Title

Convergence to monetary equilibrium: computational simulation of a trading post economy with transaction costs

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Convergence to monetary equilibrium: Computational simulation of a trading post economy with transaction costs

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Abstract:

This paper investigates the emergence of commodity money as the result of a Tatonnement adjustment in a trading post economy. A computational approach is adopted to illustrate the monetary convergence as a result of decentralized adjustment process by utility maximizing households in the economy. Starting from an arbitrary initial economy, the analysis constructs a mapping from a compact economy space to monetary equilibrium or non-monetary equilibrium. By varying the transaction costs parameters and the household endowments, the paper successfully identifies the regions of parameter space where convergence to monetary equilibrium occurs as a result of decentralized adjustment process. The reasons for non-convergence are also investigated: uniqueness of the common medium of exchange (commodity money) follows from scale economy in transaction costs as well as the endowment space asymmetry.

1. Introduction

As an effort to formulate a model to explain the endogenous emergence of a common medium of exchange, there have been mainly two different approaches: the random matching model (Kiyotaki and Wright (1989, 1993), Marimon, McGrattan, Sargent(1990)) and the trading post model (Starr (2004, 2008), Newhouse(2007)).

The trading post model differs from the random matching model in that households make trades in certain places where specific goods are traded. Unlike random

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searching model where two traders with double coincidence of wants who meet will conduct trade, trading post model does not necessarily predict this; rather the pattern of trade may be endogenously determined reflecting transaction costs. Finally, in random matching model, markets and prices don't exist – traders meet each other and directly negotiate their trade.

This paper follows from Starr (2004) which uses trading post model. We perform computer simulation to examine the conditions under which convergence to the monetary equilibrium, from an initial barter economy, occurs. Regarding the emergence of a common medium of exchange, this paper concentrates on two special economic situations: the 'Full Double Coincidence of Wants' and the 'Absence of Double Coincidence of Wants'(in short, FDCW and ADCW). The former is the case where for each household having good i and desiring good j, there is another household in the economy, desiring good i and having good j; while the latter is the case where such household does not exist.

It is a long standing tradition to attribute the emergence of a common medium of exchange to the absence of double coincidence of wants, Jevons (1875). Thus, it is somewhat counter-intuitive that even with FDCW people use the common medium of exchange. But this is very much the case in reality: Apple store employees have to pay for their MacBooks in money not in kind; Wal-Mart cashiers have to pay for their groceries in money not through exchange of their labor for groceries directly, etc. This paper will show that under some circumstances, the synchronous effects of supply-demand balance and scale economies in the transaction cost will lead even a FDCW economy to monetary equilibrium.

Specifically, this paper uses MATLAB to simulate the *tatonnement* process and analyze the convergence results in both ADCW and FDCW cases. Also, using our formalized algorithm, we investigate how decentralized decision of each household leads the economy to monetary equilibrium. This paper examines the regions in the endowment space and the transaction cost parameter space where convergence to monetary equilibrium occurs. This result will show that the asymmetry of the endowment among households and the scale economies in the transaction costs play an important role in the convergence to monetary equilibrium. Einzig (1966, p. 345), suggests "Money tends to develop automatically out of barter, through the fact that favourite means of barter are apt to arise... object[s] ... widely accepted for direct consumption." That is, Einzig suggests those goods with high trading volumes are the most liquid (presumably reflecting scale economy in transaction cost), and evolve into common media of exchange. That medium is unique because scale economies lead to 'money' as a natural monopoly.

The simulation indicates that under the same endowment space, households have the incentive to concentrate on single common medium of exchange if large trading volume reduces the transaction costs of a particular good dramatically. Monetary convergence result can be achieved under the FDCW case after the scale economy effect kicks in; on the contrary, the convergence process in the ADCW case are not that obvious because as an individual utility maximizing agent, households always fail to co-operate on which common medium of exchange to use – households have no incentive to become the indirect trader who receives lower payoff than the direct traders. The supply-demand balance drives the relative price of goods at each trading post, and it also affects the convergence results.

This paper is organized as follows: Section 1.1 discusses the relations of this paper with preceding research. Section 2 explains our model. Section 3 illustrates the detailed (Tatonnement Process) convergence algorithm with an example. Section 4 describes the result of applying the algorithm to specific economic situations. Section 5 discusses the implications of the results and draws the conclusion. The Appendix provides the MATLAB code.

1.1 Literature Review

The trading post model has been studied by Starr (2004, 2008), Newhouse (2007), Shapley & Shubik (1977), and Balasko (2007).

Shapley & Shubik (1977) and Balasko (2007) analyzed the price formation in the trading post model. Shapley & Shubik (1977) used game-theoretic approach to show how the decentralized decisions lead the economy to the equilibrium and Balasko (2007) used dynamics, different from Walras Tatonnement process, to show the price formation both in the discrete time and continuous time model. However, these papers didn't discuss how the common medium of exchange is endogenously determined in the model.

More specific discussions regarding the selection of a commodity as a common medium of exchange and the convergence to the monetary equilibrium using a unique common medium of exchange appears in Starr (2004, 2008) and Newhouse (2007).

Starr (2004, 2008) provided the sequence of examples in which the existence of monetary equilibrium is certain in a static analysis, showing that there can be a monetary equilibrium given certain endowment distribution and transaction cost parameters. Also, Starr (2004) showed that in some FDCW situation, there can be a convergence to the monetary equilibrium, using tatonnement process. Clearly, this series of examples elucidates some important intuition behind the monetary equilibrium but it couldn't provide the general analyzing tool so that the region of parameter conditions which determines the existence of the monetary equilibrium remains unknown. Also, the convergence to the locally unique monetary equilibrium in the ADCW situation hasn't been discussed. Our paper tries to develop an algorithm such that when given inputs such as endowments and transaction parameters, it outputs the result of convergence or non-convergence.

Also, Starr (2004, 2008) used certain bid prices which reveal the scale economies in the transaction costs. However, this pricing rule only allows one-to-one trading ratio in place of zero transaction cost and couldn't reveal the balance of the supply and demand in each market. This pricing rule is successful in showing how the scale economies in the transaction costs lead the economy to use only one common medium of exchange but fails to consider the supply and demand in each market. This paper, however, uses the average-cost pricing rule which can reveal both the scale economy effect and the balance of supply and demand. Thus, in our model the equilibrium is determined by the interaction of these two forces: the scale economies in the transaction costs and the relative scarcity of each good in each market.

Newhouse (2007) discussed the equilibrium selections in the trading post model, using the Markov chain. His paper shows that the initial trading volume in each trading post and the scale economies in the transaction costs play an important role in the equilibrium selection. However, this analysis is restricted to the case where there exists full double coincidence of wants.

The present paper seeks to analyze both FDCW and ADCW cases using a single algorithm.

2. Model Settings

We follow the trading post model. There is a "friction", a slow adjustment to the tatonnement process model, avoiding some forms of cyclic non-convergence. In this model, households start out doing direct trade, and then prices adjust according to supply and demand to make market clear and the bid-ask spread cover the transaction costs incurred by trading posts. After one round of price adjustment, households will make new decisions about whether to do direct trade or indirect trade, which leads to new supply and demand, according to which prices evolve in the next round. In a "frictionless" model, information is costless and all households can learn about whether their current trading pattern is optimal and switch from a less favorable trading pattern to a more favorable one immediately. However, it's possible that economy will then be stuck in cyclic price adjustment in the case of ADCW, in which all households switch from direct trade to indirect trade (or the other way around) at the same time, and the economy never converges. This result emerges because in each cycle, it is of each and every household's interest to conduct indirect trade (or direct trade), so in each cycle they find themselves alone in the trading post. Alternatively, Newhouse (2007)'s model allows only one household to make decision of whether to switch or not in each cycle. This method can slow down the adjustment process and eliminate cyclic behavior. In our paper, we use "friction" to slow down the adjustment process. Due to friction, not all the households, but instead only 20% of the them who are currently conducting less favorable trade can switch to more favorable trade pattern immediately. The reason for choosing 20% is somewhat arbitrary, but according to our experiments, results are seen consistent throughout a range of 10% to 30%.

2.1 Economy

There are N types of goods in the economy, namely 1, 2…, N (In our simulations only the cases with N=3 are included). We assume there is a continuum of households sharing a fixed amount of endowment. Each household type, composed with a continuum of households and identified by their endowment and their preference, is endowed with only one type of good, and desires another. For instance, a continuum of households form household type [i, j], which is endowed with good i and wants good j.

Instead of specifying the amount of endowments of each household unit, we specify the total amount of endowment owned by the entire household type, which corresponds to the notion of a continuum of households. This "endowment" enters into our programming code as a matrix specifying the amount of endowment of each household type. For instance, the i-j entry of endowment matrix as follows:

$$E = A \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

means that household type [i, j] is endowed with E(i, j) amount of good i, where A is the unit of goods in the economy and can be any positive number.

Here we specify A to be 1. For the FDCW case, the household type set is the following:

$$\{[i, j]: i=1, 2\ldots, N, j=1, 2\ldots, N, j \neq i\}$$

For the ADCW case, the household type set is the following:

$$\{[i, j]: i=1, 2..., N, j=i+1 \text{ if } i \neq N, j=1 \text{ if } i=N\}$$

ADCW is totally different from FDCW situation in that, in ADCW, barter equilibrium is never attainable because of the lack of double coincidence of wants. Thus, the convergence process in ADCW is a bit different from FDCW. It's typical in ADCW that every household finds no trading partner in the first cycle of the convergence process and they start to switch to the optimal indirect trading behavior from the next cycle. Monetary equilibrium in ADCW is attained when the individuals' behavior converges to using one common medium of exchange as an individually utility maximizing strategy. Otherwise, the economy remains in a process of either cyclic situation where individual trading pattern swings between barter and indirect trade through certain good or non-convergence where there is no clear trading pattern across stages. However, in FDCW, every household in the economy might agree on doing barter trade as their individually utility maximizing strategy and the convergence process to a common medium of exchange might not take place at all.

Note that although in ADCW case there is only one household type demanding each good, scale economy effect will take place if we make the endowment of this household type sufficiently large.

The utility function of household [i, j] is the following:

$$\mathrm{U(i, j) = the\ amount\ of\ good\ j\ acquired\ by\ household\ [i, j]}$$

In other words households are indifferent between possessing any amount of any good apart from its desired good.

2.2 Trading posts

Households fulfill their desire through exchange and try to maximize their utility given the prices. However they trade not directly with other households, but through trading posts. There are N(N-1)/2 trading posts in the economy, each holding trade for a pair of goods {i, j}. The transaction cost incurred by trading post {i, j} follows this equation:

$$\mathrm{C\{i, j\}} = \min\left\{\delta_i y_i^{\{i,j\}B}, \gamma_i\right\} + \min\left\{\delta_j y_j^{\{i,j\}B}, \gamma_j\right\}$$

Where $y_i^{\{i,j\}B}$ (or $y_j^{\{i,j\}B}$) denotes the amount of good i (or j) purchased by trading post {i, j} from households. Here non-convex transaction cost function is used to reflect the scale of economy effect: when trading volume is low, transaction cost is proportional to trading volume while if the trading volume exceeds a certain ceiling, the transaction cost becomes constant (i.e. γ_i or γ_j). When trading volume increases further, total transaction cost remains unchanged so per-unit transaction cost decreases, hence the scale economy effect took effect.

Each trading post is characterized by the pair of goods traded in its place, and a bid and ask price for each one of the two goods. Therefore there are altogether two independent prices and a total of four prices associated with each trading post. The bid price of good i, denoted by q(i, j) is the amount of good j that a household gets when it sells one unit of good i to this trading post; the bid price of good j, denoted by q(j, i) is the amount of good i that a household gets when it sells one unit of good j to this trading post; the ask price of good i is 1/q(j, i), the amount of good j that the trading post gets when it sells one unit of good i to a household; the ask price of good j is 1/q(i, j), the amount of good i that the trading post gets when it sells one unit of good j to a household.

For each good traded, there is a bid-ask spread associated with it, which is the difference between the bid and ask price. Trading posts use the bid-ask spread to cover the transaction costs generated from trade.

A trading post is called *active* if there is trade taking place, *inactive* if not. A commodity is a *common medium of exchange* when some or all the trading posts associated with it are active while every other trading post is not.

2.3 Pricing rule

In this paper, adjusted average-pricing rule is used which considers both the supply-demand of the market and ensures the market clearance in equilibrium. Besides, to better investigate the economic settings with the ADCW case, the paper has made slight adjustment to the pricing rule to address the pricing when there is zero volume on one or both commodities at the trading post.

At an active trading post {i, j}, the bid price of good i is defined as the difference between total trading volume of good j at trading post {i, j} minus good j's share of the total transaction cost at trading post {i, j}, and then divided by the total trading volume of good i at trading post {i, j}. That is, the bid price of a good is expressed in terms of the volume of another good relative to its own volume, with the transaction cost incurred by that good deducted.
Formally, bid price of good i at an active trading post {i, j} is calculated as follows:

$$q_i^{\{i,j\}} = \frac{vol(j,i)}{vol(i,j)} - \frac{[tc(j,i) + tc(i,j)] \times vol(j,i)}{[vol(j,i) + vol(i,j)] \times vol(i,j)}$$

Where,
vol (j, i) is the amount of good j that trading post {i, j} purchases from households;
vol (i, j) is the amount of good i that trading post {i, j} purchases from households;
$tc(j,i)$ is the total transaction cost of good j at trading post {i, j};
$tc(i,j)$ is the total transaction cost of good i at trading post {i, j}.

The bid price of good i at an inactive trading post {i, j} is calculated according to different cases. We here nominate three scenarios.

Scenario 1: $vol(j,i) = 0$; $vol(i,j) \neq 0$
This case represents the situation where at trading post {i, j} there is no one selling good j but there are people who are willing to sell good i. The demand of good i at trading post {i, j} is zero. In this case, the bid price of good j should be favorable enough to attract households to sell good j for good i. Meanwhile, households who want to sell good i for j should bear more transaction costs than they used to in an active trading post. Thus, we adjust the inactive trading post pricing so that under this scenario, those who trade good j for i incur no transaction costs while people who

trade good i for j assume both the transaction costs of good i and good j.

The adjusted pricing rule under this scenario is as follows:

$$q_i^{\{i,j\}} = \max\{1 - \delta - \frac{tc(i,j)}{vol(i,j)}, 0\}$$

$$q_j^{\{i,j\}} = 1$$

Scenario 2: $vol(i,j) = 0;\ vol(j,i) \neq 0$

Similar to scenario 1, this case represents the situation where at trading post {i, j} there is no one selling good i but there are people willing to sell good j. The demand of good j at trading post {i, j} is zero. In this case, the bid price of goods i should be favorable enough to attract households to sell good i for good j. Meanwhile, household who wants to sell good j for i should bear more transaction costs than he used to in the active trading post. Thus, we adjust the inactive trading post pricing so that under this scenario, those who trade good i for j incur no transaction cost while people who trade good j for i assume both the transaction costs of good i and good j.

The adjusted pricing rule under this scenario is as follows:

$$q_j^{\{i,j\}} = \max\{1 - \delta - \frac{tc(j,i)}{vol(j,i)}, 0\}$$

$$q_i^{\{i,j\}} = 1$$

Scenario 3: $vol(i,j) = 0;\ vol(j,i) = 0$

This case represents the situation where no household comes to the trading post {i, j} to trade either good i or good j in previous cycle. In this case, the seller and buyer of a certain good will be in the same position if they want to initiate trade at this trading post. Thus we can adopt the same set of pricing rule in scenario 3 as the pricing rule in the active trading post when the trading volume is low and supply and demand is well balanced.

The pricing rule under this scenario is as follows:

$$q_i^{\{i,j\}} = \max\{1 - \delta, 0\}$$

According to the above pricing rule, two factors determine the bid price of good i at trading post {i, j} in both active and inactive trading posts.

First, the relative supply and demand balance of good i. This effect can be shown by the ratio of trading volume of good j over good i, namely $\mathrm{vol(j,i)/vol(i,j)}$. When the demand of good i is high relative to the supply of good i, more people will sell good j for good i at trading post {i, j}, which makes the ratio $\mathrm{vol(j,i)/}$ $\mathrm{vol(i,j)}$ become larger and bids up the price of good i. Similarly, when the demand of good i is lower, less people will sell good j for good i, and the bid price of good i will be smaller.

Second is the scale economy effect. It is reflected in the total transaction cost function. When the trading volume of good i is low (i.e. vol (i, j) is small), transaction cost of good i is proportional to the trading volume; when the trading volume of good i is high (i.e. vol (i, j) is large), average transaction cost of good i decreases when the trading volume is high enough that transaction cost reaches the ceiling γ_i. Therefore as trading volume of good i increases, per-unit transaction cost decreases, which leads to a higher bid price. This can be seen clearer when rewriting the formula in the following form:

$$q_i^{\{i,j\}} = \frac{vol(j,i)}{vol(i,j)} [1 - \frac{tc(j,i) + tc(i,j)}{vol(j,i) + vol(i,j)}]$$

Interaction of this two forces decides the actual change in bid price when transaction volume changes.

2.4 Tatonnement process

Some households maximize their utility through trade at a single trading post, selling their endowments directly for the desired good. Others trade at multiple trading posts, first for some intermediate good and then trade that good for a desired consumption good. If it's more beneficial to switch to another trading pattern, only 20% of each type of the households doing unfavorable trading pattern are able to switch each time. This friction will help reducing the speed of adjustment and eliminating some cyclic behavior.

We adopt an iterative convergence *tatonnement process* from Starr (2004). The iteration goes as follows:

Step 0: each household does barter trade, i.e. all households of the type [i, j] go to trading post {i, j}.
Step 1a: Each trading post announces their prices according to the trading information collected from last period. When relative supply of good j is high, the bid price of good i is high due to two forces: supply-demand balance and the scale economy in the transaction cost
Step 1b: Each household announces their trading decision that composes of the trading posts they plan to use.
Step 2a: Each trading post adjusts their prices according to the trading information collected from last step.
Step 2b: Each household announces their trading decision that composes trading posts they plan to use.
…

The iteration continues until the two adjacent cycles generate price matrices with the difference within a limit that is considered small enough and there is a small enough proportion of households doing unfavorable trade.

2.5 Equilibrium

Equilibrium is characterized by the following conditions:

1. Material balance in two levels:

First, trading post level: Each trading post buys certain type of goods which equals to the amount it sells to household plus the amount used to cover transaction costs.

Second, household level: Total amount of each good bought by the household in the market equals to the amount of good consumed by household plus the amount used to cover the transaction costs (Newhouse, 2007).

2. Every household is maximizing their utility such that no household has the incentive to change their trading strategy given the history of other household's trading decisions in previous cycle.[1]

2.6 Monetary Equilibrium

In this study, it is interesting to find the monetary equilibria with a unique common medium of exchange. Those equilibria satisfy a third condition:

3. A single commodity is used as common medium of exchange

In fact monetary equilibrium has two kinds: equilibrium with unique money and multiple monies. Here we only consider the case where unique commodity serves as money.

3. Detailed Algorithm

The starting point is somewhat arbitrary, so in the example we computed, every household's initial trading pattern is barter. In pair-wise market, the initial price is set to equal to the average trading costs at low trading volume.

With the existence of FDCW, each household will compare their payoff between direct trade and indirect trade through other medium of exchange (sometimes such medium of exchange may involve more than one good, depending on total number of

[1] However, in programming code, we regard the state where very small portion of households doing unfavorable trade as equilibrium for programming efficiency.

good types available in the economy) based on the average pricing rule.

3.1 Convergence Result Representation

The convergence result will be achieved by the stability of pair-wise bid price of goods in every trading post. Mathematically, stability of prices can be checked if the pricing matrices in cycle t and cycle (t+1) are approximately the same. Once the bid price of good in every trading post is steadying, household in this economy will have no incentive to change their behavior in the cycle that follows. Thus, the trading pattern of each household is stabilized. This convergence result is a stable state, and the convergent state might be a single monetary equilibrium, a multiple monetary equilibrium or a barter equilibrium.

The following are some important variables defined in our programming.

Best indirect-price matrix: Given prices announced by trading posts, households make decision of whether to conduct direct trade or indirect trade. If household [i, j] uses direct trade, it will simply go to trading post {i, j}, selling the good i it has to get good j; if household [i, j] uses indirect trade, it will contemplate all possible indirect trading methods. For instance, assume there are 3 goods in the economy, household [1, 2] will consider the following trading strategy: selling good 1 for good 3 then good 3 for good 2, or good 1 directly for good 2. In an economy with a larger number of commodities, of course the possible paths of indirect trade are more varied. Among the trading methods, household [1, 2] will find the one that gives it the highest payoff. This could lead to infinite loops, so our code terminates adjustments when there are 500 cycles already taken place. Adjustment process keeps that from convergence. The best indirect-price matrix is defined to be the matrix with its (i, j) element being the highest payoff that household [i, j] can get in the current cycle after comparing the payoffs of all indirect trade options. For instance, value of the (i, j) entry in the best indirect-trade matrix being 0.75 indicates that if household [i, j] chooses indirect trade the most it can get is 0.75 unit of good j for 1 unit of good i.

Note that we always initialize the iteration using barter trade. With FDCW, this is no problem because it is indeed one plausible trading pattern; hence the initial prices we get should be within the “right” domain. However with ADCW case, this initialization could be problematic. It is possible that this will result in an unstable infinite loop and could be one of the reasons why in our programming output, money is less likely to emerge in ADCW case. Another reason for non-convergence is probably the symmetry of endowments. We discovered that when one particular commodity is more abundant convergence is more likely. Indeed, this would decrease the bid-ask spread of this commodity and increase its “saleableness”. Finally, we find that in ADCW case, once the economy is initialized at monetary trade, i.e. one household would start out doing indirect trade, the economy is not required to adjust – it just stays there. However, it is relatively more difficult to move from barter to monetary equilibrium in this case.

Flag matrix: After comparing the best indirect price with the direct bid price, each household will make a decision of whether to do indirect or direct trade. If direct trade is optimal, the program will mark "0" at the corresponding entry for this household in the "flag" matrix; if, on the other hand, indirect trade is more beneficial, the program will mark the type of good used as medium of exchange at the corresponding entry for this household. Throughout the computation, we use flag matrix to record the optimal trading pattern of each household in the economy. For instance, the (i, j) entry in the flag matrix denotes the optimal trading pattern of household [i, j]. The range of the flag matrix entry is the integer set {0,1,2,3,…,N}, where N is the total number of good types in this economy. Nonzero value m in the flag matrix entry indicates the corresponding household does indirect trade through good m, while zero value indicates corresponding household does direct trade. In the economy with three goods, a single flag matrix is enough to record all the possible indirect trading patterns. In general, in an economy with N goods, we need (N-2) flag matrices to record all the possible indirect trading patterns.

Here note the difference between optimal trading pattern and actual trading pattern. "Flag" matrix is a record of the optimal trading pattern, that is, trading pattern that households will choose without the presence of any friction. The actual trading pattern is recorded using another matrix "prop" where each entry denotes the proportion of corresponding households that is doing direct trade.

3.2 Computation results

3.2.0 Computation example (Three good, FDCW case, symmetric endowments)

Endowment matrix:

$$r = \begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$\delta = 0.5, \gamma = 0.6$

We use this example to illustrate the whole computation process. All the following examples follow similar idea, with slight adjustments to expand to 2-dimentional or 3-dimentional parameter space.

In Cycle 1:
Step 1. Price setting
At the initial stage, every household does direct trade which makes every trading post active. After learning other households' behavior in the initial stage, the price setter announces the new price for each good in cycle one. Based on the endowment space,

the bid price of each type of goods at their corresponding trading post is as follows:

$$q(1,2) = \frac{vol(2,1)}{vol(1,2)} - \frac{[tc(2,1)+tc(1,2)]\times vol(2,1)}{[vol(1,2)+vol(2,1)]\times vol(1,2)} = 1 - \frac{2\times \min\{0.5\times 3, 0.6\}\times 3}{2\times 3\times 3} = 0.8 = q(2,1)$$
$$q(1,3) = \frac{vol(3,1)}{vol(1,3)} - \frac{[tc(3,1)+tc(1,3)]\times vol(3,1)}{[vol(1,3)+vol(3,1)]\times vol(1,3)} = 1 - \frac{2\times \min\{0.5\times 2, 0.6\}\times 2}{2\times 2\times 2} = 0.7 = q(3,1)$$
$$q(2,3) = \frac{vol(3,2)}{vol(2,3)} - \frac{[tc(3,2)+tc(2,3)]\times vol(3,2)}{[vol(2,3)+vol(3,2)]\times vol(2,3)} = 1 - \frac{2\times \min\{0.5\times 1, 0.6\}\times 1}{2\times 1\times 1} = 0.5 = q(3,2)$$

And this direct pricing is shown in the q matrix, where each entry indicates the direct trade payoff (for each unit traded) of each type of household under the new price setting:

q =	*0*	*0.8000*	*0.7000*
	0.8000	*0*	*0.5000*
	0.7000	*0.5000*	*0*

In three-good case, each household faces only one indirect trade option, i.e. for household (i, j) to trade through good (6-i-j). The payoffs from the indirect trade of each household (for each unit traded) are computed as the following:

$$inq(1,2) = q(1,3) \times q(3,2) = 0.7 \times 0.5 = 0.35 = \mathrm{in}q(2,1)$$
$$inq(1,3) = q(1,2) \times q(2,3) = 0.8 \times 0.5 = 0.40 = \mathrm{in}q(3,1)$$
$$inq(2,3) = q(2,1) \times q(1,3) = 0.8 \times 0.7 = 0.56 = \mathrm{in}q(3,2)$$

The indirect trade pricing is shown in the *inq* matrix, where each entry indicates the direct trade payoff (for each unit traded) of each type of household under the new price setting:

inq =	*0*	*0.3500*	*0.4000*
	0.3500	*0*	*0.5600*
	0.4000	*0.5600*	*0*

Step 2. Decision making
After knowing the payoffs from direct and indirect trade, optimizing households will make decision to optimize their utility:

$$q(1,2) = q(2,1) = 0.8 > inq(1,2) = inq(2,1) = 0.35$$
$$q(1,3) = q(3,1) = 0.7 > inq(1,2) = inq(2,1) = 0.40$$
$$q(3,2) = q(2,3) = 0.5 < inq(3,2) = inq(2,3) = 0.56$$

Thus, optimizing, competitive households (2,3) and (3,2) have the incentive to change their trading pattern from direct trade to indirect trade through good 1 while the other types of household prefer to keep theirs unchanged.

Then this optimal trading pattern of this economy in this cycle is reflected in the *flag* matrix, where the value zero in the entry indicates that direct trade is optimal for this household type and ν value indicates that indirect trade through good ν is optimal for this household type. The diagonal entries are ignored because no household type who desire and possess the same kind of good exists in our model.

$$flag = \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}$$

However, to make our model computable for the ADCW case, only certain fraction (20% in this example) of each type of household who is in the unfavorable trading pattern is allowed to adjust their behavior for the better (We don't allow households to switch from favorable trading pattern to unfavorable). In our case, although household (2,3) and (3,2) prefer to do indirect trade, only 20% of each type is quick enough to change their trading pattern in Cycle 1, while the rest 80% will still do direct trade. As a result, if indirect trade is beneficial for households in every cycle, then at each cycle, 20% of the households still doing direct trade will switch to indirect trade and as the adjustment goes on, the newly-switching household proportion will form a geometric series.

Due to friction, the actual trading pattern in this economy at the end of Cycle 1 is as the following, where each entry in the *prop* matrix indicates the proportion of each type of households who do direct trade (as usual, the diagonal entries are ignored):

$$prop = \begin{matrix} 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.0000 & 0.8000 \\ 1.0000 & 0.8000 & 1.0000 \end{matrix}$$

In Cycle 2:

Step 1. Price setting

The procedure is exactly the same as price setting in Cycle 1. At the end of Cycle 1, 20% of household (2, 3) and (3, 2) are doing indirect trade while all the other households are doing direct trade. 20% of the households type (2, 3) who have the move will go to trading post {2,1} first, trading their units of good 2 for 0.16 (20%× r(2,3) × q(2,1)) unit of good 1, and then go to trading post {1,3} to trade this 0.16 unit of good 1 for 0.112 (0.16× q(1,3)) units of good 3. Similarly for type (3, 2) who have the move will go to trading post {3, 1} first, trading their units of good 3 for 0.14 (20%× r(3,2) × q(3,1)) unit of good 1, and then go to trading post {1, 2} to trade this 0.14 unit of good 1 for 0.112 (0.14× q(1,2)) unit of good 2.

Therefore, the trading volume at each post is as follows:

vol(1,2)=r(1,2)+0.14=3.14
vol(2,1)=r(2,1)+0.2=3.2
vol(2,3)=r(2,3)× 80%=0.8
vol(3,2)=r(3,2) × 80%=0.8
vol(1,3)=r(1,3)+0.16=2.16
vol(3,1)=r(3,1)+0.2=2.2

Hence the bid price of each good at their corresponding trading post is as follows:

$$q(1,2) = \frac{vol(2,1)}{vol(1,2)} - \frac{[tc(2,1) + tc(1,2)] \times vol(2,1)}{[vol(1,2) + vol(2,1)] \times vol(1,2)}$$
$$= \frac{3.2}{3.14} - \frac{[\min\{0.5 \times 3.2, 0.6\} + \min\{0.5 \times 3.14, 0.6\}] \times 3.2}{(3.14 + 3.2) \times 3.14}$$
$$= 0.8262$$

$$q(2,1) = \frac{vol(1,2)}{vol(2,1)} - \frac{[tc(2,1) + tc(1,2)] \times vol(1,2)}{[vol(1,2) + vol(2,1)] \times vol(2,1)}$$
$$= \frac{3.14}{3.2} - \frac{[\min\{0.5 \times 3.2, 0.6\} + \min\{0.5 \times 3.14, 0.6\}] \times 3.14}{(3.14 + 3.2) \times 3.2}$$
$$= 0.7955$$

$$q(1,3) = \frac{vol(3,1)}{vol(1,3)} - \frac{[tc(3,1) + tc(1,3)] \times vol(3,1)}{[vol(1,3) + vol(3,1)] \times vol(1,3)}$$
$$= \frac{2.2}{2.16} - \frac{[\min\{0.5 \times 2.2, 0.6\} + \min\{0.5 \times 2.16, 0.6\}] \times 2.2}{(2.16 + 2.2) \times 2.16}$$
$$= 0.7382$$

$$q(3,1) = \frac{vol(1,3)}{vol(3,1)} - \frac{[tc(3,1) + tc(1,3)] \times vol(1,3)}{[vol(1,3) + vol(3,1)] \times vol(3,1)}$$
$$= \frac{2.2}{2.16} - \frac{[\min\{0.5 \times 2.2, 0.6\} + \min\{0.5 \times 2.16, 0.6\}] \times 2.16}{(2.16 + 2.2) \times 2.2}$$
$$= 0.7116$$

$$q(2,3) = \frac{vol(3,2)}{vol(2,3)} - \frac{[tc(3,2) + tc(2,3)] \times vol(3,2)}{[vol(2,3) + vol(3,2)] \times vol(2,3)}$$
$$= 1 - \frac{2 \times \min\{0.5 \times 0.8, 0.6\} \times 0.8}{2 \times 0.8 \times 0.8} = 0.5 = q(3,2)$$

And this direct pricing is shown in the q matrix, where each entry indicates the direct trade payoff of each type of household under the new price setting:

$$q = \begin{matrix} 0 & 0.8262 & 0.7382 \\ 0.7955 & 0 & 0.5000 \\ 0.7116 & 0.5000 & 0 \end{matrix}$$

In three-good case, each household faces only one indirect trade option. The payoffs from the indirect trade of each household are computed as the following:

$$inq(1,2) = q(1,3) \times q(3,2) = 0.7382 \times 0.5 = 0.3691$$
$$\mathrm{inq}(2,1) = \mathrm{q}(2,3) \times \mathrm{q}(3,1) = 0.5 \times 0.7116 = 0.3558$$
$$inq(1,3) = q(1,2) \times q(2,3) = 0.7382 \times 0.5 = 0.4131$$
$$\mathrm{in}q(3,1) = q(3,2) \times q(2,1) = 0.5 \times 0.7955 = 0.3978$$
$$inq(2,3) = q(2,1) \times q(1,3) = 0.7955 \times 0.7382 = 0.5873$$
$$\mathrm{in}q(3,2) = q(3,1) \times q(1,2) = 0.7116 \times 0.8262 = 0.5879$$

The indirect trade pricing is shown in the *inq* matrix; each entry indicates the direct trade payoff of each type of household under the new price setting:

$$inq = \begin{matrix} 0 & 0.3691 & 0.4131 \\ 0.3558 & 0 & 0.5873 \\ 0.3978 & 0.5879 & 0 \end{matrix}$$

Step 2. Decision making

After knowing the payoffs from direct and indirect trade, optimizing household will make their decisions.

$$q(1,2) = 0.8262 > inq(1,2) = 0.3691$$
$$q(2,1) = 0.7955 > inq(2,1) = 0.3558$$
$$q(1,3) = 0.7382 > inq(1,3) = 0.4131$$
$$q(3,1) = 0.7116 > inq(3,1) = 0.3978$$
$$q(3,2) = 0.5 < inq(3,2) = 0.5879$$
$$q(2,3) = 0.5 < inq(2,3) = 0.5873$$

Then this optimal trading pattern of this economy in this cycle is reflected in the *flag* matrix, where the value zero in the entry indicates that direct trade is optimal for this household type and v value indicates that indirect trade through good v is optimal for this household type. The diagonal entries are ignored as usual.

$$flag = \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}$$

Again, only 20% of each type of household who is in the unfavorable trading pattern is allowed to adjust their behavior. Although household (2,3) and (3,2) prefer to do indirect trade, only 20% of each type who is currently doing direct trade is allowed to change their trading pattern in Cycle 2 while the remaining 80% will still do direct trade.

The actual proportion of each household type who are doing direct trade is as follows:

```
prop = 1.0000     1.0000     1.0000
       1.0000     1.0000     0.6400
       1.0000     0.6400     1.0000
```

The same goes for Cycle 3:

```
cycle = 3

q =         0     0.8447     0.7656
       0.7933          0     0.5000
       0.7186     0.5000          0

inq =       0     0.3828     0.4224
       0.3593          0     0.6074
       0.3967     0.6070          0

flag = 0     0     0
       0     0     1
       0     1     0

prop = 1.0000     1.0000     1.0000
       1.0000     1.0000     0.5120
       1.0000     0.5120     1.0000
   …
…
…
cycle = 42

q =         0     0.9043     0.8526
       0.7894          0     0.5000
       0.7371     0.5000          0

inq =       0     0.4263     0.4522
       0.3685          0     0.6730
       0.3947     0.6666          0

flag = 0     0     0
       0     0     1
       0     1     0

prop = 1.0000     1.0000     1.0000
       1.0000     1.0000     0.0001
       1.0000     0.0001     1.0000
```

Above is the final result. As can be seen, it takes 42 cycles to converge. In equilibrium, all households are doing indirect trade except for 0.01% of household type (2, 3) and (3, 2). Since it's optimal to do indirect trade for these two household types, one expects these two proportions to be zero. However in our programming, we specify that the cycle will terminate if the following two conditions hold simultaneously:

1. The difference between bid price for good i (i = 1,2,3) in this cycle and in last cycle is smaller than 0.0001, and

2. The difference between proportion of household type (i, j) (i=1, j=i+1 for i=1,2; i=3, j=1) doing direct trade in this cycle and in last cycle is smaller than 0.01%.

4 . Main Outputs

4.1 Homogeneity of degree 0 in Gamma and A

To simplify the computation, the paper focused on simulating the trading behavior of six types of representative market participants in a trading post economy with three distinct types of goods. Furthermore, the household decision as well as the relative bid price of goods remains unchanged when both Gamma and A are multiplied by the same constant c. The homogeneity of degree 0 character of parameter Gamma and A allows us to get rid of one dimension by assuming the unit of goods A to be unity without loss of generality.

Here is a brief proof of the homogeneity of degree 0 statement: first note that when households make decision, only the relative quantity comparison using one of the listed prices below is essential:

$$q_i^{\{i,\,j\}active} = \frac{y_j^{\{i,j\}} - \left(\min\left\{\delta \times y_j^{\{i,j\}}, \gamma\right\} + \min\left\{\delta \times y_i^{\{i,j\}}, \gamma\right\}\right) \times y_j^{\{i,j\}} / (y_j^{\{i,j\}} + y_i^{\{i,j\}})}{y_i^{\{i,j\}}} \quad \ldots(1)$$

$$q_i^{\{i,j\}inactive} = 1 - \delta \ldots(2)$$

$$q_i^{\{i,j\}inactive} = 1 \ \ldots \ (3)$$

$$q_i^{\{i,j\}inactive} = 1 - \delta - \min\{\delta, \gamma/y_j^{\{i,j\}}\} \ldots(4)$$

Note that the volumes of good i and j, $y_j^{\{i,j\}}$ and $y_i^{\{i,j\}}$ are linear functions of A. From the first bid price (1) described above, when we multiply some constant to both γ and A, the ratio $\frac{y_j^{\{i,j\}}}{y_i^{\{i,j\}}}$ and $\frac{y_j^{\{i,j\}}}{y_j^{\{i,j\}} + y_i^{\{i,j\}}}$ and $\frac{\left(min\left\{\delta \times y_j^{\{i,j\}}, \gamma\right\} + min\left\{\delta \times y_i^{\{i,j\}}, \gamma\right\}\right)}{y_i^{\{i,j\}}}$ aren't changed. Thus, the first bid price is not affected by the multiplication. Also, in case of $q_i^{\{i,j\}} = 1 - \delta - \min\{\delta, \gamma/y_j^{\{i,j\}}\}$, the quantity $\min\{\delta, \gamma/y_j^{\{i,j\}}\}$ is not affected by multiplying the same constant to both γ and A. Thus the overall bid price remains unchanged as well. This endorses the assumption.

4.2 Three Goods with Full Double Coincidence of Wants

Example 1 (Three goods, FDCW case, symmetric endowments, 2D parameter space)

In this example, we initialized a symmetric endowment space in this three-good economy with full double coincidence of wants. By fixing the endowment in this economy, we are able to explore the relation in the parameter space {Delta, Gamma}, setting A=1.

Example Settings:

Number of goods=3.
Household in the economy: [1,2][1,3][2,3][2,1][3,2][3,1].
Endowments: r[1,2]=r[2,1]=3A; r[1,3]=r[3,1]=2A; r[2,3]=r[3,2]=1A.
$(\delta, \gamma) \in [0, 1]\times[0, 6]$

That is, endowment matrix = A $\begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

Convergence Process:

Initially, all households do barter trade. Given this information, all trading posts summarize the supply and demand. Using the pricing rule prescribed before, all trading posts calculate new bid prices: if, for trading post {i, j}, in the last cycle both supply and demand of good i is nonzero, make bid price of good i to be q(i, j).

$$q_i^{\{i,j\}active} = \frac{y_j^{\{i,j\}} - \left(min\left\{\delta \times y_j^{\{i,j\}}, \gamma\right\} + min\left\{\delta \times y_i^{\{i,j\}}, \gamma\right\}\right) \times y_j^{\{i,j\}}/(y_j^{\{i,j\}} + y_i^{\{i,j\}})}{y_i^{\{i,j\}}}$$

If, only supply of good i is nonzero, make q (i, j)

$$q_i^{\{i,j\}inactive} = 0$$

If, only demand of good i is nonzero, make q (i, j)

$$q_i^{\{i,j\}inactive} = \max\{1 - \delta - \frac{min\{\delta \times vol(j,i), \gamma\}}{vol(j,i)}\ , 0\}$$

If, both supply and demand of good i are zero, make q (i, j)

$$q_i^{\{i,j\}inactive} = 1 - \delta$$

Given these prices, households consider two options: direct and indirect trading procedure. If, for instance, for household [i, j], indirect trade turns out to be more beneficial than direct trade, then mark the corresponding (i, j) entry of "flag" matrix the medium of exchange used by household [i, j] and specify that in next cycle, 20% of the households who is currently doing direct trade will switch to indirect trade. In case all the households still prefer direct trade, then economy will end with barter equilibrium. If not, trading posts make price adjustments again. The process will claim to converge when two conditions meet: each bid price from this cycle and last cycle is close enough, and that the proportion of households who are doing worse-than-optimal trade is small enough. After convergence is reached, the program checks whether a single good is used as common medium of exchange. What is more, the program is allowed to converge within 500 cycles else it will claim non-convergence. After the calculation based on a pair of Delta and Gamma, we expand it into a parameter space and get the graph below:

Output: Figure 1

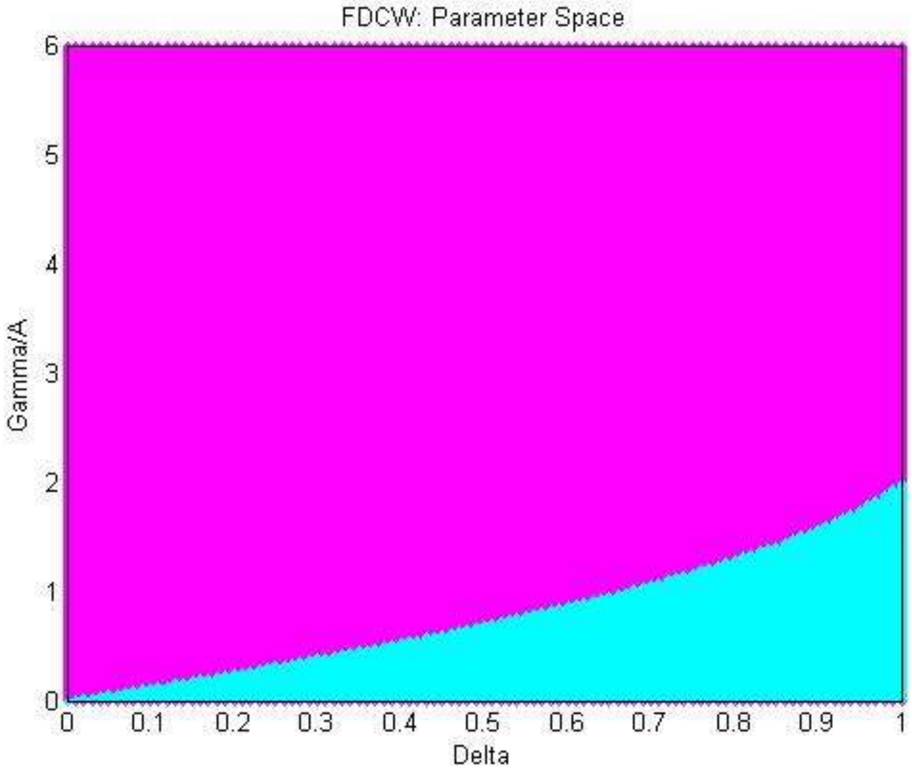

Fig. 1. Two-Dimensional Parameter Space with FDCW and Pair-wise Symmetric Endowments

Explanation:

<u>Blue area:</u> Points in the blue area of compact space {Delta, Gamma/A} correspond to the existence of monetary equilibrium.

Pink area: Points in the pink area of compact space {Delta, Gamma/A} correspond to the existence of barter equilibrium.

Interpretation:

From the graph, it is easier to achieve monetary equilibrium as the ratio of Gamma/A over Delta decreases. When the ratio of Gamma/A to Delta decreases, the scale economy effect can be reflected more easily through the total transaction cost function. There will be more incentive for the participants in the economy to concentrate on one single common medium of exchange under this condition. In this case, good 1, which has the largest amount of endowments in the economy becomes commodity money because with a relatively large trading volume, the bid price of other goods relative to good 1 becomes large, making it profitable to do indirect trade through good 1.

Example 2 (Three goods, FDCW case, asymmetric endowments, 2D parameter space)

Example Settings:
Endowment matrix:
r[1,2] = r[2,1] = 3A; r[1,3] = r[3,1] = 2A; r[2,3] = 1A; r[3,2] = 4A.

Namely, Endowment Matrix=A $\begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$

Output: Figure 2.

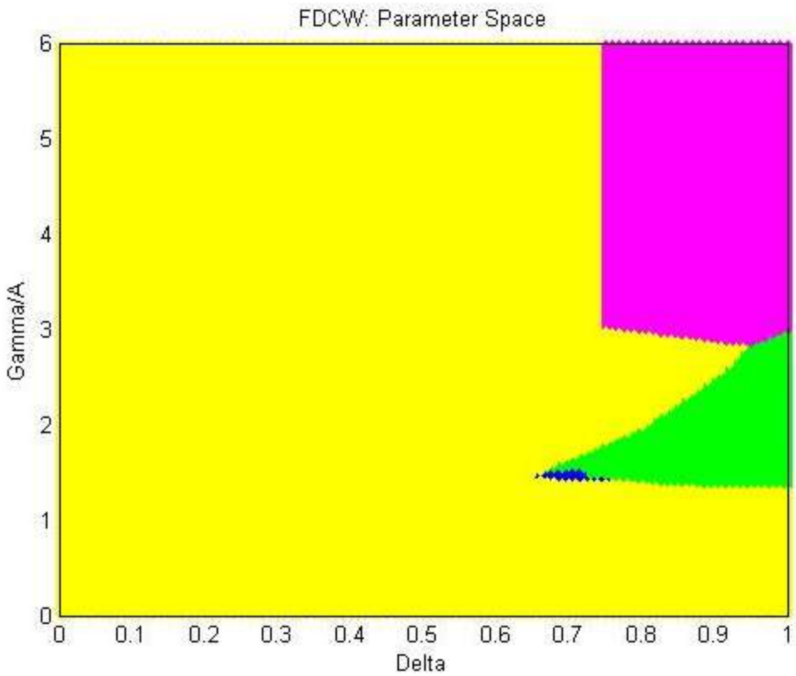

Fig. 2 Two-Dimensional Parameter Space with FDCW and Pair-wise Asymmetric Endowments

Explanation:

Green area: Points in green area correspond to the existence of monetary equilibrium where commodity 2 is used as money.
Blue area: Points in the blue area correspond to the existence of monetary equilibrium where commodity 3 is used as money.
Pink area: Points in the pink area correspond to the existence of barter equilibrium.
Yellow area: Points in the yellow area correspond to the non-convergence.

Interpretation:

Compared to the Example 1, this graph shows that when there's severe imbalance between supply and demand in the market (here, in the trading post {2, 3}, there's severe imbalance, i-j different from j-i. From the matrix's perspective, the transpose of the endowment matrix is significantly different from the original endowment matrix), it is harder to achieve monetary equilibrium. In most of the parameter space, non-convergence, triggered by profitable arbitrage, is pervasive. The problem of non-convergence is gone when Delta, the cost of the barter trade, is high and Gamma is moderately high. High Delta helps attaining monetary equilibrium because the barter trade becomes relatively less attractive to the households. When Gamma is too low, the scale economy kicks in everywhere and there's low incentive to use fewer trading posts. When Gamma is too high, the scale economy is not attainable. Only when Gamma is in the appropriate range can the economy converge to the monetary equilibrium.

Also, note that the area where commodity 2 is used as a common medium of exchange is much bigger than the area where commodity 3 is used as a common medium of exchange, despite the fact that the total volume of commodity 3 is much bigger than the total volume of commodity 2. That is, abundance alone doesn't necessarily make a good the common medium of exchange, and the adjustment process the economy goes through is highly path dependent. In the first stage, starting from barter trade, household [3, 2] has high incentive to escape from the trading post {3, 2} because of low supply of good 2 in the trading post {2, 3} and this makes the other trading posts become more active than before. Unless enough commodity 2 comes to the trading post {2, 3} so that the household [3, 2] finds it attractive to do barter trade, the economy would never converge to monetary equilibrium where commodity 3 becomes money.

Example 3 (Three goods, FDCW case, with 3D parameter space)

Example Settings:

Endowment matrix:

$$\begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} + \text{Alpha} \times \begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Alpha varies from 0 to 1. We are investigating the convex combination of the asymmetric endowment $\begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ and the symmetric endowment $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

We investigate the grid points in the 3-dimensional compact space

Delta $\times$ Gamma/A $\times$ Alpha $\in$ ([0, 1] $\times$ [0, 6] $\times$ [0, 1]) in 0.02 increments

Output: Figure 3-1, Figure 3-2

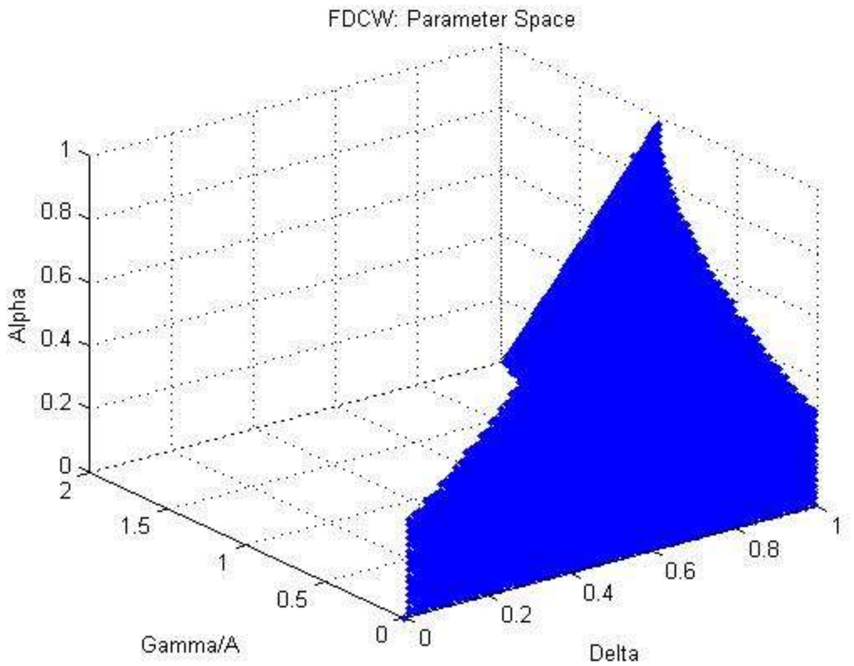

Fig. 3-1 Three-Dimensional Parameter Space with FDCW

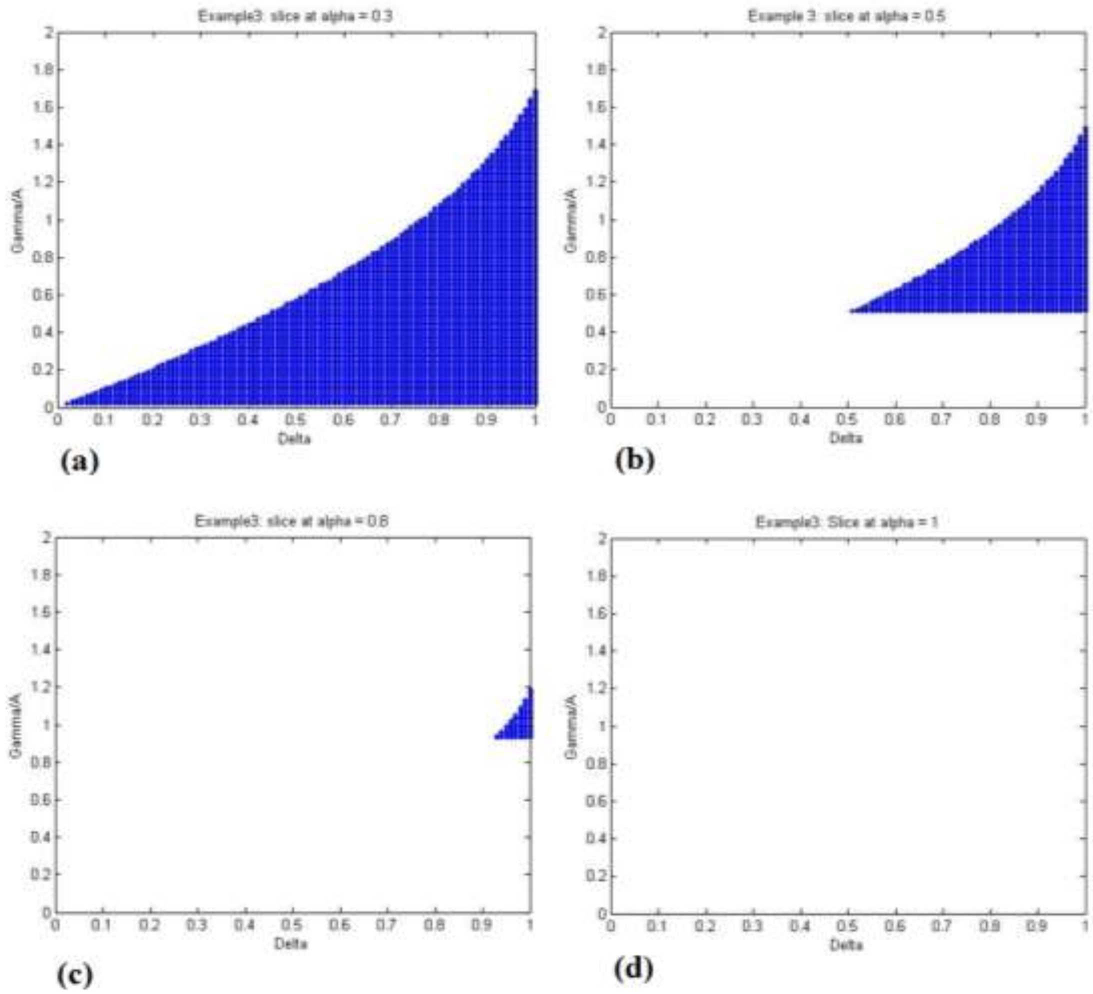

Fig. 3-2 Slice of Figure 3-1 at Alpha=0.3, 0.5, 0.8, 1, respectively

Explanation:

<u>Blue area:</u> Blue scatter points in the compact space {Gamma/A, Delta, Alpha} correspond to the existence of monetary equilibrium. In this area, good 1 is used as a common medium of exchange.
<u>White area:</u> In this area, the economy is in barter equilibrium.

Interpretation:

This 3-dimensional plot implies how the asymmetry of the endowment and the transaction cost parameters affect the result. At first, looking at the variation of the slice figure parallel to the xy-plane at different heights, it implies that if the endowment distribution is more symmetric, it’s harder to get monetary equilibrium. Intuitively speaking, if there is the same amount of each type of good in the economy, it’s harder for the economy to converge to using only one common medium of exchange. Also, looking at the variation of the slice figure parallel to yz-plane through x-axis, it implies that if the cost of direct trade is relatively high, which is represented by high Delta, there’s more incentive to use one common medium of exchange to get the benefit of scale economy.

Also, note that low Gamma/A doesn't always lead the economy to the monetary equilibrium. When the value of alpha is increased, the blue region shrinks to a point locating at Gamma/A = 1 and Delta = 1 and doesn't include the area below Gamma/A = 1. Actually, when Gamma/A is below 1, the scale economy effect kicks in everywhere. However, when Gamma/A is in the appropriate region near 1, the scale economy effect occurs only in the trading posts where commodity 1 is traded. That is, to attain the monetary equilibrium, it's important to have one good having the highest "comparative saleableness" so that the household should have incentive to switch to the indirect trade. This is an important observation in our discussion. Overall, the equilibrium result changes continuously as we vary the parameters Alpha, Gamma/A, and Delta.

4.3 Three goods with Absence of Double Coincidence of Wants

It's with ADCW case that the notion of friction really helps to get convergence (we get similar result with or without friction with FDCW case). Cyclic behavior arises without friction. Assume, for instance, that all households make decision simultaneously. Initially they do barter but find no counter trading partners exist. Then all households switch to indirect trade simultaneously, only to find, again, that no counter trading partners exist. In order to get convergence in this situation, it's needed that not all households make optimal decisions simultaneously. Hence "friction" is introduced.

Different values of proportion have been tried out and it turns out that as long as the proportion ranges from 10% to 30%, the result does not change significantly. In our examples, all proportions are set to be 20%.

Example 4 Three goods, ADCW case, 3D Parameter space

Example Settings:

Endowment matrix:

$$\begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} + \text{Alpha} \times \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Output: Figure 4-1, 4-2

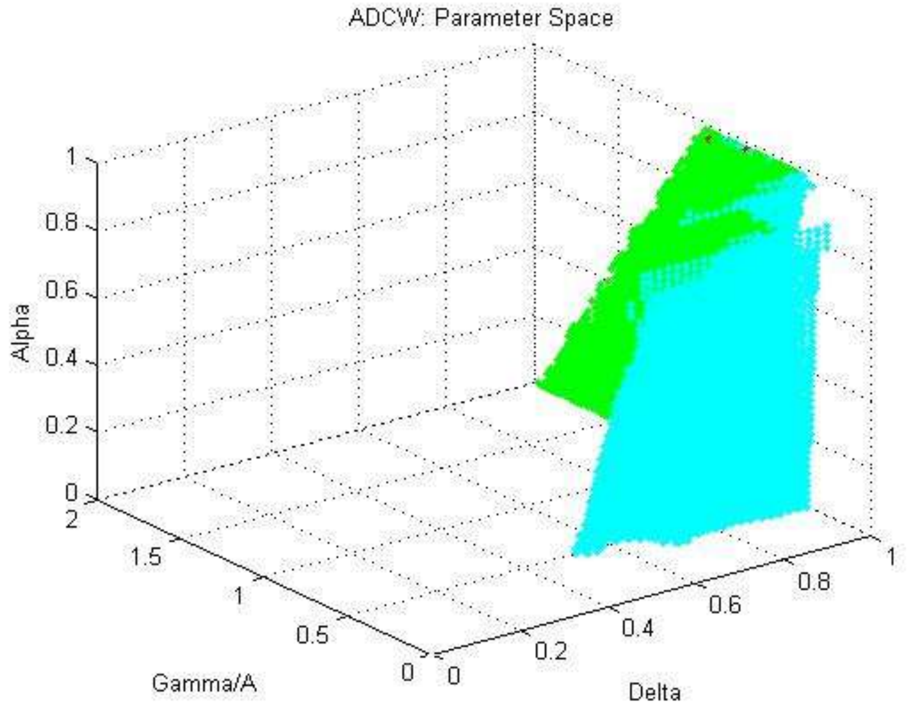

Fig. 4-1 Three-Dimensional Parameter Space with ADCW

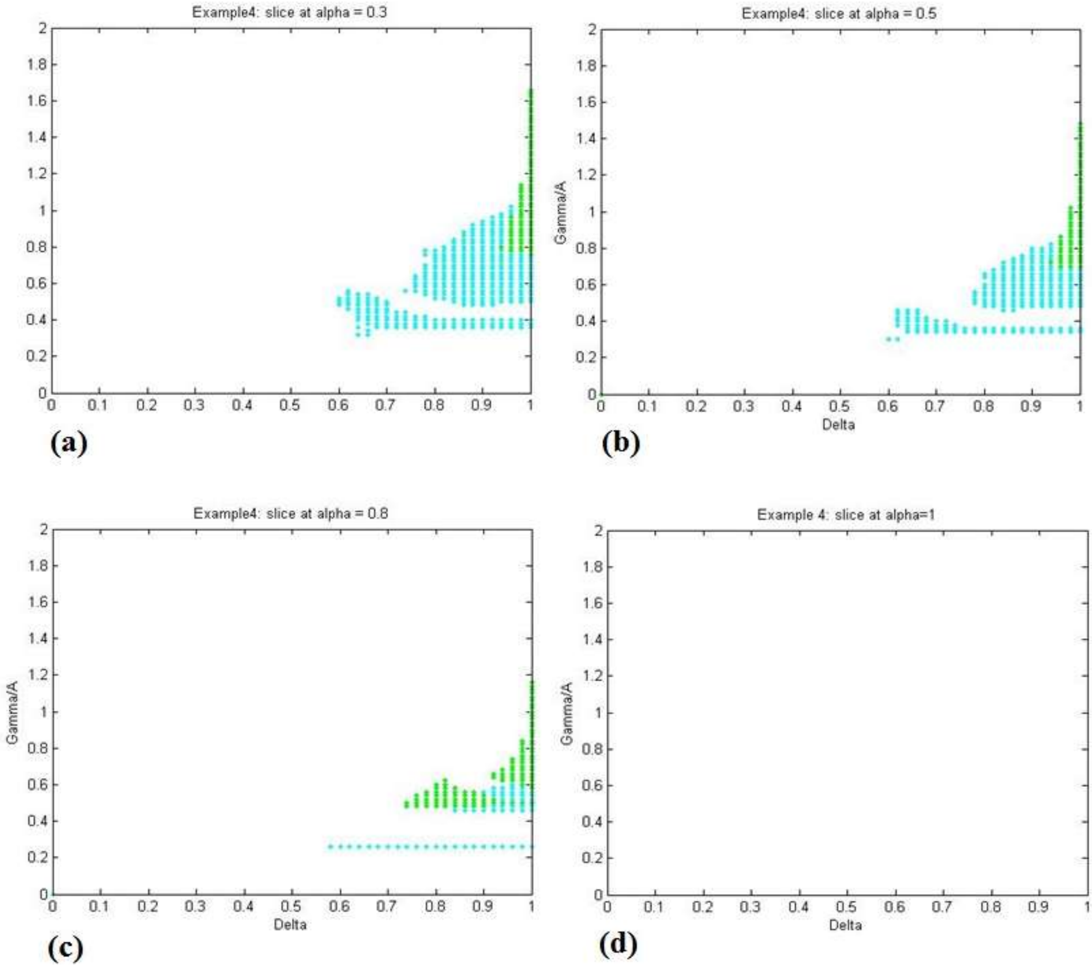

Fig. 4-2: Slice of Fig. 4-1 at Alpha=0.3, 0.5, 0.8, 1, respectively

Explanation:

Sky Blue area: The points in the sky blue area correspond to the monetary equilibrium where commodity 1 is used as money.
Green area: The points in the green area correspond to the monetary equilibrium where commodity 2 is used as money.
Blue area: The points in the blue area correspond to the monetary equilibrium where commodity 3 is used as money. Actually, only 2 points are in this area. They are (1, 0.96, 0.96) and (1, 0.74, 0.98).
Blank area: In this area, there's no convergence.

Interpretation:

Compared to the nice continuous result in FDCW case, the convergence result in ADCW case is not smooth. In many cases, there are cyclic behaviors in the economy so that the convergence to using one common medium of exchange is relatively rare. This gives some interesting comparison about the existence and the convergence result of the monetary equilibrium in ADCW case. Even though the existence of the common medium of exchange in ADCW case is accepted more naturally, in the convergence process, the convergence to the monetary equilibrium in ADCW case is harder to achieve compared to the FDCW case.

Also, looking at the variation of the slice figure parallel to the xy-plane at the different heights, we can notice that the asymmetry of the endowment distribution is not that conclusive compared to the FDCW case in the determination of the monetary equilibrium. This somewhat coincides with our previous notion of the existence of monetary equilibrium in the ADCW case. Even though there are same amount of endowments in the economy, there still exists some incentive to use the medium of exchange because of the lack of the double coincidence of wants. In FDCW, when there's a symmetric endowment, there is hardly any incentive to use one common medium of exchange regardless of the variation of the transaction cost parameters. Also, we can notice that still there's some scale economy effect in the transaction costs. That is, if the ratio of the Gamma/A and Delta is lower, there's more convergence to the monetary equilibrium. Also, as before, high Delta gives higher incentive to do indirect trade and moderate value of Gamma/A is important to achieve the monetary equilibrium.

Lastly, when alpha becomes close to 1, the economy tends to use commodity 2 instead of commodity 3 as money. Definitely, the most abundant good in the economy is commodity 3 for every alpha. However, when the difference between the total volume of commodity 3 and the total volume of commodity 2 is getting less significant, there's more chance to use less abundant good as the common medium of exchange.

5. Conclusion

The computation results above shed light on some long-standing but less formal concepts in the monetary literature.

First, in the selection of the common medium of exchange, 'comparative saleableness' of a commodity rather than 'absolute saleableness' matters. The computation results show that, in FDCW situation, when there is symmetric endowment distribution in economy, there is no convergence to monetary equilibrium at all since there's no advantage of choosing one specific commodity as money instead of the other. It is important that the scale economy effect applies selectively to attain monetary equilibrium. That is, relative superiority in saleableness is the determinant factor in convergence process. This result coincides with the idea suggested in "On the Origin of Money" (Menger, 1892),

> "**Under these circumstances, when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but, if this cannot be effected directly, for other goods also, which, while he did not want them**

> **himself, were nevertheless *more saleable than his own*. By so doing he certainly does not attain at once the final object of his trafficking, to wit, the acquisition of goods needful to himself. Yet he draws nearer to that object. *By the devious way of a mediate exchange, he gains the prospect of accomplishing his purpose more surely and economically than if he had confined himself to direct exchange.* Now in point of fact this seems everywhere to have been the case. Men have been led, with increasing knowledge of their individual interests, each by his own economic interests, without convention, without legal compulsion, nay, even without any regard to the common interest, to exchange goods destined for exchange (their "wares") for other goods equally destined for exchange, but more saleable. (…) *their superior saleableness depends only upon the relatively inferior saleableness of every other kind of commodity, by which alone they have been able to become generally acceptable media of exchange.*"**

Second, it's more likely that the most abundant good becomes the common medium of exchange. This is related with the non-convex structure in transaction costs. If scale economy kicks in due to high trading volume of this commodity, it increases the return for each household holding this commodity. If this scale economy effect is large enough to offset the additional transaction cost to go through one more trading post, then people would start using the medium of exchange to get their desirable good and this would accelerate the scale economy effect again and finally force the economy to use one common medium of exchange.

Third, the existence of monetary equilibrium and convergence to it are very distinct concepts. The existence of monetary equilibrium in ADCW situation may be easily assumed in that, without a medium of exchange, there is no way for each household to get desirable goods. However, our computation result shows that it's harder to get monetary convergence in ADCW situation than in FDCW situation in general. One explanation for this is uncertainty in market situation. Since there is active trading information in each trading post, it is easy to get information to determine optimal trading patterns. That is, if there are some economic agents who are willing to trade for the most saleable good at first, the availability of pricing information reflecting low transaction cost enables quick convergence to monetary equilibrium. In ADCW situation, in every trading post, there's no counterpart trader, that is, no reciprocal demand for each commodity in market. In the perspective of each economic agent, this means that there's little possibility that he can exchange the (prospective) medium of exchange for his desired good in the future. This makes every commodity less saleable. In that situation, the household would not try to go through additional trade using medium of exchange. Thus, unless the economy has any other factors which make one specific commodity more saleable than others, the economy may not converge to monetary equilibrium. The increase of saleableness should occur selectively. Otherwise, the economy may not converge but rather cycle indefinitely, swinging between the state where every household attempts barter and the state where every household conducts indirect trade.

References

Balasko, Y. (2007), "Out-of-equilibrium price dynamics", *Economic Theory* (2007) 33: 413-435

Einzig, P. (1966), *Primitive money in its ethnological, historical, and economic aspects,* Oxford, New York: Pergamon Press.

Howitt, P., and R. Clower (2000), "The Emergence of Economic Organization", *Journal of Economic Behavior & Organization* Vol. 41, 55-84

Kehoe, T. J., N. Kiyotaki, and R. Wright (1993), "More on Money as a Medium of Exchange", *Economic Theory*, Vol.3, No.2, pp 297-314

Kiyotaki, N., and R. Wright (1989), "On Money as a Medium of Exchange", *The Journal of Political Economy*, Vol 97, No.4, pp. 927-954

Marimon, R., E. McGrattan, and T. J. Sargent (1990), "Money as a Medium of Exchange in an Economy with Artificially Intelligent Agents", *Journal of Economic Dynamics and Control* 14 329-373

Menger, C. (1892), "On the Origin of Money", *The Economic Journal,* Vol. 2, No.6, pp 239 – 255

Newhouse, H. (2007), "Emergence of Commodity Money as a Medium of Exchange"

Shapley, L. S., and M. Shubik (1977A), "An Example of a Trading Economy with Three Competitive Equilibria", *The Journal of Political Economy*, Vol. 85, No.4, pp 873-875

Shapley, L. S., and M. Shubik (1977B), "Trade Using One Commodity as a Means of Payment", *The Journal of Political Economy*, Vol 85, No.5, pp.937-968

Starr, R. M. (2004), "The Existence and uniqueness of 'money' in general equilibrium natural monopoly in the most liquid asset", in *Assets, Beliefs and Equilibria in Economic Dynamics: Essays in Honor of Mordecai Kurz, Aliprantis, Arrow, Hammond, Kubler, Wu, and Yannelis*, eds. New York: Springer-Verlag, 2004

Starr, R.M. (2008), "Mengerian Saleableness and Commodity Money in a Walrasian Trading Post Example", *Economics Letters*, 100, pp. 35-38

Appendix:

Sample MATLAB code for three-good FDCW case

%Variable Definition:
  N: Total Number of kinds of Goods
  A: Unit of Goods
  Gamma: Ceiling Cost
  Delta: Unit Cost for Low Trading Volume
  r(i,j): Amount of Goods i in Household [i,j]
  prop(i,j): Proportion of the household {i,j} who does the direct trade
  q(i,j): Price of Selling Goods i at Trading Post {i,j}
  v(i,j): Economics of Scale Indicator
        0 - Economics of Scale Not Exist; 1 - Economics of Scale Exists
  tc(i,j): Transaction cost of good i in trading post {i,j}
  s(i,j,k,w): Amount of Goods k Sold by Household [i,j] at Trading Post {k,w}
  b(i,j,k,w): Amount of Goods k Bought by Household [i,j] at Trading Pose {k,w}
  vol(i,j): Total Trading Volumn at Trading Post {i,j}
  inq(i,j,k): Total Indirect Trade Price for Household [i,j]
  flag(i,j): Trade Method Indicator of Household [i,j]
          0 - Direct Trade;
          k - Indirect Trade using Goods k as Currency;
  dflag(i,j): Trade Method Indicator of Hosehold [i,j] in the last period
  pre_q(i,j): Price of Selling Goods i at Trading Post {i,j} in Previous Cycle
  conv: Convegence Indicator
        0 – Non-convergence; 1 – Monetary equilibrium; 2-Multiple money
  cycle: Number of Cycles to Convergence%

```
clc;

  Step 0: Initialization
r = A * [0 (3-2*Alpseq(h)) (2-Alpseq(h));
        (3-2*Alpseq(h)) 0 1;
        (2-Alpseq(h)) 1 0];
….

  Cycle 1
  Step 1: Trading Plan
(Calculating the trading volume of representative household;
And the trading volume at each trading post)

for i    = 1:N
      for j = 1:N
            prop(i,j) = 1;
```

```
    end
end

for i = 1:N
    for j = 1:N
        s(i,j,i,j) = r(i,j);
        b(i,j,i,j) = q(i,j) * s(i,j,i,j);
    end
end

 Step 2: Price Adjustment
for k = 1:N
    for w = 1:N
        vol(k,w) = sum(sum(s(:,:,k,w)));
    end
end

for i = 1:N
    for j = 1:N
        if Delta * vol(i,j) < Gamma
            tc(i,j) = Delta * vol(i,j);
        else
            tc(i,j) = Gamma;
        end
    end
end

for i = 1:N
    for j = 1:N
        if j ~= i
            if vol(i,j) == 0 && vol(j,i) == 0
                q(i,j) = max(1 - Delta, 0);
                q(j,i) = max(1 - Delta, 0);
            end

            if vol(i,j) == 0 && vol(j,i) ~= 0
                q(i,j) = 1;
                q(j,i) = max(1 - Delta - tc(j,i)/vol(j,i), 0);
            end

            if vol(i,j) ~= 0 && vol(j,i) == 0
                q(i,j) = max(1 - Delta - tc(i,j)/vol(i,j), 0);
                q(j,i) = 1;
            end
```

```
      if vol(i,j) ~= 0 && vol(j,i) ~=0
         q(i,j) = vol(j,i)/vol(i,j) - (tc(j,i) + tc(i,j))*vol(j,i)/((vol(i,j) + vol(j,i))*vol(i,j));
         q(j,i) = vol(i,j)/vol(j,i) - (tc(j,i) + tc(i,j))*vol(i,j)/((vol(i,j) + vol(j,i))*vol(j,i));
      end
            end
        end
end

for i = 1:N
        for j = 1:N
                if i ~= j
                        k = 6 - i - j;
                        inq(i,j) = q(i,k) * q(k,j);
                end
        end
end
```

Household Decision Making

```
for i = 1:N
        for j = 1:N
                if i ~= j && inq(i,j)*r(i,j) > q(i,j)*r(i,j)
                        flag(i,j) = 1;
                        prop(i,j) = prop(i,j) * 0.8;
                end
        end
end

cycle = cycle + 1;

for i = 1:N
        for j = 1:N
                dflag(i,j) = flag(i,j);
                pre_q(i,j) = q(i,j);
        end
end

if sum(sum(flag)) == 0
        conv = 1;
end
```

Cycle 2+

Step 1: Trading plan

```
while conv == 0
```

```
        s = zeros(N, N, N, N);
        b = zeros(N, N, N, N);

for i = 1:N
         for j = 1:N
                        if i ~= j
                                s(i,j,i,j) = r(i,j) * prop(i,j);
                                b(i,j,i,j) = q(i,j) * s(i,j,i,j);
                                temp = 6 - i - j;
                                s(i,j,i,temp) = r(i,j) * (1 - prop(i,j));
                                b(i,j,i,temp) = q(i,temp) * s(i,j,i,temp);
                                s(i,j,temp,j) = b(i,j,i,temp);
                                b(i,j,temp,j) = q(temp,j) * s(i,j,temp,j);
                        end
         end
end

for k = 1:N
         for w = 1:N
               vol(k,w) = sum(sum(s(:,:,k,w)));
         end
end

for i = 1:N
        for j = 1:N
               if Delta * vol(i,j) < Gamma
                       tc(i,j) = Delta * vol(i,j);
               else
                       tc(i,j) = Gamma;
               end
        end
end
```

…

(This omitted part is the price adjustment based on the new trading volume, similar as the price adjustment in cycle 1)

…

Household Decision Making

```
for i = 1:N
        for j = 1:N
               if i ~= j
                       if dflag(i,j) == 0
```

```
                if inq(i,j)*r(i,j) > q(i,j)*r(i,j)
                    flag(i,j) = 1;
                    prop(i,j) = prop(i,j) * 0.8;
                else
                    flag(i,j) = 0;
                    prop(i,j) = prop(i,j) + (1 - prop(i,j)) * 0.2;
                end
            end

            if dflag(i,j) == 1
                if inq(i,j)*r(i,j) > q(i,j)*r(i,j)
                    flag(i,j) = 1;
                    prop(i,j) = prop(i,j) * 0.8;
                else
                    flag(i,j) = 0;
                    prop(i,j) = prop(i,j) + (1 - prop(i,j)) * 0.2;
                end
            end
        end
    end
end

conv = 1;
    for i = 1:N
      for j = 1:N
        if abs(pre_q(i,j)-q(i,j)) >= 0.0001 || min(prop(i,j), 1 - prop(i,j)) >= 0.0001
          conv = 0;
        end
      end
   end


for i = 1:N
    for j = 1:N
        dflag(i,j) = flag(i,j);
        pre_q(i,j) = q(i,j);
    end
end

    cycle = cycle + 1;

    if cycle == 500
        conv = 2;
    end
```

```
    if conv == 1 && max(max((flag(:,:)))) == 1
        money = 0;
        for i = 1:N
            for j = 1:N
                if flag(i,j) == 1 && money == 0
                    money = 6 - i - j;
                end
                if flag(i,j) == 1 && money ~= 0
                   if money ~= 6 - i - j
                    conv = 3; multiple common medium of exchange
                   end
                end
            end
        end
    end

if conv == 1
     row = row + 1;
end
if conv == 3
     mrow = mrow + 1;
end
end
…
```

(The final part uses the plot function to draw a 3-D parameter space, and identify the single monetary equilibrium)

(Full version of the other MATLAB code is available upon request.)