Title
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Publication Date
1984-07-01

Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

Working Paper No. 295

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July 1984

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1. Introduction

It is tempting to characterize the burgeoning literature on option value as a label in search of a contents. By this I do not mean to imply that there is no substance to the notion of option value. To the contrary, there is an abundance of significant and useful concepts which have been associated with the label "option value." But it is a peculiarity of this piece of intellectual history that the label became firmly established in the economics literature well in advance of any agreement as to the definition of the concept. In consequence, there has been a certain amount of confusion as to the precise meaning of the term, and several distinct notions have borne this same label. As the emphasis has shifted in recent years from theoretical speculation to empirical measurement and applied cost/benefit analysis of actual environmental policies, the need for a taxonomy of concepts and a delineation of their interrelationships has increased. This is the goal of the present paper.

The literature on option value was initiated by Weisbrod (1964), who coined the term and dramatized it with a striking parable. Suppose an action is being considered whose consequence is the destruction of a national park. Suppose, further, that the park is operated by a private management and that, when used for recreation, the costs of running the park exceed the revenues. In that case, Weisbrod wrote, "it is certainly true that a profit-maximizing entrepreneur would cease operating if all costs could not be covered. . . . But it may be socially unsound for him to do so. To see why, the reader need recognize the existence of people who anticipate purchasing the commodity (visiting the park) at some time in the future, but who, in fact, never will purchase (visit) it. Nevertheless, if these consumers behave as 'economic men,' they will be willing to pay something for the option to consume the commodity in the future. This 'option value' should influence the decision of whether or not to close the park and turn it to an alternative use."

Weisbrod's argument seemed persuasive as well as novel. However, the precise definition of this option value was unclear, and soon after his paper appeared a debate began as to what it really meant and how it could be measured. Two different interpretations have emerged from this literature. The first, presented originally by Cicchetti and Freeman (1971) and refined by Schmalensee (1972), Bohm (1974), and Graham (1981) interprets option value as something akin to a risk premium arising from uncertainty as to the future value of the park if it were preserved. This concept, which I shall refer to as Schmalensee-Bohm-Graham (SBG) option value, stresses the uncertainty associated with the value of the park but does not involve the passage of time in any special way. The second interpretation, advanced independently by Arrow and Fisher (1974) and Henry (1974), focuses explicitly on the intertemporal aspects of the problem and the irreversibility of any decision to close the park and convert it to alternative uses. I will refer to this as AFH option value.

As the literature matured, some important differences in the properties of these two concepts of option value have been identified. It was long recognized that the SBG concept was tied to the notion of risk aversion; and it was assumed at first that, as with a risk premium, the SBG option value must be positive for a risk-averse individual. However, first Schmalensee and then Bohn and Graham proved that this option value could be negative even for a risk-averse individual. By contrast, Arrow and Fisher (1974) and Henry (1974) proved that their concept must be positive regardless of the individual's risk preferences. Therefore, the two concepts seemed to be entirely different in nature.

This conclusion is indeed correct: the two concepts of option value deal with different aspects of uncertainty. However, in this paper I want to show that they can be reconciled. The issues addressed by SBG option value also arise in the context of AFH option value, and one can actually construct an analog of the SBG concept within the AFH framework. I should add that a similar argument has recently been made by Måler (1984), and our two papers are complementary. Unlike Mäler, I will focus exclusively on a single individual and will not deal with the aggregation of option values across individuals. However, in addition to showing how the SBG and AFH concepts of option value can be integrated, I will investigate the link between SBG option value and the concept of a risk premium. I show that there are some subtle differences between the two concepts. This discussion of SBG option value is presented in section 2. In section 3, AFH option value is discussed, and its relation to SBG option value is explained.

2. Schmalensee-Bohnm-Graham Option Value

As Ulph (1982) has emphasized, the SBG option value arises out of a distinction between two alternative approaches to deriving money measures of the expected benefits of some policy action in the face of uncertainty which Ulph identifies with Harsanyi (1955) and Hammond's (1981) distinction between ex ante and ex post welfare. Alternatively, SBG option value has been characterized as a kind of risk-aversion premium associated with this uncertainty. In this section, I will elaborate on Ulph's interpretation and then examine how it relates to the concept of a risk premium. I will also comment on a special case of SBG option value, called "supply side option value," to which Bishop (1982) has drawn attention.

The theoretical set-up is as follows. An individual's utility depends on his income, y, and the consequences of a binary-valued decision, represented by the variable, d. In the environmental literature, d typically measures the availability of some environmental good; $d = 0$ signifies that the good is available (i.e., the natural environment is preserved), and $d = 1$ signifies that d is not available. From a formal point of view, however, all that matters is that d is binary valued; whether or not the decision is irreversible is immaterial. In addition, there may be some monetary costs that are incurred if the environment is preserved ($d = 0$), denoted k_0, and other costs, denoted k_1 that are incurred if the environment is not preserved. For the time being, I assume only that utility is increasing in net income and decreasing in d. In some versions, the utility function contains other shift variables, z. The utility function may be a conventional direct utility function or it may be an indirect one arising after the individual has optimized with respect to some choice variables. Similarly, this may be a single-period utility function or, as in Chavas and Bishop (1983), it may be a multiperiod utility function. Whatever the temporal framework, all that matters is that the individual faces some uncertainty concerning his welfare.

The literature on SBG option values has formulated this uncertainty in various ways. These may be classified into three types: (i) uncertainty about y, k_0, and/or k_1 (i.e., uncertainty about net income), (ii) uncertainty about preferences (i.e., state-dependent utility), and (iii) uncertainty about the value of the shift variable, z. Type (ii) uncertainty appears in Bohm (1974); types (i) and (iii) appear in Hartman and Plummer (1982); and combinations of types (i) and (ii) appear in Henry (1974, pp. 89 and 90), Graham (1981), and Anderson (1981). All of these formulations lead to a version of SBG option value.

For simplicity, I will adopt the second formulation and write the utility function as $u(y, d; s)$, where $s = 1, \ldots, S$ represent the state of nature which occurs with probability π_s. The decision at hand is whether to preserve or destroy the environment ($d = 0$ or 1). Measured in utility units, the expected net benefit of preservation over development is

$$(1) \quad \Delta_u \equiv \Sigma\, \pi_s\, u(y - k_0, 0; s) - \Sigma\, \pi_s\, u(y - k_1, 1; s),$$

and the decision rule is: set $d = 0$ if $\Delta_u \geq 0$. Suppose that one wishes to measure these same benefits in money units instead. There are two different ways to do this. One method is to look at the individual's ex post willingness to pay to have $d = 0$ rather than $d = 1$. If state s occurs, the individual would be willing to pay the quantity C_s which satisfies

$$(2) \quad u(y - k_0 - C_s, 0; s) = u(y - k_1, 1; s);$$

if $C_s \geq 0$, he will prefer to have $d = 0$ when state s occurs.[1] But since the true state of nature is not known at the time when the decision on d is made, one takes the ex ante expectation of C_s to form what is called (somewhat awkwardly) the ex post net benefit,

$$(3) \quad C_{EP} = \Sigma\, \pi_s\, C_s.$$

Using this criterion, the decision rule is: set d = 0 if $C_{EP} \geq 0$. The second measure is the amount of money that the individual would commit himself to paying ahead of time to ensure that d = 0 instead of d = 1. This is the ex ante willingness to pay, C_{EA}, which satisfies

$$(4) \quad \Sigma\, \pi_s\, u(y - k_0 - C_{EA}, 0; s) = \Sigma\, \pi_s\, u(y - k_1, 1; s).$$

Using this criterion, the decision rule is: set d = 0 if $C_{EA} \geq 0$. The quantity C_{EA} is generally referred to as the "option price" that the individual would be willing to pay to avoid development, and the SBG option value is simply defined as the difference between the two measures of expected willingness to pay,

$$(5) \quad \text{SBG option value} \equiv C_{EA} - C_{EP}.$$

It follows from (1) and (4) that sign $\{\Delta_u\}$ = sign $\{C_{EA}\}$. Hence, these two criteria lead to the same development decision. However, in general, it is not true that $C_{EA} = C_{EP}$ nor even that sign $\{C_{EA}\}$ = sign $\{C_{EP}\}$. Thus, the ex ante and ex post measures of willingness to pay may lead to conflicting development decisions. A special case is where the utility function is restricted so that, for all y and s,

$$(6) \quad u(y - k_0, 0; s) \geq u(y - k_1, 1; s);$$

this restriction is imposed by Anderson (1981), Bishop (1982), Bohm (1974), and Hartman and Plummer (1982). It follows from (6) that

$$\text{sign}\ \{C_s\} = \text{sign}\ \{C_{EP}\} = \text{sign}\ \{C_{EA}\} > 0.$$

In this case the ex post and ex ante money measures both imply the same decision (do not develop), but they yield <u>different</u> estimates of the benefits of this decision. A sufficient (and, in general, necessary) condition for C_{EA} and C_{EP} to concide--and for SBG option value to be zero--is that the marginal utility of money, $\partial u/\partial y$, be constant (i.e., independent of both y and s). Otherwise, as Bohm (1974) and Graham (1981) have shown, C_{EA} can be larger or smaller than C_{EP} even if $\partial^2 u/\partial y^2 < 0$. Therefore, even with what one conventionally thinks of as risk aversion, SBG option value can be positive or negative. This should come as no surprise since, as Karni (1980, 1983) and others have pointed out, the conventional definition of risk aversion does not readily carry over to multivariate utility functions; there is no longer a simple correspondence between the concavity of the utility function in money and a positive risk premium.

The SBG option value is often linked in the literature to the idea of a risk premium for uncertainty; it may be useful to explore this connection. For this purpose, it is necessary to consider a different scenario in which the individual's utility depends solely on his income and is state independent: $u = u(y)$. I assume that u is increasing and concave in y. In this case, we know that the concept of a risk premium is well defined, and we can avoid the complications introduced by multivariate utility. I will show that the SBG concept of option value can be reproduced in this context and that it is <u>not</u> strictly the same as a risk premium.

Suppose that the individual faces uncertainty concerning his income: in state s he will receive y_s'. Now, as the result of some policy, this changes and his income will be y_s''. Does the change make him better off? How much would he be willing to pay to secure the change? The ex post way to measure this willingness to pay is to calculate the contingent willingness to pay for the change conditional on the state of nature being s, C_s, where

(2') $u(y_s' - C_s) = u(y_s'')$,

and then calculate the expectation, $C_{EP} \equiv \Sigma\ \pi_s\ C_s$. By the ex post criterion, the change renders the individual better off if $C_{EP} < 0$. The ex ante measure of willingness to pay is the quantity C_{EA} that satisfies

(4') $\Sigma\ \pi_s\ u(y_s' - C_{EA}) = \Sigma\ \pi_s\ u(y_s'')$,

and the change is judged an improvement only if $C_{EA} < 0$. By analogy with (5), one can define the SBG option value of the change as

(5') SBG option value $\equiv C_{EA} - C_{EP}$.

In this case, however, it is possible to measure the ex ante willingness to pay for the change in another way. Ex ante, the value of the original income stream to the individual may be measured by its certainty equivalent, $\hat{y}'$, which satisfies

(7a) $u(\hat{y}') = \Sigma\ \pi_s\ u(y_s')$.

Similarly, the certainty equivalent for the new income stream, $\hat{y}''$, satisfies

(7b) $u(\hat{y}'') = \Sigma\ \pi_s\ u(y_s'')$.

The alternative ex ante measure of the value of the change is the difference between the two certainty equivalents,

(8) $C^*_{EA} \equiv \hat{y}' - \hat{y}''$,

and the change is judged an improvement only if $C^*_{EA} < 0$.

What is the relation between the ex post and the two ex ante measures of willingness to pay? Since

$$C^*_{EA} \gtrless 0 \iff \Sigma\, \pi_s\, u(y'_s) \gtrless \Sigma\, \pi_s\, u(y''_s) = \Sigma\, \pi_s\, u_s(y'_s - C_{EA}),$$

it follows that

$$\text{sign}\,\{C^*_{EA}\} = \text{sign}\,\{C_{EA}\} = \text{sign}\,\{\Delta_u\},$$

where

(1') $\Delta_u = \Sigma\, \pi_s\, u(y'_s) - \Sigma\, \pi_s\, u(y''_s)$.

Thus, both ex ante measures lead to the same qualitative evaluation of the change. However, unless $u(y)$ is linear (i.e., the individual is risk neutral), C_{EA} and C^*_{EA} are generally different in magnitude. Moreover, the ex post measure, C_{EP}, can differ from the two ex ante measures not only in magnitude but also in sign. To see this, observe that, from (4'), $C_S = y'_s - y''_s$; therefore,

(9) $C_{EP} = \Sigma\, \pi_s(y'_s - y''_s) = \Sigma\, \pi_s\, y'_s - \Sigma\, \pi_s\, y''_s \equiv \bar{y}' - \bar{y}''$.

Moreover, denoting the risk premium associated with $\{y'_s\}$ and $\{y''_s\}$ by P' and P'', we have

(10) $\hat{y}' = \bar{y}' - P'$, $\hat{y}'' = \bar{y}'' - P''$;

if the individual is risk averse, as assumed above, then $P' \geq 0$ and $P'' \geq 0$. Substituting into (8),

$$C^*_{EA} = (\bar{y}' - P') - (\bar{y}'' - P'')$$
$$= C_{EP} + (P'' - P').$$

Hence,

$$(11) \quad C^*_{EA} - C_{EP} = P'' - P'.$$

It follows that the difference between the ex ante measure, C^*_{EA}, and the ex post measure, C_{EP}, depends on the difference in risk premia. Suppose that the new income stream, $\{y''_s\}$, is more risky than the original stream, $\{y'_s\}$. Given that the individual is risk averse, $P'' > P'$ and $C^*_{EA} - C_{EP} > 0$. In this case, the ex ante rule based on C^*_{EA} is a more conservative criterion for judging the change than the ex post rule based on C_{EP}; it can happen that $C^*_{EA} > 0$ while $C_{EP} < 0$. What about the comparison between C_{EP} and the ex ante measure, C_{EA}, whose difference constitutes the SBG option value? We know that, if $C^*_{EA} > 0$, then $C_{EA} > 0$. But it does not necessarily follow that $C_{EA} - C_{EP} > 0$. The point is that C^*_{EA} and C_{EA} are different quantities. Therefore, even in this simplified, univariate utility model, it is not strictly true to say that SBG option value has anything to do with risk premia. Nevertheless, the difference between C^*_{EA} and C_{EA}--and, hence, between (5') and (11)--is clearly small and involves second-order affects (this is illustrated in the diagram in Figure 1). Thus, it is approximately accurate to characterize SBG option value as being related to the concept of a risk premium.

Figure 1

Returning to the earlier multivariate utility model, I would like to discuss the case of supply-side option value which was introduced by Bishop (1982). This can be regarded as a special case of the type (ii) uncertainty analyzed above in which there are only two states of the world, $k_0 = k_1 = 0$, and the utility function is restricted by assuming that, for all y,

$$(12) \quad u(y, 0; 1) = u(y, 1; 1) = u(y, 0; 2) > u(y, 1; 2).$$

Thus, it does not matter to the individual whether d = 0 or 1 if state 1 occurs, but it does matter if state 2 occurs; moreover, the individual is indifferent between states 1 and 2 if d = 0 but not if d = 1. Bishop (1982) proves that, under these conditions, if u(y, 0; 1) and u(y, 1; 2) are each concave in income, then the SBG option value must be positive. This result has received considerable attention. For example, Brookshire, Eubanks, and Randall (1983) cite it in emphasizing the importance of the distinction between demand uncertainty and supply uncertainty. They argue that, whereas the sign of SBG option value is ambiguous in the case of demand uncertainty, it is unambiguously positive (assuming risk aversion) in the case of supply uncertainty. Therefore, they imply (page 3), the concept of option value is meaningful and relevant in the presence of supply uncertainty although the same may not be true for demand uncertainty.

Why are the restrictions in (12) interpreted as supply uncertainty? Because Bishop motivates them with a specific example in which d = 1 is a decision to continue dumping toxic wastes into a body of water, d = 0 is a decision to cease dumping; and the two states of nature correspond to different degrees of assimilative capacity in the water. In state 1, the water can naturally assimilate any wastes that are discharged; hence it does not matter whether or not the dumping is halted. In state 2, the water cannot assimilate wastes. If there is no dumping, the individual is as well off as in state 1; but if dumping continues, he is worse off. Thus, the uncertainty pertains to the state of the environment, not to the individual's preferences. However, one can invent an exactly analogous scenario of demand uncertainty. Suppose that, in state 1, the individual does not visit the lake which in state 2 he does visit. Then, in state 1, it does not matter whether the dumping is halted; in state 2, if the dumping ceases, he is as well off as in state 1; but if it continues, he is worse off. Thus, the restrictions in (12) apply, and SBG option value must be positive.

Perhaps one could argue that demand uncertainty should preclude the equality of u(y, 0; 1) and u(y, 0; 2) so that (12) cannot apply. The main point I would like to make is that (12) is a highly restrictive characterization of supply uncertainty. There are surely many instances of uncertainty about the state of the natural environment in which the equality restrictions in (12) do not hold. In my view, labeling (12) as supply uncertainty is somewhat misleading and unfortunate. Certainly, if supply uncertainty is conceived more broadly than in (12), it would not be correct to assert that it always generates a positive SBG option value. A more meaningful way to distinguish between demand and supply uncertainty would be to impose some structure on the utility model, for example, by assuming that

$$(13) \quad u(y, d; s) = \max_{t} \hat{u}(y, z),$$

subject to z = f(t, d). This is a household production model in which $\hat{u}(\cdot)$ is the utility function, $f(\cdot)$ is a production function, and z is a generalized commodity, such as recreation experience, for which the state of the natural environment is an input. If $\hat{u}(\cdot)$ depends on the state of nature, s, there is demand uncertainty; if $f(\cdot)$ depends on s, there is supply uncertainty. However, without defining an underlying structural model, I do not believe that it is fruitful to attempt to distinguish between demand and supply uncertainty simply by inspecting the properties of the reduced-form utility function, $u(y, d; s)$.

Finally, it may be worth speculating on why there sometimes appears to be a feeling of discomfort in the face of the fact that SBG option value can, in general, be a negative as well as a positive quantity. I think that this arises from the notion, which goes back to Weisbrod's (1964) original paper, that option value is a value--a distinct type of benefit which should be included (but often is not) in the benefit cost analysis of environmental projects. As noted in the introduction, it is unclear whether Weisbrod (1964) was actually thinking of the SBG concept of option value as opposed to the AFH concept. It clearly would be better to regard SBG option value as a correction factor that is to be applied when benefits are measured in money units and that serves to convert from an ex post into an ex ante measure of expected money benefits. Seen in this light, the ambiguity in its sign becomes less disturbing. At the same time, the concept itself becomes of less importance to environmental benefit cost analysis. The crucial question is which is the relevant welfare measure in the presence of uncertainty--ex post or ex ante willingness to pay. Bohm (1974), for example, argues that the ex ante measure (the option price) is the only relevant measure. Graham (1981) agrees that the ex post measure is irrelevant and argues that one should use the expected value of what he calls the "fair bet point" if the risks involved are insurable or complete contingent claims markets exist and the ex ante measure otherwise. However, Ulph (1982) holds that there are circumstances in which the ex post and ex ante measures are each justified, and he suggests that an optimal strategy might be to employ some combination of both of them. To resolve this issue lies beyond the scope of this paper. I would merely like to observe that, as an empirical proposition, it is probably easier to measure either C_{EA} or C_{EP} directly; measuring one of them and then SBG option value in order to derive the other is probably a more cumbersome and less practical procedure.

3. Arrow-Fisher-Henry Option Value

In this section, I describe the AFH concept of option value, show how it differs from the SBG concept, and explain how one can construct an analog of the SBG concept within the AFH framework. Because an analysis of AFH option value appears in Fisher and Hanemann (1983), my discussion here will be brief.

The setting of the AFH concept is explicitly intertemporal. There are two time periods, t = 1, 2, and two decision variables, d_1 and d_2; these variables can be interpreted as the amount of land developed during period d = 1, 2. The units of measurement are chosen so that the maximum possible level of development is unity,

(14) $d_1 \leq 1,\ d_1 + d_2 \leq 1.$

The key assumption is that any development is irreversible,

(15) $d_1 \geq 0,\ d_2 \geq 0.$

In addition, I assume that development is a binary decision--either develop fully during a period ($d_t \equiv 1$) or do not develop at all ($d_t = 0$). Associated with any development program is some overall level of net benefits,

$$(16)\quad B = B_1(d_1) + B_2(d_1 + d_2, d_2; \theta).$$

$B_1(\cdot)$ is the benefits accruing during the first period which depend on the amount of land developed during that period, d_1. The second-period benefits, $B_2(\cdot)$, depend both on the <u>total</u> amount of land developed over the two periods, $d_1 + d_2$, and on the incremental amount developed specifically in the second period, d_2. These benefits also involve an element of uncertainty here represented by the random variable, θ (this corresponds to the state-of-nature representation, s, employed in the previous section). There may also be uncertainty about the benefits of the first period so that

$$(17)\quad B_1 = B_1(d, \eta),$$

where η is a random variable; but this is immaterial to the argument that follows.[2] Finally, the benefits may be measured in money or units of utility. At first, I will not specify which units are used, but I will return to this distinction later.

The social decision involves the maximization of expected benefits, $E\{B\}$, with respect to d_1 and d_2 subject to (14), the irreversibility constraints (15), and the constraint that $d_t = 0$ or 1, t = 1, 2. The decision on d_1 is made at the start of the first period, but there are two possible scenarios for the decision on d_2. One scenario is that d_2 is also determined at the start of the first period or, equivalently, it is determined at the start of the second period, but no more information about Θ is available then than before. The other scenario is that the specific value of Θ is known at the start of the second period, and the choice of d_2 can be postponed until that time in order to incorporate this information.[3] As far as the choice of initial development is concerned, under the first scenario, the decision is to maximize $V^*(d_1)$,

$$(18)\quad V^*(d_1) = B_1(d_1) + \max_{\substack{d_2 \\ d_1+d_2\leq 1 \\ d_2=0 \text{ or } 1}} [E\{B_2(d_1 + d_2, d_2; \Theta)\}],$$

subject to $d_1 = 0$ or 1. Similarly, under the second scenario, the decision is to maximize $\hat{V}(d_1)$,

$$(19)\quad \hat{V}(d_1) = B_1(d_1) + E\{\max_{\substack{d_2 \\ d_1+d_2<1 \\ d_2=0 \text{ or } 1}} [B_2(d_1 + d_2, d_2; \Theta)]\},$$

subject to $d_1 = 0$ or 1. Thus, the difference between the scenarios depends on the difference between the two value functions, $V^*(d_1)$ and $\hat{V}(d_1)$. Each of these functions measures the expected benefits over <u>both</u> periods as a function of the <u>initial</u> amount of development, d_1, given that the amount of development in the second period is <u>optimally</u> chosen subject to the irreversibility constraint and the limitation of the information structure in the scenario. It is readily shown that, for all d_1,

(20) $\hat{V}(d_1) \geq V^*(d_1)$.

Let d^*_1 be the solution to the maximization of (18) and $\hat{d}_1$ the solution to the maximization of (19). Thus,

(21) $$d_1^* = \begin{cases} 0 & \text{if } V^*(0) - V^*(1) \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

and

(22) $$\hat{d}_1 = \begin{cases} 0 & \text{if } \hat{V}(0) - \hat{V}(1) \geq 0 \\ 1 & \text{otherwise,} \end{cases}$$

where, from (16),

(23) $$V^*(0) = B_1(0) + \max\,[E\{B_2(0,\ 0;\ \Theta)\},\ E\{B_2(1,\ 1;\ \Theta)\}]$$

(24) $$\hat{V}(0) = B_1(0) + E\{\max[B_20,\ 0;\ \Theta),\ B_2(1,\ 1;\ \Theta)]\},$$

while

(25) $$\hat{V}(1) = V^*(1) = B_1(1) + E\{B_2(1,\ 0;\ \Theta)\}.$$

The reason for (25) is that, with full development in the initial period, given the irreversibility assumption, it is not possible to adjust the stock of developed land during the second period so that it makes no difference whether or not new information becomes available at the beginning of the second period. Define $V^* \equiv V^*(d_1^*)$ and $\hat{V} \equiv \hat{V}(\hat{d}_1)$; these are the expected benefits with an optimal choice of development in <u>both</u> periods. Their difference, $\hat{V} - V^*$, is known in the literature on decision theory as the expected value of perfect information (EVPI). It follows from (20) that EVPI = $\hat{V} - V^* \geq 0$.

In Hanemann (1983) and Fisher and Hanemann (1983), I have shown that the AFH concept of option value, denoted OV, is given by

$$(26)\quad OV = [\hat{V}(0) - \hat{V}(1)] - [V^*(0) - V^*(1)].$$

It is, thus, the difference in the advantage of initial preservation ($d_1 = 0$) over initial development ($d_1 = 1$) as between the scenario where information about the future consequences of development becomes available and the scenario where there is no new information. This can be motivated in the following manner. Suppose that, in contemplating whether to permit development, a decision-maker behaves according to (21) ignoring the possibility of improved information and setting d_1 and d_2 on the basis of his current expectation of the future benefits and costs of development. If it is, in fact, possible to wait to determine d_2 after the value of Θ is known, this not an optimal decision. The inefficiency can be corrected, in principle, by introducing a "shadow tax" on development, τ, so that, instead of comparing $V^*(0)$ and $V^*(1)$, a decision-maker will compare $V^*(0)$ and $[V^*(1) - \tau]$. This tax must satisfy the condition that

$$(27)\quad V^*(0) - [V^*(1) - \tau] = \hat{V}(0) - \hat{V}(1).$$

A comparison of (26) and (27) reveals that $\tau = OV$.

Thus, AFH option value can be characterized as a correction factor, or a tax on development required to correct the misallocation of resources which arises if current development decisions ignore the possibility of acquiring information which could be used in future development decisions, that is based on $V^*(d_1)$ instead of $\hat{V}(d_1)$. An alternative but equivalent characterization of this concept of quasi-option value can be obtained by substituting (25) into (26),

$$(28)\quad OV = \hat{V}(0) - V^*(0)$$
$$= E\{\max[B_2(0, 0; \Theta), B_2(1, 1; \Theta)]\} - \max[E\{B_2(0, 0; \Theta)\}, E\{B_2(1, 1; \Theta)\}].$$

The right-hand side of (28) will be recognized from decision theory as a formula for the expected value of perfect information. It is, in fact, a conditional value of perfect information: it is the gain from information with respect to the choice of d_2 conditional on $d_1 = 0$. Using (28) and (20), it can be shown that

$$(29)\quad OV \geq EVPI \geq 0.$$

To summarize, the AFH concept of option value grows out of the contrast between decisions with two different information structures. It pertains specifically to the difference between $E\{\max B_2(\cdot; \Theta)\}$ and $\max E\{B_2(\cdot; \Theta)\}$, a difference that is positive irrespective of whether preferences are risk neutral or nonneutral and irrespective of whether benefits are measured in units of utility or money. However, the question of units of measurement does arise in the AFH context, and it involves the same issues about the correct money measure--ex ante or ex post--that were discussed in the previous section. By virtue of this, one can, indeed, construct an analog of the SBG option value within the intertemporal context of AFH.

For this purpose, it is convenient to rewrite the formula for AFH option value as

(30) $OV = OB - OD,$

where

$$OB \equiv E\{\max[B_2(0,\ 0;\ \Theta),\ B_2(1,\ 1;\ \Theta)]\} - E\{B_2(1,\ 0;\ \Theta)\}$$

and

$$OD \equiv \max[E\{B_2(0,\ 0;\ \Theta)\},\ E\{B_2(1,\ 1;\ \Theta\}] - E\{B_2(1,\ 0;\ \Theta)\}.$$

It is shown in Hanemann (1983) that the quantities OB and OD can each be interpreted as correction factors. The needed correction factor is OB when the decision-maker proposes to determine the initial level of development, d_1, <u>solely</u> by reference to the <u>first-period</u> benefits and costs of development, $B_1(0) - B_1(1)$ ignoring the fact that there are future consequences of development which depend, in part, on the choice of d_1, i.e., ignoring the fact that the choice should properly be based on $\hat{V}(0) - \hat{V}(1)$. Similarly, OD is the correction factor needed when no future information is forthcoming and the choice of d_1 is based myopically on $B_1(0) - B(1)$ instead of $V^*(0) - V^*(1)$. In fact, the quantity OB was originally presented as a form of option value by Bernanke (1983); in Hanemann (1983), I discuss how it differs from the AFH concept, OV. Here, however, my concern is to show how one can construct ex ante and ex post money measures of OB and OD in order to derive corresponding money measures of AFH option value via (30). I assume that the underlying utility function takes the form

(31) $u = u_1(y_1, d_1) + u_2(y_2, d_1 + d_2; \Theta)$,

were y_t is the individual's income in period t = 1, 2. This is a natural generalization of the present explicitly intertemporal framework of the utility model introduced in the previous section. If $u_1(\cdot)$ in (31) contains a random element so that $u_1 = u_1(y, d_1; \eta)$, this corresponds to the case where there is also uncertainty concerning the benefits of the first period as in (17). However, it can be seen from (30) that the expected first-period benefits, whether measured in utility or money units, do not enter into the formula for AFH option value; therefore, I will concentrate on deriving money measures of OB and OD from $u_2(y_2, d_1 + d_2; \Theta)$.

To complete the mapping from the utility-theoretic formulation (31) and the earlier analysis based on $B_2(d_1 + d_2, d_1; \Theta)$, if $d_1 + d_2 = 0$,the individual pays the amount k_{20} in the second period representing his share of the costs of preserving the natural environment. If $d_1 = 1$, he pays k_{21} during the second period while, if $d_1 = 0$ but $d_2 = 1$, he pays $k_{21} + C_2$; k_{21} can be thought of as the operating costs and C_2 as the capital costs for any development that has occurred. In utility units, the quantities OB and OD are measured, respectively, by

$$(32)\quad \hat{\Delta}_u = E\{\max[u_2(y_2 - k_{20}, 0; \Theta) - u_2(y_2 - k_{21}, 1; \Theta), \\ u_2(y_2 - k_{21} - c_2, 1; \Theta) - u_2(y_2 - k_{21}, 1; \Theta)]\}$$

and

$$(33)\quad \Delta_u^* = \max[E\{u_2(y_2 - k_{20}, 0; \Theta) - u_2(y_2 - k_{21}, 1; \Theta)\},$$
$$E\{u_2(y_2 - k_{21} - C_2, 1; \Theta) - u_2(y_2 - k_{21}, 1; \Theta)\}].$$

In money units, the ex post measures of these quantities are

$$(34)\quad \hat{C}_{2,EP} = E\{\max[C_\Theta, -C_2]\}$$

$$(35)\quad C^*_{2,EP} = \max[E\{C_R\}, -C_2],$$

where C_Θ is the state-contingent, ex post willingness to pay defined by

$$u_2(y_2 - k_{20} - C_\Theta, 0; \Theta) = u_2(y_2 - k_{21}, 1, \Theta).$$

The ex ante money measure of OB is the quantity $\hat{C}_{2,EA}$ which satisfies

$$(36)\quad E\{\max[u_2(y_2 - k_{20} - \hat{C}_{2,EA}, 0; \Theta), u_2(y_2 - k_{21} - C_2 - \hat{C}_{2,EA}, 1; \Theta)]\}$$
$$= E\{u_2(y_2 - k_{21}, 1; \Theta)\},$$

while the ex ante money measure of OD is

$$(37)\quad C^*_{2,EA} = \max[C^*, - C_2],$$

where C* satisfies

$$(38)\quad E\{u_2(y_2 - k_{20} - C^*, 0; \Theta)\} = E\{u_2(y_2 - k_{21}, 1; \Theta)\}.$$

Combining these formulas, AFH option value, (26), is measured in utility units by

(39) $OV_u \equiv \hat{\Delta}_{2,u} - \Delta^*_{2,u}$

and in money units by either the ex ante measure,

(40) $OV_{EA} = \hat{C}_{2,EA} - C^*_{2,EA}$,

or the ex post measure,

(41) $OV_{EP} = \hat{C}_{2,EP} - C^*_{2,EP}$.

Accordingly, the analog of the SBG concept in the AFH context is the difference between these two money measures of AFH option value,

(42) SBG analog = $OV_{EA} - OV_{EP}$.

It is shown in the Appendix that OV_u, OV_{EA}, and OV_{EP} have the same sign--they are all positive as one would expect from (29). However, unless $\partial u_2/\partial y_2$ is constant (i.e., independent of both y_2 and Θ), OV_{EA} and OV_{EP} are different in magnitude and, even if $\partial^2 u_2/\partial y_2^2 < 0$, OV_{EA} can be larger or smaller than OV_{EP}. Thus, although AFH option value is always positive, the analog, the SBG concept representing the difference between two alternative measures of AFH option value (42), can be positive or negative. It may be useful to illustrate this argument with a simple numerical example. Suppose there are two possible states of the world: $\Theta = \Theta'$ with probability π and $\Theta = \Theta''$ with probability $(1 - \pi)$. Let the second-period utility function be given by

$$
(43)\quad \begin{aligned} u_2(y_2, d_1 + d_2; \Theta') &= (2 + d_2) \ln y \\ u_2(y_2, d_1 + d_2; \Theta'') &= (2 - d_2) \ln y, \end{aligned}
$$

which satisfies $\partial u_2/\partial y_2 > 0$ and $\partial^2 u_2/\partial y_2^2 < 0$. Suppose that $y_2 = \$10$, $k_{20} = \$2.00$, $k_{21} = \$1.00$, and $C_2 = \$1.50$. Suppose, also, that $\pi = 0.75$. Application of (34) through (41) yields the following values:

$$\hat{C}_{2,EA} \simeq \$0.218, \; C^*_{2,EA} = \$-1.50$$

$$\hat{C}_{2,EP} = \$0.125, \; C^*_{2,EP} = \$-1.50.$$

Thus, the ex ante measure of option value is $OV_{EA} = \$1.718$ while the ex post measure is \$1.625. Hence, the analog of the SBG concept for AFH option value, (42), is positive. However, let $\pi = 0.25$ while retaining the same concave utility functions in (43); one now obtains

$$\hat{C}_{2,EA} = \$3.75, \; C^*_{2,EA} = \$2.804$$

$$\hat{C}_{2,EP} = \$3.375, \; C^*_{2,EP} = \$-1.00.$$

Thus, the ex ante measure of option value is $OV_{EA} = \$0.696$ while the ex post measure is \$4.375. Hence, despite the concavity of the utility functions, the analog of the SBG concept is negative.

What can we conclude from this analysis? The AFH and SBG concepts of option value clearly deal with different aspects of decision making under uncertainty. AFH option value grows out of the distinction between two scenarios for the temporal resolution of uncertainty; in effect, it is a correction factor to be applied when decisions are based on $V^*(d_1)$ instead of $\hat{V}(d_1)$

irrespective of whether these value functions are measured in units of utility or money. By contrast, SBG option value arises only when benefits are measured in money, and it grows out of the distinction between two different approaches to mapping from utility to money in the face of uncertainty. However, as soon as one attempts to calculate AFH option value in money units, the same distinction between ex ante and ex post measurement will arise; the fundamental value judgment as to the appropriate welfare measure in the face of uncertainty cannot be avoided.

Appendix

It is immediately obvious from (32), (33), and (39) that $OV_u \geq 0$ and, from (34), (35), and (41), that $OV_{EP} \geq 0$. It may be less obvious that $OV_{EA} \geq 0$. This can be proved by contradition. Suppose that $C^* \geq -C_2$ so that $OV_{EA} = \hat{C}_{2,EA} - C^*$. Combining (36) and (39) yields the inequality,

(A.1) $E\{u_2(y_2 - k_{21} - C^*, 0; \Theta)\} \geq E\{u_2(y_2 - k_{20} - \hat{C}_{2,EA}, 0; \Theta)\}$.

However, if $OV_{EA} < 0$, i.e., $\hat{C}_{2,EA} < C^*$, since $\partial u_2/\partial y > 0$, one obtains

$$E\{u_2(y_2 - k_{21} - C^*, 0; \Theta)\} < E\{u_2(y_2 - k_{20} - \hat{C}_{2,EA}, 0; \Theta)\},$$

which contradicts (A.1). Alternatively, let $C^* < -C_2$ so that $OV_{EA} = \hat{C}_{2,EA} + C_2$. From (36), one obtains the inequality,

(A.2) $E\{u_2(y_2 - k_{21}, 1; \Theta)\} \geq E\{u_2(y_2 - k_{21} - C_2 - \hat{C}_{2,EA}, 1; \Theta)\}$.

However, if $OV_{EA} < 0$, i.e., $\hat{C}_{2,EA} + C_2 < 0$,

$$E\{u_2(y_2 - k_{21}, 1; \Theta)\} < E\{u_2(y_2 - k_{21} - C_2 - \hat{C}_{2,EA}, 1; \Theta)\},$$

which contradicts (A.2). Therefore, $OV_{EA} \geq 0$.

Footnotes

[1]This is a compensating surplus measure of benefits; the following analysis could also be conducted in terms of equivalent surplus with similar results.

[2]If there is uncertainty regarding first-period benefits, as implied by (17), the term $B_1(d_1)$ would be replaced by $E\{B_1(d_1, \eta)\}$ without affecting any of the analysis.

[3]In the terminology of Dreze and Modigliani (1972), the first scenario involves a "temporal" prospect while the second involves a "timeless" prospect. It is important to emphasize that the information about Θ in the second scenario is independent of the level of development in the first period, d_1. For a different type of model involving endogenous information, see Miller and Lad (1984).

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