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by

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Abstract

This paper presents a model that endogenizes housing returns, housing construction, mortgage loan terms, and household maintenance behavior within the market for single family homes. The model is based upon the premise that the value of a home, unlike the value of many other financial assets, depends upon the care its owner exerts on upkeep. Banks respond to this moral hazard problem by restricting the size of the loans they are willing to issue. As a result people bid what the can for housing, rather than what they may wish to. This in turn ties housing prices to changes in the endowment process which are both predictable and time varying. When endowments are growing quickly (a city with a rapidly growing economy) housing prices exhibit above market expected returns. Conversely, when endowments are shrinking housing returns will drop to below market rates. Developers in the model are fully cognizant of these facts and react accordingly. When housing prices are expected to increase faster than the rate of interest, developers delay construction in order to earn the economic rents on their land holdings. Thus, during periods of rapid expected economic growth housing construction ceases until one reaches the crest whereupon development booms. The model also produces the empirically verified prediction that current mortgage loan to value ratios can be used to forecast future housing returns.

U.S. Census Bureau figures show that 1997 expenditures on new single family housing construction equaled 164.4 billion dollars (Bureau of the Census (October 1998)) . Simultaneously, the U.S. economy spent another 100.6 billion dollars on home maintenance and improvements that year (Bureau of the Census (Fourth Quarter 1997)). This paper explores the implications that home maintenance has on the housing market by recognizing that it introduces a moral hazard problem between lenders and borrowers. Lenders want borrowers to maintain the their residence since it collateralizes the loan. Borrowers on the other hand, will cut back on maintenance to the degree that the rewards for such activities go to the bank instead of themselves. With this tension as a backdrop the model endogenously derives a family's effort to maintain their home, the loan to value requirements used by banks, mortgage rates, and the dates on which builders will develop new housing. The fundamental result is that single family housing returns within this setting are cyclical and predictable. This in turn induces building cycles. These cycles are characterized by construction slumps, and an accompanied decline in the total housing supply, during expected price run ups. Conversely, when expected returns are low, building resumes which then replenishes the housing stock.

In order to understand how moral hazard can drive housing prices consider the mortgage problem faced by a bank. While the value of a stock or a bond may not depend upon on the owner's devotion to its upkeep, the same cannot be said for housing. An unkept share of stock that has been wrinkled, washed and torn has the same value as a share in pristine condition. In contrast, a neglected house will be worth much less than its well kept neighbor. This simple fact implies that banks will not make unlimited loans to families wishing to purchase homes. If a bank makes a large loan (relative to the home's current value) then it must demand very high mortgage payments to offset its capital costs. However, a high loan to value ratio implies that in

all probability any care invested by the family simply goes to the bank in the form of a more valuable asset. Consequently, this reduces the homeowner's financial incentive to invest in the home's upkeep resulting in a deterioration of the bank's collateral. Banks therefore restrict loan to value ratios which suggests that some families will be credit constrained.

Credit constraints in housing markets and the associated moral hazard issues appear to be economically important. Papers by Duca and Rosenthal (1991), Rosenthal, Duca and Gabriel (1991), Ambrose, Pennington-Cross and Yezer (1998), and Haurin, Hendershott, and Wachter (1997) show that at the interest rates charged by banks there exist families that would like to borrow money in order to purchase a house, but are nevertheless turned away. In short, interest rates and housing prices do not appear to clear the mortgage market. In terms of residential maintenance, Williams (1993b) provides evidence that when people do not have a stake in their home's value (renters) they do not expend much effort on care which naturally leads to increased degradation of the physical plant.

If banks restrict their loans to induce care buyers may not be able to bid expected housing returns down to the current discount rate. As a result, housing returns become cyclical with the economy. Imagine that an area is about to undergo rapid economic growth, and that everybody in the area knows that it will happen. In this situation expected housing returns will rise because the future population in the area will be considerably wealthier than the current population. Alas, the credit constraints faced by the local population will prevent them from bidding up housing prices to the point where they reflect this future value. In this case bidding for the housing stock in the current period stalls out at a level low enough that those moving into the city expect to earn a positive expected abnormal return on their real estate purchase. Thus, the model may help to

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explain the empirical findings found in Case and Shiller (1989), Meese and Wallace (1994), and

Capozza and Seguin (1996). Both Case and Shiller (1989) and Meese and Wallace (1994) find

that housing prices do not follow a random walk, a fact consistent with the model presented here.

However, even more closely related is Capozza and Seguin's (1996) finding that housing prices

"over-react" to income growth since income growth is the mechanism that the model posits as

the driving force behind the time varying expected returns in housing.

Since credit rationing leads to time varying expected returns, home builders face a rather

unusual problem. If a construction firm owns an empty lot it can either develop it now, or wait a

period. Absent the moral hazard issue, this is the standard problem in the real options literature.

By waiting the developer both earns the return on housing and retains the option to delay

construction into the yet more distant future. But, if returns vary over time, developers will then

find it optimal to postpone development during periods of high expected real estate returns, and

to construct housing during periods of low expected real estate returns. This makes development

counter cyclical with real economic growth in an area. If development is counter cyclical, then

so is the aggregate supply of single family housing. At the very least fire, floods and natural

pests such as termites will destroy part of the housing stock each period. Absent an offsetting

level of construction activity the housing stock also becomes counter cyclical.

Since the model endogenously derives the decisions of developers, families and banks it

also produces predictions regarding cycles in the observed set of mortgage contracts. In the

model high rates of expected economic growth lead to high expected housing returns. This helps

to solve the moral hazard problem between the bank and homeowners, since for a given LTV

ratio the homeowner will have a greater incentive to care for the house if the expected return to

housing is high. Conversely, low expected housing returns exacerbate the moral hazard problem.

Thus, high growth leads to high expected returns and thus high LTV ratios, low growth to the

inverse. Empirically this should imply that LTV ratios can be used to forecast housing returns,

and in fact this prediction has been verified in Lamont and Stein (1999). They correlate leverage

with future housing returns under a variety of empirical specifications, and in each case the

leverage measure is positively associated with subsequent housing returns.

The paper is organized as follows: The next section presents a literature review. After that

section 2 presents the theoretical model. Section 2.1 lays out the model's representation of the

housing stock and the level of economic activity. Section 2.2 describes the agents within the

model, and section 2.3 the problems faced by a family that moves into the city. Section 2.4

combines all of these elements to calculate the equilibrium actions taken by the model's agents

and the resulting supply and price of housing. Section 2.5 lays out the model's equilibrium

properties. Subsection 2.5.1 describes the return process, mortgage rates, and equilibrium LTVs

while subsection 2.5.2 describes the housing supply. Section 3 discusses a number of extensions

to the model, with heterogenous wealth levels covered in subsection 3.1, and heterogenous

housing quality in subsection 3.2. Finally section 4 contains the paper's conclusions, and the

Appendix all of the proofs.

1 Literature Review

Generally, papers on housing construction treat the problem as a subset of the real options

literature and seek to determine the optimal stopping time. Papers within this literature include

Titman (1985), Majd and Pindyck (1987), Williams (1991), Capozza and Li (1994), Capozza

and Sick (1991, 1994), Grenadier (1995), Bar-Ilan and Strange (1996a), and Bar-Ilan and Strange (1998). These models are in a sense purely financial in that they can be applied to any asset that can be described as an option to invest within an environment where the developer is a price taker. Noncompetitive counterparts to this literature can be found in models by Williams (1993a), Grenadier (1996) and Williams (1997). Other papers by Capozza and Helsley (1990), Capozza and Sick (1991), and Bar-Illan and Strange (1996b) modify the real options problem to allow for the possibility that a home's distance from a city's center will influence its desirability. What distinguishes the current paper is the introduction of a moral hazard problem between home owners and their lenders which then induces a price process with time varying returns that may include periods with above market expected returns. In contrast, housing prices in the above models can never experience expected above or below market returns since prices always adjust to eliminate such rents.

This paper builds upon Spiegel and Strange (1991), and to some extent Williams (1993b). Both of these articles presume that housing maintenance plays an important role. However, they differ from the current paper in that they do not focus on price and construction dynamics. Instead, Williams (1993) seeks to explain why single family housing tends to be owner occupied while larger complexes are run by a landlord that rents out the units. In his model, people either rent units or own them outright. Large complexes are then rented out because landlords have a maintenance technology that is relatively inefficient when applied to small complexes and relatively efficient when applied to large complexes. Note that, in contrast to the current paper, the moral hazard problem in Williams' (1993) does not impact owner occupied housing. Spiegel and Strange (1991) examine a single period model in which the moral hazard issue between

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mortgage lenders and their borrowers leads to credit rationing and potentially above market expected returns. In this model the home's terminal value derives from an exogenously specified price process which then drives the initial transaction price. The current paper extends the analysis in a number of directions by allowing for construction, a dynamic infinite horizon setting, and the endogenous derivation of all prices.

Another related article is by Stein (1995). In his paper potential homeowners are faced with an exogenously specified down payment requirement when bidding on a house. He then uses this model to analyze on how credit constraints influence the joint determination of housing prices and trading volume. As his analysis shows, when a home increases in value the owner has additional equity that can be used to bid for additional housing. Thus, an increase in housing prices leads to an increase in demand which in turn leads to yet further increases in housing prices. Conversely, a decline in prices tightens the down payment constraint and thus stifles demand. These duel impacts lead to a very volatile housing price series in which trading volume is positively correlated with price increases. In contrast, the current paper focuses more on the development of the housing stock, and the correlation between the equilibrium mortgage contract and the local economic environment.

2 The Model

2.1 Geography and the Level of Economic Activity

For simplicity, assume that land within commuting distance of the business district exists in finite supply, and that only *n* single family homes can be built upon it. For now also assume

that all homes are identical.[1] Thus, a lot is always in one of two states: it either contains a

deteriorated unsaleable house that must be rebuilt or occupied by a saleable house. Anyone that

cannot afford a single family house is accommodated elsewhere, perhaps in a rental apartment or

another city. Based upon these assumptions the model should be viewed as applying to built up

cities such as New York rather than younger metropolises that are still expanding their borders.

The model has neither a beginning or ending date. What differentiates one period from

the next are two economic state variables w_t and s_t. The variable w_t captures the general level of

economic activity in the economy with higher values representing a more prosperous society.

The variable s_t captures the rate at which the economy is currently growing and interacts with w_t

via the following dynamic equation:

$$w_{t+1} = s_t w_t \delta_{t+1} \tag{1}$$

where δ_t represents a forecast error. Throughout, the paper assumes that the δ_t and the s_t are

independent across time periods with time independent distribution functions of $f(\delta)$, and $q(s)$

respectively. Since every variable in the model will be tied to the economic activity variable w_t it

is natural to restrict both w and δ to values between zero and plus infinity.   Finally, to cut down

on the notational burden, the expected values of both random variables are normalized to one.

2.2 Agents

Cities are dynamic places where people move in and out, and where homes are

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[1]The model abstracts from the price gradient issue for simplicity. But in reality
commuting costs will determine an effective boundary. This issue really comes down to one of
heterogenous housing quality, since homes further from the center can be thought of as lower
quality. Section 3.2 of the paper discusses how heterogeneous housing quality influences the
primary results.

constructed and fall apart. The model seeks to capture these processes via a stylized sequence of events. At the start of period t, the generation of inhabitants that moved in the previous period moves out and a new generation moves in. In real life, people move in and out of a city for any number of reasons that have nothing to do with the housing market. For example, they may get married, change jobs, or even die. Thus, the model takes the decision to move as an exogenous event. However, for the model's basic predictions to hold it is only necessary that every period some people move for reasons unrelated to housing prices.

Families possess a Cobb-Douglas utility function over housing and wealth of the following form:

$$U(\theta, W_{t+1}) = \theta W_{t+1} - c_t.$$ (2)

In the equation θ represents an indicator variable that equals one if the family lives in a house and zero if it does not, while W_{t+1} equals the family's wealth when it moves out of town. This functional form has the advantage of implying that above all else families prefer to live in single family housing rather than apartments or another city.[2] However, given the family's residence it prefers more wealth to less.[3] The variable c_t equals the utility spent by the family to care for their

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[2]Implicitly most spatial models in the real options literature employ a similar assumption. Papers by Capozza and Helsley (1990), Capozza and Sick (1991), and Bar-Illan and Strange (1996b) all assume that housing demand is completely inelastic with each resident of the city demanding one unit.

[3]Section 3.2 discusses the impact on the model's results that would arise from generalizing θ from a discrete variable to a continuous variable that reflects the utility a family receives from a house of a particular quality.

housing purchase. A family can vary its level as they see fit in response to their economic environment.

Since c_t does not come out of the budget constraint one should interpret it as representing the personal time and effort that people must put into keeping a home in shape if they are to prevent it from deteriorating. The model pursues this route for simplicity since adding c_t to the budget constraint does not alter its qualitative conclusions. Quantitatively, however, changing c_t from effort to cash strengthens the model's predictions regarding the existence and influence of credit constraints. If c_t represents cash instead of effort then the banks will further restrict their loans to ensure that the owners of the house have both the incentive and funds to care for it. Under the model's current assumptions the banks do not need to worry about the latter issue.

When members of the old generation move out of the city they put their homes on the market. The new generation then bids on these homes, with each house going to the highest bidder. In case of a tie among the bidders the winner is selected via a random draw that gives each of the high bidders an equal chance to win. Each incoming family arrives with a wealth level of $W_t = w_t$ (recall w_t is just a state variable, thus this definition just normalizes it to equal the current wealth of the incoming generation). In an attempt to purchase a home they can combine their wealth with a bank loan. Banks offer mortgages in which they lend an amount ℓ in exchange for a commitment from the borrower to repay m the next period when the family moves out of the city. The model assumes that mortgages are nonrecourse loans, and thus if the borrower defaults the bank can take possession of the house, but cannot otherwise punish the borrower. Relaxing this assumption will only alter some of the model's quantitative predictions,

and not its qualitative qualities. To obtain the paper's results one only needs that borrowers can, to some degree, default on their loan commitments.[4]

In the event of default the bank takes possession of the house and can either sell it immediately, rent it out, or leave it vacant. If either of the latter two options are employed then it can then try to sell the house at some later date. Since casual observation indicates that empty homes tend to deteriorate rather quickly the model assumes that an unoccupied home receives an amount of care equal to zero.

While the model assumes that only occupants can care for a residence, one can modify the model to allow for care by the lender as well. The model's basic qualitative properties will remain so long as lenders are less efficient than occupants at home maintenance. This seems like the natural assumption since care by the lender is likely to be less timely and involve other inefficiencies. Furthermore, contractual difficulties may arise leading to costly litigation regarding what care the lender is responsible for and what care is the resident's responsibility.[5] One can model the relative inefficiency of lender maintenance by assuming that lender care involves a fixed cost and that at the margin the benefits of a dollar of lender care are less than the benefits from a dollar of residential care. In this case, if the fixed costs are high enough then in equilibrium the banks will not chose to care for the house, and the model's results will remain

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[4]While the paper only examines traditional loans, it can be easily generalized to include others types such as equity participation loans. For the model's results to hold one only needs that larger mortgage repayment provisions further reduce the owner's payoff in some states of nature. So long as this holds, owner provided care of the house will fall as the repayment provisions increase, and the model's qualitative results will remain unchanged.

[5]Given the extensive litigation over this issue in condominiums and co-operatives it is apparently very difficult to write precise contracts regarding the definition and limits of home maintenance.

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unchanged. For lower fixed costs, the bank will expend some resources on care, but it will also restrict the amount it lends to induce care by the residents too. The important point is that even in this case credit constraints for the purpose of inducing residential care will arise.

Banks are assumed to be profit maximizing, competitive, risk neutral institutions. They have an infinite capacity to issue mortgages and will do so if they believe that the expected return on the loan equals or exceeds the current interest rate r. In addition to banks, the city also contains developers. Developers can build homes on empty lots at a cost of $hw_t s_t$. The assumption that housing construction costs depend upon the state variables captures the idea that in a wealthy or fast growing economy local resource demands go up and this in turn adds to building costs.[6] While the model's qualitative results do not depend on this assumption altering it will reduce its tractability.[7] For simplicity, also assume that a payment of $hw_t s_t$ dollars in period t allows the developer to produce a home in that same period. While incorporating a lag time between the start and conclusion of construction brings up a number of interesting issues, they are beyond the scope of this analysis. Qualitatively, however, one expects that construction lags to have the same influence in the model presented here as it does in the models devoted to this subject such as Majd and Pindyck (1987), Capozza and Li (1994) and Bar-Ilan and Strange (1998).

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[6]This point has been verified empirically by Somerville (1999).

[7]Most papers in the literature following Capozza and Helsley (1990) assume a constant cost of production independent of the state of nature. The truth probably lies somewhere between the proportionality assumption employed here and the independence assumption generally used. Later on the paper discusses how altering this assumption will modify the paper's results.

While, in principle, the banks and developers are separate entities, it is easier to treat them as one and the same. Thus, the discussion in the paper will not attempt to separate out the two and the term bank and developer will be used interchangeably as appropriate. As will become clear, this essentially implies that when a bank forecloses on an empty lot, it turns it over to its development office instead of selling it to a separate corporation. This assumption has absolutely no bearing on the paper's results, and simply reduces amount of notation needed to construct the model.

2.3 The Incoming Family's Problem

When moving into the city families are initially faced with the problem of bidding for a house. After that, if they successfully purchase a house they must then decide how much care they should put into its maintenance. Higher maintenance levels cost the family more in lost utility, but increase the future value of the house. Since, for now, the model assumes that the housing stock is of homogenous quality, care can only influence the probability that the recently purchased house will remain standing next period. One can thus think of care as representing the family's effort at maintaining the home's structural integrity. This simplifying assumption, regarding the impact care has on the housing stock, allows one to work within an environment in which homes are of homogenous quality while still allowing costly care to make a difference. Section 3.2 of the paper discusses the impact of relaxing this assumption and instead allowing care to influence a home's deterioration rate, rather then the probability it remains in saleable condition.

Based upon the above discussion a homeowner's optimization problem can be written as

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$$\max_{c} \quad g(c/s_t w_t) \int_{\psi} [P_{t+1} - m] f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1}$$

$$+ \ [1 - g(c/s_t w_t)] \int_{\varphi} [V_{t+1} - m] f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} - c. \tag{3}$$

In equation (3) ψ represents the set of states such that $P_{t+1} \geq m$ when the home remains saleable, and φ the set of states where the homeowner remains solvent even though the home is no longer saleable. The function V represents the value of the unsaleable house. Naturally, both P and V are functions of the state variables s and w.

The model's basic premise lies in the idea that the moral hazard problem between the bank and the homeowner induces housing returns and construction to exhibit a number of patterns that are atypical of most financial assets. For the model to capture this, it is necessary that the level of care produced by a family have some impact upon the expected value of the house that they live in. To guarantee that this is true it is necessary to impose a number of restrictions on g.

The first restriction on g is that it must be sensitive enough to the level of care that agents believe their actions will materially impact its value. For example, suppose that a home's probability of remaining saleable does not in any way depend upon the level of care expended by the homeowner. Then obviously homeowners will not expend utility resources on care and lenders will not restrict their lending practices in order to induce homeowners to engage in care either. A second restriction on g is that it must allow care to be sufficiently cost effective that

there exist some conditions under which a family will not set it equal to zero. Otherwise one is once again faced with a situation where the bank's lending practices cannot influence the behavior of its borrowers. Third, g must not make care "too cheap." If care is too cost effective then homeowners will simply set it to the point where the probability that the house becomes unsaleable equals zero and the moral hazard problem will once again fade away.

Based upon the above three criteria the moral hazard issue will matter so long as one assumes that g has the following properties:

1. $g(0)=0$,

2. $g'(0)>1$, and

3. for c such that $g = 1$, $g' = 0$.

The first condition states that if the homeowner does not care for the house it becomes unsaleable with probability one. The second condition states that at the point where care equals zero a small amount of care will return more in expected housing value than it costs. The third condition states that at the margin if the probability that the house will remain saleable equals one, the marginal benefit from additional care equals zero. Obviously this must be true for the right derivative, but the condition also imposes it for the left derivative. Thus, an alternative (but slightly more restrictive) assumption would be that g is twice continuously differentiable for all levels of care.

Within g (see equation (3)) the care level is divided by the current state of the economy to allow for the idea that when the economy is doing well and growing quickly a family's

opportunity costs increase making it more costly for them to produce the same quality of care.[8]

Another interpretation is that as the economy grows the family must put more into the house in order to keep it attractive relative to other housing alternatives in the area. This particular functional form has the advantage of both capturing these effects while simultaneously adding to the model's overall tractability.

2.4 Equilibrium

The paper assumes that the level of care set by the family cannot be imbedded in the mortgage loan agreement. Thus, mortgage loans contain simply an amount the bank will lend and a balloon payment that the borrower will owe the following period. Under these circumstances the homeowner's first order condition can be written as

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\begin{aligned}
&g' \frac{1}{s_t w_t} \int_\psi [P_{t+1} - m] f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} \ - \\
&g' \frac{1}{s_t w_t} \int_\varphi [V_{t+1} - m] f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} - 1 \ = \ 0.
\end{aligned}
\tag{4}
$$

Due to the moral hazard problem posed by the homeowner's care decision banks may chose to restrict the size of the loan they will extend to any one family. This in turn restricts the amount which a family can bid for a house. However, prior to formulating the bank's problem one must address the issue of whether or not a home that has been repossessed will always be

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[8]One can relax the assumptions regarding how the state of nature alters the effectiveness of a unit of care without altering most of the model's qualitative assumptions. The primary impact will be on the relationship between equilibrium housing prices and the state of nature. With this functional form, prices will turn out to be homogenous of degree one in both s_t and w_t. While other functional forms will not produce this relationship they will cause housing prices to increase in the two state variables which is all the model really needs to drive its results. Such generalizations are discussed at the end of section 2.5.2.

sold to a family that wishes to occupy it. This not only affects the bank's problem but also the

distribution of the price processes. If, in some states of nature, speculators that wish to buy and

hold homes unoccupied can outbid families that wish to occupy them, then this must be

accounted for. Fortunately, none of these concerns arise in equilibrium since one can prove that

if a home is in saleable condition the highest bidder will always be a family that wishes to take

possession. Or equivalently, if a bank forecloses on a house it immediately resells it to a family.

Proposition 1: Developers and banks never rent or leave vacant any home in saleable condition.

Proposition 1 shows that the paper's results hold even though there exist wealthy

individuals or institutions that can potentially purchase and hold housing in order to speculate on

its return. The difficulty speculators face is that while owner occupied housing may have an

above market return, in any particular period, unoccupied and rental housing does not. Thus,

they cannot profitably bid homes away from the rest of the people that wish to live in the area

and thus do not influence the equilibrium price process. Along with the standard tax arguments

(that are not covered in this paper), this may help to explain why 85.5% of all single family

detached homes were owner occupied in 1990 (U.S. Census, 1990).

From Proposition 1 it immediately follows that: (i) families will always outbid

developers for a house in saleable condition, and (ii) if a bank forecloses on a house it

immediately resells it to a family that intends to take possession. Since (i) and (ii) imply that

banks always resell homes that they have repossessed, the equilibrium mortgage contract must

solve

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$$
\begin{aligned}
\max_{m} \quad & g(c/s_t w_t)\left\{\int_{\neg\psi} P_{t+1} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} \;+\; m\int_{\psi} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1}\right\} \\
& + \; [1-g(c/s_t w_t)]\left\{\int_{\neg\varphi} V(s_{t+1}, w_{t+1}) f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} \;+\; m\int_{\varphi} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1}\right\}.
\end{aligned}
\tag{5}
$$

The region of integration $\neg\psi$ represents the set of states where bankruptcy occurs and the home remains saleable ($P_{t+1} < m$). If the price of the house falls below the mortgage payment the family will default and the bank will take possession of the house and then sell it. This is represented by the first integral. The second integral covers those states of nature where the value of the home exceeds the mortgage payment due to the bank. Whenever this happens the bank expects the family to repay the loan and keep the house. The second term in braces represents the value to the bank when a house has deteriorated to the point of being unsaleable. In the region of integration $\neg\varphi$ the bank obtains an unsaleable house during foreclosure, while in the region φ the homeowner prefers to pay off the loan.

With the assumptions behind the formulation of equation (5) now verified one can differentiate it with respect to m. Setting the result equal to zero produces the following first order condition

$$g' \frac{dc}{dm} \frac{1}{s_t w_t} \left\{ \int_{-\psi} \int P_{t+1} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} + m \int_{\psi} f(\delta_{t+1}) q(s_{t+1}) d\delta ds \right\}$$

$$+ \ g \int_{\psi} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1}$$

$$- \ g' \frac{dc}{dm} \frac{1}{s_t w_t} \left\{ \int_{-\varphi} \int V_{t+1} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} + m \int_{\varphi} f(\delta_{t+1}) q(s_{t+1}) d\delta ds \right\}$$

$$+ [1-g] \int_{\varphi} f(\delta_{t+1}) q(s_{t+1}) d\delta_{t+1} ds_{t+1} \ = \ 0$$

(6)

that must characterize the mortgage which yields the maximum loan amount. To help keep the

notation compact g' represents the derivative of g. The dc/dm term represents the fact that as the

bank demands an ever higher mortgage payment the level of care taken by the homeowner will

change. The next Proposition indicates that c weakly decreases in m.

Proposition 2: The family's optimal level of care weakly decreases in the required mortgage

payment. Formally, $dc/dm \leq 0$.

In equilibrium the conjectures held by both the bank and the city's families regarding the

distribution of future housing prices must be accurate. Following standard practice, the solution

to the model is found by initially conjecturing a solution to the price process. This conjecture is

then used to solve for the actions of each player and in turn their actions are used to derive the

dynamics of the price process. If the conjecture holds true, in that the prices produced by the

players matches the conjectured price process then an equilibrium has been found.

Within the current setup the appropriate conjectures are that the banks will lend an

amount $\ell_t = w_t s_t k_\ell$, and in return will require a mortgage repayment of $m_t = w_t s_t k_m$. Should the

bank find itself in possession of an unsaleable house then its expected profit from building and

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selling a new house at the optimal date will equal $V_t = w_t s_t k_V$. Finally, in response to both the bank's lending practices and the resulting housing price process homeowners will set their care level to $c = w_t s_t k_c$, where k_ℓ, k_m and k_c are constants.

Since families will spend whatever is necessary to move into a house, they will bid up housing prices until they equal the sum of the current loan amount and family wealth level. Thus, $P_t = \ell_t + w_t$ and given the conjectured solution to ℓ_t one can rewrite this as $P_t = w_t(s_t k_\ell + 1)$. Increasing the index one period yields $P_{t+1} = w_{t+1}(s_{t+1} k_\ell + 1)$ and since $w_{t+1} = w_t s_t \delta_{t+1}$ this implies $P_{t+1} = w_t s_t \delta_{t+1}(s_{t+1} k_\ell + 1)$.

Substituting in the conjectured solution into the homeowner's first order condition, equation (4) becomes

$$
\begin{aligned}
&g'\int_\psi (k_\ell s_{t+1} + 1)\delta_{t+1} - k_m)f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} - \\
&g'\int_\varphi (k_v s_{t+1}\delta_{t+1} - k_m)f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} - 1 = 0
\end{aligned}
\tag{7}
$$

where the $s_t w_t$ terms in the integral have been canceled out with those arising from the differentiation of g. Similarly, the first order condition characterizing the equilibrium mortgage payment (6) now equals

$$g'\frac{dc}{dm}\left[\int_{\neg\psi}(k_l s_{t+1}+1)\delta_{t+1}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} + k_m\int_{\psi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}\right]$$
$$+ g\int_{\psi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}$$
$$- g'\frac{dc}{dm}\left[\int_{\neg\varphi}k_V s_{t+1}\delta_{t+1}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} + k_m\int_{\varphi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}\right]$$
$$+ (1-g)\int_{\varphi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} = 0. \tag{8}$$

In addition to equations (7) and (8) in equilibrium banks must earn a zero expected profit on their loans. Thus, the condition

$$k_l = \frac{1}{1+r}\left\{g\left[\int_{\neg\psi}(k_l s_{t+1}+1)\delta_{t+1}f(\delta_{t+1})q(s_{t+1})d\delta ds_{t+1} + k_m\int_{\psi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}\right]\right.$$
$$\left. + (1-g)\left[\int_{\neg\varphi}k_V s_{t+1}\delta_{t+1}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} + k_m\int_{\varphi}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}\right]\right\} \tag{9}$$

must also be satisfied, where r is the appropriate discount rate.

Equations (7), (8), and (9) describe the equilibrium conditions for the homeowner and the bank given the valuation function describing the bank's payoff when in receipt of an unsaleable home. Given the proposed conjectures the valuation function must satisfy

$$k_V = \frac{1}{1+r}\left\{\int_{\xi}\left[(k_l s_{t+1}+1)\delta_{t+1}-hs_{t+1}\delta_{t+1}\right]f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1} + \right.$$
$$\left. \int_{\neg\xi}k_V s_{t+1}\delta_{t+1}f(\delta_{t+1})q(s_{t+1})d\delta_{t+1}ds_{t+1}\right\} \tag{10}$$

where ξ represents the set of future states under which the bank builds and then sells a house on the lot, and ¬ξ the set of states where the bank postpones construction until at least the next period. As will be shown later the optimal rule for the bank has it construct and sell a house if s_{t+1} lies below some critical bound. Based upon the above set of equilibrium equations one can now solve for k_c, k_ℓ, k_m, and k_V. Furthermore, the conjectures will be self fulfilling since the level of care, the size of the mortgage loans, the required balloon payment, and the implicit value function (V) can all be obtained independently of w_t or s_t's current value.

2.5 Properties of the Equilibrium

2.5.1 Expected Housing Returns, Mortgage Rates, and LTVs

Given solutions to k_c, k_ℓ, k_m, and k_V one can now calculate the expected return to a house over the next period as

$$\frac{E(P_{t+1})}{P_t} = \frac{E\left[(k_\ell s_{t+1}+1)s_t\delta_{t+1}\right]}{k_\ell s_t+1} = \frac{(k_\ell+1)s_t}{k_\ell s_t+1} \tag{11}$$

where the last equality follows from the assumption that $E(s_{t+1})$ and $E(\delta_{t+1})$ equal 1. This equation represents the typical definition used by people that construct real estate price indices (for example, repeat sales indices such as those found in Case and Shiller (1989), or hedonic models such as Meese and Wallace (1994)). Since it does not include either the cost of maintenance or the probability that the house will become unsaleable next period it does not represent the actual expected return to say a homeowner with a 100% equity position in the house. Nevertheless, this is the proper definition for somebody holding a lot without a saleable home on it that wishes to know if construction should take place this period or next, which is the

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issue under study in this paper.   However, with relatively little difficulty the results regarding

expected returns can be modified to account for other perspectives.

Equation (11) implies that the return to housing over the next period lies between 0 and

$(k_\ell + 1)/k_\ell$ as s_t varies between zero and plus infinity (assuming k_ℓ is nonnegative). The next

proposition refines this range by finding restrictions on the value of k_ℓ.

Proposition 3: The value of k_ℓ is strictly greater than 0 and less than 1/r. It equals 1/r only when

there does not exist a moral hazard problem.

Based upon equation (11) the variation in expected housing returns depends critically upon the

equilibrium value of k_ℓ.   The value of k_ℓ in turn depends upon the importance of the moral hazard

problem. Since (11) is strictly increasing in s_t the expected return to housing must range from 0,

when s_t equals 0, to a high of $(k_\ell+1)/k_\ell$ as s_t goes to infinity. Thus, based upon Proposition 3

when care is irrelevant housing returns vary from 0 to r, while in an environment where care is

important expected returns need not have any upper bound. These results are summarized in the

next Proposition.

Proposition 4: Expected housing returns are strictly increasing in the local rate of economic

growth, $d\{E(P_{t+1})/P_t)\}/ds_t > 0$. At $s_t = 0$ the expected return to housing equals zero, as s_t goes to

infinity the expected return to housing approaches $(k_\ell+1)/k_\ell$.

The above results extend to a multiperiod setting Spiegel and Strange's (1992) conclusion

that moral hazard issues can produce housing prices that exhibit predictable excess returns.

However, by deriving this result within a dynamic model the current analysis goes further by

showing that predictable excess housing returns can occur in *any* economy where care matters if

s_t happens to be large enough in that period. Conversely, the dynamic setting also shows that for

s_t small enough housing prices will exhibit predictable *below* market returns. All of this implies

that expected housing returns will depend more upon the current rate of growth in the local

economy (s_t) than the current interest rate.

Since housing prices within the model are determined by the equilibrium mortgage

contract, the model also provides predictions regarding how this contract varies with expected

economic growth.

Proposition 5: In equilibrium mortgage interest rates are invariant over time since $m_t/\ell_t = k_m/k_\ell$,

while LTVs vary pro-cyclically with expected rates of economic growth since $\ell_t/P_t = s_t k_\ell/(s_t k_\ell + 1)$.

This proposition indicates that mortgage contracts will adjust to changing economic conditions in

the local economy via the LTV demanded by banks rather than through the mortgage interest

rate. Within the model this occurs because the LTV acts as a mechanism via which the bank can

induce the homeowner to care for the house. As shown in Proposition 4 high rates of expected

economic growth lead to high expected housing returns. But, higher housing returns allow the

bank to relax the LTV constraint.[9] For a given LTV the higher the expected returns in the

housing market the greater the homeowner's incentive to care for the house. The mortgage

interest rate stays constant within the model since the bank's required return on its loans always

equals r and this number does not vary with local economic conditions.

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[9]As noted in the introduction, the model's prediction that LTV ratios forecast expected
housing returns is consistent with the empirical findings in Lamont and Stein (1999).

2.5.2 Housing Supply and Construction

New housing can only be built on empty lots (or equivalently on lots with unsaleable homes), and lots only become empty when they deteriorate due to a lack of care. Thus, the next proposition lays out the probability that a particular lot will become unsaleable.

Proposition 6: In equilibrium homeowners set c such that g<1. Thus, lots become vacant with some positive probability.

The model thus produces the realistic result that over time the general housing stock will deteriorate and thus needs replacing. Of course, the more interesting question concerns the conditions under which that replacement will take place. In a typical market when firms forecast an increase in the consumer's willingness to pay for an item production goes up. However, housing is not typical since it is both an investment and consumption good. As the next proposition shows this duality leads to building cycles that move in the opposite direction as income growth.

Proposition 7: *Housing construction takes place when income growth (s_t) falls below some critical bound s^*.*

Proposition 7 shows that housing construction occurs when local economic growth is slow, and stops when the local economic growth is high. Housing construction thus occurs counter cyclically in the model. This is the opposite result one obtains in most real options papers on housing construction. Typically models assume that prices evolve exogenously via a random walk. In such models the implicit assumption is that unexpectedly large improvements in the local economy result in unexpected increases in the price of housing. Developers then react to the elevated price level by building new homes on any vacant lots they own, thereby

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driving prices back down. However, in the current model the endogenous credit constraint

causes this reasoning to break down. Since families cannot bid as much as they like, prices at

any one moment in time reflect what people can bid, and not what they would like to bid. As a

result future price changes are not independent of the current state of nature, even though that

state is common knowledge. Developers know this and react accordingly.

To understand the impact of the developer's problem consider the optimal strategy when

people believe that housing prices will increase at a rate that exceeds the interest rate r, say r_h. If

the developer waits one period and then builds he will expect to earn r_h from the increase in the

lot's value. Since this return exceeds the interest rate, it must be a good idea to delay

development. However, one can of course do even better by only building in the next period if it

is optimal to do so. In this case even if r_h falls slightly below the interest rate r, it will still pay

the developer to delay construction. Now consider what this implies for the housing stock. From

Proposition 6, in equilibrium, homeowners never produce enough care to prevent their home

from becoming unsaleable with probability one. Thus, in any period without new construction

the available housing stock will shrink. Combining this with the response that the builders have

to local economic growth leads to the result that when the local economy grows sufficiently

quickly housing prices will rise accordingly, new development will cease, and the supply of

housing will decline. These conclusions are summarized in the following proposition.

Proposition 8: When the growth rate of the local economy exceeds a critical bound new

construction ceases and thus the supply of saleable housing declines.

In the model presented here housing construction takes place when housing prices are not

expected to grow too quickly. On the other hand, the price level itself has no impact on the rate

of construction. Conversely, models based upon Capozza and Helsley (1990) (the "stochastic cities" literature for short) find that housing construction takes place when prices are high, but that expected returns have no impact on the construction rate. These contrasting results are due in part to the technological assumptions used by the two models. In this paper construction costs rise proportionately with the level of economic activity, while in the stochastic cities literature construction costs are independent of such activity. More realistically, one would like to allow for costs that rise somewhat with economic activity but at a rate that is less than proportional. Generalizing the model in this manner would result in housing construction rates that increase with the price level but decrease with the expected rate of return in housing prices. In contrast, this dual dependance on both the price level and expected returns will not arise in the stochastic cities literature since one of the equilibrium conditions is that the expected return to housing equals the interest rate.

3 Extensions

3.1 Heterogenous Wealth Levels

Allowing for heterogenous wealth levels does not, in principle, pose any difficulties. As in almost any model with market clearing, the marginal purchaser sets the price which in this case will be the richest family that cannot buy a house. Unfortunately, however, incorporating heterogenous wealth into a multiperiod setting makes it impossible to obtain a closed form solution.[10] With heterogenous wealth levels the current supply of housing now impacts the

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[10]In contrast, Spiegel and Strange's (1992) single period setting does allow for an explicit solution, since it their model it coincides with the solution for a population with homogenous wealth endowments.

equilibrium since it impacts the wealth of the marginal family. This adds another state variable and one whose distribution depends on the recent history of the economy. After several periods of high economic growth the housing stock will shrink. Thus, the wealth of the marginal family will have grown both due to the expanding economy and the shrinking housing supply. Nevertheless, since the marginal family is always credit constrained (or they would bid more in order to move into a house) they still set the equilibrium price, and therefore the paper's general qualitative conclusions must hold.

3.2 Heterogenous Housing Quality

Heterogenous housing quality in the absence of heterogenous wealth does not alter the model in a meaningful way. Suppose that the variable θ in the family's utility function (equation (2)) is now continuous, and represents the quality of the house owned by the family. Since the marginal family will still bid up to the limit of their credit constraint the housing price process will not change. In order for heterogenous housing quality to have a significant impact on the model it must be accompanied by heterogenous wealth. While a closed form solution is now unavailable one can still characterize the equilibrium bids within a particular period.

Clearly, with heterogenous housing quality and wealth the equilibrium bids will sort wealthier families into better homes since they can outbid their poorer neighbors. Sort the homes by increasing quality $1, \ldots, k$ and the people by decreasing wealth $1, \ldots, n$ with $n > k$ (in fact the model requires that n exceed the number of lots zoned for single family housing). Family $k+1$ represents the wealthiest family that cannot afford to purchase a house, and thus family k need only outbid them to obtain residence k. Since family $k+1$ just misses out on a house they must bid to the point where their credit constraint binds, and thus the price of the kth home equals the

wealth of the *k*th family plus the maximal amount they can borrow. Now consider the price of home *k*-1. In order to move into this house family *k*-1 must outbid family *k*. This means that the price of the *k*-1 home will depend both upon the quality difference between it and house *k* and the wealth of families *k* and *k*-1. Suppose the quality difference is large. Then the price of house *k* will equal the most family *k*+1 can bid, while the price of house *k*-1 will equal the most family *k* can bid. In this case both prices look like those found within any one period in a model of homogenous housing quality and family wealth. On the other hand suppose the quality difference is relatively small. Then family *k* will only bid to the point where it is indifferent between house *k* and *k*-1. Thus, family *k*-1 need only bid enough to make family *k* just barely worse off in house *k*-1. How do you know that family *k*-1 will do this? Since family *k*-1 has a larger wealth endowment, it can borrow less than family *k* and still win the house. Because family *k*-1 can buy the house while borrowing less, that family will take better care of the house (a straightforward conclusion that derives from equation (4)) and thus it will provide them with a higher expected utility. This implies that family *k*-1 will always outbid family *k* for home *k*-1. However, in terms of the housing price process the important thing to note is that the price of the *k*th home plays a pivotal role in determining the price of house *k*-1. Since the price of house *k* depends upon the credit limit faced by family *k*+1, the price of the higher quality homes is also influenced by it and therefore one expects the general qualitative conclusions of this paper to hold under these conditions also.

4 Conclusion

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This paper develops a model that enogenizes the equilibrium construction decisions by homebuilders, the mortgage contracts offered by banks, and the bidding and care decisions by families wishing to move into a neighborhood. When these elements are combined, housing prices become linked with the rate of economic growth in the local economy. This linkage implies that developers will look to the current state of the economy before they decide to build new housing.

The model provides a number of empirical predictions, many of which have already been shown to hold in the data. In equilibrium, the model predicts that LTV ratios will forecast future housing returns, with high ratios preceding high housing returns. A prediction that was recently confirmed in Lamont and Stein's (1999) empirical study. Another prediction that appears to hold in the data (for example Case and Shiller (1989)) is that housing returns will experience periods in which they predictably yield above or below market returns. However, the model also makes the as yet untested prediction that these periods will be linked to the rate of growth in local economic activity. In terms of new construction, the model indicates that it will occur in cycles. If correct, then within the data there will exist periods where relatively little construction takes place (despite high housing returns) followed by periods of intense activity. Finally, the model also helps to explain why relatively few individuals and institutions rent out single family homes. As the model shows, potential buyers can make offers that are sufficiently attractive to potential renters that it pays potential landlords to sell the house rather than rent it out.

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6 Appendix

Proposition 1: Developers and banks never rent or leave vacant any home in saleable condition.

Proof: To prove the proposition first note that if a family moves into a rental house they expend zero effort on care, since their return for doing so equals zero. Thus, vacant and rental homes receive the same level of care, zero. This implies that the expected value of the house in the following period is same whether the owner rents it out or leaves it vacant. Thus, renting must dominate (since the owner earns a fee in the meantime) and no homes are allowed to stand vacant between sales.

To show that banks always sell rather than rent saleable homes in their possession it is only necessary to show that there exists some mortgage contract that allows a family to pay more for the house then the bank can get in expected value from its rental. As noted above a rented house receives no care, and thus with probability one will become unsaleable next period. Thus, the value to either a bank or developer of renting a currently saleable home for one period is the present value of owning an unsaleable home one period hence plus the rental fee. Clearly the rental fee must be less than or equal to the current wealth level (w_t) possessed by a family, which implies the rental value is strictly less than what a family can offer to buy the house. To see why consider a suboptimal mortgage with m set to infinity. From equation (4) a family with this mortgage will set c to zero and thus with probability one the bank will take possession of an unsaleable house one period hence. Thus, this mortgage contract has the same value as a currently saleable home that is rented for w_t. However, a family can make at least a weakly higher bid since, due to the moral hazard problem, a mortgage contract with m equal to infinity will not necessarily maximize the available loan size. Q.E.D.

Proposition 2: The family's optimal level of care weakly decreases in the required mortgage payment. Formally, dc/dm≤0.

Proof: Differentiate equation (4) with respect to m and then rearrange to show that $dc/dm \le 0$ after recalling that since c maximizes (4) the second derivative with respect to c must be negative. Q.E.D.

Proposition 3: The value of k_ℓ is strictly greater than 0 and less than 1/r. It equals 1/r only when there does not exist a moral hazard problem.

Proof: The proof begins by showing that k_ℓ is strictly positive. Rearrange equation (9) to solve for k_ℓ producing

$$k_\ell \left[1 - \frac{1}{1+r} g \int_{\neg\psi} s_{t+1} \delta_{t+1} fqd(\delta_{t+1})d(s_{t+1}) \right] =$$
$$\frac{1}{1+r} \left\{ g \left[\int_{\neg\psi} k_v s_{t+1}\delta_{t+1} fqd(\delta_{t+1})d(s_{t+1}) + k_m \int_\psi fqd(\delta_{t+1})d(s_{t+1}) \right] + \right. $$
$$\left. (1-g) \left[\int_{\neg\varphi} k_v s_{t+1}\delta_{t+1} fqd(\delta_{t+1})d(s_{t+1}) + k_m \int_\varphi fqd(\delta_{t+1})d(s_{t+1}) \right] \right\}. \qquad (12)$$

Since r>0, g≤1, and the expected values of s_{t+1} and δ_{t+1} both equal 1 the term in square brackets on the left hand side of the equation must be strictly positive. Since the terms on the right hand side sum to a strictly positive number, one has that $k_\ell > 0$.

To find the upper bound on k_ℓ consider the conditions that would maximize the amount a bank would lend. The largest possible period t+1 payoff to the bank would occur if at m equal to infinity, the home always remained saleable. In this case the bank repossess the home in period t+1 for sure and thereby acquires its full value. Clearly, the actual loan must be based on the bank's expectation that it will get less than this. Setting m to infinity and g to one in equation (9)

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yields $k_\ell < (k_{\hat{}}+1)/(1+r)$, solving for k_ℓ yields $k_\ell \leq 1/r$. To prove that the inequality is strict in the presence of moral hazard, note that setting k_m such that $g = 1$ cannot be optimal. If $g = 1$, then raising k_m will allow the bank to lend strictly more and thus increasing maximal bid by a homeowner. Q.E.D.

Proposition 6: In equilibrium homeowners set c such that g<1. Thus, lots become vacant with some positive probability.

Proof: The proof is by contradiction. Suppose that $g = 1$. By assumption when $g=1$, its derivative $g' = 0$. Thus, if $g = 1$ the left hand side of the equilibrium equation (8) reduces to the probability that $P_{t+1} > m$. For the first order conditions to hold this probability must equal zero. However, if P_{t+1} never exceeds m then the solution to the homeowner's first order condition must set care to zero. But if care equals zero then $g = 0$, a contradiction. Q.E.D.

Proposition 7: Housing construction takes place when income growth (s_t) falls below some critical bound s^*.

Proof: Given the current value of the state variables s_t and w_t it pays to construct a new house in the current period if

$$[k_\ell s_t+1]w_t - h s_t w_t \geq$$
$$\frac{1}{1+r}\left\{\int_{\xi}[[k_\ell s_{t+1}+1]\delta_{t+1} - h s_{t+1}\delta_{t+1}]s_t w_t f q d\delta_{t+1} ds_{t+1} + \int_{\neg\xi} k_v s_{t+1}\delta_{t+1} s_t w_t f q d\delta_{t+1} ds_{t+1}\right\} \quad (13)$$

where the left hand side of the equation equals the value obtained from immediate construction and the right hand side the value from delaying construction one period and then building optimally. To prove the proposition one must first sign the following expression

$$\Xi = (h-k_\ell)(1+r) + \int_{\xi}\left[(k_\ell s_{t+1}+1)\delta_{t+1} - hs_{t+1}\delta_{t+1}\right]fqd\delta_{t+1}ds_{t+1} + \int_{\neg\xi}k_v s_{t+1}\delta_{t+1}fqd\delta_{t+1}ds_{t+1}. \tag{14}$$

The last two terms in (14) are proportional to the value from delaying construction one period and then building optimally. Now consider the suboptimal strategy of waiting one period and then building next period in all states of nature. Since this strategy, by definition, must have at least a weakly lower payoff one has that

$$\Xi \geq (h-k_\ell) + \int\left[(k_\ell s_{t+1}+1)\delta_{t+1} - h\delta_{t+1}s_{t+1}\right]fqd\delta_{t+1}ds_{t+1} =$$
$$(h-k_\ell)(1+r)+k_\ell+1-h =$$
$$1+r(h-k_\ell). \tag{15}$$

From Proposition 3, the value of k_ℓ lies below $1/r$ and therefore $\Xi \geq rh \geq 0$. Having signed Ξ, one can now rearrange (13) to show that the developer will build on a vacant lot if $s_t \leq (1+r)/\Xi$.

Q.E.D.