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# Information, Diversification, and Cost of Capital <sup>\*†</sup>

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## **Abstract**

### **Information, Diversification, and Cost of Capital**

We investigate the effects of information and diversification on cost of capital in a noisy rational expectations model. Assuming a factor structure for risky asset payoffs and two classes of investors, informed and uninformed, we show that in large economies the APT (Ross, 1976) holds and i) information from private signals about idiosyncratic shocks has no effect on cost of capital and ii) information from private signals about systematic factors affects cost of capital only through factor risk premiums; there is no effect on factor loadings. These results imply that there are no cross-sectional effects of information on cost of capital within large economies.

# 1 Introduction

This paper considers the cost of capital effects of information in a noisy rational expectations model in which risky asset payoffs obey a factor structure. Information takes the form of private signals with components related to systematic factors as well as idiosyncratic shocks underlying those payoffs. At issue is the interplay between information and diversification. We derive the following results in the limiting case where the number of risky assets goes to infinity: the APT (Ross, 1976) holds and i) information about idiosyncratic shocks has no effect on cost of capital and ii) information about systematic factors reduces cost of capital only through factor risk premiums; it has no effect on factor loadings (betas). These results imply that there are no cross-sectional effects of private information in large economies.

The information structure allows for asymmetry between informed investors who receive private signals and uninformed investors who can only draw inferences from asset prices. Exploiting this structure in the limiting case, we further show that greater information asymmetry on systematic factors as measured by a smaller proportion of informed to uninformed investors implies higher factor risk premiums. Interestingly, the APT holds in this case notwithstanding that the information structure encompasses heterogeneous beliefs.

These results are intuitive. Information on idiosyncratic shocks changes expectations of future risky asset payoffs, but in the presence of full diversification does not affect expected returns on those assets. While information on idiosyncratic shocks resolves uncertainty on such factors, thereby diminishing risk, this does not matter given the risk is not priced. Information on systematic factors does affect expected returns by resolving uncertainty about risks that are priced even with full

diversification. As the economy expands, the information about systematic factors conveyed by a private signal pertaining to a particular risky asset becomes sufficiently small such that it has no effect on betas in the limit, yet in the aggregate the information reduces the factor risk premiums.

Allowing for asymmetry of information on systematic factors implies resolution of uncertainty through both private signals received by informed investors and inferences drawn by uninformed investors through partially revealing equilibrium prices. More informed investors imply greater resolution of systematic factor uncertainty and, therefore, a lower factor risk premium.

The absence of a cross-sectional prediction regarding information, including asymmetric information, and cost of capital is important because it calls into question the interpretation of recent empirical studies' findings of cost of capital effects of cross sectional variations in proxies thought to capture the information environments of different firms (e.g., Botosan, 1997; Botosan and Plumlee, 2002; Botosan, Plumlee, and Xie, 2004; Healy, Hutton, and Palepu, 1999; Francis, LaFond, Olsson, and Schipper, 2002; and Easley, Hvidkjaer, and O'Hara, 2002). Although lacking theoretical underpinning, one possible explanation for these findings is that investors are under-diversified. Another possible explanation is that differences in information environments have direct effects on asset payoffs, not just on inferences.<sup>1</sup>

While our principal results pertain to the case of imperfect information about systematic factors in the sense of finite aggregate posterior precision, other cases yield intuitively complementary results. At one extreme, if private signals are simply risky asset payoffs plus noise (e.g., Admati, 1985), then in the limit as the

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<sup>1</sup>Aboody, Hughes, and Liu, 2004, consider these potential explanations in their empirical study of the cost of capital effects of asymmetric information.

economy expands the factor becomes perfectly revealed through aggregation implying factor risk premiums equal to zero. Thus, although factor loadings in asset risk premiums (betas) are affected by information in this case, there is no cross-sectional prediction with respect to cost of capital. At the other extreme, if private signals are uninformative about systematic factors (e.g., Easley and O'Hara, 2004), then there is no resolution of uncertainty about those factors and neither betas nor factor risk premiums are affected, again, implying no cross-sectional prediction.

An aspect of the information structure that we impose is the presence of a component related to systematic factors in private signals at the firm level. Consistent with this information structure, Seyhun (1992) and Lakonishok and Lee (2001) provide evidence that corporate insiders are able to time the market. This is not surprising given that firm level financial data typically includes measures such as revenues, earnings, and cash flows that are affected by systematic factors as well as idiosyncratic shocks; evidence dates back to Ball and Brown (1968). These data also include descriptions of business risk factors and management's discussion and analysis of prospective performance. While it is likely that firm-level financial data are far more informative of idiosyncratic shocks, even an infinitesimally small amount of information on systematic factors extracted from private signals for each firm in large economies, when aggregated, can have a finite effect on factor risk premiums. The further aspect that many investors become informed by private signals of many firms captures the notion that professional traders and financial analysts constitute a large body of sophisticated investors who access similar financial data.

Our analysis provides a modeling contribution in that we solve for equilibrium prices and risk premiums explicitly; something infeasible under Admati's (1985) diverse information structure. We begin by characterizing an equilibrium for a

finite number of risky assets and demonstrate the effects of information conveyed by private signals on asset betas and factor risk premiums. In this case, idiosyncratic information affects factor risk and asset betas. Accordingly, firms with the same betas but different information environments have different costs of capital implying the presence of cross-sectional effects. These cross-sectional effects disappear when, next, we take the limit as the number of risky assets goes to infinity. Moreover, having an explicit pricing solution for the limiting case allows us to examine how changes in signal precision and the proportion of informed investors affect factor risk premiums.

Another modeling contribution, supported by the empirical findings of Chordia, Roll, and Subrahmanyam (2000) and Huberman and Hulka (2001), is the introduction of a systematic component to the random supply of risky asset shares that commonly serves as the source of noise in rational expectations models. Without this systematic component or some alternative structure, prices in large economies will eliminate asymmetry in information by fully revealing private signals. In other words, when the number of assets and related signals is large, only systematic noise can prevent information, idiosyncratic or systematic, from being perfectly inferred from prices by uninformed investors.

Like us, Admati (1985) considers the interplay between information and diversification in a noisy rational expectations framework.<sup>2</sup> Rather than a factor structure, Admati's principal analysis assumes asset payoffs are distributed normally and satisfy a general variance-covariance matrix. Admati (1982) recognizes the advantages of a factor structure in characterizing a systematic component of information, but, as previously noted, an explicit solution in the case of diverse information is difficult to derive due to mathematical complexities.

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<sup>2</sup>Brennan and Cao (1997) employ a similar structure.

Our study is an extension of Easley and O’Hara (2004). They also examine the effects of asymmetric information on cost of capital in a noisy rational expectations framework with multiple assets very similar to ours. The differences are that they provide for public as well as private signals<sup>3</sup>, they assume all information pertains to idiosyncratic risk, and they only consider the case in which the number of assets is finite. Holding the role of public signals in their model aside, we offer two perspectives on their characterization of risk premiums. First, our characterization of asset risk premiums in the finite assets case reduces to their characterization when factor loadings in our model are set equal to zero. Taking the limit as the number of assets goes to infinity in this case results in no factor risk premium. Alternatively, one might interpret the assets in Easley and O’Hara as analogous to the systematic factors in our model. Removing all idiosyncratic risks and assuming factor independence in our model would then result in equivalent factor risk premiums.

The rest of the paper is organized as follows: Section 2 studies an economy with a finite number of risky assets; Section 3 studies the limit of a large economy as the number of risky assets goes to infinity; and Section 4 concludes the paper.

## 2 The Finite Economy

In this section, we present a noisy rational expectation model in which the asset payoffs, signals, and the random supply of the assets all have factor structures. We

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<sup>3</sup>Specifically, Easley and O’Hara (2004) assume that there are a fixed set of signals, some of which are public and the rest private. Our model does not consider public signals; all signals are private to informed investors. Accordingly, unlike Easley and O’Hara (2004), our comparative statics do not encompass the case in finite economies where the proportions of signals that are public or private are allowed to change.



solve the equilibrium in closed form. We then give some examples.

## 2.1 The Setup

We assume that there is a risk-free asset, in perfectly elastic supply, with return  $R_f$ . There are  $N$  risky assets that have payoff  $\nu$  which is generated by a factor structure of the form

$$\nu = \bar{\nu} + \beta F + \Sigma^{1/2}\epsilon. \quad (1)$$

The mean of asset payoffs  $\bar{\nu}$  is an  $N \times 1$  constant vector, the factor  $F$  is a  $K \times 1$  vector of mean normal random variables with covariance matrix  $\Sigma_F$ , the factor loading  $\beta$  is an  $N \times K$  constant matrix, and the idiosyncratic risk  $\epsilon$  is a vector of standard normal random variables.

The supply of risky assets,  $x$ , is a vector of  $N \times 1$  random variables with mean vector  $\bar{x}$  and covariance matrix  $\Sigma_x$  and  $\eta_x$  is a standard normal random variable:

$$x = \bar{x} + \beta_x F_x + \Sigma_x^{1/2}\eta_x. \quad (2)$$

The noisiness of the supply is necessary in our setting to prevent prices from fully revealing the informed investors' private signal (defined below) and can be interpreted as caused by trading for liquidity reasons. The presence of a systematic component is based on the reasonable view that liquidity trading is influenced by market-wide forces that may or may not correspond to factors influencing risky asset payoffs. If we interpret the random supply as due to a liquidity effect, then our assumption of systematic components in random supply is supported by empirical studies that find there are systematic components of liquidity; for example, Chordia, Roll, and Subrahmanyam (2000) and Huberman and Hulka (2001). IPO waves are also suggestive of systematic components. Without a systematic component in the random supply, then in the limiting case, as the number of risky assets

becomes large (implying an infinite number of independent asset specific signals), prices would still be fully revealing of the informed investors' private signals. In other words, noisy supply is necessary but not sufficient to ensure that asymmetric information is not a moot issue in large economies; there also needs to be a systematic component. We further assume for simplicity that  $F_x$  is independent of the factors generating asset payoffs. We will comment on effects of relaxing this assumption later.

We assume that there are two classes of investors, informed and uninformed, with each class containing an infinite number of identical agents. The informed investors all receive private signal  $s$  on asset payoffs and the uninformed can only (imperfectly) infer the signal from market prices. This specification is used by Grossman and Stiglitz (1980) and Easley and O'Hara (2004). In Admati (1985), there are infinitely many agents each of whom receives independent signals. It can be argued that our assumption and Admati's are two special cases of a general information structure where investors have both diverse and asymmetric information: while we emphasize asymmetry, Admati emphasizes diversity. Technically speaking, the correlation between the private signals across informed investors is perfect in our model and zero in Admati's model. While in our analysis price will be a function of informed investors' private information, price is a function of the actual asset payoffs in Admati's case when the number of assets is infinite due to the elimination of signal noise through aggregation of signals across assets.

Noisy rational expectation equilibrium models with many assets having a factor structure in asset payoffs, but not in the random supply of risky assets, have been considered in Caballe and Krishnan (1994); Daniel, Hirshleifer, and Subrahmanyam (2001); Kodres and Pritsker (2002); and Pasquariello (2004).

We assume all investors have mean-variance utility

$$U = \mu - \frac{A}{2}\sigma^2, \quad (3)$$

where  $\mu$  is the mean portfolio payoff,  $\sigma^2$  is the variance of the portfolio payoff, and  $A$  is the investor's absolute risk aversion coefficient. Each investor faces a budget constraint:

$$W_1 = W_0 R_f + D'(\nu - R_f p), \quad (4)$$

where  $W_0$  is the investor's initial wealth,  $W_1$  is the investor's terminal wealth, and  $D$  is a vector containing the numbers of shares invested in risky assets. Given mean-variance utility, the investors' portfolio choice problem is

$$\begin{aligned} \max_D \quad & E[W_1|J] - \frac{A}{2}\text{var}[W_1|J], \\ \text{s.t.} \quad & W_1 = W_0 R_f + D'(\nu - R_f p), \end{aligned}$$

where  $J$  represents the investor's information set. The first-order condition implies optimal demand takes the following form:

$$D_J^* = \frac{1}{A} \Sigma_{\nu|J}^{-1} E[\nu - R_f p|J]. \quad (5)$$

When asset payoffs do not depend on systematic factors,  $\beta = 0$ , it is easy to show investors' demands for securities are increasing in expected asset payoffs and the precision of information about asset payoffs, and decreasing in relative risk aversion. In the more general case where asset payoffs do depend on systematic factors,  $\beta \neq 0$ , the demand for asset  $i$  depends not only on investors' posterior precision of beliefs on payoffs for asset  $i$ , but also on their posterior beliefs on payoffs for other assets. The informed and the uninformed have differential demand schedules because they condition on different information sets  $J$ .

## 2.2 The Informed Investors

The informed investors receive private signal  $s$  which takes the form

$$s = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2}\eta. \quad (6)$$

The  $N \times 1$  constant vector  $b$  reflects the relative information content of the signal with respect to the systematic factors and  $\eta$  is an  $N \times 1$  standard normal random variable. To conform with the interpretation of factor models, we will assume that  $F$ ,  $\epsilon$ ,  $\eta$ , and  $\eta_x$  are jointly normal and independent and the matrices  $\Sigma_s$  and  $\Sigma_x$  are diagonal.

Our specification of asset payoffs is distinct from an alternative specification where asset payoffs do not follow a factor structure, but satisfy a general variance-covariance matrix (e.g., Admati, (1985)). Though a factor structure such as (1) implies a specific variance-covariance matrix, a general variance-covariance matrix does not imply a corresponding factor structure. Admati (1985) entertains such constructions and concludes that a factor structure is a natural way to capture the idea that asset payoffs have both systematic and idiosyncratic components upon which private signals may be obtained. However, mathematical complexities prevented her from achieving an explicit solution.

The signal  $s$  for each risky asset specified in equation (6) is a linear combination of information about the systematic components of the asset's payoff, information about the idiosyncratic component of that payoff, and noise. The signal  $s$  can also be interpreted as a combination of two signals: a signal about the idiosyncratic component of asset payoffs,  $s_1 = \Sigma^{1/2}\epsilon + (bF + \Sigma_s^{1/2}\eta)$ , where  $bF + \eta$  as a whole can be interpreted as noise; and a signal about the systematic components,  $s_2 = bF + (\Sigma^{1/2}\epsilon + \Sigma_s^{1/2}\eta)$ , where  $(\Sigma^{1/2}\epsilon + \Sigma_s^{1/2}\eta)$  is interpreted as noise. The assumption that informed investors receive information not only about the

idiosyncratic component, but also about the systematic components of risky asset payoffs, although uncommon in the theoretical literature, is intuitive. Informed investors such as corporate insiders are likely to know more than the general public about the firm’s fundamentals such as earnings and cash flows. To the extent that the fundamentals are generated by a factor structure, the private information is likely to contain both components. Consistent with this assumption, Seyhun (1992) and Lakonishok and Lee (2001) show that aggregated trading by corporate insiders is predictive of future market returns.

Our specification of signals differs from Admati’s (1982). Besides signals in our model being perfectly correlated across informed investors, the “two signals” constructively received by informed investors in our model are correlated with covariance matrix  $\Sigma_s$  conditional on  $\nu$  and  $F$ , whereas the two signals for a given investor in Admati (1982) are uncorrelated. Assuming the two signals are uncorrelated, as in Admati’s signal specification, changes the expressions, but does not affect either the structure of the explicit solution or the qualitative results that follow from that solution<sup>4</sup>.

To calculate the conditional expectations and covariance matrixes, we need to derive the joint density function of  $\nu$  and  $F$  conditional on information  $s$ .

**Remark 1** *The moments of the joint distribution of  $\nu$  and  $F$  conditional on signal  $s$  are*

$$\begin{aligned} \mathbf{E}[\nu|s, F] &= \bar{\nu} + \beta F + \Sigma_{\nu|s, F} \Sigma_s^{-1} (s - bF), \\ \mathbf{E}[F|s] &= \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s, \\ \Sigma_{\nu|s, F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1} \end{aligned}$$

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<sup>4</sup>For example, we could assume one signal about systematic factors is received by all informed traders rather assume that signals for each asset include a systematic component.

$$\begin{aligned}\Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b'(\Sigma + \Sigma_s)^{-1}b \\ \hat{\Sigma}_s &= \Sigma + b\Sigma_F b' + \Sigma_s.\end{aligned}$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal  $s$ , the payoff is of the form

$$\nu = \bar{\nu} + \Sigma_{\nu|s,F}\Sigma_s^{-1}s + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)F + \Sigma_{\nu|s,F}^{1/2}\epsilon_{\nu|s,F}, \quad (7)$$

where, conditional on  $s$  and  $F$ ,  $\epsilon_{\nu|s,F}$  is a standard normal random variable. We note that from the perspective of an informed investor the factor loading on the systematic factors has become  $\beta_s = \beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b$ . The precision matrix of the factors has increased from  $\Sigma_F^{-1}$  to  $\Sigma_{F|s}^{-1} = \Sigma_F^{-1} + b'(\Sigma + \Sigma_s)^{-1}b$ .

From equation (7), the expectation of  $\nu$  conditional on  $s$  is

$$E[\nu|s] = \bar{\nu} + \Sigma_{\nu|s,F}\Sigma_s^{-1}s + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}b'(\Sigma + \Sigma_s)^{-1}s \quad (8)$$

and the variance of  $\nu$  conditional on  $s$  is

$$\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'. \quad (9)$$

Equations (8) and (9) can be substituted into the demand function to calculate the investors' demand  $D_s^*$  for risky assets:

$$D_s^* = \frac{1}{A}\Sigma_{\nu|s}^{-1}(\bar{\nu} + \Phi_s s - R_f p), \quad (10)$$

where

$$\Phi_s = \Sigma_{\nu|s,F}\Sigma_s^{-1} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}b'(\Sigma + \Sigma_s)^{-1}.$$

## 2.3 The Uninformed Investors

The uninformed investors do not observe the signal  $s$ , but can imperfectly infer  $s$  from the equilibrium price.

We conjecture that the equilibrium prices have the following form:

$$p = C + B(s - \lambda(x - \bar{x})),$$

where  $C$  is an  $N \times 1$  vector and  $B$  and  $\lambda$  are  $N \times N$  matrices. We will assume that  $B$  is invertible. Therefore, observing the price  $p$  is equivalent to observing  $\theta$  which is defined as

$$\theta = B^{-1}(p - C) = s - \lambda(x - \bar{x}).$$

Substituting equations (2) and (6), we can write

$$\theta = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2}\eta + \lambda\beta_x F_x + \lambda\Sigma_x^{1/2}\eta_x. \quad (11)$$

Therefore, we can interpret  $\theta$  as another signal which has sensitivity  $b$  to the factor  $F$  and "idiosyncratic" shocks with covariance matrix  $\Sigma_\theta$ , where

$$\Sigma_\theta = \Sigma_s + \lambda(\beta_x \Sigma_{F_x} \beta_x' + \Sigma_x) \lambda'.$$

Note that signal  $\theta$  is less informative than signal  $s$ , i.e., its conditional variance-covariance matrix is larger than that of  $s$ , i.e.,  $\Sigma_\theta = \Sigma_s + \lambda \Sigma_x \lambda' \geq \Sigma_s$ . We should remark that  $\lambda$  is in general non-diagonal; the "idiosyncratic" shocks  $\Sigma_s^{1/2}\eta + \lambda \Sigma_x^{1/2}\eta_x$ , although independent of  $F$ , are not independent of each other.

When systematic factors in the random supply are uncorrelated with systematic factors in asset payoffs, as we assumed, the signal  $s$  is a sufficient statistic for  $(s, \theta)$  ( $\theta$  is a garbling of  $s$ ). However, it is plausible that the two systematic factors are correlated. In this case, the signal  $s$  is no longer a sufficient statistic for  $(s, \theta)$ . While the uninformed will continue to condition on only  $\theta$ , the informed will now condition on both  $s$  and  $\theta$ , a departure from the above analysis in which the informed only conditioned on  $s$ . We assume independence for tractability. Nonetheless, we are confident that our analysis can be extended to accommodate

the case of correlated factors and that our results are robust with respect to the relaxation of the independence assumption. The crucial aspect for cost of capital to be affected by asymmetric information is whether the informed investors learn more about systematic factors that influence asset payoffs than uninformed investors in equilibrium; this can be modeled with or without the correlation between the two classes of systematic factors.

To calculate the conditional expectations and covariance matrixes, we need to derive the moments of the joint density function of  $\nu$  and  $F$  conditional on information  $\theta$ .

**Remark 2** *The moments of the joint distribution of  $\nu$  and  $F$  conditional on the signal  $\theta$  are*

$$\begin{aligned} \mathbb{E}[\nu|\theta, F] &= \bar{\nu} + \beta F + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} (\theta - bF), \\ \mathbb{E}[F|\theta] &= \Sigma_{F|\theta} b' (\Sigma + \Sigma_{\theta})^{-1} \theta, \\ \Sigma_{\nu|\theta, F}^{-1} &= \Sigma^{-1} + \Sigma_{\theta}^{-1} \\ \Sigma_{F|\theta}^{-1} &= \Sigma_F^{-1} + b' (\Sigma + \Sigma_{\theta})^{-1} b \\ \hat{\Sigma}_{\theta} &= \Sigma + b \Sigma_F b' + \Sigma_{\theta}. \end{aligned}$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal  $\theta$ , the payoff is of the form

$$\nu = \bar{\nu} + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} \theta + (\beta - \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b) F + \Sigma_{\nu|\theta, F}^{1/2} \epsilon_{\nu|\theta, F}, \quad (12)$$

where  $\epsilon_{\nu|\theta, F}$  is a standard normal random variable. We note that from the perspective of an uninformed investor the factor loading on the systematic factors has become  $\beta + \Sigma_{\nu|\theta, F} \Sigma_{\theta}^{-1} b$ . The precision matrix of the factors has increased from  $\Sigma_F^{-1}$  to  $\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + b' (\Sigma + \Sigma_{\theta})^{-1} b$ .



From equation (12), the expectation of  $\nu$  conditional on  $\theta$  is

$$E[\nu|\theta] = \bar{\nu} + \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}\theta + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}b'(\Sigma + \Sigma_{\theta})^{-1}\theta \quad (13)$$

and the variance of  $\nu$  conditional on  $\theta$  is

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|\theta,F} + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}(\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)'. \quad (14)$$

Equations (13) and (14) can be substituted into the demand function to calculate the uninformed investors' demand  $D_J^*$  for risky assets:

$$D_{\theta}^* = \frac{1}{A}\Sigma_{\nu|\theta}^{-1}(\bar{\nu} + \Phi_{\theta}\theta - R_f p), \quad (15)$$

where

$$\Phi_{\theta} = \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1} + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}b'(\Sigma + \Sigma_{\theta})^{-1}.$$

## 2.4 The Equilibrium

Imposing the market clearing condition that the total demand from the informed and the uninformed investors equals the supply, we obtain the following equation:

$$x = \frac{\mu}{A}\Sigma_{\nu|s}^{-1}(\bar{\nu} + \Phi_s s - R_f p) + \frac{1-\mu}{A}\Sigma_{\nu|\theta}^{-1}(\bar{\nu} + \Phi_{\theta}\theta - R_f p),$$

where  $\mu$  is the proportion of informed investors. Defining  $\bar{\Sigma}_{\nu} = \left(\mu\Sigma_{\nu|s} + (1-\mu)\Sigma_{\nu|\theta}\right)^{-1}$ , we derive the following expression for the prices of risky assets:

$$\begin{aligned} p &= \frac{1}{R_f} \left( \bar{\nu} + \bar{\Sigma}_{\nu} \left( \mu\Sigma_{\nu|s}^{-1}\Phi_s s + (1-\mu)\Sigma_{\nu|\theta}^{-1}\Phi_{\theta}\theta - Ax \right) \right) \\ &= \frac{1}{R_f} (\bar{\nu} - \bar{\Sigma}_{\nu} A \bar{x}) + \frac{1}{R_f} \bar{\Sigma}_{\nu} \mu \Sigma_{\nu|s}^{-1} \Phi_s \left( s - \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s \right)^{-1} A (x - \bar{x}) \right) \\ &+ \frac{1}{R_f} \bar{\Sigma}_{\nu} (1-\mu) \Sigma_{\nu|\theta}^{-1} \Phi_{\theta} (s - \lambda(x - \bar{x})). \end{aligned} \quad (16)$$

Comparing the above expression to the conjectured form of the price  $p$ , it must be true that

$$\lambda = (\mu \Sigma_{\nu|s}^{-1} \Phi_s)^{-1} A. \quad (17)$$

Note that  $\lambda$  is solved in terms of the parameters of the model. The matrices  $\Sigma_{\nu|\theta}$ ,  $\Phi_\theta$ , and  $\bar{\Sigma}_\nu$  are expressed in terms of  $\lambda$  as well as the parameters of the model; they are solved once  $\lambda$  is solved.

**Theorem 1** *Given that informed investors receive a private signal,  $s$ , that is informative about both idiosyncratic and systematic components of asset payoffs, a partially revealing noisy rational expectations equilibrium exists, and prices of risky assets satisfy*

$$p = \frac{1}{R_f} \bar{\nu} - \frac{1}{R_f} \bar{\Sigma}_\nu A \bar{x} + \frac{1}{R_f} \bar{\Sigma}_\nu \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right) (s - \lambda(x - \bar{x})) \quad (18)$$

*This equation confirms the conjectured form of the price*

$$p = C + B(s - \lambda(x - \bar{x})),$$

*where  $C = \frac{1}{R_f} (\bar{\nu} - \bar{\Sigma}_\nu A \bar{x})$  and  $B = \frac{1}{R_f} \bar{\Sigma}_\nu \left( \mu \Sigma_{\nu|s}^{-1} \Phi_s + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \Phi_\theta \right)$ .*

*The ex ante price,  $\bar{p}$ , is*

$$\bar{p} = \frac{1}{R_f} (\bar{\nu} - A \bar{\Sigma}_\nu \bar{x});$$

*and the ex ante risk premium of assets satisfies*

$$\bar{\nu} - R_f \bar{p} = A \bar{\Sigma}_\nu \bar{x} = A \left( \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} \bar{x}. \quad (19)$$

**Proof:** The price  $p$  and the expressions for  $B$  and  $C$  are derived by combining the equations (16) and (17). The ex ante price (i.e., the price before the signal  $s$  is revealed) is obtained by taking the unconditional average of equation (18).

The equation for the ex ante risk premium immediately follows. Note that the posterior precisions  $\Sigma_{\nu|s}^{-1}$  and  $\Sigma_{\nu|\theta}^{-1}$  do not depend on realizations of signals  $s$  and  $\theta$ , respectively.

The first term in the price  $p$  is the expected payoff without signals discounted by the risk-free return. This is the price if investors are risk-neutral ( $A = 0$ ) and there are no signals in the economy. The second term is the average discount in price associated with risk when there are no signals in the economy. The sum of the first two terms,  $\frac{1}{R_f}\bar{p} - \frac{1}{R_f}\bar{\Sigma}_\nu A\bar{x}$ , is the average price. The third term is associated with signals and noisy supply. The price of an asset will be higher than its average if either there is a positive signal ( $s > 0$ ) or a below-average supply ( $x < \bar{x}$ ).

The ex ante risk premium (referred to as the risk premium) is determined by the geometric average of the covariance matrices of asset payoffs conditional on  $s$  and  $\theta$ ,  $\Sigma_{\nu|s}$  and  $\Sigma_{\nu|\theta}$ . That is, the risk premium compensates the average of the risks conditional on  $s$  and  $\theta$ . Two properties of the risk premium follow. First, from equation (9),  $\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'$  and similarly for  $\Sigma_{\nu|\theta}$ , the average risk includes idiosyncratic risk  $\Sigma_{\nu|s,F}$  and  $\Sigma_{\nu|\theta,F}$ . Therefore, idiosyncratic risks are priced. Second, the average covariance matrix,  $\bar{\Sigma}_\nu$  depends on  $\beta$  nonlinearly, thus the risk premium depends on  $\beta$  nonlinearly.

To present more concrete picture of the equilibrium properties, we next provide some examples.

## 2.5 Special Cases

### 2.5.1 No Information: $\mu = 0$

There are only uninformed investors when  $\mu = 0$ . In this case,  $\lambda \rightarrow \infty$ , the inferred signal  $\theta$  is infinitely more noisy than  $s$  and thus is not informative at all.

It follows immediately that the covariance matrix conditional on  $\theta$ ,  $\Sigma_{\nu|\theta}$ , is the same as  $\Sigma$  and the factor covariance matrix conditional on  $\theta$ ,  $\Sigma_{F|\theta}$ , is the same as  $\Sigma_F$ . Furthermore, beta conditional on  $\theta$  does not change. From Theorem 1, the risk premium is

$$\bar{\nu} - R_f \bar{p} = \Sigma_{\nu|\theta} A \bar{x} = \left( \Sigma + \beta \Sigma_F \beta' \right) A \bar{x}. \quad (20)$$

The above is the risk premium in an economy with no signals, private or contained in price, and thus no updating of beliefs, as expected. The first term in the parentheses is the risk premium for idiosyncratic risk and the second term is the risk premium for the systematic risk. In this case, the idiosyncratic risk is priced but  $\beta$  appears linearly in the risk premium.

### 2.5.2 Symmetric Information: $\mu = 1$

All investors are informed when  $\mu = 1$ . An application of Theorem 1 implies that the risk premium in this case is

$$\bar{\nu} - R_f \bar{p} = \Sigma_{\nu|s} A \bar{x} = \left( \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b) \Sigma_{F|s} (\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b)' \right) A \bar{x}.$$

Similar to the previous case, the idiosyncratic risk is priced. However, the risk premium depends on the beta conditional on  $s$ ,  $\beta - \Sigma_{\nu|s,F} \Sigma_s^{-1} b$ . In such an economy, an econometrician who observes the return but not the signal will conclude that the risk premium depends on  $\beta$  as well as some firm specific characteristics,  $\Sigma_{\nu|s,F} \Sigma_s^{-1} b$ . Thus, firms with the same  $\beta$  but different  $\Sigma_{\nu|s,F} \Sigma_s^{-1} b$  may have different expected returns. This economy seems potentially to provide a theory for the empirical findings of Daniel and Titman (1998).

### 2.5.3 Identically Distributed Risky Asset Payoffs

We now allow for the presence of both informed and uninformed investors and assume the following: identically distributed risky asset payoffs and related signals: i.e., the covariance matrices of the payoffs, signals, and the random supply are all proportional to the identity matrix; the betas of all risky asset payoffs are equal; and the sensitivities of the signals to the factor (we assume for convenience that the number of factors is one) are equal. The case of identically distributed risky asset payoffs allows explicit computations while preserving most of the intuition applicable to more general cases.

Let  $1_{N \times M}$  denote the  $N \times M$  matrix with all elements being 1 and  $I_N$  denote the identity matrix of dimension  $N$ . We will abuse the notation and denote:  $\bar{\nu} = \bar{\nu}1_{N \times 1}$ ,  $\beta = \beta 1_{N \times 1}$ ,  $b = \frac{k}{\sqrt{N}}1_{N \times 1}$ ,  $\beta_x = \beta_x 1_{N \times 1}$ ,  $\Sigma = \sigma^2 I_N$ ,  $\Sigma_s = \sigma_s^2 I_N$ ,  $\Sigma_x = \sigma_x^2 I_N$ ,  $\Sigma_F = \sigma_f^2$ . In this example, we expressed parameters with some scaling by powers of  $N$  for use later in taking the large  $N$  limit. For example, in the large  $N$  limit, we expect  $\beta$  to be independent of  $N$ , but that  $b$  will go to zero as  $\frac{1}{\sqrt{N}}$  does.

We require various formulae for the identical asset case. The derivation of these formulae are given in the Appendix as part of the proof of Corollary 1 below.

The beta conditional on  $s$  is  $\beta_s = \beta - \frac{\sigma^2}{\sigma^2 + \sigma_s^2} b$ , which can be larger or smaller than  $\beta$ , depending on the sign of  $b$ . The covariance matrices conditional on  $s$  are

$$\begin{aligned}\Sigma_{\nu|s,F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\ \Sigma_{F|s} &= (\sigma_f^{-2} + Nb^2(\sigma^2 + \sigma_s^2)^{-1})^{-1} = (\sigma_f^{-2} + k^2(\sigma^2 + \sigma_s^2)^{-1})^{-1} \equiv \sigma_{fs}^2.\end{aligned}$$

The conditional covariance matrix  $\Sigma_{\nu|s,F}$  is the same as the one in the standard case of no correlation. The covariance matrix of the factor,  $\Sigma_{F|s}$ , conditional on  $s$ , is smaller than the factor covariance matrix without information,  $\sigma_f^2$ . Keeping  $b$

fixed, the larger the number of assets, the smaller the conditional factor covariance matrix. The explicit  $N$  dependence is due to the assumption of identical assets.

The covariance matrix of the asset payoff conditional on  $s$ ,  $\Sigma_{\nu|s}$ , is given by

$$\begin{aligned}\Sigma_{\nu|s} &= (\sigma^{-2} + \sigma_s^{-2})^{-1}I_N + (\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1}\sigma_s^{-2}b)^2\Sigma_{F|s}1_{N \times 1}1_{1 \times N} \\ &= S_0I_N + S_11_{N \times N},\end{aligned}$$

where

$$\begin{aligned}S_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1}, \\ S_1 &= \frac{\left(\beta - \frac{1}{\sqrt{N}}\frac{\sigma^2}{\sigma^2 + \sigma_s^2}k\right)^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1}k^2} = \sigma_{f_s}^2\beta_s^2.\end{aligned}$$

$S_0$  is the idiosyncratic variance conditional on  $s$  and  $(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1}k^2)^{-1}$  is the factor variance conditional on  $s$ . Both are smaller than their counterparts with no information. However, the total systematic risk which is the product of the beta and factor risk conditional on  $s$ , given by  $S_1 = \sigma_{f_s}^2\beta_s^2$ , can be greater than the total systematic risk without information, given by  $\sigma_f^2\beta^2$ , if the  $\beta_s^2 \geq \beta^2$ . For example, this happens if  $\beta = 0$  but  $k \neq 0$ . In this case, there is no factor risk in the payoffs and thus the systematic risk is zero without information; the signal introduces factor risk into the payoffs conditional on the signal if the signal has a factor component.

One can verify that the matrix  $\Phi_s$  is given by

$$\Phi_s = \frac{\sigma^2}{\sigma^2 + \sigma_s^2}I_N + \frac{1}{\sqrt{N}}\sigma_{f_s}^2\beta_s k(\sigma^2 + \sigma_s^2)^{-1}1_{N \times N}.$$

The  $\lambda$  matrix is given by

$$\lambda = A\mu^{-1}\Phi_s^{-1}\Sigma_{\nu|s} = \frac{1}{N}\lambda_0I_N + \frac{1}{\sqrt{N^3}}\lambda_1I_{N \times N},$$

where

$$\begin{aligned}\lambda_0 &= NA\mu^{-1}\sigma_s^2 = \gamma\mu^{-1}\sigma_s^2 \\ \lambda_1 &= \gamma\mu^{-1}\frac{\beta_s(\sigma^2 + \sigma_s^2) - \frac{1}{\sqrt{N}}\sigma_s^2 k}{k + \frac{1}{\sqrt{N}}\frac{\sigma^2}{\sigma_{fs}^2\beta_s}}.\end{aligned}$$

We used the notation  $\gamma = \frac{A}{N}$ . The signal  $\theta$  can be written as

$$\begin{aligned}\theta &= s - \lambda(x - \bar{x}) \\ &= s - \left(\frac{\lambda_0}{N} + \frac{\lambda_1}{\sqrt{N}}\right)\beta_x 1_{N \times 1} F_x - \left(\frac{\lambda_0}{N}\sigma_x \eta_x + \frac{\lambda_1}{\sqrt{N}}\sigma_x \hat{\eta}_x I_{N \times 1}\right),\end{aligned}$$

where  $\hat{\eta}_x \equiv \frac{1'\eta_x}{N}$  is the sample average of  $\eta_x$ . The covariance matrix of  $\theta$  is given by

$$\Sigma_\theta = \sigma_{\theta 0}^2 I_N + \frac{1}{N}\sigma_{\theta 1}^2 1_{N \times N},$$

where

$$\begin{aligned}\sigma_{\theta 0}^2 &= \sigma_s^2 + \frac{\lambda_0^2}{N^2}\sigma_x^2; \\ \sigma_{\theta 1}^2 &= \left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}}\right)^2 \sigma_{fx}^2 \beta_x^2 + \left(\frac{\lambda_1^2}{N} + \frac{2\lambda_0\lambda_1}{N^{3/2}}\right)\sigma_x^2.\end{aligned}$$

Note that  $\sigma_{\theta 0}^2$ , the idiosyncratic variance conditional on  $(\theta, F)$ , is larger than  $\sigma_s$  because of the idiosyncratic random supply term in  $\theta$ ,  $-\frac{\lambda_0}{N}\sigma_x \eta_x$ , and that  $\Sigma_\theta$  is not diagonal due to the systematic component in the random supply. The covariance matrix of the payoff conditional on  $(\theta, F)$  also has systematic terms,  $\frac{1}{N}\sigma_{\theta 1}^2 1_{N \times N}$ , due to the systematic component,  $F_x$ , in the random supply which gives rise to  $\left(\lambda_1 + \frac{\lambda_0}{\sqrt{N}}\right)^2 \sigma_{fx}^2 \beta_x^2$ , as well as the idiosyncratic component,  $\eta_x$ . The idiosyncratic component  $\eta_x$  creates a correlation between assets from the variance of  $\hat{\eta}_x = \frac{1'}{N}\eta_x$  (the  $\frac{\lambda_1^2}{N}\sigma_x^2$  term) and its correlation with  $\eta_x$  (the  $-\frac{2\lambda_0\lambda_1}{N^{3/2}}\sigma_x^2$  term).

The covariance matrix of payoffs conditional on  $(\theta, F)$  is

$$\Sigma_{\nu|\theta, F} = (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left( I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2 (\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} 1_{N \times N} \right).$$

This covariance matrix is also non-diagonal due to the systematic component in the random supply. The factor covariance conditional on  $\theta$  is

$$\Sigma_{F|\theta} = (\Sigma_F^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b)^{-1} = \left( \sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^{-1} \equiv \sigma_{f\theta}^2.$$

Because  $\sigma_s^2 < \sigma_{\theta 0}^2$ , the variance of the payoff factor conditional on  $\theta$  is larger than that conditional on  $s$ , as expected. The beta conditional on  $\theta$  is

$$\beta_\theta = \beta - \Sigma_{\nu|\theta, F} \Sigma_\theta^{-1} b = \left( \beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1}.$$

We point out that  $\beta_\theta$  can be smaller or greater than  $\beta$ , depending on the sign of  $k$ . Furthermore, the absolute magnitude of  $\beta_\theta - \beta$  is smaller than  $\beta_s - \beta$  because the uninformed are less confident about the signal than the informed. The covariance matrix of the asset payoffs conditional on  $\theta$  is given by

$$\Sigma_{\nu|\theta} = \Theta_0 I_N + \Theta_1 I_{N \times N},$$

where

$$\begin{aligned} \Theta_0 &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1}, \\ \Theta_1 &= \sigma_{f\theta}^2 \left( \beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^2 + \frac{1}{N} \frac{\sigma^4 \sigma_{\theta 1}^2}{(\sigma^2 + \sigma_{\theta 0}^2)(\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)}. \end{aligned}$$

Note that the idiosyncratic covariance conditional on  $\theta$ ,  $\Theta_0$ , is greater than the idiosyncratic covariance conditional on  $s$ ,  $S_0$ , as expected. However, the systematic covariance conditional on  $\theta$ ,  $\Theta_1$ , may be smaller or greater than the systematic covariance conditional on  $s$ , even though the factor risk conditional on  $\theta$ ,  $\sigma_{f\theta}^2$ , is always greater than the factor risk conditional on  $s$ .

**Corollary 1** *Given identically distributed risky asset payoffs, the risk premium is*

$$\begin{aligned} \bar{\nu} - R_f \bar{p} &= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\times \left( 1 + N \frac{\mu S_0^{-1} \left( \frac{S_0}{N S_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{N \Theta_1} + 1 \right)^{-1}}{\mu S_1^{-1} \left( \frac{S_0}{N S_1} + 1 \right)^{-1} + (1 - \mu) \Theta_1^{-1} \left( \frac{\Theta_0}{N \Theta_1} + 1 \right)^{-1}} \right) \bar{x} I_{N \times 1}. \end{aligned}$$



The proof is given in the Appendix. As in the two previous cases, idiosyncratic risks are priced. Furthermore, the average risk premium depends on  $\beta$  in a highly non-linear way.

Figures 1 and 2 plot the risk premium against the fraction of the informed investors for various numbers of risky assets and variances of the systematic component of the random supply, respectively. The risk premium decreases with  $N$  due to the diversification effect. The risk premium is more sensitive to the fraction of informed investors for a larger number of assets. When the fraction of informed investors is increased, the risk premium is decreased, implying that more information in the sense of more investors receiving the private signal always reduces the cost of capital. This latter point is not completely obvious. Although the factor covariance conditional on a signal that is informative about the factor is smaller than the factor covariance without a signal, the beta conditional on a signal can be larger than the beta without a signal. Accordingly, the systematic risk conditional on a signal, which is the product of the factor covariance and the beta, can be greater than its counterpart without a signal. Nevertheless, the risk premium conditional an informative signal is always smaller than the risk premium without a signal.

In an economy with a finite number of assets, we conclude that: idiosyncratic as well as systematic risk is priced; information on idiosyncratic shocks reduces idiosyncratic risk and hence the risk premium; information can increase or decrease beta; the risk premium depends on beta nonlinearly; and information on the systematic factor reduces systematic risk and hence the risk premium. As we will show next, in the limit, as the number of assets goes to infinity, only the last property survives.

### 3 The Diversification Limit

In this section we study the effects of asymmetric information on price and the cost of capital when the economy is large in the sense that the number  $N$  of the risky assets is large; i.e., in the limit as  $N \rightarrow \infty$ , which we call the diversification limit. We consider various scenarios in which information does or does not affect the cost of capital in the diversification limit. In particular, we will show that only systematic risk is priced, only information associated with the systematic factor reduces the risk premium, and beta is unaffected by information in this limit.

In order to address caveats concerning the implications of constant absolute risk aversion for risk premiums in the diversification limit, we will consider the case of identically distributed asset payoffs and no information, i.e.,  $\mu = 0$ . From equation (20), we get

$$\bar{v} - R_f \bar{p} = (\sigma^2 + \beta^2 \sigma_f^2 N) A \bar{x}.$$

There are two problems with the above risk premium. First, the risk premium depends on the idiosyncratic volatility  $\sigma^2$ ; in other words, the idiosyncratic risk is priced. The more serious problem is that the risk premium goes to infinity as  $N \rightarrow \infty$ . Given that investors are assumed to have constant absolute risk aversion, investors in this case are so risk averse that even the “small” idiosyncratic risk is priced with a non-zero risk premium and the “big” systematic risk is priced with an infinite risk premium.

Alternatively, one could assume constant relative risk aversion. However, as pointed out by Dybvig (1983) and Grinblatt and Titman (1983), investors with constant relative risk aversion price idiosyncratic risk demanding a finite risk premium. For idiosyncratic risk not to be priced, the absolute risk aversion coefficient of the investors should approach zero when the wealth is high. Ross (1976) deals

with this issue by assuming that relative risk aversion is bounded. We achieve a similar result in our formulation by assuming the absolute risk aversion coefficient  $A$  to be inversely proportional to  $N$ ,

$$A(N) = \gamma/N,$$

where the constant  $\gamma$  is proportional to the relative risk aversion coefficient. Under this assumption, the risk premium for the case of no information is given by

$$\bar{\nu} - R_f \bar{p} = \gamma \beta^2 \sigma_f^2 \bar{x}.$$

The above risk premium does not price the idiosyncratic risk and produces a finite premium in the diversification limit.

### 3.1 Information on Only Idiosyncratic Components of Asset Payoffs

We now study the case when the informed investors receive imperfect private information about just the idiosyncratic components of risky asset payoffs. In this case,  $b = 0$  and the signal can be written as

$$s = \nu - \bar{\nu} - \beta F + \Sigma_s^{1/2} \eta. \tag{21}$$

Note that when  $\beta \neq 0$ , the asset payoffs are correlated. In the special case where all asset payoffs are uncorrelated, i.e.,  $\beta = 0$ , this structure reduces to the setting considered by Easley and O'Hara (2004). It is easy to see that, for finite  $N$ , the information asymmetry about idiosyncratic factors matters because of the terms  $\Sigma_{\nu|J,F}$ . This result, similar to that of Easley and O'Hara (2004), is not surprising because all idiosyncratic risk matters if we do not take the diversification limit.

We have the following proposition summarizing the limiting behavior of the risk premium:

**Proposition 1** *Given that the informed investors receive a private signal only about the idiosyncratic components of asset payoffs, in the limit as  $N \rightarrow \infty$ , the risk premium satisfies*

$$\bar{v} - R_f \bar{p} = \gamma \beta \Sigma_F \beta' \bar{x} / N. \quad (22)$$

The proof is given in the Appendix. Note that  $\beta' \bar{x}$  is of order  $N$  and hence  $\beta' \bar{x} / N$  is of order 1 when  $N \rightarrow \infty$ . Thus, we have a finite risk premium. Note that the proposition holds notwithstanding systematic components in the random supply,  $\beta_x \sigma_{fx} \neq 0$ . This result is quite intuitive: Even if all the agents are informed,  $\mu = 1$ , there is no resolution of uncertainty about the factor that affects asset payoffs, implying that the random supply of assets is irrelevant for asset pricing. The risk premium in this case is the same as the risk premium without information,  $\mu = 0$ ,  $\gamma \beta \Sigma_F \beta' \bar{x} / N$ , implying that this is the risk premium for all  $\mu$ .

The above proposition shows that when the private signal is only informative about the idiosyncratic component of a risky asset's payoff, the asset's risk premium is unaffected by the information asymmetry. In other words, the risk posed by asymmetric information on purely idiosyncratic shocks is fully diversifiable. It is easy to verify that the risk premium in (22) is no different from the risk premium obtained in the standard setting where investors have homogeneous beliefs. Furthermore, in the setting studied by Easley and O'Hara (2004),  $\beta = 0$ , and the average risk premium is reduced to zero, i.e.,  $\bar{v} - R_f p = 0$ .

More generally, we expect that the same results will hold as long as  $b'(\Sigma + \Sigma_s)^{-1} b \rightarrow 0$  and  $b'(\Sigma + \Sigma_\theta)^{-1} b \rightarrow 0$  when  $N \rightarrow \infty$ . Intuitively, diversification works at the power of  $1/N$ , implying that if the systematic component of the signal has a power less than  $1/N$ , then it will be eliminated by diversification.

We observe that although ex ante the information in this case does not change

the risk premium, ex post information does affect the price of an asset as the price is linear in signal  $s$ . Similarly, the information also affects the portfolio holdings and expected utility of both informed and uninformed investors.

### 3.2 Information on Total Risky Asset Payoffs

We now consider the case when the informed investors receive a private signal about total asset payoffs. In this case,  $b = -\beta$  and the signal can be written as

$$s = \nu - \bar{\nu} + \Sigma_s^{1/2}\eta. \quad (23)$$

This is the special case of Admati (1985), when the covariance matrix of the assets has the form of a factor structure and the signals between different assets are uncorrelated.

In this case,  $\Sigma_{F|s}^{-1} = \Sigma_F^{-1} + \beta'(\Sigma + \Sigma_s)^{-1}\beta$ , which goes to infinity as  $N \rightarrow \infty$ . Therefore, we have

$$\Sigma_{F|s} = 0.$$

Similarly,  $\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + (\beta + \beta_x)'(\Sigma + \Sigma_\theta)(\beta + \beta_x)$ , which also goes to infinity as long as  $\beta + \beta_x$  goes to a constant as  $N \rightarrow \infty$ ; thus we also have

$$\Sigma_{F|\theta} = 0.$$

It is easy to show that the above two equations imply that the average risk premium is zero.

The intuition here is also clear. Infinitely many signals about asset payoffs reveal the systematic factor  $F$  completely and thus eliminate the risk associated with that factor. Therefore, conditional on  $s$  or  $\theta$ , all the risks are idiosyncratic and the cost of capital in this case is the risk-free rate. More generally, as long as

$b'(\Sigma + \Sigma_s)b \rightarrow \infty$  and  $b'(\Sigma + \Sigma_\theta)b \rightarrow \infty$ , the cost of capital will be the risk-free rate.

### 3.3 Information on Systematic and Idiosyncratic Components of Risky Asset Payoffs

We have considered the cases where  $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b) \rightarrow 0$  and  $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b) \rightarrow \infty$ . We now consider the cases where the limit of  $(b'(\Sigma + \Sigma_s)^{-1}b, b'(\Sigma + \Sigma_\theta)^{-1}b)$  is a non-zero finite constant; what we call finite aggregate precision. This happens, for instance, if the elements of  $\sqrt{N}b$  go to a non-zero constant when  $N \rightarrow \infty$ .

In this case, in addition to information about the idiosyncratic component of a firm's asset payoffs, informed investors also receive firm-level information about the systematic factor. We will show that the risk premium will be affected by the latter information. Intuitively, the private signal is informative about both the systematic and the idiosyncratic components of asset payoffs. While any risk associated with the private information about the idiosyncratic component is fully diversified, the private information about the systematic factor has an impact on the risk premium in equilibrium. Since the effect on the equilibrium risk premium attributable to the informed investors is different from the effect attributable to the uninformed investors, the fraction of the informed investors in the economy plays an important role in the determination of that risk premium.

#### 3.3.1 Special(ID)Case: Identically Distributed Risky Asset Payoffs

For concreteness, we will first revisit the (ID) case of identically distributed risky asset payoffs. Recall that in this case, all risky asset payoffs have the same  $\beta$ , same

sensitivities  $b$  and  $\beta_x$ , and same idiosyncratic variance. Therefore, all risky asset payoffs have the same distribution; however, these distributions are not independent if either  $\beta \neq 0$ , or  $b \neq 0$ , or  $\beta_x \neq 0$ . We can take the  $N \rightarrow \infty$  limit using the results of subsection 2.5.3. In this case,

$$\begin{aligned}\Sigma_{\nu|s,F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\ \Sigma_{F|s} &= (\sigma_f^{-2} + k^2(\sigma^2 + \sigma_s^2)^{-1})^{-1},\end{aligned}$$

Therefore,

$$\Sigma_{\nu|s} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N + \frac{\beta^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2} 1_{N \times N}.$$

It follows that

$$\Phi_s = \frac{\sigma^2}{\sigma^2 + \sigma_s^2} I_N + \frac{1}{\sqrt{N}} \frac{\beta k}{(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2)(\sigma^2 + \sigma_s^2)} 1_{N \times N}.$$

Therefore,

$$\lambda = \frac{1}{N} \gamma \mu^{-1} \sigma_s^2 I_N + \frac{1}{\sqrt{N^3}} \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} I_{N \times N}.$$

The signal  $\theta$  can be written as

$$\theta = s - \frac{1}{\sqrt{N}} \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \beta_x F_x 1_{N \times 1},$$

where terms involving  $\eta_x$  become negligible. The covariance matrix conditional on  $\theta$  is

$$\Sigma_\theta = \sigma_s^2 I_N + \frac{1}{N} \left( \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \right)^2 \sigma_{fx}^2 \beta_x^2 I_{N \times N}.$$

Therefore, the uncertainty of  $\theta$  due to idiosyncratic components is the same as that of  $s$ . The covariance matrix of payoffs conditional on  $(\theta, F)$  is given by

$$\Sigma_{\nu|\theta,F} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N.$$

This covariance matrix is diagonal since the non-diagonal elements due to the systematic component disappear.

The factor covariance conditional on  $\theta$  is

$$\Sigma_{F|\theta} = \left( \sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_s^2 + \left( \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \beta_x \sigma_{fx} \right)^2} \right)^{-1}.$$

Note that the variance of the payoff factor conditional on  $\theta$  is larger than that conditional on  $s$ , as expected. The beta conditional on  $\theta$  is  $\beta$ ; that is, it is unchanged in the diversification limit. The covariance matrix of risky asset payoffs conditional on  $\theta$  is

$$\Sigma_{\nu|\theta} = (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N + \left( \sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_s^2 + \left( \gamma \mu^{-1} \frac{\beta(\sigma^2 + \sigma_s^2)}{k} \right)^2 \sigma_{fx}^2 \beta_x^2} \right)^{-1} \beta^2 \mathbf{1}_{N \times N}.$$

The risk premium in the large  $N$  limit is

$$\begin{aligned} & \gamma \sigma_f^2 \beta^2 \bar{x} \left( 1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2} \left( \mu + \frac{1 - \mu}{1 + (\sigma^2 + \sigma_s^2) \left( \frac{\gamma \beta \sigma_{fx} \beta_x}{\mu k} \right)^2} \right) \right)^{-1} \mathbf{1}_{N \times 1} \\ &= \gamma \sigma_f^2 \beta^2 \bar{x} \left( 1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2} \right)^{-1} \left( \mu + (1 - \mu) \frac{1 + \frac{\sigma_f^2 k^2}{(\sigma^2 + \sigma_s^2) + (\sigma^2 + \sigma_s^2)^2 \left( \frac{\gamma \beta \sigma_{fx} \beta_x}{\mu k} \right)^2}}{1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2}} \right)^{-1} \mathbf{1}_{N \times 1}. \end{aligned}$$

The first factor,  $\gamma \sigma_f^2 \beta^2 \bar{x}$ , is the risk premium without information. It depends on the risk aversion, the beta, and the factor variance.

The risk premium when all investors are informed is given by  $\gamma \sigma_f^2 \beta^2 \bar{x} \left( 1 + \frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2} \right)^{-1}$ ; it decreases with the systematic sensitivity  $k$  of the signal to the factor and increases with the variance of the payoff  $\sigma^2$  and variance of the signal  $\sigma_s^2$ . In the non-ID case, the term  $\frac{\sigma_f^2 k^2}{\sigma^2 + \sigma_s^2}$  corresponds to  $k'(\Sigma + \Sigma_s)^{-1}k$ , which is the aggregate information on the factor  $F$ .



The factor  $\mu + \frac{1-\mu}{1+(\sigma^2+\sigma_s^2)\left(\frac{\gamma\beta\sigma_{fx}\beta_x}{\mu k}\right)^2}$  determines the effect of asymmetric information, and  $\mu$  and  $\beta_x\sigma_{fx}$  only affect this term. Therefore, when either  $\mu$  or  $\beta_x\sigma_{fx}$  changes while keeping other parameters fixed, only the asymmetries of the information structure are affected while the risk premiums for the cases where no investors are informed ( $\mu = 0$ ) and all investors are informed ( $\mu = 1$ ) do not change.

If  $\beta_x\sigma_{fx} = 0$ , the risk premium reduces to that of the case with  $\mu = 1$ . This is due to the fact that the price fully reveals the signal,  $s$ , in the large  $N$  limit if there are no systematic components in the random supply. Note also that  $\sigma_x$  does not appear in the formula, because the idiosyncratic component of the random supply is diversified away.

When  $k = 0$ , the information is only idiosyncratic and the risk premium reduces to that of the case with no information,  $\gamma\sigma_f^2\beta^2\bar{x}$ , even if  $\beta_x\sigma_{fx} \neq 0$ .

As expected, the risk premium decreases with the fraction of the informed  $\mu$ , the accuracy of the information,  $1/\sigma_s^2$ , and the strength of the signal  $k^2$ ; it increases with  $\beta$ , the volatility  $\sigma$ , risk aversion  $\gamma$ ,  $\beta_x$ , and  $\sigma_{fx}$ . Again, we find the results to be intuitive. Although parameters that characterize the idiosyncratic properties of risky asset payoffs or information, such as  $\sigma^2$  or  $\sigma_s^2$ , enter into the factor risk premium, neither is idiosyncratic risk priced, nor does the resolution of uncertainty regarding idiosyncratic shocks per se affect the risk premium.

### 3.3.2 General Case

For the general case with finite aggregate precision, the risk premium is given by the following proposition.

**Proposition 2** *Given that informed investors receive private signals informative about both idiosyncratic and systematic components of asset payoffs and finite ag-*

gregate precision, in the limit as  $N \rightarrow \infty$ , the (ex ante) risk premium is

$$\gamma\beta \left( \mu\Sigma_{F|s}^{-1} + (1 - \mu)\Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta'\bar{x}}{N}$$

and the factor risk premium is

$$\gamma \left( \mu\Sigma_{F|s}^{-1} + (1 - \mu)\Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta'\bar{x}}{N}.$$

The proof is given in the Appendix.

We can arrange the risk premium as follows

$$\beta \left( \mu + (1 - \mu)\Sigma_{F|s}\Sigma_{F|\theta}^{-1} \right)^{-1} (I_K + \Sigma_F k'(\Sigma + \Sigma_s)^{-1}k)^{-1} \left( \gamma\Sigma_F \frac{\beta'\bar{x}}{N} \right).$$

Similarly to the ID case, the term  $\Sigma_F^{-1}\gamma\frac{\beta'\bar{x}}{N}$  determines the risk premium without information; it does not contain parameters that characterize information structure. The term  $(I_K + \Sigma_F k'(\Sigma + \Sigma_s)^{-1}k)^{-1}$  determines the effects of information in reducing the factor risk premium; it does not contain parameters that characterize the asymmetric structure of the information. The term  $\left( \mu + (1 - \mu)\Sigma_{F|s}\Sigma_{F|\theta}^{-1} \right)^{-1}$  determines the effects of information asymmetry in increasing the risk premium.

As in the ID case, in the limit as the number of risky assets goes to infinity, neither is idiosyncratic risk priced, nor does information on idiosyncratic shocks affect the risk premium. Rather, the risk premium is entirely determined by beta. In fact, it is proportional to beta, as is the risk premium in the case without information; i.e., the APT (Ross, 1976) holds. Information about the systematic factors in asset payoffs and the systematic component in the random supply affect the covariances of the factors. The risk premium is proportional to the geometric average of the factor covariance matrices conditional on  $s$  and  $\theta$ ,  $\Sigma_{F|s}$  and  $\Sigma_{F|\theta}$ . Since  $\Sigma_{F|s} < \Sigma_{F|\theta}$ , the cost of capital decreases with the fraction of the informed; in particular, the cost of capital with information is always smaller than the cost

of capital without information. As made evident within the proof of Proposition 2 contained in the Appendix, without a systematic component in the random supply, the conditional factor covariance matrices would be equal, consistent with our earlier claim that prices would then fully reveal the informed investors' private signal. Hence, the systematic component in the random supply plays a crucial role in our extension of the APT to a case with heterogeneous beliefs. We further observe that beta is not affected by information (asymmetric or symmetric) in the large  $N$  limit.

### 3.4 Cross-sectional Predictions

The analysis in foregoing subsections 3.1-3.3 leads to the following proposition on the cross-sectional relationship between information and cost of capital in the diversification limit:

**Proposition 3** *In the limit as  $N \rightarrow \infty$ , there are no cross-sectional effects of information on cost of capital.*

The intuition for each of the special cases encompassed by the above result is straightforward. When as in 3.1, information on only idiosyncratic components of risky asset payoffs is conveyed by private signals and no information about the systematic factor, then neither beliefs about the systematic factor variance nor about betas are affected by the information. Accordingly, there are no cross-sectional effects of information on cost of capital.

When, as in 3.2, information about the systematic factors conveyed by private signals is such that aggregate precision is infinite, beliefs about both the systematic factor variance and betas by each class of investors are affected by the information. However, because of infinite aggregate precision, the factor is fully revealed to both

informed and uninformed investors. Thus, despite different beliefs about betas, the factor risk premium is zero, implying that the cost of capital is zero. Again there are no cross-sectional effects of information on cost of capital.

When, as in 3.3, information about the systematic factor conveyed by the private signals is such that aggregate precision is finite, i.e., the systematic component of each risky asset's signal is infinitesimally small, the beliefs of both informed and uninformed investors about betas are not affected by the information. Because the aggregate precision of information about the systematic factor is finite, the factor risk premium is reduced, though not to zero. Thus there is an effect on cost of capital, but only through the factor risk premium, not beta; once more implying there are still no cross-sectional effects of information on cost of capital.

Proposition 3 suggests that in order to detect cross-sectional effects of information in large economies, one needs to examine markets that are effectively segmented, such as the financial markets of developing countries. However, a number of recent studies that employ only U.S. data claim to have found cross-sectional effects of information asymmetry. Some studies, e.g., Botosan (1997) and Botosan, Plumlee, and Xie (2004), find cost of capital is correlated with a firm's information characteristics such as AMIR disclosure score or analysts' coverage. Barring the possibility that these studies omitted risk factors correlated with the firm's information characteristics, these findings are consistent with our model only when the number of assets is small. Alternatively, given the large number of assets in the U.S. economy, these findings may suggest that investors may not fully diversify due to frictions or behavioral reasons.

Other studies, such as Easley, Hvidkjaer, and O'Hara (2002) and Aboody, Hughes and Liu (2004), construct an information risk factor and find that it is priced. However, the specifications in these studies are not consistent with our

model prediction because in our model information affects pricing through factor risk premiums; information asymmetry does not create a new risk factor. A plausible way to accommodate these findings might be to extend our model to incorporate real investment and production so that information not only affects investors' beliefs but also risky asset payoffs. Thus, it is conceivable that information risk could enter the factor structure of risky asset payoffs in a systematic way.

One study that indirectly relates cost of capital to information environments across distinct markets is Bhattacharya and Daouk (2002). Using international data, their evidence indicates that cost of capital is lower in countries in which insider trading laws exist and are enforced.

These empirical findings notwithstanding, at the minimum, our study suggests that empirical studies that purport to detect cross-sectional effects of information should carefully consider their theoretical underpinnings; the cross-sectional predictions in settings with a single risky asset or a small number of risky assets do not carry over to the case when the number of risky assets is large.

## 4 Conclusion

In this paper we provide an explicit solution to a noisy rational expectations model that characterizes the interplay between information and diversification effects on expected returns. Risky asset payoffs in this model obey a factor structure. Information takes the form of private signals that are informative of systematic factors as well as idiosyncratic shocks. Our principal result is that, in large economies where the number of risky assets goes to infinity, the APT (Ross, 1976) holds and information (symmetric or asymmetric) affects cost of capital only through

reducing factor risk premiums. There are no cross-sectional effects of information because information does not affect asset betas. The absence of a cross-sectional information effect implies that one must look beyond the phenomena captured by our model to explain cross-sectional associations between a firm's information environment and its cost of capital in the diversification limit.

On the effects of information asymmetry per se, we show that a higher proportion of informed to uninformed investors leads to a greater resolution of uncertainty as manifested by a smaller aggregate posterior factor covariance matrix and, hence, a lower cost of capital. This result depends on the presence of a systematic component in the random supply of risky assets. Eliminating the systematic component of the random supply would remove the asymmetry of information between informed and uninformed investors by causing prices to become fully revealing of private signals, but would not affect our results for symmetric information.

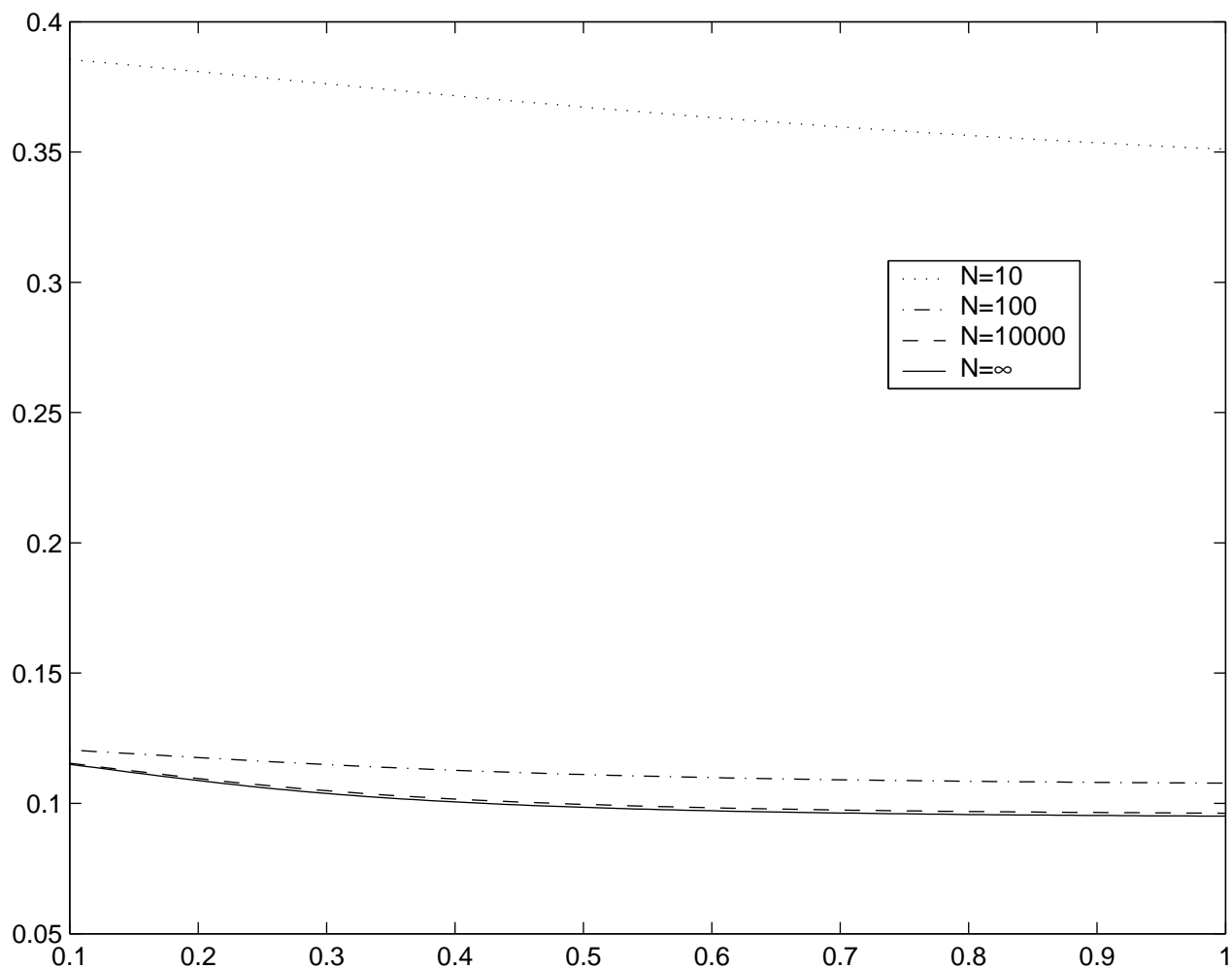
Our intuition suggests that as long as the precision of aggregate posterior beliefs about systematic factors is finite (ensuring only partial revelation of systematic factors when information is aggregated over individual assets), the information supplied by the private signal for an individual risky asset about systematic factors must be small. In turn, the effect of such information on beliefs with respect to any aspect of an individual risky asset's payoff including its beta must be small, approaching zero in the limit as the number of assets goes to infinity<sup>5</sup>. We have confirmed this intuition when distributions are normal, utility functions are CARA with absolute risk aversion decreasing in the number of assets, and there are two classes of investors that can be ordered by statistical sufficiency of their information

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<sup>5</sup>We point out that a similar though less interesting result would be obtained if, in place of a systematic component to the signal for each asset that became less informative as the economy expands, we assumed one signal about the systematic factors with finite precision was received by all informed investors.

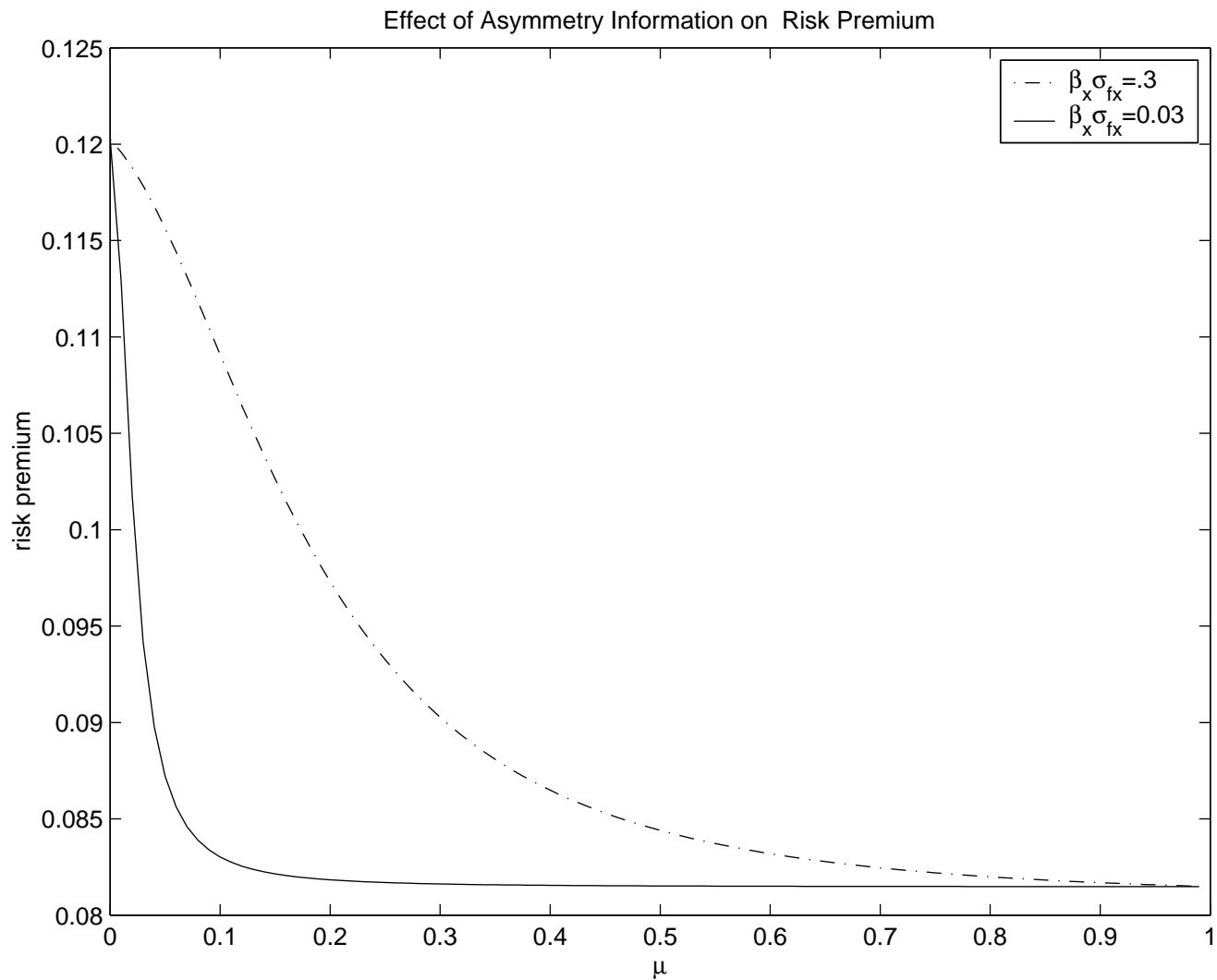
with respect to systematic factors. We conjecture that the absence of an effect of information on beta in large economies would hold up in more general cases where distributions and utility functions depart from these assumptions.

We note that our results in the case of asymmetric information establishes the validity of the APT in a setting that has heterogeneous expectations. If, as conjectured above, the effects of information supplied by private signals in large economies can be reduced to heterogeneous posterior beliefs on systematic factors, while preserving homogeneous beliefs on betas for those factors, then we further conjecture as does Ross (1976) that the APT holds in such economies with private signals, provided that the conditions for the APT to hold in the absence of private signals are met. These conjectures, if true, imply that the diversification limit in large economies eliminates cross-sectional effects of information in a far broader context than the setting for our model.



**Figure 1. The Risk Premium.** This graph plots the average risk premium in the ID case against the fraction of informed investors, for various numbers of risky assets,  $N$ . The parameters are:  $\gamma = 3$ ,  $\sigma = 30\%$ ,  $\beta = 1$ ,  $\sigma_f = 20\%$ ,  $\sigma_s = 25\%$ ,  $\sigma_{fx} = 30\%$ ,  $\beta_x = 1$ ,  $\sigma_x = 30\%$ , and  $k = -1$ .





**Figure 2. The Risk Premium.** This graph plots the average risk premium in the ID case against the fraction of informed investors for different values of  $\sigma_{fx}\beta_x$ . The other parameter values are:  $N = \infty$ ,  $\gamma = 3$ ,  $\sigma = 30\%$ ,  $\beta = 1$ ,  $\sigma_f = 20\%$ ,  $\sigma_s = 25\%$ , and  $k = -1$ .

## References

- [1] Aboody, D., J. Hughes, and J. Liu. "Earnings quality, insider trading and cost of capital." Working paper, UCLA, 2004.
- [2] Admati, A. "On models and measures of information asymmetries in financial markets." PhD thesis, Yale University, 1982.
- [3] Admati, A. "A noisy rational expectations equilibrium for multi-asset securities markets." *Econometrica*, 53 (1985): 629-58.
- [4] Ball, R., and P. Brown. "An empirical evaluation of accounting income numbers." *Journal of Accounting Research*, 6 (1968): 159-178.
- [5] Bhattacharya, U., and H. Daouk. "The world price of insider trading." *The Journal of Finance*, 57 (2002): 75-108.
- [6] Botosan, C. "Disclosure level and the cost of equity capital." *The Accounting Review*, 72 (1997): 323-49.
- [7] Botosan, C., and M. Plumlee. "A re-examination of disclosure level and the expected cost of equity capital." *Journal of Accounting Research*, 40 (2002): 21-40.
- [8] Botosan, C., M. Plumlee, and Y. Xie. "The Role of Private Information Precision in Determining Cost of Equity Capital," *Review of Accounting Studies*, (2004): forthcoming.
- [9] Brennan, M., and H. Cao. "Information, trade, and derivative securities." *Review of Financial Studies*, 9 (1997): 163 - 208.

- [10] Caballe, J., and M. Krishnan, “Imperfect Competition in a Multi-Security Market with Risk Neutrality,” *Econometrica*, 62 (1994): 695-704.
- [11] Chordia, T., R. Roll, and A. Subrahmanyam. “Commonality in liquidity.” *Journal of Financial Economics*, 56 (2000): 3-28.
- [12] Daniel, K., D. Hirshleifer, and A. Subrahmanyam. “Overconfidence, Arbitrage, and Equilibrium Asset Pricing,” *Journal of Finance*, 56 (2001): 921-965.
- [13] Daniel, K., and S. Titman. “Characteristics or Covariances?” *Journal of Portfolio Management*, 24 (1998): 24-33.
- [14] Dybvig, P. “An explicit bound on individual assets’ deviations from APT pricing in a finite economy.” *Journal of Financial Economics*, 12(1983): 483-496.
- [15] Easley, D., S. Hvidkjaer, and M. O’Hara. “Is information risk a determinant of asset returns?” *Journal of Finance*, 57 (2002): 2185-221.
- [16] Easley, D., and M. O’Hara. “Information and the cost of capital.” *Journal of Finance*, 59 (2004): 1553-1583.
- [17] Francis, J., R. LaFond, P. Olsson, and K. Schipper. “The market pricing of earnings quality.” Working paper, Duke University and University of Wisconsin, 2002.
- [18] Grinblatt, M., and S. Titman. “Factor pricing in a finite economy.” *Journal of Financial Economics*, 12 (1983): 497-507.
- [19] Grossman, S., and J. Stiglitz. “On the impossibility of informationally efficient markets.” *Journal of Economic Theory*, 22 (1980): 477-498.

- [20] Healy, P., A. Hutton, and K. Palepu. "Stock performance and intermediation changes surrounding sustained increases in disclosure." *Contemporary Accounting Research* 16 (1999): 485-520.
- [21] Huberman, G., and D. Hulka. "Systematic Liquidity." *Journal of Financial Research*, 24 (2001): 161-178.
- [22] Kodres, L., and M. Pritsker, "A Rational Expectations Model of Financial Contagion." *Journal of Finance*, 57 (2002): 769-799.
- [23] Lakonishok, J., and I. Lee. "Are insider trades informative?" *Review of financial studies*, 14 (2001): 79-111.
- [24] Pasquariello, P. "Imperfect Competition, Information Heterogeneity, and Financial Contagion." Working paper, University of Michigan, 2004.
- [25] Ross, S. "The arbitrage pricing theory of capital asset pricing." *Journal of Economic Theory*, 13 (1976): 341-60.
- [26] Seyhun, H. "Why does insider trading predict future stock returns?" *Quarterly Journal of Economics*, 107 (1992): 1303-32.

## Appendix

In the Appendix, we will use the following identity extensively:

$$(\Sigma + \beta\Omega\beta')^{-1} = \Sigma^{-1} - \Sigma^{-1}\beta(\Omega^{-1} + \beta'\Sigma^{-1}\beta)\beta'\Sigma^{-1}.$$

## The Proof of Remark 1.

We solve for the filtering rule, given signal  $s$ . Our assumptions have specified the distribution functions  $f(\nu|F, s)$ ,  $f(\nu|F)$ , and  $f(F)$ . Therefore,

$$f(\nu, F, s) = f(s|\nu, F)f(\nu|F)f(F).$$

We can rewrite the above as

$$f(\nu, F, s) = f(\nu|s, F)f(F|s)f(s).$$

Focusing on the exponential terms of the joint normal distribution densities, we obtain

$$\begin{aligned} -\ln f(\nu, F, s) &= -\ln f(s|\nu, F) - \ln f(\nu|F) - \ln f(F) \\ &= \frac{1}{2}(s - (\nu - \bar{\nu} - \beta F) - bF)' \Sigma_s^{-1} (s - (\nu - \bar{\nu} - \beta F) - bF) \\ &\quad + \frac{1}{2}(\nu - \bar{\nu} - \beta F)' \Sigma^{-1} (\nu - \bar{\nu} - \beta F) + \frac{1}{2}F' \Sigma_F^{-1} F \\ &= \frac{1}{2}(\nu - \bar{\nu} - \beta F)' (\Sigma^{-1} + \Sigma_s^{-1}) (\nu - \bar{\nu} - \beta F) - (\nu - \bar{\nu} - \beta F)' \Sigma_s^{-1} (s - bF) \\ &\quad + \frac{1}{2}(s - bF)' \Sigma_s^{-1} (s - bF) + \frac{1}{2}F' \Sigma_F^{-1} F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F]) \Sigma_{\nu|s, F}^{-1} (\nu - \mathbb{E}[\nu|s, F]) \\ &\quad + \frac{1}{2}(s - bF)' (\Sigma + \Sigma_s)^{-1} (s - bF) + \frac{1}{2}F' \Sigma_F^{-1} F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F]) \Sigma_{\nu|s, F}^{-1} (\nu - \mathbb{E}[\nu|s, F]) + \frac{1}{2}s' (\Sigma + \Sigma_s)^{-1} s \\ &\quad + \frac{1}{2}(bF)' (\Sigma + \Sigma_s)^{-1} bF - s' (\Sigma + \Sigma_s)^{-1} bF + \frac{1}{2}F' \Sigma_F^{-1} F \\ &= \frac{1}{2}(\nu - \mathbb{E}[\nu|s, F]) \Sigma_{\nu|s, F}^{-1} (\nu - \mathbb{E}[\nu|s, F]) \\ &\quad + \frac{1}{2}(F - \mathbb{E}[F|s])' \Sigma_{F|s}^{-1} (F - \mathbb{E}[F|s]) + \frac{1}{2}s' \hat{\Sigma}_s^{-1} s \\ &= -\ln f(\nu|s, F) - \ln f(F|s) - \ln f(s), \end{aligned}$$

The distribution functions  $f(\nu|s, F)$ ,  $f(F|s)$ , and  $f(s)$  can then be identified from the above equation, with

$$\begin{aligned} \mathbf{E}[\nu|s, F] &= \bar{\nu} + \beta F + \Sigma_{\nu|s, F} \Sigma_s^{-1} (s - bF), \\ \mathbf{E}[F|s] &= \Sigma_{F|s} b' (\Sigma + \Sigma_s)^{-1} s, \\ \Sigma_{\nu|s, F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1}, \\ \Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b' (\Sigma + \Sigma_s)^{-1} b, \\ \hat{\Sigma}_s &= \Sigma + b' \Sigma_F b + \Sigma_s. \end{aligned}$$

## The Proof of Remark 2.

The structure of the filtering rule, given signal  $\theta$ , is the same as that for  $s$ . The proof proceeds in exactly the same fashion.

## The Proof of Corollary 1.

The beta conditional on  $s$  is  $\beta_s = \beta - \frac{\sigma^2}{\sigma^2 + \sigma_s^2} b$ . We have

$$\begin{aligned} \Sigma_{\nu|s, F} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N; \\ \Sigma_{F|s} &= (\sigma_f^{-2} + k^2 (\sigma^2 + \sigma_s^2)^{-1})^{-1}. \end{aligned}$$

Therefore,

$$\begin{aligned} \Sigma_{\nu|s} &= (\sigma^{-2} + \sigma_s^{-2})^{-1} I_N - (\beta + (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} b)^2 \Sigma_{F|s} \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N} \\ &\equiv S_0 I_N + S_1 \mathbf{1}_{N \times N}, \end{aligned} \tag{24}$$

where

$$\begin{aligned} S_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1}; \\ S_1 &= \frac{\left( \beta - \frac{1}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_s^2} k \right)^2}{\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2}. \end{aligned}$$

It follows that

$$\begin{aligned}
\Phi_s &= (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} I_N \\
&\quad + (\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} b) (\sigma_f^{-2} + b^2 (\sigma^2 + \sigma_s^2)^{-1} N)^{-1} b (\sigma^2 + \sigma_s^2)^{-1} 11' \\
&\equiv \phi_0 I_N + \frac{\phi_1}{\sqrt{N}} 1_{N \times N}, \tag{25}
\end{aligned}$$

where

$$\begin{aligned}
\phi_0 &= (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} = \frac{\sigma^2}{\sigma^2 + \sigma_s^2}; \\
\phi_1 &= \frac{\left( \beta - \frac{1}{\sqrt{N}} (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} k \right) k}{(\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2) (\sigma^2 + \sigma_s^2)}.
\end{aligned}$$

One can readily verify that

$$\Phi_s^{-1} = \phi_0^{-1} \left( I_N - (\sqrt{N} \phi_0 \phi_1^{-1} + N)^{-1} 1_{N \times N} \right).$$

Therefore,

$$\begin{aligned}
\lambda &= A\mu^{-1} \Phi_s^{-1} \Sigma_{\nu|s} \\
&= A\mu \phi_0^{-1} S_0 \left( I_N + \left( S_0^{-1} S_1 - \frac{1 + S_0^{-1} S_1 N}{\sqrt{N} \phi_0 \phi_1^{-1} + N} \right) I_{N \times N} \right) \\
&= A\mu^{-1} \phi_0^{-1} S_0 \left( I_N + \left( S_0^{-1} S_1 - \frac{1 + S_0^{-1} S_1 N}{\sqrt{N} \phi_0 \phi_1^{-1} + N} \right) I_{N \times N} \right) \\
&= A\mu^{-1} \phi_0^{-1} S_0 \left( I_N + \frac{\sqrt{N} S_0^{-1} S_1 \phi_0 \phi_1^{-1} - 1}{N + \sqrt{N} \phi_0 \phi_1^{-1}} I_{N \times N} \right) \\
&= \frac{1}{N} \lambda_0 I_N + \frac{1}{\sqrt{N^3}} \lambda_1 I_{N \times N},
\end{aligned}$$

where

$$\begin{aligned}
\lambda_0 &= NA\mu^{-1} \sigma_s^2 = \gamma\mu^{-1} \sigma_s^2 \\
\lambda_1 &= \sqrt{N^3} A\mu^{-1} \frac{\frac{\sqrt{N} (\beta - (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} b) (\sigma^2 + \sigma_s^2)}{\sqrt{N} b} - \sigma_s^2}{N + \sqrt{N} \phi_0 \phi_1^{-1}} \\
&= \gamma\mu^{-1} \frac{\left( \beta - \frac{1}{\sqrt{N}} (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} k \right) (\sigma^2 + \sigma_s^2) - \frac{1}{\sqrt{N}} \sigma_s^2 k}{k + \frac{1}{\sqrt{N}} \frac{\sigma^2 (\sigma_f^{-2} + (\sigma^2 + \sigma_s^2)^{-1} k^2)}{\left( \beta - \frac{1}{\sqrt{N}} (\sigma^{-2} + \sigma_s^{-2})^{-1} \sigma_s^{-2} k \right)}}.
\end{aligned}$$

We used the notation  $\gamma = \frac{A}{N}$ . The signal  $\theta$  can be written as

$$\begin{aligned}\theta &= s - \lambda(x - \bar{x}) = s - \left( \frac{\lambda_0}{N} I_N + \frac{\lambda_1}{\sqrt{N^3}} I_{N \times N} \right) (\beta_x 1_{N \times 1} F_x + \sigma_x \eta_x) \\ &= s - \left( \frac{\lambda_0}{N} + \frac{\lambda_1}{\sqrt{N}} \right) \beta_x 1_{N \times 1} F_x - \left( \frac{\lambda_0}{N} \sigma_x \eta_x + \frac{\lambda_1}{\sqrt{N}} \sigma_x \hat{\eta}_x I_{N \times 1} \right),\end{aligned}\quad (26)$$

where  $\hat{\eta}_x \equiv \frac{1'}{N} \eta_x$  is the sample average of  $\eta_x$ . So we have

$$\begin{aligned}\Sigma_\theta &= \sigma_s^2 I_N + \frac{1}{N} \left( \lambda_1 + \lambda_0 / \sqrt{N} \right)^2 \sigma_{f_x}^2 \beta_x^2 1_{N \times N} + \frac{\lambda_0^2}{N^2} \sigma_x^2 I_N + \left( \frac{\lambda_1^2}{N^2} + 2 \frac{\lambda_0 \lambda_1}{N^{5/2}} \right) \sigma_x^2 I_{N \times N} \\ &= \left( \sigma_s^2 + \frac{\lambda_0^2}{N^2} \sigma_x^2 \right) I_N + \frac{1}{N} \left( \left( \lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{f_x}^2 \beta_x^2 + \left( \frac{\lambda_1^2}{N} + \frac{2 \lambda_0 \lambda_1}{N^{3/2}} \right) \sigma_x^2 \right) I_{N \times N} \\ &= \sigma_{\theta 0}^2 I_N + \frac{1}{N} \sigma_{\theta 1}^2 I_{N \times N},\end{aligned}\quad (28)$$

where

$$\begin{aligned}\sigma_{\theta 0}^2 &= \sigma_s^2 + \frac{\lambda_0^2}{N^2} \sigma_x^2; \\ \sigma_{\theta 1}^2 &= \left( \lambda_1 + \frac{\lambda_0}{\sqrt{N}} \right)^2 \sigma_{f_x}^2 \beta_x^2 + \left( \frac{\lambda_1^2}{N} + \frac{2 \lambda_0 \lambda_1}{N^{3/2}} \right) \sigma_x^2.\end{aligned}\quad (29)$$

The covariance matrix of payoffs conditional on  $(\theta, F)$  is

$$\begin{aligned}\Sigma_{\nu|\theta, F} &= \left( \sigma^{-2} I_N + \left( \sigma_{\theta 0}^2 I_N + \frac{1}{N} \sigma_{\theta 1}^2 1_{N \times N} \right)^{-1} \right)^{-1} \\ &= \left( \sigma^{-2} I_N + \sigma_{\theta 0}^{-2} \left( I_N + \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma_{\theta 0}^2} 1_{N \times N} \right)^{-1} \right)^{-1} \\ &= \left( \sigma^{-2} I_N + \sigma_{\theta 0}^{-2} \left( I_N - N^{-1} \left( \frac{\sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} 1_{N \times N} \right) \right)^{-1} \\ &= \left( (\sigma^{-2} + \sigma_{\theta 0}^{-2}) I_N - N^{-1} \left( \frac{\sigma_{\theta 0}^4}{\sigma_{\theta 1}^2} + \sigma_{\theta 0}^2 \right)^{-1} 1_{N \times N} \right)^{-1} \\ &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left( I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2 (\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} 1_{N \times N} \right).\end{aligned}$$



The matrix  $\Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}$  can be written as

$$\begin{aligned}
\Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1} &= (\Sigma^{-1} + \Sigma_{\theta}^{-1})^{-1}\Sigma_{\theta}^{-1} = (I_N + \Sigma_{\theta}\Sigma^{-1})^{-1} \\
&= \left( I_N + \frac{\sigma_{\theta 0}^2}{\sigma^2}I_N + \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2}1_{N \times N} \right)^{-1} \\
&= \left( 1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left( I_N - \frac{1}{N} \left( \frac{\sigma^2 + \sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} 1_{N \times N} \right) \\
&= \left( 1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left( I_N - \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} 1_{N \times N} \right).
\end{aligned}$$

The factor covariance, conditional on  $\theta$ , is

$$\begin{aligned}
\Sigma_{F|\theta} &= (\Sigma_F^{-1} + b'(\Sigma + \Sigma_{\theta})^{-1}b)^{-1} = \left( \sigma_f^{-2} + \frac{k^2}{N}1_{1 \times N} \left( (\sigma^2 + \sigma_{\theta 0}^2)I_N + \frac{\sigma_{\theta 1}^2}{N}1_{N \times N} \right)^{-1} 1_{N \times 1} \right)^{-1} \\
&= \left( \sigma_f^{-2} + \frac{k^2}{N(\sigma^2 + \sigma_{\theta 0}^2)}1_{1 \times N} \left( I_N + \frac{\sigma_{\theta 1}^2}{(\sigma^2 + \sigma_{\theta 0}^2)N}1_{N \times N} \right)^{-1} 1_{N \times 1} \right)^{-1} \\
&= \left( \sigma_f^{-2} + \frac{k^2}{N(\sigma^2 + \sigma_{\theta 0}^2)}1_{1 \times N} \left( I_N - \left( \frac{(\sigma^2 + \sigma_{\theta 0}^2)N}{\sigma_{\theta 0}^2} + N \right)^{-1} 1_{N \times N} \right) 1_{N \times 1} \right)^{-1} \\
&= \left( \sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2} \left( 1 - \left( \frac{\sigma^2 + \sigma_{\theta 0}^2}{\sigma_{\theta 1}^2} + 1 \right)^{-1} \right) \right)^{-1} = \left( \sigma_f^{-2} + \frac{k^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^{-1} \equiv \sigma_{f\theta}^2.
\end{aligned}$$

The beta, conditional on  $\theta$ , is

$$\begin{aligned}
\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b &= \beta 1_{N \times 1} - \frac{k}{\sqrt{N}} \left( 1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left( I_N - \frac{1}{N} \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} 1_{N \times N} \right) 1_{N \times 1} \\
&= \beta 1_{N \times 1} - \frac{k}{\sqrt{N}} \left( 1 + \frac{\sigma_{\theta 0}^2}{\sigma^2} \right)^{-1} \left( 1 - \frac{\sigma_{\theta 1}^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1} = \left( \beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right) 1_{N \times 1}.
\end{aligned}$$

The covariance matrix of the asset payoffs, conditional on  $\theta$ , is

$$\begin{aligned}
\Sigma_{\nu|\theta} &= (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1} \left( I_N + \frac{1}{N} \frac{\sigma^2 \sigma_{\theta 1}^2}{\sigma_{\theta 0}^2(\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2)} 1_{N \times N} \right) + \sigma_{f\theta}^2 \left( \beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta 0}^2 + \sigma_{\theta 1}^2} \right)^2 1_{N \times N} \\
&= \Theta_0 I_N + \Theta_1 I_{N \times N},
\end{aligned}$$

where

$$\Theta_0 = (\sigma^{-2} + \sigma_{\theta 0}^{-2})^{-1},$$

$$\Theta_1 = \sigma_{f\theta}^2 \left( \beta - \frac{k}{\sqrt{N}} \frac{\sigma^2}{\sigma^2 + \sigma_{\theta_0}^2 + \sigma_{\theta_1}^2} \right)^2 + \frac{1}{N} \frac{\sigma^4 \sigma_{\theta_1}^2}{(\sigma^2 + \sigma_{\theta_0}^2)(\sigma^2 + \sigma_{\theta_0}^2 + \sigma_{\theta_1}^2)}.$$

The average of inverse covariance matrix of the asset payoffs is

$$\begin{aligned} & \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \\ &= \mu S_0^{-1} \left( I_N - \left( \frac{S_0}{S_1} + N \right)^{-1} I_{N \times N} \right) + (1 - \mu) \Theta_0^{-1} \left( I_N - \left( \frac{\Theta_0}{\Theta_1} + N \right)^{-1} I_{N \times N} \right) \\ &= (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}) I_N - \left( \mu S_0^{-1} \left( \frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{\Theta_1} + N \right)^{-1} \right) I_{N \times N}. \end{aligned}$$

The geometric average of the covariance matrices is

$$\begin{aligned} \bar{\Sigma}_\nu &= \left( \mu \Sigma_{\nu|s}^{-1} + (1 - \mu) \Sigma_{\nu|\theta}^{-1} \right)^{-1} \\ &= \left( (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}) I_N - \left( \mu S_0^{-1} \left( \frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{\Theta_1} + N \right)^{-1} \right) I_{N \times N} \right)^{-1} \\ &= (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\quad \times \left( I_N - \left( -\frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left( \frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{\Theta_1} + N \right)^{-1}} + N \right)^{-1} I_{N \times N} \right). \end{aligned}$$

The average risk premium is

$$\begin{aligned} \bar{\nu} - R_f \bar{p} &= A \bar{\Sigma}_\nu \bar{x} 1_{N \times 1} \\ &= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\quad \times \left( 1 - \left( -\frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left( \frac{S_0}{S_1} + N \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{\Theta_1} + N \right)^{-1}} + N \right)^{-1} N \right) \bar{x} I_{N \times 1} \\ &= A (\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1})^{-1} \\ &\quad \times \left( 1 - \left( -\frac{\mu S_0^{-1} + (1 - \mu) \Theta_0^{-1}}{\mu S_0^{-1} \left( \frac{S_0}{NS_1} + 1 \right)^{-1} + (1 - \mu) \Theta_0^{-1} \left( \frac{\Theta_0}{N\Theta_1} + 1 \right)^{-1}} + 1 \right)^{-1} \right) \bar{x} I_{N \times 1}. \end{aligned}$$

Note that

$$\begin{aligned}
& 1 - \frac{\mu S_0^{-1} + (1 - \mu)\Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}} \\
&= \frac{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1} - \mu S_0^{-1} - (1 - \mu)\Theta_0^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}} \\
&= -\frac{1}{N} \frac{\mu S_1^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_1^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}}{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}}.
\end{aligned}$$

The risk premium can be written as

$$A \left( \mu S_0^{-1} + (1 - \mu)\Theta_0^{-1} \right)^{-1} \left( 1 + N \frac{\mu S_0^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_0^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}}{\mu S_1^{-1} \left(\frac{S_0}{NS_1} + 1\right)^{-1} + (1 - \mu)\Theta_1^{-1} \left(\frac{\Theta_0}{N\Theta_1} + 1\right)^{-1}} \right) \bar{x} I_{N \times 1}.$$

## Proof of Proposition 1.

Because  $b = 0$ , we have

$$\begin{aligned}\Sigma_{F|s} &= \Sigma_F; \\ \Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta\Sigma_F\beta'; \\ \Sigma_{F|\theta} &= \Sigma_F; \\ \Sigma_{\nu|\theta} &= \Sigma_{\nu|\theta,F} + \beta\Sigma_F\beta'.\end{aligned}$$

Intuitively, the matrices  $\Sigma_{\nu|s}$  and  $\Sigma_{\nu|\theta}$  differ only in the idiosyncratic matrices  $\Sigma_{\nu|s,F}$  and  $\Sigma_{\nu|\theta,F}$  which do not matter for the risk premium and thus should produce the risk premium  $\gamma\beta\Sigma_F\beta'\bar{x}$ . The formal proof is as follows. From

$$\Sigma_\theta = \Sigma_s + \lambda(\beta_x\Sigma_{Fx}\beta'_x + \Sigma_x)\lambda' \geq \Sigma_s,$$

we know that

$$\Sigma \geq (\Sigma^{-1} + \Sigma_\theta^{-1})^{-1} = \Sigma_{\nu|\theta,F} \geq (\Sigma^{-1} + \Sigma_s^{-1})^{-1} = \Sigma_{\nu|s,F}.$$

It follows that

$$\begin{aligned}\Sigma + \beta\Sigma_F\beta' &= (\mu(\Sigma + \beta\Sigma_F\beta') + (1 - \mu)(\Sigma + \beta\Sigma_F\beta'))^{-1} \\ &\geq (\mu(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta') + (1 - \mu)(\Sigma_{\nu|\theta,F} + \beta\Sigma_F\beta'))^{-1} = \bar{\Sigma}_\nu \\ &\geq (\mu(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta') + (1 - \mu)(\Sigma_{s|\theta,F} + \beta\Sigma_F\beta'))^{-1} = \Sigma_{\nu|s,F} + \beta\Sigma_F\beta'.\end{aligned}$$

Hence,

$$\lim_{N \rightarrow \infty} \frac{1}{N}(\Sigma + \beta\Sigma_F\beta') = \lim_{N \rightarrow \infty} \frac{1}{N}\beta\Sigma_F\beta' \geq \lim_{N \rightarrow \infty} \frac{1}{N}\bar{\Sigma}_\nu \geq \lim_{N \rightarrow \infty} \frac{1}{N}\Sigma_{\nu|s,F} + \beta\Sigma_F\beta' = \lim_{N \rightarrow \infty} \frac{1}{N}\beta\Sigma_F\beta'.$$

Therefore, the average risk premium is

$$\bar{\nu} - R_f\bar{p} = \gamma\frac{1}{N}\bar{\Sigma}_\nu\bar{x} \rightarrow \gamma\frac{1}{N}(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta')\bar{x} \rightarrow \gamma\frac{1}{N}\beta\Sigma_F\beta'\bar{x}.$$

## Proof of Proposition 2.

For the case of non-identically distributed risky asset payoffs, the leading order terms in the large  $N$  limit are

$$\begin{aligned}\Sigma_{\nu|s,F} &= (\Sigma^{-1} + \Sigma_s^{-1})^{-1}, \\ \Sigma_{F|s} &= \left( \Sigma_F^{-1} + \frac{1}{N} k'(\Sigma + \Sigma_s)^{-1} k \right)^{-1}.\end{aligned}$$

The variance of  $\nu$  conditional on  $s$

$$\begin{aligned}\Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta \Sigma_{F|s} \beta' + O(N^{-1/2}), \\ \Phi_s &= \Sigma_{\nu|s,F} \Sigma_s^{-1} + \frac{1}{\sqrt{N}} \beta \Sigma_{F|s} k'(\Sigma + \Sigma_s)^{-1} + O(N^{-1}).\end{aligned}$$

Both first terms in the above equations are diagonal matrices. The second terms are due to factors. We use  $O(N^\alpha)$  to denote matrices with all of their elements generally non-zero and of order  $N^\alpha$ . In the case of identical assets,  $O(N^\alpha) \propto N^\alpha \mathbf{1}_{N \times N}$ . These terms will be negligible, in the large  $N$  limit, as far as the risk premium is concerned. The  $\Phi_s^{-1}$  matrix is

$$\begin{aligned}\Phi_s^{-1} &= \Sigma_s \left( I_N + \frac{1}{\sqrt{N}} \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \right)^{-1} \Sigma_{\nu|s,F}^{-1} \\ &= \Sigma_s \left( I_N - \Sigma_{\nu|s,F}^{-1} \beta \left( \sqrt{N} \Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \right) \Sigma_{\nu|s,F}^{-1}\end{aligned}$$

and

$$\begin{aligned}\Phi_s^{-1} \Sigma_{\nu|s} &= \Sigma_s \left( I_N - \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} \left( \sqrt{N} I_K + k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} \right)^{-1} k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \right) \\ &\quad \times \left( I_N + \Sigma_{\nu|s,F}^{-1} \beta \Sigma_{F|s} \beta' \right) \\ &= \Sigma_s \left( I_N - \Sigma_{\nu|s,F}^{-1} \beta \left( \sqrt{N} \Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \right. \\ &\quad \left. - \Sigma_{\nu|s,F}^{-1} \beta \left( \sqrt{N} \Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1} \Sigma + I_N)^{-1} \Sigma_{\nu|s,F}^{-1} \beta \right)^{-1} \sqrt{N} \beta' \right) \\ &\rightarrow \Sigma_s \left( I_N + \frac{1}{\sqrt{N}} \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma^{-1} \beta \right)^{-1} \beta' \right).\end{aligned}$$

Therefore,

$$\lambda = \frac{\gamma}{\mu N} \Phi_s^{-1} \Sigma_{\nu|s} = \frac{1}{\sqrt{N^3}} \gamma \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma^{-1} \beta \right)^{-1} \beta'.$$

The signal  $\theta$  is now

$$\theta = s - \frac{1}{\sqrt{N}} \gamma \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma^{-1} \beta \right)^{-1} \frac{\beta' \beta_x}{N} F_x \equiv s - \frac{1}{\sqrt{N}} \Lambda F_x,$$

with  $\Lambda = \gamma \mu^{-1} \Sigma_s \Sigma_{\nu|s,F}^{-1} \beta \left( \frac{1}{N} k' \Sigma^{-1} \beta \right)^{-1} \frac{\beta' \beta_x}{N}$ . The idiosyncratic component of the random supply disappears; it is diversified away. The covariance matrix of the payoffs, conditional on  $\theta$ , is

$$\Sigma_\theta = \Sigma_s + \frac{1}{N} \Lambda \beta_x \Sigma_{F_x} \beta_x' \Lambda'.$$

Note that  $\Sigma_s$  is a diagonal matrix while  $\Lambda \beta_x \Sigma_{F_x} \beta_x' \Lambda'$  is a matrix with all of its matrix elements being of order 1. Therefore, when  $\Sigma_\theta$  is multiplied by a vector of 1's from the right, the second term has the same order of magnitude as the first term. We can show that

$$\Sigma_{\nu|\theta,F} = \Sigma_{\nu|s,F} + O(N^{-1}).$$

As will be shown later, the contribution of such terms to the risk premium goes to zero in the limit as  $N \rightarrow \infty$ . The factor covariance matrix, conditional on  $\theta$ , is

$$\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + \frac{1}{N} k' \left( \Sigma + \Sigma_s + \frac{1}{N} \Lambda \beta_x \Sigma_{F_x} \beta_x' \Lambda' \right)^{-1} k.$$

Note that, when multiplied by vectors of 1's from left and from right, the term  $\frac{1}{N} \Lambda \beta_x \Sigma_{F_x} \beta_x' \Lambda'$  produces a  $K \times K$  matrix with elements of order  $N$ , the same as matrix  $\Sigma + \Sigma_s$ .

The variance of  $\nu$ , conditional on  $\theta$ ,

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|s,F} + \beta \Sigma_{F|\theta} \beta'.$$

The matrix  $\Sigma_{\nu|s,F}$  is diagonal, while all the elements of the matrix  $\beta\Sigma_{F|\theta}\beta'$  are of order 1. The terms neglected earlier produce matrices with all elements of order  $N^{-1}$ .

From the identity,

$$\begin{aligned} & \mu\Sigma_{\nu|s}^{-1} + (1 - \mu)\Sigma_{\nu|\theta}^{-1} \\ = & \Sigma_{\nu|s,F}^{-1} - \Sigma_{\nu|s,F}^{-1}\beta \left( \mu \left( \Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \right)^{-1} \right) \beta'\Sigma_{\nu|s,F}^{-1} \end{aligned}$$

we can write

$$\left( \mu\Sigma_{\nu|s}^{-1} + (1 - \mu)\Sigma_{\nu|\theta}^{-1} \right)^{-1} = \Sigma_{\nu|s,F} + \beta M^{-1}\beta',$$

where

$$\begin{aligned} M &= \left( \mu \left( \Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \right)^{-1} \right)^{-1} - \beta'\Sigma_{\nu|s,F}\beta \\ &= \left( \mu \left( \Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \right)^{-1} \right)^{-1} \\ &\times \left( \mu \left( \Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left( \Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \right)^{-1} \Sigma_{F|\theta}^{-1} \right). \end{aligned}$$

In the large  $N$  limit,  $\beta'\Sigma_{\nu|s,F}^{-1}\beta$  is of order  $N$ , therefore,  $\Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \rightarrow \beta'\Sigma_{\nu|s,F}^{-1}\beta$ .

Similarly,  $\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \rightarrow \beta'\Sigma_{\nu|\theta,F}^{-1}\beta$ , so

$$\begin{aligned} M &\rightarrow \beta'\Sigma_{\nu|s,F}^{-1}\beta \left( \mu \left( \beta'\Sigma_{\nu|s,F}^{-1}\beta \right)^{-1} \Sigma_{F|s}^{-1} + (1 - \mu) \left( \beta'\Sigma_{\nu|\theta,F}^{-1}\beta \right)^{-1} \Sigma_{F|\theta}^{-1} \right) \\ &= \mu\Sigma_{F|s}^{-1} + (1 - \mu)\Sigma_{F|\theta}^{-1}. \end{aligned}$$

The risk premium is given by

$$\gamma\beta \left( \mu\Sigma_{F|s}^{-1} + (1 - \mu)\Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta'\bar{x}}{N}$$

and the factor risk premium is given by

$$\gamma \left( \mu\Sigma_{F|s}^{-1} + (1 - \mu)\Sigma_{F|\theta}^{-1} \right)^{-1} \frac{\beta'\bar{x}}{N}.$$