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Abstract

We study the second-price offer feature of eBay auctions in which the seller has multiple units. Perhaps surprisingly, the opportunity to make second-chance offers can *reduce* seller profit. This happens if her marginal cost function is sufficiently steep. Hence, sellers should be wary about the possibility that buyers anticipate second-chance offers, and it may be advantageous for a seller to acquire a reputation for not making second-chance offers. (*JEL* D44, D82)

Keywords: Second-price auction, second-chance offer, eBay, optimal auction

1 Introduction

eBay has become one of the most important market places for retail goods worldwide. Yet important aspects of the strategic bidding incentives in eBay auctions remain unexplored. eBay's main sales mechanism is closely related to a second-price auction, which has been extensively studied. There are, however, important dissimilarities to a standard second-price

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auction. In particular, the eBay auction allows the seller to sell multiple units of the same good.[1] While the seller is committed to sell the first unit to the highest bidder at essentially the second-highest bid, she also retains the option to make "second-chance offers". She may offer a second unit to the second-highest bidder at the second-highest bid, a third unit to the third-highest bidder at the third-highest bid, and so on.

We show that the opportunity to make second-chance offers might *reduce* the seller's profit. This happens if the seller's marginal cost function is sufficiently steep. For example, suppose that the seller has two units of a rare collectable item, and her cost arises from the lost opportunity of selling in a different market. It may well be that, due to downward sloping demand in the other market, her cost of selling the first unit in the auction is considerably smaller than her cost of selling the second unit in the auction. Then the opportunity to make second-chance offers can harm her.

We conclude that sellers should be wary about the possibility that buyers anticipate second-chance offers. In fact, it may be advantageous for a seller to acquire a reputation for not making second-chance offers.

To see why second-chance offers may reduce the seller's expected profit, observe that the anticipation of a second-chance offer reduces the bidders' incentives to bid aggressively. Because the seller cannot commit to a minimum bid threshold for second-chance offers, the bid reduction effect can dominate so that the seller's overall profit is reduced.

We can find only two papers which address the topic of second-chance offers in second-price auctions from a game-theoretic perspective, Joshi et al. (2005) and Salmon and Wilson (2008). Both of these works consider a generalized second-stage mechanism in which the seller is not restricted to charge the losing bidder's stage-one bid. This allows the seller to use the information contained in losing bidders' bids in order to price-discriminate. A second-chance offer on eBay, in contrast, has to equal the second-highest bid, allowing no price discrimination.

Our goal is to stay as close as possible to eBay rules and environment. We consider a

[1] Another important difference is that eBay allows sequential bids. The sequentiality, however, does not play a crucial role in the private-value environments that we consider.

setting in which buyers have symmetric independent private values for a single unit.[2] The seller may have a second unit for sale, and the bidders may be uncertain about the seller's endowment (to make our point, it is not necessary to consider more then two units). The seller may set a minimum bid for her auction, and decides about a second-chance offer depending upon the observed bids.

There exists a symmetric and strictly increasing bidding equilibrium. The seller makes a second-chance offer if and only if the second-highest bid exceeds her opportunity cost of selling the second-unit.

We show that if the marginal cost of the second unit is high, then the seller's expected profit is lower than in the equilibrium of the game in which she cannot make a second-chance offer. This holds for any number of bidders, for any distribution over values, and whether or not the seller uses a minimum bid.[3]

An important benchmark is given by the seller's profit maximizing sales mechanism. Under the assumption that the environment is regular in the sense of Myerson (1981), the seller should allocate her units to the bidders in decreasing order of their valuations, as long as virtual valuations exceed marginal costs (Maskin and Riley, 1989).

Hence, the seller can achieve her optimal allocation by using the eBay auction with second-chance offers if she has *constant* marginal costs; it suffices that the minimum bid is set such that its virtual valuation equals her marginal cost. A different auction format in which the seller is not bound to auction bids, but can freely choose second-chance offer prices (Joshi et al., 2005, Salmon and Wilson, 2008) will typically be harmful to the seller because of the resulting distortion of the allocation away from the optimal one.

In order to achieve the seller optimal allocation in environments with increasing marginal costs, a separate minimum bid is needed for selling the second unit (possibly higher than the minimum bid for the first unit), another minimum bid for the third unit, and so on. Adapting the eBay rules accordingly would yield an optimal mechanism, however this would

[2]The symmetry assumption fits well to the largely anonymous trading environment in which eBay auctions take place. Also, the assumption that each buyer demands at most one unit is reasonable in many contexts.

[3]We do not have statistics on the prevalence of minimum bids. There is some evidence that high minimum bids discourage participation in auctions (see Bajari and Hortacsu, 2003) and this may influence their use by sellers.

require eBay's enforcement of the minimum bid schedule. Without third-party enforcement or some other commitment mechanism, optimum revenue is not obtainable.

2 The model and preliminary results

Consider a seller who has one or two units of an indivisible good. The seller has quasi-linear risk-neutral preferences. Her cost of selling one unit is denoted $c_1 \geq 0$; her marginal cost of selling the second unit (if she possesses two units) is denoted $c_2 \geq c_1$. Let $\lambda > 0$ denote the probability that the seller has two units. There are $n \geq 2$ potential buyers with single-unit demand and quasi-linear risk-neutral preferences. Buyer i's ($i = 1, \ldots, n$) valuation for the good is independently distributed according to a cumulative distribution function F with positive and Lipschitz continuous density f on $[0, 1]$. Let X_i denote the random variable for buyer i's value.

The buyers participate in a variant of a second-price auction in which the seller may offer her second unit at the second-highest bid to the second-highest bidder ("second-chance offer"). The seller can announce a minimum bid $r \geq 0$. The buyers/bidders expect the seller to make a second-chance offer if and only if the second-highest bid is not smaller than some number $p \geq r$. Our analysis will be based on the following equilibrium result.

Proposition 1. *There exists a symmetric bidding equilibrium. All types $x < r$ stay out of the auction. The equilibrium bid function $\beta : [r, 1] \to [r, 1]$ is strictly increasing. All types $x \in [r, p]$ bid their values $\beta(x) = x$, and, for all $x \in [p, 1]$,*

$$\begin{aligned} 0 &= (x - \beta(x))(n-1)F(x)^{n-3}f(x)(F(x)(1-\lambda) + \lambda(1-F(x))(n-2)) \\ &\quad -\lambda(n-1)(1-F(x))F(x)^{n-2}\beta'(x). \end{aligned} \tag{1}$$

All types $x \in (p, 1)$ submit bids below their values, $\beta(x) < x$.

Any second-chance offer is accepted.

The proof combines standard equilibrium arguments for first-price and second-price auctions and is relegated to the Appendix.

Using (1) one can show that, the higher the probability buyers put on the event that the seller has two units, the less they will bid. This reveals a fundamental tradeoff: while second-chance offers allow the seller to sell additional units of her good, they also reduce the bidders' incentives to compete.

Because all bidders use the same strictly increasing bid function, we have the following

Corollary 1. *In equilibrium, one unit of the good is assigned to the buyer with the highest valuation, as long as this valuation is at least r; with probability λ, a second unit is assigned to the buyer with the second-highest valuation, provided that valuation is not lower than p.*

Recall from Myerson (1981) that the environment is *regular* if the virtual valuation function $\psi(x) = x - (1 - F(x))/f(x)$ is strictly increasing. The following result provides an important benchmark.

Corollary 2. *Suppose the environment is regular. Then the second-price auction with the minimum bid $r = \psi^{-1}(c_1)$ and the second-chance offer threshold*

$$p = \psi^{-1}(c_2) \tag{2}$$

yields the profit-maximizing allocation for the seller.

Proof. If it were commonly known that the seller has only one unit ($\lambda = 0$), then optimality would follow from Myerson (1981). If it is commonly known that the seller has two units ($\lambda = 1$), then optimality follows from Maskin and Riley (1989). From the irrelevance result of Mylovanov and Tröger (2012), these allocations remain optimal if the buyers are uncertain about the seller's endowment. *QED*

The fundamental problem with second-chance offers is that the seller cannot commit to the threshold (2). Instead, the buyers expect the seller to make a second-chance offer whenever it is profitable,

$$p = \max\{c_2, r\}. \tag{3}$$

In environments in which the seller's ex-ante profit-maximizing threshold (2) differs from her interim-optimal threshold (3), the second-price auction with second-chance offer does *not* yield the seller-optimal allocation.

Corollary 3. *In a regular environment, the second-price auction with optimal minimum bid* $r = \psi^{-1}(c_1)$ *and interim-optimal second-chance offer threshold (3) is a profit-maximizing mechanism if and only if*

$$c_2 \;=\; c_1. \tag{4}$$

Elsewhere (Joshi et al., 2005, Salmon and Wilson, 2008) a different second-chance offer mechanism is analyzed, in which the seller is free to make any second-chance offer, as a take-it-or-leave-it price, after seeing the second-highest bid. In a regular environment in which (4) holds, this freedom to choose a price can never be advantageous for the seller. A buyer would get no rent from the trade of the second unit if her bid revealed her value; as a result, the buyer typically has an incentive to randomize her bidding (cf. Salmon and Wilson, 2008), so that the resulting final allocation is distorted away from the seller-optimal allocation.

3 The costs of second-chance offers

In this section, we focus on environments in which c_2 is large, that is, in which the marginal cost function is steep. By Corollary 3, then, the second-price auction with second-chance offers is not an optimal mechanism for the seller. Our point is that even a standard second-price auction *without* second-chance offers can then be better for the seller.

Applying standard envelope arguments (e.g., Milgrom and Segal, 2002), the seller's expected profit from a second-price auction with minimum bid r and second-chance offer threshold $p \geq r$ is given by

$$\Pi^{\text{SCO}}(r,p) \;=\; \int_r^1 (\psi(y) - c_1) f^{(1)}(y)\mathrm{d}y + \lambda \int_p^1 (\psi(y) - c_2) f^{(2)}(y)\mathrm{d}y, \tag{5}$$

where

$$f^{(1)}(x) = nF(x)^{n-1} f(x), \qquad f^{(2)}(x) = n(n-1)F(x)^{n-2}(1 - F(x))f(x) \tag{6}$$

are the densities of the highest and second-highest order statistics of the random variables $X_1, \ldots, X_n$.

For comparison, the seller's expected profit from a standard second-price auction with minimum bid r is given by

$$\Pi^{\text{SPA}}(r) \;=\; \int_r^1 (\psi(y) - c_1) f^{(1)}(y)\mathrm{d}y. \tag{7}$$

Our result is that, if the marginal cost of the second unit is sufficiently close to the highest possible valuation, then the opportunity to make a second-chance offer is harmful for the seller, given that she uses her interim-optimal threshold (3). This holds for arbitrary minimum bids, arbitrary numbers of buyers, and arbitrary distributions of valuations.[4]

Proposition 2. *Consider any minimum bid $r < 1$. If $c_2 < 1$ is sufficiently close to 1, and p is given by (3), then $\Pi^{SCO}(r,p) < \Pi^{SPA}(r)$.*

Comparing (5) and (7), and using that $p = c_2$ if c_2 is close to 1, we have to show that

$$\int_{c_2}^1 (\psi(y) - c_2) f^{(2)}(y)\mathrm{d}y \;<\; 0. \tag{8}$$

[4]Note that we keep the minimum bid r fixed. Alternatively, one may assume that for all c_2, the minimum bid is chosen to maximize $\Pi^{\text{SCO}}(r,p)$, with p given by (3). Because the optimal r stays bounded away from 1 as $c_2 \to 1$, the proof still goes through.

The integrand is negative if $y < \psi^{-1}(c_2)$, and positive if the opposite inequality holds. At y close to 1, ψ is approximately linear with slope 2. Hence, if the density $f^{(2)}(y)$ were constant in this area, then the integrations over the two sub-areas would cancel out. However, because $f^{(2)}$ is the density for the *second*-highest (rather than highest) order statistics, it is decreasing at all points close to 1, implying that the integration over the sub-area in which the integrand is negative dominates (cf. Figure 1). The details of the proof can be found in the Appendix.

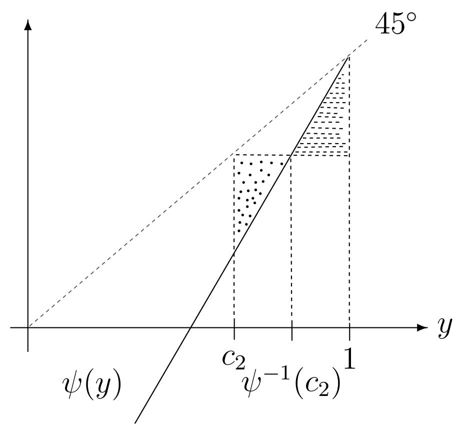

Figure 1: In the dotted area, the marginal cost of selling the second unit exceeds the buyer's virtual valuation; this represents the cost of making a second-chance offer. In the striped area, the buyer's virtual valuation exceeds the marginal cost of selling the second unit; this represents the benefit of making a second-chance offer. If c_2 is close to 1, the left area's negative contribution to the integral (8) dominates the right area's negative contribution because the density of the second-highest order statistics is decreasing at points y close to 1.

4 Illustration

Suppose $n = 3$ and F is uniform on $[0, 1]$. Figure 2 shows the percent increase or decrease in seller revenue that is created by the introduction of the second-chance offer for three minimum bid and second-chance offer threshold scenarios and two choices of λ.[5] The left panels of Figure 2 show that if an arbitrary reserve is chosen by the seller (we use the

[5]Details of the calculations are found in the appendix.

marginal cost of the first unit), then seller revenue is reduced by the introduction of the second-chance offer. This shows that the requirements on c_2 implied by Proposition 2 can be be very weak indeed. Here, seller revenue is reduced for all $c_2 \geq c_1$, although this is not true for larger n. The right panel shows that even when the seller is able to set an optimal minimum bid, seller revenue is reduced when the marginal cost of the second unit is high enough. This is the case mentioned in footnote 4. The figure shows that the seller can mitigate, but not eliminate, the potentially negative impact of the second-chance offer by choosing an optimal minimum bid.

5 Appendix

Proof of Proposition 1. Consider a buyer (say, buyer 1) of type $x \in [0,1]$ who believes that everybody else uses the strictly increasing and continuous bid function β with $\beta(r) = r$ and all types $< r$ staying out. Her expected payoff from bidding $b \in [r, \beta(1)]$ is

$$\begin{aligned}\Pi(b,x) &= E[\mathbf{1}_{\max_{i\neq 1}\beta(X_i)\leq b}(x - \max\{r, \max_{i\neq 1}\beta(X_i)\})] \\ &\quad +\lambda \mathbf{1}_{b\geq p}(x-b)(F^{1,n-1}(\beta^{-1}(b)) - F^{2,n-1}(\beta^{-1}(b))),\end{aligned}$$

where $F^{k,n-1}$ denotes the c.d.f. for the kth largest among $n-1$ values that are drawn i.i.d. according to F.

For all types $x \in [r,p]$ the expected payoff is maximized by value-bidding, for the same reason as in a standard second-price auction.

Consider then $x > p$. Any bid $b < p$ is suboptimal because $\Pi(b,x) < \Pi(p,x)$ for the same reason as in a standard second-price auction.

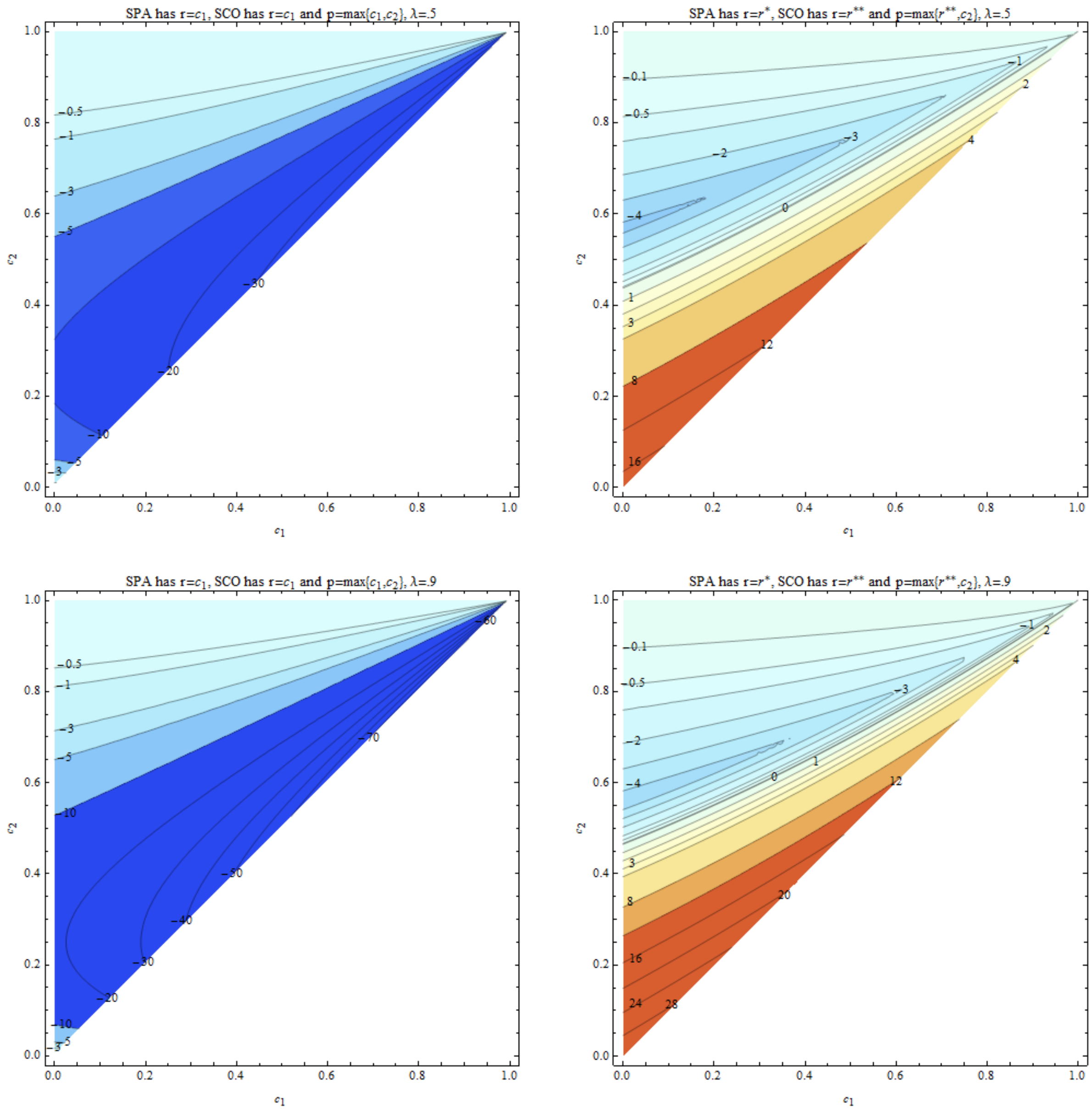

Figure 2: Levels are the percent increase or decrease in seller revenue that results from the introduction of a second-chance offer to the second-price auction. SPA and SCO refer to second-price auction without and with a second-chance offer, respectively. $r^* = (1 + c_1)/2$ is the optimal minimum bid for the second-price auction without a second-chance offer. r^{**} is the minimum bid that maximizes $\Pi^{\mathrm{SCO}}(r, max\{r, c_2\})$, as defined in (16).

For any $b \in [p, \beta(1)]$, we can write the expected payoff as

$$\begin{aligned}\Pi(b,x) &= F^{1,n-1}(r)(x-r) + \int_r^{\beta^{-1}(b)} (x - \beta(y))\mathrm{d}F^{1,n-1}(y) \\ &\quad +\lambda(x-b)(F^{1,n-1}(\beta^{-1}(b)) - F^{2,n-1}(\beta^{-1}(b)))\end{aligned}$$

$$
\begin{aligned}
&= F(r)^{n-1}(x-r) + \int_r^{\beta^{-1}(b)} (x-\beta(y))(n-1)F(y)^{n-2}f(y)\mathrm{d}y \\
&\quad +\lambda(x-b)(n-1)(1-F(\beta^{-1}(b)))F(\beta^{-1}(b))^{n-2}.
\end{aligned}
$$

The payoff change from a marginal bid increase is

$$
\begin{aligned}
\frac{\partial \Pi}{\partial b} &= (\beta^{-1})'(b)(x-b)(n-1)F(\beta^{-1}(b))^{n-2}f(\beta^{-1}(b))(1-\lambda) \\
&\quad +\lambda(x-b)(n-1)(1-F(\beta^{-1}(b)))(n-2)F(\beta^{-1}(b))^{n-3}f(\beta^{-1}(b))(\beta^{-1})'(b) \\
&\quad -\lambda(n-1)(1-F(\beta^{-1}(b)))F(\beta^{-1}(b))^{n-2}.
\end{aligned}
$$

Because this function is increasing in x, the same argument as for a standard first-price auction shows that Π is quasi-concave in b. Hence, to show the optimality of the bid $b = \beta(x)$, it is sufficient to verify the first-order condition

$$
\begin{aligned}
0 &= \left.\frac{\partial \Pi}{\partial b}\right|_{b=\beta(x)} \\
&= \frac{x-\beta(x)}{\beta'(x)}(n-1)F(x)^{n-3}f(x)(F(x)(1-\lambda) \\
&\quad +\lambda(1-F(x))(n-2)) - \lambda(n-1)(1-F(x))F(x)^{n-2}.
\end{aligned}
$$

We have to solve the differential equation (1) for $x \in [p, 1]$, with the boundary condition $\beta(p) = p$. Because the differential equation is linear in β and β', a unique solution exists.

We use the equation (1) in order to show that $\beta'(x) > 0$ for all $x \in (p, 1)$, implying that β is strictly increasing, thus justifying the use of the inverse above.

The differential equation (1) has the form $(x - \beta(x))h(x) = k(x)\beta'(x)$, where $h(x) > 0$ and $k(x) > 0$ for all $x \in [p, 1)$.

Fix any $\overline{x} < 1$. First we show that

$$
\arg\min_{x\in[p,\overline{x}]} x - \beta(x) = \{p\}. \tag{9}
$$

Suppose otherwise. Then there exists $y \in (p, \overline{x}]$ where $x - \beta(x)$ is minimized, implying

$1 - \beta'(y) \leq 0$ by the standard first-order condition (we write "≤ 0" instead of "$= 0$" to include the possibility of a minimum at the right boundary $\overline{x}$). Hence, $\beta'(y) > 0$. Thus, using the differential equation, $(y - \beta(y))h(y) = k(y)\beta'(y) > 0$, implying $y - \beta(y) > 0$. Because y is a minimizer, we conclude that $p - \beta(p) \geq y - \beta(y) > 0$, a contradiction.

From (9) and $p - \beta(p) = 0$ it follows that $x - \beta(x) > 0$ for all $x \in (p, 1)$. Hence, $\beta'(x) > 0$ by (1). *QED*

Proof of Proposition 2. First we show that

$$\int_{c_2}^{1} (\psi(y) - (2y-1)) f^{(2)}(y) \mathrm{d}y \quad = \quad o((1-c_2)^3), \tag{10}$$

where $o(x)$ stands for any function such that $o(x)/x \to 0$ as $x \to 0$.

To see (10), observe that

$$\begin{aligned} \int_{c_2}^{1} (\psi(y) - (2y-1)) f^{(2)}(y) \mathrm{d}y \quad &= \quad \int_{c_2}^{1} (\psi(y) - (2y-1)) f(y) \frac{f^{(2)}(y)}{f(y)} \mathrm{d}y \\ &= \quad \int_{c_2}^{1} (f(y)(1-y) - (1 - F(y))) \frac{f^{(2)}(y)}{f(y)} \mathrm{d}y. \end{aligned}$$

By the fundamental theorem of calculus, $1 - F(y) = F(1) - F(y) \geq (1-y) \min_{\xi \in [c_2, 1]} f(\xi)$. Hence, we can continue

$$\begin{aligned} \ldots \quad \leq \quad & \int_{c_2}^{1} (f(y) - \min_{\xi \in [c_2,1]} f(\xi))(1-y) \frac{f^{(2)}(y)}{f(y)} \mathrm{d}y \\ &= (f(y') - \min_{\xi \in [c_2,1]} f(\xi)) \frac{f^{(2)}(y')}{f(y')} \int_{c_2}^{1} (1-y) \mathrm{d}y \end{aligned}$$

for some $y' \in [c_2, 1]$, by the mean value theorem. The last integral can be easily evaluated as

$$\int_{c_2}^{1} (1-y) \mathrm{d}y \quad = \quad \frac{1}{2}(1-c_2)^2. \tag{11}$$

Moreover, using the Lipschitz constant L for f,

$$f(y') - \min_{\xi \in [c_2,1]} f(\xi) \quad \leq \quad L(1-c_2), \tag{12}$$

and, using (6),

$$\frac{f^{(2)}(y')}{f(y')} \leq n(n-1)(1-F(y')) \leq n(n-1) \max_{\xi \in [c_2,1]} f(\xi)(1-c_2).$$

Combining this with (11) and (12), we have an upper bound for the left-hand side of (10) that is $o((1-c_2)^3)$. A lower bound that is $o(1-c_2)$ is obtained in a similar way, showing (10).

Observe that $f^{(2)}$ is Lipschitz continuous because f is Lipschitz continuous. Hence, for Lebesgue almost-every $x \in [0,1]$, the derivative $(f^{(2)})'(x)$ exists and, using standard differentiation rules, if $n \geq 3$,

$$\begin{aligned}(f^{(2)})'(x) \quad &= \quad n(n-1)(1-F(x))F^{n-3}(x)\left((n-2)f(x)^2 + F(x)f'(x)\right) \\ &\quad\; -n(n-1)F^{n-2}(x)f(x)^2.\end{aligned}$$

Thus, because $|f'(x)|$ is bounded by the Lipschitz constant for f,

$$(f^{(2)})'(x) \quad \leq \quad -\delta, \quad \text{where } \delta \overset{\text{def}}{=} \frac{1}{2}n(n-1)f(1)^2,$$

for all x sufficiently close to 1. The same conclusion holds if $n = 2$.

Hence, if c_2 is sufficiently close to 1, then, for all y with $c_2 \leq y \leq \frac{c_2+1}{2}$,

$$\begin{aligned}f^{(2)}(y) - f^{(2)}(1-(y-c_2)) \quad &= \quad \int_y^{1-(y-c_2)} -(f^{(2)})'(x)\mathrm{d}x \\ &\geq \quad (1-2y+c_2)\delta.\end{aligned} \tag{13}$$

Finally, we evaluate

$$\begin{aligned}
&\int_{c_2}^{1}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y \\
= &\int_{c_2}^{(c_2+1)/2}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y + \int_{(c_2+1)/2}^{1}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y
\end{aligned}$$

The change of variables $x = 1-(y-c_2)$ in the second integral yields that

$$\int_{(c_2+1)/2}^{1}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y = \int_{c_2}^{(c_2+1)/2}(1+c_2-2x)f^{(2)}(1-(x-c_2))\mathrm{d}x$$

Plugging this into the above evaluation yields

$$\begin{aligned}
\int_{c_2}^{1}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y &= \int_{c_2}^{(c_2+1)/2}(2y-1-c_2)\left(f^{(2)}(y)-f^{(2)}(1-(y-c_2))\right)\mathrm{d}y \\
&\overset{(13)}{\leq} \int_{c_2}^{(c_2+1)/2}(2y-1-c_2)(1-2y+c_2)\delta\mathrm{d}y \\
&= -\int_{c_2}^{(c_2+1)/2}(1-2y+c_2)^2\mathrm{d}y\delta \\
&= -\frac{1}{6}(1-c_2)^3\delta.
\end{aligned}$$

Combining this with (10), we conclude that

$$\begin{aligned}
&\int_{c_2}^{1}(\psi(y)-c_2)f^{(2)}(y)\mathrm{d}y \\
= &\int_{c_2}^{1}(\psi(y)-(2y-1))f^{(2)}(y)\mathrm{d}y + \int_{c_2}^{1}(2y-1-c_2)f^{(2)}(y)\mathrm{d}y \\
< &\ 0 \quad \text{if } c_2 \text{ is close to 1,}
\end{aligned}$$

implying (8). *QED*

Calculations for figure 2. Optimal seller revenue of the second-price auction without second-chance offers is given by

$$\frac{1}{2} - c_1 + \frac{1}{32}(1+c_1)^4. \tag{14}$$

The figures shown in the middle panel require optimal seller revenue in the auction with second-chance offers. By (3) and (5), we can solve for the optimal seller revenue in the auction with second-chance offers, for each marginal cost pair (c_1, c_2) with $0 \leq c_1 \leq c_2 \leq 1$, by maximizing, with respect to r,

$$\Pi^{\text{sco}}(r,p) = \frac{1}{2} - c_1 + (1+c_1)r^3 - \frac{3}{2}r^4 - \lambda(1-p)^2(c_2 + 2c_2 p - 3p^2), \tag{15}$$

subject to $p = max\{r, c_2\}$ and evaluating it at optimized r. To trace out the solutions, start off with c_1 fixed and consider $c_2 = c_1$. From Corollary 3 we know that $r = \psi^{-1}(c_1)$ and $p = max\{c_2, \psi^{-1}(c_1)\}$ maximizes (15). Note also that for the case of F uniform on $[0,1]$, $\psi^{-1}(c_1) = \frac{1+c_1}{2} > c_1$. Hence, the initial solution has $r^* = \frac{1+c_1}{2}$ and $p = \frac{1+c_1}{2}$.

Let

$$r^{**} = argmax_r \frac{1}{2} - c_1 + (1+c_1)r^3 - \frac{3}{2}r^4 - \lambda(1-r)^2(c_2 + 2c_2 r - 3r^2). \tag{16}$$

and let

$$r^{+} = argmax_r \frac{1}{2} - c_1 + (1+c_1)r^3 - \frac{3}{2}r^4 - \lambda(1-c_2)^2(c_2 - c_2^2), \tag{17}$$

where the latter maximization is subject to the constraint $r^+ \leq c_2$. As c_2 increases we have to compare revenue $\Pi^{\text{sco}}(r^{**}, r^{**})$ to $\Pi^{\text{sco}}(r^+, c_2)$. However, for any $c_1 < c_2 \leq \frac{1+c_1}{2}$, we know that $r^+ = c_2$ (since $\Pi^{\text{sco}}(r, c_2)$ is increasing in r up to its maximum at $r = \frac{1+c_1}{2}$). But, (c_2, c_2) is a permissible solution to (16). Hence, $\Pi^{\text{sco}}(r^{**}, r^{**})$ is a valid solution for $c_2 \leq \frac{1+c_1}{2}$.

Next consider $c_2 > \frac{1+c_1}{2}$. Now we have to compare $\Pi^{\text{sco}}(r^+, c_2)$ to $\Pi^{\text{sco}}(r^{**}, r^{**})$ with $r^{**} > c_2$. If $\frac{1+c_1}{2} < c_2 \leq r^{**}$ and $\Pi^{\text{sco}}(r^{**}, r^{**}) \geq \Pi^{\text{sco}}(r^+, c_2)$, then $\Pi^{\text{sco}}(r^{**}, r^{**})$ is the solution. If $\frac{1+c_1}{2} < c_2 \leq r^{**}$ and $\Pi^{\text{sco}}(r^{**}, r^{**}) < \Pi^{\text{sco}}(r^+, c_2)$, then $\Pi^{\text{sco}}(r^+, c_2)$ is the solution. Finally, if $r^{**} > c_2$, then $\Pi^{\text{sco}}(r^+, c_2)$ is the solution.

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