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Imperfect Competition in Securities Markets with Diversely Informed
Traders

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Abstract

We show that the infinite regression problem in models with differentially informed traders can be solved using a fixed point method which we use to derive the dynamic equilibrium in a multi-auctions model with diversely informed traders. We find that when the informed traders' signals are not perfectly correlated, their private information will be revealed to the market gradually so that the market is only semi-strong form efficient and not strong-form efficient. Market depth in the continuous auctions model initially increases with time but decreases to zero at the end. Our results are in contrast to the results of Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) (HS-FV) who showed that when auctions occur frequently and informed traders have perfect information, the information is revealed to the market almost immediately. However, when the correlation in the private signals goes to 1, our model converges to the HS-FV model.

1 Introduction

A trader with long-lived information faces a dilemma as to whether to trade heavily in the short run in the hope of getting a good price before the information leaks out, or whether to spread his trades to minimize the price impact. Kyle (1985) develops a model in which a single privately informed trader with long-lived information optimally exploits his monopoly power over time. The informed trader, and noise traders who trade randomly,¹ submit order quantities to risk-neutral market makers who set the price equal to their expectation of the risky asset payoff. In equilibrium, the informed trader trades in a gradual manner so that his information is incorporated into the price at a linear rate, and in a continuous auction economy the expected profit of the informed trader is twice that in the single auction economy. The financial market in this model equilibrium is semi-strong form efficient but not strong-form efficient.

When there are more than one informed trader who trades strategically, competition between them causes information to be revealed more rapidly. Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) (HS-FV) have found that when there is more than one perfectly informed trader who trades strategically, there exists a unique linear equilibrium in which the informed traders trade very aggressively.² The entry of even one additional informed trader causes nearly all of the private information to be revealed to the market maker extremely rapidly so that the depth of the market becomes extremely large almost immediately. Consequently, a market with more than one informed trader is close to perfectly strong-form efficient, in that the security prices reflect virtually all available information.

HS-FV make the strong assumption that informed traders have perfect information. A more general assumption is that the private signals of the informed traders are noisy and less than perfectly correlated.³ However, while models of informed investors with diverse signals have been extensively studied in the literature on competitive noisy rational expectations equilibrium, their use in non-competitive dynamic trading models has been limited by recognition of an infinite regression problem⁴. In this paper, we develop a fixed point

¹Spiegel and Subrahmanyam (1992) replace the noise traders in the Kyle model with strategic, utility maximizing hedgers who trade to hedge endowment shocks.

²HS assumed normality of the risky asset value while FV allowed for elliptically contoured distributions. FV went on to show that the expected profits of the informed traders vanish as trading occurs more frequently.

³Even if the informed traders buy information from the same information seller, the information seller may want to add diversified noise to the signal to the information buyers. For a discussion of diversified signals and value of information, see Admati and Pfleiderer (1986).

⁴See Townsend (1983) and He and Wang (1993) for a discussion of the infinite regression problem in dynamic rational expectations models. Spiegel and Subrahmanyam discuss a model with diversely informed

method for solving the infinite regression problem and use it to analyze a model in which the information structure is symmetric, but informed traders receive different signals.

In a single auction model in which the informed traders have diverse signals and there are more than 3 informed traders, market depth decreases with the variance of the uncorrelated element of informed investors' private signals when that is small. Moreover, the profits of the informed traders do not vanish as their number increases to infinity. However, when the private signals of the informed traders are perfectly correlated, the total profits for the informed traders go to zero as the number of the informed traders increases.⁵

In a dynamic multiperiod auction model, each informed trader learns about the private information of other informed traders through the time series of prices and revise his expectation of the value of the risky asset accordingly. It is natural then to suppose that informed traders will restrict their trading in the early rounds in order to conserve their private information advantage, as in Kyle (1985). This intuition is confirmed in a linear symmetric equilibrium. We find that as long as the private signals of the informed traders are not perfectly correlated, they trade conservatively in order to limit the leakage of their private information into the price. The expected profits of the informed traders do not vanish as the market opens more frequently as happens when the private signals are perfectly correlated. Market depth initially increases over time but decreases towards the final auction. When auctions proceed continuously, all private information is revealed by the end.

The intuition for our results is that when traders have diverse information, the idiosyncratic error in each informed trader's private signal makes him act like a monopolist of his own private signal and causes him to trade more conservatively. For example, suppose that there are two informed traders who observe imperfectly correlated noisy signals and that there exists an equilibrium in which traders trade so aggressively in the first round that virtually all private information is incorporated into the price. We show in the following arguments that informed traders will deviate from such a proposed equilibrium. Since the first informed trader trades very aggressively in the first trading session, his information will be almost fully incorporated into the price after the first trading session. Then the second informed trader will be better off not to trade in the first session but wait until the later rounds where he can trade as an information monopolist. By not trading in the first round, the second trader manipulates the beliefs of both the first informed trader and the market maker. The market maker, believing that the price aggregates all private information almost perfectly (which is false when the second trader deviates from the equilibrium), will make the price insensitive to the order flow in later trading rounds. The first informed trader,

traders in a one period non-competitive securities market.

⁵See also Foster and Viswanathan (1993).

drawing the wrong inference from the price, has incorrect expectations about the value of the risky asset and believes that the price is very close to the conditional expectation given all private information. As a result, the first informed trader will trade a very small amount while the second informed trader will earn large profits from trading in later rounds in the off-equilibrium strategy, which breaks the proposed equilibrium. Consequently, informed traders trade conservatively so that prices incorporate the private information gradually.

However, if the two informed traders both observe a perfectly informative signal as in HS-FV,⁶ i.e., the value of the risky asset, the case will be different. In their perfect revealing equilibrium, if the second informed trader deviates by not trading in the first round, the second informed trader can manipulate the belief of the market maker but not the belief of the first informed trader. In the subsequent trading rounds the first informed trader again competes with the second informed trader on the same signal and drives both traders' profits from later rounds close to 0. Therefore, the proposed equilibrium is robust to this kind of deviation.⁷

Section 2 presents our single period model with diverse signals. Section 3 extends the model to a multiple-period setting. Section 4 discusses a limiting result when the number of auctions goes to infinity. Section 5 presents the numerical examples for both the discrete time model and the continuous auction model. Section 6 concludes the paper.

2 The Single Auction Model

In Subsection 2.1, we describe the economic structure of the model and discuss an infinite regression problem that has been identified by Townsend (1983). We provide a general solution for the infinite regression problem using a fixed point argument. In Subsection 2.2, we use the fixed point method to construct an equilibrium in a setting in which the information structure is symmetric.

⁶The same argument goes through when the two informed traders observe the same noisy signal. Both traders will trade very aggressively on the difference between price and the expectation of the risky asset value given the private signal

⁷However, the market maker will incur very large costs if the second trader deviates from the equilibrium by mistake.

2.1 The Economic Structure and the Infinite Regression Problem

Following Kyle (1985), a single risky asset is traded by three types of traders: risk neutral informed traders who possess private information about the liquidation value of the risky asset, v , liquidity traders whose demands are exogenous, and a competitive risk neutral market maker. The market maker absorbs the net demands that others trade and sets the price equal to the expected liquidation value of the risky asset given the order flow. We assume that each of M informed traders, $i, i = 1, \dots, M$, observes a signal of the form

$$z_i = v + \delta + \epsilon_i. \quad (1)$$

The total random demand by the noise traders is denoted u and v, δ, ϵ_i, u are normally and independently distributed with mean 0.⁸ The variances of v, δ, ϵ_i, u are $\sigma_v^2, \sigma_\delta^2, \sigma_{\epsilon_i}^2, \sigma_u^2$ respectively. The informed traders are assumed to be risk neutral.

In this model, informed traders have different but correlated private signals. This gives rise to a potential infinite regression problem, which may be thought of as follows. Consider the case of two informed traders. Let x_i denote the optimal trading strategy of informed trader i and $E_i[\cdot]$ denote his expectation. The first informed trader's optimal trading strategy, x_1 , depends on his own private signal, z_1 , and his expectation of market maker's price which depends on the total order flow. Therefore, x_1 depends on z_1 , and the first informed trader's expectation of the second investors' optimal trading strategy, $E_1[x_2]$. Similarly, the second informed trader's optimal trading strategy also depends on his own private signal, z_2 and his expectation about the first informed trader's optimal strategy, $E_2[x_1]$. This implies that the first informed trader's optimal trading strategy depends on his private signal, z_1 , his expectation of the second informed trader's private signal, $E_1[z_2]$, and his expectation about the second informed trader's expectation of his optimal strategy, $E_1[E_2[x_1]]$. Returning to the second informed trader, his optimal trading strategy now depends on $z_2, E_2[z_1], E_2[E_1[z_2]], E_2[E_1[E_2[x_1]]]$. In this way, there appears to be a problem of infinite regression.

This problem of infinite regression can be solved if we can show that there exists an equilibrium in which the form of each informed trader's demand function, expressed in his private information and public information, is common knowledge.⁹ Conforming to the

⁸The model can be easily extended to include more general correlation structure among v, δ, ϵ_i as long as the information structure among the informed traders is symmetric in which case the average of their private signals is a sufficient statistic of all private information.

⁹This approach is similar to that in Townsend (1983), p569-575.

literature on non-competitive trading, we restrict our analysis to linear Cournot equilibria in which each informed trader's equilibrium demand is optimal given others' equilibrium demand.¹⁰ We then show that the infinite regression problem reduces to a fixed point problem.

We conjecture that in equilibrium the optimal order of informed trader i , x_i , is given by¹¹

$$x_i = \beta_i z_i. \quad (2)$$

The aggregate order of the informed traders is denoted by $X = \sum_{i=1}^M x_i$ and in the conjectured equilibrium the market maker sets the price, p , according to the linear rule.

$$p = \lambda(X + u). \quad (3)$$

The vector $(\beta_1, \dots, \beta_M, \lambda)$ is common knowledge and, given the assumption that other informed traders follow the conjectured equilibrium strategy, informed trader i 's optimization problem is to maximize his expected profit given his private signal z_i . Let π_i denote the profit from trading for informed trader i , his expected profit from trading, $E[\pi_i|z_i]$, is given by

$$E[\pi_i|z_i] = E[x_i(v - p)|z_i] = E[x_i(v - \lambda\{x_i + \sum_{j \neq i} \beta_j z_j\})|z_i].$$

The first order condition of the problem is

$$E[v - \lambda(x_i + \sum_{j \neq i} \beta_j z_j)|z_i] - \lambda x_i = 0.$$

Since the prior means of v, δ, ϵ_i are 0, multivariate normality implies that both $E[v|z_i]$ and $E[z_j|z_i, j \neq i]$ are proportional to z_i as conjectured, therefore, the optimal demand of informed trader i is of the form given by (2) and β_i is a function of $\lambda, \beta_j, j \neq i$ written as

$$\beta_i = f_i(\lambda, \beta_1, \dots, \beta_{i-1}, \beta_{i+1}, \dots, \beta_M).$$

The market efficiency condition, that the market maker sets price equal to the expectation of the liquidation value of the risky asset given the order flow, gives (3) where λ is also a function of β_1, \dots, β_M written as

¹⁰There are potentially many Stackelberg equilibria, with multiple leaders and followers and there could also exist many non-linear equilibria.

¹¹We are implicitly using the condition that the prior mean of v is 0. If the prior mean is p_0 , the demand function should be $x_i = \beta_j(z_i - p_0)$, where p_0 is the prior mean of v .

$$\lambda = g(\beta_1, \dots, \beta_M).$$

The infinite regression problem then reduces to the problem of finding a fixed point of the mapping (f_1, \dots, f_M, g) that maps the Euclid space \mathbb{R}^{M+1} into itself.

A similar analysis carries through in a dynamic model with N auctions. We assume that the liquidity traders trade Δu_n at the n th auction and conjecture a linear equilibrium in which the optimal order of informed trader $i, \Delta x_{ni}, \forall i$ at period $n, n = 1, \dots, N$ is given by

$$\Delta x_{ni} = L_{ni}(z_i, p_1, \dots, p_{n-1}).$$

The total order flow ΔD_n at the n th auction is

$$\Delta D_n = \sum_{i=1}^M \Delta x_{ni} + \Delta u$$

and market maker sets the price according to

$$p_n = L_{nm}(\Delta D_1, \dots, \Delta D_n)$$

where $L_{ni}(\cdot), L_{nm}(\cdot)$ are linear functions in \mathbb{R}^n . What we need to show is that given the price function $L_{nm}(\cdot)$ and other informed traders' equilibrium demand functions, $L_{nj}(\cdot), j \neq i$, $L_{ni}(\cdot)$ is the optimal trading strategy for trader i at n th auction. Then, given the informed traders' equilibrium demand functions, $p_n = L_{nm}(\cdot)$ is the expected value of the risky asset for the market maker at the n th auction. The infinite regression problem again reduces to a fixed point problem.

In the next subsection, we use this approach to solve the infinite regression problem in the case where the information structure is symmetric. When the informed traders have a symmetric information structure, the $M + 1$ -dimensional fixed point problem is reduced to a 2-dimensional fixed point problem. Whether the fixed point problem can be solved where the information structure is asymmetric, and more generally, the necessary and sufficient condition for the fixed point problem described above to be solved are beyond the scope of this simple paper.

2.2 The Single Auction Model with Symmetric Diverse Signals

Throughout the analysis we set the variance $\sigma_{\epsilon_i}^2 = M\sigma_\epsilon^2$ so that the precision of the private signals is identical for each informed trader. To derive the equilibrium, it is convenient to

define a sufficient statistic of the informed traders' private information, Y , which is equal to the average of the private signals. Y is related to the liquidation value of the risky asset v and investor i 's private signal, z_i , as follows,

$$Y = \frac{\sum_{i=1}^M z_i}{M} = v + \delta + \frac{\sum_{i=1}^M \epsilon_i}{M}$$

$$E[v|Y] = \frac{\sigma_v^2}{\Sigma_0} Y = k_1 Y$$

$$E[Y|z_i] = \frac{\Sigma_0}{\Sigma_0 + (M-1)\sigma_\epsilon^2} z_i = s_0 z_i,$$

where

$$\Sigma_0 = \text{var}[Y] = \sigma_v^2 + \sigma_\delta^2 + \sigma_\epsilon^2$$

$$k_1 = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2 + \sigma_\delta^2}$$

$$s_0 = \frac{\Sigma_0}{\Sigma_0 + (M-1)\sigma_\epsilon^2}.$$

We set the variance of ϵ_i to be proportional to M so that the variance of the sufficient statistic of the private information does not change with M . In Theorem 1, we describe a linear symmetric Cournot equilibrium.

Theorem 1 *There exists an equilibrium in which the order of informed trader i , x_i , and the equilibrium price p are given by*

$$x_i(z_i) = \beta k_1 z_i, \quad X = \sum_{i=1}^M x_i, \quad P = \lambda(X + u) \quad (4)$$

where

$$\beta = \frac{s_0}{\lambda(1 + Ms_0)} \quad (5)$$

$$\lambda = \frac{Mk_1^2\beta\Sigma_0}{M^2k_1^2\beta^2\Sigma_0 + \sigma_u^2} \quad (6)$$

Proof: For trader i to maximize his expected profit, $E[x_i(v-p)|z_i]$, he has to estimate $v-p$ using his private information and the assumption that other informed traders follow the

equilibrium strategy. Now

$$\begin{aligned}
E[v - p|z_i] &= E[v - \lambda(x_i + \beta k_1 \sum_{j \neq i} z_j + u)|z_i] \\
&= E[E[v - \lambda M \beta k_1 Y|Y] - \lambda(x_i - \beta k_1 z_i)|z_i] \\
&= E[k_1 Y - M \beta \lambda k_1 Y - \lambda(x_i - \beta k_1 z_i)|z_i] \\
&= (1 - M \beta \lambda) k_1 s_0 z_i - \lambda(x_i - \beta k_1 z_i)
\end{aligned}$$

Let π_i denote the profit for informed trader i , his expected profit is $E[\pi_i|z_i] = E[x_i(v - p)|z_i]$, maximizing this with respect to x_i ; we get,

$$(k_1 - M \beta k_1 \lambda) s_0 z_i - \lambda(x_i - \beta k_1 z_i) - x_i \lambda = 0,$$

which is equivalent to

$$x_i = \frac{s_0}{\lambda(1 + M s_0)} k_1 z_i.$$

Therefore, β is given by (5). The market efficiency condition implies that λ is the regression coefficient of v on $\sum_{i=1}^M x_i + u$. Multivariate normality implies that the regression is linear, so we have

$$\lambda = \frac{M k_1^2 \beta \Sigma_0}{M^2 k_1^2 \beta^2 \Sigma_0 + \sigma_u^2}.$$

Q.E.D.

Solving (5) and (6) explicitly for β and λ , we have and λ

$$\begin{aligned}
\beta &= \sqrt{\frac{s_0}{M \Sigma_0}} \frac{\sigma_u}{k_1} \\
\lambda &= \frac{\sqrt{s_0 M \Sigma_0} k_1}{(1 + M s_0) \sigma_u} = \frac{\sigma_v^2}{\sigma_u} \frac{\sqrt{(\sigma_v^2 + \sigma_\delta^2)/M + \sigma_c^2}}{(1 + 1/M)(\sigma_v^2 + \sigma_\delta^2) + 2\sigma_c^2}.
\end{aligned} \tag{7}$$

In this market the ex ante profit of informed trader i before he gets his signal is

$$E[\pi_i] = E[x_i(v - p)] = \frac{1}{M} \lambda \sigma_u^2.$$

The total expected profits for the informed traders are

$$\Pi \equiv \sum_{i=1}^M \pi_i = \lambda \sigma_u^2, \tag{8}$$

where λ is given by (7).

The total expected profits of the informed traders depend only on λ and σ_u^2 . For simplicity, we analyze the comparative statics of λ , a monotonic transformation of $\ln \lambda$. We then have

$$\begin{aligned}\frac{d \ln \lambda}{d \sigma_c^2} &= M \frac{(M-3)(\sigma_v^2 + \sigma_\delta^2) - 2M\sigma_c^2}{2(\sigma_v^2 + \sigma_\delta^2 + M\sigma_c^2)[(M+1)(\sigma_v^2 + \sigma_\delta^2) + 2M\sigma_c^2]} \\ \frac{d \ln \lambda}{d \sigma_\delta^2} &= - \frac{(M+1)(\sigma_v^2 + \sigma_\delta^2) + 2M^2\sigma_c^2}{2(\sigma_v^2 + \sigma_\delta^2 + M\sigma_c^2)[(M+1)(\sigma_v^2 + \sigma_\delta^2) + 2M\sigma_c^2]} \\ \frac{d \ln \lambda}{d \sigma_v^2} &= \frac{\sigma_\delta^2 + \sigma_v^2/2}{\sigma_v^2(\sigma_v^2 + \sigma_\delta^2 + M\sigma_c^2)} + \frac{M\sigma_c^2[2M\sigma_c^2 + (M+1)\sigma_\delta^2 + 2\sigma_v^2]}{\sigma_v^2[(M+1)(\sigma_v^2 + \sigma_\delta^2) + 2M\sigma_c^2]} \\ \frac{d \ln \lambda}{d M} &= - \left(\frac{\sigma_v^2 + \sigma_\delta^2}{M} \right)^2 \frac{1 - 1/M}{2[(\sigma_v^2 + \sigma_\delta^2)/M + \sigma_c^2][(1 + 1/M)(\sigma_v^2 + \sigma_\delta^2) + 2\sigma_c^2]}.\end{aligned}$$

When $M > 3$, λ increases in the variance of the uncorrelated signal noise, σ_c^2 for small σ_c^2 . The maximum value is achieved when

$$\sigma_c^2 = \frac{M-3}{2M}(\sigma_v^2 + \sigma_\delta^2).$$

After achieving the maximum point, λ decreases with the variance of the uncorrelated signal noise.

In contrast, λ decreases monotonically with respect to M and σ_δ^2 and increases monotonically with σ_v^2 . As in Kyle (1985), the quantity $1/\lambda$ measures the depth of the market. The discussion above may be summarized by the following proposition for the market depth.

Proposition 1 *Market depth always increases with M and σ_δ^2 and decreases with σ_v^2 . When $M \leq 3$, market depth always increases with σ_c^2 . When $M > 3$, market depth initially decreases with σ_c^2 and then increases with σ_c^2 after reaching the maximum point at $\sigma_c^2 = (M-3)(\sigma_v^2 + \sigma_\delta^2)/(2M)$.*

The adverse selection problem faced by the market maker, decreases with σ_δ^2 , the variance of the correlated element of the error in the private signals, and increases with σ_v^2 , the variance of the liquidation value of the risky asset. Consequently market depth increases with σ_δ^2 and decreases with σ_v^2 . Competition between the informed traders causes more information to be revealed to the market and therefore increases the liquidity of the risky asset. As a result, market depth increases with the number of informed traders. However σ_c^2 , the variance of the uncorrelated element of the private signals affects both the adverse selection problem

faced by the market maker and the competition between the informed traders. On one hand, an increase in σ_ϵ^2 reduces the adverse selection problem faced by the market maker. On the other hand, it decreases the correlation between informed traders' private signals and lessens the competition between them. When M is small, the effect on reducing adverse selection dominates the effect on reducing competition and market depth always increases with σ_ϵ^2 . However, when M is large and σ_ϵ^2 small, the effect on reducing competition dominates the effect on reducing adverse selection, and market depth decreases with σ_ϵ^2 . Finally, for a given M , when σ_ϵ^2 is sufficiently large, the effect on adverse selection again dominates the effect on competition and market depth increases with σ_ϵ^2 .

The variance of the uncorrelated signal noise σ_ϵ^2 has a profound effect on the nature of the equilibrium as $M \rightarrow \infty$. Thus, when $\sigma_\epsilon^2 > 0$, λ converges to

$$\lambda = \frac{\sigma_\epsilon \sigma_v^2}{\sigma_u(\sigma_v^2 + \sigma_\delta^2 + 2\sigma_\epsilon^2)}.$$

Equation (8) shows that the total expected profits of the informed traders are proportional to λ . Therefore the total profits of the informed traders diminish to a positive limit for any strictly positive σ_ϵ^2 .¹² However, from equation (7), when $\sigma_\epsilon^2 = 0$

$$\lambda = \frac{\sqrt{M}\sigma_v^2}{\sigma_u} \frac{\sqrt{\sigma_v^2 + \sigma_\delta^2}}{(M+1)(\sigma_v^2 + \sigma_\delta^2)}.$$

Thus when M goes to infinity, λ decreases at the rate $\sqrt{1/M}$ when $\sigma_\epsilon^2 = 0$ and approach zero for large M . The private signal noise ϵ_i makes each informed trader an information monopolist and causes him to act strategically even when their number is large. It is only when $\sigma_\epsilon^2 = 0$ that they behave competitively and the price, p , fully reveals the private information in the limit.

¹²A related issue concerns the value of the private information and the source of the diverse element ϵ_i . Consider a highly risk averse information seller who commits not to trade in the risky asset and observes $v+\delta$. Suppose that the information seller can sell his information to M traders in the form $v+\delta+\epsilon_i$. In equilibrium, the payment to the information seller equals the total profits for the informed traders. Therefore, information seller maximize total profits for the information buyers and from equation (8), this is equivalent to maximize λ . By Proposition 1, for $M > 3$, information seller will add personalized noise to the traders and optimally set $\sigma_\epsilon^2 = (M-3)(\sigma_v^2 + \sigma_\delta^2)/(2M)$ and from equation (7), λ is given by $\sqrt{M}/[8(M-1)]\sigma_v^2/\sigma_u$. For $M \leq 3$, the information seller will sell the information as it is, i.e., set $\sigma_\epsilon^2 = 0$, and cause λ to be $\sqrt{M}/(M+1)\sigma_v^2/\sigma_u$. Clearly, λ decreases with M and λ is maximized when $M = 1$. Therefore, the information seller should sell his information to only one trader. For a similar analysis in a non-competitive market where information seller can only sell identical signals to the traders, see Admati and Pfleiderer (1988).

Consider finally the efficiency of the price in aggregating private information as measured by Σ/Σ_0 , the variance of Y given the price divided by the variance of Y before auction starts,

$$\Sigma/\Sigma_0 \equiv \text{var}[Y|X + u]/\text{var}[Y] = \frac{(\sigma_v^2 + \sigma_\delta^2)/M + \sigma_\epsilon^2}{(1 + 1/M)(\sigma_v^2 + \sigma_\delta^2) + 2\sigma_\epsilon^2}. \quad (9)$$

To determine how the idiosyncratic error, ϵ_i , in informed traders' private signals affects the efficiency of price in aggregating private information, we have

$$\frac{d\Sigma/\Sigma_0}{d\sigma_\epsilon^2} = \frac{(1 - 1/M)(\sigma_v^2 + \sigma_\delta^2)}{[(1 + 1/M)(\sigma_v^2 + \sigma_\delta^2) + 2\sigma_\epsilon^2]^2}.$$

For $M = 1$, the price always reveals half of the private information. For $M > 2$, the market will be less efficient in aggregating the private information when σ_ϵ^2 increases. This is because traders with less than perfectly correlated signals will not trade as aggressively as they would if they had perfectly correlated signals. In the limit for $\sigma_\epsilon^2 > 0$, when $M \rightarrow \infty$, from (9), Σ/Σ_0 is given by

$$\Sigma/\Sigma_0 = \frac{\sigma_\epsilon^2}{\sigma_v^2 + \sigma_\delta^2 + 2\sigma_\epsilon^2}.$$

Σ/Σ_0 is strictly positive for any $\sigma_\epsilon^2 > 0$.¹³ Therefore, as M goes to infinity, the sufficient statistic for the private information is not fully revealed to the market. However, when $\sigma_\epsilon^2 = 0$, (9) becomes

$$\Sigma/\Sigma_0 = \frac{1}{M + 1}.$$

Σ/Σ_0 goes to zero with the rate of $1/(M + 1)$. As M goes to infinity, all information are revealed and this explains why informed traders' profits goes to 0 in the limit.

When $\sigma_\delta^2 = \sigma_\epsilon^2 = 0$, in multiple auction models, Holden and Subrahmanyam (1992), and Foster and Viswanathan (1993) have demonstrated that the total expected profits to informed traders will go to 0 if the market opens more frequently in any given time interval. To examine how the noise of the informed traders' private signal affects their results, we extend our model to a multiple auction setting.

¹³Notice that this result is obtained when σ_ϵ^2 is fixed. If $\lim_{M \rightarrow \infty} \sigma_\epsilon^2 = 0$, i.e., $v + \delta$ can be perfectly estimated from the private information in the limit, then from (7) and (9), it is easy to show that both λ and Σ/Σ_0 converges to 0. $\sigma_\epsilon^2 = 0$ and be viewed as a special case of the condition $\lim_{M \rightarrow \infty} \sigma_\epsilon^2 = 0$. In this case the competitive and imperfectly competitive markets yield equilibria with identical informational properties in the limit. However, as long as $\lim_{M \rightarrow \infty} \sigma_\epsilon^2 > 0$, both λ and Σ/Σ_0 will be positive. See also Kyle (1989) for a model with limit orders.

3 The Multiple Auction Model

Continuing with the asset payoff and information structure described in Section 2, we now assume that there are N auctions starting at time 0 and ending at time 1. Following Kyle (1985), let Δt_n denote the time interval between the n th auction and the previous auction. Let Δu_n be the aggregate order submitted by noise traders at the n th auction. We assume that Δu_n is serially uncorrelated and normally distributed with mean zero and variance of $\sigma_u^2 \Delta t_n$. Let ΔX_n denote the total order submitted by the informed traders and Δx_{ni} denote the order submitted by informed trader i . Finally, let π_{ni} denote the total profits of the i th informed trader from positions acquired at all future auctions n, \dots, N . Trading takes place through competitive risk neutral market makers who observe the combined order flow $\Delta X_n + \Delta u_n$ and set the price equal to their expectation given the order flow. The market price of the risky asset at the n th auction is denoted p_n . Since the prior mean of v is 0, we set the price before the auctions start, p_0 , to be 0.

Let informed trader i conjecture that all other informed traders submit orders of the form $\beta_n(k_1 z_j - p_{n-1})$, $j \neq i$. Given this conjecture, we derive the optimal strategy of informed trader i , and show that he also submits an order of the form $\beta_n(k_1 z_i - p_{n-1})$. Since β_n is independent of i , the price which depends on the aggregate order flow is a noisy signal of the sufficient statistic of the private information. Due to the symmetric structure of the informed traders' private signals, each informed trader's estimation of the aggregate trades by the informed traders is reduced to the estimation of the sufficient statistic of the private signals in the market. Therefore, the optimal trading strategy of informed trader i depends only on the series of prices and his private signal. This permits a solution of the infinite regression problem. We show in the next theorem that a linear equilibrium exists in this model.

Theorem 2 *There exists a recursive linear symmetric equilibrium in which the demand for the informed traders and the market price are as described below*

$$\Delta x_{ni} = \beta_n(k_1 z_i - p_{n-1})\Delta t_n, \quad \Delta X_n = \sum_{i=1}^M x_{ni}, \quad \Delta p_n = \lambda_n(\Delta X_n + \Delta u_n) \quad (10)$$

$$\Sigma_n \equiv \text{var}[Y | \Delta x_1 + \Delta u_1, \dots, \Delta x_n + \Delta u_n] \quad (11)$$

$$E[\pi_{ni} | z_i, p_1, \dots, p_{n-1}] = \alpha_{n-1}(k_1 z_i - p_{n-1})^2 + \delta_{n-1} \quad (12)$$

$$s_{n-1} \equiv \frac{\Sigma_{n-1}}{\Sigma_{n-1} + (M-1)\sigma_\epsilon^2} \quad (13)$$

$$\beta_n \Delta t_n = \frac{s_{n-1} - \gamma_n \lambda_n}{\lambda_n [1 + (1 - \gamma_n \lambda_n) s_{n-1} M]} \quad (14)$$

$$\alpha_{n-1} = \beta_n s_{n-1} \Delta t_n (1 - \lambda_n M \beta_n \Delta t_n) + \alpha_n (1 - \lambda_n M \beta_n s_{n-1} \Delta t_n)^2 \quad (15)$$

$$\gamma_{n-1} = [\gamma_n + \beta_n \Delta t_n (1 + \{1 - \gamma_n \lambda_n\} s_{n-1} M)] [1 - \beta_n \Delta t_n \lambda_n (M - 1)] (1 - M \beta_n \Delta t_n \lambda_n) \quad (16)$$

$$\eta_{n-1} = \frac{[1 - (M - 1) \beta_n \Delta t_n \lambda_n]^2}{4 \lambda_n (1 - \eta_n \lambda_n)} \quad (17)$$

$$\lambda_n = M k_1^2 \Sigma_n \beta_n / \sigma_u^2 \quad (18)$$

$$\Sigma_n = (1 - M \beta_n \Delta t_n \lambda_n) \Sigma_{n-1} \quad (19)$$

$$\delta_{n-1} = \delta_n + \alpha_n \lambda_n^2 \Delta t_n \sigma_u^2 + \alpha_n \lambda_n^2 M^2 (M - 1) \beta_n^2 k_1^2 s_{n-1} \sigma_c^2 \Delta t_n^2 \quad (20)$$

for all auctions $n = 1, \dots, N$ and for all informed traders $i = 1, \dots, M$, subject to the boundary conditions

$$\alpha_N = 0, \quad \gamma_N = 0, \quad \eta_N = 0, \quad \delta_N = 0 \quad (21)$$

$$\Sigma_N = \frac{\Sigma_{N-1}}{1 + M s_{N-1}}, \quad \lambda_N = \frac{k_1 \sqrt{s_{N-1} \Sigma_{N-1} M}}{\sigma_u \sqrt{\Delta t_N (1 + M s_{N-1})}}, \quad \beta_N = \sqrt{\frac{s_{N-1}}{M \Delta t_N \Sigma_{N-1}}} \frac{\sigma_u}{k_1} \quad (22)$$

and the second order condition¹⁴

$$\lambda_n (1 - 2 \eta_n \lambda_n) > 0 \quad (23)$$

Proof: Consider a symmetric linear equilibrium, let $I_{n,i}$ denote the information set of informed trader i before the n th auction, and F_n denote the information set of the market maker at the n th auction. First notice that in the proposed equilibrium, $\Delta X_n + \Delta u_n = M \beta \Delta t_n (k_1 Y - p_{n-1}) + \Delta u_n$, so that the aggregate order flow is a noisy signal of the sufficient statistic of the private signals Y . Given the trading strategies of the informed traders, the market maker set the price $p_n = E[v|F_n] = E[E[v|Y]|F_n]$, which is linear in the aggregate order flow due to the assumption of multivariate normality. From the n th auction price, informed trader i can estimate the total expected trading by other informed traders in the next trading session through his updated estimate of Y . Since the only variable informed trader i needs to estimate is Y , the infinite regression problem is easily solved in this model.

Given the proposed equilibrium described in the theorem, each informed trader uses price to update his estimate of Y , informed trader i may have an incentive to deviate from the proposed equilibrium so that other traders estimate the wrong Y from the price, while trader i who estimates Y correctly gains an advantage over other traders. Therefore it remains to

¹⁴The parameters γ_n, η_n relate to the off-equilibrium expected profits for the representative informed trader and is discussed in the following proof.

be shown that informed traders have no incentive to deviate from the equilibrium. This is shown in Appendix A.

By the market efficiency assumption of market makers, λ_n is a regression coefficient of v on $\Delta X_n + \Delta u_n$, given I_n . Normality assumption implies that

$$\lambda_n = \frac{M \beta_n k_1^2 \Sigma_{n-1}}{M^2 k_1^2 \beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} \quad (24)$$

and

$$\Sigma_n = \frac{\sigma_u^2 \Sigma_{n-1}}{M^2 \beta_n^2 k_1^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} \quad (25)$$

(18), (19) can be derived from equations (24), (25). Boundary condition (21) essentially means that there will be no more profits for the informed traders after the auction is completed. Boundary condition (22) can be derived in a single auction model presented in subsection 2.2. The second order condition (23) is derived in Appendix A.

Q.E.D.

In the next proposition, we provide a procedure to solve the recursive system described above. The technique is similar to the recursive method used in Holden and Subrahmanyam (1992). Let $q_n \equiv \gamma_n \lambda_n$. we have

Proposition 2 *The recursive system of the equilibrium described in Theorem 2 can be solved starting from $q_N = 0$ and a conjecture of s_{N-1} and iterating backward for $q_{N-1}, \dots, q_1, s_{N-2}, \dots, s_0$ using the following equations.*

$$M(s_{n-1} - q_{n-1})s_{n-2}^2 + [1 - M(s_{n-1} - q_{n-1})]s_{n-2} - s_{n-1} = 0 \quad (26)$$

$$M s_{n-2} \left(\frac{\Delta t_{n-1}}{\Delta t_n} \right) q_{n-1}^3 - (1 + M s_{n-2}) \left(\frac{\Delta t_{n-1}}{\Delta t_n} \right) q_{n-1}^2 - C_n q_{n-1} + C_n s_{n-2} = 0 \quad (27)$$

where

$$C_n = \frac{s_{n-1}^2 [1 + M q_n (1 - s_{n-1})] [1 + s_{n-1} + (M - 1 - M s_{n-1}) q_n]^2}{(s_{n-1} - q_n) [1 + (1 - q_n) M s_{n-1}]^2}$$

If the resulting s_0 derived from the recursive solutions is different from the initial condition, s_{N-1} is revised until the s_0 derived from the recursive solutions is close to the initial condition.

Given the solution of $q_n, s_{n-1}, n = 1, \dots, N$, each of the following variables can be obtained by the following expressions.

$$\Sigma_n = \frac{(M - 1) \sigma_\epsilon^2 s_n}{1 - s_n} \quad (28)$$

$$\lambda_n = \left(\frac{M k_1^2 \Sigma_n (s_{n-1} - q_n)}{\Delta t_n \sigma_u^2 [1 + (1 - q_n) M s_{n-1}] } \right)^{\frac{1}{2}} \quad (29)$$

$$\beta_n = \frac{\lambda_n \sigma_u^2}{M k_1^2 \Sigma_n} \quad (30)$$

4 A Limiting Result

When informed traders have perfect information about the liquidation value of the risky asset v , HS-FV have shown that the market approaches strong form efficiency as trading occurs frequently. We have argued informally in the introduction that when informed traders have idiosyncratic errors in their signals, they would prefer to trade smoothly so that price gradually aggregates private information. For each informed trader, the idiosyncratic error in his signal causes him to act like a monopolist of his own signal as in Kyle (1985). As shown by the numerical results in the next Section, when $N \rightarrow \infty$, λ_n and Σ_n converge to smooth functions of calendar time t . We now examine the analytic limits of the above recursive system when the number of auctions goes to infinity. We set $t_n = n/N$. Let $\Sigma_n, \lambda_n, \beta_n, \gamma_n, \delta_n, \alpha_n, s_{n-1}$ be defined as continuous function $\Sigma(t), \lambda(t)$, etc., by the convention $\lambda(t) = \lambda_{n-1}$ for all $t \in [t_{n-1}, t_n)$, etc. We have the following theorem:

Theorem 3 *Consider a sequence of sequential equilibria such that $\max_n |\Delta t_n| \rightarrow 0$. Then the limiting values of $\lambda(t), \beta(t), \Sigma(t), \alpha(t), \delta(t)$ are given by*

$$\lambda(t) = \frac{k_1}{\sigma_u} \sqrt{a} \Sigma(t)^{2-2/M} e^{(1-1/M)\sigma_\epsilon^2/\Sigma(t)} \quad (31)$$

$$\beta(t) = \frac{\sigma_u}{M k_1} \sqrt{a} \Sigma(t)^{1-2/M} e^{(1-1/M)\sigma_\epsilon^2/\Sigma(t)} \quad (32)$$

$$\int_{\Sigma(t)}^{\Sigma_0} \sigma^{(4/M-4)} e^{2(1/M-1)\sigma_\epsilon^2/\sigma} d\sigma = at \quad (33)$$

$$\alpha(t) = \frac{\sigma_u}{M k_1 \sqrt{a}} \int_0^{\Sigma(t)} \frac{[\sigma + (M-1)\sigma_\epsilon^2] e^{(1/M-1)\sigma_\epsilon^2/\sigma}}{[\Sigma(t) + (M-1)\sigma_\epsilon^2]^2 \sigma^{2-2/M}} d\sigma \quad (34)$$

$$\delta(t) = k_1^2 \int_0^{\Sigma(t)} \alpha(\sigma) d\sigma = \frac{k_1 \sigma_u}{M \sqrt{a}} \int_0^{\Sigma(t)} \frac{[\Sigma(t) - \sigma] e^{(1/M-1)\sigma_\epsilon^2/\sigma}}{[\Sigma(t) + (M-1)\sigma_\epsilon^2] \sigma^{2-2/M}} d\sigma, \quad (35)$$

where

$$a = \int_0^{\Sigma_0} \sigma^{4/M-4} e^{2(1/M-1)\sigma_\epsilon^2/\sigma} d\sigma.$$

Proof: We take it as given that in the limit $N \rightarrow \infty$, the limit function $\Sigma(t), \lambda(t)$, etc., converges and is differentiable almost everywhere.

From equations (14)-(20) we obtain

$$s_{n-1} - \gamma_n \lambda_n = O(\Delta t_n) \quad (36)$$

$$\frac{\alpha_n - \alpha_{n-1}}{\Delta t_n} = \beta_n s_{n-1} (2\alpha_n \lambda_n M - 1) + O(\Delta t_n) \quad (37)$$

$$\frac{\gamma_n - \gamma_{n-1}}{\Delta t_n} = \beta_n [M s_{n-1} (\gamma_n \lambda_n - 1) - 1 + (2M - 1) \gamma_n \lambda_n] + O(\Delta t_n) \quad (38)$$

$$1 - 2\eta_n \lambda_n = O(\sqrt{\Delta t_n}) \quad (39)$$

$$\frac{\delta_n - \delta_{n-1}}{\Delta t_n} = -\alpha_n \lambda_n^2 \sigma_u^2 \quad (40)$$

$$\frac{\Sigma_n - \Sigma_{n-1}}{\Delta t_n} = -M \beta_n \lambda_n \Sigma_{n-1} \quad (41)$$

Standard convergence results for converting difference equations into differential equations allow us to conclude that the solution for the continuous time limit functions should satisfy.¹⁵

$$s(t) = \gamma(t) \lambda(t) \quad (42)$$

$$\alpha'(t) = \beta(t) s(t) [2\alpha(t) \lambda(t) M - 1] \quad (43)$$

$$\gamma'(t) = \beta(t) [M s(t) \gamma(t) \lambda(t) + (2M - 1) \gamma(t) \lambda(t) - M s(t) - 1] \quad (44)$$

$$2\eta(t) \lambda(t) = 1 \quad (45)$$

$$\delta'(t) = -\alpha(t) \lambda(t)^2 \sigma_u^2 \quad (46)$$

$$\Sigma'(t) = -M \beta(t) \lambda(t) \Sigma(t) \quad (47)$$

The equations (13), (18) in the limit become

$$s(t) = \frac{\Sigma(t)}{\Sigma(t) + (M - 1) \sigma_c^2} \quad (48)$$

$$\lambda(t) = M k_1^2 \Sigma(t) \beta(t) / \sigma_u^2 \quad (49)$$

¹⁵The difference equation system around $t = 1$ behave so badly that standard convergence theorems cannot be applied. Nevertheless, for all $\epsilon > 0$, the difference system behaves well in $[0, 1 - \epsilon]$ and converges to the differential equations (47)-(54). We then take ϵ to 0 and obtain the differential equations in the limit.

As shown in appendix C, the system of differential equations (47)-(54) can also be derived directly in a continuous-time auction model. In appendix D, we solve for the differential equations to yield

$$\Sigma'(t) = -a\Sigma(t)^{4-4/M}e^{2(M-1)\sigma_c^2/M\Sigma(t)}. \quad (50)$$

Use the boundary condition $\Sigma(1) = 0$,¹⁶ equation (50) can be solved directly, yielding a closed form solution (33).

$\lambda(t), \beta(t)$ can then be obtained by solving the simultaneous equation (37), (39), which gives (31), (32). (43) now becomes the standard first order Bernoulli equation, and we get (34). Finally, integration of (44) and a little algebra gives the expression for $\delta(t)$ in (35). of $\Sigma(t)$

Q.E.D.

Combining (39) and (40) and evaluate before informed traders receive their private signals at time 0, we get the total ex ante profits for the informed traders

$$\Pi \equiv E\left[\sum_{i=1}^M \pi_i(0)\right] = \frac{k_1 \sigma_v}{\sqrt{a}} \int_0^{\Sigma_0} \sigma^{2/M-2} e^{(1/M-1)\sigma_c^2/\sigma} d\sigma. \quad (51)$$

Notice that from the expression of the integration coefficient a in Theorem 3, it is clear that when $M > 1, \sigma_c^2 = 0$, a goes to infinity and the solution is not well defined. This corresponds to HS-FV's results.

When $M = 2$, (31), (32) and (33) can be simplified to obtain explicit expressions of $\lambda(t), \beta(t), \Sigma(t)$,

$$\lambda(t) = \frac{\sigma_v^2}{\sigma_u \sqrt{1-t} \sigma_c^2 - \Sigma_0 \ln(1-t)} \frac{\sigma_c}{\sigma_c} \quad (52)$$

$$\beta(t) = \frac{\sigma_u \Sigma_0}{2\sigma_v^2 \sigma_c \sqrt{1-t}} \quad (53)$$

$$\Sigma(t) = \frac{\Sigma_0 \sigma_c^2}{\sigma_c^2 - \Sigma_0 \ln(1-t)}. \quad (54)$$

Notice that $\beta(t)$ is inversely proportional to σ_c . $\beta(t)$ measures the aggressiveness of the trading strategy of the informed traders. When σ_c is small, the informed traders trade very

¹⁶For $M > 2$, the rationale for this boundary condition is the following: (i), numerical examples show that when $N \rightarrow \infty$, Σ_N will converge to 0; (ii), if we denote the boundary condition to be $\Sigma(1) = \theta$, then the ex ante informed trader's profit decreases in θ . Therefore, every informed trader will agree to trade according to the equilibrium strategy where $\Sigma(1) = 0$; (iii), only the boundary condition $\Sigma(1) = 0$ is consistent with the boundary condition $\gamma(1) = 0$. The case where $M = 1$ is described in Kyle (1985).

aggressively all the time. As a result, when $\sigma_c \rightarrow 0$, price aggregates private information very efficiently. Therefore $\Sigma(t)$ goes to zero and market depth goes to infinity for all $t > 0$.

5 Characterization of the Equilibrium

In this section we present numeric results of the mutiperiod auction model in Figures 1-6 and numeric results of the continuous auction model in Figures 7-15.

As in Kyle (1985), the parameters Σ_n and λ_n are inverse measures of price efficiency and market depth, respectively. To compare the case with diverse signals and the case with perfectly correlated signals we present a series of numerical examples. We assume that $\Sigma_0 = 1, \sigma_c^2 = 0.5, \sigma_u^2 = 1, \sigma_f^2 = 0$ and $\Delta t_n = 1/N, n = 1, \dots, N$ unless otherwise stated.

Figure 1 plots λ_n for the cases of $N = 2, 4, 8, 16, 64, \infty$. M is set at 2 in Figure 1. As can be seen from Figure 1, λ_n initially decreases with time but then increases with time at the end of the auction. The curve at $N = \infty$ is plotted using the limiting expression of λ_n in the last section. It is clear that the discrete time solution quickly converges to the continuous time solution.

Figure 2 plots Σ_n for the cases of $N = 2, 4, 8, 16, \infty$. Notice that Σ_n quickly converges to the continuous auction solution as N increases. Moreover, at $N = \infty$, the Σ_n starts as a convex curve but becomes concave near the end of auction. This is due to the fact that near the end of the auction, each informed trader's private signal becomes highly correlated with the private signals of other informed traders. Competition between the informed traders then causes the traders to trade very aggressively and increases the rate of reduction of Σ_n .

Figures 3 and 4 plot λ_n and Σ_n respectively for the cases of $M = 1, 2, 4, 20$ fixing the number of auctions N at 256. As M increases, both λ_n and Σ_n initially decrease faster. However, λ_1 increases with M while $\lambda_{N/2}$ decreases with M . Notice that Σ_n at $M = 4$ is not much different from that at $M = 20$. As will be shown later, in the continuous auction model, when $M \rightarrow \infty$, both λ and Σ will converge to a smooth curve.

Figures 5 and 6 demonstrate the effect of changing σ_c^2 on λ_n and Σ_n . We set $N = 256$, $\Sigma_0 = 1$, $\sigma_c^2 = 0, 0.02, 0.1, 0.5$ respectively. Notice that λ_1 decreases with σ_c^2 while $\lambda_{N/2}$ increases with σ_c^2 . At $\sigma_c^2 = 0.02$, λ_n starts very high but quickly decreases to zero. Similarly at $\sigma_c^2 = 0.02$, Σ_n goes to zero very quickly. These results indicate that when $\sigma_c^2 \rightarrow 0$, our results converge to HS-FV's results at $\sigma_c^2 = 0$.

Figures 7 and 8 give the graph of $\Sigma(t), \lambda(t)$, for different M . Imperfect competition causes $\lambda(t)$ to rise above the monopolistic case near the beginning and the end of the auction and fall below the monopolistic case in the middle of the auction. Since $\lambda(t)^2 \sigma_u^2 dt$ also measures the expected loss of liquidity traders between time t and $t + dt$, Figure 7 indicates that the expected loss of liquidity traders is higher in the two ends of the auction and lower in the middle of the auction when private information spreads evenly among more than one informed trader. Notice that when $M \rightarrow \infty$, due to the assumption that the total private information remains constant, the liquidity parameter $\lambda(t)$ and the variance of the remaining private information $\Sigma(t)$ decrease smoothly over time.

Figures 9 and 10 give the graphs of $\Sigma(t)$ and $\lambda(t)$ when $M = 2$ with different σ_ϵ^2 . In these figures, we set $\Sigma_0 = 1, \sigma_\epsilon^2 = 0.02, 0.1, 0.5$. Notice that $\Sigma(t)$ goes down faster when σ_ϵ^2 is smaller. Moreover, $\lambda(0)$ is proportional to the inverse of $1/\sigma_\epsilon$ while for t sufficiently large $\lambda(t)$ decrease with σ_ϵ^2 . When $t = 0$, From (52), (54), we have $\Sigma(0) = \Sigma_0, \lambda(0) = \sigma_v^2 / \sigma_\epsilon \sigma_u$. However, for any t strictly positive, when $\sigma_\epsilon^2 \rightarrow 0, \Sigma(t) \rightarrow 0, \lambda(t) \rightarrow 0$.¹⁷ Our result is consistent with the results in HS-FV, where they showed in a discrete time model, $\lambda_1 \rightarrow \infty, \lambda_n \rightarrow 0, \Sigma_n \rightarrow 0$, when $N \rightarrow \infty$, and $n/N > \tau$ for any $\tau > 0$.

The effects of σ_ϵ^2 on $\lambda(0), \lambda(t), \Sigma(t)$ is illustrated in Figures 11 and 12. In Figures 11 and 12, we set $M = 2, \sigma_v^2 = 1$, and draw the graph of $\lambda(0), \lambda(0.5)$ with respect to σ_ϵ^2 . Since at $\sigma_\epsilon^2 = 0$ the solution is not well defined, we set the value at $\sigma_\epsilon = 0$ to be the limit of our solutions when $\sigma_\epsilon^2 \rightarrow 0$. Clearly, $\lambda(0)$ decreases with σ_ϵ^2 , and $\lambda(0.5)$ first increases with σ_ϵ^2 , and then decreases with σ_ϵ^2 . On the contrary, as shown in Figure 12, $\Sigma(0.5)/\Sigma_0$ starts at 0 and increases monotonically with σ_ϵ^2 .

In Figures 13-15, we set $\sigma_v^2 = 1$ and examine the effects of σ_ϵ^2 on the total expected profits of informed traders from trading. In Figure 13, we compare the total expected profits of the informed traders for $M = 1, 2, 4, \infty$. For $M = 1$, $\Pi(0)$ always decreases with σ_ϵ^2 , while for $M > 1$, $\Pi(0)$ starts at 0 and initially increases with σ_ϵ^2 and then decreases with σ_ϵ^2 . Notice that $\Pi(0)$ decreases with M but even at $M = \infty$, $\Pi(0)$ is finite for all strictly positive σ_ϵ^2 . The total profits for the informed traders decreases with M . It would be better for the informed traders to form a mutual fund and trade as a group with all of their private information if the informed traders agree not to trade on their own. When σ_ϵ is large, the total profits for the case with $M = 1$ is very close to the case where $M = \infty$. Therefore,

¹⁷More generally, for $M > 1, t > 0$, it can be shown that When $\sigma_\epsilon^2 \rightarrow 0, \lambda(0) = O(\sigma_\epsilon^{4/M-3}), \lambda(t) = O(\sigma_\epsilon), \Sigma(t) = O(\sigma_\epsilon^2)$. Moreover, $\beta(0) = O(\sigma_\epsilon^{4/M-3}), \beta(t) = O(\sigma_\epsilon^{-1})$. This implies that, when σ_ϵ is small, each informed trader will trade very aggressively on the difference between the expected value of the risky asset conditional only on his own signal and the expected value of the risky asset conditional only on the public signal extracted from the price all the time.

when the informed traders have information of low quality, the incentive for them to form a mutual fund is very weak. Let Π^I denote their profits when they trade individually and Π^M denote their profits when they pool their information together and trade through a mutual fund. Figure 14 illustrates the ratio of Π^I over Π^M with respect to σ_ϵ^2 , for $M = \infty$. This ratio starts at zero and quickly increases to 0.91 at $\sigma_\epsilon^2=2$ and stabilizes for larger σ_ϵ^2 .

Finally, we compare the expected profits of the informed traders in the single auction model and the continuous auction model at $M = 2, \sigma_v^2 = 1$. Let Π^c and Π^s denote the total expected profits from trading in the continuous auction and the single auction economy respectively. As illustrated in Figure 15, when σ_ϵ^2 is small, the informed traders earn higher expected profits in the single auction economy. However, for σ_ϵ^2 sufficiently large, the informed traders' expected profits are higher in the continuous auction economy. This is different from the monopolist case where in the continuous auction economy the informed trader always earns twice of the expected profit he gets in the single auction economy.

6 Conclusion

We have shown that when informed traders' private signals are not perfectly correlated, their total expected profits will not vanish when market opens frequently. Furthermore, informed traders trade smoothly in the beginning of the auctions and the private information is incorporated into prices gradually. Market depth initially increases but decreases dramatically in the end. This is in contrast to the results of HS(1992)-FV(1993). They showed that, when the informed traders know the exact value of the risky asset payoff and trading occurs frequently with a constant influx of liquidity traders, the informed traders will compete so aggressively in the first auction that almost all the private information will be revealed instantaneously. In this sense, the market is strong-form efficient in continuous auctions. The results presented in this paper indicates that the HS-FV result are sensitive to the assumption that the informed investors' signals are perfectly correlated. When the variance of the uncorrelated element $\epsilon_i, M\sigma_\epsilon^2$, goes to zero, our results converge to HS-FV's results.¹⁸ There are many extensions of the current work. We have assumed that the informed traders receive private signals only in the first period and there are no public signals

¹⁸The results in this paper are also sensitive to the assumption that each informed trader is the only recipient of his private signal. Consider a model in which there are M private signals as described in this paper. But there are MI Informed traders ($I > 1$) and each of the M private signals is shared by I different informed traders. Then it can be shown numerically that when trading occurs frequently, all private information is revealed to the market almost immediately. This indicates that when there is competition on exactly the same signal, the information content in that signal will be revealed very rapidly.

besides the price. It is straightforward to extend our model to the case in which public signals are released before trading starts and informed traders receive private signals of the same precision in every period.¹⁹ An alternative model structure is to let informed traders have perfect information of different elements of the risky asset.²⁰ Another extension of the current paper is to let the informed traders be risk averse. In our solution of continuous trading, the price become extremely sensitive to the order flow at the end of the auction. This is due to the risk-neutrality of the informed traders, which implies that they may take very large positions in the risky asset.²¹ Near the end of the auction, the private signals of informed traders become highly correlated with the signals of other informed traders' and they will compete very aggressively to extract profits from the remaining information. When informed traders are risk averse, they will not take very large positions in the risky asset. As a result, when trading occurs frequently, the price will not reveal all the private information and the price will not be very volatile near end of the auction.²² Moreover, the current work may be extended to more general distributions of the risky asset value and to include limit orders in the informed traders' strategy.²³

In this paper, as in most existing literature, we assume that the informed traders receives privated signals at the same time.²⁴ It would be interesting to determine how the timing of private information acquisition is going to affect the trading strategy of the informed traders. In the continuous trading economy, consider the case of two informed traders in which the early informed trader acquires a private signal at time 0 and the late informed

¹⁹He and Wang (1993), and Brennan and Cao (1994) have developed such models in a competitive noisy rational expectations framework.

²⁰For example, we can assume that the asset payoff is given by $v = \sum_{i=1}^M v_i$, where v_i is identical and independently normally distributed and each v_i is observed by informed trader i . However, we can write $Mv_i = v + (M-1)v_i - \sum_{j \neq i} v_j = v + \epsilon_i$ where $\epsilon_i \equiv (M-1)v_i - \sum_{j \neq i} v_j$. Notice that ϵ_i is independent of v . Therefore, this extension is a variant of the model presented here except that ϵ_i s are correlated among the informed traders. The results obtained in the two models are essentially the same.

²¹In the continuous auction economy, let dX denote the total quantity of informed trading and $E[dX/dt|Y]$ denote the total expected rate of informed trading. Then it can be show that $E[dX/dt|Y] = \sigma_u \sqrt{a} \Sigma(t)^{1-1/M} e^{(1-1/M)\sigma_u^2/\Sigma(t)} / M \Sigma_0^{1/M}$. For $M = 1$, the expected rate of informed trading is a constant. However, for $M > 1$, the total expected rate of informed trading initially decreases and then increases to infinity near the end of auction.

²²The intuition that market depth initially increases due to the reduction of adverse selection, as private information is revealed through the time series of prices, and decreases near the end of the auction due to more intensive competition between the informed traders still applies to the model with risk averse informed investors. It is likely that for informed traders with small risk aversion, $\lambda(t)$ is non-monotonous in t .

²³Foster and Vishwanathan (1993) extended the model to the class of elliptically contoured distributions and Back (1992) extended the continuous time version of the Kyle (1985) model to general distributions of the risky asset value. Rochet and Vila (1994) discussed an extension of Kyle (1989) model in which traders submit limit orders to general distributions of the risky asset value.

²⁴An exception is provided by Hirshleifer, Subrahmanyam, and Titman (1993).

trader acquires a private signal at time $1/2$. When the two traders observe the same signal, clearly the optimal strategy is for the early informed trader to reveal the signal at time $1/2$ since the late informed trader will compete with the early informed trader to reveal the private signal instantaneously in later trading rounds. However, if the two informed traders observe different signals, it is likely that the early informed trader will not trade so aggressively as to reveal his signal at time $1/2$ since he wants to earn additional profits from the noisy traders in later trading rounds. The intuition that the informed traders will trade less competitively when their signals are less correlated still applies in this case.

In order to make the analysis of our paper tractable, we have made the assumption that all the informed traders' signals have the same precision. It would be worthwhile to analyze the model in which there is asymmetry in the structure of informed traders' private signals. In principle, the fixed point technique described in Section 2.1 may offer a general solution to the infinite regression problem. However, even if the solution to the fixed point problem exists, it would probably be difficult to find the fixed point in multiple trading sessions or extend the result to continuous time trading.

Appendix A

Suppose that informed trader i follows a strategy different the proposed equilibrium strategy. In this case the off-equilibrium derived profit function will differ from equation (15) in the proposed equilibrium. Suppose that trader i trades $\bar{x}_{ni}, \forall n$, and denote the resulting prices by $\bar{p}_n, \forall n$. Since every trader agree on the prior mean of the risky asset, we set $\bar{p}_0 \equiv p_0 = 0$. Other informed traders still follow the equilibrium strategy and their demand functions are denoted $\Delta \bar{x}_{nj} = \beta_n \Delta t_n (k_1 z_j - \bar{p}_{n-1})$. We conjecture that the off-equilibrium derived profit function for informed trader i is described by the following equation.

$$E\{\bar{\pi}_{(n+1)i}|F_{ni}\} = \alpha_n (k_1 z_i - p_n)^2 + \gamma_n (k_1 z_i - p_n)(p_n - \bar{p}_n) + \eta_n (p_n - \bar{p}_n)^2 + \bar{\delta}_n. \quad (55)$$

To prove this conjecture, we proceed by backward induction,

$$\begin{aligned} E[\bar{\pi}_{ni}|F_{(n-1)i}] &= \max_{\Delta \bar{x}_{ni}} E\{(v - \bar{p}_n)\Delta \bar{x}_{ni} + \alpha_n (k_1 z_i - p_n)^2 \\ &\quad + \gamma_n (k_1 z_i - p_n)(p_n - \bar{p}_n) + \eta_n (p_n - \bar{p}_n)^2 + \bar{\delta}_n | F_{(n-1)i}\}. \end{aligned}$$

From the proposed equilibrium, we have

$$\Delta \bar{p}_n \equiv \bar{p}_n - \bar{p}_{n-1} = \lambda (\Delta \bar{x}_{ni} + \sum_{j \neq i}^M \Delta \bar{x}_{nj} + \Delta u_n).$$

In order to form optimal demands, the deviating trader need to estimate the random variables $v, \Delta p_n$ and $\Delta \bar{p}_n$ assuming that others follow the equilibrium strategy.

$$\begin{aligned} E[\Delta p_n | F_{(n-1)i}] \lambda_n \sum_{i=1}^M \Delta x_{ni} &= E[\lambda_n \beta_n \Delta t_n \{M k_1 Y - M p_{n-1}\} | F_{(n-1)i}] \\ &= \lambda_n \beta_n \Delta t_n M s_{n-1} (k_1 z_i - p_{n-1}) \end{aligned}$$

$$\text{var}[\Delta p_n | F_{(n-1)i}] = \lambda_n^2 \sigma_u^2 \Delta t_n + \lambda_n^2 \beta_n^2 M^2 k_1^2 \Sigma_{n-1} \Delta t_n^2$$

$$\begin{aligned} E[\Delta \bar{p}_n | F_{(n-1)i}] &= \lambda_n (\Delta \bar{x}_{ni} + \sum_{j \neq i} \Delta \bar{x}_{nj}) \\ &= \lambda_n (\Delta \bar{x}_{ni} + E[\beta_n \Delta t_n (M k_1 Y - k_1 z_i - (M-1)\bar{p}_{n-1}) | F_{(n-1)i}]) \\ &= \lambda_n [\Delta \bar{x}_{ni} + \beta_n (M s_{n-1} - 1)(k_1 z_i - p_{n-1}) + \beta_n \Delta t_n (M-1)(p_{n-1} - \bar{p}_{n-1})] \end{aligned}$$

$$E[v - p_{n-1} | F_{(n-1)i}] = s_{n-1} (k_1 z_i - p_{n-1})$$

$$\begin{aligned}
E[\bar{\pi}_n | F_{(n-1)i}] &= \max_{\Delta \bar{x}_{ni}} \Delta \bar{x}_{ni} \{ (s_{n-1}(k_1 z_i - p_{n-1}) + p_{n-1} - \bar{p}_{n-1} \\
&- \lambda_n [\Delta \bar{x}_{ni} + \beta_n \Delta t_n [(M s_{n-1} - 1)(k_1 z_i - p_{n-1}) + (M - 1)(p_{n-1} - \bar{p}_{n-1})]) \} \\
&+ \alpha_n \{ [1 - \lambda_n \beta_n \Delta t_n (M s_{n-1})]^2 (k_1 z_i - p_{n-1})^2 + \lambda_n^2 \sigma_u^2 \Delta t_n + \lambda_n^2 \beta_n^2 M^2 k_1^2 \sum_{n-1} \Delta t_n^2 \} \\
&+ \gamma_n (1 - M \beta_n \Delta t_n \lambda_n s_{n-1}) (k_1 z_i - p_{n-1}) \\
&\times \{ p_{n-1} - \bar{p}_{n-1} - \lambda_n \Delta \bar{x}_{ni} + \beta_n \Delta t_n \lambda_n [k_1 z_i - p_{n-1} - (M - 1)(p_{n-1} - \bar{p}_{n-1})] \} \\
&+ \eta_n \{ p_{n-1} - \bar{p}_{n-1} - \lambda_n \Delta \bar{x}_{ni} + \beta_n \Delta t_n \lambda_n [k_1 z_i - p_{n-1} - (M - 1)(p_{n-1} - \bar{p}_{n-1})] \}^2. \quad (56)
\end{aligned}$$

The first order condition for the above problem is

$$\Delta \bar{x}_{ni} = \frac{A_n (k_1 z_i - p_{n-1}) + B_n (p_{n-1} - \bar{p}_{n-1})}{\lambda_n [2 - 2\eta_n \lambda_n]}, \quad (57)$$

where

$$A_n = s_{n-1} (1 - M \beta_n \Delta t_n \lambda_n) + \lambda_n \beta_n \Delta t_n - \lambda_n \gamma_n - 2\eta_n \lambda_n^2 \beta_n \Delta t_n + \gamma_n \lambda_n^2 M \beta_n \Delta t_n s_{n-1} \quad (58)$$

$$B_n = (1 - 2\eta_n \lambda_n) [1 - (M - 1) \beta_n \Delta t_n \lambda_n]. \quad (59)$$

The optimal demand function (57) is a linear combination of $k_1 z_i - p_{n-1}$ and $p_{n-1} - \bar{p}_{n-1}$. From (56), trader i 's expected profit function is a quadratic function of $\Delta \bar{x}_{ni}$, $k_1 z_i - p_{n-1}$ and $p_{n-1} - \bar{p}_{n-1}$. Consequently, trader i 's expected profit function is of the form conjectured in (55). In equilibrium, $p_{n-1} = \bar{p}_{n-1}$, the second term in (57) drops out and (14) can be derived from (57) and (58). Substitute $\Delta \bar{x}_{ni}$ in (56) using (57), (58), (59), recursive equations (15)-(17),(20) can be derived easily. Since $p_0 = \bar{p}_0 = 0$, from (57), it follows that in the first auction, trader i will follow the equilibrium strategy and therefore $\bar{p}_1 = p_1$. Proceeding inductively, informed trader i will follow equilibrium strategy in every period and we have $\bar{p}_n = p_n, \forall n$.

The second order derivative of (56) with respect to $\Delta \bar{x}_{ni}$ is given by $2\eta_n \lambda_n^2 - \lambda_n$, and this gives the second order condition (23). In all the numerical examples provided in the next section, the second order condition is satisfied.

Appendix B

In this appendix, we derive the recursive method used to solve the discrete time equilibrium. Define $q_n \equiv \gamma_n \lambda_n$. From (14) and (16), we have

$$\gamma_{n-1} = \frac{s_{n-1}}{\lambda_n} \left[1 - \frac{(M-1)(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right] \left[1 - \frac{M(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right],$$

or

$$q_{n-1} = \frac{s_{n-1}\lambda_{n-1}}{\lambda_n} \left[1 - \frac{(M-1)(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right] \left[1 - \frac{M(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right],$$

implying that

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{s_{n-1}}{q_{n-1}} \left[1 - \frac{(M-1)(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right] \left[1 - \frac{M(s_{n-1} - q_n)}{1 + (1 - q_n)s_{n-1}M} \right]. \quad (60)$$

Now, from (16) we also have

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}} \frac{\Sigma_n}{\Sigma_{n-1}}$$

and, from (19), this is equivalent to

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}} [1 - M\beta_n\lambda_n\Delta t_n]. \quad (61)$$

Multiply both sides of (61) by λ_n/λ_{n-1} and Substituting for β_n from (14) to (61), we have

$$\frac{\lambda_n^2}{\lambda_{n-1}^2} = \frac{(s_{n-1} - q_n)(1 - Mq_n(1 - s_{n-1}))}{[1 + (1 - q_n)s_{n-1}M]^2} \frac{1 + (1 - q_{n-1})s_{n-2}M}{s_{n-2} - q_{n-1}} \frac{\Delta t_{n-1}}{\Delta t_n}. \quad (62)$$

Squaring the RHS of (60), equating the resulting expression to the RHS of (62) and rearranging yields the cubic equation (27) in Proposition 2. Equation (26) can be derived from (13), (14) and (19). From (13) we have

$$\frac{\Sigma_n}{\Sigma_{n-1}} = \frac{s_n(1 - s_{n-1})}{s_{n-1}(1 - s_n)} \quad (63)$$

From (14) and (19) we have

$$\frac{\Sigma_n}{\Sigma_{n-1}} = \frac{1 - Mq_n(1 - s_{n-1})}{[1 + (1 - q_n)s_{n-1}M]}. \quad (64)$$

Equating the RHS of (63) and (64) we get (26).

Appendix C

In this section we discuss a model in which trading takes place continuously. Using the similar technique as that in Kyle (1985), we assume that a linear equilibrium with a structure analogous to the discrete time model exists as described below,

$$dx_i(t) = \beta(t)(k_1 z_i - p(t)) \quad (65)$$

$$dx(t) = \sum_{i=1}^M dx_i(t) \quad (66)$$

$$dp(t) = \lambda(t)(dx(t) + du). \quad (67)$$

As discussed in the case of discrete time model, we need to define the optimal strategy of informed trader i , when he deviates from the proposed equilibrium trading. Let $\bar{x}(t)$ denote the optimal strategy when trader i 's trade deviates from proposed in the equilibrium due to an error or price manipulation in early trading rounds. we restrict the strategy for the informed trader in the market to the diffusion processes. Specifically we assume that $d\bar{x}_i(t) = dx_i(t) + (p(t) - \bar{p}(t))dy$, where $dy = \mu(t)dt + \sigma(t)dw$ follows a diffusion process. This particular form of $d\bar{x}_i$ is chosen so that when $\bar{p}(t) = p(t)$, it is optimal form informed trader i to follow the trading strategy in the proposed equilibrium. Let F_{ii} be the information filtration for trader i . We assume that the off equilibrium maximized profit function have the following form analogous to the discrete time model,

$$E[\bar{\pi}(t)|F_{ii}] = \alpha(t)(k_1 z_i - p(t))^2 + \gamma(t)(k_1 z_i - p(t))(p(t) - \bar{p}(t)) + \eta(t)(p(t) - \bar{p}(t))^2 + \delta(t). \quad (68)$$

Since $\bar{x}_i(t)$ is the optimal strategy of informed trader i , $\bar{x}_i(t)$ must be chosen so that the following the Bellman equation is satisfied.

$$E[\bar{\pi}_t dt + \bar{\pi}_p dp + \bar{\pi}_{\bar{p}} d\bar{p} + \frac{1}{2}\bar{\pi}_{pp}(dp)^2 + (v - \bar{p})d\bar{x}_i + \bar{\pi}_{p\bar{p}} dp d\bar{p} + \frac{1}{2}\bar{\pi}_{\bar{p}\bar{p}}(d\bar{p})^2 | F_{ii}] = 0. \quad (69)$$

This basically means that the instantaneous profit is exactly offset by the expected change in $\bar{\pi}$ when an optimal policy is followed.

Since

$$\bar{\pi}_t = \alpha'(t)[k_1 z_i - p(t)]^2 + \gamma'(t)[k_1 z_i - p(t)][p(t) - \bar{p}(t)] + \eta'(t)[p(t) - \bar{p}(t)]^2 + \delta'(t)$$

$$\bar{\pi}_p dp = -2\alpha(t)[k_1 z_i - p(t)] - \gamma(t)[p(t) - \bar{p}(t) - k_1 z_i - p(t)] + 2\eta(t)[p(t) - \bar{p}(t)]$$

$$\bar{\pi}_{\bar{p}}d\bar{p} = -\gamma(t)[k_1 z_i - p(t)] - 2\eta(t)[p(t) - \bar{p}(t)]$$

$$\bar{\pi}_{pp} = 2\alpha(t)$$

$$\bar{\pi}_{p\bar{p}} = \gamma(t) - 2\eta(t)$$

$$\bar{\pi}_{\bar{p}\bar{p}} = 2\eta(t)$$

$$E[d\bar{p}|F_{it}] = M\beta(t)\lambda(t)s(t)[k_1 z_i - p(t)]$$

$$E[d\bar{p}|F_{it}] = \lambda(t)\{d\bar{x}_i + \beta(t)(Ms(t) - 1)[k_1 z_i - p(t)] + \beta(t)(M - 1)[p(t) - \bar{p}(t)]\}$$

$$E[(d\bar{p})^2|F_{it}] = E[(dpd\bar{p})|F_{it}] = E[(d\bar{p})^2|F_{it}] = \lambda(t)^2\sigma_u^2 dt$$

$$E[v - \bar{p}(t)|F_{it}] = s(t)(k_1 z_i - p(t)) + p(t) - \bar{p}(t).$$

Plug these expressions back to the Bellman equation (69) and collect the terms, we have

$$\begin{aligned} & \{[\alpha'(t) - \beta(t)s(t)(2\alpha(t)\lambda(t)M - 1)][k_1 z_i - p(t)]^2 + [\gamma'(t) - \beta(t)[M\gamma(t)\lambda(t)s(t) \\ & + (2M - 1)\gamma(t)\lambda(t) - Ms(t) - 1][k_1 z_i - p(t)][p(t) - \bar{p}(t)] + [\eta'(t) \\ & - \eta(t)\lambda(t)[2\beta(t)(M - 1) + \lambda(t)\sigma(t)^2]][p(t) - \bar{p}(t)]^2 + \delta'(t) + \alpha(t)\lambda(t)^2\sigma_u^2\}dt \\ & - \{[\gamma(t)\lambda(t) - s(t)][k_1 z_i - p(t)] + [1 - 2\eta(t)\lambda(t)][p(t) - \bar{p}(t)]\}d\bar{v}x_i = 0. \end{aligned} \quad (70)$$

Set the coefficients of the (69) to be 0 gives the following system of differential equations.

$$\alpha'(t) = \beta(t)s(t)(2\alpha(t)\lambda(t)M - 1) \quad (71)$$

$$\gamma'(t) = \beta(t)[M\gamma(t)\lambda(t)s(t) + (2M - 1)\gamma(t)\lambda(t) - Ms(t) - 1] \quad (72)$$

$$\eta'(t) = 2\eta(t)\lambda(t)\beta(t)(M - 1) - \eta(t)\lambda(t)^2\sigma(t)^2 \quad (73)$$

$$\delta'(t) = -\alpha(t)\lambda(t)^2\sigma_u^2 \quad (74)$$

$$\gamma(t)\lambda(t) = s(t) \quad (75)$$

$$2\eta(t)\lambda(t) = 1. \quad (76)$$

The system of differential equations (71), (72), (74)-(76) is identical to (43)-(46), (42). (47) and (49) can be derived using the appropriate Kalman filtering. (48) obtains by definition. Therefore the solutions derived from differential equations (42)-(49) and $\sigma(t)$ obtained from (73) satisfy the Bellman equation (69) and give the continuous time trading equilibrium.

Appendix D

In this appendix, we solve for the differential equation described in Section 4.

From (42), (44), (49) we can get

$$\left(\frac{s(t)^2}{\lambda(t)^2}\right)' = \frac{2\sigma_w^2}{Mk_1^2\Sigma(t)}[Ms(t)^2 + (M-1)s(t) - 1]. \quad (77)$$

From (47) and (49) we have

$$\lambda(t)^2 = -k_1^2\Sigma'(t)/\sigma_w^2. \quad (78)$$

Substitute $\lambda(t)^2$ in (78) to (77) and substitute $s(t)$ from (47), we get

$$\frac{\Sigma''(t)}{\Sigma'(t)} = \left[\frac{4M-4}{M\Sigma(t)} - \frac{2(M-1)\sigma_c^2}{M\Sigma(t)^2}\right]\Sigma'(t),$$

which implies that

$$[\ln(-\Sigma'(t))] = \frac{4M-4}{M} \ln[\Sigma(t)] + \frac{2(M-1)\sigma_c^2}{M\Sigma(t)} + c.$$

It is immediate that (50) follows.

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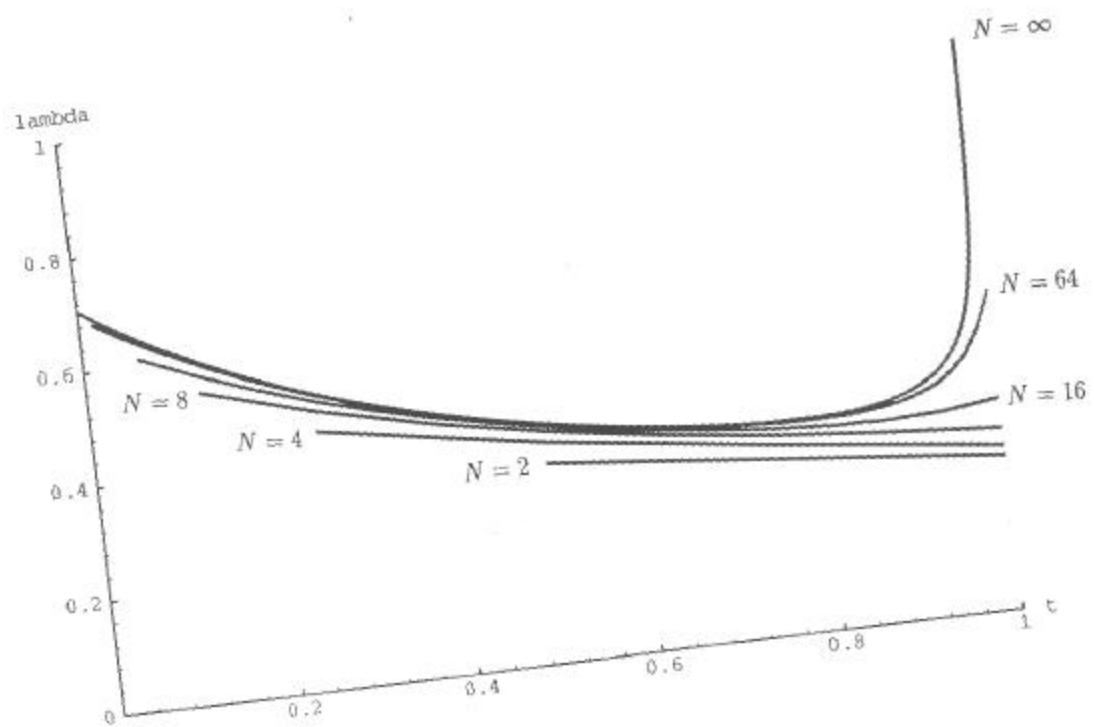


Figure 1

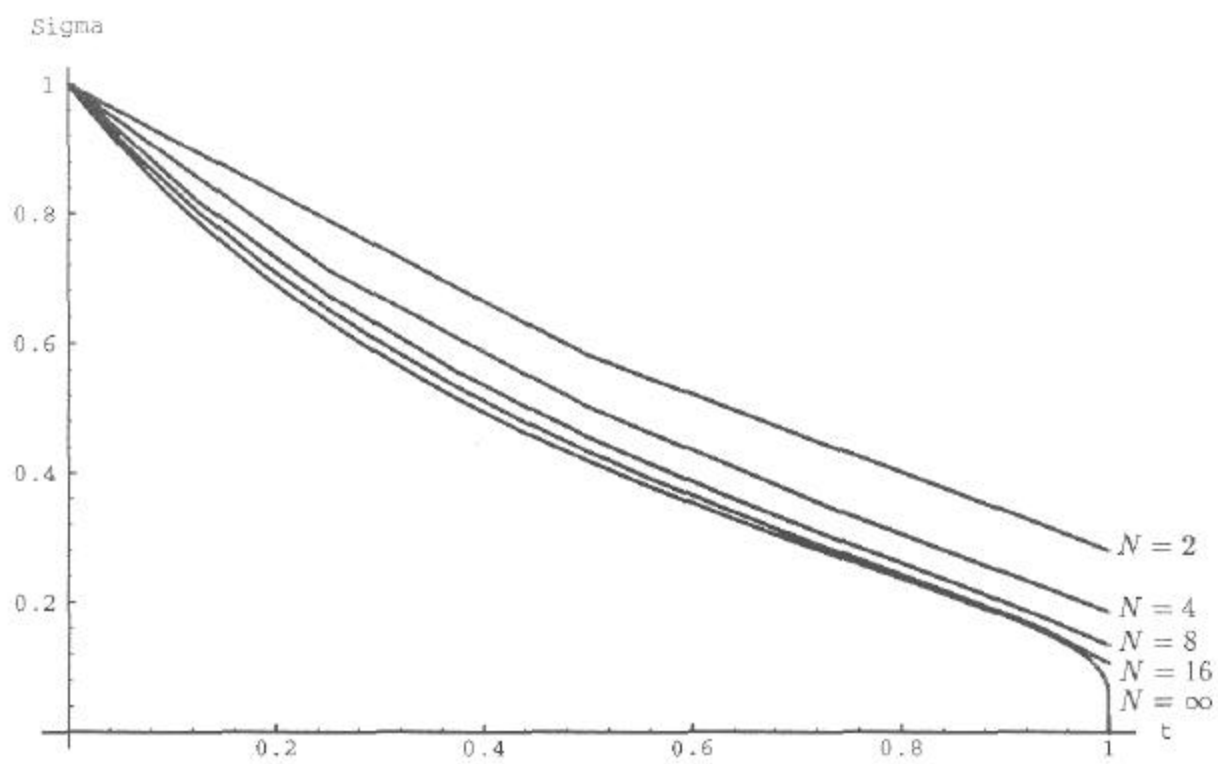


Figure 2

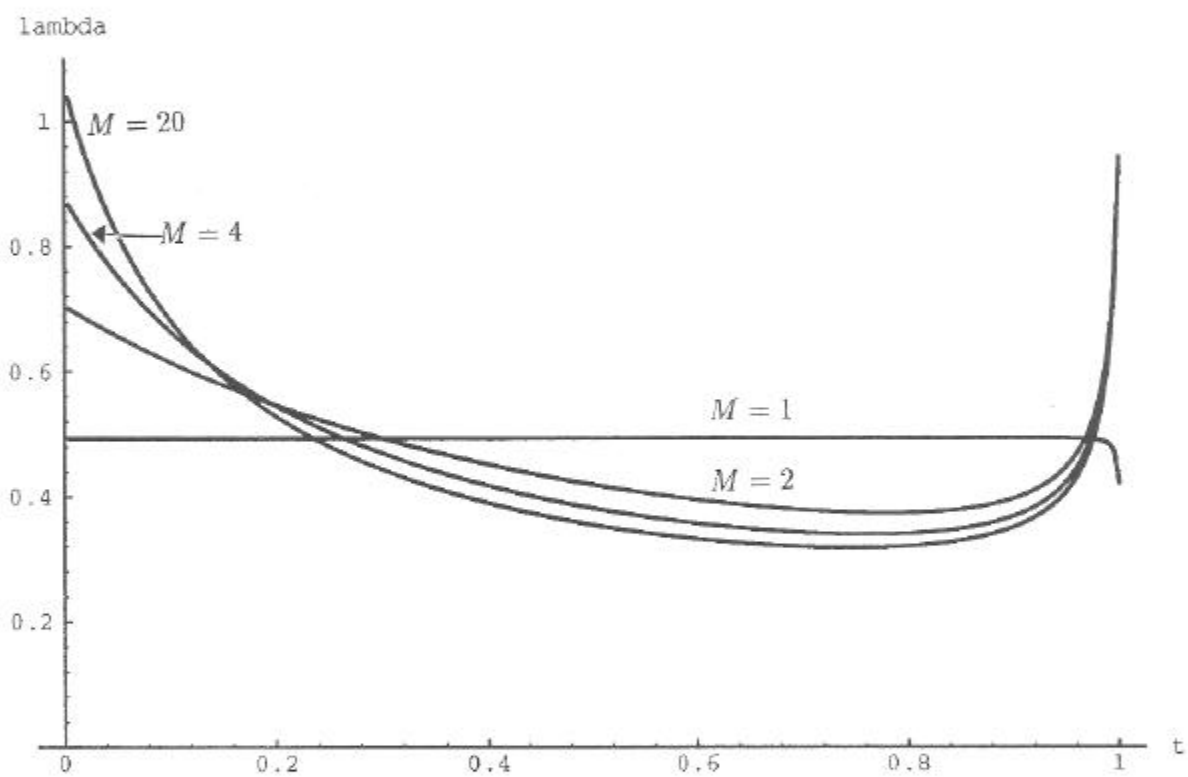


Figure 3

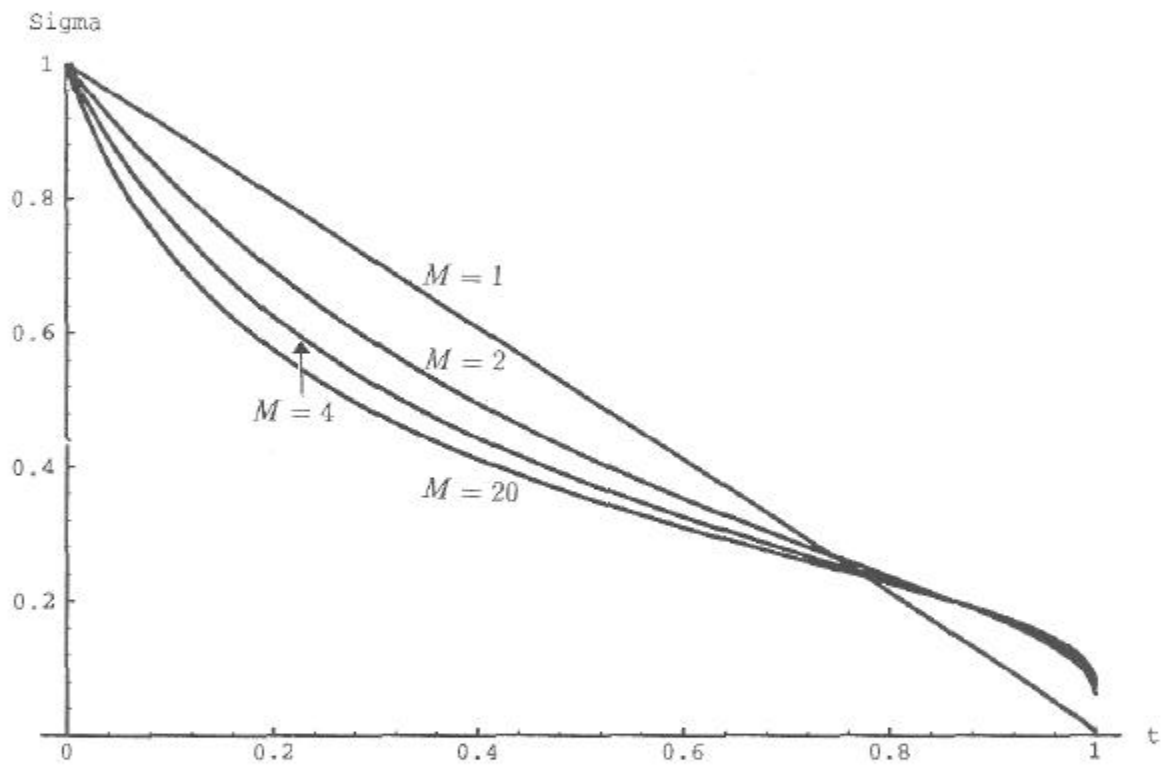


Figure 4

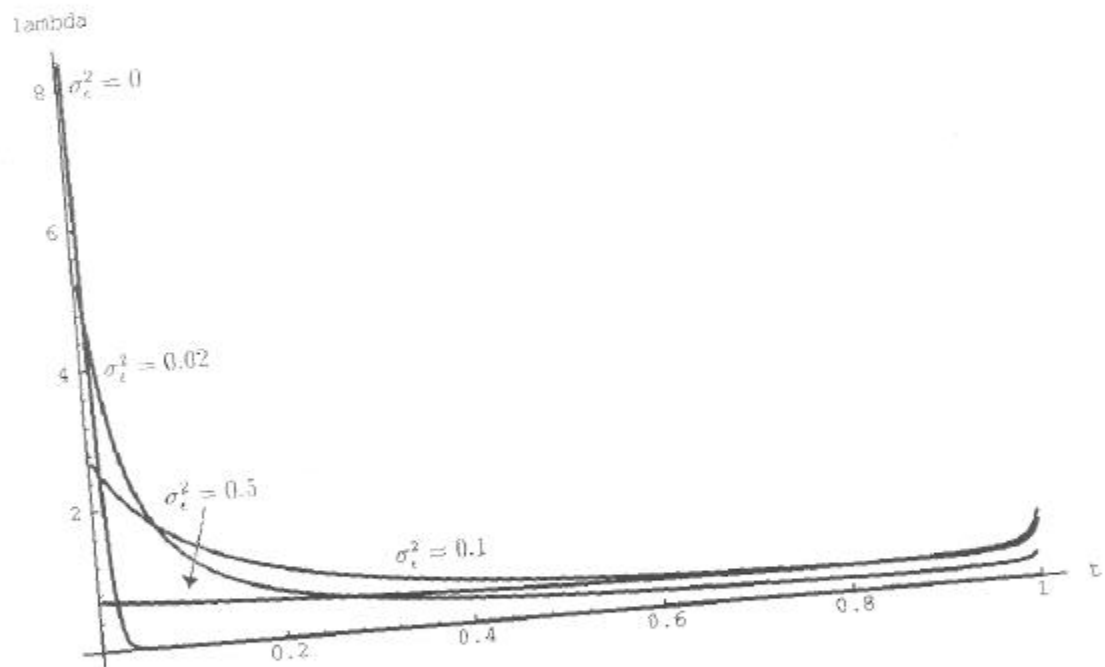


Figure 5

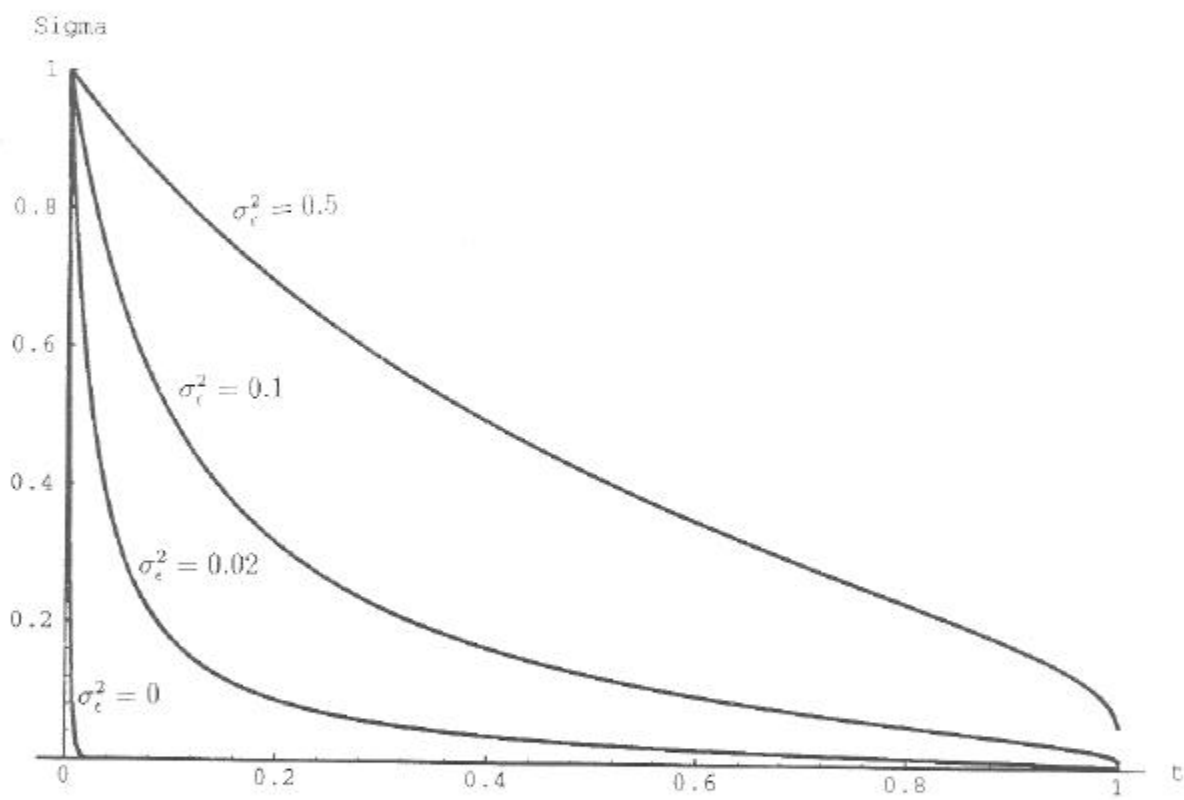


Figure 6

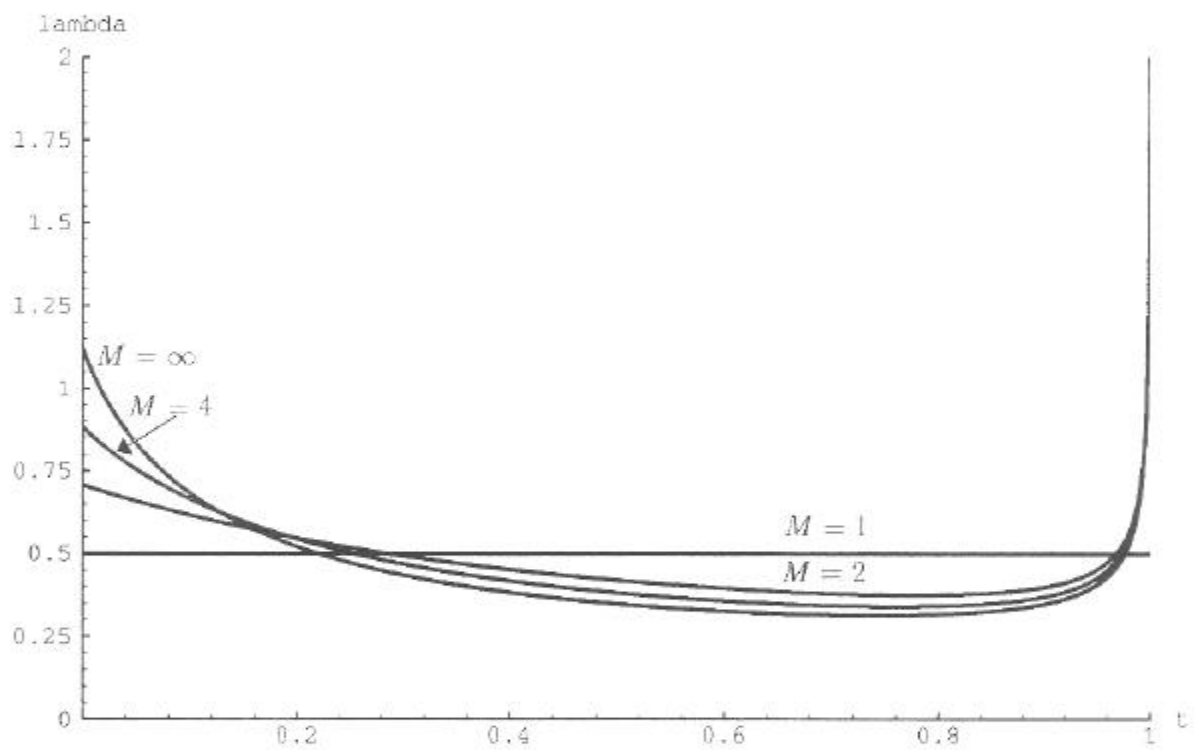


Figure 7

