category of peasants, who now acquire a stake in the asset base of the local economy.

The decision to enter into or exit from the pool depends on the outside options of the individuals. It is reasonable to assume that an agent's outside option is likely to be determined by her wealth level. In particular, if the realized payoff is too low, then the agent will not participate. Let $u(w_i)$ denote the value of the outside option for an agent with wealth w_i where $u(w_i)$ is strictly increasing in w_i for $w_i > 0$. Consider a

---

[9]Asymmetric equilbria can exist and will have the property if $x_i > x_j$, then $k_i > k_j$.

community of $\bar{n}$ individuals. For simplicity, assume that there are three given levels of wealth in the population, w_p, w_m and w_r and that a fraction p of the population have wealth w_p, a fraction q have wealth w_m and the remaining fraction $(1 - p - q)$, have wealth w_r. Let $\tilde{F}(w) = (p, q, 1 - p - q)$ denote all possible distributions of wealth with the same mean $w = pw_p + q\ w_m + (1 - p - q)w_r$. Redistribution constitutes changes in p and q (with the support remaining the same) so as to maintain the same mean wealth level w :

$$(w_r - w_p)dp + (w_r - w_m)dq = 0$$

This implies that a decrease in the size of the poor class must be accompanied by an increase the size of the middle class and a decrease in the size of the rich class. This is for simplicity only - we could allow for more general redistributive strategies such as changing the levels of wealth.

This allows us to consider the pure public good problem of section 2 and pure commons problem within the same framework. Let $X^*(\lambda_p, \lambda_m, \lambda_r)$ denote the equilibrium level of the relevant collective action (e.g., size of the provision of the public good, or the total amount of utilization of the commons) when the mean wealth level is w and a measure λ_p of the poor, λ_m of the middle class, and λ_r of the rich participate. Let $\pi^*(w_i, X^*(\lambda_p, \lambda_m, \lambda_r))$ denote the equilibrium payoff level of a player with wealth w_i. A player participates if and only if

$$\pi^*(w_i, X^*(\lambda_p, \lambda_m, \lambda_r)) \geq u(w_i).$$

An important property of $\pi^*(w_i, X^*)$ is that it is concave in w_i for both types of collective action problems. Consider the pure public goods problem first.

$$\pi^*(w_i, X^*) = w_i^\alpha (X^*)^\beta - x_i^*.$$

By the envelope theorem, $\frac{\partial \pi^*}{\partial w_i}$ is $\alpha w_i^{\alpha-1} (X^*)^\beta$. Also, by the second-order condition derivative of $\alpha w_i^{\alpha-1} (X^*)^\beta$ with respect to x_i^* is negative, while x_i^* is increasing in w_i. Given that π^* is a concave function of w_i ignoring the dependence through x_i^* this establishes that π^* is concave in w_i. An identical argument proves this for the commons case. A second property of $\pi^*(w_i, X^*)$ is that it takes the value 0 for $w_i = 0$.

We are going to make an important assumption regarding the property of $u(w_i)$: (a) it is flat for low levels of wealth : i.e., $u(w_i) = \bar{u}$ for $w \in [0, w^*]$ and (b) for $w > w^*$, $u(w_i)$ is (weakly) convex is w_i. Also, define w_1 such that for $w > w_1, u(w_i) > \pi^*(\bar{w}, X^*)$ and w_0 such that for $w_i < w_0, u(w_i) > \pi^*(w_i, X^*)$. Two alternative possible configurations of the function are shown in figure 1. This simplifies the analysis of the equilibrium considerably.

Since redistribution is potentially costly to carry out, and the costs increasing with the amount of redistribution, the optimal policy would be to redistribute among adjacent groups (e.g. the rich to the middle class and not the poor). Consider the following three cases :

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(a) $\pi^*(\ w_r, X^*(p, q, 1 - p - q)) \geq u(\ w_r)$.

(b) $\pi^*(w_m, X^*(p, q, 0)) \geq u(w_m)$.

(c) $\pi^*(w_p, X^*(p, 0, 0)) \geq u(w_p)$.

Notice that in these three cases the value of X^* can be ranked : since X^* is increasing in the total amount of wealth among participating members, this total amount of wealth is strictly decreasing from case (a) to case (c).

First take the public goods problem. This property implies that so long as $w_p \geq w_0$, if (a) holds all groups will participate, if (b) holds only the middle class and the poor will participate, and in (c) only the poor will participate. Moreover, these cases are mutually exclusive. In the first case a mean-preserving redistribution within the pool will improve efficiency by Proposition 1. In the second case,

$$X^* = \beta^{\frac{1}{1-\beta}} \left(\frac{p}{p+q} w_p^{\frac{\alpha}{1-\beta}} + \frac{q}{p+q} w_m^{\frac{\alpha}{1-\beta}} \right).$$

A redistribution policy that reduces the size of the rich class and increases that of the middle class (i.e., an increase in q and a decrease in p) will improve efficiency since

$$\frac{dX^*}{dq} = \beta^{\frac{1}{1-\beta}} \frac{1}{(p+q)^2} \left(w_m^{\frac{\alpha}{1-\beta}} - w_p^{\frac{\alpha}{1-\beta}} \right) \left(p + q \frac{w_r - w_m}{w_r - w_p} \right) > 0$$

where we have used the fact that $(w_r - w_p)dp + (w_r - w_m)dq = 0$ for the class of redistributive strategies we are focusing on. In the third case, with redistribution from the rich to the poor that pulls some of the poor out of the pool and into the middle class, the level of efficiency within the pool will remain unchanged (because $X^* = \beta^{\frac{1}{1-\beta}} \left(\frac{1}{n} \sum_{i=1}^n w_i^{\frac{\alpha}{1-\beta}} \right) = \beta^{\frac{1}{1-\beta}} w_p^{\frac{\alpha}{1-\beta}}$ in this case). If $w_p < w_0$ then the poor do not participate at all. Redistribution will increase efficiency within the pool by expanding the size of the middle class at the expense of the rich (because both the amount of the public good will increase and the allocation of the private input will be more efficient).

Second, take the commons problem. Here a higher value of X^* implies lower profits for any given level of wealth (except for $w_i = 0$) in contrast with the previous case. Again start with the situation where $w_p > w_0$. If (a) holds then (b) and (c) cannot be equilibria since if the rich prefer to participate in the worst case scenario (i.e., everyone else does) then they will prefer to do so in all other cases. Similarly, if (b) holds (but not (a)) then (c) cannot be an equilibrium configuration. So in this case too the three cases are mutually exclusive. If the equilibrium is (a) then intra-pool redistribution may increase efficiency if the initial level of inequality is more than the optimal degree of inequality (as characterized by Proposition 2). If the equilibrium is type (b), then a redistributive strategy that reduces the size of the rich and the poor class and increases that of the middle class will accentuate the congestion externality within the pool and hence reduce efficiency. If the equilibrium is of type (c) then

a redistributive strategy of the kind we are considering will alleviate the congestion externality within the pool by reducing the number of users. If $w_p \leq w_0$ then the poor never participate. In that case, if initially only the middle class was in the pool, then our redistributive strategy will lead to greater congestion and lower efficiency. If initially both the middle and the rich classes were in the pool, then redistribution may or may not increase efficiency because the trade off described in Proposition 2 will emerge. We summarize our findings in this section in :

Proposition 3 *A redistributive strategy that reduces the size of the rich and the poor class and increases that of the middle class will affect the pool of participants in a way that will : (a) improve efficiency in the pure public good problem; (b) improve or reduce efficiency in the CPR problem depending on whether it increases the number of users in the pool so long as the rich are not in the pool. If the rich are in the pool, the trade off described in Proposition 2 will emerge.*

The implication of this analysis is that whether land reform type redistributive policies that change the composition and size of the pool of participants in collective action problems increase or decrease efficiency depends crucially on whether the collective action problem relates to contributions towards a public good or utilization of a CPR. In the former case, in general redistributive policies are favorable for efficiency. In the latter case they are typically not because they accentuate congestion unless the gains from reducing the misallocation of the private input are very strong.

5   Concluding Remarks

In this paper we have analyzed the effects of wealth inequality on the provision of collective goods (public goods and CPR) when there are market imperfections in inputs that are complementary in production to the collective good. We showed that for public goods inequality impedes efficiency, while for use of CPRs there is an inverse U-shaped relationship between inequality and efficiency. As we have noted these theoretical results have implications for redistributive policies and their impact on the structure of incentives. It can be extended in the following directions :

(a) Many real world collective goods have features of both public goods and congestion externality. A more general hybrid model with the pure public good and pure commons as limiting cases may yield richer results.

(b) In some situations of asset asymmetry the Stackelberg model may provide a more appropriate equilibrium concept.

(c) In a dynamic setting where both the distribution of wealth and the efficiency of collective action problems are endogenous. For example, it is possible to have

multiple stationary states with high (low) wealth inequality leading to low (high) incomes to the poor due to low (high) level of provision of public goods, which via low (high) mobility can sustain an unequal (equal) distribution of wealth.

6 Appendix

The second-order condition in the commons problem : We have

$$
\begin{aligned}
h_1(x_i, X) &= -\beta\theta x_i^{\beta\theta-2} \, X^{\beta(\gamma-\theta)-1} \left[(1-\beta\theta)X + \beta(\theta-\gamma)x_i\right] \\
h_2(x_i, X) &= -\beta(\theta-\gamma)x_i^{\beta\theta-1} \, X^{\beta(\gamma-\theta)-2} \left[\beta\theta X - \{1+\beta(\theta-\gamma)\}x_i\right].
\end{aligned}
$$

and so $h_1(x_i, X) + h_2(x_i, X)$
$= -\beta x_i^{\beta\theta-2} \, X^{\beta(\gamma-\theta)} \left[\theta(1-\beta\theta) + 2\beta\theta(\theta-\gamma)z_i - (\theta-\gamma)\{1+\beta(\theta-\gamma)\}z_i^2\right]$
where $z_i = \frac{x_i}{X} \in [0,1]$. The expression $\theta(1-\beta\theta) + 2\beta\theta(\theta-\gamma)z_i - (\theta-\gamma)\{1+\beta(\theta-\gamma)\}z_i^2$ has the quadratic form $az_i^2 + bz_i + c$ where $c > 0$. If $\theta > \gamma$ then $a < 0, b > 0$ and otherwise, $a > 0, b < 0$. In the former case the minimum value the expression can take is for $z_i = 0$ or $z_i = 1$ as it reaches an interior maximum at $\frac{\beta\theta}{1+\beta(\theta-\gamma)}$. For $z_i = 0$ it is clearly positive. For $z_i = 1$, the expression is $\gamma(1-\beta\gamma) > 0$. If $\theta < \gamma$ then the expression is a convex function of z_i with a positive value at $z_i = 0$. Now there are no real roots of the quadratic $az_i^2 + bz_i + c = 0$ if $b^2 < 4ac$, i.e., $4\beta^2\theta^2(\theta-\gamma)^2 < 4\theta(1-\beta\theta)(\theta-\gamma)\{1+\beta(\theta-\gamma)\}$, or, $\beta\gamma < 1$ which is true. Since this the same condition as the minimum value of the expression is positive, we have proved that $\theta(1-\beta\theta) + 2\beta\theta(\theta-\gamma)z_i - (\theta-\gamma)\{1+\beta(\theta-\gamma)\}z_i^2 > 0$ for all values of z_i and so the second order conditions are satisfied.

The strategic stability condition in the commons problem:

Let $\frac{x_1}{X} = z$ so that $\frac{x_2}{X} = 1-z$. Then the expression in (8) is equal to

$$
\beta^2\theta x_1^{\beta\theta-2} x_2^{\beta\theta-2} \, X^{2\beta(\gamma-\theta)} \{(1-\beta\theta) + \beta(\theta-\gamma)z\}\{(1-\beta\theta) + \beta(\theta-\gamma)(1-z)\}
$$
$$
\left[\theta + (\theta-\gamma)\left\{(1-z)\frac{\beta\theta - \{1+\beta(\theta-\gamma)\}(1-z)}{(1-\beta\theta)+\beta(\theta-\gamma)(1-z)} + z\frac{\beta\theta - \{1+\beta(\theta-\gamma)\}z}{(1-\beta\theta)+\beta(\theta-\gamma)z}\right\}\right]
$$

Let $\phi(z) \equiv \frac{\beta\theta - (1+\beta(\theta-\gamma))z}{1-\beta\theta+\beta(\theta-\gamma)z}$. Now $\phi(0) = \frac{\beta\theta}{1-\beta\theta}$ and $\phi(1) = -1$. Also, $\phi(\frac{1}{2}) = \frac{\beta(\theta+\gamma)-1}{2-\beta(\theta+\gamma)}$.

$$
\phi'(z) = -\frac{1-\beta\gamma}{\{1-\beta\theta+\beta(\theta-\gamma)z\}^2}.
$$

Also

$$
\phi''(z) = \frac{2(1-\beta\gamma)\beta(\theta-\gamma)}{\{1-\beta+\beta(\theta-\gamma)z\}^3} > 0.
$$

Now the derivative of the function $\psi(z) \equiv z\phi(z) + (1-z)\phi(1-z)$ is $\{\phi(z) - \phi(1-z)\} + [z\phi'(z) - (1-z)\phi'(1-z)]$. For $z = \frac{1}{2}$ this is 0. Also, $\psi(z)$ is symmetric around

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$\frac{1}{2}$. The second derivative of $\psi(z)$ is $2\left[\phi'(z) + \phi'(1-z)\right] + z\phi''(z) + (1-z)\phi''(1-z)$. It is readily checked that

$$2\phi'(z) + z\phi''(z) = -\frac{1-\beta}{\beta(\theta-\gamma)}\phi''(z) < 0.$$

So it is strictly concave for all z. The derivative of $\psi(z)$ at $z = 0$ is $\frac{1}{1-\beta\theta} + \frac{1}{1-\beta\gamma} > 0$. Hence $\psi(z)$ reaches a global maximum at $z = \frac{1}{2}$. As a result, a sufficient condition for the strategic stability condition to be satisfied is

$$\theta + (\theta - \gamma)\frac{\beta(\theta+\gamma) - 1}{2 - \beta(\theta+\gamma)} > 0.$$

Since this expression is equal to $(\theta + \gamma)\frac{1-\beta\gamma}{2-\beta(\theta+\gamma)}$, this condition is satisfied so long as $\beta \le 1, \gamma \le 1$ and $\theta \le 1$ with at least one inequality being strict. That is indeed our retained assumption.

The above result easily extends to the case where $n > 2$ if we make the following additional assumption

$$\beta(\theta + \gamma) > 1.$$

Recall that the sign of $h_2(x_i, X)$ is the opposite of that of $\{\beta\theta X - \{1 + \beta(\theta - \gamma)\}x_i\}$. Now suppose that $\max_i x_i < \frac{\beta\theta}{1+\beta(\theta-\gamma)}X$. Then $h_2(x_i, X) < 0$ and since $h_1(x_i, X) < 0$, $g_2(w_i, X) = -\frac{h_2(x_i, X)}{h_1(x_i, X)} < 0$. Hence (7) is satisfied for any i and j from the set of players $N = \{1, 2, .., n\}$. Suppose instead $\max x_i > \frac{\beta\theta}{1+\beta(\theta-\gamma)}X$. So long as $\frac{\beta\theta}{1+\beta(\theta-\gamma)} > \frac{1}{2}$, or, $\beta(\theta+\gamma) > 1$, there cannot exist a $x_j \ne x_i$ such that $x_j > \frac{\beta\theta}{1+\beta(\theta-\gamma)}X$. Then $h_2(x_j, X) < 0$, and $g_2(w_j, X) < 0$ for all $j \ne i$. Since by the second-order condition $h_1(x_i, X) + h_2(x_i, X) < 0$ for all i, we have $h_2(x_i, X) < -h_1(x_i, X)$, or $0 < g_2(w_i, X) < 1$ even for the maximum value of x_i. Since $g_2(w_i, X) < 0$ for all other x_i the condition (7) is satisfied for redistributions between any two players. If the assumption $\beta(\theta+\gamma) > 1$ is not satisfied, then the result goes through for redistributions between two players i and j such that either $\max\{x_i, x_j\} < \frac{\beta\theta}{1+\beta(\theta-\gamma)}X$ or, $\max\{x_i, x_j\} > \frac{\beta\theta}{1+\beta(\theta-\gamma)}X > \min\{x_i, x_j\}$.

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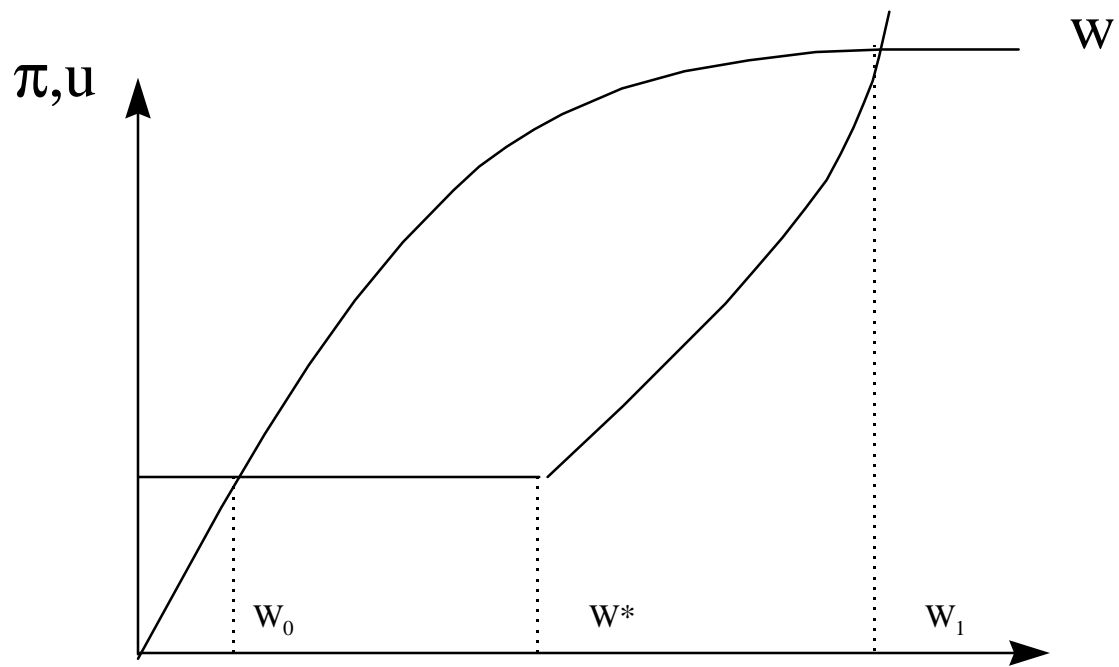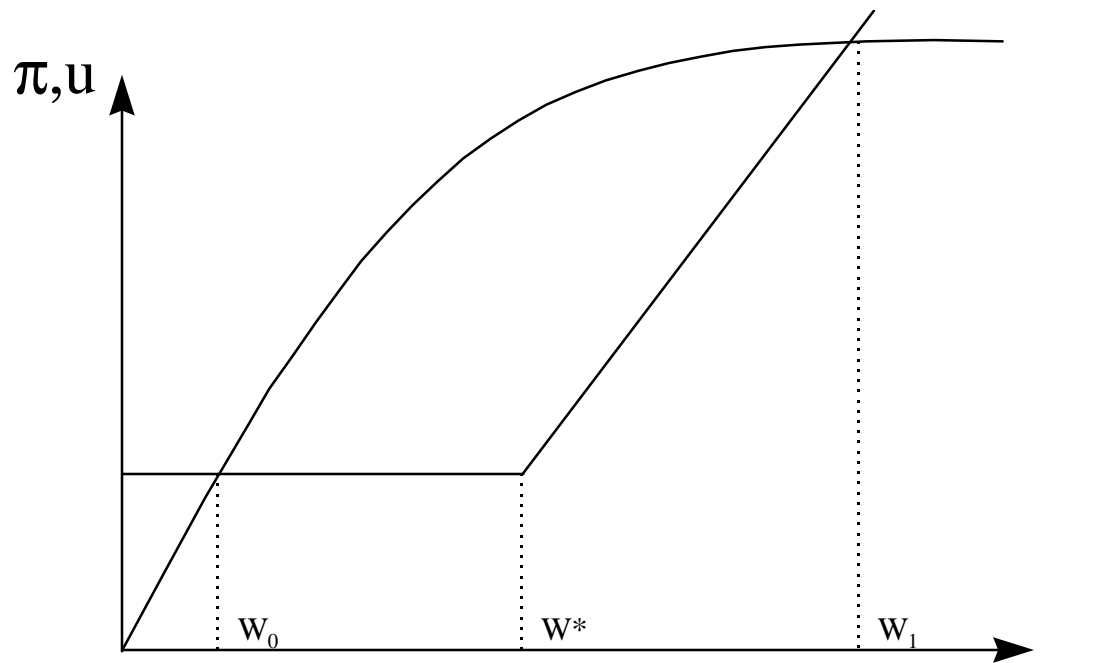
Figure 1