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by

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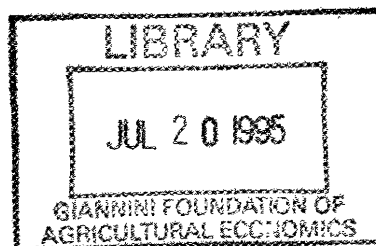
NATURAL RESOURCE PRICES: WILL THEY EVER TURN UP?

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Proposed running head: Natural Resource Prices

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ABSTRACT

Hotelling's theory predicts that natural resource rents should increase over time. However, technical progress in resource extraction, environmental constraints, or great natural abundance could result in stagnant or declining product prices. Thus, there is no theoretical reason to believe that product prices will rise in the near future. The prediction of product prices by time-series methods is shown to depend critically upon whether the series are modeled as differenced or trend stationary. Dickey-Fuller and Lagrange Multiplier tests are used to show that the series are differenced stationary. Long- and short-sample series are tested. Trend-stationary modeling strongly predicts rising resource prices. The result from differenced-stationary modeling is that there is only a weak supposition that natural resource prices will rise.

NATURAL RESOURCE PRICES: WILL THEY EVER TURN UP?

According to Hotelling's [6] theory, natural resource rents (the prices of resources before extraction) should be increasing at the rate of interest. If the increase in rents is reflected in increased prices for natural resource products, then the scarcity of natural resources could dampen economic activity.[1] The hypothesis that resource prices increase was first systematically examined by Barnett and Morse [1]. They found quite the opposite; resource prices fell. More recently, Slade [13] examined resource prices and found reason to predict that prices would rise. Since Slade's examination of the price data, 12 years have elapsed. Armed with a new, longer data series and some very different time-series techniques, we re-examine the scarcity (price increase) hypothesis and estimate the likelihood that the real prices of semi-processed natural resources will increase.

The resources considered are aluminum, coal, copper, iron, natural gas, petroleum, silver, zinc, and, briefly lead. The series used are all real-price series, and they update Manthy's series to 1991, except for iron where the last datum is 1985.

In the second section of the paper, we discuss the pure theory of natural resource prices and emphasize the cases where resource prices can be expected to be stagnant or fall. Barnett and Morse [1], Fisher [4], Slade [13], and Livernois and Uhler [8], among many others, have explained how technical progress in extraction and processing, possibly along with depletion of grade, can offset the increase in resource rents and lead to declining product prices. Slade shows how the progress effect can lead to a u-shaped product price path. Using Hartwick's [5] model of many grades with technical progress, rather than the Livernois-Uhler model, an example of declining price is given. A second example where price is stagnant is great natural abundance. With enough resource, or effective enough recycling, resource price would be near zero. A third example is the case of environmental constraint, which seems to

have been first mentioned by Barnett and Morse [1] and then by Fisher [4]. Briefly, if the use of coal requires clean air and one runs out of clean air before one runs out of coal, then coal could have a resource rent of exactly zero. Any combination of these arguments would lead to declining or stagnant real-price paths for resource products.

The third section begins the econometric inquiry with a theoretical discussion of the properties of time series of prices. The reasons why natural resource price series should be expected to be differenced stationary (DS), rather than trend stationary (TS), are reviewed, and the known consequences of choosing the wrong methods of time-series analysis are spelled out. A general statistical model is then presented, and it is shown how the model of Slade and an ARIMA model can be realized as a restriction of that model. Tests that can be used to help choose among the plausible model restrictions are then described. Finally, the section deals with an ancillary issue raised by Smith [14]—the possibility that the data series are not of one piece and that the best predictions can be obtained by using shorter series. The major choices set out in the section are between using short-series (SS) or long-series (LS) and between using TS or DS methods.

In the fourth section, results on TS and DS models are presented. Adding 15 years' worth of data or choosing a later starting date are not found to make a qualitative change in the predictions from a TS model.

Tests for a unit root, which are used to choose between TS and DS models are then presented, as is a DS estimation. The optimal length of series for DS models is also addressed.

In the fifth and final section, results and conclusions are presented. The impatient reader will find the conclusions summarized in Table X which gives the authors' beliefs about the likelihood that prices will rise. The likelihood has two parts: the point estimate of future price and the confidence interval about that point estimate. The change of technique from TS to DS is shown to make more of a difference for the

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size of the confidence interval than for the point estimate. In short, we would predict rising prices but be much less surprised about being wrong than were the previous authors.

I. THE THEORY OF RESOURCE PRICES

The usual way to square the observation of decreasing natural resource prices with Hotelling's theory is to assume that technical progress in mining and refining drives marginal cost down quickly. Considered in this section are three cases in which prices can be declining or stagnant: the depletion and progress case, the great abundance case, and the environmental constraint case. The depletion and progress case is an extension of Hartwick's model, which is different from the continuous distribution of ore model used by Slade [13] and Livernois and Uhler [8]. That resource rents would be low with great natural abundance seems obvious. Meadows *et al.* [9] were among the first to suggest that environmental constraints are more important than resource constraints. The consequences for resource rent are brought out in this section.

Turning to the first of these reasons for stagnant or declining prices, the standard model with many grades of ore (Hartwick) is that the ith price-taking firm maximizes its present value profits from a fixed and known stock of ore, x_{i0}. Technical progress is added to Hartwick's model at rate z so that the cost of extracting from the ith body at time t is $c_i(t) = c_{i0}e^{-zt}$. Since all of the bodies are subject to the same rate of progress, the ranking of costs does not change over time and the bodies are extracted in order of increasing c_{i0}, exactly as in Hartwick's original model. The prices faced by the firm are $p(t)$, while the interest rate is r. Hartwick has shown that, in an equilibrium plan, the ith body is exhausted just as its price reaches the price at which the i + 1st body begins to be exploited. Let T_{i+1} be the time at which that happens so

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that the ith body is exploited during the interval $[T_i, T_{i+1}]$. The present value profit-maximizing problem faced by the ith firm is

$$\max_{h(t)} \int_{T_i}^{T_{j+1}} (p\,h - h\,c_i(t))\,e^{-rt}\,dt \text{ s.t. } \dot{x}_i = -h_i \text{ and } x_i(0) \text{ given.} \qquad (1)$$

Letting λ_i be the present value costate variable associated with the Hamiltonian from problem (1), it is well known that λ_i increases at the rate of interest. Assuming that there is positive extraction during all of $[T_i, T_{i+1}]$, the usual maximum principle manipulations yield the differential equation for the equilibrium price path. In general form,

$$\dot{p} = r(p - mc) + \frac{dmc}{dt}, \qquad (2)$$

where marginal cost (mc) is c_i. The sign of $\dot{p}$ is unknown: $\dfrac{dmc}{dt}$ is negative, while the $p - mc$ is positive because of the assumption that there is extraction. Thus, price may rise or fall while rent increases at an exponential rate.

Making the substitutions of the particular form of the cost function, the solution to this differential equation is given by

$$p(t) = (p_{i0} - c_{i0})\,e^{rt} + c_{i0}e^{-zt}, \qquad (3)$$

where p_{i0} is the price when ore body i begins to be exploited. The determination of p_{i0} will be discussed below. It can be verified that (3) is the solution to (2) by substituting it into that differential equation.

In this model prices decrease whenever $d/dt\, p < 0$, which occurs when $(r + z)\, c_{i0} > r\, p_{i0} > r\, c_{i0}$. Indeed, $d^2/dt^2(p) = r^2(p_{i0} - c_{i0})\,e^{rt} + z^2\, c_{i0}e^{-zt} > 0$. Thus, price either falls at the beginning of the exploitation of a particular grade or it never falls during the exploitation of that grade.

Rearranging the condition for decreasing prices gives

$$z/r > (p_{i0} - c_{i0})/c_{i0}, \tag{4}$$

or the ratio of technical progress to interest exceeds the percent markup on cost. It is easy to imagine a progress rate of 3 percent, a real interest rate of 6 percent, and costs accounting for 3/4 of price.

While grade depletion per se can never drive prices down, it contributes to making $d/dt\ p$ negative by raising c_{i0}. At the beginning of each ore body's exploitation, c_{i0} increases to c_{i+10}, lowering the markup and making it more likely that prices will fall. The form of the equation also makes it clear that, without grade depletion, prices will eventually rise. Most particularly, if there are two grades, the second taking a long time to exhaust, one can get Slade's u-shaped price curves. In conclusion, stagnant prices (or u-shaped price paths) can come from technical progress and switching grades.

There are, of course, much simpler explanations of stagnant prices. Great abundance is the most obvious. While the ith ore body is being mined, equation (3) gives the price path up to the unknown constant, p_{i0}. That constant is found from equilibrium in the flow markets for the resource and the exhaustion conditions. The amount supplied at any instant must equal the amount demanded, $Q(p)$, where Q is a demand curve. The conditions for exhaustion are

$$x_i(0) = \int_{T_i}^{T_{i+1}} Q(p(t, i))\ dt. \tag{5}$$

Optimality also requires that price be continuous at the switch times, T_i. Assuming that the integral can be solved or approximated, it can be turned about to get

$$p_{i0} = f_i(x_1(0) \ldots x_n(0)). \tag{6}$$

There are valuation functions, f_i, that give value as a function of existing stocks of the resource.

It can be shown that p decreases as any x_i increases. In fact, if Q is the constant elasticity demand equation and there is only one grade of ore, it is not difficult to make x so large that p = mc for all practical purposes. Recycling has precisely the same effect; it adds to the stock and decreases rent. In this view of great natural abundance (or near costless recycling), resource rent still rises at the rate of interest. It starts at such a low value that its increase cannot be observed with the naked eye in the mere few thousand years of recorded history. With great abundance, observationally, prices are determined by the vicissitudes of the economy and costs, not by the relentless increase in rents.

Another case in which resource rent is zero is given by Meadows *et al.* [9][2] They consider a world with pollution and resources and simulate "alternative futures" by making most variables grow at their current rates. Economic systems are unlikely to grow in such a consistent way. Prices, for instance, will restrain resource usage, and governments will restrain pollution. The conclusion from the Meadows *et al.* model is that the world chokes to death before it exhausts its resources.

To be concrete, if a little obvious, assume that there is one grade of a resource, the resource market is competitive, and every unit of resource consumed produces exactly one unit of pollution. There are precisely n identical firms. The prototypical resource users problem is

$$\max_{h(t)} \int_0^\infty (p\, h - c(h, t))\, e^{-rt}\, dt \text{ s.t. } \dot{x} = -h \text{ and } x(0) \text{ given.} \tag{7}$$

The government dislikes bad air, which grows according to $\dot{b} = + n\, h$. Being a government, it decrees that there shall be no more bad air than amount B—an amount set so as not to fully exhaust the resource. The producer is one of n similar producers, so each of them views clean air as an open-access resource. If they produce less resources, the only result will be that another will produce more. In short, air has a private cost of zero—the standard pollution story. The social problem, of course, is

quite different. It is the same as imposing a terminal value constraint on the producer's problem. When b reaches B [which, by assumption, is less than x(0)], there is still stock left over. Thus, the transversality condition is that $\lambda = 0$. Of course, air has reached a constraint, so its social value, γ, is not zero. Provided that the government separates the air rights from the mineral rights, the natural resource will have no value.

II. ECONOMETRIC ISSUES

There are two reasons why time series of natural resource prices are expected to be DS, rather than TS. The price of a resource is the sum of the marginal cost of extraction and the resource rent. Economic theory predicts that the rent series, which is an asset series, should not be TS. The marginal cost series should depend upon the prices of inputs used in the extraction process. Schwert [12] and others have shown, empirically, that nearly all price series are DS. Thus, marginal cost (which is a function of other prices) should also be DS. These theoretical and empirical explanations give good reason to expect that natural resource prices are DS.

While the interesting econometric issue is TS versus DS, there is an ancillary issue regarding the length of the series. The TS and DS models both assume that the price series is generated by the same process from its first to its last year. Simple econometric tests are used to determine whether or not this is true. The hypothesis that there is one long consistent series is termed the LS hypothesis. The alternative is the SS hypothesis.

Taken together, the choice of TS or DS and LS or SS gives rise to four different models. The econometrics of choosing among them is discussed in this section. It begins with a demonstration that asset prices (the same as the price of unextracted coal—here, called the resource rent) cannot be TS. In the next subsection, a statistical model that nests the relevant TS and DS alternatives is provided and how to test for

the relevant alternative is shown. Provided in the last subsection is further information on choosing the length of the series to be used.

A. *Theory: Stochastic Trend*

The DS property of rents comes directly from resources being assets. Equilibrium in an asset market implies that, to a good first approximation, there are no gains to be made from predicting the price of the asset.[3] Asset prices are expected to be DS, not TS.

In discrete time, the simplest version of the asset-pricing rule is that $\lambda_t/\lambda_{t-1} = (1 + r) v_t$, where r is the risk-adjusted interest rate and v is a log-normal random variate. Taking the natural log of the pricing rule gives $d \ln(\lambda_t) = \ln(1 + r) + \varepsilon_t$, where $d \ln(.)$ is the first difference of the natural logs and $\varepsilon = \ln v$ is a normal random variate. Using the rule twice gives $\ln(\lambda_t/\lambda_{t-2}) = 2 \ln(1 + r) + \varepsilon_t + \varepsilon_{t-1}$. The formula shows that any gain in price above that expected at time t - 1 persists in time t. Since ε persists, a positive ε shifts the entire future price path up by ε. Although the expected slope of the price path is always the same, the expected level of the price path depends upon the entire history of the error terms, ε. Such a series is called a stochastic trend. The formula also shows that the log difference of the prices form a stationary sequence. [That is, inter alia, all of the values of $d \ln(\lambda)$ have the same mean and variance.] Hence, the model is called DS.

In contrast, consider $\ln(\lambda_t/\lambda_0) = t \ln(1 + r) + \varepsilon_t$. In this second formula, λ grows exponentially from its initial value and ε_{t-1} has no effect on λ_t. Knowing the value of λ_{t-2} or ε_{t-1} is not helpful if λ_0 is already known. The observed values of λ are normally distributed around a fixed exponential time trend. Hence, it is called a deterministic trend. Since $\ln(\lambda_t)$ less a fixed function of time is a stationary series, it is called TS. Of course, $\ln(\lambda_t)$ could just be stationary. The literature generally

includes this case as the limiting case of TS (the trend is zero), and the same econometric tests are used to distinguish it from DS.

An asset price cannot follow a TS series in equilibrium. Consider two periods and, for the sake of argument, assume that ε_1 has a positive realization. The expected log rate of return for holding the asset from period one to period two is $\ln(\lambda_2/\lambda_1) = \ln(1 + r) - \varepsilon_1$. Since other assets in the same risk class provide a log return of $\ln(1 + r)$, there is no incentive for anyone to hold the asset in question which is expected to earn ε_1 less than $\ln(1 + r)$. Since no one would hold the asset, the asset market cannot be in equilibrium with a TS asset-price path. In contrast the DS asset-pricing equation always yields $1 + r$ in expectation and can always be an equilibrium path. Thus, it is expected that a sequence of asset prices form a DS time series, not a TS time series.[4]

There is also a practical reason to expect resource prices to be DS. A wide array of economic time series have been tested for DS (Schwert [12]). Almost all of them are DS. Since resource prices are the sum of marginal costs and rents, prices could also have a DS component because marginal costs (a function of other prices in the economy) are DS. Adding a I(1) and an I(0) process gives an I(1) process, so a process that is the sum of other processes, some of which are integrated, DS, is itself likely to be DS.

B. A General Statistical Model

The statistical models (both TS and DS) for natural resource prices are contained within the transfer function model

$$y_t = a + b\, t + c\, t^2 + \phi(L)/\theta(L)\; \varepsilon_t, \tag{8}$$

where t is time; a, b, and c are parameters; ε_t is a white-noise error process; and ϕ and θ are polynomials in the lag operator L.

The model that Slade chose to use was the general model with the following restrictions: $\phi(L) = 1$ and $\theta(L) = 1 - \rho L$. Applying these restrictions to the general model gives

$$y_t = \rho\, y_{t-1} + a' + b'\, t + c'\, t^2 + \varepsilon_t, \tag{9}$$

where $a' = (1 - \rho)\, a + (b - c)\, \rho$, $b' = (1 - \rho)\, b + 2c\rho$, and $c' = (1 - \rho)\, c$. Equation (9) can be estimated as an AR model (for instance, by the Cochrane-Orcutt method) as long as ρ is less than one. This is the quadratic TS alternative. The same model can be estimated with the entire data series or with any sub-series, so the natural test for LS versus SS is a "Chow" test for the constancy of the coefficients of the regression across two sub-periods.

The case that has received the greatest interest in the macro time-series literature is the DS case where there is a unit root and no time trend. For a (single) unit root, let $\theta(L) = (1 - L)\, \Theta(L)$. The model is

$$(1 - L)\, y_t = b - c + c2\, t + \phi(L)/\Theta(L)\, \varepsilon_t, \tag{10}$$

which is a model in the first difference of y and is linear in t, because differencing a quadratic yields a linear equation. Simplifying the notation a bit, let Δy be $(1 - L)\, y$, $g = b - c$, and $h = 2c$, so

$$\Delta y_t = g + h\, t + \phi(L)/\Theta(L)\, \varepsilon_t. \tag{10'}$$

When $h = 0$, (10') is just an ARIMA (p, 1, q) model with a constant

$$\Delta y_t = g + \phi(L)/\Theta(L)\, \varepsilon_t. \tag{11}$$

ARIMA models, such as (11), can be estimated on the SS or the LS.

One last case deserves mention, and it is the case of a stationary model. In that case $b = c = 0$ and the roots of $\phi(L)$ lie outside of the unit circle. Such a model is

said to be invertible if the roots of $\Theta(L)$ lie outside of the unit circle. The model is an ARMA model. It, too, can be estimated on either choice of series length.

The choice among models (8) through (11) (or the stationary model) requires the use of unit-root tests. If there is a unit root, then only models (10) and (11) are possible. If there is a unit root and no time trend, then only model (11) is possible. Unfortunately, successful estimation of (9) provides little information about the choice between (9) and the other models.

Even when the true model is DS, a TS (or a stationary) model can be estimated "successfully" but with potentially disastrous consequences for inference and prediction. For instance, if the data were truly generated by a random walk, about 80 percent of the time it would be possible to fit an ordinary least-squares regression of price on time to the data and obtain a significant coefficient for time (Nelson and Kang [10]). This spurious regression gives the wrong prediction (that prices will rise or fall) when the true best prediction is that they will stay the same. It also underestimates the prediction error, particularly the prediction error for distant times. Given a true random walk, the prediction error grows as the square root of time. The spurious regression has a slower growing prediction error. From the point of view of testing whether prices are now expected to rise, this understatement of prediction error is crucial. It can (and does) lead to statements such as "prices will rise with 95 percent confidence" when the true statement is closer to "prices will rise with 52 percent confidence." The latter statement is quite similar to "either they will rise or fall," while the former is "proof" that resource rents are now the driving force in resource prices. Fitting an adequate time-trend model is not good evidence that the true model is DS.

Since the DS model arises from a first difference, tests must be done to ascertain that there is actually a unit root. There are two tests performed. The Lagrange-Multiplier (LM) test of Schmidt and Phillips [11] has as its maintained

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hypothesis the existence of a quadratic-trend and ARIMA errors. The LM test is used to show that a unit root cannot be rejected, and this result is buttressed by Monte-Carlo work. This will lead to the conclusion that an equation such as (10) or (11) is more appropriate than the general model (8), quadratic model (9), or stationary ARMA model.

The standard piece of evidence for the stable (11), rather than the possibly explosive (10'), is provided by the extended Dickey-Fuller (DF) [3] test. The DF test gives a joint test of a unit root and $h = 0$ in model (10').

To discriminate between the SS and LS hypothesis, a natural test is, again, a Chow-like test on the constancy of coefficients. But this test assumes that both sub-series are estimable with the same ARIMA model, which is untrue. A simpler, but still problematic test, is to estimate the transfer model (10') and test for $h = 0$. Although this tests only whether the constant term in the ARIMA model is unchanged across time periods (it ignores the changes in the coefficients in the MA and AR parts of the model), it will be sufficient to decide between a long- or short-period ARIMA representation. Lest the test is interpreted as evidence for the model (10'), the transfer function estimate is repeated on an SS and the coefficient is compared to the coefficient from the LS.

Thus, three tests are used for model selection: The LM test, the DF test, and the (asymptotic) t-test of $h = 0$.

III. ESTIMATION

The natural beginning point for estimation of natural resource price models is Slade's model—the quadratic-trend model with auto-correlation correction. In that study, there were 11 natural resource price series with data beginning as early as 1870 and ending in the mid-1970s. At the 95 percent level of confidence, 9 of the 11 series have a significant upward quadratic trend. One of the remaining two series

has an upward quadratic trend significant at the 90 percent level. Only lead appears to be trendless. Slade's [13] empirical (TS) model formed the basis of the belief that resource prices may have begun to rise. This section contains a re-estimation of the TS model with LS and SS, tests for unit roots, and estimation of DS models.

A. The TS Model: A Quadratic Trend

Figs. 1a-1d show the result of repeating the TS analysis with forecasts to 1991 on a 1976 base. The open circles are the actual data in sample, while the closed diamonds are the actual data out of sample. The solid line is the predicted values, and the dotted lines show the bounds given by plus and minus two standard errors. Lead remains trendless no matter how the data are analyzed, so it is generally omitted from further discussion. A satisfactory continuation of the nickel series was not found, so it is omitted also.

The figures differ from those in Slade's paper in four ways: (1) the out-of-sample data through 1991 are included for comparison; (2) the method of estimation is maximum likelihood, rather than a method that drops the first observation; (3) error bands are included that show plus and minus two standard-deviation prediction errors; and (4) the plotted predictions are shown including, rather than excluding, the effect of the auto-correlation term. That is, the predictions include ρ times the price of the prior period. Of course, this makes the predictions conform to the data, and it also accounts for the difficulty in discerning a quadratic shape.

Slade's [13] estimation was also repeated, with the same modifications described above, using data through 1991 and predicting to the year 2000. For reasons of space, those results are summarized in Table I, rather than including plots. The table gives the value in the base year, 1991; the predicted value for year 2000; the standard error of prediction; and the probability (using normality of prediction error)

that the 2000 level would be above the 1991 level (i.e., the probability price will increase). The prediction of price increase is remarkably strong.

The predictions for petroleum, based on data to 1976, completely miss the upward spike around 1980. The predictions based on data through 1991 trace the spike quite well. The difference in fit between these two estimation periods reveals how important the AR term is in these estimates. The major difference between these models is the availability of lagged values to predict the spike. This dependence on the AR term prompts the suspicion that it is the AR term that is driving the fit and that a DS model might be more appropriate.

There is nothing extraordinary about the statistics and values for most of the trend equations. The coefficient on time squared was positive in all equations and significant in all but natural gas and coal. The equation for coal and natural gas had estimated ρ over .9 and no other significant coefficients, which is again suggestive of an integrated process.

If the model that generated Table I were, indeed, a reasonable approximation to the true model, then the conclusion that natural resource prices have turned the corner and are now increasing should stand.

However, the quadratic TS model (9) is sensitive to the time period specified. Examining Figs. 1a-1c, it is quite clear that the series for aluminum, copper, and petroleum decrease quite sharply at their beginnings. In the late 1970s, all of the series showed an upward trend and some (e.g., coal) quite dramatically so. The combination of the upward trend in the late 1970s and the early down slope is what gives these series their quadratic-like shape. In fact, the bottoms of these u-shaped curves are too flat to be from a quadratic. For example, petroleum falls from a beginning price of 10 to 5 by 1880, remains near 5 until the 1970s, and then rises to over 10 again.

One test of the quadratic nature of these curves is a test of the constancy of the coefficients over time. The TS equations are estimated with AR errors by the maximum-likelihood technique. An appropriate test is a likelihood ratio test of the hypothesis that the coefficients of the regression are the same from one sub-period to the next. Reported in Table II are the results of this exercise for the sub-periods: beginning of series to 1919 and 1920 to 1976 or 1991.

The asymptotic chi-squared distribution reported holds even with auto-correlated errors, while the small sample F-statistic (which will also reject) is sensitive to this assumption. The more general Brown-Durbin-Evans test also requires serially uncorrelated errors (Smith [14] showed that it also rejects).

These tests were repeated with the break positioned at 1940 and 1950 with similar results. Breaking the series at 1950 and ending in 1976 yields a rejection at .01 for all but iron. The natural-gas series starts later than the others, so the break in the natural-gas series is at 1940 for the tests in the table.

The likelihood ratio test resoundingly rejects the hypothesis that the quadratic TS model is a reasonable approximation to the data.

There remains the possibility that the TS model is a reasonable approximation to the last part of the data. In this case, for the period 1950 to 1976, one would get significant coefficients of the expected sign on time and its square for all commodities, except copper, aluminum, and iron. Changing the starting dates of the series yields slightly different results. Thus far, it seems that a short TS series might be an adequate representation of the data, but the shortened representation does not imply nearly the significant upward trend exhibited in the LS.

B. Unit Roots: The DS Alternative

The trend exhibited in the quadratic auto-regression might well be an artifact caused by a unit root. For instance, based on the 500 point Monte Carlo experiment,

half of the time one can fit a significant quadratic-trend to an ARIMA (1, 1, 0) model with AR parameter .8. The chance of getting all coefficients in the trend model spuriously significant in an I(1) model is 30 percent. It is well known (Nelson and Kang [10]) that fitting a linear trend can be spuriously accomplished on an I(1) model 80 percent of the time. Thus, unit roots can easily lead to completely incorrect inferences. At the very least, in the presence of a unit root, all of the usual statistical inferences about significance of trend are misleading. Even if a (spurious) TS model forecasts reasonable predictions, it would understate the size of the prediction confidence intervals. Despite the good results from the trend regression, one must test for a unit root.

The appropriate test for a unit root in the general model (1) is the LM test of Schmidt and Phillips [11]. The test maintains a quadratic-trend and ARIMA errors in both null and alternate hypothesis, so it corresponds to the general model.

Briefly, the test is a corrected τ-test. First, the undifferenced series is regressed on its own lag, time and time squared—the TS model. A correction for ARIMA errors is then calculated from the auto-covariances (up to about lag 9) of the errors of this regression. This correction is called ω^2. The next step is to calculate a t-ratio. The differenced series is regressed against time, and the residuals are saved. Then, the differenced series is regressed on time and the cumulative sum of the residuals from the previous regression. The t-statistic on the cumulative sum term is then divided by ω^2, and the resulting statistic is the LM test statistic. The 10 percent critical value of -3.31 is tabulated in Schmidt and Phillips [11]. Table III gives the results of this unit-root test for varying time periods; the asterisk indicates a rejection of the null hypothesis of a unit root at 10 percent.

A unit root is not rejected for any of the SS except silver. All of the series display the characteristic auto-correlation functions of a nonstationary series; all would be differenced by a practitioner of Box-Jenkins identification.

The power of unit-root tests is known to be relatively low. Schmidt and Phillips [11] provide evidence on power for their tests and for DF tests. While there are situations in which the power is less than the size, the general conclusion is that, for initial conditions and growth rates common to economic series, the power against an AR term of .80 is 73 percent and the power against an AR term of .70 is 95 percent. In other words, if the true process were the general model with $\rho = .70$ (or less), then the tests performed would almost surely reject a unit root. To get a further sense of the power of the LM test, a Monte Carlo experiment was run. The estimated aluminum model (TS, LS) was used as the data-generating process and generated 500 samples. Then, both DF tests were run and LM tests were run on these TS series. The DF test rejected a unit root 13 percent of the time and rejected a trendless unit root 18 percent of the time. The LM test rejected a unit root 70 percent of the time. The size of all tests was nominally 10 percent. This confirms that the LM test is the test of choice.

Taken from either the vantage of 1991 or of 1976, the evidence greatly supports differencing the data. The most questionable cases would seem to be petroleum and silver. Both of these resources have had spectacular price run-ups attributable to less-than-competitive activity, followed by a price senescence as more normal market conditions prevailed. In the case of silver, the Hunt brothers attempted corner happens only in the 1991 price series, so the series ending in 1976 seems a more natural representation of ordinary events. For petroleum, there are several abnormal events corresponding to warfare in the Middle East. These events dominate the price series and seem to exhibit mean reversion. With these qualifications, whether differenced models actually exhibit a time trend was considered. In model (10'), is the coefficient on time actually zero?

The DF null hypothesis is model (11)—DS with no time trend. The extended DF test provides an F-like test of the hypothesis that there is one unit root and that

the time-trend term is zero. The DF regression depends upon approximating $\phi(L)/\Theta(L)$ by a short ($O(^3\sqrt{T})$) AR polynomial, $\xi(L)$, and estimating

$$\Delta y_t = (\rho - 1)\, y_{t-1} + g + h\, t + 1/\xi(L)\, \varepsilon_t. \qquad (12)$$

The F-like test of interest is ($\rho = 1$ and $h = 0$). Since the LM tests reduce the models under serious consideration to a DS form, the real remaining question for the DF tests is whether h is zero. If it is zero, then the model is simply an ARIMA model with constant.

In the SS, the DF tests do not reject an ARIMA representation, except for aluminum (see Table IV). In the LS (beginning in 1920), there are several rejections, depending upon the ending date. From the vantage point of 1976, petroleum and natural gas would be modeled as having a trend and iron and zinc might include a trend. From the vantage point of 1991, aluminum is the only series that would not be obviously modeled as an ARIMA process without time trend. Putting this together, the DF tests (except for aluminum) suggest that there was no trend in the latter part of the period even if there was one in the earlier part of the period.

C. The ARIMA (DS) Estimation

Assuming that it is correct to difference the series, which leads to consistent, if not efficient, estimates in all circumstances, the constancy of the constant term in the ARIMA representation of the data can be tested. The same set of regressions serve to estimate the value of the time-trend parameter in (10') and give a standard t-test as to whether it is, indeed, zero. The LS, ending in 1976 and 1991, were estimated by standard Box-Jenkins techniques. The auto-correlation function suggests differencing for all the series as do the Leung-Box-Pierce (LBP) Q statistics. Using the auto-correlogram and partial auto-correlogram, low-order ARIMA (p, 1, q) processes were identified. Several alternatives were then estimated for each series, generally by

increasing and decreasing p and q from the identified value, and the resulting models were compared. The model with an LBP statistic that did not reject white-noise residuals, which had the lowest Akaike information criteria (AIC), was then selected. The selected models were then re-estimated as a transfer function model with time as the added variable. Under the maintained hypothesis that the model is an ARIMA model, the coefficient on time should be zero.

Table V shows the transfer model (LS) coefficient on time and its t-ratio. Using the series ending in 1976, four of the series have a nonstationary mean and one other very likely has a nonstationary mean. Silver cannot be estimated in this form. Natural gas does not have a statistically significant time trend, but the large value of the time coefficient and its large standard error make it unlikely that the mean is constant over time. It is difficult to determine which way it will go. Zinc is less clearly unstable. In the series ending in 1991, time appears less important. However, since instability over any sub-period is enough to establish instability, the 1976 data are sufficient to show nonconstancy of coefficients. The same model was run with shorter time periods (e.g., beginning in 1940), and these shorter-term DS representations do not have a significant time trend. The result of this statistical experiment is to favor SS series. That is, the conclusion is similar to that of the DF tests. The last part of the series has no significant trend, even if the series as a whole does. This suggests that the earlier part of the series had a marked trend but that the two parts of the series are different.[5]

Given that the series are likely to be SS and DS, they can be modeled as ARIMA processes. Standard Box-Jenkins techniques were used to identify and estimate models for each of the series. The chosen models differ with the end time. All models are differenced, because the auto-correlograms all suggest first differencing. Appendix Table I gives the estimated models for the period ending in 1976, while Appendix Table II gives the models for the period ending in 1991. There are a few

notes about these models. The models have a low R^2 which reflects an inability to explain what is left of the series after differencing. Such series are very nearly I(1). Models different from the chosen model by either p or q +/- 1 were also examined. For example, if (p, 1, q) was chosen, (p + 1, 1, q) was also estimated. The final model was the one with the lowest AIC, provided that the LBP statistic at lag 6 for that model did not reject white-noise residuals. For purely technical reasons regarding econometric software, models that are best estimated as I(1) have been represented as (1, 1, 0). In general, the highest-order MA and AR terms have coefficients significantly different from zero at the 95 percent level, though lower-order terms may not be significant.

On the suggestion of a referee, a set of SS ARMA models were also estimated, and they are reported in Appendix Tables III and IV. These models assume that the series are stationary, while the standard interpretation of the test results is that the models are most likely not stationary. The estimates of two of these models included AR terms with roots inside of the unit circle (i.e., non-stationary), and one of these models was noninvertible. As the results section attests, these models have excellent predictive performance, despite their apparent misspecification.

IV. RESULTS

The forecasting results of the stationary ARMA, TS, and DS models estimated with SS (i.e., from 1940 forward) and from LS (the entire series) are presented in Tables VI, VII, and VIII. The tables contain: (1) the model's prediction using 1940 to 1976 data for 1991; (2) the standard error of that forecast (this standard error is likely much too small because of both the inherent pretest nature of Box-Jenkins methods and the additional pretesting done in this paper); (3) the probability that the forecast will exceed the 1976 value of the series, based on the printed standard errors; (4) the

model's forecast using beginning of sample to 1976 data; (5) the forecast standard error for the long series; (6) the 1991 actual price; (7) the percent error of the forecast for the SS; and (8) the percent error of the forecast from the long series. The tables also contain the root mean square (RMS) errors of the 1991 forecast, the averaging being over series. Figs. 2a-2d contain the SS, TS fit and forecast; Figs. 3a-3d contain the SS, ARMA fit and forecast; and Figs. 4a-4d contain the SS, DS fit and forecast.

Comparing the three tables, note that the DS representation gives a much lower probability of prices increasing than the TS representation (lower in all but one case and much lower in three cases), while the ARMA models find decreasing prices more probable in all but one case, and strongly so in half of the series. Three factors contribute to the dramatically different probabilities. First, if one believes that there is an element of spurious correlation in the TS representation, then the work of Nelson and Kang [10] would leave one to believe that the standard errors in the TS alternative are too low, which would help explain the high apparent confidence of prediction. The ratio of the RMS standard error of the DS models to the TS alternative is about 6/5. Much of the difference between the TS and DS models, however, lies in the TS models predicting generally higher prices. Both the TS and DS models overpredict the 1991 values in SS form. Lastly, because of the mean reversion properties of the stationary ARMA models, those models predict that prices will fall back toward their historical average (with the exception of the nonstationary ARMA models for aluminum and natural gas).

The DS predictions are more accurate than the TS predictions, having only about 1/2 of the RMS percent prediction error. The ARMA predictions are even better, having less than half of the RMS of the DS predictions. The lower prediction errors are almost exclusively a function of lower predictions. Still, only one ARMA SS model, two ARMA LS models, and three LS DS models underpredict the data.

The RMS percent error of prediction of 1991 from a 1976 base largely favors the ARMA models. This period was characterized by an extreme run-up in prices followed by a senescence. For some resources, this occurred somewhat later than for others. Because the ARMA models strongly revert to their mean levels, these models did well in predicting the fall in coal, iron, and silver prices following their run-up during the 1970s. However, these models missed the continuing rise of natural gas and petroleum which fell somewhat later. In all cases the ARMA models ended up predicting the final value quite well. When the intermediate predictions are also counted, the ARMA model still outpredicted the DS model for just four of eight commodities and, overall, had about 50 percent of the RMSE (taken across commodities) of the DS model. The difference between the RMSE of the DS and ARMA models comes mostly from poor DS predictions for silver and coal.

Because the last decade seems so unusual, the predictions of DS and ARMA models were also compared for estimation periods ending in 1963, 1968, 1973, 1978, and 1983. Estimates were made using 44 years of data for each period, and predictions were made for 10 years forward. Here, the DS model did best in the early period; the DS and ARMA were tied in the next three periods; and much like the 1976 base year model, the ARMA model outperformed the DS in the latest period. The ARMA models significantly outperformed the trend model in all periods, while the DS models did so in all but the last. Except for the last decade, the ARMA model seems very comparable to the DS model in terms of forecasting performance. Predictions and prediction errors were also calculated using the first half of the sample to forecast the second half. In this exercise the forecasts were from 1951 to 1991. Again, the ARMA models performed best and had fewer forecasts outside of the two standard-error bound than the DS model. The TS model performed least well. The results of these models are in Appendix Figs. 1a-3d. Insofar as forecast error is the criterion, ARMA models, even non-invertible and non-stationary, seem to be the (unorthodox) choice.

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Tables I, IX, and X give the predictions for the year 2000, and the story is quite plain. The evidence against the TS LS model is overwhelming, so there is no need to consider the forecasts from that model. Similarly, there is considerable evidence against the DS LS and ARMA LS models, though either would appear to be acceptable for natural gas and zinc. Since there is no real gain from using the LS models, the choices are the SS ARMA, DS, and TS models. If the TS version is believed, prices will almost surely rise. Using the LM test, only silver has a reasonable chance of not having a unit root. Thus, the TS prediction for silver might be given some weight. Countering that inclination is the very peculiar circumstances for the pricing of silver during the Hunt brothers corner on the market. Also, another point against using any of the TS models is that they are extremely poor in sample prediction. Thus, ARMA or DS models should be used for prediction.

The choice between ARMA and the DS ARIMA models is less easily made. All of the resources' price series other than silver may have unit roots (though is there conflicting evidence on whether aluminum has a trendless unit root). Series with unit roots ought to be estimated as integrated processes. However, the sample forecast evidence favors ignoring the unit-root problem and estimating the series as ARMA models, regardless of whether the estimates are non-invertible or non-stationary. Fortunately, the answer for the future of resource prices is much the same.

In summation, the evidence for unit roots shows that natural resource prices do inherit the unit-root property of asset prices, but the evidence for the other property of asset prices (increasing prices) is unclear. The best that can be said is that the chances of increasing prices are 7 in 10 for zinc and copper. The remainder are between 45 percent and 60 percent. The use of DS methods combined with SS, both dictated by reasonable tests, reduce the predictions of price increase from near certainty to maybe. If it is believed that the ARMA models are closer to the truth, increasing prices are even less likely.

FOOTNOTES

[1]Fisher [4] argues, as did Barnett and Morse [1], for the importance of product prices. As long as the refined product is inexpensive, it matters little to the economy that it was made from an increasingly expensive raw material. Brown and Field [2] argue that the price of interest is the resource rent—the price of an unextracted resource. If Hotelling's theory (that resource rents increase) is tested, Brown and Field are undoubtedly right. Clearly, both prices are of interest. The data, however, are overwhelmingly on product prices, so that is and must be the emphasis of empirical analysis.

[2]Much the same conclusion could be forced by assuming, as did the former Secretary of Interior, James Watt, that the world would end happily before all resources were exhausted. (A pessimist who believes in imminent nuclear destruction would share Watt's views concerning resources. There is nothing special about a happy imminent end.) He used this argument to push for faster than market-driven extraction of resources on public lands.

[3]A more accurate statement is that the marginal gain to predicting prices is equal to the marginal cost of increasing the precision of prediction. For a market with a great deal of underlying value (such as the market for Fortune 500 stocks), there would be a large payoff to even slightly improved precision. Thus, a large effort in predicting prices is expected, which results in a market with nearly perfect information. Some natural resources (such as standing trees) offer much less return to prediction activity, so the publicly available information set might enable an analyst to improve upon the precision in predicting prices. See Leroy [7] for a summary of efficiency tests.

[4]The argument that, in an efficient market, asset prices are not TS originated in the random walk theory but was significantly generalized by Samuelson (see Leroy [7]). The best statement, if not the earliest, from the theory of finance is in Leroy [7].

He shows that, with risk neutrality and efficient capital markets, the discounted-to-time-zero value of a mutual fund that reinvests its dividends follows a martingale. In the case of a natural resource, which pays no dividends, it would be discounted rent that follows a martingale. If resource rent were TS, that is of the form $\lambda_t = f(t) + \varepsilon_t$, where the error term was mean zero with positive variance, then $v_{t+1} = \lambda_{t+1}/(1 + r)^{-t-1}]$, would not be a martingale. Direct substitution shows that $E[v_{t+1}] = v_t$ for only one possible realization of ε_t. A DS series does lead to a martingale, but a martingale is more general than the DS models considered here. Much of the modern finance literature surveyed by Leroy is about the apparent empirical violations of assumptions about variance needed for asset prices to follow a martingale. For these purposes, it is enough that efficient markets admit DS, not TS, alternatives.

[5]The model with the AR and MA coefficients held constant could not be estimated, but the time-trend variable allowed to take one value in the early years and another in the later years. Therefore, there is no definitive test for this statement.

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TABLE I

Probability of Increasing Price in the Quadratic-Trend Model

Resource	1991 real price*	2000 predicted	Standard error	Prob. of increase
Aluminum	0.167	0.51	0.30	87.26%
Coal	6.13	7.28	1.35	80.22%
Copper	30.75	38.21	10.90	75.35%
Iron	68.93	105.79	13.44	99.70%
Natural gas	46.12	59.03	13.36	83.31%
Petroleum	4.64	7.41	1.75	94.35%
Silver	113.61	382.54	127.14	98.28%
Zinc	14.85	17.42	3.84	74.88%

*Iron ends in 1985.

TABLE II

Test Against Constancy of Coefficients in the Quadratic-Trend Model

Resource	Chi-squared (4) end = 1976	Significance	Chi-squared (4) end = 1991	Significance
Aluminum	111.10	4.16E-23	126.60	2.66E-26
Coal	45.23	3.56E-09	41.87	1.77E-08
Copper	36.16	2.68E-07	34.60	5.50E-07
Iron	25.56	3.87E-05	28.98	7.88E-06
Natural gas	19.14	7.38E-04	20.79	3.49E-04
Petroleum	67.51	7.59E-14	10.29	3.57E-02
Silver	10.35	3.49E-02	59.00	4.70E-12
Zinc	64.67	3.01E-13	62.89	7.37E-13

Note: The sub-periods are beginning of data to 1920 and 1921 to end.

TABLE III

Lagrange-Multiplier Test Statistics for Unit Roots for Different Time Periods

Resource	Begin-1976	1920-1976	1940-1976	Begin-1991	1920-1991	1940-1991
Aluminum	-2.24	-3.56*	-2.11	-2.70	-3.62*	-2.68
Copper	-0.76	0.10	0.23	-2.73	-2.96	-1.61
Iron	-1.45	-1.25	-1.13	-1.06	0.33	0.59
Lead	-3.27	-1.26	-1.22	-2.57	-0.98	-1.04
Zinc	-17.07*	-2.55	-1.15	-14.17*	-1.91	-1.64
Petroleum	-3.54*	-3.61*	-1.25	-3.31*	-1.80	-0.94
Coal	-2.46	-2.88	-2.09	-3.12	-2.56	-2.51
Nat. gas	-2.62	-2.62	-1.95	-2.97	-2.97	-3.04
Silver	-1.64	-1.15	0.15	-1.78	-3.92*	-3.47*

*Rejected at the 10 percent level of significance.

TABLE IV

Dickey-Fuller Tests for Trendless Unit Root for Different Time Periods*

Resource	1920-1976	1940-1976	1920-1991	1940-1991
Aluminum	2.02	18.56	2.58	14.07
Copper	3.56	4.73	3.02	2.05
Iron	4.67	1.26	9.95	2.01
Lead	N/A	N/A	N/A	N/A
Zinc	4.66	1.54	7.78	2.67
Petroleum	6.82	1.54	3.50	1.73
Coal	1.89	1.63	5.40	4.17
Natural gas	6.82	2.18	3.52	3.76
Silver	2.61	2.31	4.27	3.18

*The 10 percent critical value is 5.34.

TABLE V

Transfer Function Models: Significance of Time in Long-Series Models

Resource	1991 end time*		1976 end time	
	Time	t-statistic	Time	t-statistic
Aluminum	9.00E-04	2.03	1.20E-03	2.04
Coal	4.00E-05	0.03	3.48E-03	1.60
Copper	6.17E-03	1.15	1.37E-02	2.54
Iron	2.07E-02	5.95	2.67E-02	9.62
Natural gas	1.16E-01	0.20	1.13E-01	0.21
Petroleum	2.18E-03	1.31	2.98E-03	2.58
Silver	-1.90E-03	0.01	N/A	N/A
Zinc	1.38E-03	0.95	2.32E-03	1.00

*Iron ends in 1985.

TABLE VI

ARMA Model Results: Predicting 1991 Values From 1976 Base

Resource	Short series			Long series		1991 actual price*	Short series % Forecast error	Long series % Forecast error
	1991 forecast	Forecast std. error	Prob. of increase	1991 forecast	Forecast std. error			
Aluminum	0.20	0.12	0.42	0.23	0.76	0.17	19.30*	35.18
Coal	8.31	2.47	0.18	8.80	2.05	6.13	35.49	43.62
Copper	34.00	8.34	0.41	39.51	12.88	30.75	10.58	28.52
Iron	69.07	15.08	0.02	83.29	15.04	68.93	0.19	20.83
Natural gas	49.97	13.29	0.91	25.32	7.67	46.12	8.36*	45.11
Petroleum	3.32	0.70	0.05	3.16	1.27	4.64	28.51	31.82
Silver	146.98	63.91	0.08	157.88	52.41	113.61	29.37	38.97
Zinc	15.65	2.77	0.04	15.40	3.84	14.85	5.42	3.71
						RMS % error	20.97	33.48

*The iron-series prediction errors are calculated based on the prediction for 1985. The short-series models use data from 1940 forward, and the long-series models use data from the beginning of the sample.

TABLE VII

Quadratic-Trend Model Results: Predicting 1991 Values From 1976 Base

Resource	Short series			Long series		1991 actual price*	Short series % Forecast error	Long series % Forecast error
	1991 forecast	Forecast std. error	Prob. of increase	1991 forecast	Forecast std. error			
Aluminum	0.45	0.17	0.91	0.77	0.40	0.17	168.51	362.23
Coal	18.60	3.78	0.98	12.05	1.85	6.13	203.36	96.57
Copper	50.78	13.53	0.82	48.25	11.72	30.75	65.16	56.94
Iron	118.43	19.56	0.84	97.45	12.63	68.93	71.81	41.37
Natural gas	65.94	9.06	0.99	49.12	7.67	46.12	42.98	6.51
Petroleum	5.87	1.30	0.86	5.99	1.62	4.64	26.47	29.07
Silver	447.29	60.68	0.99	329.0	37.57	113.61	293.71	189.59
Zinc	31.03	6.10	0.96	17.28	4.06	14.85	108.97	16.40
						RMS % error	149.93	151.08

*The iron-series prediction errors are calculated based on the prediction for 1985. The short-series models use data from 1940 forward, and the long-series models use data from the beginning of the sample.

TABLE VIII

ARIMA Model Results: Predicting 1991 Values From 1976 Base

| Resource | Short series | | | Long series | | 1991 actual price* | Short series % Forecast error | Long series % Forecast error |
	1991 forecast	Forecast std. error	Prob. of increase	1991 forecast	Forecast std. error			
Aluminum	0.17	0.12	0.33	-0.12	0.58	0.17	4.15	174.30
Coal	15.58	4.19	0.88	11.57	2.38	6.13	154.11	88.65
Copper	54.05	5.11	0.99	37.82	11.99	30.75	75.79	23.01
Iron	105.69	12.89	0.70	86.66	15.62	68.93	53.32	25.72
Natural gas	49.94	14.03	0.90	38.53	10.17	46.12	8.29	16.45
Petroleum	5.26	1.23	0.74	3.55	1.62	4.64	13.31	23.55
Silver	249.58	47.69	0.59	291.55	69.41	113.61	119.68	156.62
Zinc	22.90	5.84	0.66	17.64	4.37	14.85	54.23	18.83
						RMS % Error	78.95	90.24

*The iron-series prediction errors are calculated based on the prediction for 1985. The short-series models use data from 1940 forward, and the long-series models use data from the beginning of the sample.

TABLE IX

ARMA Model Results (Estimation Period is 1940-1991)

Resource	1991 actual price*	2000 predicted	Standard error	Prob. of increase
Aluminum	0.17	0.25	0.05	0.95
Coal	6.13	6.11	1.73	0.49
Copper	30.75	31.99	7.80	0.56
Iron	68.93	64.98	13.92	0.39
Natural gas	46.12	31.70	18.57	0.22
Petroleum	4.64	4.18	1.87	0.40
Silver	113.61	173.44	156.9	0.65
Zinc	14.85	15.31	3.01	0.56

*Iron ends in 1985.

TABLE X

ARIMA Model Results (Estimation Period is 1940-1991)

Resource	1991 actual price*	2000 predicted	Standard error	Prob. of increase
Aluminum	0.17	0.18	0.06	56.19%
Coal	6.25	6.26	2.64	50.19%
Copper	31.34	36.25	8.57	71.64%
Iron	68.93	75.04	22.50	60.70%
Natural gas	45.58	42.25	24.60	44.60%
Petroleum	4.73	4.91	3.05	52.41%
Silver	115.83	140.15	256.00	53.78%
Zinc	15.14	18.35	5.66	71.46%

*Iron ends in 1985.

SOURCES

DEFLATOR:

Producer Price Index: all commodities.

ALUMINUM, COPPER, ZINC, SILVER:

Aluminum Copper and Zinc are in cents/per pound. Silver is in cents per ounce. Sources: U.S. Bureau of Mines, *Nonferrous Metal Prices in the United States through 1988*. Then, Aluminum: American Bureau of Metal Statistics, Engineering, Mining Journal, and Metals Week. Copper: Bureau of Mines Minerals Yearbook, Engineering and Mining Journal, American Metal Market, and Metals Week. Zinc: 1870-1875: R. Manthy, *Natural Resource Commodities, A Century of Statistics*. 1875-1904: New York Price, Ingalls, W.R., *Lead and Zinc in the United States*. 1905-1914 and 1919-1970: St. Louis prices, *American Metal Market/Metal Statistics* and E/MJ Metal and Mineral Markets 1915-1918: St. Louis price calculated from slides by producers. 1971-1990: *Metals Week*. 1991: London Metal Exchange Silver: *Bureau of Mines Minerals Yearbook*.

COAL, NATURAL GAS, PETROLEUM:

Coal is dollars/short ton and petroleum is dollars/barrel. Sources are *United States Statistical Abstract* and, *Annual Energy Review 1991* by the Energy Information Administration.

PIG IRON:

Producer price index where 1972-74= 100. Index was discontinued in 1985.

ADDITIONAL NOTE:

Observations for 1992 and 1993 were obtained from the appropriate commodity specialist at the U.S. Bureau of Mines and the Economic Research Service.

FIGURE LEGENDS

Fig. 1a-1d. Long-series, trend-stationary models through 1976, forecast to 1991.

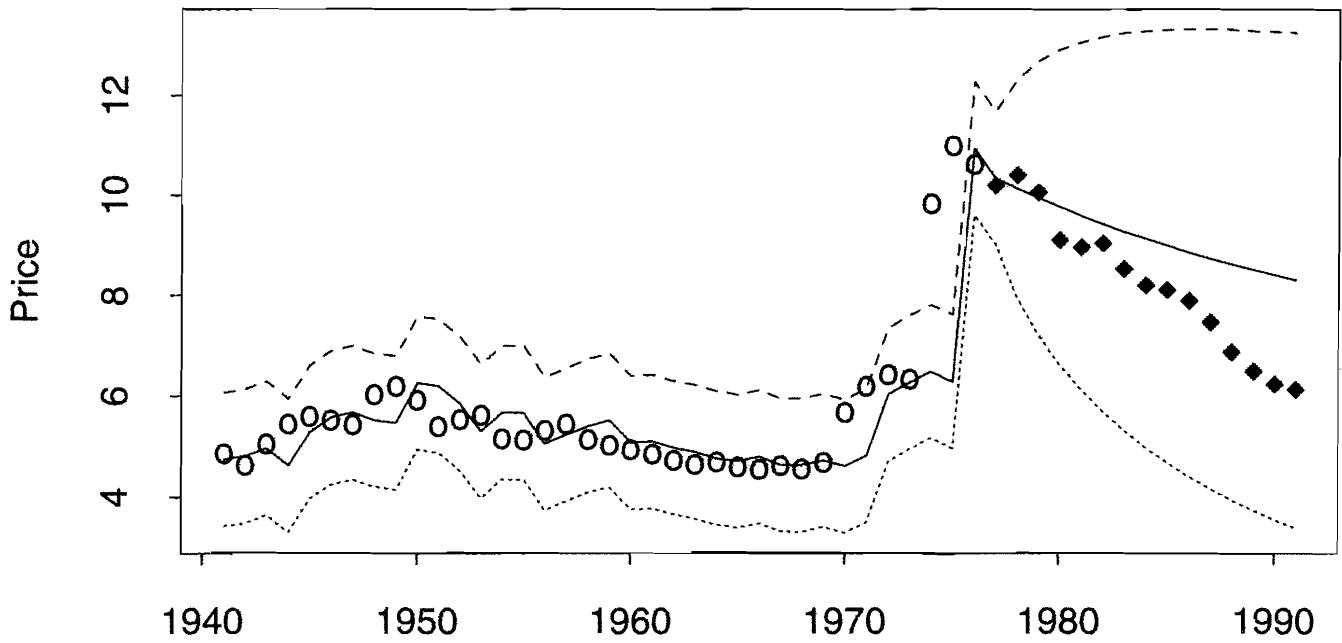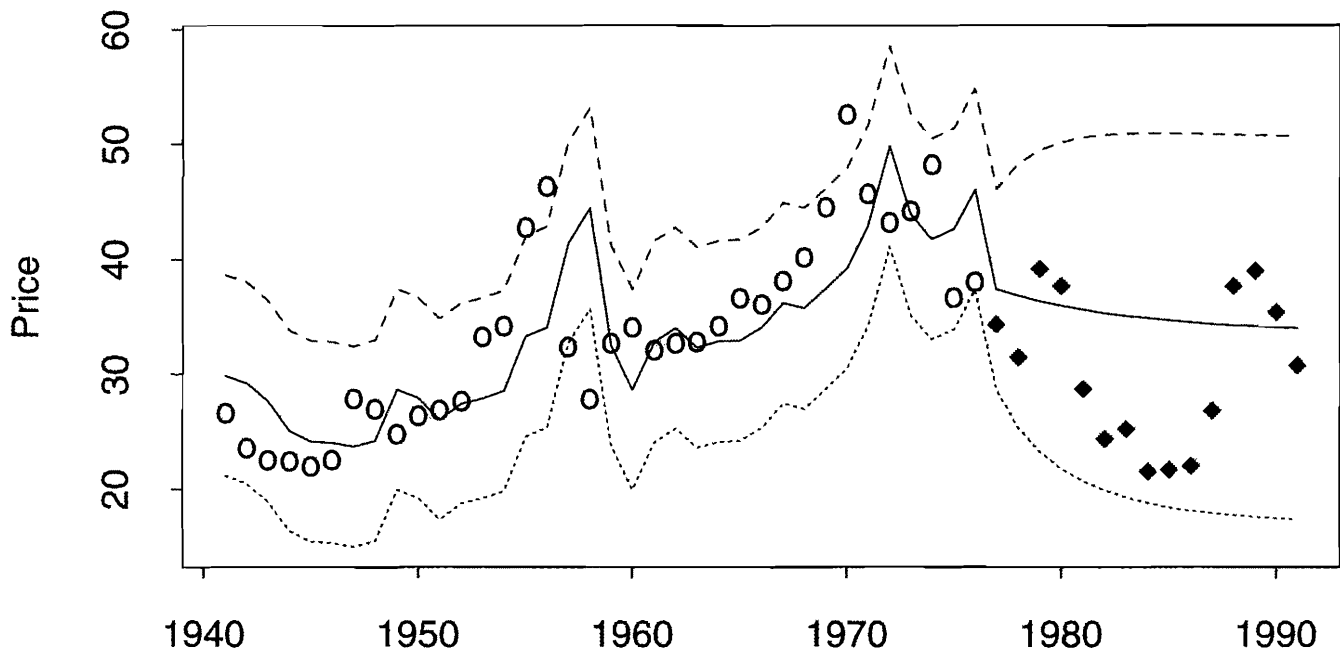Fig. 2a-2d. Short-series, trend-stationary models through 1976, forecast to 1991.

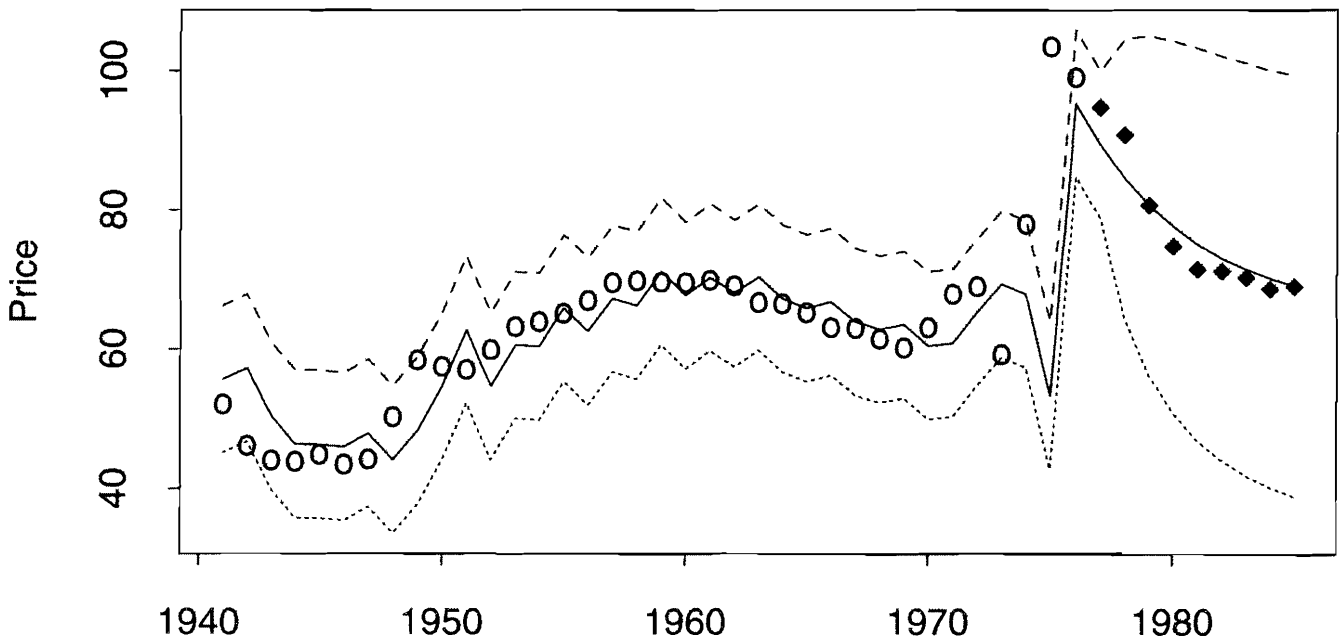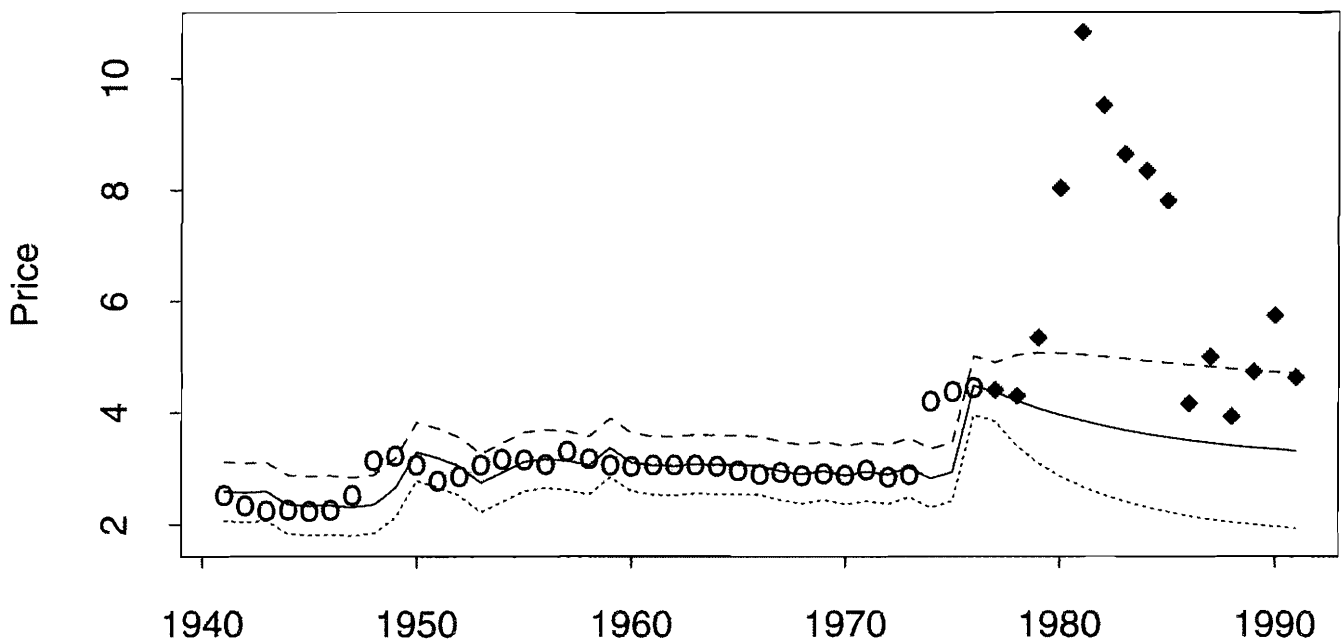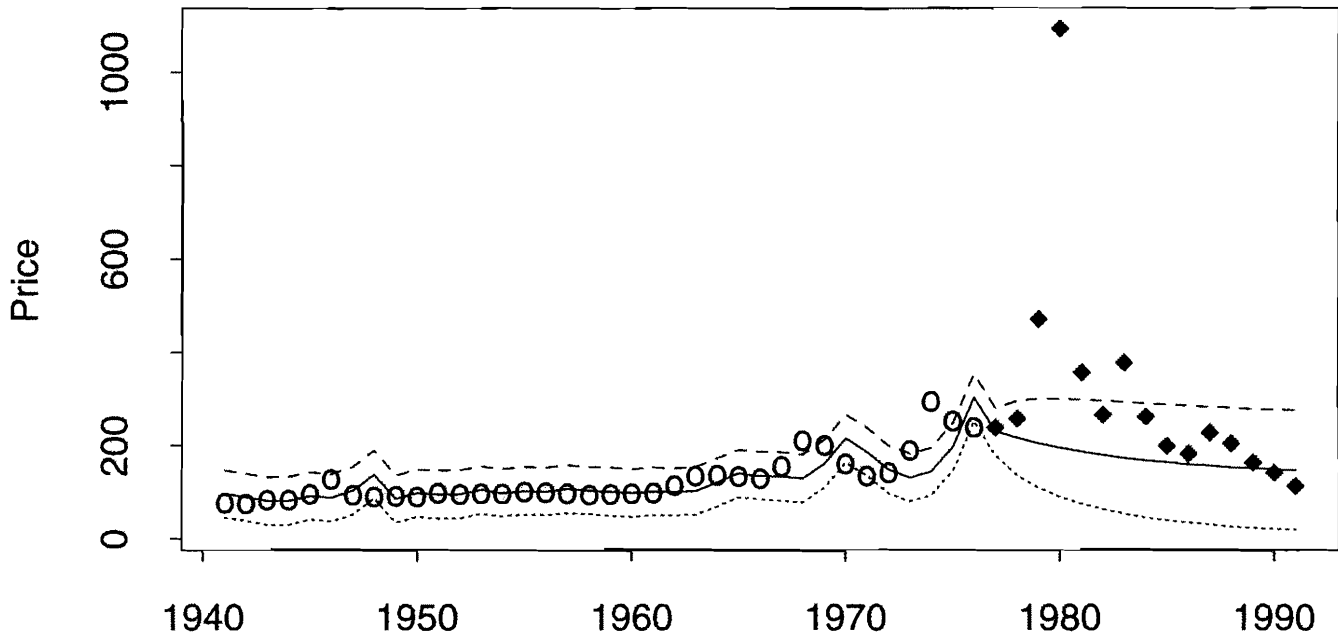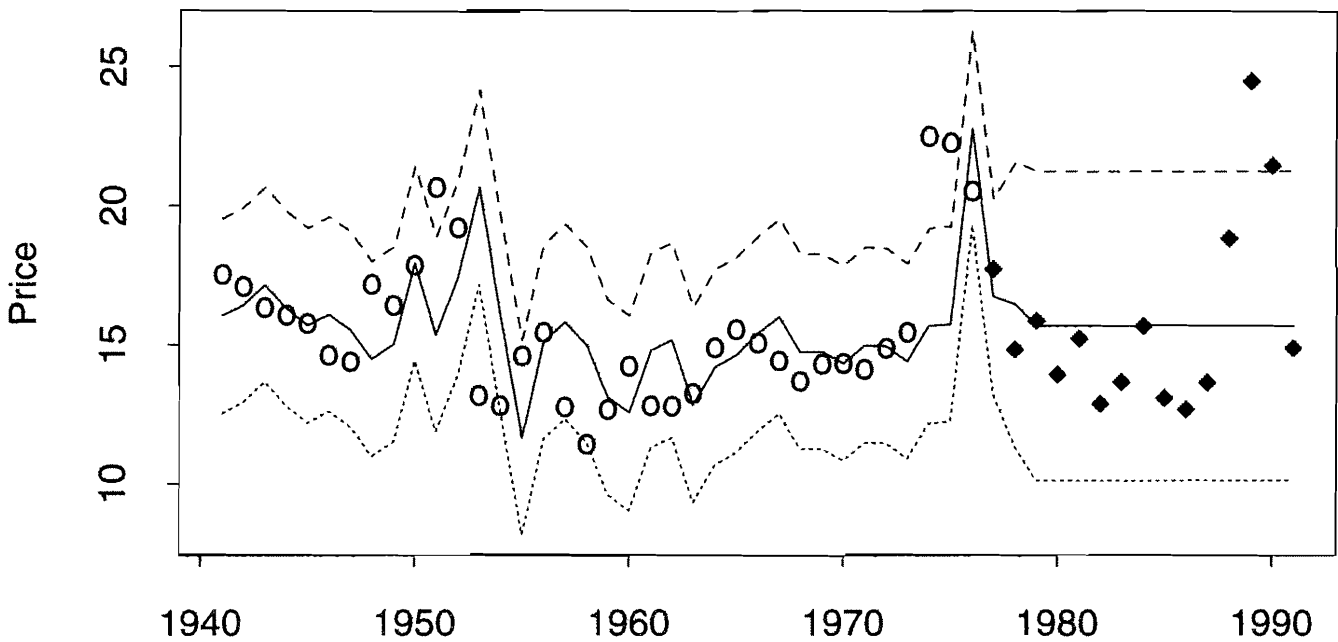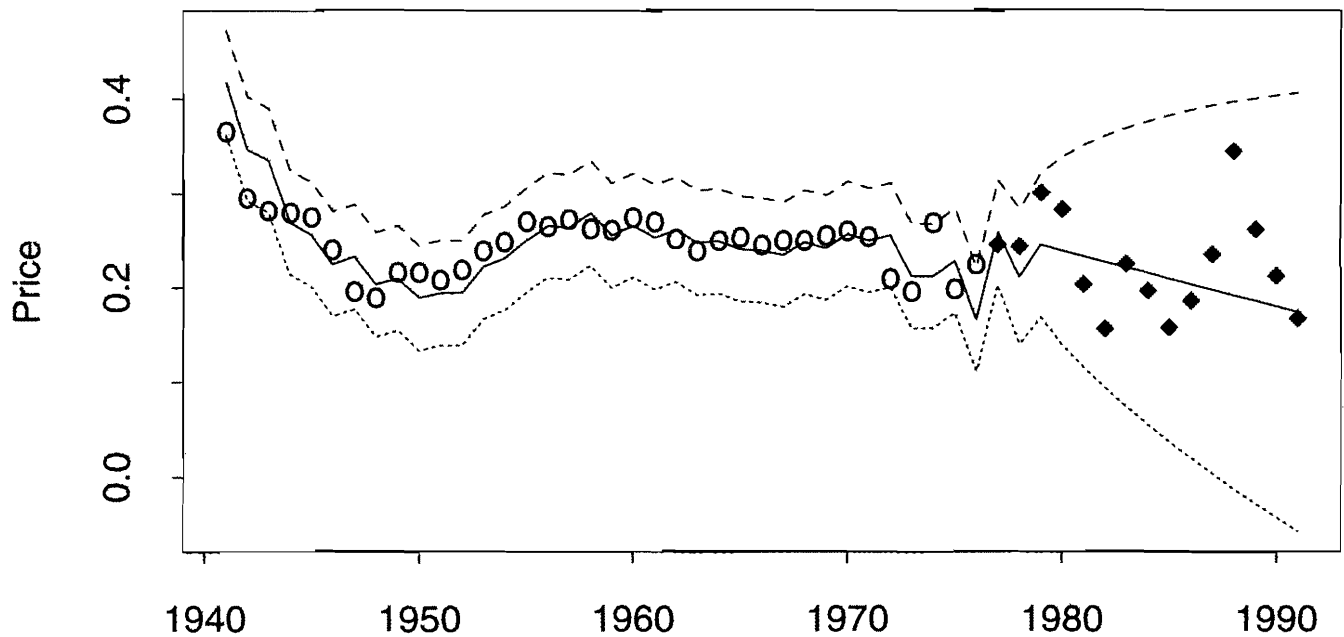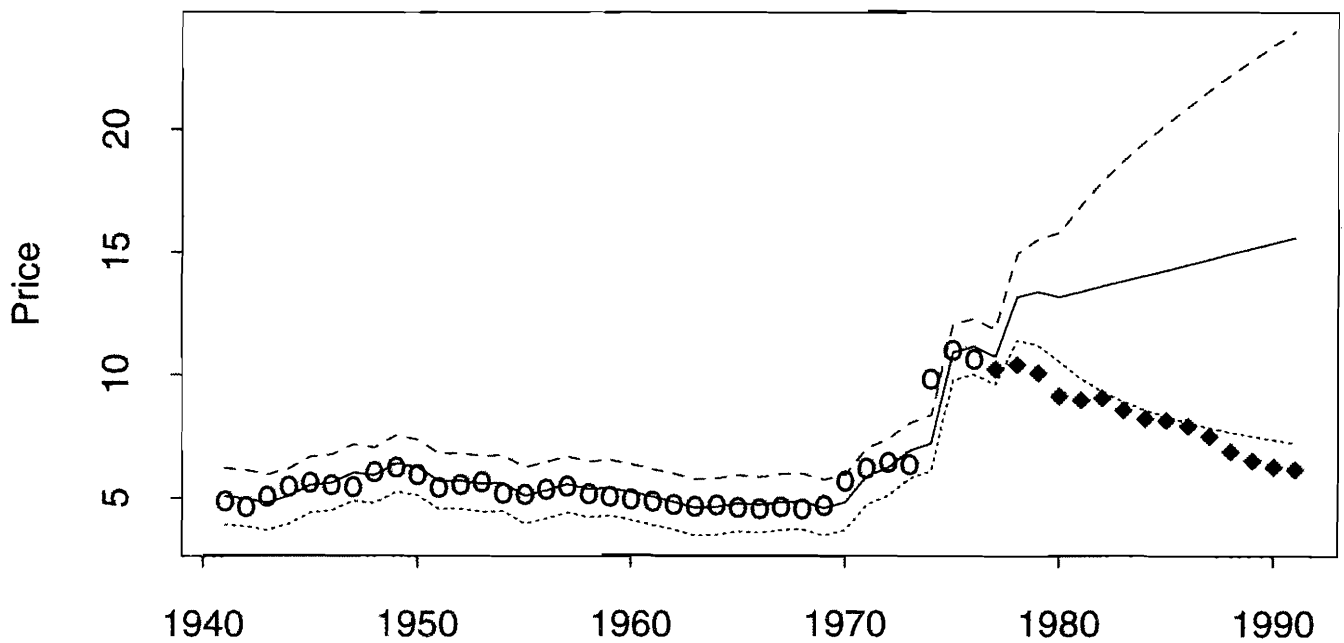Fig. 3a-3d. Short-series, ARMA models through 1976, forecast to 1991.

Fig. 4a-4d. Short-series, differenced-stationary models through 1976, forecast to 1991.

Aluminum
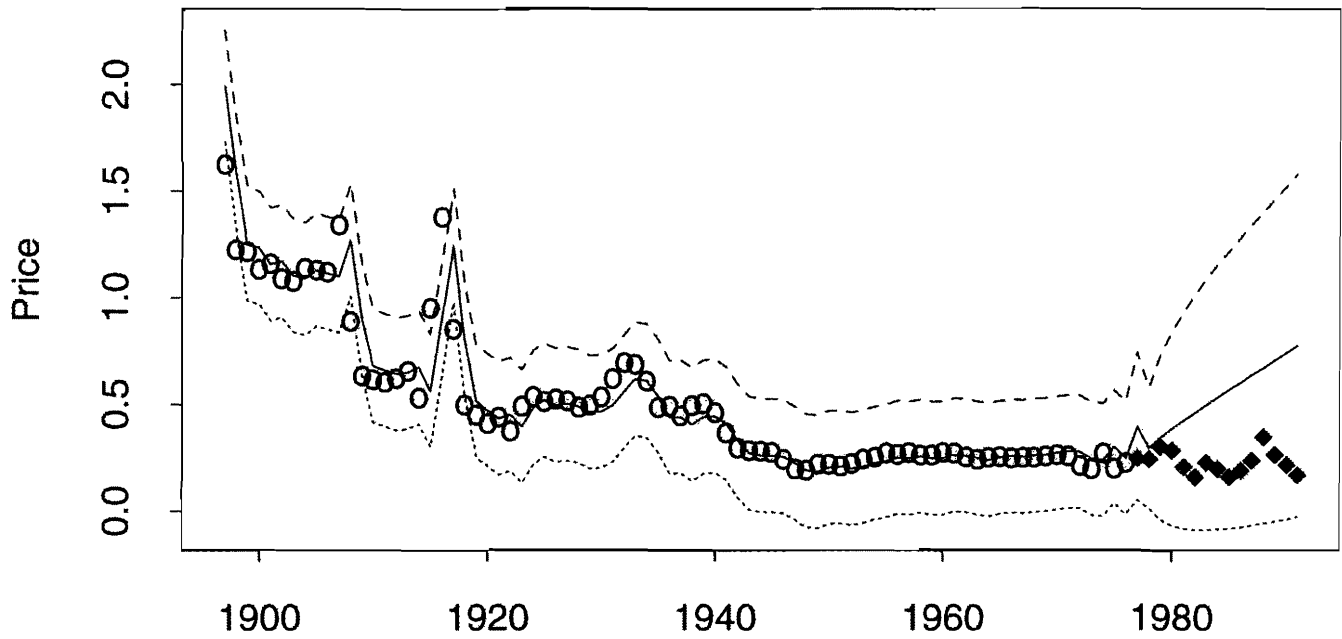
Coal
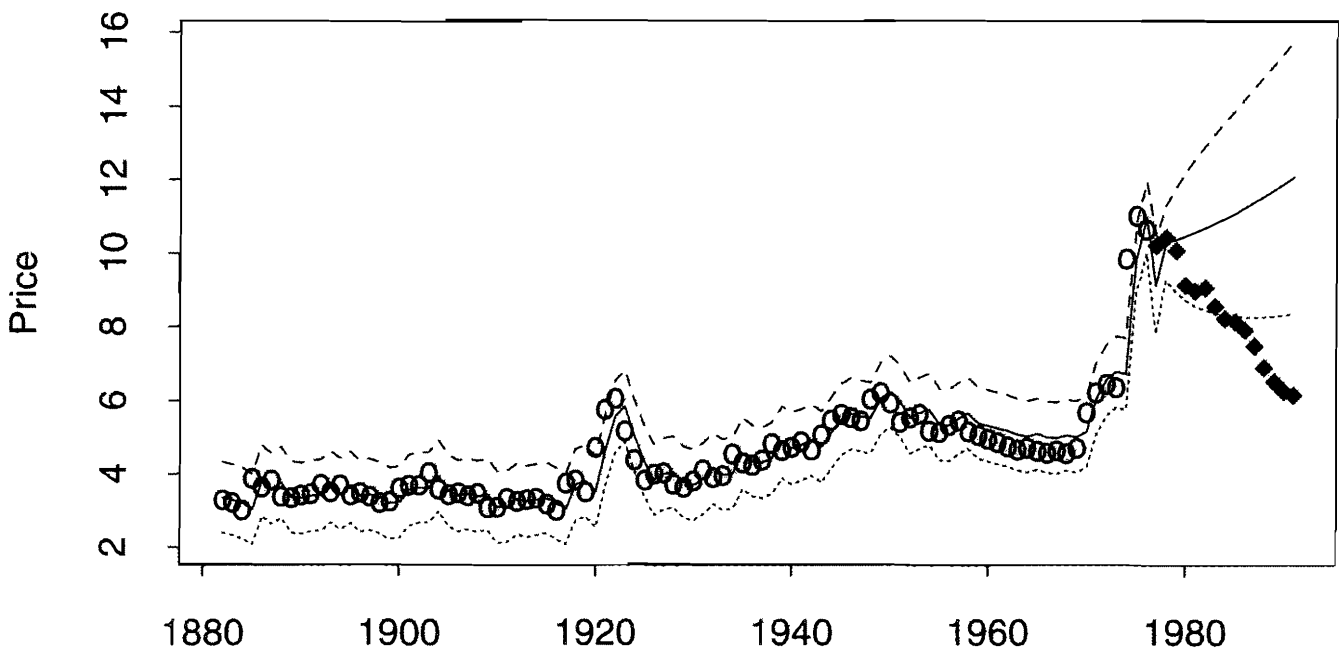
FIG. 1a. Long-series, trend-stationary models through 1976, forecast to 1991.

Copper
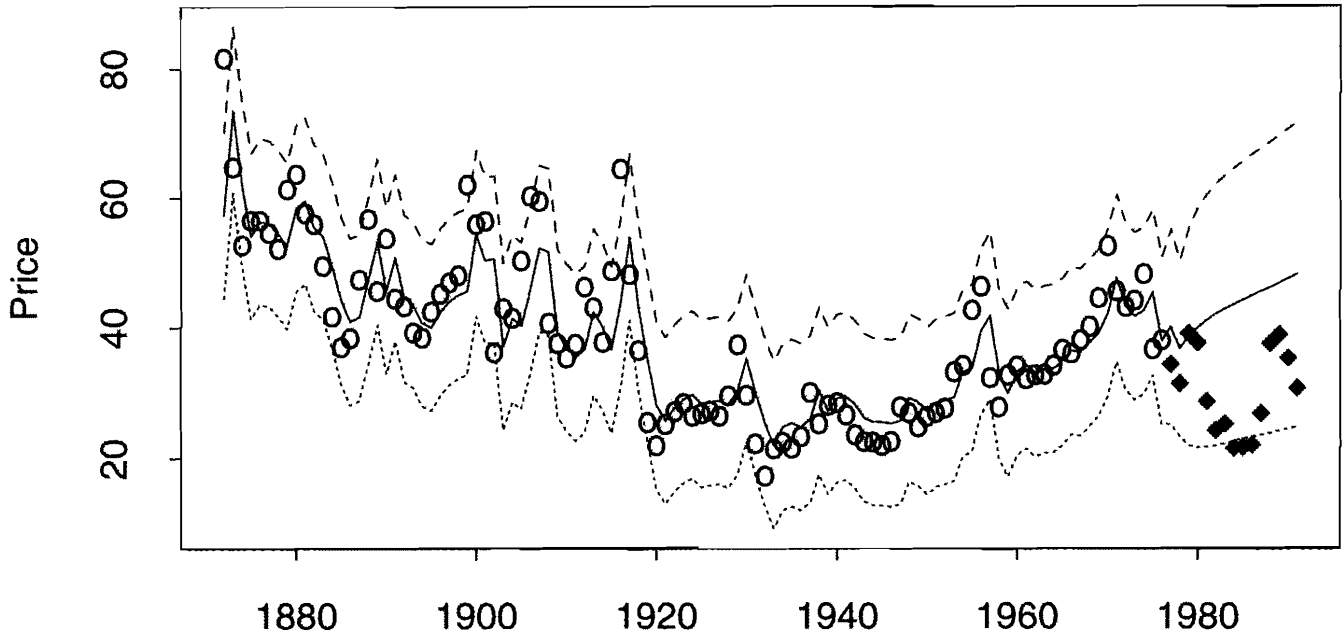
Iron
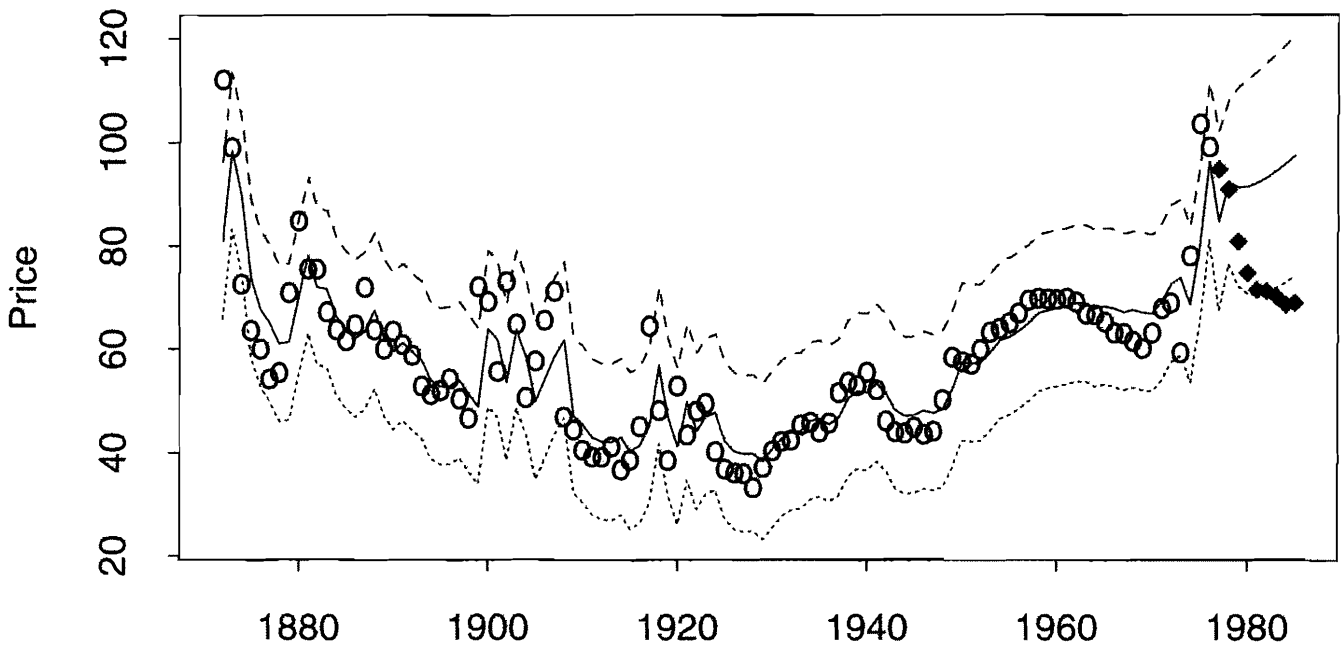
FIG. 1b. Long-series, trend-stationary models through 1976, forecast to 1991.

Natural Gas
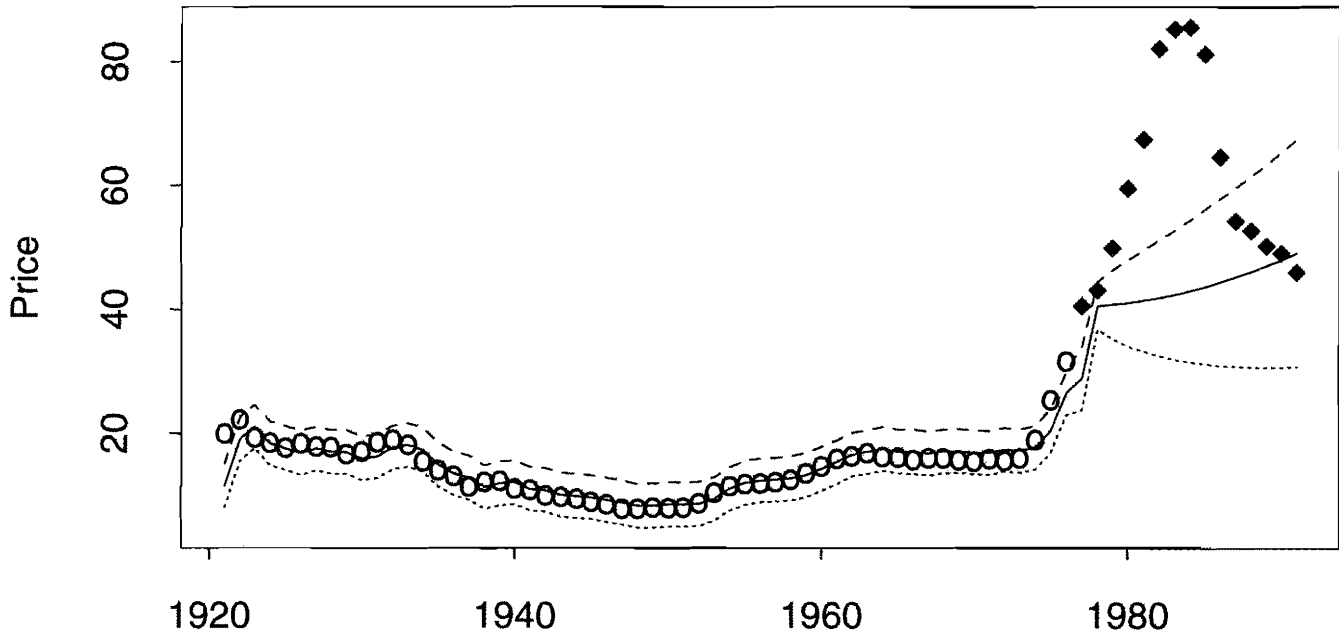
Petroleum
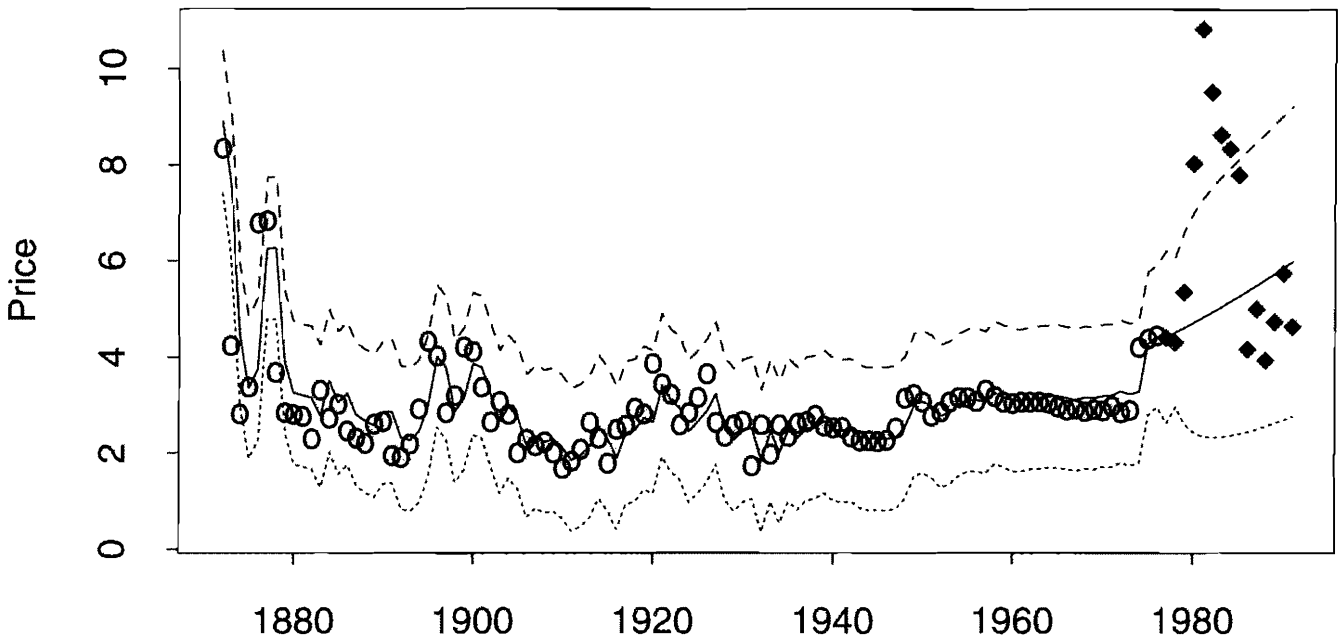
FIG. 1c. Long-series, trend-stationary models through 1976, forecast to 1991.

Silver
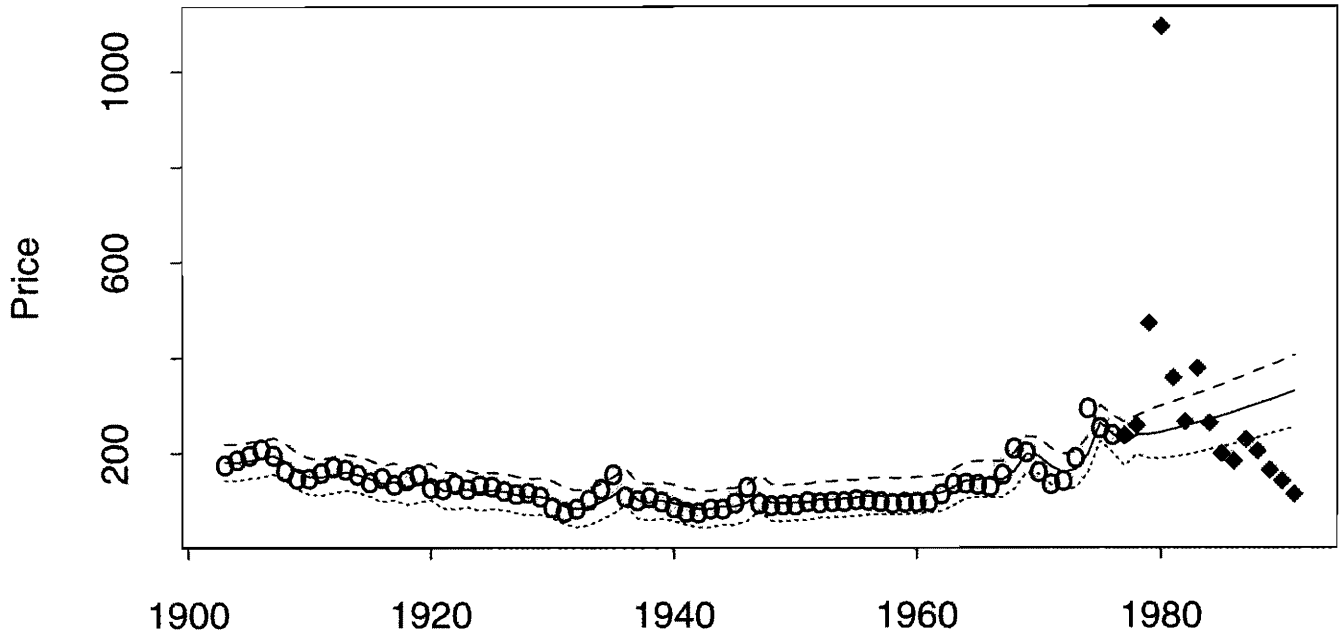
Zinc
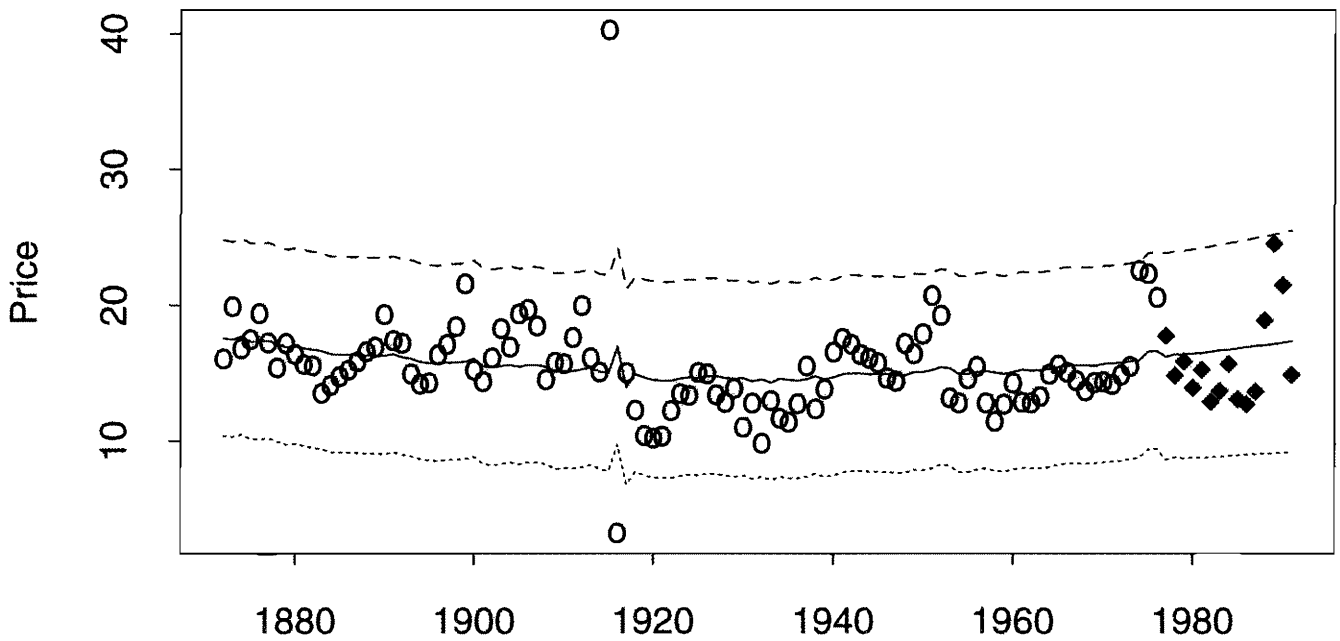
FIG. 1d. Long-series, trend-stationary models through 1976, forecast to 1991.
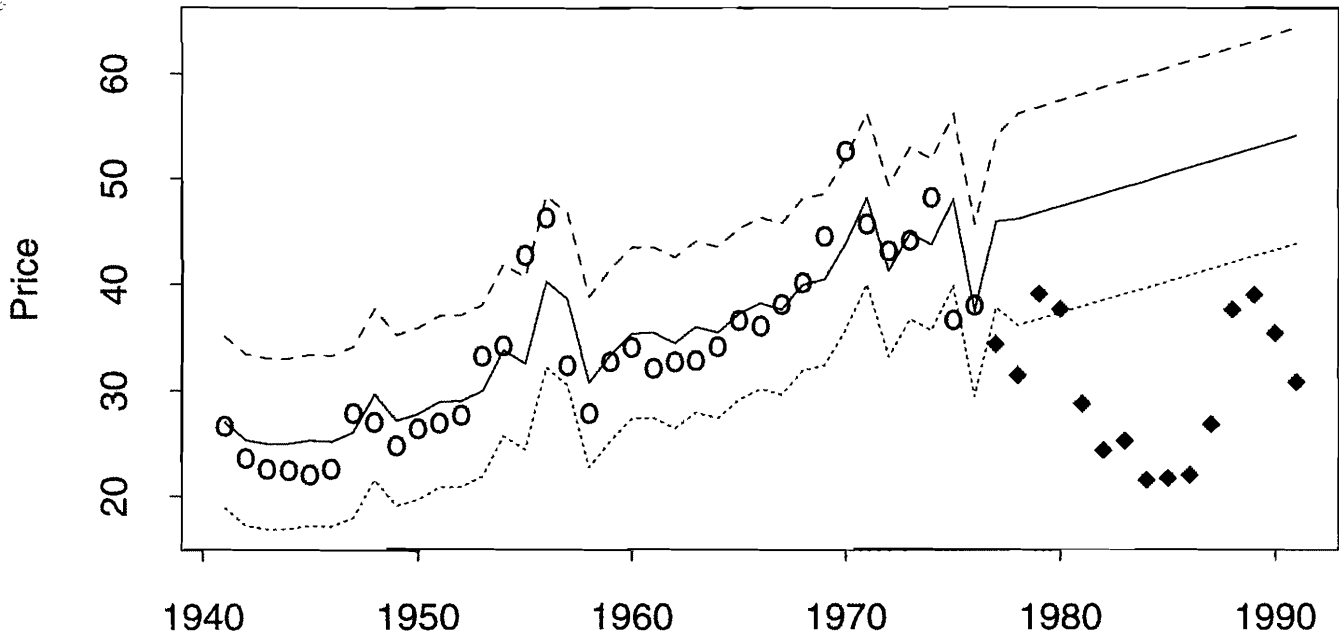
Aluminum
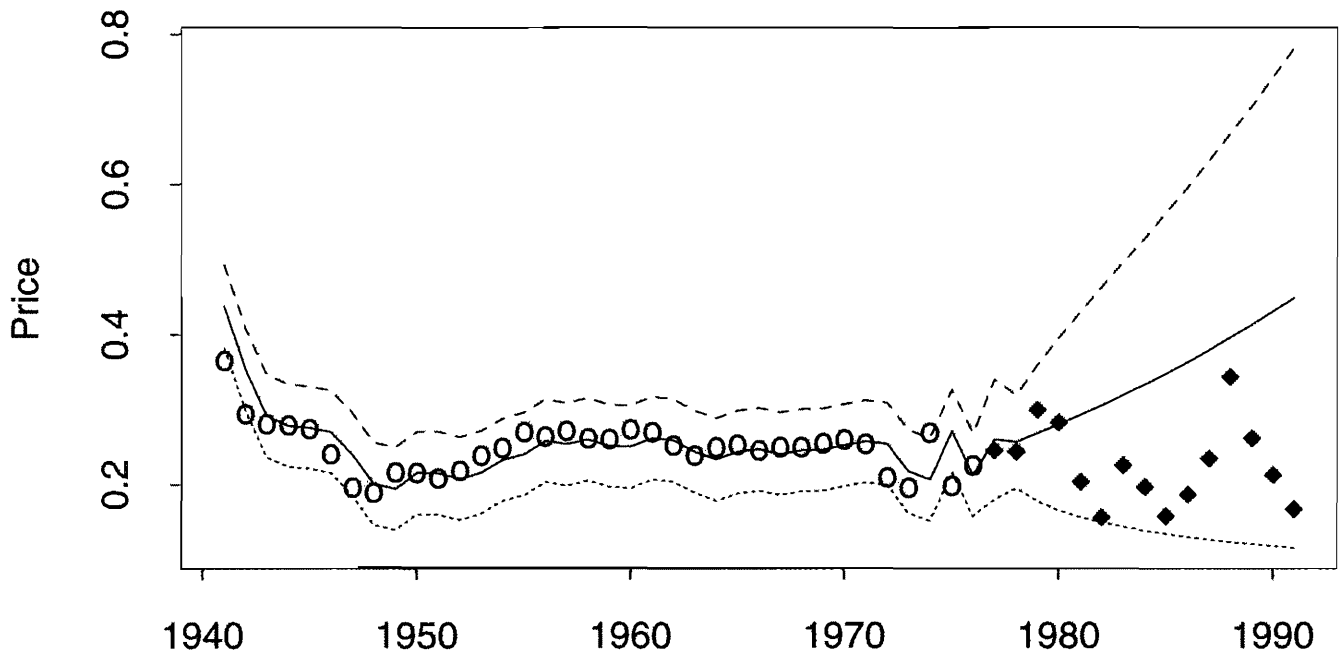
Coal
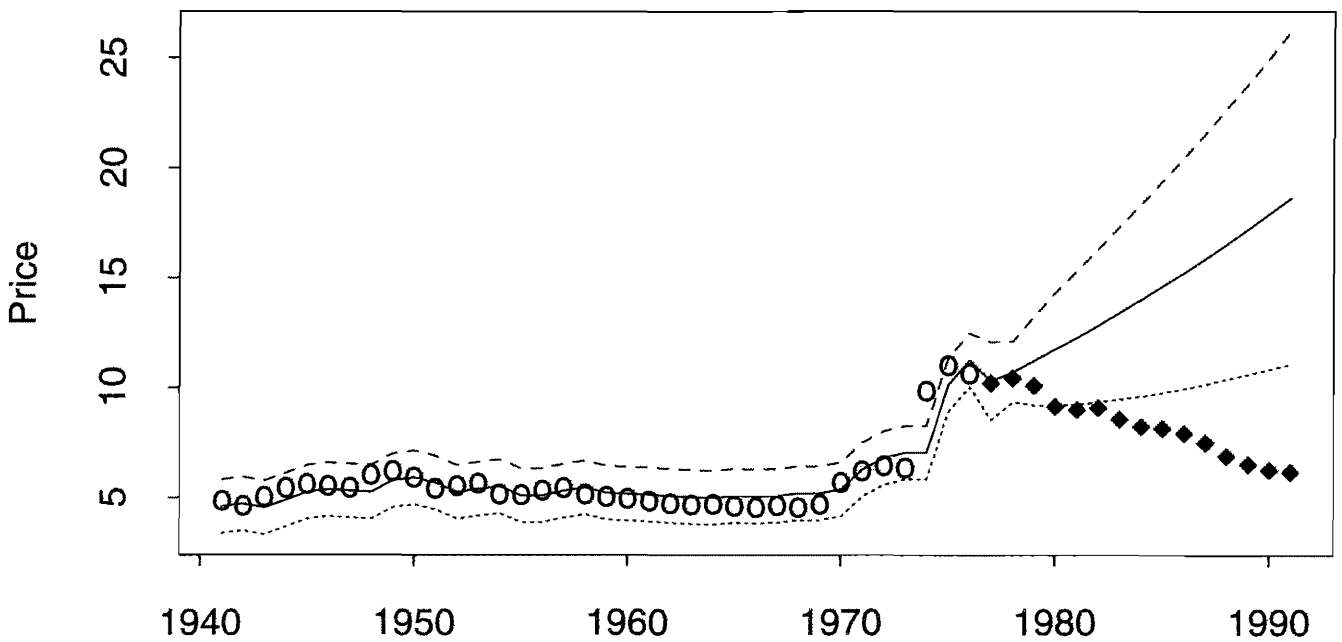
FIG. 2a. Short-series, trend-stationary models through 1976, forecast to 1991.
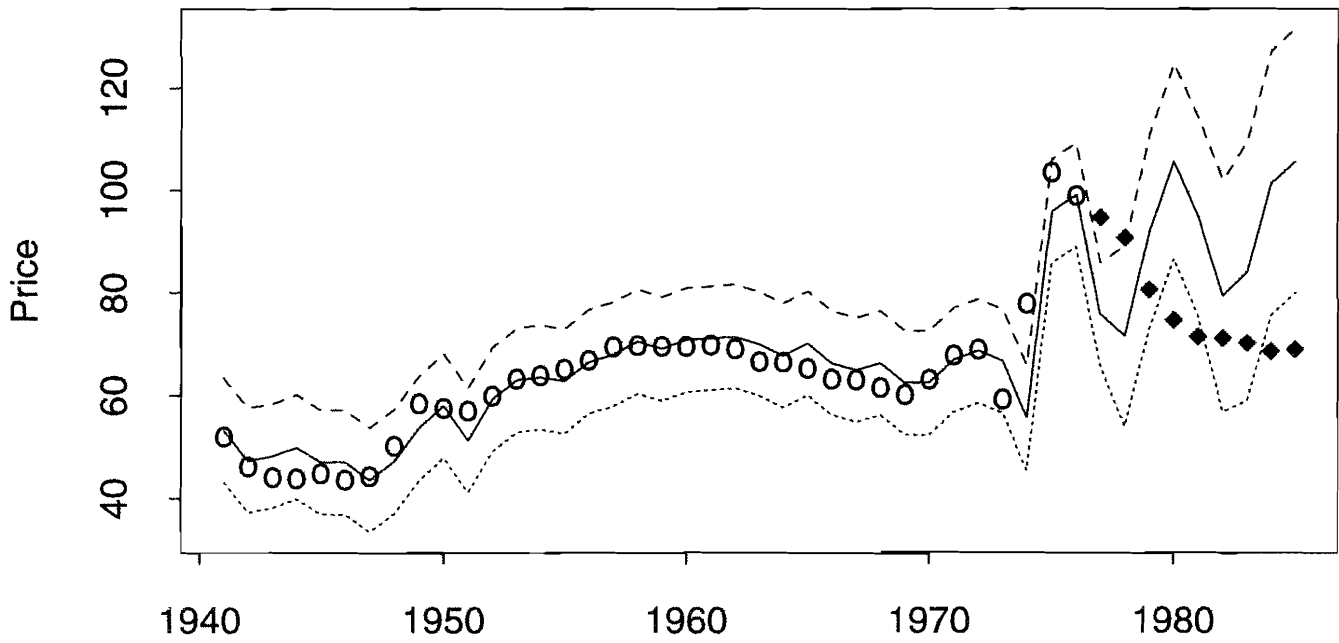
Copper
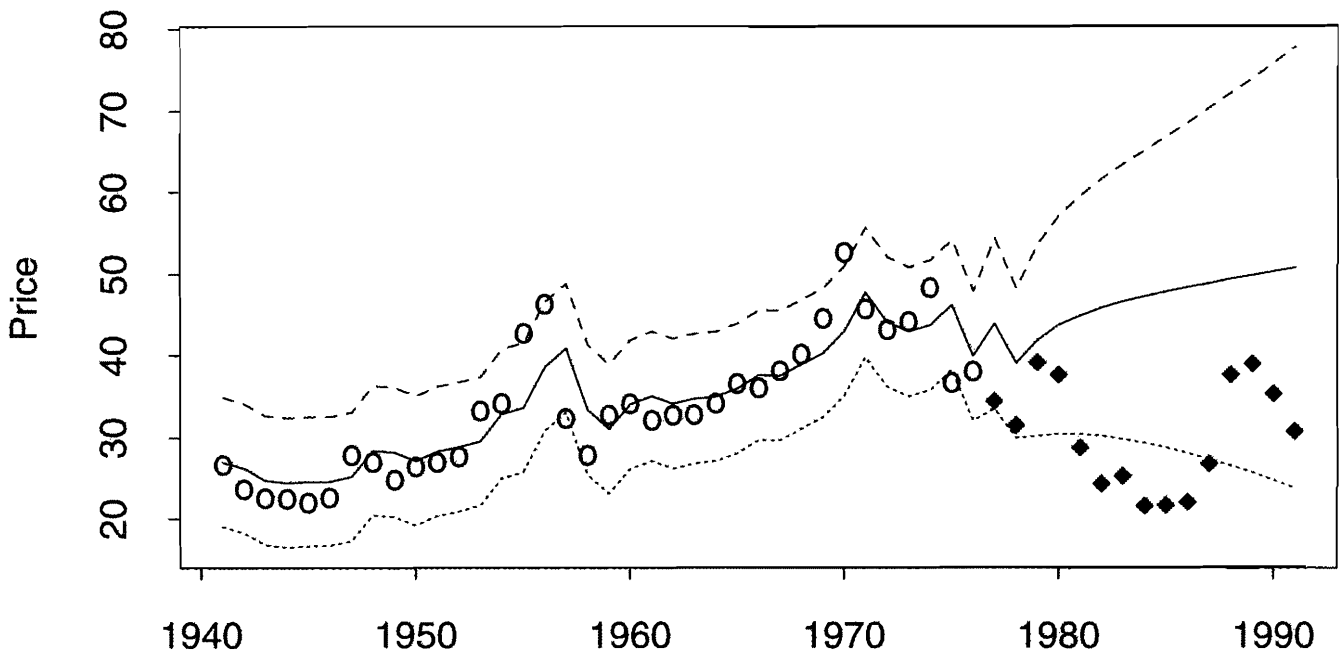
Iron
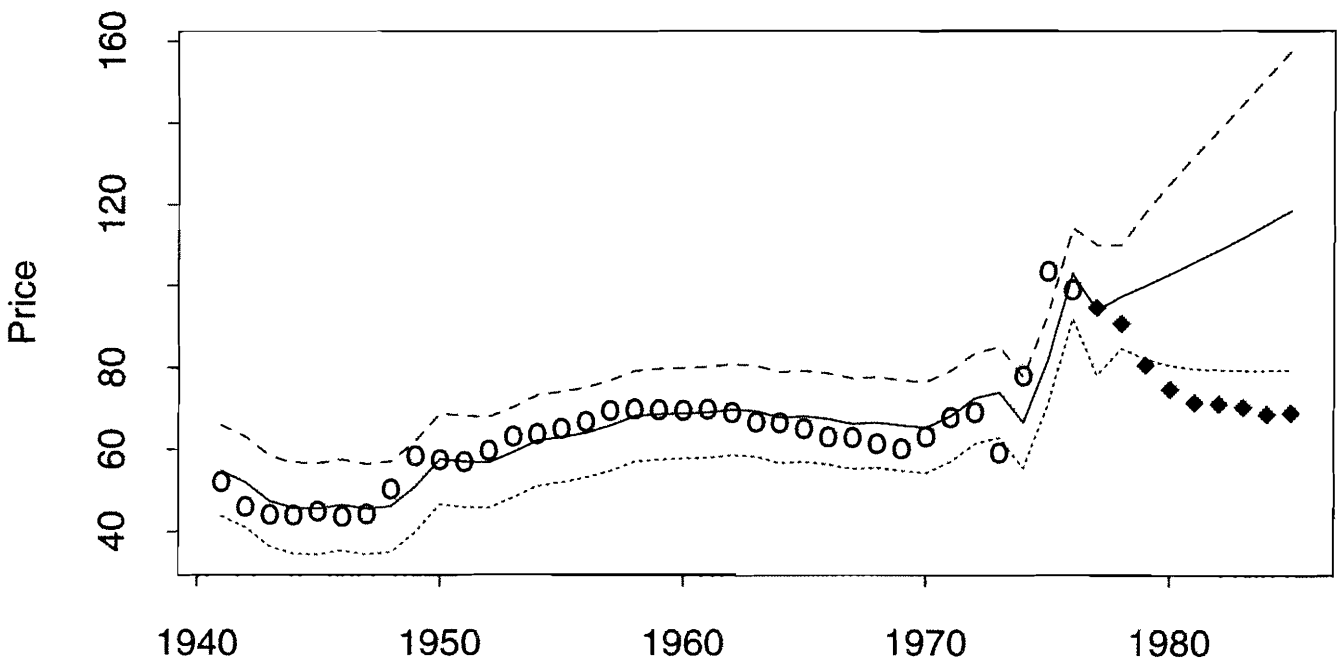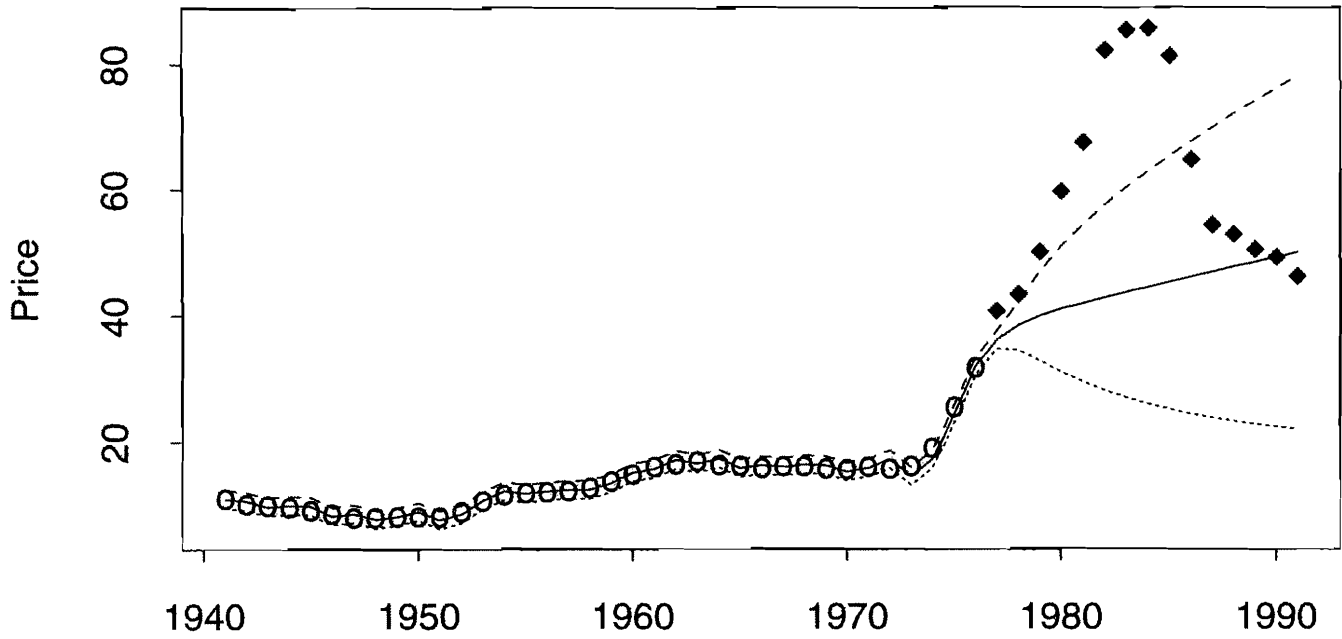

FIG. 2b. Short-series, trend-stationary models through 1976, forecast to 1991.

Natural Gas
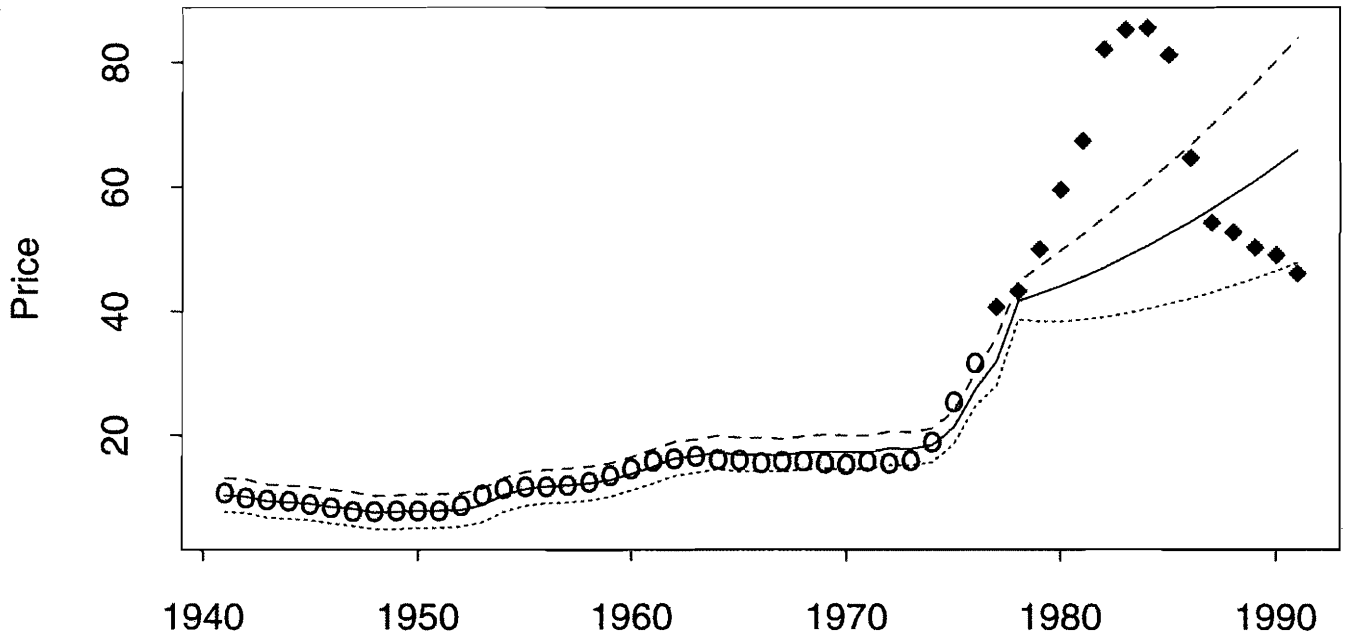
Petroleum
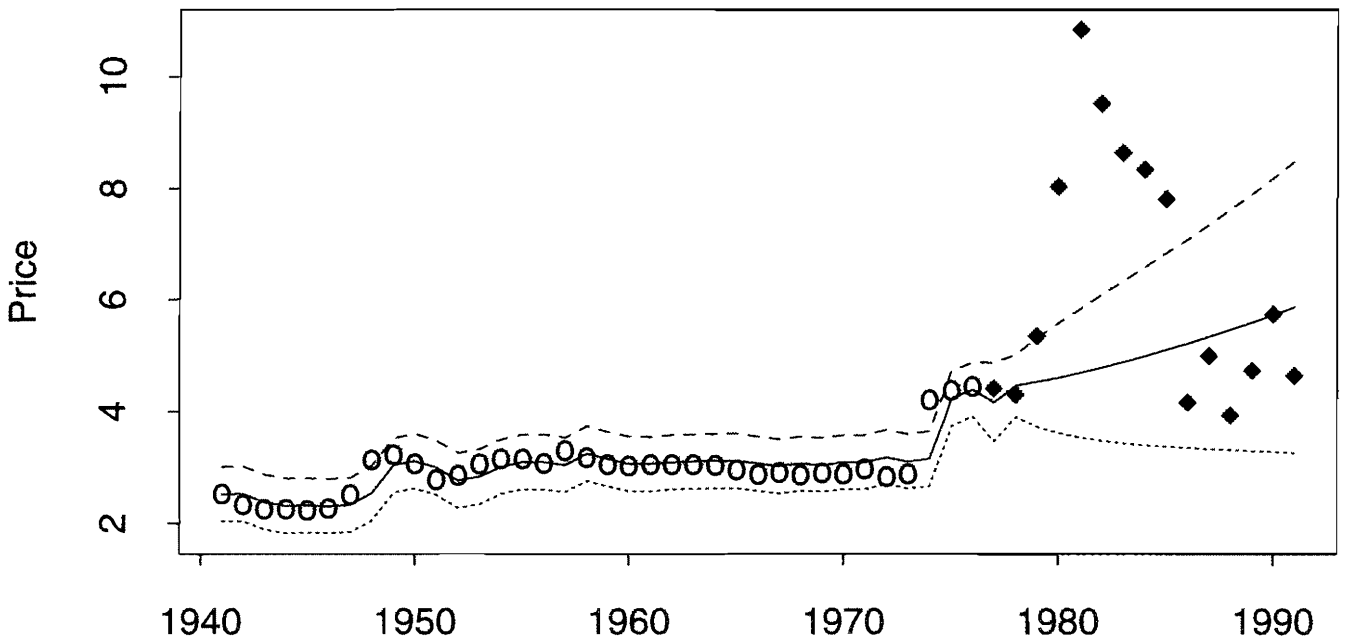
FIG. 2c. Short-series, trend-stationary models through 1976, forecast to 1991.

Silver
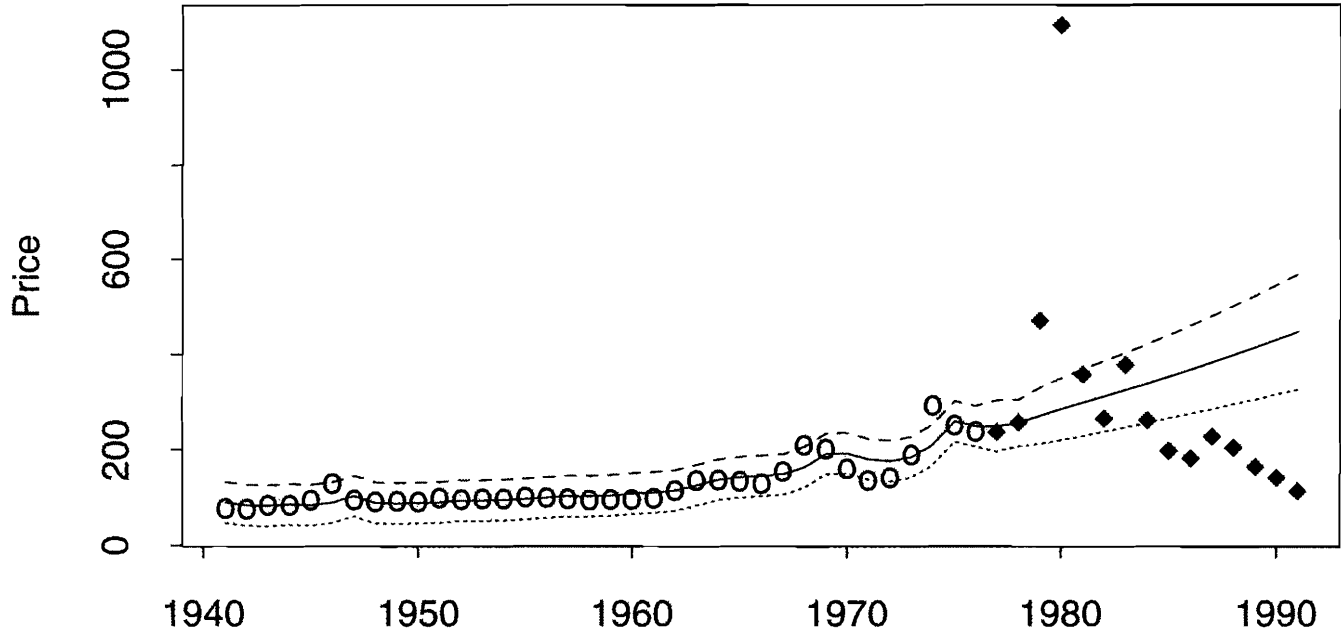
Zinc
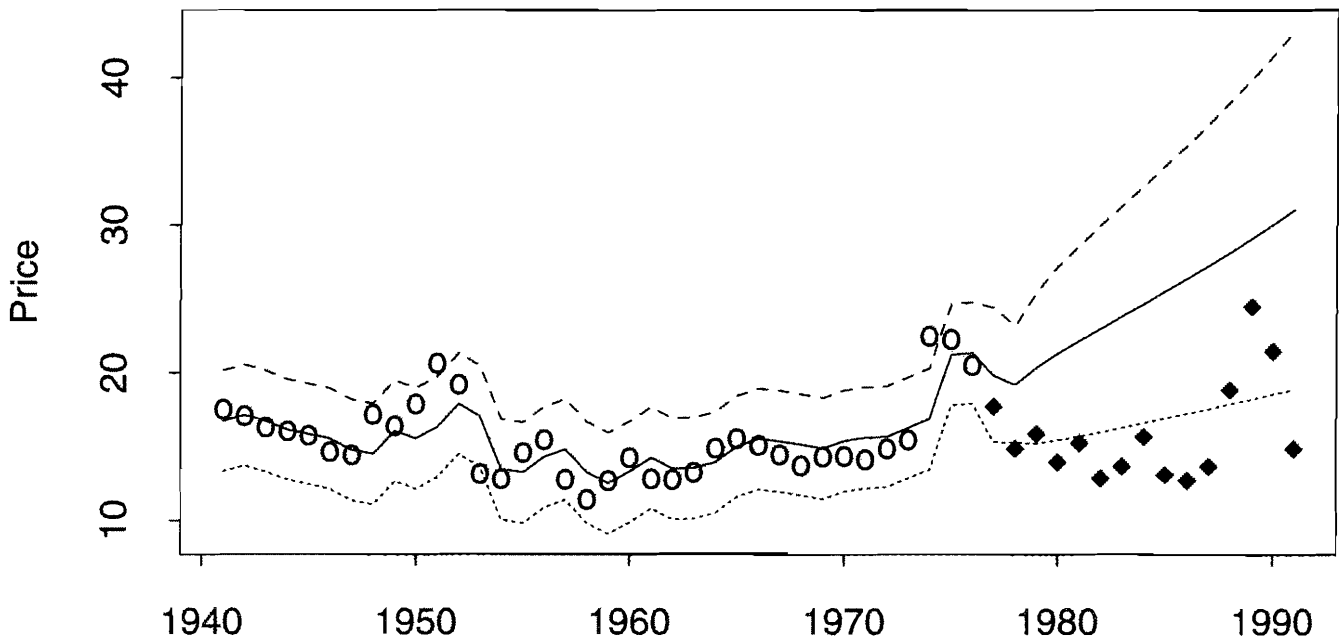
FIG. 2d. Short-series, trend-stationary models through 1976, forecast to 1991.

Aluminum
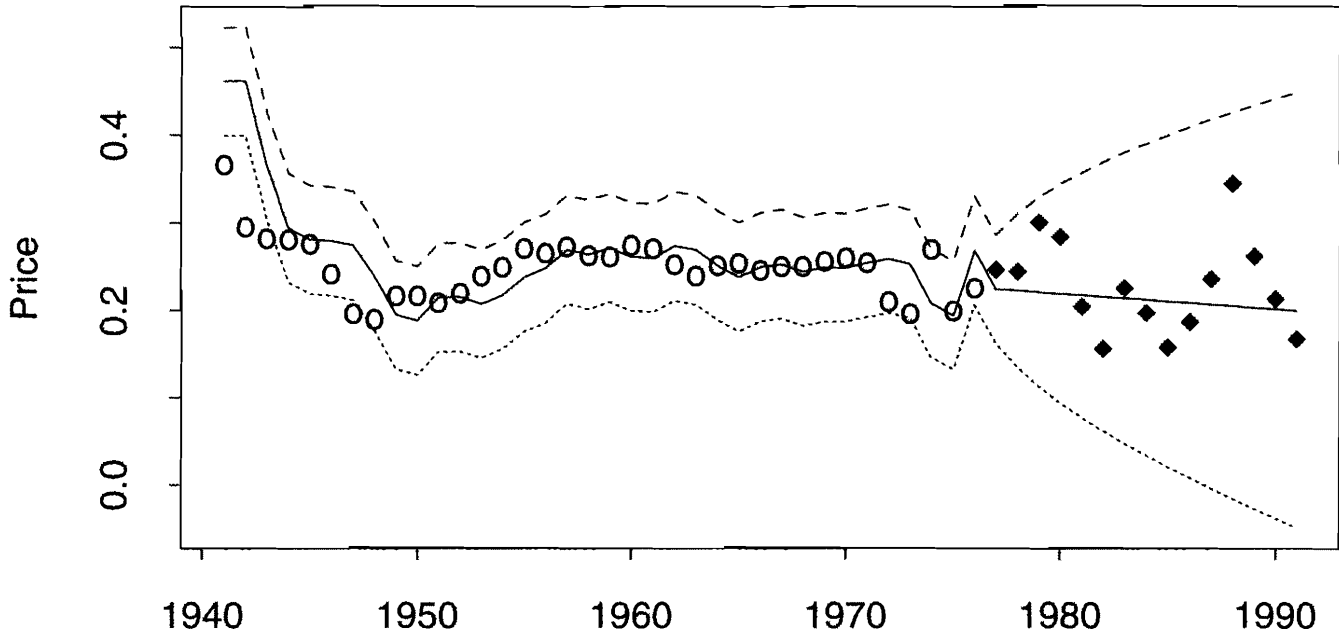
Coal

FIG. 3a. Short-series, ARMA models through 1976, forecast to 1991.

— 49 —

Copper

Iron

FIG. 3b. Short-series, ARMA models through 1976, forecast to 1991.

Natural Gas
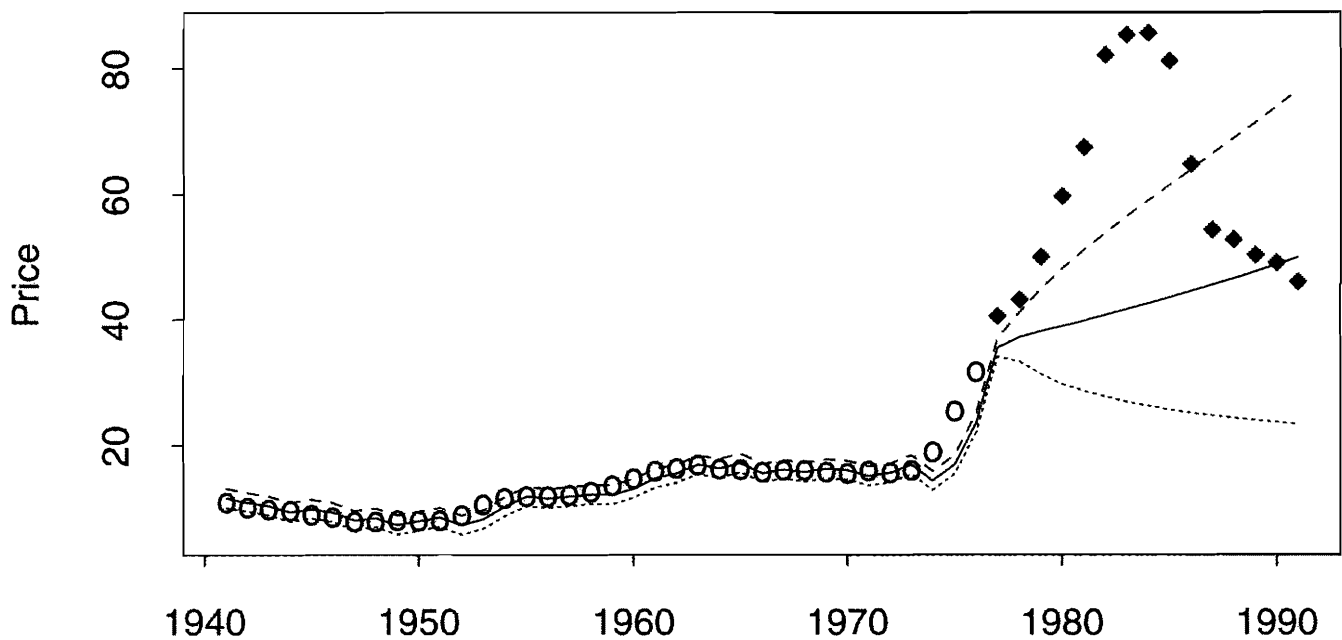
Petroleum

FIG. 3c. Short-series, ARMA models through 1976, forecast to 1991.

Silver

Zinc

FIG. 3d. Short-series, ARMA models through 1976, forecast to 1991.

Aluminum

Coal

FIG. 4a. Short-series, differenced-stationary models through 1976, forecast to 1991.

Copper

Iron

FIG. 4b. Short-series, differenced-stationary models through 1976, forecast to 1991.
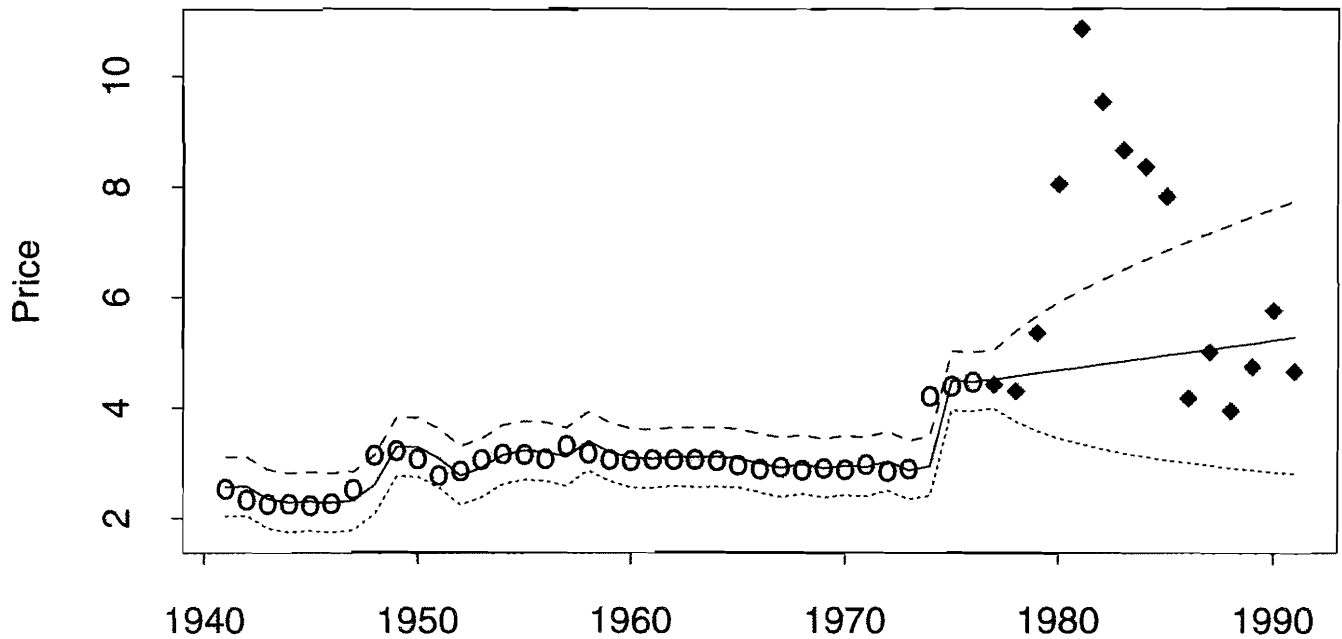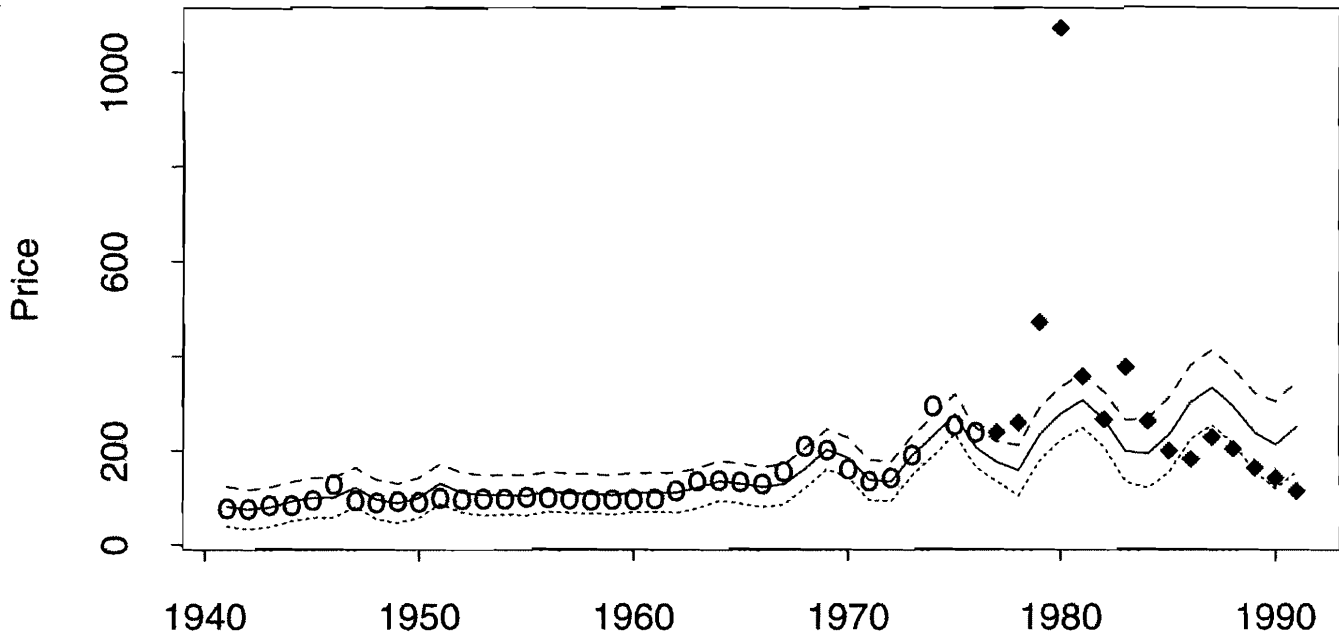
Natural Gas

Petroleum

FIG. 4c. Short-series, differenced-stationary models through 1976, forecast to 1991.
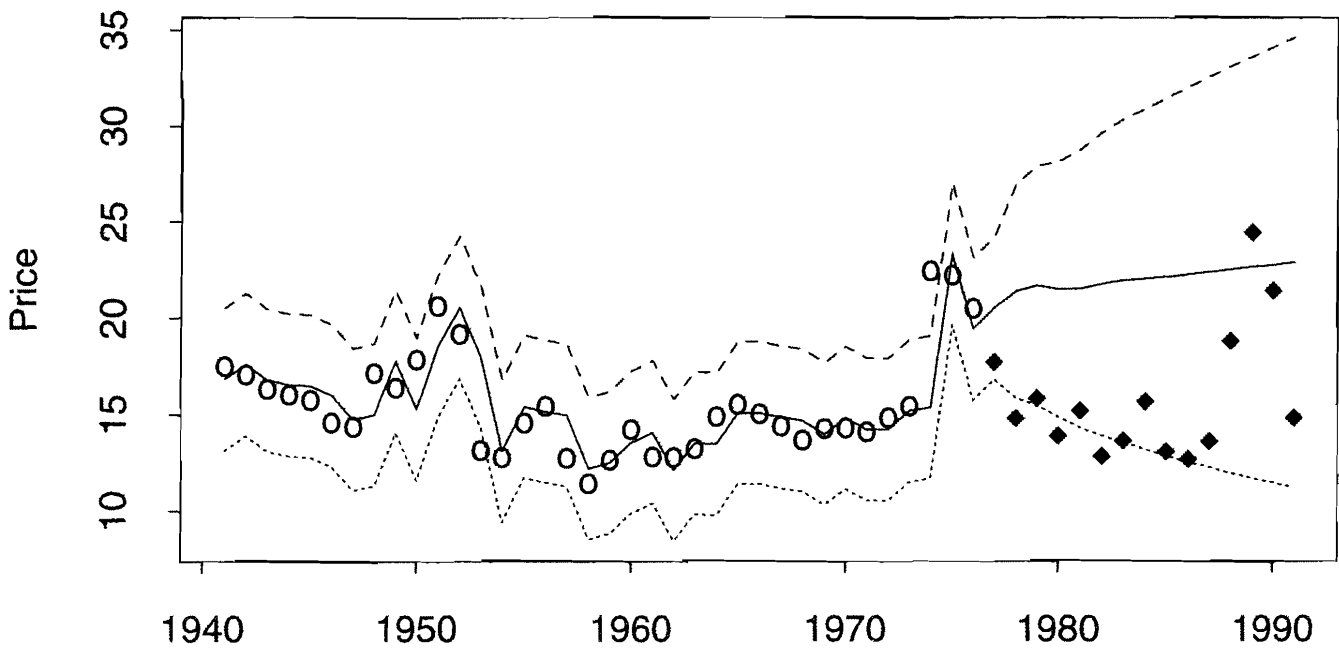
Silver

Zinc

FIG. 4d. Short-series, differenced-stationary models through 1976, forecast to 1991.