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Submitted to Physics Letters B

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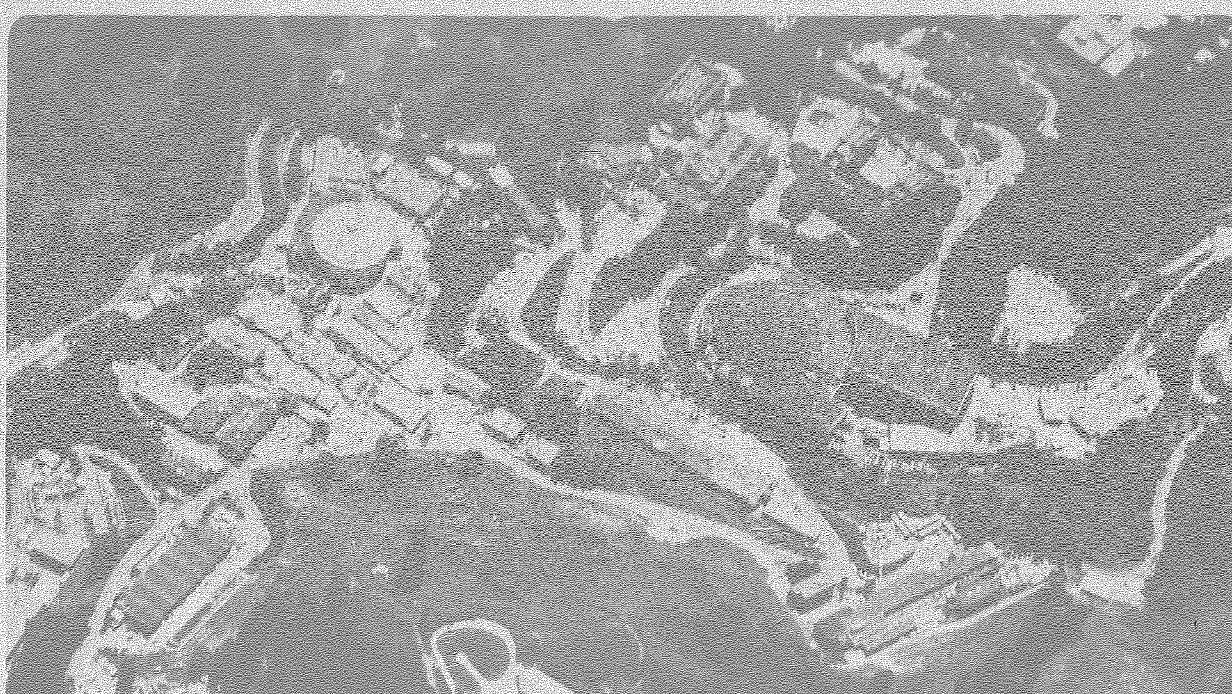
September 1980



Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

September 1980 LBL-11551

NON-PERTURBATIVE EFFECTS IN HEAVY QUARKONIA

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ABSTRACT

The effect of a non-zero vacuum gluon-condensate on heavy quarkonia is discussed. As a function of the quark mass, it is determined which low lying levels of the spectrum are dominated by the one-gluon-exchange potential.

A useful feature of the observed heavy quarkonium states is that the energy gap between the ground state and the manifest flavor threshold is much smaller than the resonance mass itself. This implies a picture of non-relativistic heavy quarks whose interactions should be adequately described by potential models. In QCD, due to asymptotic freedom, it is well known that the short distance part of the potential is dominated by one gluon exchange, giving rise to a calculable 1/R potential. For the long-distance part of the interaction, various phenomenological potentials have been postulated that reproduce the observed heavy hadronic spectrum. One of the hopes is that for very heavy quarks the bound state radius will be of a size that only samples the known short-distance part of the potential, allowing unambiguous theoretical calculations. A crude back-of-the-envelope estimate of how massive the quarks must be to see only the "color coulomb interaction" is made by requiring that the coulomb-like binding energy is much greater than some hadronic energy scale

$$\left(\frac{4}{3}\alpha_s\right)^2 \frac{m_Q}{4} > 1 \text{ GeV} \qquad (1)$$

choosing the strong interaction scale parameter λ to be ≈ 500 MeV, and the effective coupling constant to be at the scale of the bound state Bohr radius yields $m_Q > 25$ GeV.

It is now possible to determine more rigorously which low lying levels of the heavy quark bound state spectrum are dominantly coulombic, as a function of the quark mass. The procedure will be to calculate the non-perturbative power corrections to the 1/R potential for large quark masses where these power corrections are small, and then determine how small the quark masses can become before the coulomb approximation breaks down. The method for studying these non-perturbative effects is due to the pioneering work of Shifman, Vainshtein, and Zacharov [1]. Their technique is to extract the long distance behavior of internal lines in Feynman diagrams systematically, and parameterize this dynamical contribution with experimentally determined quantities. As applied to a heavy quark-antiquark bound state, the procedure is to take the lowest order perturbative diagrams of Fig. 1 for gluon exchange within the $Q\bar{Q}$ bound state and allow each gluon line to go soft individually. The soft line is cut, and the cut ends of the long wavelength line are allowed to propagate into the vacuum yielding the set of diagrams illustrated by Fig. 2. Note that the complete set of diagrams of Fig. 2 is exactly the set of diagrams considered by Peskin in determining the gauge invariant coupling of long wavelength gluons to a color-singlet heavy $Q\bar{Q}$ bound state [2]. The well known result gives the first term in an operator product expansion

$$\text{(Fig. 2)} = \frac{1}{6}\left\langle\Phi\left|r^i\,\frac{1}{H_8-H_1}\,r^j\right|\Phi\right\rangle\left\langle 0\left|g^2G_a^{io}G_a^{jo}(0)\right|0\right\rangle + \frac{1}{24}\left\langle\Phi\left|\sigma^i\,\frac{1}{H_8-H_1}\,\sigma^j\right|\Phi\right\rangle\left\langle 0\left|g^2G_a^{kl}G_a^{mn}(0)\right|0\right\rangle\frac{\varepsilon^{ikl}}{2}\,\frac{\varepsilon^{jmn}}{2} \tag{2}$$

where r_i is the $Q\bar{Q}$ separation in the bound state Φ, $H_1(H_8)$ is the Hamiltonian of the $Q\bar{Q}$ in a color singlet (octet) state, $\underset{\sim}{\sigma}$ is the Pauli spin matrix, and the gluon field strength, $G_a^{\mu\nu}$, is evaluated at the origin of the bound state. The energy denominator can be further simplified by noting that for one gluon exchange $H_8 - H_1 = \frac{3g^2}{8\pi r}$. By defining $E_a^i \equiv G_a^{oi}$ and $B_a^i \equiv \frac{1}{2}\varepsilon^{ijk}G^{jk}$, and choosing Φ to be a spin zero

state, the expression simplifies to

$$\text{(Fig. 2)} = \frac{\langle\Phi|r^3|\Phi\rangle}{27\alpha_s}\langle 0|g^2\underset{\sim}{E}^a\cdot\underset{\sim}{E}^a|0\rangle + \frac{\langle\Phi|r|\Phi\rangle}{9\alpha_s m_Q^2}\langle 0|g^2\underset{\sim}{B}^a\cdot\underset{\sim}{B}^a|0\rangle. \tag{3}$$

Shifman et al have determined the vacuum expectation vacuum expectation value of the square of the gluon field strength tensor from remarkably successful charmonium sum rules [3]. They find

$$\frac{g^2}{4\pi^2}\langle 0|G^a_{\mu\nu}G^{\mu\nu a}|0\rangle \equiv M_o^4 \cong (330\text{MeV})^4 \tag{4}$$

which implies

$$\frac{g^2}{\pi^2}\langle 0|\underset{\sim}{B}^a\cdot\underset{\sim}{B}^a|0\rangle = -\frac{g^2}{\pi^2}\langle 0|\underset{\sim}{E}^a\cdot\underset{\sim}{E}^a|0\rangle = M_o^4 \tag{5}$$

We can now rewrite eq. 3 as

$$\text{(Fig. 2)} = h^E + h^M \tag{6a}$$

with

$$h^E \equiv -\frac{\langle\Phi|r^3|\Phi\rangle}{27\alpha_s}\pi^2M_o^4$$
$$h^M \equiv \frac{\langle\Phi|r|\Phi\rangle}{9\alpha_s m_Q^2}\pi^2M_o^4 \tag{6b}$$

To determine how this long wavelength "vacuum gluon-condensate" affects the bound state Hamiltonian, we will calculate the bound state propagator of the $Q\bar{Q}$ system as illustrated by Fig. 3.

$$\text{(Fig. 3)} = \lim_{T\to\infty}\int_0^T dt\, e^{-i(H_1-\varepsilon_1)t}\left\{1+\int_0^t idt_1 h^E+\ldots\right\} \times\left\{1+\int_0^t idt_1 h^M+\ldots\right\} \tag{7}$$

with ε_1 the color singlet bound state energy. Using the identity

$$\int_0^t idt_1 A \int_0^{t_1} idt_2 A \cdots \int_0^{t_{n-1}} idt_n A = \frac{(itA)^n}{n!} \tag{8}$$

we can exponentiate the contributions of h^E and h^m to find the corrections to the color singlet Hamiltonian

$$H_1^{s=0} \rightarrow H_1^{s=0} = -\frac{4\alpha_s}{3r} - \left(\frac{\pi^2 M_o^4}{9\alpha_s m_Q^2}\right) \langle \Phi | r | \Phi \rangle + \left(\frac{\pi^2 M_o^4}{27\alpha_s}\right) \langle \Phi | r^3 | \Phi \rangle \tag{9a}$$

for the spin-zero bound state. Going back to eq. 2, we can do similar manipulations for the spin-one bound state, yielding

$$H_1^{s=1} \rightarrow H_1^{s=1} = -\frac{4\alpha_s}{3r} - \left(\frac{\pi^2 M_o^4}{27\alpha_s m_Q^2}\right) \langle \Phi | r | \Phi \rangle + \left(\frac{\pi^2 M_o^4}{27\alpha_s}\right) \langle \Phi | r^3 | \Phi \rangle \tag{9b}$$

We are now in a position to determine when the coulombic approximation is a valid one for a given quark mass, and a specified energy level. First note that the "magnetic" term proportional to $\langle \Phi | r | \Phi \rangle$ is always much less than the "electric" term proportional to $\langle \phi | r^3 | \Phi \rangle$ for $\alpha_s < 1$. Thus, to determine when the coulomb approximation is valid, we can define the ratio

$$R \equiv \frac{\left(\frac{\pi^2 M_o^4}{27\alpha_s}\right) \langle \Phi | r^3 | \Phi \rangle}{\langle \Phi \left| \frac{4\alpha_s}{3r} \right| \Phi \rangle} \tag{10}$$

which is the ratio of the energy of the non-perturbative power corrections to the coulombic binding energy. If $R \ll 1$, the state Φ can be well described by a coulomb wavefunction. In Fig. 4 we plot R as a function of quark mass for the n = 1, 2, 3 levels of the coulomb spectrum. The coupling constant in the expression for R is

normalized to be $\alpha_s(m_Q{=}1.5\text{ GeV}) = .3$, as determined from potential model fits to charmonium [4], and it's scale is the bound state Bohr radius. If, for example, we decided that $R < .2$ implies a reasonable coulomb dominance, the 1s level would be coulombic for $m_Q \geqslant 10$ GeV, the 2P levels for $m_Q \geqslant 50$ GeV, the 2S level for $m_Q \geqslant 60$ GeV, etc.

To estimate the accuracy of these predictions, we must address two points. The first is a guess of the size of the contribution from higher dimensional operators in the operator product expansion. Dimensionally we expect higher order operators ($D_\mu G^a_{\mu\alpha} D_\nu G^a_{\nu\alpha}$, $f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma} G^c_{\sigma\mu}$, ...) to contribute with corresponding additional powers of M_o, and the coefficient functions to contribute corresponding powers of the Bohr radius, a_o. The effective expansion parameter is $(M_o a_o) \simeq .15$ for $m_Q \sim 20$ GeV, and decreases as $1/m_Q$. Secondly, we must determine how the uncertainty in M_o affects our results. Shifman et al estimate that $(M_o)^4$ is known to within a factor of two. Shifting the normalization of our curves for R by a factor of two induces an uncertainty in our determinations of m_Q of roughly $\pm$ 25%.

Thus we see that one-gluon-exchange dominance occurs for quark masses substantially larger than present energies. This is as expected from the simple estimate of eq. 1, but our new estimates are much more quantitative with a firm theoretical foundation.

ACKNOWLEDGMENTS

I thank M. Suzuki for useful discussions. This research was supported by the High Energy Physics Division of the U.S. Department of Energy under contract No. W-7405-ENG-48.

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4. G. Bhanot and S. Rudaz, Phys. Lett. 78B (1978) 119.

FIGURE CAPTIONS

Figure 1: Lowest order perturbative diagrams for gluon exchange within a $Q\bar{Q}$ bound state.

Figure 2: Sum of diagrams generated by cutting the soft gluon lines of Fig. 1.

Figure 3: Vacuum gluon condensate contribution to the $Q\bar{Q}$ propagator.

Figure 4: The quantity R, as defined in eq. 10, as a function of m_Q for the n = 1, 2, 3 levels of the coulomb spectrum.

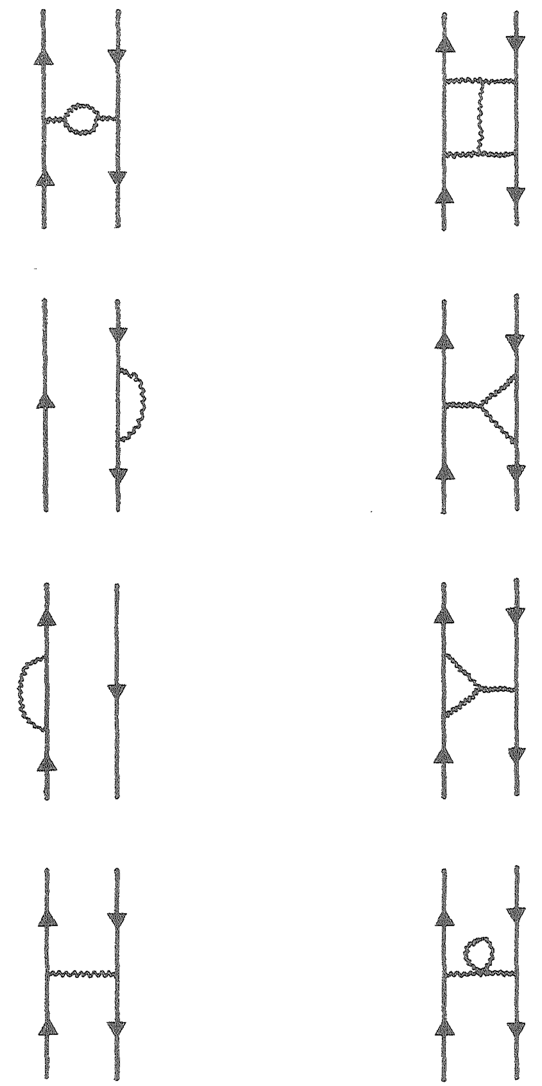

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FIG. 1

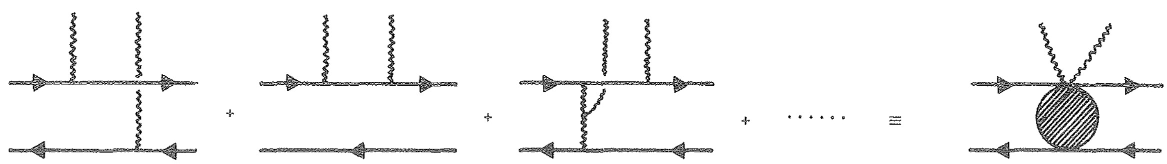

FIG. 2

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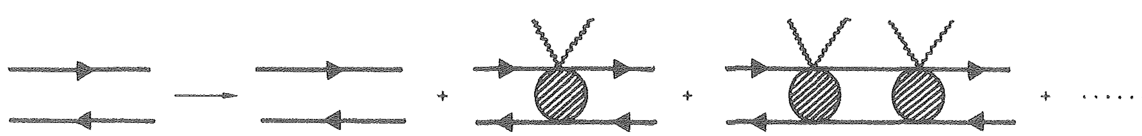

FIG. 3

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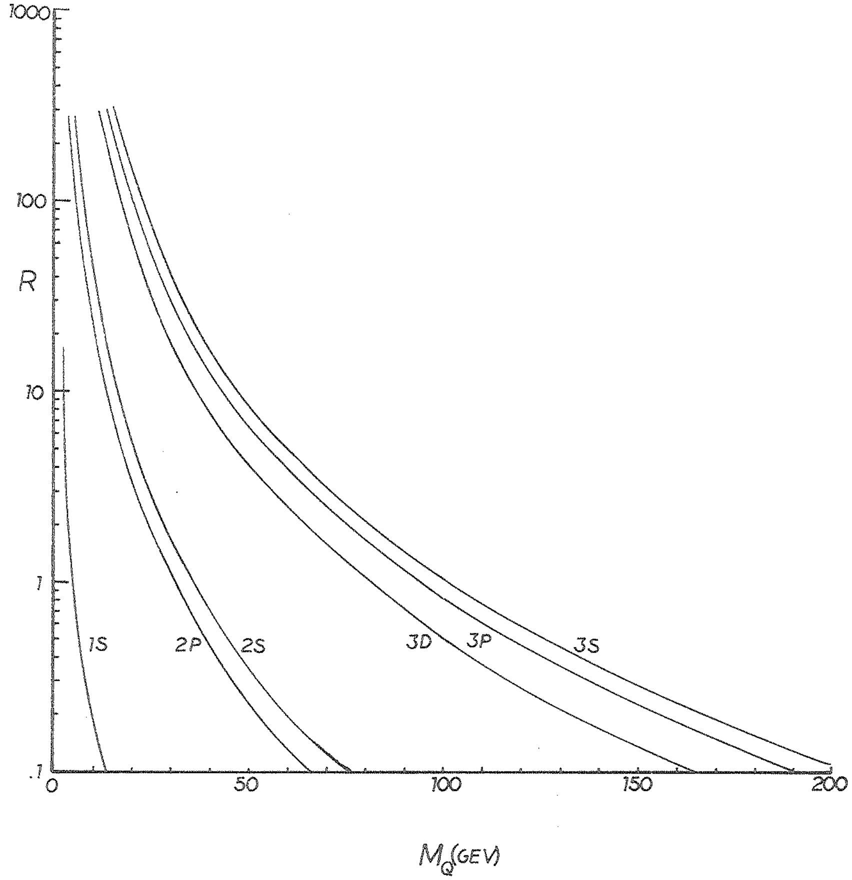

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FIG. 4