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String Stability Of Interconnected Systems: An Application To Platooning In Automated Highway Systems

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D.V.A.H.G. Swaroop

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Abstract

String Stability of Interconnected Systems:
An Application to Platooning in Automated Highway Systems

by

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Automated Highway System (AHS) is primarily aimed at improving the traffic flow capacity of the highways while ensuring safety. Central to successful deployment, of AHS is the development of Automated Vehicle Control Systems (AVCS). The longitudinal control aspect of AVCS deals with automatically controlling the intervehicular spacing of close-vehicle formations called platoons. This dissertation investigates various platooning strategies and their impact on the performance of the platoon

String stability of a vehicle platoon is the primary performance parameter. Intuitively. string stability of a vehicle platoon ensures that the intervehicular spacing errors of all the vehicles are hounded uniformly in time provided the initial spacing errors of all the vehicles are bounded. In this dissertation. we design various decentralized control algorithms and characterize their performance in terms of the minimum attenuation of the maximum spacing errors that can be guaranteed from vehicle to vehicle in the platoon.

Parametric uncertainties degrade the platoon performance. In order to improve the robustness of a string stable control algorithm, a direct adaptive cont'rol algorithm that guarantees improved performance is designed.

The concept of string stability is extended to general nonlinear dynamical systems. We derive sufficient conditions for ensuring stability for a countably infinite interconnection of exponentially stable nonlinear systems. We also show that under the same conditions, string stability is preserved for structural and singular perturba-

tions. Then. we present a decentralized adaptive controller to improve the robustness in the presence of parametric uncertaint'ies for the same class of systems.

The contributions of this dissertation are twofold: From an application point of view. this dissertation proposes " practical" platooning strategies. From a theoretical point of view. this study extends the concepts of stability to a countably infinite interconnection of general nonlinear dynamical systems and introduces techniques for analysis and design of decentralized control laws for them.

J. K. Hedrick. Thesis Committee Chair.

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by

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Dedicated to all my teachers

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Chapter 1

Introduction

This dissertation addresses the platoon control problem, i.e.. the problem of designing decentralized controllers that maintain a desired intervehicular spacing in a vehicle string in the presence of uncertainties and disturbances and in the light of various available feedforward/feedback information. The main difficulty encountered in designing such algorithms is to ensure that the spacing errors (deviation from the desired intervehicular spacing) do not amplify from vehicle to vehicle along the platoon. This problem is generalized to investigating the string stability of nonlinear interconnected systems. Intuitively, string stability guarantees the uniform boundedness of all the states of the interconnected system, if the initial states are all uniformly hounded.

Three spacing policies - constant, separation (spacing). constant headway time and constant safety factor - can be implemented for vehicle follower systems. In constant separation policy. the intervehicular spacing is independent of the velocity of the string of vehicles. In constant headway time policy. intervehicular spacing increases linearly with an increase in the velocity of the controlled vehicle, the constant of proportionality being the headway time. In constant safety policy, the desired intervehicular spacing is a safety factor (greater than 1) times the stopping distance of the following vehicle at that speed. In other words, the desired intervehicular spacing varies quadratically with vehicle speed.

For the constant, spacing policy. Levine and Athans [22] used optimal control theory to propose control laws for regulating high speed vehicle strings. The control force on each vehicle depended on the spacing errors of the entire string. which required the position and velocity data of all the vehicles in the string. This posed a burdensome data handling problem. especially when the string is long.

In order to overcome such problems, Wilkie [55] proposed a moving cell scheme. in which a fictitious moving cell on the road acts as a reference to each vehicle. The control effort is a function of the motion of the vehicle relative to the moving cell. The disadvantage of this scheme is that information of the neighboring vehicles and the string is ignored. Vehicles in this scheme communicate with the wayside computers instead.

Decentralized controllers offer a compromise between the above two extremes. The information available dictates the structure of the decentralized controller and the spacing policy that is to he adopted. Since vehicles interact through their dynamics and the feedback control laws (which incorporate the information structure and the desired spacing policy). it is necessary to evaluate the performance of such decentralized controllers quantitatively.

One of the earliest schemes in decentralized cont'rol laws for constant spacing policy was proposed by Levine. Athans and Levis [22], Caudill and Garrard [4], Peppard [23] and Fenton [3]. Although the dynamics of ground vehicle is highly nonlinear [2], [25], linear analysis is usually performed to determine qualitative/quantitative effects of information structure and spacing policy on the string stability.

Caudill and Garrard. and Peppard have used inertial vehicle models to analyze the string stability. Caudill and Garrard fed back the spacing and velocity error measurements to their controller. Peppard added integral of the spacing error information to the above controller. They obtained the transfer functions relating the spacing error of every controlled vehicle relative to that of its predecessor. Since reference (lead) vehicle information was lacking. they proved that the magnitude frequency response of the transfer function has a peak greater than unity and thus. concluded that) the constant spacing policy (without reference vehicle information) is not string stable.

Shladover [38] introduced lead vehicle information in the control law and demonstrated string stability. Sheikholeslam and Desoer [35] used a nominal third order nonlinear vehicle model and used feedback linearization to obtain a triple integrator model for the vehicle. He then fed back the acceleration and the velocity of the lead vehicle to obtain string stability. Hedrick et al., [13] developed a nonlinear vehicle model and used a sliding mode control algorithm that incorporates the information of the lead vehicle and experimentally implemented the algorithm.

With respect to the other two spacing policies, Caudill and Garrard have shown that the control algorithms . which use spacing and velocity error information: are string stable. Chiu [6] has shown. by simulation: that constant headway time policy requires a very high bandwidth actuators for a smaller headway time. Chien and Ioannou [17] have proposed an autonomous time headway control strategy which guarantees attenuation of spacing errors.

Prior to Chu's work [19], interaction of vehicles and propagation of disturbances along the string was investigated in a cursory sense. Analysis was mainly restricted to special information structures and only disturbances from the ends of the string were studied.

Chu considered infinite vehicles in the string (real and fictitious) that are indesed by k. $-\infty < k < \infty$. Every vehicle has a predetermined reference (see figure 1.1) and the position deviation of the k-th vehicle from its predetermined reference is denoted by $x_k(t)$. The dynamics of k-th vehicle is assumed to be

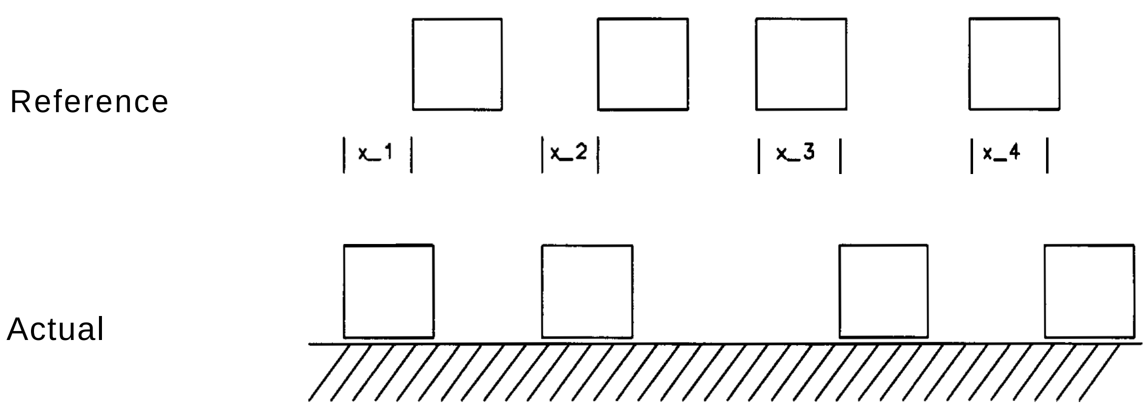


Figure 1.1: Vehicles and their reference positions

$$\begin{bmatrix} \dot{x}_k \\ \ddot{x}_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k \qquad (1.1)$$

where $\alpha \geq 0$ represents the linearized drag force/unit mass and $u(t)$ is the control force/unit mass.

He considers the following set of plausible inputs for the controller of the k-th vehicle:

1. Position error of j-th vehicle. $x_j(t)$;
2. Velocity error of j-th vehicle: $\dot{x}_j(t)$;
3. Relative Position error between the j-th vehicle and the k-th vehicle. $x_j(t) - x_k(t)$;
4. Relative Velocity error between the j-th vehicle and the k-th vehicle. $\dot{x}_j(t) - \dot{x}_k(t)$;

The total information data for the k-th vehicle is denoted by a vector $y_k(t)$. The components of $y_k(t)$ are assumed to he any of (1) to **(4)** for various indices. Mathematically, $y_k(t)$ can be expressed linearly as:

$$y_k(t) = \sum_{j=-\infty}^{\infty} h_{k-j} \begin{bmatrix} x_j(t) \\ \dot{x}_j(t) \end{bmatrix} \qquad (1.2)$$

where matrices h_i (constants) equal zero for sufficiently large $|i|$ or approach zero exponentially as $i \rightarrow \pm\infty$. This condition implies that the interaction between any two vehicles diminish as more and more vehicles come between them (if the controller is designed properly). Identically structured linear feedback control of the form

$$u_k(t) = F y_k(t) \qquad (1.3)$$

is applied to all the vehicles.

By the use of bilateral z-transformation. the dynamics of every vehicle in the string is aggregated into a lumped form. Using the z-transformation technique. the dependence on the vehicle index is eliminated,

$$\dot{X}(z,t) = AX(z,t) + BU(z,t) \quad (1.4)$$

$$Y(z,t) = H(z)X(z,t) \quad (1.5)$$

$$U(z,t) = FY(z,t) \quad (1.6)$$

where , for any $p_k(t)$, $P(z,t) := \sum_{j=-\infty}^{\infty} p_k(t)z^{-k}$ and $X(z,0) = \sum_{k=-\infty}^{\infty} \begin{bmatrix} x_k(0) \\ \dot{x}_k(0) \end{bmatrix} z^{-k}$ is the initial condition for the above system.

The closed loop dynamics of the string is given by:

$$X(z,t) = D(z,F)X(z,t) \quad (1.7)$$

where $D(z,F) = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} FH(z)$. After integrating the above differential equation,

$$X(z,t) = exp(D(z,F)t)X(z,0) \quad (1.8)$$

The resultant time-space relation of each vehicle can be obtained through inverse z-transformation:

$$\Big| \quad \Big| \quad \sum \oint exp(D(z,F)t)z^{k-j}dz \Big| \quad \Big| \quad (1.9)$$

Chu defined string stability as follows:

Definition: A string of vehicles is stable with a feedback control structure if $x_k(0), \dot{x}_k(0)$ are bounded for all **k,** implies that, for all k. $x_k(t)$ is bounded and $x_k(t) \rightarrow 0$ as $t \rightarrow \infty$.

The following theorem is then used to characterize string stability:

Theorem **(Chu)** : A string of vehicles described by 1.4 is stable if and only if all the eigenvalues of $D(z, F)$ have negative real parts for all $|z| = 1$.

In order to optimally choose F, the following performance index is then chosen.

$$J = \frac{1}{2}\int_0^{\infty} \sum_{k=-\infty}^{\infty} q(x_{k-1} - x_k)^2 + p(\dot{x}_{k-1} - \dot{x}_k)^2 + u_k^2 dt \quad (1.10)$$

where $q, p \geq 0$. The first two terms penalize relative spacing and velocity error respectively between the k-th and **k-1** st vehicles. The last term penalizes the control effort to make the ride comfort as good as possible. Using bilateral z-transform. this performance index can he converted to

$$X(z^{-1}, t)Q(z)X(z, t) + U(z^{-1}, t)U(z, t)dt \tag{1.11}$$

with $Q(z) = Q(z^{-1})$ and is given by $Q(z) = 2 - z - z^{-1}$. Using equations 1.4. the performance index is minimized via Lyapunov like equations in $D(z. \mathbf{F})$. For more details, see [19].

This dissertation consists of six chapters. In chapter 2. we introduce the notion of "string stability" for a platoon. Our definition of string stabilit'y considers relative spacing errors $(x_{k-1}(t) - x_k(t))$ and relative velocity errors $(\dot{x}_{k-1}(t) - \dot{x}_k(t))$ instead of $x_k(t)$ and $\dot{x}_k(t)$. We design various platooning strategies and analyze them for string stability. Our analysis of string stability differs from Chu's analysis in two respects:

1. We include the acceleration information (feedforward information) in our cont'rol laws.

2. First' the dependence on time is eliminated by the use of input-output norm relationships. Then, string stability reduces to examining a difference inequality. In Chu's approach. the dependence on vehicle index is eliminated by the use of a bilateral z-transform. Stability of the string is then examined by studying the stability of a differential equation.

It is found that: for the constant intervehicular spacing strategy. only "weak" string stability can be guaranteed if all the vehicles in the platoon do not have a reference vehicle information. Intuitively. weak string stability requires that the intial spacing and velocity errors be absolutely summable, if the spacing and velocity errors of all the vehicles in the platoon have to be bounded at all times. Furthermore: weak string stability is not robust to singular perturbations such as parasitic actuator dynamics and signal processing lags. For constant headway time strategy, weak string stability can only be guaranteed when all the vehicles in the platoon do not avail of a reference vehicle information. In this case, string stability robustness decreases with decreasing headway time.

In chapter 3. we investigate the effects of uncertainty on a constant inter-vehicular spacing algorithm. We present a gradient parameter adaptation law to guarantee that the estimated parameters and the spacing errors of all the vehicles are uniformly bounded in time: if the initial parameter estimation errors and spacing errors of all vehicles are uniformly bounded.

In chapter 4, the platoon problem is generalized to investigating the string stability of a countably infinite interconnection of identical nonlinear exponentially stable systems. 11-e derive sufficient conditions that guarantee string stability of the interconnected system. Under the same sufficient conditions, we prove that string stability is robust to small singular perturbations. The above results help in designing/decoupling the interconnections for countably infinite feedback linearizable nonlinear systems, while ensuring string stability.

Contributions of the thesis and relations to previous work:

The contributions of this dissertation to the area of longitudinal control of vehicle platoons are as follows:

1. We precisely define the "string stability" requirement for satisfactory functioning of the platoon in time domain. Without explicitly defining string stability. Caudill and Garrard. Peppard, Shladover. Sheikholeslam prescribe frequency domain conditions for string stability (i.e. the infinity norm of the transfer function that relates the spacing error of the i-th vehicle to that of the i-1st vehicle he less than unity). However. this is a necessary but not a sufficient condition for string stability.

2. We show that, if the initial spacing errors are zero. the maximum spacing error decreases geometrically in vehicle index in the platoon (vehicle ID) for anp lead vehicle maneuver with the availability of lead vehicle relative position information to every controlled vehicle. The question arises as to how every controlled vehicle obtains lead vehicle relative position information. Spacing error of every

controlled vehicle relative to the lead vehicle is the spacing error of its preceding vehicle relative to the lead vehicle plus the spacing error of the controlled vehicle relative to its predecessor. Since relative position error information of the controlled vehicle relative to its predecessors is obtained by onboard sensors such as radar and sonar. it is sufficient that every controlled vehicle broadcast its error information relative to the lead vehicle to its successor. Broadcasting such information as relative position information, lead vehicle acceleration and velocity is possible with the current state of radio communications technology.

We also examine the effect of availability of "r" vehicle look ahead information. knowledge of vehicle ID on the string stability of the platoon for both constant intervehicular spacing and constant headway time strategies. Without reference vehicle information. string stability of a platoon for constant spacing strategies is not robust to singular perturbations like parasitic actuator dynamics and robustness in string stability to singular perturbations decreases with decreasing headway time for constant headway strategies as documented in Chiu, Stupp and Brown [6]. Knowledge of controlled vehicle's ID helps build the error information of all the preceding vehicles in the platoon and realizability of such a control scheme depends on the availability of preceding vehicle acceleration information. This is the basis for a semi-autonomous cruise control strategy. Previous work in this area have not addressed these issues.

3. We develop a direct, adaptive longitudinal control law that increases the robustness of the platoon in the presence of uncertainty in the parameters such as aerodynamic drag. mass of the vehicle and simulations demonstrate its effectiveness. Sheikholeslam, [36], proposed an indirect adaptive controller. The advantage of a direct scheme is its ease in implementation.

In addition, we propose hybrid platooning strategies (mixed constant spacing and constant headway time strategies). We also propose a two-time scale information update to facilitate implementation - on board sensors update their information on a faster time scale and lead vehicle information is updated on a slower time scale.

The contributions of this dissertation to the area of cont'rol of interconnected dynamical systems are as follows:

1. Research in this area is heavily concentrated on the stability of finite interconnection of systems. See references [26], [20], [5], [36], [51], [41], [@]. In this dissertation. we introduce the notion of string stabilit'y for a countably infinite interconnection of identical systems. A version of string stability called γ-stability was introduced by Chang [5], in reference to infinite circuit networks. In the context of vehicle following applications, Chu [19] defined string stability as seen earlier. String stability is a generalization of Lyapunov stability for the above class of interconnected systems. We derive sufficient conditions to determine if a closed loop interconnected system is string stable. This provides some guideline for designing controllers for interconnected systems that guarantee string stability.

2. We investigate the robustness of string stable interconnected systems. Like exponentially stable nonlinear systems, the class of interconnected systems considered are shown to be robust to structural and singular perturbations.

3. We present a gradient-based parameter adaptation law for a class of interconnected systems.

Chapter 2

Vehicle Model

In this chapter: a vehicle model based on Cho and Hedrick, [7], is developed and validated. Control algorithms that explicitly address the issue of string stability for constant spacing and headway control strategies will be developed in the later chapters, based on this model. A three state variable lumped parameter longitudinal model of a vehicle based on the following assumptions is developed for simulating the response of the vehicles in the platoon:

1. Ideal gas law holds in the intake manifold.
2. Temperature of the intake manifold is a constant'.
3. The drive axle is rigid.
4. The torque converter is locked.
5. The brakes obey first order dynamics.

X simple model for the intake manifold dynamics is given by:

$$\dot{m}_a = \dot{m}_{ai} - \dot{m}_{ao} \tag{2.1}$$

$$P_m V = m_a R_g T_m \tag{2.2}$$

where m_a is the mass of air in the intake manifold and $\dot{m}_{ai}$ and $\dot{m}_{ao}$ are the mass flow rates through the throttle valve and into the cylinders, respectively. A schematic of the engine is shown in Figure 2.1 . P_m, V and T_m are the intake manifold pressure, volume and temperature respectively. R_g is the gas constant for air. Assumptions 1 and 2 enable us to obtain an algebraic relationship between the manifold pressure. P_m (which is sensed) and the mass of air in the manifold: m_a. The empirical relationship used for $\dot{m}_{ai}$, ['i],is :

$$\dot{m}_{ai} = MAX \cdot TC \cdot PRI(\frac{P_m}{P_a}) \tag{2.3}$$

where MAX is a constant dependent on the size of the throttle body. $TC(\alpha)$ is the throttle characteristic which is the projected area the flow sees as a function of the throttle angle, α. PRI is the pressure influence function which describes the choked flow relationship which often occurs through the throttle valve. P_a is the atmospheric pressure. $\dot{m}_{ao}$ is the mass air flow rate into the combustion chamber and is a nonlinear function of the intake manifold pressure P_m and the engine speed w_e. For vehicle position control applications. the intake manifold dynamics is much faster than the engine speed dynamics, so that we can assume

$$\dot{m}_{ao}(w_e, m_a) = \dot{m}_{ai} = MAX \cdot TC \cdot PRI(\frac{P_m}{P_a}) \tag{2.4}$$

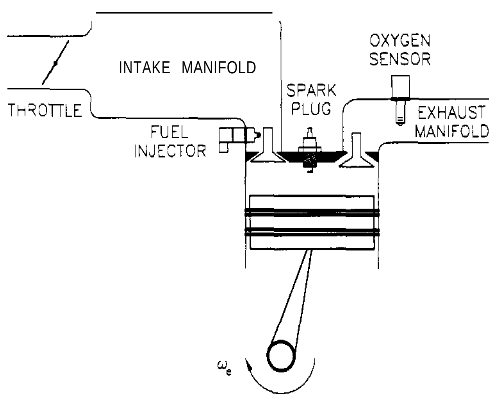


Figure 2.1: Schematic of an Engine

A free body diagram of the vehicle is shown in Figure 2.2 . Assumptions 3 and 4 ensure that the wheel speed is proportional to the engine speed: w_e. The rotational dynamics of the engine is described by:

$$\dot{w}_e = \frac{T_{net} - R(hF_{tr} + T_{br} + hF_f)}{I_e} \tag{2.5}$$

where T_{net} is the net combustion torque (indicated torque - friction torque). It is also a nonlinear function of w_e and P_m. $\dot{m}_{ao}$ and T_{net} are provided as tabular functions by the engine manufacturers. R is the gear ratio, h is the tire radius and I_e is the effective rotational inertia of the engine when the inertia of the wheel is also referred to the engine side. T_{br} is the brake torque at the wheels and F_{tr} is the tractive force. The tractive force F_{tr} is given by [54]:

$$F_{tr} = K_r sat(\frac{i}{i_{max}}) \tag{2.6}$$

where K_r is the longitudinal tire stiffness. The saturation function, $sat(x)$ is defined as follows:

$$sat(x) = \begin{cases} 0 & x \in [-1, 1] \\ -1 & x \leq -1 \\ 1 & x \geq 1 \end{cases}$$

The slip, i, between the tire and the ground is defined as:

$$i = 1 - \frac{v}{Rhw_e} \tag{2.7}$$

where v is the longitudinal velocity of the vehicle. The dynamics of the brake is given by:

$$\dot{T}_{br} = \frac{T_{bc} - T_{br}}{\tau_b} \tag{2.8}$$

where T_{bc} is the commanded brake torque and τ_b is the time constant for the brake actuator. A detailed dynamic model for brake can be found in [8]. Finally. the equation for longitudinal vehicle velocity is given by:

$$\dot{v} = \frac{F_{tr} - c_a v^2}{M} \tag{2.9}$$

where c_a is the drag coefficient, F_f models the energy loss (rolling resistance) and M is the effective mass of the vehicle.

The model developed above is used for simulation. We apply the same throttle input to the actual vehicle and the simulation mode. The maneuvers that have been chosen to validate the simulation model do not require braking. Hence, the simulation model is only partially valid. The simulated and experimental responses for the constant speed and variable speed trajectory tests are shown in Figure 2.3 . The two responses agree quite well.

2.1 Simplified model for control

The design of the controller is made simpler by a "NO SLIP" assumption. i.e $v = Rhw_e$. With this assumption. equations 2.5 and 2.9 reduce to

$$\dot{w}_e = \frac{T_{net} - c_a R^3 h^3 w_e^2 - R(hF_f + T_{br})}{J_e} \tag{2.10}$$

J_e is the effective rotational inertia of the engine when the vehicle mass and the wheel inertias are referred to the engine side. Equations 2.4, 2.8. 2.10 describe the model for the controller

Algorithms for spacing and headway control strategies have the same feedback structure except for the information that is fed back. The feedback structure is developed using the 1/0 linearization technique. [16]. 1/0 linearization is best suited for this problem considering the nonlinearities in the engine model. It is assumed that every controlled vehicle has access to its state variables such as velocity. brake torque. acceleration. and that the parameters such as aerodynamic drag, rolling resistance friction: effective engine inertia. gear ratios and tire radii are known exactly. The desired output, "y", is the longitudinal position of the j-th following vehicle, x_j.

$$y = x_j \tag{2.11}$$

$$\dot{y} = \dot{x}_j = v_j = Rhw_e \tag{2.12}$$

$$\ddot{y} = \ddot{x}_j = (\frac{Rh}{J_e}[T_{net} - c_a R^3 h^3 w_e^2 - R(hF_f + T_b r)])_j \tag{2.13}$$

Choose

$$(T_{net})_j = [c_a R^3 h^3 w_e^2 + R(hF_f + T_{br})] + \frac{J_e}{Rh} u_{jsl} \tag{2.14}$$

where $(T_{net})_j$ is the desired net engine torque and u_{jsl} is chosen to make the closed loop system satisfy certain performance objectives. Knowing the desired net engine torque and the actual speed. the desired manifold pressure. P_{md} can be found from the table-look up map. Using equation 2.4. the desired throttle angle. α_d can be calculated as follows:

$$\alpha_d = TC^{-1}[\frac{\dot{m}_{ao}(w_e, P_{md})}{MAX.PRI(\frac{P_m}{P_a})}] \tag{2.15}$$

We can simplify computations by combining $T_{net}(w_e, P_m)$ and equation 2.4 to yield $T_{net}(w_e, \alpha)$. If $\alpha \leq \alpha_o$, the minimum allowable throttle angle, then braking should occur. in which case. the desired brake torque T_{bd} is given by:

$$T_{bd} = \frac{J_e}{R^2 h} u_{jsl} - \frac{c_a R^3 h^3 w_e^2 + R h F_f}{R} \tag{2.16}$$

In order to close the loop for the brake dynamics, we define another synthetic output y_b so that

$$y_b = T_{br} - T_{bd} \tag{2.17}$$

$$\dot{y}_b = \dot{T}_{br} - \dot{T}_{bd} = \frac{T_{bc} - T_{br}}{\tau_b} - \dot{T}_{bd} \tag{2.18}$$

Choose T_{bc} such that

$$T_{bc} = T_{br} + \tau_b(\dot{T}_{bd} - \lambda_b(T_{br} - T_{bd})) \tag{2.19}$$

To simplify implementation, $\dot{T}_{bd}$ is obtained by numerically differentiating the desired brake torque signal. λ_b is chosen sufficiently high so that the use of throttle and brake control approximates the vehicle (plant) model as

$$\ddot{x}_j = u_{jsl} \tag{2.20}$$

The choice of u_{jsl} reflects the platooning strategy that is considered. Every platooning strategy is analyzed for robustness to parasitic actuator lags (like the brake dynamics).

In contrast to the control model developed here: Sheikholeslam [35] uses a nominal third order nonlinear model and uses exact linearization to obtain a triple integrator model for a vehicle.

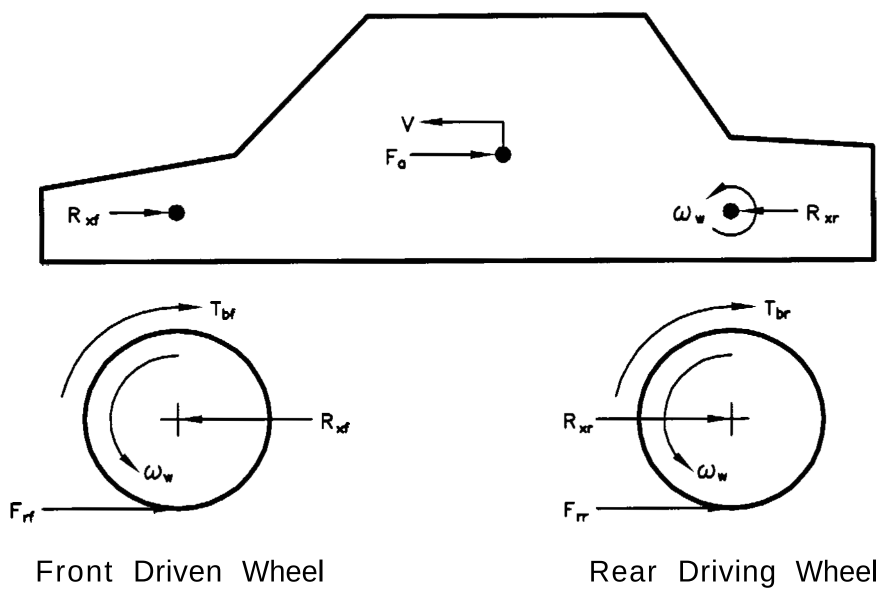


Figure 2.2: Forces acting on a moving vehicle

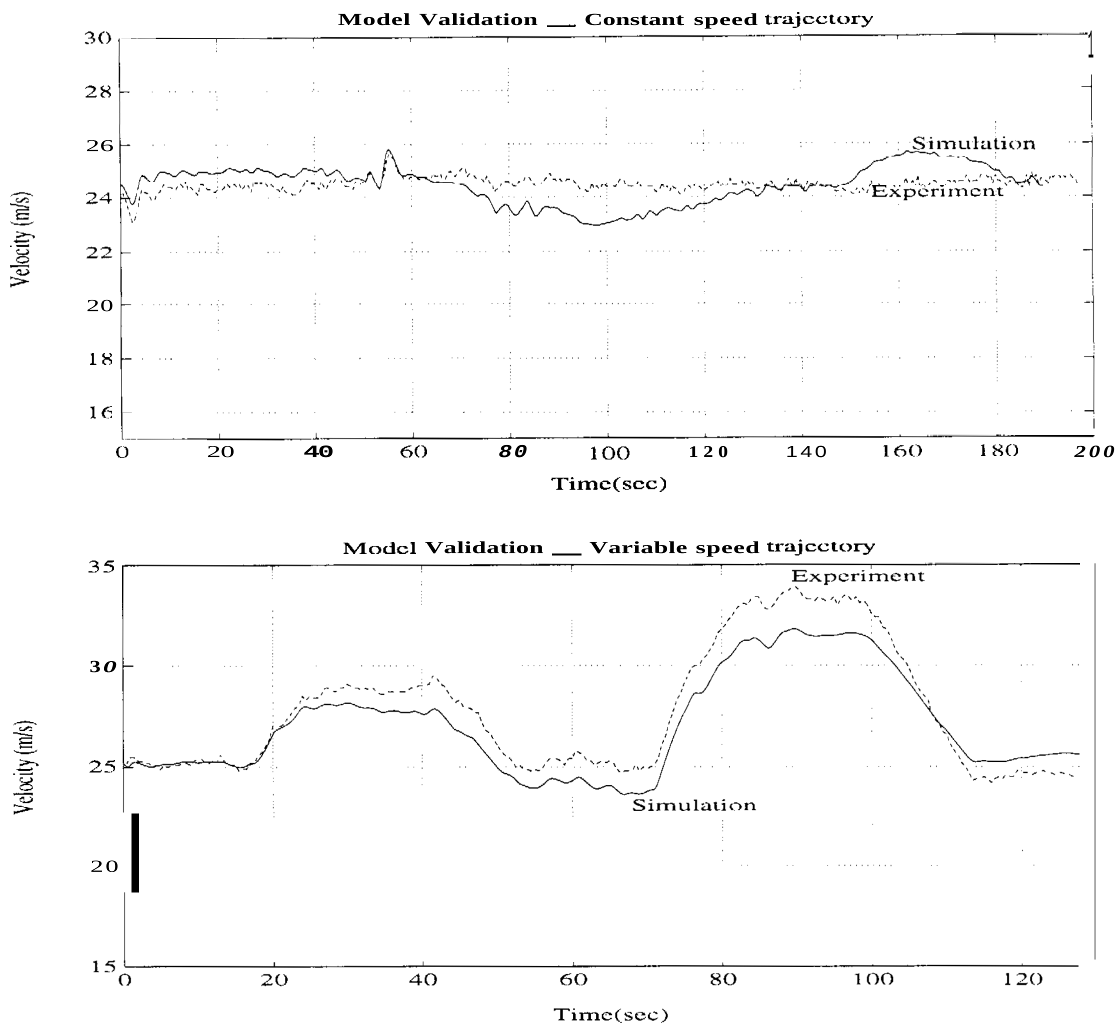


Figure 2.3: Simulation Model Validation

Chapter 3

Platooning Strategies

3.1 Introduction

Platoon control strategies directly affect traffic flow capacities. Analysis of different platooning control strategies serves two purposes: 1) Based on the information available. the most effective platooning strategy can be chosen. 2) In case of sensor failure, it provides a back-up control strategy. The effectiveness of a platoon control strategy can be gauged by the maximum traffic flow capacity. the attenuation of spacing errors. that it can guarantee and the amount of information that is needed to implement the strategy in real-time.

In this chapter. we consider the following platooning strategies

1. Constant Spacing control strategies : In these strategies, the desired intervehicular spacing is independent of the velocity of the controlled vehicle. The tracking requirement is stringent. since every controlled vehicle has to match its position, velocity and acceleration with the vehicle ahead. As a consequence, these strategies require more information to guarantee performance. The achievable traffic capacity is very high in a constant spacing control strategy. We consider the following constant spacing strategies:

   Control with information of reference vehicle information only.

   Autonomous and serni-autonomous control.

Semi-Autonomous control with vehicle indes information.

Control with information of preceding and reference vehicles

Control with information of "r" immediately preceding vehicles

Mini-platoon control.

Mini-platoon control with lead vehicle information.

2. Variable Spacing control strategies : The desired intervehicular spacing varies with the velocity of the controllecl vehicle in these platooning strategies. The tracking requirement, is not as stringent as the previous case. Some of the variable spacing control strategies can. therefore. be implemented with onboard sensors. However. the achievable traffic capacity is limited. We consider the following variable spacing strategies in this dissertation:

   Autonomous Intelligent Cruise Control (XICC).

   Control with information of "r" immediately preceding vehicles.

3. Hybrid strategies : Constant spacing and variable spacing strategies can be combined to develop strategies that, are compatible with the given information and to guarantee robustness. These strategies. however. are not analyzed in this dissertation.

We also determine their performance limits in terms of string stability. The method of analysis involves the use of linear input-output norm relationships to convert the problem of the stability of a string of moving vehicles into the stability of linear difference equation with constant coefficients. Sufficient conditions to ensure string stability are derived. The limitations/effectiveness of these schemes is clemonstrated by simulation results.

3.2 String Stability

The following figure illustrates the definitions of spacing errors in the platoon:

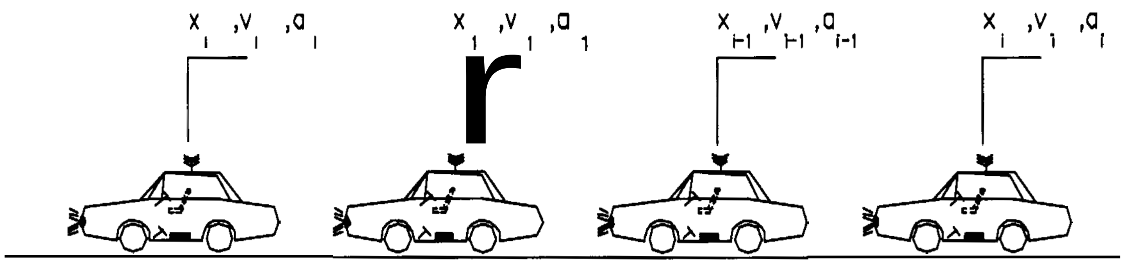


Figure 3.1: Spacing errors in a platoon

The spacing error in the i-th vehicle, ϵ_i is given by $\epsilon_i = x_i - x_{i-1} + L_i$, where L_i is the desired intervehicular spacing. The following are the platooning specifications:

1. Individual vehicle stability: The ability of any vehicle in the platoon to track any bounded acceleration and velocity profile of its predecessor with a bounded spacing and velocity error.

2. String Stability: It is required to ensure that the spacing errors do not amplify upstream from vehicle to vehicle in a platoon.

3. Zero Steady state spacing error: Irrespective of the lead vehicle maneuvers. it is required that, every controlled vehicle maintain the desired spacing in the steady state. This is desirable to maintain a reliable traffic capacity and for safety.

Before defining string stability precisely, we need the following notations: We use $||f_i(\cdot)||_\infty$ or simply $||f_i||_\infty$ to mean $sup_{t\geq 0}|f_i(t)|$, and $||f_i(0)||_\infty$ or simply $||f(0)||_\infty$ to mean $sup_i|f_i(0)|$. Similarly $||f_i(\cdot)||_1$ or $||f_i||_1$ means $\int_0^\infty |f_i(\tau)|d\tau$, and $||f_i(0)||_1$

denotes $\sum_1^\infty |f_i(0)|$.

Definition 1: A platoon is string stable if. given $\gamma > 0$. $\exists\delta > 0$ such that whenever

$$\max \quad [||\epsilon_i(0)||_\infty, ||\dot{\epsilon}_i(0)||_\infty, ||\sum_1^i \epsilon_j(0)||_\infty, ||\sum_1^i \dot{\epsilon}_j(0)||_\infty] < \delta \Rightarrow \sup_i \quad ||\epsilon_i||_\infty < \gamma$$

Definition 2: A platoon is string stable in the weak sense if, given $\gamma > 0$, 30 > 0 such that whenever

$$\max \quad [||\epsilon_i(0)||_1, ||\dot{\epsilon}_i(0)||_1] < \delta \Rightarrow \sup_i \quad ||\epsilon_i||_\infty < \gamma$$

Definition 3: A platoon has uniformly bounded spacing errors if given 0 > 0.$\exists\gamma > 0$ such that

$$\max \quad [||\epsilon_i(0)||_\infty, ||\dot{\epsilon}_i(0)||_\infty, ||\sum_1^i \epsilon_j(0)||_\infty, ||\sum_i^i \dot{\epsilon}_j(0)||_\infty] < \delta \Rightarrow \sup_i \quad ||\epsilon_i||_\infty < \gamma$$

There is an underlying difference equation which relates the maximum spacing error of the i-th vehicle with the maximum spacing errors of the vehicles preceding it. If this difference equation has all its roots inside the unit circle, then the platoon is string stable. If the difference equation has a simple root on the unit circle. then the platoon is weak sense string stable. It is clear that, every platoon of finite number of stable vehicles is string stable. Although. in practice, no platoon has infinite vehicles in it. it is necessary that platooning specifications be satisfied independent of the size of the platoon to prevent actuator saturation.

A vehicle model for control based on [7], [25] and in the previous chapter is given by:

$$\ddot{x}_i = \frac{u_i - c_i\dot{x}_i^2 - f_i}{M_i} \tag{3.1}$$

where u_i is the effective control torque (net engine/brake torque). c_i is the effective aerodynamic drag coefficient. f_i is the effective tire drag, and M_i is the effective mass of the i-th controlled vehicle. The control effort u_i is chosen to be

$$u_i = c_i\dot{x}_i^2 + f_i + M_i u_{isl} \tag{3.2}$$

so that

$$\ddot{x}_i = u_{isl} \tag{3.3}$$

where the synthetic input u_{isl} is based on the information that is available for feedback and is chosen to satisfy the performance objectives. The spacing error dynamics of vehicles in a platoon depends on the choice of u_{isl}. The following preliminaries are used to analyze the string stability of a platoon.

3.2.1 Preliminaries :

Fact 1 : If $\hat{H}(s)$ is a stable, proper transfer function, with $|\hat{H}(0)| = \alpha$, and if $h(t) = \mathcal{L}^{-1}(\hat{H}(s))$ is its impulse response, then $||h||_1 = \alpha \iff h(t)$ does not change sign.

Proof : $|\hat{H}(0)| = |\int_0^\infty h(t)dt| \leq \int_0^\infty |h(t)|dt$ and from Calculus, $\int_0^\infty |h(t)|dt = |\int_0^\infty h(t)dt| \iff h(t)$ does not change sign.

Fact 2 : If $h(t) > 0$. then all the Input/Output induced norms are equal.

Proof : Let γ_p he the p-th induced norm from input to output. From Linear System Theory, [9],

$|\hat{H}(0)| \leq ||\hat{H}(jw)||_\infty \leq \gamma_p \leq ||h||_1$

If $h(t) > 0$, then $|\hat{H}(0)| = ||h||_1 \Rightarrow \gamma_p = ||h||_1$.

Fact 3 : Define $P_r(z) = z^r - \sum_1^r \alpha_j z^{r-j}$; where $\alpha_j > 0$. $j = 1, 2. ...., r$. Then all the roots of $P_r(z)$ lie inside the unit disc if $\sum_1^r \alpha_j < 1$.

Proof : This is proved by contradiction. Suppose there exists a z_0 such that $P_r(z_0) = 0$ and $|z_0| \geq 1$. Then

$$1 = \sum_1^r \alpha_j z_0^{-j} \leq \sum_1^r \alpha_j |z_0|^{-j} \leq \sum_1^r \alpha_j < 1$$

which is a contradiction.

Fact 4 : If $\sum_1^r \alpha_j = 1$, then all the roots of $P_r(z)$ lie inside the closed unit disc and all the roots that lie on the unit disc are simple.

Proof : This fact is also proved by contradiction. If $P_r(z)=0$ and $|z|>1$. then

$$1=\alpha_j z^{-j}\leq\sum_1^r\alpha_j|z|^{-j}<\sum_1^r\alpha_j=1$$

This is a contradiction. This establises that all the roots of $P_r(z)$ lie inside the closed unit disk.

Let $|z_0|=1$ such that $P_r(z_0)=0$.

Claim : $P_r^{'}(z_0)\neq 0$.

Proof of the Claim : Otherwise.

$$P_r^{'}(z_0)=rz_0^{r-1}-\alpha_1(r-1)z_0^{r-2}-\ldots-\alpha_{r-1}=0$$

$$\Rightarrow 1=\sum_1^{r-1}\alpha_j\frac{r-j}{r}z_0^{-j}\leq\sum_1^{r-1}\alpha_j|z_0|^{-j}\frac{r-j}{r}<\sum_1^r\alpha_j\leq 1$$

and this is also a contradiction. Since $P_r^{'}(z_0)\neq 0$, the roots that' lie on the unit circle are simple.

Fact **5** : If $h_j(t)$ is the impulse response of $H_j(s)$ and $\alpha_j := ||h_j||_1$, $\sum_1^r\alpha_j = \sum_1^r|\hat{H}_j(0)| \iff h_j(t)$does not change sign.

Proof : $\alpha_j\geq|\hat{H}_j(0)|$ and from Fact 1, $\alpha_j=|\hat{H}_j(0)| \iff h_j(t)$ does not change sign.

Fact **6** : If $\sum_1^r H_j(s)\equiv 1$. then

- $\sum_1^r\alpha_j\geq 1$
- $\sum_1^r\alpha_j=1 \iff \hat{H}_j(s)\equiv\alpha_j$, a constant.

Proof : (1) follows from a fact from Linear System Theory. [9], that $||h||_1\geq||\hat{H}(jw)||_\infty$.

From Fact 5. each $h_j(t)$ should not change sign. But $\sum_1^r h_j(t)=\delta(t)\Rightarrow$ Every $h_j(t)$ is a scaled impulse. Therefore, $H_j(s)=\alpha_j$, a constant.

Usually the spacing error dynamics of any vehicle is a function of the spacing error dynamics of "r" vehicles ahead of it. where "r" is a constant. In the platooning strategies discussed in this dissertation, the associated spacing error dynamics is a special case of the general case presented here. The spacing error dynamics of vehicles in a platoon usually satisfy the following set of differential equations:

$$\ddot{\epsilon}_1 + q_1\dot{\epsilon}_1 + q_2\epsilon_1 = 0 \tag{3.4}$$

$$\ddot{\epsilon}_i + q_1\dot{\epsilon}_i + q_2\epsilon_i = \sum_{j=1}^{r} k_{aj}\ddot{\epsilon}_{i-j} + k_{vj}\dot{\epsilon}_{i-j} + k_{pj}\epsilon_{i-j} \tag{3.5}$$

with $\epsilon_{i-j} \equiv 0 \;\; \forall \;\; i \leq j$.

Define

$$\hat{H}_j(s) := \frac{k_{aj}s^2 + k_{vj}s + k_{pj}}{s^2 + q_1 s + q_2}; \qquad \alpha_j := ||\mathcal{L}^{-1}(\hat{H}_j(s))||_1 = ||h_j||_1$$

$$\Delta(s) = s^2 + q_1 s + q_2; \qquad w_i(t) = \sum_{1}^{i} \epsilon_j(t) = (x_i(t) - x_l(t) + \sum_{j=1}^{i} L_j);$$

where $\mathcal{L}^{-1}(\hat{F}(s))$ denotes the inverse Laplace Transform of $\hat{F}(s)$ and is given by $f(t)$. $h_j(t)$ is the impulse response of $H_j(s)$ and $||h_j(\cdot)||_1$ is given by $\int_0^\infty |h_j(\tau)|d\tau$.

Define $P_r(z) = z^r - \sum_1^r \alpha_j z^{r-j}$. Platoon performance can be described by the spectral radius associated with this characteristic polynomial. $P_r(z)$. i.e

$$\rho = \max\{|z| : P_r(z) = 0\} \tag{3.6}$$

Usually. if all the vehicles in the platoon do not have the same (reference) vehicle information. the transfer functions, $\hat{H}_j(s)$. are such that $\sum_{j=1}^{r} H_j(s) \equiv 1$ for a constant spacing strategy and $\sum H_j(0) = 1$ for a variable spacing strategy. These constraints pose performance limitations. since $\sum \alpha_j \geq \sum \hat{H}_j(0) = 1$ and hence. $\rho \geq 1$. If $h_j(t)$ does not change sign. then $\alpha_j = H_j(0)$ and $\rho = 1$. The design problem for such strategies reduces to choosing the control gains such that, the corresponding impulse response of the transfer functions $H_j(s), j = 1, \ldots, r$, does not change sign. If, $\rho > 1$ for a strategy that has to be used, the following safety precautions can be observed:

1. Choose the intervehicular spacing, $L_i = \rho^{i-1} L_1$.

2. Limit the number of vehicles in a platoon.

The above precautions limit the traffic capacity. Input (throttle) saturation can occur. since no effort has been macle to guarantee the attenuation of spacing errors.

In the proposition that follows, we give sufficient conditions on the the gains. α_j, of the transfer functions. $H_j(s)$.

Proposition : Small Transfer Function Gain Theorem for String Stability of the Platoon

1. If $\sum_1^r \alpha_j < 1$, the platoon is string stable. and $\sup_i \ ||w_i||_\infty$ is bounded.
2. If $\sum_1^r \alpha_j = 1$, the platoon is string stable in the weak sense.
3. If $\sum_1^r \alpha_j < 1$, then given $\gamma > 0, \exists\delta > 0$, such that $\max\ [||\dot{\epsilon}_i(0)||_\infty, ||\ddot{\epsilon}_i(0)||_\infty, ||\dot{w}_i(0)||_\infty, ||\ddot{w}_i(0)||_\infty] < \delta \Rightarrow \max\{\sup_i\ ||\dot{\epsilon}_i||_\infty, \sup_i\ ||\dot{w}_i||_\infty\} < \gamma$, i.e the relative velocity errors are bounded in vehicle index and time.
4. $\epsilon_i(t) \longrightarrow 0$ asymptotically for all vehicles in the platoon.

Proof:

1. Let $\gamma > 0$ be given. From equation 3.5, it follows that

$$\hat{\epsilon}_i(s) = \sum_{j=1}^{r} \hat{H}_j(s)\hat{\epsilon}_{i-j} + \frac{(s+q_1)\epsilon_i(0) + \dot{\epsilon}_i(0) - \sum_{j=1}^r (k_{aj}s + k_{vj})\epsilon_{i-j}(0) + k_{aj}\dot{\epsilon}_{i-j}(0)}{\Delta(s)}$$

There exists $c_j, d_j \geq 0$. $j = 0, 1.2.\ldots.r$ such that

$$||\epsilon_i||_\infty \leq \sum_{j=1}^{r} \alpha_j ||\epsilon_{i-j}||_\infty + \sum_{j=0}^{r} c_j|\epsilon_{i-j}(0)| + d_j|\dot{\epsilon}_{i-j}(0)|$$

From Fact 3, $P_r(z)$, has all its roots inside the unit circle. Therefore. $\exists M > 0, 0 < \lambda < 1$, such that

$$||\epsilon_i||_\infty \leq M\lambda^{i-1}||\epsilon_1||_\infty + M\sum_{j=1}^{i-1}\lambda^{i-j}(\sum_{k=0}^{r} c_k||\epsilon_i(0)||_\infty + d_k||\dot{\epsilon}_i(0)||_\infty)$$

which implies that

$$||\epsilon_i||_\infty \leq M||\epsilon_1||_\infty + \frac{M}{1-\lambda}\sum_{k=0}^{r} c_k||\epsilon_i(0)||_\infty + d_k||\dot{\epsilon}_i(0)||_\infty$$

From equation 3.4, $||\epsilon_1||_\infty \leq c_1^{'}|\epsilon_1(0)| + c_2^{'}|\dot{\epsilon}_1(0)|$ for some $c_1^{'}, c_2^{'} > 0$. Let

$$\delta = \min \quad [\frac{\gamma}{2c_1^{'}}, \frac{\gamma}{2c_2^{'}}, \frac{\gamma(1-\lambda)}{2Mc_1^{'}(1-\lambda) + 2M\sum_0^r c_j}, \frac{\gamma(1-\lambda)}{2Mc_2^{'}(1-\lambda) + 2M\sum_0^r d_j}]$$

Clearly, $\sup_i \quad ||\epsilon_i||_\infty < \gamma$.

w_i satisfies equations **3.4** and **3.5.** Hence, by the same argument as above.

$$\max [||\sum_1^i \dot{\epsilon}_j(0)||_\infty, ||\sum_1^i \epsilon_j(0)||_\infty] < \delta \Longrightarrow ||\sum_1^i \epsilon_j||_\infty < \gamma$$

2. If $\sum \alpha_j$ = **1.** from Fact 4, $P_r(z) = 0$ has simple roots on the unit circle and all other roots are within the unit circle. Therefore.

$$||\epsilon_i||_\infty \leq ||\epsilon_1||_\infty + \sum_{j=0}^r c_j||\epsilon_i(0)||_1 + d_j||\dot{\epsilon}_i(0)||_1 \leq (c_1^{'} + c_2^{'} + \sum_0^r c_j + d_j) \max [||\epsilon_i(0)||_1, ||\dot{\epsilon}_i(0)||_1]$$

Hence: the platoon is string stable in the weak sense. Similarly, it can he shown that

$$||w_i||_\infty \leq (c_1^{'} + c_2^{'} + \sum_{j=0}^r c_j + d_j) \max [||w_i(0)||_1, ||\dot{w}_i(0)||_1]$$

3. Let $p_i = \dot{\epsilon}_i$ and p_i satisfies equations **3.4** and **3.5.** Hence! by part (a). the result follows.

4. From equation 3.5 and the final value theorem,

$$\epsilon_{iss} = \sum_{j=1}^r \hat{H}_j(0)\epsilon_{(i-j)ss}$$

where ϵ_{iss} is the steady state spacing error in the i-th vehicle. Since. $\sum_{j=1}^r |H_j(0)| \leq \sum_{j=1}^r \alpha_j \leq$ **1.** there exists some $M_1 > 0$ such that $|\epsilon_{iss}| \leq M_1|\epsilon_{1ss}|$. Since $\epsilon_{1ss} = 0$. it follows that E,,, $= 0$.

Sheikholeslam [35] uses a triple integrator control model for a vehicle and considers two cases:

- Autonomous case : In this case. the only external information fed back in the control law is from the on-board radar (spacing and velocity error).
- Control with information of lead vehicle : Lead vehicle velocity and accleration infornmtion is fed back in addition to the on-board radar information

In both the cases. the spacing error dynamics of the i-th vehicle is dependent, on the spacing error dynamics of its preceding vehicle. i.e.,

$$\hat{\epsilon}_i(s) = \hat{H}(s)\hat{\epsilon}_{i-1}(s)$$

$$\hat{\epsilon}_1(s) = G(s)\hat{a}_l(s)$$

In contrast to the conditions for string stability given here. Sheikholeslam requires the following conditions for string stability :

- $\hat{H}(s), \hat{G}(s)$ should be proper stable transfer functions.
- $G(0) = 0$ for zero steady state spacing errors.
- $||H(jw)||_\infty \leq 1$ for attenuating spacing errors along the platoon.
- It is desirable that the impulse response, $h(t)$, of $\hat{H}(s)$ be positive so that, the spacing errors do not exhibit oscillatory behavior.

Since me are more concerned with the amplification of maximum spacing error from vehicle to vehicle along the platoon. it is logical to use the $\mathbf{x} - \infty$ induced norm, $||h||_1$, of the spacing error propagation transfer function. $\hat{H}(s)$ instead of $2 - 2$ induced norm: $||\hat{H}(jw)||_\infty$. In fact. by use of Fact 1, the last two conditions proposed by Sheikholeslam try to make $||h||_1$ as close to $||H(jw)||_\infty$ as possible and they are equal when $h(t) > 0$. In this dissertation. we impose conditions on $||h||_1$ which automatically includes the last two conditions proposed by Sheikholeslam.

3.3 Constant spacing control strategies

3.3.1 Control with information of reference vehicle information only

It is insightful to look into the advantages of having reference vehicle information.

Control Law :

Consider the following control law

$$u_{isl} = \ddot{x}_l - c_v(\dot{x}_i - \dot{x}_l) - c_p(x_i - x_l + \sum_{1}^{i} L_j)$$

Henceforth, x_l refers to the position of the lead vehicle in the platoon

Spacing Error Dynamics :

The spacing error dynamics for all strategies is obtained using the following equation :

$$\ddot{\epsilon}_i = \ddot{x}_i - \ddot{x}_{i-1} = u_{isl} - u_{(i-1)sl}$$

From the above two equations, we obtain

$$\ddot{\epsilon}_i + c_v\dot{\epsilon}_i + c_p\epsilon_i = 0$$

Comparison of the spacing error dynamics with equation 3.5 yields $\sum_1^r \alpha_j = 0$ and the corresponding stability polynomial associated with this strategy is $z = 0$. This is the "best" achievable platoon performance. It is unsafe since it does not take the information of the preceding vehicle into consideration.

3.3.2 Autonomous control

In this strategy. control law is based only on the on-hoard sensor measurements.

Control Law :

$$u_{isl} = -k_v\dot{\epsilon}_i - k_p\epsilon_i$$

Spacing Error **Dynamics** :

$\ddot{\epsilon}_i = u_{isl} - u_{(i-1)sl} \quad \forall \ i \geq 2$

$\Rightarrow \ddot{\epsilon}_i + k_v \epsilon_i + k_p \epsilon_i = k_v \dot{\epsilon}_{i-1} + k_p \epsilon_{i-1}$

$\hat{\epsilon}_i(s) = \hat{H}(s) \hat{\epsilon}_{i-1}(s)$

where

$$\hat{H}(s) = \frac{k_v s + k_p}{s^2 + k_v s + k_p}$$

$$\Rightarrow |\hat{H}(jw)| = \frac{k_p^2 + k_v^2 w^2}{(k_p - w^2)^2 + k_v^2 w^2}$$

For $i = 1$.

$\ddot{\epsilon}_i + k_v \dot{\epsilon}_i + k_p \epsilon_i = -\ddot{x}_l$

For all $k_v > 0. k_p > 0.$ $|\hat{H}(jw)| \geq 1$ for sufficiently small frequencies. A sinusoidal lead vehicle acceleration profile at that frequency results in errors amplifying along the platoon. Consequently. $\rho > 1$ and the stability polynomial associated with this strategy-, $z = \rho$, is unstable.

3.3.3 Semi-Autonomous control

In this control strategy. preceding vehicle's acceleration information is assumed to be available or estimated accurately.

Control Law :

$u_{isl} = k_a \ddot{x}_{i-1} - k_v \dot{\epsilon}_i - k_p \epsilon_i$

Spacing Error **Dynamics** :

$\ddot{\epsilon}_i + k_v \epsilon_i + k_p \epsilon_i = k_a \ddot{\epsilon}_{i-1} + k_v \dot{\epsilon}_{i-1} + k_p \epsilon_{i-1} \quad \forall \ i \geq 2$

$\hat{\epsilon}_i(s) = \hat{H}(s) \hat{\epsilon}_{i-1}(s)$

where

$\hat{H}(s) = \dfrac{k_a s^2 + k_v s + k_p}{s^2 + k_v s + k_p}$

For $i = 1$,

$\ddot{\epsilon}_i + k_v \dot{\epsilon}_i + k_p \epsilon_i = (k_a - 1)\ddot{x}_l$

If $k_a > 1$, $|\hat{H}(jw)| > 1$ for w sufficiently high. Hence, any lead vehicle acceleration at such a high frequency results in errors getting amplified along the platoon. If $k_a < 1$, then for all $k_v > 0, k_p > 0$, $|\hat{H}(jw)| \geq 1$ for sufficiently small frequencies. Therefore, for string stability, $k_a = 1$ and $H(s) \equiv 1$. Potentially, weak string stability can be guaranteed.

Robustness to Signal Processing/ Actuator Lags : As a result of signal processing/actuator lags, the control effort, u_{isl}, is the output of a filter

$\tau \dot{u}_{isl} + u_{isl} = k_a \ddot{x}_{i-1} - k_v \dot{\epsilon}_i - k_p \epsilon_i$

The perturbed transfer function $H_p(s) = \frac{s^2 + k_v s + k_p}{\tau s^3 + s^2 + k_v s + k_p}$ and $|\hat{H}_p(jw)| > 1$ for all $\tau > 0$ and for sufficiently small frequencies. Referring to equation 3.6, $\rho > 1$ and the stability polynomial associated with this strategy, $z = \rho$, is unstable and therefore, this scheme cannot be used for platooning.

3.3.4 Control with information of lead and preceding vehicles

With lead vehicle acceleration, velocity and position information, [13], define S_i as

$$S_i = \dot{\epsilon}_i + q_1 \epsilon_i + q_3(v_i - v_l) + q_4(x_i - x_l + \sum_{j=1}^{i} L_j)$$

S_i incorporates the information of the lead and preceding vehicles. It is chosen in such a way that the spacing error dynamics on the surface $S_i = S_{i-1} = 0$ is string stable.

Spacing **Error** Dynamics :

$$S_i - S_{i-1} = (1 + q_3)\dot{\epsilon}_i + (q_1 + q_4)\epsilon_i - \dot{\epsilon}_{i-1} - q_1 \epsilon_{i-1}$$

Laplace transforming the above equation,

$$\hat{\epsilon}_i(s) = \frac{s+q_1}{(1+q_3)s+(q_1+q_4)}\hat{\epsilon}_{i-1}(s) + \frac{(S_i - S_{i-1}) + ((1+q_3)\epsilon_i(0) - \epsilon_{i-1}(0))}{(1+q_3)s+(q_1+q_4)}$$

In this case, $\hat{H}(s) = \frac{s+q_1}{(1+q_3)s+(q_1+q_4)}$. $h(t) = \mathcal{L}^{-1}(\hat{H}(s)) > 0$ if $q_1q_3 \geq q_4$. Since $||h||_1 = |H(0)|$ whenever $h(t)$ does not change sign. $\alpha_1 = \frac{q_1}{41+44} < 1$. and therefore. this platooning strategy is string stable. $q_1q_3 \geq q_4$ indicates that sufficient damping (q_3) is required to make $\alpha_1 = \frac{q_1}{41+44}$.

$$\Rightarrow ||\epsilon_i||_\infty \leq \frac{q_1}{q_1+q_4}||\epsilon_{i-1}||_\infty + \frac{||S_i - S_{i-1}||_\infty + (2+q_3)||\epsilon_i(0)||_\infty}{q_1+q_4}$$

if $q_1q_3 \geq q_4$. Clearly. if all the sliding surfaces. S_i and the spacing errors are uniformly bounded at the initial time and the sliding surfaces are chosen such that $S_i\dot{S}_i \leq 0$. then the sliding surfaces, the spacing and velocity errors of all the vehicles are uniformly bounded at all time. Therefore, we choose the control, u_{isl}, to make S; $+\lambda S_i = 0$ and is given by :

Control **Law** :

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} + q_3\ddot{x}_l - (q_1+\lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i - (q_4+\lambda q_3)(v_i - v_l) - \lambda q_4(x_i - x_l + \sum_{j=1}^{i} L_j)]$$

The condition that $\sum_{j=0}^{i}\epsilon_j(0), \sum_{j=0}^{i}\dot{\epsilon}_j(0)$ be bounded is required to show that the control effort and the sliding surface is bounded at all times. Boundedness of $\epsilon_i(t), \dot{\epsilon}_i(t)$ alone does not guarantee the boundedness of sliding surface and the control effort. In order to prove boundedness of the sliding surface and the control effort: we need to show that $\dot{w}_i := (v_i(t) - v_l(t))$ and $w_i := (x_i(t) - x_l(t) + \sum L_j)$ is bounded for all vehicles at all time. Using the definition of sliding surface S_i.

$$S_i - S_1 = (1+q_3)\dot{w}_i + (q_1+q_4)w_i - \dot{w}_{i-1} - q_1 w_{i-1}$$

By the same argument as above, it follows that $\dot{w}_i, w_i$ is hounded at all time for all vehicles.

Robustness to Actuator/Signal processing lags

With actuator/signal processing lags. the actual input (throttle/brake) input to the system is a first order filtered output. u_f. of u_i. Hence,

$$\ddot{x}_i = \frac{u_f - c_i\dot{x}_i^2 - f_i}{M_i}$$

where

$$\tau \dot{u}_f + u_f = c_i \dot{x}_i^2 + f_i + M_i u_{isl}$$

$$\Longrightarrow \tau \frac{d}{dt}\ddot{x}_i + \ddot{x}_i\left(1 + \frac{2c_i\dot{x}_i}{M_i}\right) = u_{isl}$$

Since $\frac{2c_i\dot{x}_i}{M_i} \approx \frac{2*.05*35}{220} \approx 0.0175 << 1$, we neglect that term to get the following equations:

$$\ddot{\epsilon}_1 + \left(\lambda + \frac{q_1+q_4}{1+q_3}\right)\dot{\epsilon}_1 + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_1 = -\tau\frac{d}{dt}\ddot{x}_l$$

and for $i \geq 2$.

$$\tau\frac{d}{dt}\ddot{\epsilon}_i + \ddot{\epsilon}_i + \left(\lambda + \frac{q_1+q_4}{1+q_3}\right)\dot{\epsilon}_i + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_i = \frac{1}{1+q_3}[\ddot{\epsilon}_{i-1} + (q_1+\lambda)\dot{\epsilon}_{i-1} + q_1\lambda\epsilon_{i-1}]$$

Proposition : Robustness to Actuator/ Signal Processing lag

If $q_1 > \frac{q_1+q_4}{1+q_3}$, then $\exists \tau_l > 0$ such that $\forall\ \tau < \tau_l$, the platoon has uniformly bounded spacing errors for all bounded acceleration manuevers of the lead vehicle.

Proof It is easy to see that

$$\tau\frac{d}{dt}\ddot{\epsilon}_1 + \ddot{\epsilon}_1 + \left(\lambda + \frac{q_1+q_4}{1+q_3}\right)\dot{\epsilon}_1 + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_1 = -\tau\frac{d}{dt}\ddot{x}_l$$

It suffices to prove that the gain of the perturbed transfer function, $\hat{H}_p(s)$, is less than unity: where $H_p(s)$ is given by:

$$\hat{H}_p(s) = \frac{1}{1+q_3}\frac{(s+q_1)(s+\lambda)}{(\tau s^3 + s^2 + (\lambda + \frac{q_1+q_4}{1+q_3})s + \frac{\lambda(q_1+q_4)}{1+q_3})}$$

Let $\beta_1 = \min\{\frac{q_1+q_4}{1+q_3}, \lambda\}$ and $\beta_2 = \max\{\frac{q_1+q_4}{1+q_3}, \lambda\}$. Figure 3.2 illustrates how the poles of the perturbed transfer function change with τ. Let $\beta_1' < \beta_2' < \beta_3'$ be the poles of the perturbed transfer function. From the figure, it is clear that given any $\nu > 0$, $\exists \nu_1, \nu_2 > 0$. such that $\nu_1, \nu_2 < \nu$ and $\beta_1' = \beta_1 - \nu_1$, $\beta_2' = \beta_2 + \nu_2$. $\beta_3' = \frac{1}{\tau} - \beta_1 - \beta_2 + \nu_1 - \nu_2$. Define $\tau' = \frac{1}{\beta_3'}$.

$$H_p(s) = \frac{1}{1+q_3}\frac{\tau'}{\tau}\frac{(s+q_1)(s+\lambda)}{(\tau' s+1)(s+\beta_1')(s+\beta_2')}$$

$$\overline{s+\beta_1' \quad s+\beta_2' \quad s+\beta_3' \, \tau(1+q_3}$$

where

$$A_1 = \frac{(q_1-\beta_1')(\lambda-\beta_1')}{(1-\tau'\beta_1')(\beta_2'-\beta_1')}$$

$$A_2 = \frac{(q_1-\beta_2')(\lambda-\beta_2')}{(1-\tau'\beta_2')(\beta_1'-\beta_2')}$$

$$A_3 = \frac{(1-\tau' q_1)(1-\tau'\lambda)}{(1-\tau'\beta_1')(1-\tau'\beta_2')}$$

If $\lambda = \beta_1, \frac{q_1+q_4}{1+q_3} = \beta_2$. for sufficiently small τ, $A_1, A_2, A_3 > 0$ and hence. $h(t) > 0$. If $\lambda = \beta_2, \frac{q_1+q_4}{1+q_3} = \beta_1$, A_2 is of the order of ν. $A_1, A_3 > 0$ and since $\beta_2 > \beta_1$, $e^{-\beta_2 t}$ decays faster than $e^{-\beta_1 t}$. For sufficiently small τ and consequently. for sufficiently small ν, $h(t) > 0$. Therefore. $\rho = \frac{q_1}{q_1+q_4}$. This result establishes that the string stability property is not lost for a small actuator lag.

3.3.5 Semi-Autonomous control with vehicle ID knowledge

It is desirable to guarantee string stabilit'y or at least uniform boundedness of spacing errors with as little external information as possible. In such a case. autonomous/ semi-autonomous implementation is possible. In this section. we will investigate such a scheme.

Modifying the control law from the earlier subsection.

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} - (q_1+\lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i - (q_4+\lambda q_3)(v_i - v_l) - \lambda q_4(x_i - x_l + \sum_{j=1}^{i} L_j)]$$

Notice that the lead vehicle acceleration information is not utilized in this scheme. If the lead vehicle velocity and position information can. somehow, be reconstructed knowing vehicle index. then a semi-autonomous implementation is possible.

Spacing Error Dynamics :

With this control law. the spacing error dynamics is given by:

$$\ddot{\epsilon}_1 + (\frac{q_1+q_4}{1+q_3} + \lambda)\dot{\epsilon}_1 + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_1 = -\frac{q_3}{1+q_3}\ddot{x}_l$$

$$\ddot{\epsilon}_i + (\frac{q_1+q_4}{1+q_3} + \lambda)\dot{\epsilon}_i + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_i = \frac{1}{1+q_3}[\ddot{\epsilon}_{i-1} + (q_1+\lambda)\dot{\epsilon}_{i-1} + q_1\lambda\epsilon_{i-1}]$$

$$\hat{H}(s) = \frac{1}{1+q_3}\frac{(s+q_1)}{(s+\frac{q_1+q_4}{1+q_3})}$$

Hence, $\epsilon_{i-j} = \hat{H}^{-j}(s)\epsilon_i$.

Control **Law** :

The control law is implemented as follows:

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} - (q_1+\lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i - [1 + \hat{H}^{-1} + .. + \hat{H}^{-(i-1)}]\{(q_4+\lambda q_3)\dot{\epsilon}_i + \lambda q_4\epsilon_i\}]$$

Knowing ϵ_i, $\dot{\epsilon}_i$. $\ddot{x}_{i-1}$ and i. the vehicle index: we can guarantee that $\rho = \frac{q_1}{q_1+q_4}$. Knowledge of $\ddot{x}_{i-1}$ is essential: otherwise. $\hat{H}(s)$, is strictly proper and $\hat{H}^{-1}$ is not realizable. This scheme is attractive, since it requires the minimum information to guarantee uniform boundedness of spacing errors for a constant spacing strategy. Autonomous implementation is dependent on how smooth the signals $\ddot{x}_i$ and $\dot{\epsilon}_i$ are. to allow for estimating $\ddot{x}_{i-1}$ via numerical differentiation. Drawbacks include requiring very accurate signals of ϵ_i, *t;* and requiring q_4 << yl. The number of vehicles in the platoon is constrained. since our performance objective is to let $||H(jw)|| \leq 1$. While estimating the lead vehicle information, inversion of this transfer function is necessary. which leads to amplification of the noise.

Another way to utilize the information of vehicle ID is to vary the gains in the controller depending on the vehicle index However, in order to guarantee uniform boundedness for this case. the gains have to increase at least linearly. This results in increased control effort at the tail of the platoon, leading to saturation in the throttle angle input in the corresponding vehicles.

Real Time implementation: In order to estimate the velocity and position error relative to the lead vehicle in the platoon using ϵ_i and $\dot{\epsilon}_i(t)$ measurements. the following state space realization is utilized:

$$\dot{z}_1 = -q_1 z_1 + z_2$$

$$\dot{z}_2 = -q_1 z_2 + z_3$$

$$\dot{z}_{i-1} = -q_1 z_{i-1} + (q_4 + \lambda q_3)\dot{\epsilon}_i + \lambda q_4$$

$$y_{out} = \sum_{r=0}^{i-1}\sum_{j=0}^{r} \begin{pmatrix} r \\ j \end{pmatrix} (1+q_3)^r (q_4 - q_1 q_3)^j z_{i-r+j}$$

where

$$u_{isl} = \text{———} \; \ddot{\epsilon}_i - (q_1 + \lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i - y_{out}$$

Assuming that the sampling time is 'T', implementation in discrete time is given by,

$$\begin{bmatrix} z_1(k+1) \\ \vdots \\ z_{i-1}(k+1) \end{bmatrix} = e^{-q_1 T} \begin{bmatrix} 1 & T & \frac{T^2}{2!} & & \frac{T^{i-1}}{(i-1)!} \\ 0 & 1 & T & & \frac{T^{i-2}}{(i-2)!} \\ & & & & \\ 0 & 0 & & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1(k) \\ \vdots \\ z_{i-1}(k) \end{bmatrix} +$$

$$\begin{bmatrix} 1 - e^{-q_1 T}[1 + q_1 T + \frac{(q_1 T)^2}{2!} + \ldots + \frac{(q_1 T)^{i-1}}{(i-1)!}] \\ 1 - e^{-q_1 T}[1 + q_1 T + \frac{(q_1 T)^2}{2!} + \ldots + \frac{(q_1 T)^{i-2}}{(i-2)!}] \\ \vdots \\ 1 - e^{-q_1 T} \end{bmatrix} (q_4 + \lambda q_3)\dot{\epsilon}_i + \lambda q_4 \epsilon_i$$

$$y_{out}(k) = \sum_{r=0}^{i-1}\sum_{j=0}^{r} \begin{pmatrix} \\ \end{pmatrix} (1+q_3)^r (q_4 - q_1 q_3)^j z_{i-r+j}(k)$$

3.3.6 Control With Information of "r" Vehicles Ahead

If a platoon consists of a large number of vehicles. the communication delays of information from the lead vehicle to the end of the platoon could degrade the platoon performance significantly. In order to circumvent such delays, a platoon strategy in which every controlled vehicle requires only the information of vehicles in its vicinity. is desirable. However, the string stability aspect, of such a scheme has to be analyzed.

Control Law :

Consider the following control strategy where every controlled vehicle has the information of its "r" preceding vehicles:

$$u_{isl} = \sum_{j=1}^{r} k_{aj}\ddot{x}_{i-j} - k_{vj}(v_i - v_{i-j}) - k_{pj}(x_i - x_{i-j} + \sum_{k=\max[0,i-j+1]}^{i} L_k)$$

with $x_{i-j} \equiv x_l \quad \forall\ i \leq j$.

Spacing Error Dynamics :

$$\ddot{\epsilon}_1 + \sum_{j=1}^{r}(k_{vj}\dot{\epsilon}_1 + k_{pj}\epsilon_1) = 0 \tag{3.7}$$

$$\ddot{\epsilon}_i + \sum_{j=1}^{r} k_{vj}\epsilon_i + k_{pj}\epsilon_i = \sum_{j=1}^{r} k_{aj}\ddot{\epsilon}_{i-j} + k_{vj}\dot{\epsilon}_{i-j} + k_{pj}\epsilon_{i-j} \quad \forall\ i \geq 2 \tag{3.8}$$

which implies that

$$\hat{H}_j(s) = \frac{k_{aj}s^2 + k_{vj}s + k_{pj}}{s^2 + \sum_{j=1}^{r}(k_{vj}s + k_{pj})}$$

Xrguing in a manner similar to that in section **3.3.3,**

$$|\sum_{j=1}^{r} \hat{H}_j(jw)| \leq 1 \iff \sum k_{aj} = 1 \iff \sum_{j=1}^{r} \hat{H}_j(s) \equiv 1$$

From Fact 6, $\sum_1^r \alpha_j \geq 1$ and $\sum_1^r \alpha_j = 1 \iff \hat{H}_j(s) = \alpha_j$, a constant.

From the small transfer function gain theorem for string stability of a platoon. we can guarantee string stability in the "weak" sense only. The platoon performance. in terms of string stability. is limited by the above inequality.

Robustness to Actuator/ Signal Processing Lag :

Weak sense string stability is achieved only when $H_j(s)$ is a constant. In order to achieve the best possible performance, weak sense string stability, $h_j(t) = \alpha_j\delta(t)$. When $h_j(t)$ is perturbed (by. say. actuator lag), it changes sign and hence. $\sum_{j=1}^{r} \alpha_j > 1$. Therefore. this scheme also lacks robustness in string stability.

For example, if the actuator were to have parasitic dynamics or if there were signal processing lags; then

$$\tau \frac{d}{dt}\ddot{x}_{ip} + \ddot{x}_{ip} = u_{islp}$$

With zero initial spacing and velocity errors,

$$\hat{\epsilon}_{ip}(s) = \sum_{j=1}^{r} \hat{H}_{jp}\hat{\epsilon}_{(i-j)p}(s)$$

$$\hat{H}_{jp}(s) = \frac{k_{aj}s^2 + k_{vj}s + k_{pj}}{\tau s^3 + s^2 + q_1 s + q_2}$$

where the subscript "p" denotes the corresponding variable for the perturbed system. Therefore.

$$\sum_{j=1}^{r} \hat{H}_{jp}(s) = \frac{s^2 + q_1 s + q_2}{\tau s^3 + s^2 + q_1 s + q_2}$$

At sufficiently low frequency. w_0, $|\sum_{j=1}^{r} \hat{H}_{jp}(jw_0)| > 1 \Rightarrow \sum_{j=1}^{r} \alpha_{jp} > 1$.

3.3.7 Mini-platoon control strategy

String stability is guaranteed in the constant spacing control strategies proposed in [13]. because every controlled vehicle receives information from a reference (lead) vehicle. The mini-platoon control strategy uses the idea that feeding back information from a reference vehicle improves the robustness properties of the string. while reducing the effects of communication delays associated with transmitting the lead vehicle information. In this control strategy. every platoon is divided into mini-platoons and the last vehicle of a mini-platoon becomes the reference vehicle for the following mini-platoon. Figure *3.3* show how the information is transmitted between vehicles in the platoon. The controller given in [13] is modified as follows:

$$S_i = \dot{\epsilon}_i + q_1\epsilon_i + q_3(v_i - v_{ref}) + q_4(x_i - x_{iref} + \sum_{j=iref+1}^{i} L_j)$$

where the subscript ref refers to the index of the reference vehicle for the controlled vehicle (i.e.. index of the leader of the mini-platoon to which the controlled vehicle belongs). For the sake of analysis. we assume that every mini-platoon consists of "r" vehicles.

Control **Law** :

The control input u_{isl} is chosen to make $\dot{S}_i + \lambda S_i = 0$. The corresponding control input is given below:

For $nr + 1 \leq i < (n+1)r$, $\forall\ n = 0,1,2,\ldots$ where n denotes the n-th mini-platoon.

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} + q_3\ddot{x}_{nr} - (q_1+\lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i$$

$$-(q_4+\lambda q_3)(v_i - v_{nr}) - \lambda q_4(x_i - x_{nr} + \sum_{j=nr+1}^{i} L_j)] \tag{3.9}$$

For $i = nr$, $\forall\ n = 1,2,$

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} + q_3\ddot{x}_{(n-1)r} - (q_1+\lambda)\dot{\epsilon}_i - q_1\lambda\epsilon_i$$

$$-(q_4+\lambda q_3)(v_i - v_{(n-1)r}) - \lambda q_4(x_i - x_{(n-1)r} + \sum_{j=(n-1)r+1}^{i} L_j)] \tag{3.10}$$

Spacing Error Dynamics :

From the above equations, we obtain: For $nr + 2 \leq i \leq (n+1)r$,

$$\ddot{\epsilon}_i + (\frac{q_1+q_4}{1+q_3} + \lambda)\dot{\epsilon}_i + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_i = \frac{1}{1+q_3}[\ddot{\epsilon}_{i-1} + (q_1+\lambda)\dot{\epsilon}_{i-1} + \lambda q_1\dot{\epsilon}_{i-1}]$$

For $i = nr + 1$,

$$\ddot{\epsilon}_{nr+1} + (\frac{q_1+q_4}{1+q_3} + \lambda)\dot{\epsilon}_{nr+1} + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_{nr+1} =$$

$$\frac{1}{1+q_3}[\ddot{\epsilon}_{nr} + (q_1+\lambda)\dot{\epsilon}_{nr} + \lambda q_1\dot{\epsilon}_{nr}] + \frac{1}{1+q_3}[q_3\ddot{D}_n + (q_4+\lambda q_3)\dot{D}_n + \lambda q_4 D_n] \tag{3.11}$$

where $D_n = x_{nr} - x_{(n-1)r} + \sum_{j=(n-1)r+1}^{nr} L_j$, describes the spacing error between the lead vehicles of n-th and n-1th mini-platoons. The right hand side of the above equation has terms involving D_n, which describe how the mismatch between two successive reference vehicle's information affects the spacing error in the first follower of every mini-platoon. With zero initial spacing and velocity errors :

For $nr + 2 \leq i \leq (n+1)r$,

$$\hat{\epsilon}_i(s) = \hat{H}(s)\hat{\epsilon}_{i-1}(s)$$

and

$\hat{\epsilon}_{nr+1}(s) = \hat{\epsilon}_{(n-1)r+1}(s)$

where $H(s) = \frac{s+q_1}{(1+q_3)s+(q_1+q_4)}$. Since the roots of the polynomial. $P_r(z) = z^r - 1 = 0$. are simple and lie on the unit circle. this platooning strategy is string stable in the "weak" sense. Due to signal processing lags. the perturbed polynomial is given by $z^r - \alpha = 0$ where $\alpha > 1$. The roots of the perturbed polynomial are all outside the unit circle and their magnitude is $\alpha^{\frac{1}{r}}$. Hence, the spacing errors of the first followers increase with mini-platoon index. The magnitude of the perturbed roots gives the average attenuation/amplification upstream from vehicle to vehicle. The spacing errors attenuate geometrically within the mini-platoon. Since the number of vehicles in the platoon is finite. in practice, the maximum number of vehicles in a mini-platoon is determined by the hardware/communication limitations.

The advantage of this scheme is that we need to focus our attention only on the leaders of the mini-platoon. One could treat the dynamics of a platoon by the dynamics of its leader in a higher level of control for automated highway systems. If every controlled vehicle in the platoon has the information of its "r" preceding vehicles. we have seen that constant spacing strategy yields to limited robustness. Instead. we should organize the platoon into miniplatoons of "r" vehicles. At least. vehicles in the mini-platoon exhibit geometric attenuation and good robustness property to actuator/sensor lags.

3.3.8 Mini-platoon control with lead vehicle information

The motivation for this scheme is to improve the robustness property of the leaders of the mini-platoon by making lead vehicle information available to the leaders of the mini-platoon. Consider a scheme in which the leader of every mini-platoon gets information from its preceding vehicle and the leader of the platoon and all the vehicles in the mini-platoon get information from their predecessors and the leader of the mini-platoon. For the sake of analysis. we assume that, every mini-platoon has "r" vehicles in it. For real-time implementation, it is envisaged that the lead vehicle information is updated on a slower time scale compared to the other information that is required for feedback control law.

Control Law:

Consider the following control law:

$$u_{jsl} = \frac{1}{1+q_3}[\ddot{x}_{jsl} + q_3\ddot{x}_{ir+1} - (q_1+\lambda)\dot{\epsilon}_j - q_1\lambda\epsilon_j - (q_4+\lambda q_3)(\dot{x}_j - \dot{x}_{ir+1})$$

$$-\lambda q_4(x_j - x_{ir+1} + \sum_{ir+2}^{j} L_q)] \;\; \forall \;\; ir+1 < j \le ir+r, i = 0, 1, 2, ..$$

and

$$u_{ir+1} = \frac{1}{1+q_3}[\ddot{x}_r + q_3\ddot{x}_1 - (q_1+\lambda)\dot{\epsilon}_{ir+1} - q_1\lambda\epsilon_{ir+1} - (q_4+\lambda q_3)(\dot{x}_{ir+1} - \dot{x}_1)$$

$$-\lambda q_4(x_{ir+1} - x_1 + \sum_{2}^{ir+1} L_j)] \;\; \forall \;\; i = 1, 2, 3, .. \qquad (3.12)$$

Spacing **Error** Dynamics :

With this control law, we can show that

$$\hat{\epsilon}_j(s) = \hat{H}(s)\hat{\epsilon}_{j-1}(s) \;\; \forall \;\; ir+1 < j \le (i+1)r$$

where

$$\hat{H}(s) = \frac{1}{1+q_3}\frac{(s+q_1)(s+\lambda)}{s^2 + (\frac{q_1+q_4}{1+q_3}+\lambda)s + \frac{\lambda(q_1+q_4)}{1+q_3}}$$

Therefore, the maximum spacing errors decrease geometrically within a miniplatoon. The spacing error dynamics of the leaders of mini-platoon is given by the following equations:

$$\ddot{x}_{ir+1} - \ddot{x}_{(i-1)r+1} = \frac{1}{1+q_3}[\ddot{x}_{ir} - \ddot{x}_{(i-1)r} - (q_1+\lambda)\dot{\epsilon}_{ir+1} + (q_1+\lambda)\dot{\epsilon}_{(i-1)r+1}$$

$$-\lambda q_1\epsilon_{ir+1} - (q_4+\lambda q_3)(\dot{x}_{ir+1} - \dot{x}_{(i-1)r+1}) - \lambda q_4(x_{ir+1} - x_{(i-1)r+1})]$$

which implies that'

$$\ddot{\epsilon}_{ir+1} + (\frac{q_1+q_4}{1+q_3}+\lambda)\dot{\epsilon}_{ir+1} + \frac{\lambda(q_1+q_4)}{1+q_3}\epsilon_{ir+1} = \frac{1}{1+q_3}[\ddot{\epsilon}_{(i-1)r+1} + (q_1+\lambda)\dot{\epsilon}_{(i-1)r+1} + \lambda q_1\epsilon_{(i-1)r+1}$$

$$-q_3[\ddot{\epsilon}_{ir} + \ldots + \epsilon_{(i-1)r+2}] + (q_4 + \lambda q_3)[\dot{\epsilon}_{ir} + \ldots + \dot{\epsilon}_{(i-1)r+2}] + \lambda q_4[\epsilon_{ir} + \ldots + \epsilon_{(i-1)r+2}]$$

Simplifying the above equation and using the definition of $\hat{H}(s)$,

$$\hat{\epsilon}_{ir+1} = \hat{H}^r(s)\hat{\epsilon}_{(i-1)r+1}$$

Therefore, the maximum spacing errors of the leaders of the mini-platoon attenuate with the same geometric ratio.

It is hoped that the maximum spacing errors of the leaders of the mini-platoon do not amplify with a slower time scale update of the lead vehicle information. A detailed two scale analysis of this strategy is necessary to implement this strategy.

3.4 Variable Spacing Control Strategies

3.4.1 Autonomous Intelligent Cruise Control (AICC)

It is worthwhile considering the effect of feeding back controlled vehicle's velocity on the platooning specifications. Consider the following control law:

$$u_{isl} = k_a\ddot{x}_{i-1} - k_v\dot{\epsilon}_i - k_p\epsilon_i - k_1\dot{x}_i$$

The spacing error dynamics is given by:

$$\ddot{\epsilon}_1 + (k_v + k_1)\dot{\epsilon}_1 + k_p\epsilon_1 = (k_a - 1)\ddot{x}_l - k_1\dot{x}_l$$

$$\ddot{\epsilon}_i + (k_v + k_1)\dot{\epsilon}_i + k_p\epsilon_i = k_a\ddot{\epsilon}_{i-1} + k_v\dot{\epsilon}_{i-1} + k_p\epsilon_{i-1}$$

From the above equations. it is clear that non-zero steady state errors result from a step change in lead vehicle's velocity. The magnitude of the error is given by $-\frac{k_1}{k_p}\Delta v$, where Δv is the step change in velocity. The negative sign indicates that the vehicles fall hack whenever Δv is positive. The spacing between vehicles is higher at higher speeds. It is also clear that k_1 is required to be zero for zero steady state spacing error for any step change in lead vehicle velocity. However, in order to ensure string stability: $k_1 \neq 0$. Hence. for the autonomous case: zero steady state spacing error and string stability requirements are at odds with each other. It is intuitive that, the magnitude of steady state spacing errors that can be tolerated relates directly to the robustness of this scheme to actuator/signal processing lags. If the zero steady state spacing error specification is relaxed. we can define a generalized spacing error of the i-th vehicle, δ_i, as follows:

$$\delta_i = x_i - x_{i-1} + L_i + h_w \dot{x}_i$$

where h_w is the desired constant headway time [1] to be maintained. This is the basis for AICC law proposed by Chien. Ioannou and Hauser, [17].

Control Law :

Consider the following law, which requires on-board information only [17] :

$$u_{isl} = -\frac{\dot{x}_i - \dot{x}_{i-1} + \lambda\epsilon_i}{h_w} \tag{3.13}$$

Generalized Spacing Error Dynamics :

The generalized spacing error dynamics is given by:

$$h_w\ddot{\delta}_i + (1 + \lambda h_w)\dot{\delta}_i + \lambda\delta_i = \dot{\delta}_{i-1} + \lambda\delta_{i-1}$$

and

$$h_w\ddot{\delta}_1 + (1 + \lambda h_w)\dot{\delta}_1 + \lambda\delta_1 = 0$$

From the above equations,

$$H(s) = \frac{\delta_i}{\delta_{i-1}}(s) = \frac{1}{h_w s + 1}$$

Clearly, $\rho = 1$ for all $h_w > 0$. This is a very attractive feature of this strategy considering that no lead vehicle information is fed back. There are two drawbacks of this scheme:

1. The control effort is inversely proportional to the desired headway time. For maintaining a small desired headway time, the brake and engine torques may saturate. This fact is also documented in [6].

[1] Headway time is defined as the time it takes the vehicle i to cover a distance $x_i - x_{i-1} + L_i$

2. X small desired headway time implies larger traffic capacity. Hence. there is a limit on the maximum traffic capacity that is achievable.

Robustness to Actuator/Signal processing lags:

As seen earlier. actuator/signal processing lags can be modelled as

$$\tau \frac{d}{dt}\ddot{x}_i + \ddot{x}_i = u_{isl}$$

From equation 3.13 and the above equation. we get.

$$\tau h_w \frac{d}{dt}\ddot{\delta}_i + h_w \ddot{\delta}_i + (1 + \lambda h_w)\dot{\delta}_i + \text{AS;} = \dot{\delta}_{i-1} + \lambda \delta_{i-1}$$

$$\Longrightarrow \frac{\hat{\delta}_i}{\hat{\delta}_{i-1}} := H_p(s) = \frac{s+\lambda}{\tau h_w s^3 + h_w s^2 + (1 + \lambda h_w)s + \text{X}}$$

Claim:

1. For sufficiently small τ. $||\mathcal{L}^{-1}(\hat{H}_p(s))||_1 = 1$
2. $||\mathcal{L}^{-1}(H_p(s))||_1 = 1 \Rightarrow \tau \leq \frac{h_w}{2}$.

Proof

1.From an argument' similar to that in Proposition of section 3.3.4. the result follows.
2.A necessary condition for $||L^{-1}(H_p(s))||_1 = 1$ is that $|\hat{H}_p(jw)| \leq 1 \ \forall \ w$. Therefore.

$$w^2 + \text{A}' \leq (\text{X} - h_w w^2)^2 + (1 + \lambda h_w - \tau h_w w^2)^2 w^2$$

$$\Longrightarrow 0 \leq \tau^2 h_w^2 w^4 + (h_w^2 - 2\tau h_w(1 + \lambda h_w))w^2 + \lambda^2 h_w^2 \ \ \forall \ \ w$$

From theory of quadratic equations. the above inequality holds if and only if one of the following inequalities hold:

1. $h_w^2 - 2\tau h_w(1 + \lambda h_w) \geq 0$

   or

2. $(h_w^2 - 2\tau h_w(1 + \text{Ah,.}))' \ \ - 4\lambda_2 h_2^{\acute{}}{}_{\tau} {}^2 h_w^{\acute{}} \leq 0$

Both the conditions are satisfied only if $\tau \leq \frac{h_w}{2}$.

This result establishes that the robustness in string stability at a small time headway is limited.

3.4.2 Constant headway time control strategy with information of "r" vehicles ahead

Feeding back controlled vehicle's velocity results in non-zero steady state errors for most lead vehicle maneuvers. However, it impros-es the robustness in string stability to actuator lags and signal processing lags. The degree of robustness in string stability depends on the magnitude of non-zero steady state errors that can he tolerated. This scheme has also been proposed independently by Green and Ren [30].

Control Law :

Consider the following control law:

$$u_{isl} = \sum_{j=1}^{r}[k_{aj}\ddot{x}_{i-j} - k_{vj}(v_i - v_{i-j}) - k_{pj}(x_i - x_{i-j} + \sum_{k=\max[0,i-j+1]}^{i} L_k)] - k_v\dot{x}_i$$

with $x_{i-j} \equiv x_l \quad \forall \ i \leq j$. This results in a state state spacing error **in** i-th following vehicle given by $\epsilon_{iss} = \frac{k_v}{jk_{pj}}\Delta v$ where Δv is any step change in the lead vehicle's velocity. In this strategy. the desired intervehicular spacing varies as $L_i + h_w v_i$, where $h_w = \frac{k_v}{jk_{pj}}$.

Generalized Spacing **Error** Dynamics

A generalized spacing error for this strategy is, therefore. given by:

$$\delta_i = x_i - x_{i-1} + L_i + h_w\dot{x}_i$$

δ_i satisfies the following set of equations:

$$\ddot{\delta}_1 + (\sum_{j=1}^{r} k_{vj} + k_v)\dot{\delta}_1 + \sum_{j=1}^{r} k_{pj}\delta_1 = \sum_{j=1}^{r}[h_w k_{aj}\dddot{x}_l + (k_{aj} - 1)\ddot{x}_l] \tag{3.14}$$

$$\ddot{\delta}_i + (\sum_{j=1}^{r} k_{vj} + k_v)\dot{\delta}_i + \sum_{j=1}^{r} k_{pj}\delta_i = \sum_{j=1}^{r}[k_{aj}\ddot{\delta}_{i-j} + k_{vj}\dot{\delta}_{i-j} + k_{pj}\delta_{i-j}]$$

Therefore.

$$\sum_{j=1}^{r}\hat{H}_j(s) = \frac{\sum_{j=1}^{r}(k_{aj}s^2 + k_{vj}s + k_{pj})}{s^2 + \sum_{j=1}^{r}[(k_{vj} + k_v/r)s + k_{pj}]}$$

Since, $\sum_{j=1}^{r} H_j(0) = 1 \Rightarrow \sum_{j=1}^{r} \alpha_j \geq 1$. Therefore, this control strategy can, at best, guarantee "weak" string stability. For $||\sum_{j=1}^{r} h_j||_1 = 1 \Rightarrow \sum_{j=1}^{r} k_{vj} \geq \frac{1}{h_w}$. To guarantee string stability and to maintain a small headway time, the closing rate gains have to be chosen sufficiently high. There is an upper bound on these gains, which is determined by the input bandwidth/saturation constraints, and hence, there is a lower limit on h_w. The limiting case of the headway control strategy, $h_w = 0$, is the constant spacing control strategy, which clearly lacks robustness to parasitic dynamics in the actuator and signal processing lags as seen in section 3.3.6. Therefore, an arbitrarily small headway time cannot be maintained. From equation 3.11, the steady state spacing errors are dependent on the lead vehicle maneuver and may not decay exponentially to zero. In addition to the above limitations, this strategy requires the information of all the "r" vehicles ahead of it, which may put a serious burden on the communication system.

Consider a hybrid mini-platoon strategy in which the leaders of the mini-platoon follow AICC while the vehicles in the mini-platoon follow constant' spacing strategy (with the information of the leader of mini-platoon). The Headway control strategy with information of "r" preceding vehicles is inferior to this strategy in two ways:

- Robustness to lags : Follower vehicles in the mini-platoon are robust to actuator/signal processing lags. Robustness of the leaders of the mini-platoon is governed by the time headway they have to maintain.

- Traffic Capacity : The average time headway for hybrid mini-platoon strategy is h_w/r where h_w is the time headway the leaders of the mini-platoon maintain and r is the number of vehicles in the mini-platoon.

In other words, for the same traffic throughput, the leaders of the mini-platoon can maintain "r" times the time headway each vehicle maintains in the other strategy. Since robustness is inversely proportional to time headway, hybrid mini-platoon strategy guarantees better robustness properties.

3.5 Simulation Results

In this section: we will briefly summarize the salient features of all the strategies and show their corresponding simulation results. For all the simulation plots. a 10 vehicle platoon is considered. In all the simulation plot's. the number n on the plot represents the n-th following vehicle in the platoon. All the vehicles start with a velocity of $24.5m/s$ and they are positioned in such a way that the initial spacing error is zero. The plant' model of the vehicle has a throttle angle saturation rate of $1000°/s$ and a brake saturation limit of $8000N - m$.

Figure 3.4 shows the velocity and acceleration profile of the lead vehicle used in the simulations.

Figure 3.5 demonstrates the behavior of spacing errors under semi-autonomous constant spacing control. In this strategy, every controlled vehicle requires the acceleration information of its preceding vehicle in addition to on-board sensor information like the spacing and velocity error from the radar. As seen earlier in this chapter. every platooning strategy has an associated string stability polynomial and the spectral radius of the polynomial is a measure of the effectiveness (in terms of string stability) for the strategy. For string stability. the spectral radius of the polynomial should be less than unity. X platoon is string stabile in the weak sense if the spectral radius is equal to unity. For this strategy, the string stability polynomial is $z = 1$ and this strategy is string stable in the weak sense. The gains used for this simulation are: $k_a = 1; k_v = 2: k_p = 1$. As expected, due to mismatchecl uncertainties in the plant (discretization etc..). the spacing errors and consequently. the control effort grow with vehicle index. Figure 3.6 shows the effect of signal processing lags on the spacing errors. The throttle angle of all the vehicles behind the fourth following vehicle is saturated. In all the above simulations. accelerations of the preceding vehicle is assumed to be available or estimated accurately. With any signal processing or actuator lags. the string stability polynomial is $z = \rho$ where $\rho > 1$ and this scheme is not robust'. Clearly, semi-autonomous constant spacing strategy cannot be used for platooning.

Figure 3.7 shows the effect, of availability of lead vehicle velocity and acceleration information to every controlled vehicle. on the platoon performance. The spacing errors decrease with vehicle index in this case. The gains selected are as follows: $q_1 = 1.0, q_3 = 0.5, \lambda = 1$. Figure 3.8 shows the effect of signal processing /actuator lags. The spacing errors, in the presence of signal processing lag of 50ms, arc larger in magnitude. The throttle angle increases initially with vehicle index due to the feedback from the lead vehicle. Since the masimum spacing error decreases with vehicle indes. the spacing error relative to the lead vehicle remains the same for all the vehicles at the tail of the platoon. As a result. the throttle angle (control effort) is the same for all the vehicles at the tail of the platoon. Although the associated string stability polynomial for this strategy is $z = 1$, the string stability polynomial is robust to actuator/signal processing lags.

In obtaining the simulation result shown in Figure 3.9. we have assumed that every controlled vehicle in the platoon has the information of lead vehicle's relative position information. The following gains are chosen: $q_1 = 0.8$: $q_3 = 0.5$: $q_4 = 0.4$: $\lambda = 1$. Clearly, the spacing errors decrease with a geometric ratio given by $\frac{q_1}{q_1+q_4}$. The associated string stability polynomial for this strategy is $z = \frac{q_1}{q_1+q_4}$. The string stability polynomial is robust to signal processing/ actuator lags. In the presence of small signal processing/actuator lags, although the magnitude of spacing errors is high. the attenuation ratio remains constant. Figure 3.10 shows this behavior.

In order to obtain the relative position information of the lead vehicle relative to the controlled vehicle. we plan to do the following:

1. Integrate numerically the velocity of the controlled vehicle relative ot the lead vehicle. We assume that the lead vehicle information is continually broadcast.

2. Every vehicle is required to broadcast' its position relative to the lead vehicle to all its following vehicles. Hence, the position of the j-th vehicle relative to the lead vehicle can he obtained by adding the position of the j-th vehicle relative to the j-1st vehicle (which is available from sensors like radar) and the position of the j-1st vehicle relative to the lead vehicle. Estimate using 1 is updated by this estimate to get a better estimate of the controlled vehicle's position relative to the lead vehicle.

Xlthough there are delays/lags associated with obtaining such estimates, all the simulations do not incorporate such features other than signal processing/actuator lag. It is recommended, for the constant spacing strategy, that lead (reference) vehicle information be utilized as much as possible for platooning.

Knowledge of vehicle ID helps attenuate maximum spacing errors.

'It is desirable to utilize as little external information as possible to guarantee the attenuation of maximum spacing errors. Esternal information in the form of knowledge of vehicle ID and the preceding vehicle information helps attenuate maximum spacing errors if the vehicle controller model is accurate. The idea behind this strategy is to reconstruct lead vehicle's relative velocity and position information from the spacing and velocity error information of the controlled vehicle and feed it back into the control lam. Very roughly speaking, knowing the controlled vehicle's ID in the platoon, we build an observer for the error dynamics of every vehicle preceding the controlled vehicle in the platoon. Figure 3.11 shows the behavior of spacing errors in the platoon with information of knowledge of vehicle ID and preceding vehicle's acceleration. In Figure 3.11. the spacing errors are an order of magnitude larger than the spacing errors in the earlier strategy using lead vehicle information. This is due to two reasons. First. lead vehicle acceleration information is not available/utilized. Second, we assume that every vehicle is 1/0 linearized so that there is an exact transfer function relationship between the errors of consecutive vehicles. Although, this rarely is the case, lead vehicle information is reconstructed using the spacing and velocity error measurements and the spacing error attenuation is guaranteed. The other disadvantages of this strategy are : the controller computations for the vehicles at the tail of the platoon gets complex with vehicle ID and the amount, of spacing error attenuation that can be guaranteed is limited.

Figure 3.12 shows the behaviour of the platoon with every controlled vehicle in the platoon having the information of 5 vehicles ahead. The motivation for this strategy is to investigate how the platoon performance is affected if every controlled vehicle has the information of "r" vehicles in its vicinity. The string stability polynomial corresponding to this strategy is $z = 1$. In the presence of any signal processing lags, the string stability polynomial gets perturbed to $z = \rho$ where $\rho > 1$. The first five vehicles in the platoon behave exactly the same way as in the previous case. In this platooning strategy, the maximum spacing errors of the following vehicles are guaranteed to be less than or equal to the maximum spacing error of the first follower in the platoon. Since the maximum spacing error at the tail of the platoon is approximately equal to the spacing error in the first vehicle. the throttle/control effort increases with vehicle index. This causes saturation of the throttle at the tail of the platoon. Furthermore. with signal processing lags. this scheme cannot ensure weak string stability. This scheme is not recommended for platooning.

Figure 3.13 depicts the behaviour of the spacing errors in the platoon under miniplatoon cont'rol strategy. The rationale behind this strategy is that feeding back reference vehicle information improves the string stability and robustness properties. In this strategy, every platoon is divided into mini-platoons of "r" vehicles each. Within the mini-platoon, every controlled vehicle is assumed to have access to the mini-platoon leader's information. The leaders of the mini-platoon have only the information of the vehicle ahead. As one would expect: the spacing errors in the miniplatoon decrease geometrically with vehicle index in the mini-platoon and the leaders of the miniplatoon experience larger errors due to the lack of lead vehicle information. The spacing errors of the leaders of the miniplatoon increase with miniplatoon index. in the same way the spacing errors increase when only the preceding vehicle's information is available. as shown in [13]. Miniplatoon can be modeled as a single vehicle when a constant intraplatoon spacing is maintained. The control input increases with miniplatoon index. limiting the number of miniplatoons allowable per platoon. If every controlled vehicle has the information of it's "r" preceding- vehicles. mini-platoon strategy should be employed, so that improved robustness is obtained.

If we feed the information of the lead vehicle in the platoon to the leaders of the miniplatoons. it is shown in section 3.3.8 that the performance of the platoon is similar to the case when every controlled vehicle in the platoon utilizes the lead vehicle information. A two time scale update is suggested for implementing the nliniplatoon control algorithm. in which all the vehicles in the miniplatoon get the information from their respective leaders on a faster time scale and the leaders of the miniplatoon get the information of the leader broadcast on a slower time scale. However. further analysis is required to study the string stability of this scheme.

In order to improve the robustness in string stability: feeding back the velocity of controlled vehicle at the expense of non-zero steady state spacing errors and consequently. traffic capacity is necessary. In AICC strategy. velocity of the controlled vehicle is fedback in addition to the on-board information from radar. The advantage of this strategy is that external informat'ion is not required. The disadvantage of this straegy is that the control effort is inversely proportional to the desired time headway and the robustness to actuator lag decreases with decreasing time headway. Figure 3.14 illustrates the behavior of the generalized spacing errors and throttle angles of vehicles in the platoon. The maximum generalized spacing error and throttle angle decreases with vehicle index. This strategy does not require maintaining a constant spacing between vehicles. Consequently. the controlled vehicle is not required to track the preceding vehicles' acceleration profiles exactly, which leads to a reduction in control effort. From simulations, a headway time of at least 0.2 sec is necessary to maintain smooth throttle angles and accelerations.

It is no coincidence that all the (proposed) control strategies which do not avail of the lead (reference) vehicle information can: at best, guarantee only weak sense string stability. Since they do not use the lead (reference) vehicle information. their associated string stability polynomials have at least a simple root at $z = 1$. Furthermore. they are not robust to signal processing lags/actuator lags. The advantage of using reference vehicle information is shown in section 3.1 . Therefore. it is imperative to have a single reference vehicle information for platooning.

If the lead vehicle information is not available to all the vehicles in the platoon. the mini-platoon strategy has some benefits over the other platooning strategy discussed in section 3.3.6 and 3.4.2. Firstly: it can guarantee geometric attenuation of the spacing errors within the platoon. Secondly, for a medium size platoon of 20-30 vehicles, which can be split into *3* or 4 mini-platoons, me need to focus only on the first follower in each mini-platoon. The first follower in every mini-platoon can be made to maintain a relatively large spacing compared to the nominal intervehicular spacing. Another alternative is to treat every mini-platoon as a vehicle and make the reference vehicles follow an AICC law. As a result. the traffic capacity achievable is much higher than the other strategy discussed in section 3.3.6 and 3.4.2.

The results of this chapter are summarized in Table 3.1 .

3.6 Steady State traffic Capacity Calculations and Evaluation of platooning strategies

Consider a platoon of N vehicles maintaining a distance L_p from its preceding one. Let L_v be the inter-vehicle spacing in the platoon and L_c be the vehicle length. The ideal (steady state) traffic capacity. [39], [53], is given by :

$$\phi_{id} = \frac{3600v}{L_v + L_c + \frac{L_p}{N}} veh/lane/hr$$

where v is the velocity of the platoon. For the case of spacing control strategies, $L_v = L_0$. a constant. For the case of headway control strategies, $L_v = L_0 + h_w v$ where h_w is the desired headway time. In order to account for merge and lane changing, the steady state traffic capacity is derated by 20%. L_p is estimated assuming that no collisions are allowed when the platoons are moving at $v_c m/s$ and when the lead vehicle platoon decelerates at $d_1 m/s^2$ and the following platoon decelerates at $d_2 m/s^2$, Δt sec after the lead vehicle platoon has started decelerating.

$$L_p = v_c \Delta t + \frac{v_c^2}{2}[\frac{1}{d_1} - \frac{1}{d_2}]$$

Typical values of these parameters: $v_c = 30m/s; \Delta t = 0.3sec; d_1 = 4m/s^2; d_2 = 10m/s^2; L_0 = 1m; L_p = 76.5m$. For spacing control strategy:

$$\phi_{act} = \frac{2880v}{6 + \frac{76.5}{N}}$$

For headway control strategy,

$$\phi_{act} = \frac{2880v}{6 + h_w v + \frac{76.5}{N}}$$

From the above formula. it is clear that a lower headway time yields higher lane capacity. The lane capacities for both the schemes. given by the two equations above. are shown in Figure 3.15, where N in the plot refers to the corresponding

Spacing Strategy	ρ	*Robustness*	*Implementation Requirements*	*Capacity*
Constant Spacing				High
With reference vehicle info. only	0	Unsafe. Does not consider preceding vehicle info.	Reference vehicle info. broadcast requires radio	
Autonomous Control	>1	Sot robust		
Semi-.Autonomous Control	$= 1$	Not robust	preceding vehicle info. required	
Control with knowledge of vehicle ID	< 1	$\rho \approx 1$	Require preceding vehicle info. and accurate vehicle model	
Control with lead and preceding vehicle info.	≤ 1	Robust	Require preceding and lead vehicle info.	
Control with info. of "r" vehicles ahead	≥ 1	Not Robust	Info. of "r" vehicles should he broadcast	
Miniplatoon Control	$= 1$	Not robust entirely	Info. of preceding and lead vehicles in the miniplatoon required	
Variable Spacing				Low to Medium
AICC	$= 1$	Proportional to time headway		
Headway Control with info. of "r" vehicles	$= 1$	Proportional to time headway	Info. of "r" vehicles should be broadcast	

Table 3.1: Summary of Platooning Strategies

platoon size. From the previous section, a headway of 0.2 sec is chosen for comparison. It can be seen that the spacing control has atleast 30% more traffic capacity than the headway control.

Recommendations: Higher traffic capacity is achievable only by constant spacing strategy. In order to guarantee string stability performance, lead vehicle acceleration. velocity and relative position information should be broadcast. This strategy is recommended for it's robustness property: guaranteed string stability performance and achievable high traffic capacity.

In the absence of any external information, AICC strategy is recommended, since it is the only strategy that guarantees weak string stability. If information of "r" preceding vehicles is available only, a hybrid mini-platoon strategy is recommended. In this strategy. "r" vehicles form a mini-platoon. The leaders of the mini-platoon follow XICC while the followers in the mini-platoon follow a constant spacing strategy with lead and preceding vehicle information. The advantage of this strategy over other strategies is that the leaders of the mini-platoon follow weak string stable dynamics and the spacing errors attenuate geometrically within the mini-platoon.

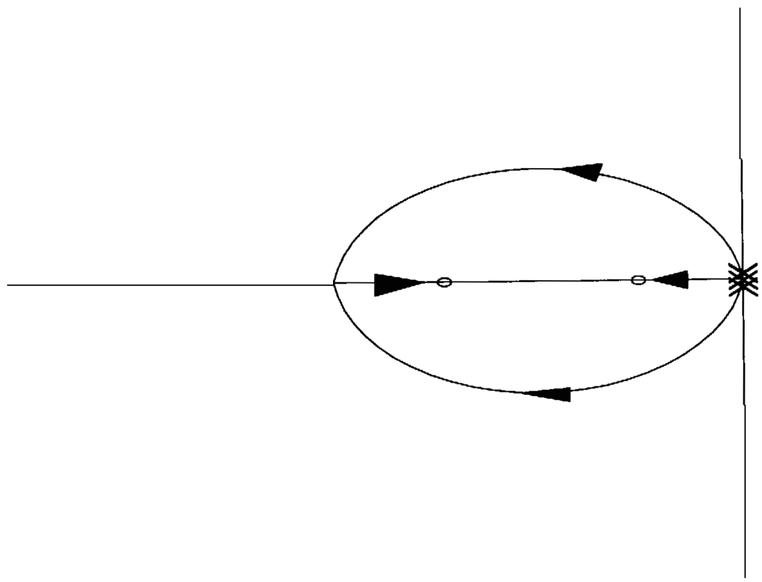


Figure 3.2: Root, locus of the poles of $\hat{H}_p(s)$ with variation in actuator lag, τ

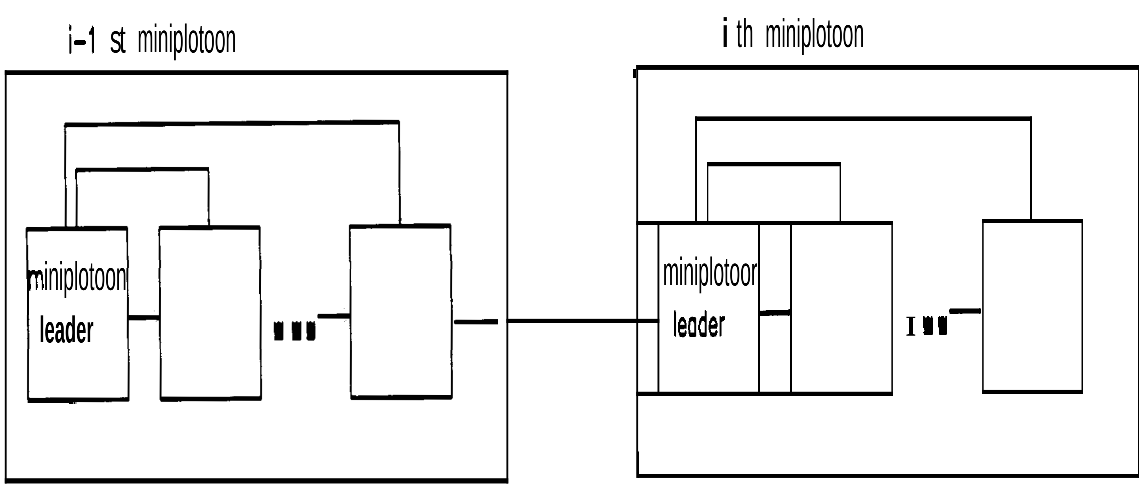


Figure **3.3:** Mini-platoon information structure

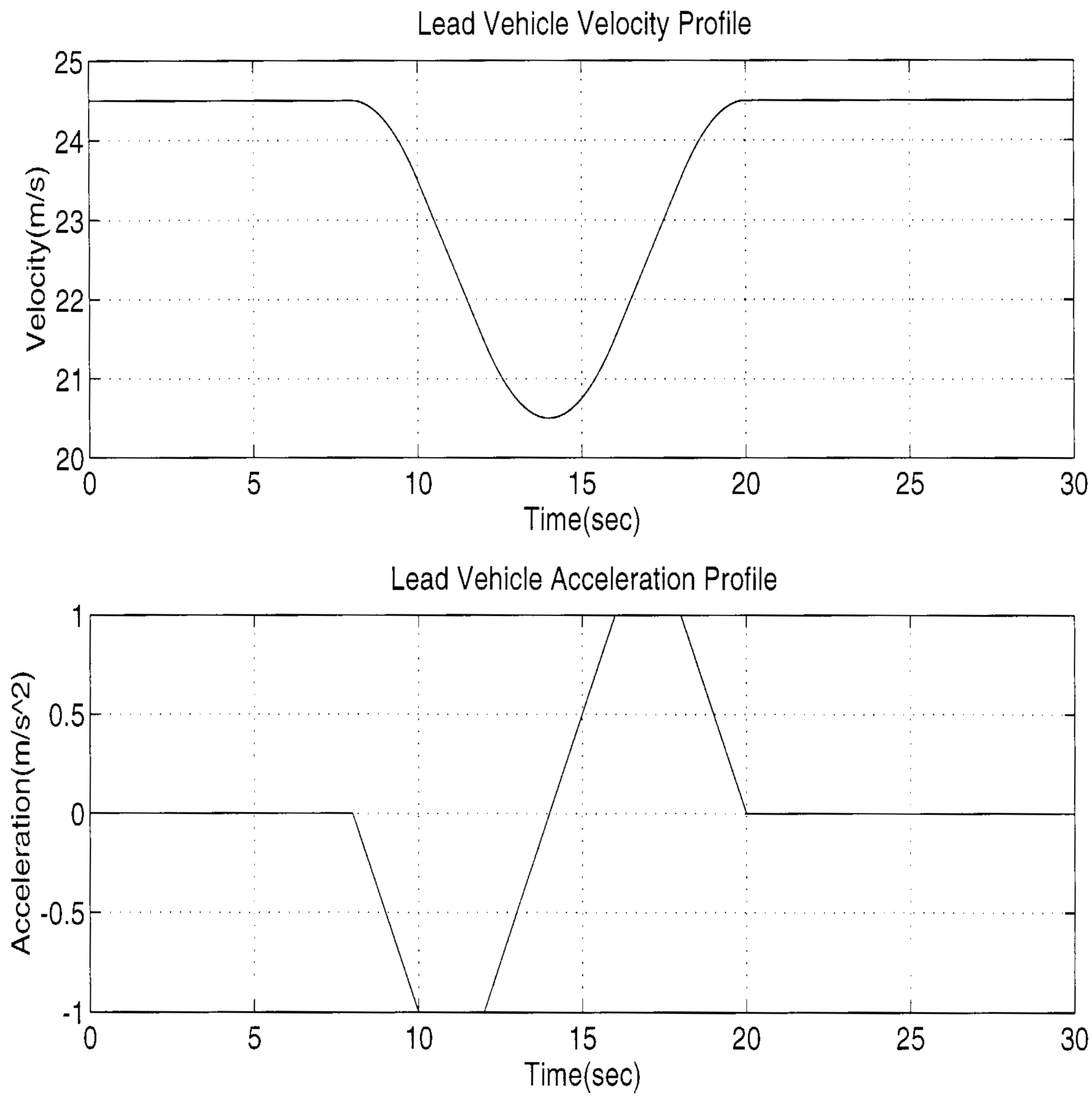


Figure **3.4:** Lead vehicle velocity and acceleration profiles for simulations/experiments

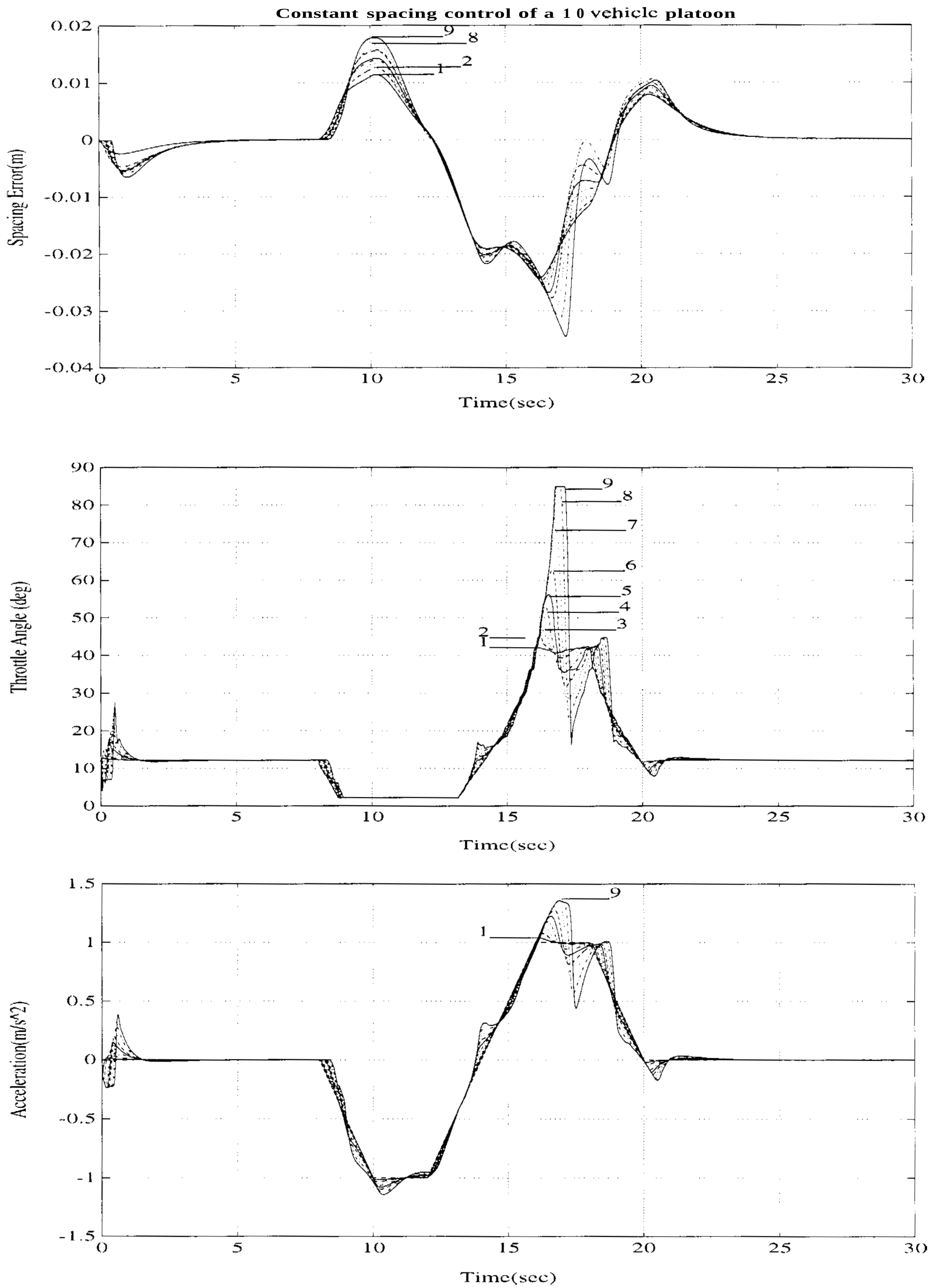


Figure *3.3:* Constant spacing semi-autonomous control of a 10 vehicle platoon.

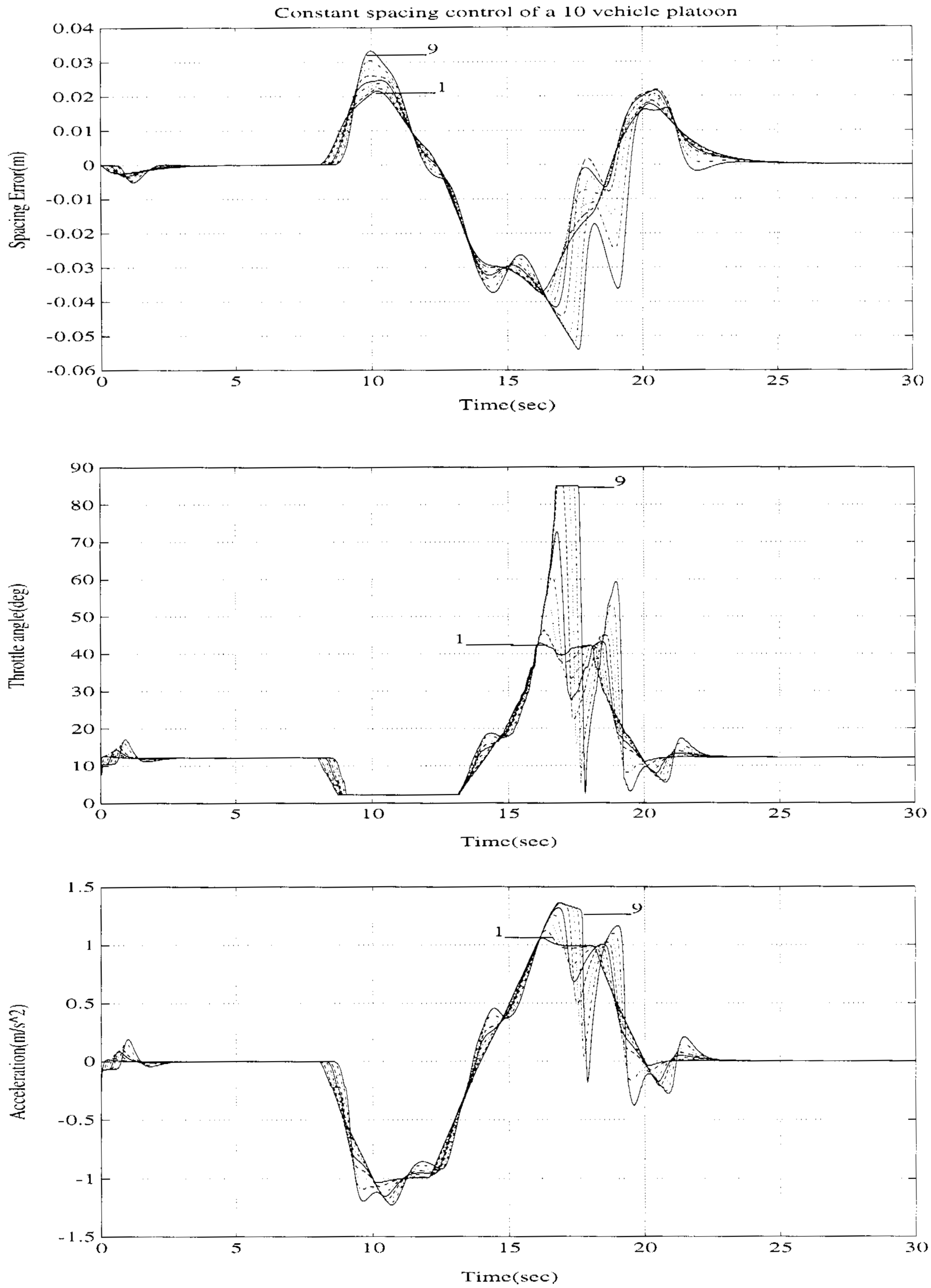


Figure 3.6: Semi-autonomous control with signal processing lag of 50ms.

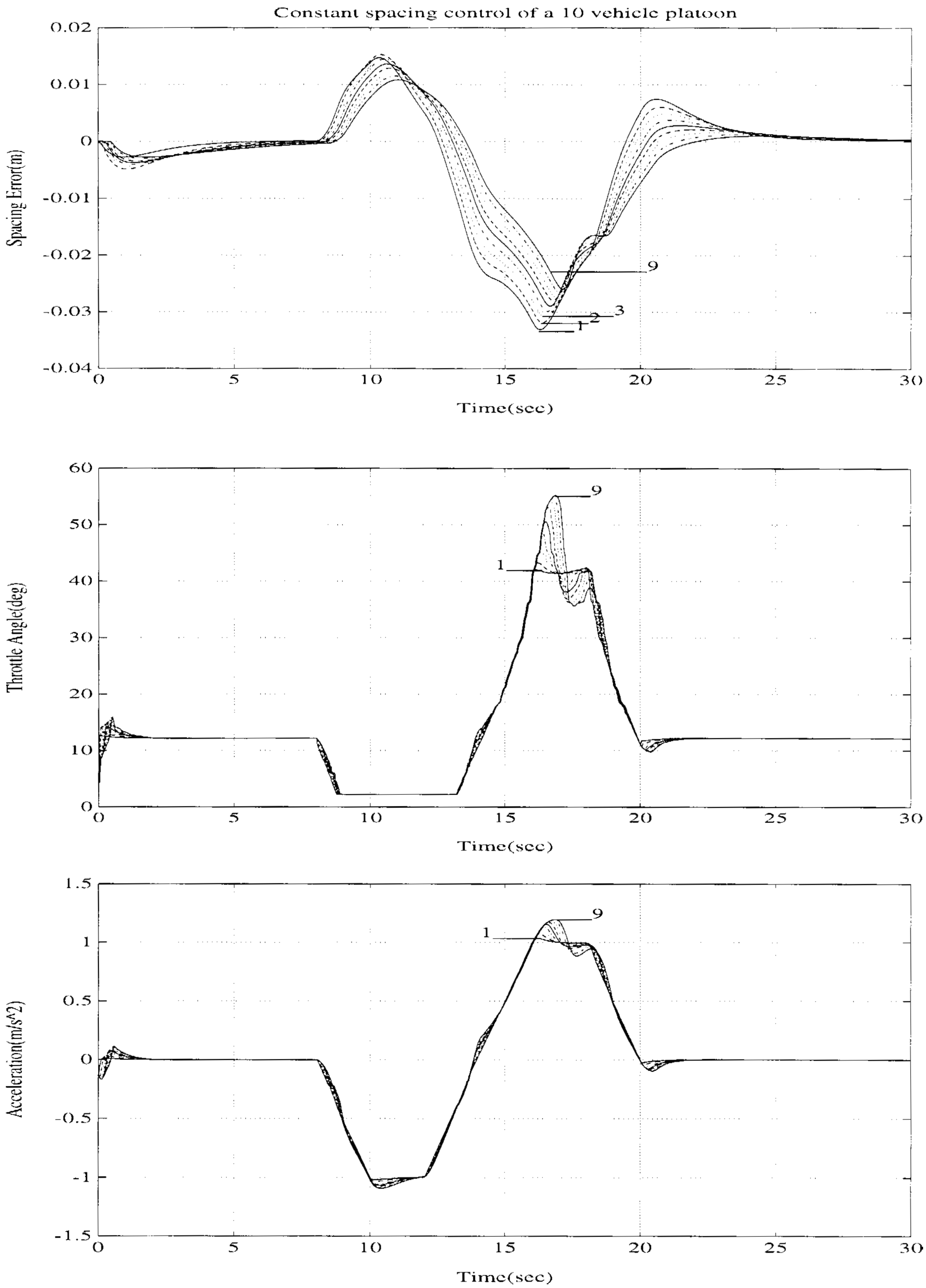


Figure 3.7: Constant spacing control of a 10 vehicle platoon with lead vehicle velocity and acceleration information.

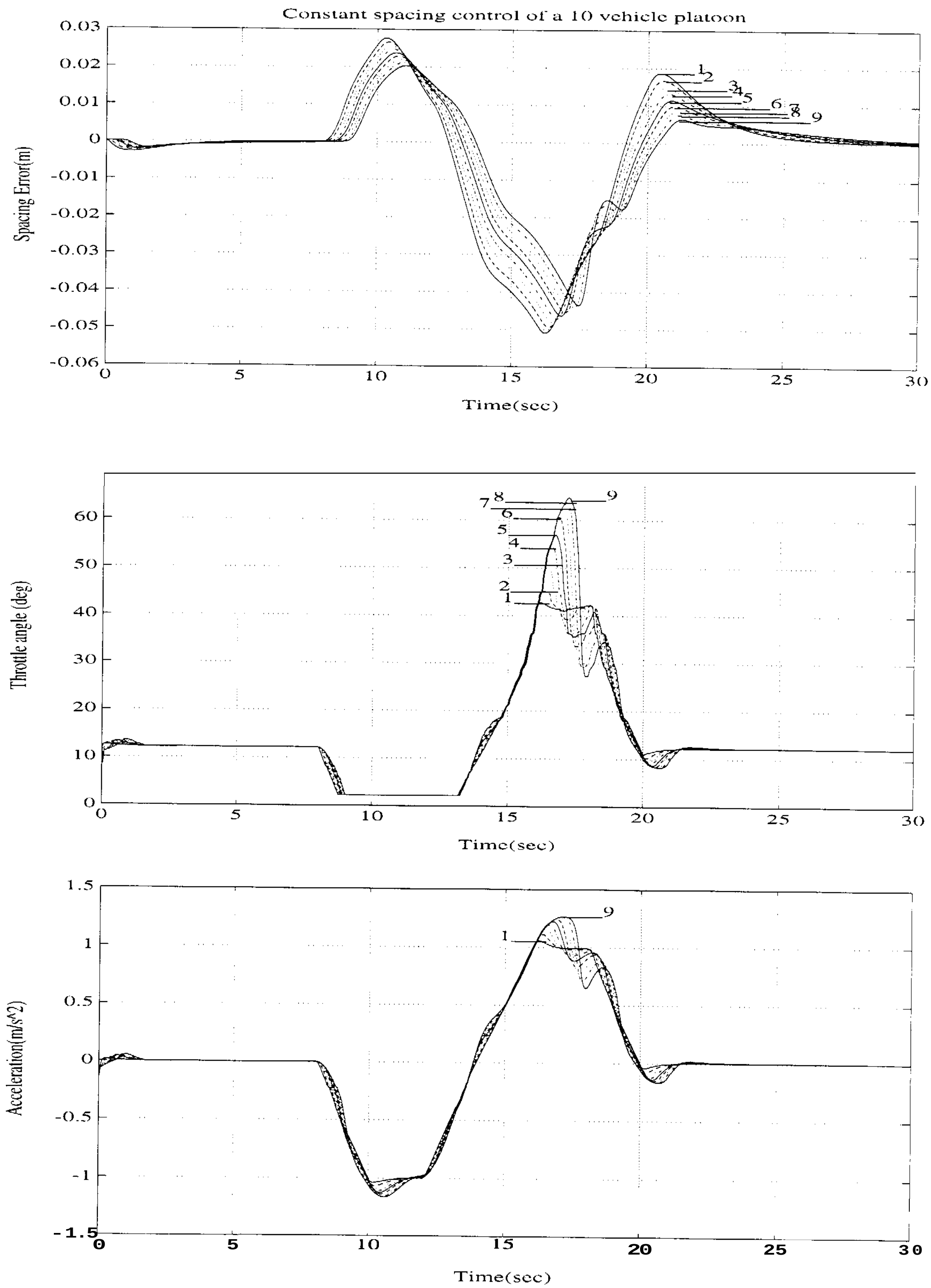


Figure 3.8: Constant spacing control of a 10 vehicle platoon with lead vehicle velocity and acceleration information and with a signal processing lag of 30 ms.

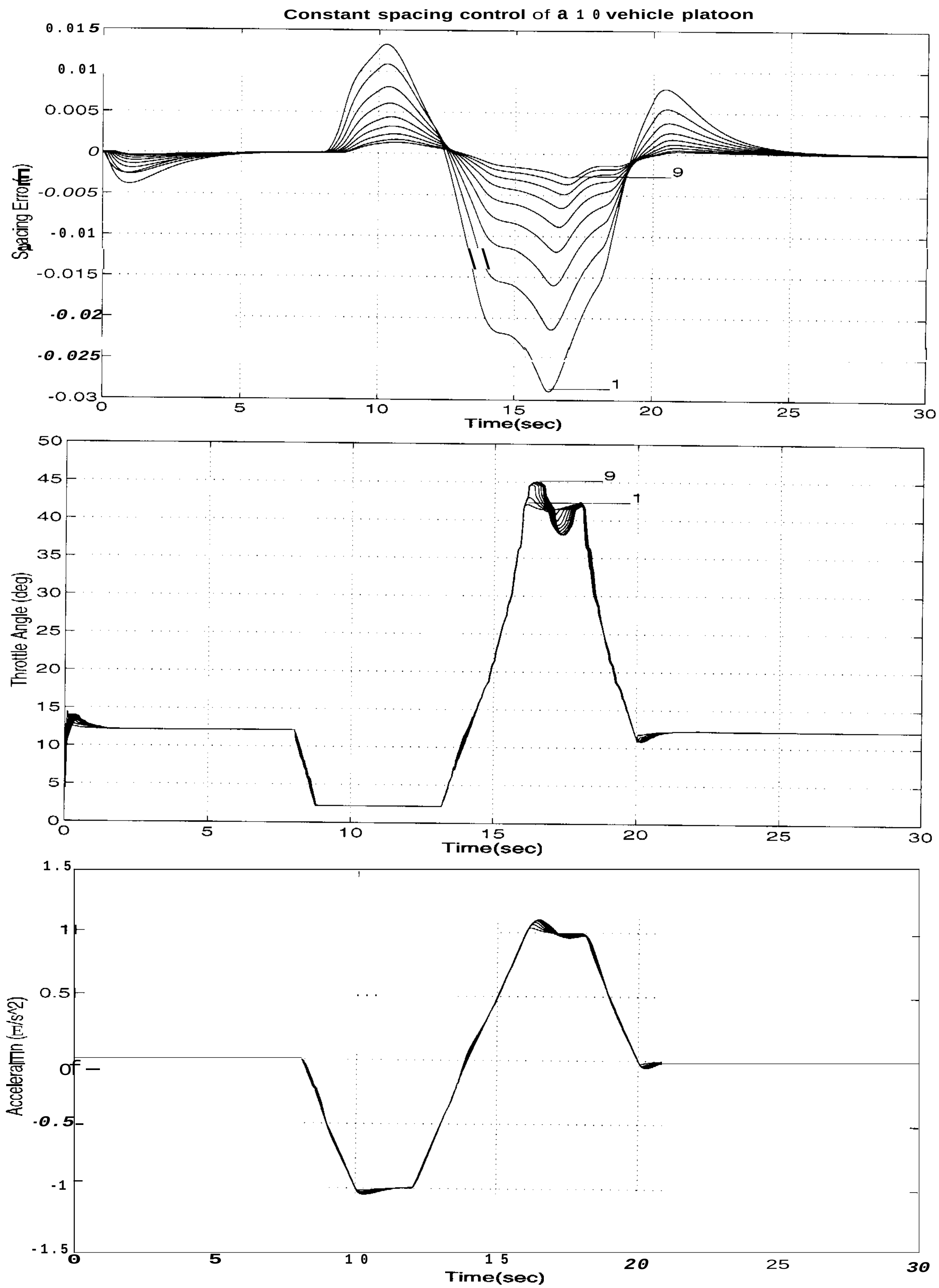


Figure 3.9: Constant spacing control of a 10 vehicle platoon with lead vehicle acceleration, velocity and position information.

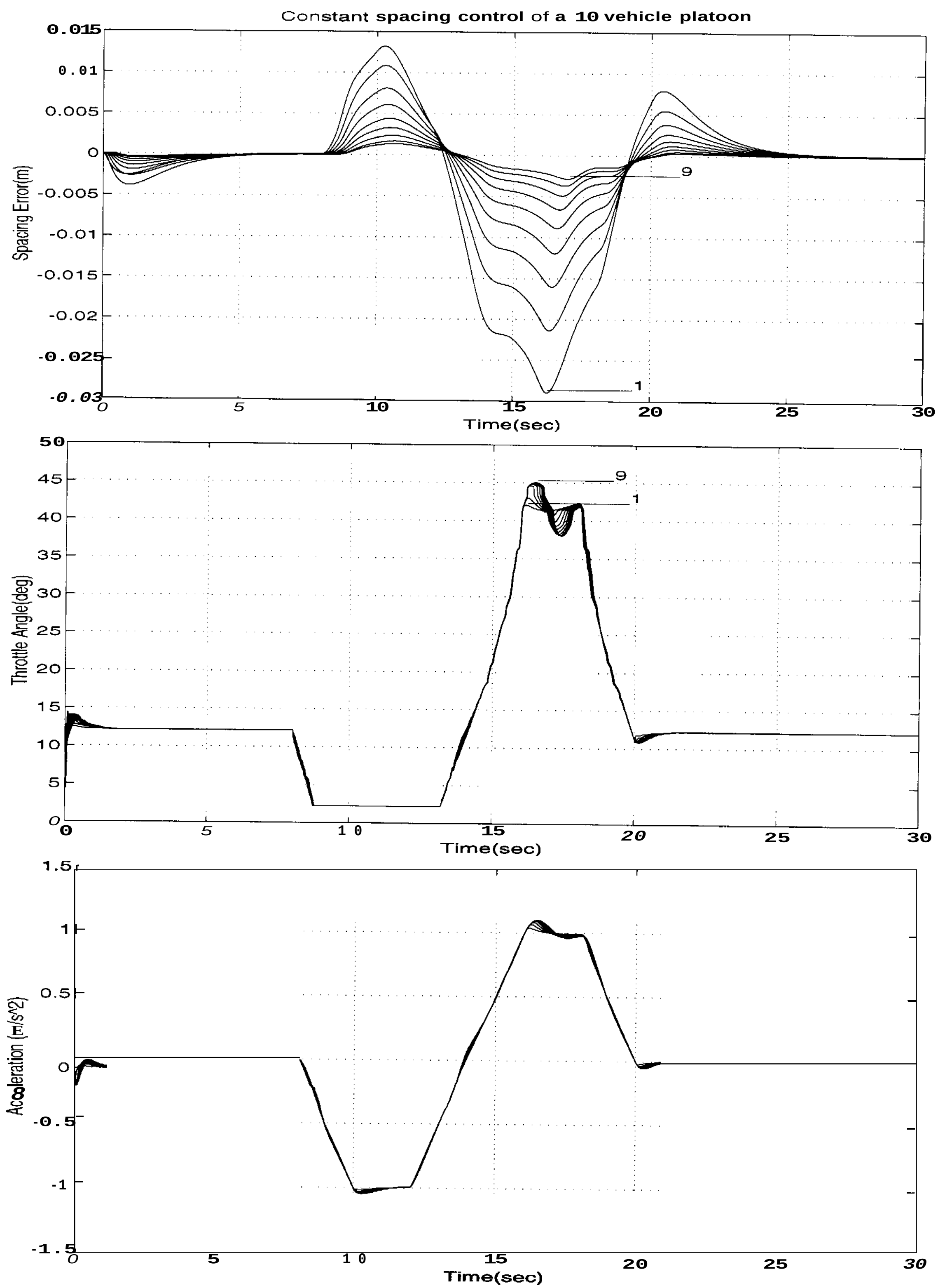


Figure 3.10: Constant spacing control of a 10 vehicle platoon with lead vehicle acceleration. velocity and position information and with a signal processing lag of 50ms.

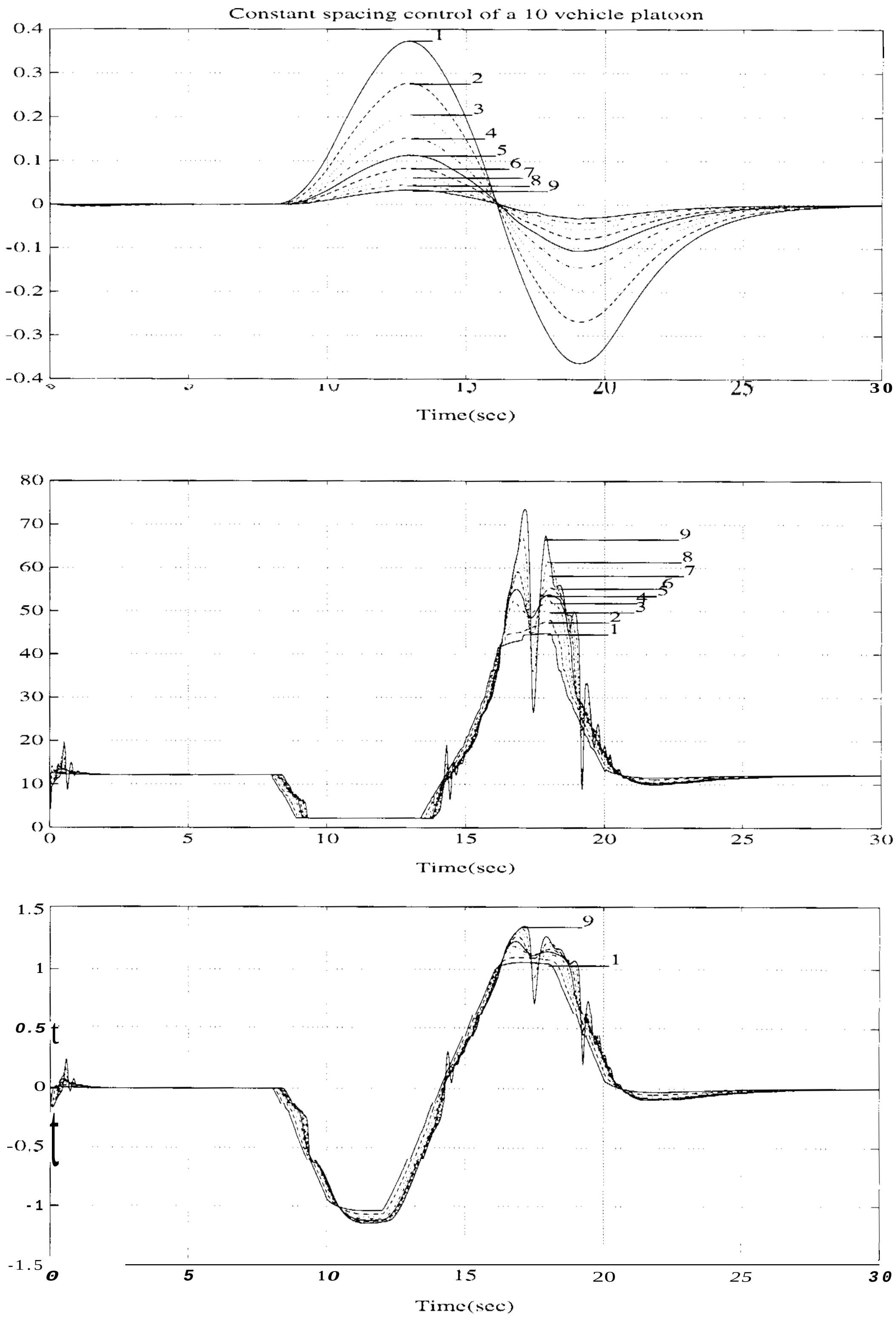


Figure 3.11: Constant spacing control of a 10 vehicle platoon with knowledge of vehicle ID in the platoon and preceding vehicle acceleration

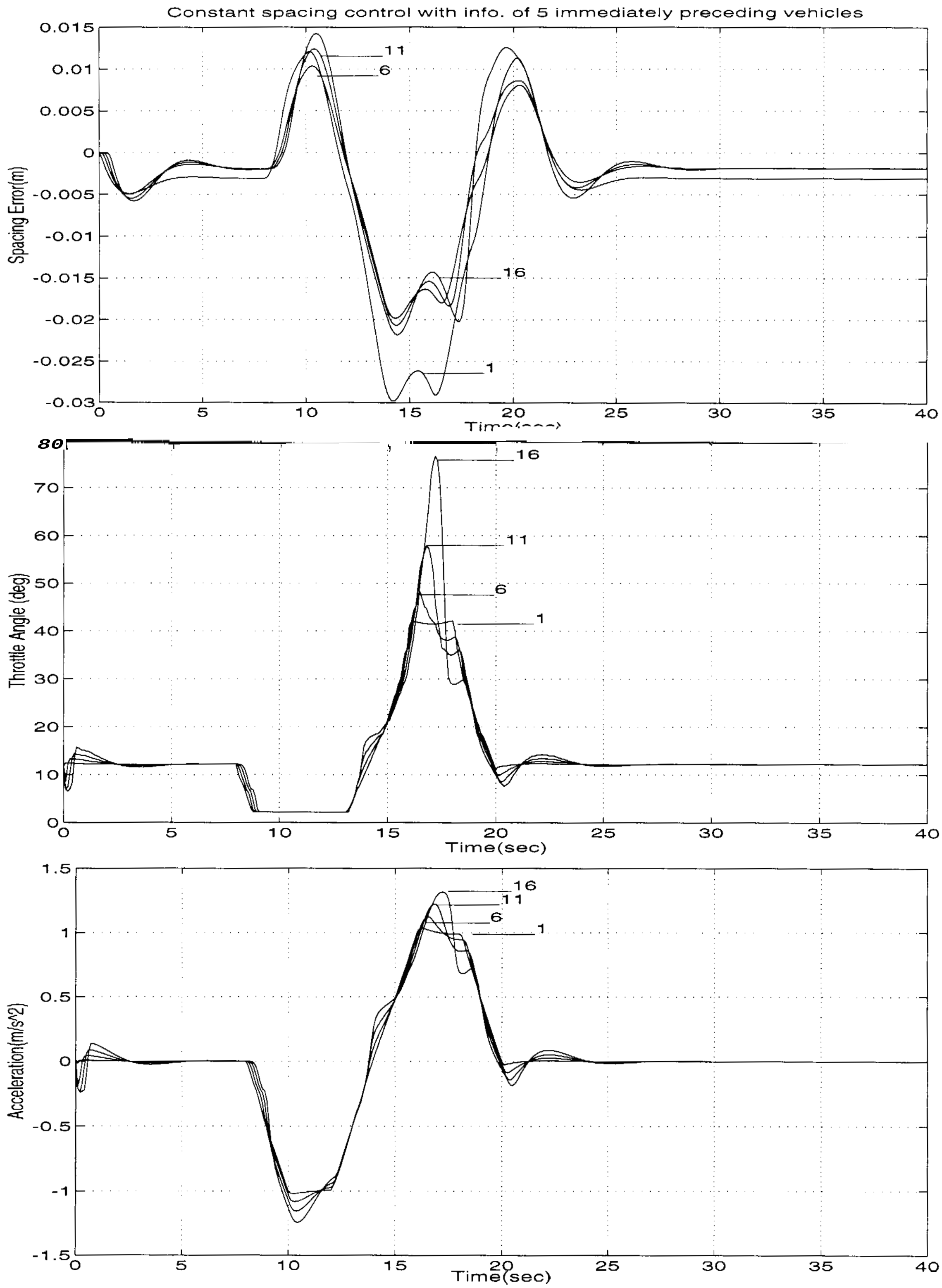


Figure 3.12: Constant spacing control with information of 3 vehicles ahead

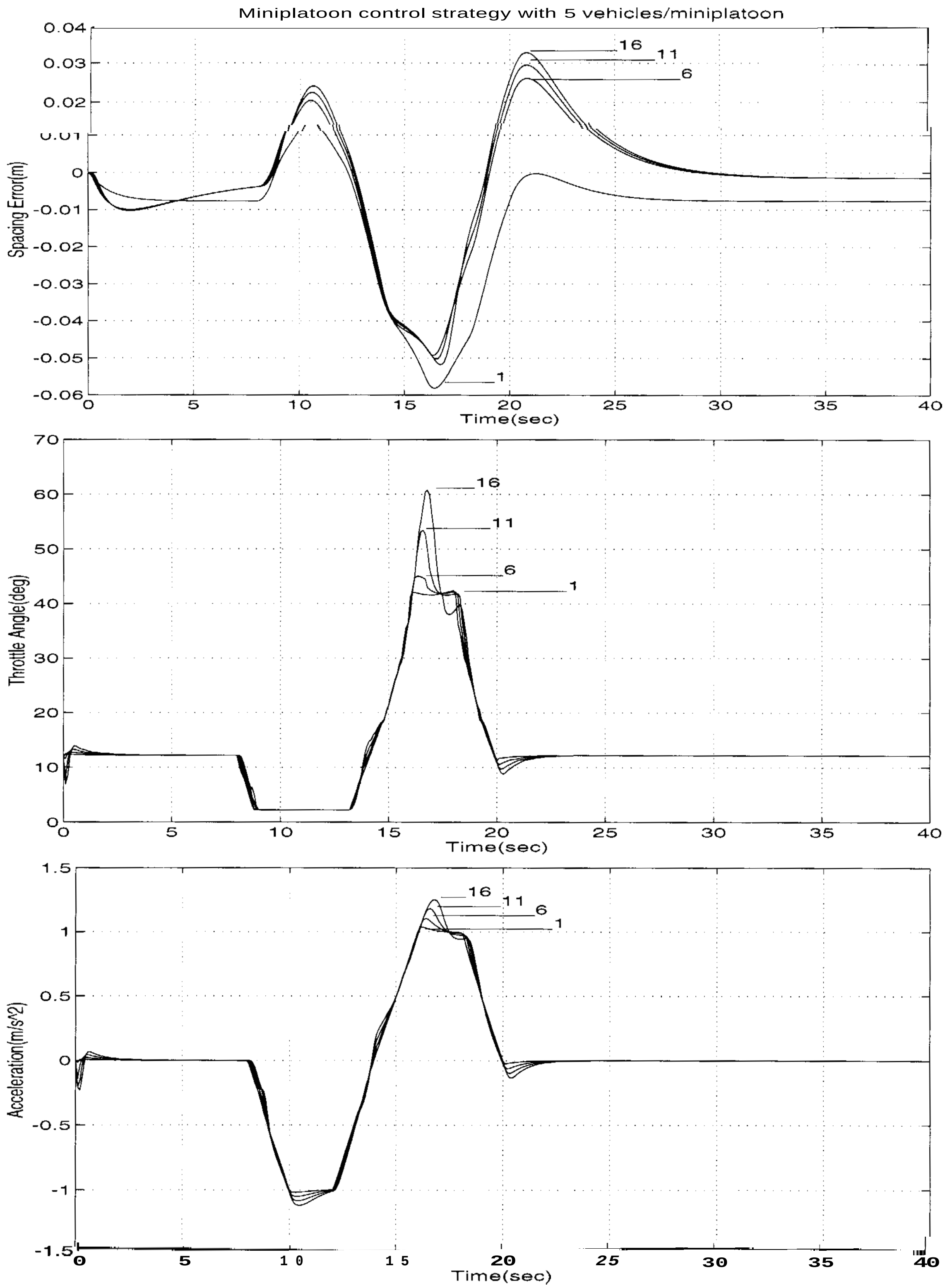


Figure 3.13: Miniplatoon control strategy

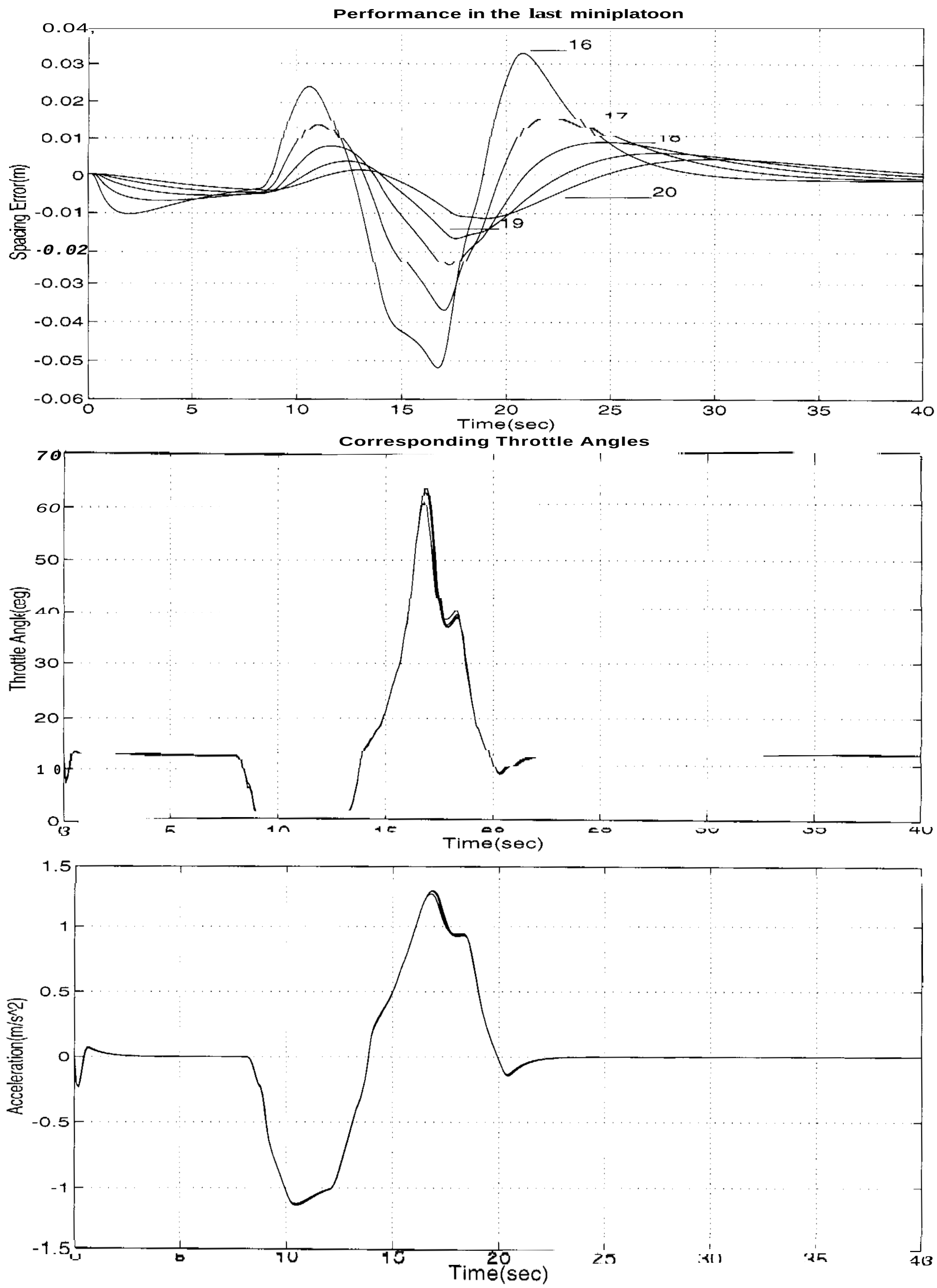


Figure **3.14:** Behavior of the vehicles in the last hliniplatoon

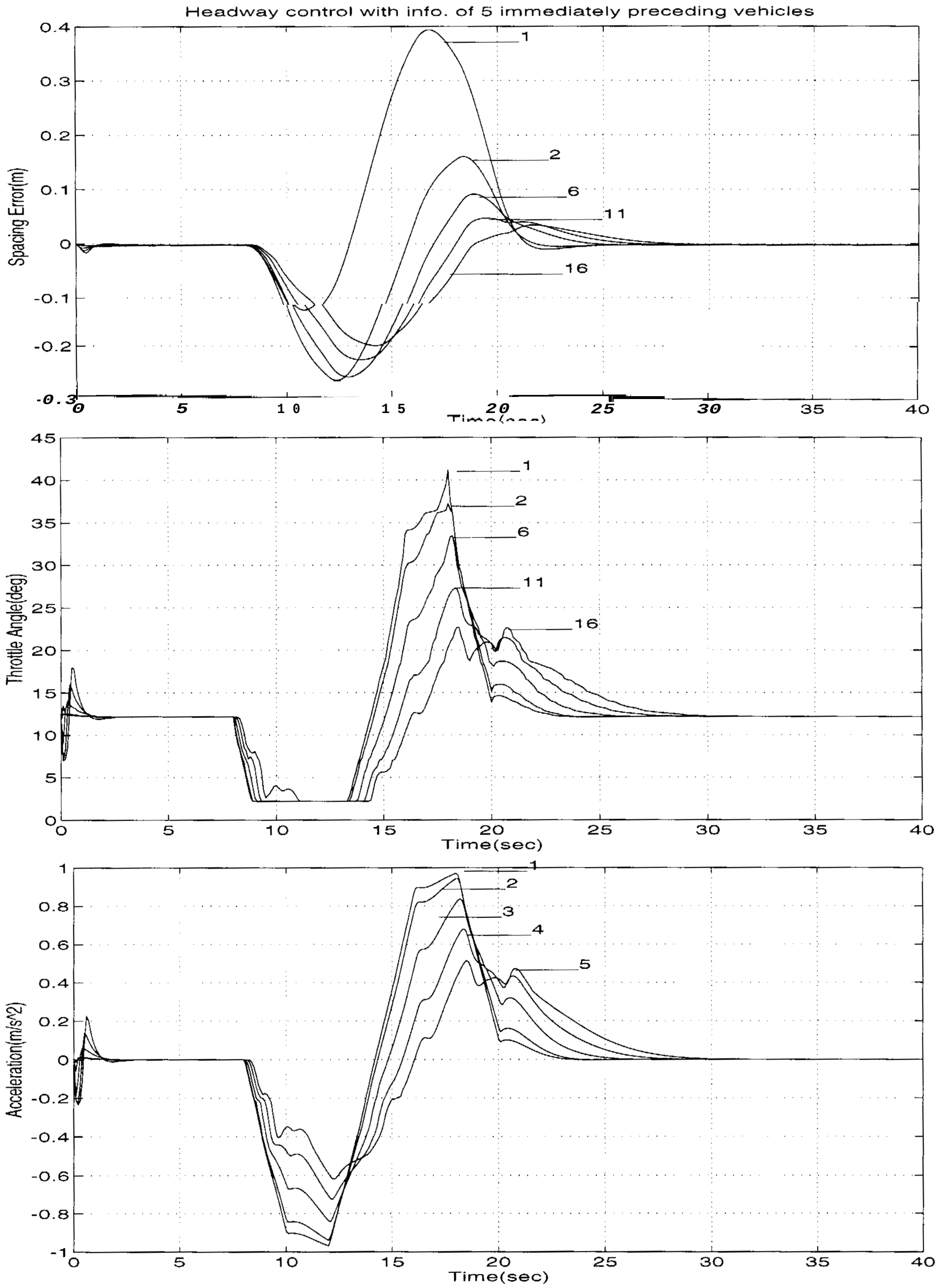
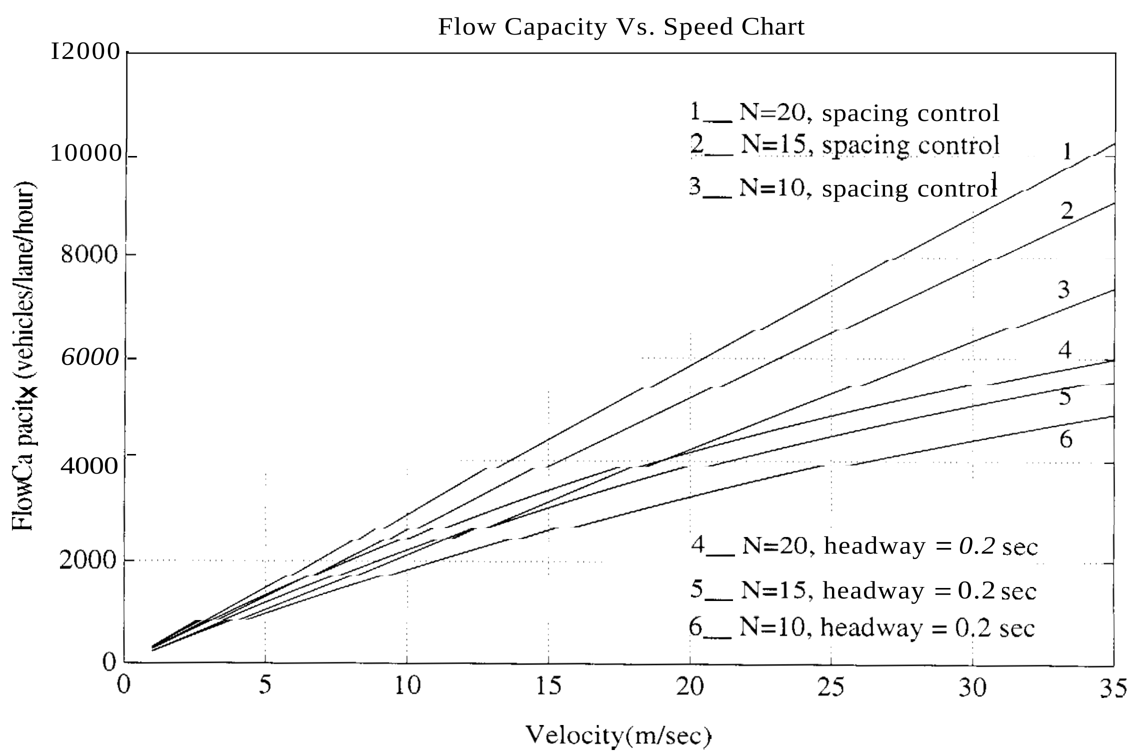


Figure 3.15: Headway control strategy with information of 3 vehicles ahead


Flow Capacity Vs. Speed Chart
FlowCa pacity (vehicles/lane/hour)
Velocity(m/sec)
1__ N=20, spacing control
2__ N=15, spacing control
3__ N=10, spacing control
4__ N=20, headway = 0.2 sec
5__ N=15, headway = 0.2 sec
6__ N=10, headway = 0.2 sec
12000
10000
8000
6000
4000
2000
0
0
5
10
15
20
25
30
35
1
2
3
4
5
6

Chapter 4

Adaptive Longitudinal Control of Vehicle Platoons

Platooning of vehicles with close intervehicular spacing requires a high performance longitudinal controller. Such controllers should not only ensure the stability of every individual vehicle in the platoon, but also the uniform boundedness of spacing errors (i.e that the spacing errors of all the vehicles in the platoon is bounded by a constant which relates to safety in the platoon), even in the presence of parametric uncertainties. Since safety is a prime concern: uniform boundedness of spacing errors should atleast be guaranteed. Sheikholeslam [37] presents an indirect decentralized adaptive control algorithm for a platoon of vehicles. In this chapter, we present, a direct decentralized adaptive control algorithm which satisfies the same performance objectives. The advantage of such a direct scheme is its ease in on-line implementation. This chapter is organized as follows :

In section 4.1. we investigate the effect of parametric uncertainty on the platoon performance. In section 4.2, we present the adaptive control algorithm. In section 4.3, we examine the platoon performance with the adaptive longitudinal controller and discuss the conditions for parameter convergence. In section 4.4. we discuss the simulation results.

4.1 Effect of parametric uncertainty on the platoon performance

From equation 3.2 . a model for control for the member vehicles in the platoon is given by:

$$\ddot{x}_i = \frac{u_i - c_i\dot{x}_i^2 - f_i}{M_i} \tag{4.1}$$

where x_i, u_i, c_i, f_i, M_i are the position, control effort, effective aerodynamic drag coefficient. rolling resistance friction, effective inertia of the i-th following vehicle respectively.

Using the control law described in section 3.3.4, we cannot, in general. guarantee string stability in the presence of parametric uncertainty. To ensure uniform boundedness of spacing errors is to guarantee a weaker version of string stability. Decentralized controllers should be designed to ensure string stability in the absence of parametric uncertainty, so that with parameter adaptation. uniform boundedness of spacing errors can, at least, be guaranteed. In this section, we investigate the effect of uncertainty on the performance of the platoon.

4.1.1 Effect of uncertainty in mass of the vehicle

We consider the control law described in section 3.3.4. This control law incorporates the lead and preceding vehicle information and guarantees string stability in the absence of parametric uncertainty. The sliding surface S_i in this control law is chosen such that $S_i = S_{i-1}$ yields string stable dynamics. Consider the sliding surface described in section 3.3.4 :

$$S_{1i} = \dot{\epsilon}_i + q_1\epsilon_i + q_3(v_i - v_l) + q_4(x_i - x_l + \sum_{j=1}^{i} L_j) \tag{4.2}$$

The control effort u_i is chosen to make $\dot{S}_{1i} + \lambda_1 S_{1i} = 0$ and is given by:

$$u_i = c_i\dot{x}_i^2 + f_i + M_i u_{isl} \tag{4.3}$$

$$u_{isl} = \frac{1}{1+q_3}[\ddot{x}_{i-1} + q_3\ddot{x}_l - q_1\dot{\epsilon}_i - q_4(v_i - v_l) - \lambda S_{1i}] \tag{4.4}$$

For individual vehicle stability, $q_1, q_3, q_4, \lambda > 0$. Henceforth, $\hat{M}_i, \hat{c}_i, \hat{f}_i$ represent the estimates of mass of the vehicle, effective aerodynamic drag coefficient, and the effective tire drag for the controlled vehicle respectively. Similarly, $\tilde{M}_i, \tilde{c}_i, \tilde{f}_i$ represent, the estimation errors (difference between estimated and true parameter value) of the mass of the vehicle, aerodynamic drag coefficient, tire drag respectively. In the presence of uncertainty in the mass of the vehicle,

$$u_i = c_i \dot{x}_i^2 + f_i + \hat{M}_i u_{isl} \tag{4.5}$$

so that

$$\ddot{x}_i = \frac{\hat{M}_i}{M_i} u_{isl} \tag{4.6}$$

The transfer function that relates the spacing errors is given by:

$$G(s) := \frac{\epsilon_1}{\hat{a}_l}(s) = \frac{\alpha - 1}{s^2 + \alpha[(\lambda + \frac{q_1+q_4}{1+q_3})s + \frac{\lambda(q_1+q_4)}{1+q_3}]}$$

$$\hat{H}(s) := \frac{\hat{\epsilon}_i}{\hat{\epsilon}_{i-1}}(s) = \frac{\alpha}{(1+q_3)} \frac{(s+q_1)(s+\lambda)}{(s^2 + \alpha[(\frac{q_1+q_4}{1+q_3} + \lambda)s + \frac{\lambda(q_1+q_4)}{1+q_3}])} \tag{4.7}$$

where $\alpha = \frac{\hat{M}_i}{M_i}$.

Proposition 1 : Given $\hat{H}(s)$, and that $q_1 > \frac{q_1+q_4}{1+q_3}$, $\lambda \neq q_1$, $\lambda \neq \frac{q_1+q_4}{1+q_3}$, there exist two constants $\beta_l < 1, \beta_h > 1$ such that $\forall\ \alpha \in [\beta_l, \beta_h]$, $||h||_1 = \frac{q_1}{q_1+q_4}$.

The intuition is that the impulse response, $h(t)$, is a continuous function of α. If $h(t) > 0$ for $\alpha = 1$, then $h(t) > 0$ in some neighborhood of $\alpha = 1$. Therefore, $||h||_1$ does not change for small perturbations in α around unity. The poles of $\hat{H}(s)$ should be simple so that $h(t) > 0\ \forall\ t \geq 0$. We can also guarantee that $\epsilon_i \to 0$ asymptotically whenever the lead vehicle reaches a steady velocity after a maneuver in finite time. Proof of this proposition is given in [44].

Proposition 2 : Robustness to mass variation

If $q_1 > \frac{q_1+q_4}{1+q_3}$, $\lambda \neq q_1$, $\lambda \neq \frac{q_1+q_4}{1+q_3}$, then the decentralized control given by equation 4.3 ensures uniform boundedness of spacing errors of member vehicles, for all bounded acceleration maneuvers of the lead vehicle in the platoon.

Proof Let

$$\Delta(s) = s^2 + \alpha[(\frac{q_1 + q_4}{\quad} + \qquad + \frac{\lambda(q_1 + q_4)}{1 + q_3}]$$

Let β_1, β_2 be the minimum and maximum absolute values of the zeros of $\Delta(s)\ \forall\ \alpha \in [\beta_l, \beta_h]$. Let

$$C_1 = \epsilon_i(0) - \frac{\alpha}{1 + q_3}\epsilon_{i-1}(0)$$

$$C_2' = \dot{\epsilon}_i(0) - \frac{\alpha}{1 + q_3}\dot{\epsilon}_{i-1}(0) + \alpha((\frac{q_1 + q_4}{1 + q_3} + \lambda)\epsilon_i(0) - \frac{q_1 + \lambda}{1 + q_3}\epsilon_{i-1}(0))$$

$$\hat{\epsilon}_i(s) = \hat{H}(s)\hat{\epsilon}_{i-1}(s) + \frac{sC_1 + C_2'}{\Delta(s)}$$

Define

$$C_2 = C_2' - \beta_2 C_1$$

$$\hat{\epsilon}_i(s) = \hat{H}(s)\hat{\epsilon}_{i-1}(s) + \frac{(s + \beta_2)C_1(\epsilon_i(0), \dot{\epsilon}_i(0)) + C_2(\epsilon_i(0), \dot{\epsilon}_i(0))}{\Delta(s)}$$

It now follows that.

$$||\epsilon_i||_\infty \leq \frac{q_1}{q_1 + q_4}||\epsilon_{i-1}||_\infty + \frac{|C_1|}{\beta_1} + \frac{(1 + q_3)|C_2|}{\alpha\lambda(q_1 + q_4)}$$

Since C_1, C_2 are linear functions of $\epsilon_i(0)$ and $\dot{\epsilon}_i(0)$, uniform houndedness of spacing errors can be guaranteed.

The error in the first following vehicle is governed by $\hat{G}(s)$. Because of parameter mismatch. maximum error in the first vehicle is dependent, on the magnitude and frequency content of the lead vehicle maneuver and string stability cannot' be assured.

β_l, β_h indicate the degree of robustness in string stability to variations in mass. If $q_1 = 3, q_3 = 1, q_4 = 1, \lambda = 4$, $\beta_l < 0.9, \beta_h \geq 1.166$. With this choice of control gains. we have robustness in string stability to a 10% variation in mass. The proof of uniform boundedness relies on the fact that $q_4 \neq 0$. i.e, the availability of lead vehicle's relative position to every controlled vehicle.

4.1.2 Effect of uncertainty in rolling resistance and mass of the vehicle

With uncertainty in mass of the vehicle and rolling resistance moment. the control effort u_i is given by:

$$u_i = \hat{M}_i u_{isl} + c_i \dot{x}_i^2 + \hat{f}_i \tag{4.8}$$

Hence, for $i \geq 2$,

$$\hat{\epsilon}_i(s) = \hat{H}(s)\hat{\epsilon}_{i-1}(s) + \frac{\hat{\phi}_{1i}(s)}{\Delta(s)} \tag{4.9}$$

$$\Delta(s) = s^2 + \alpha[(\frac{q_1 + q_4}{1 + q_3} + \lambda)s + \frac{\lambda(q_1 + q_4)}{1 + q_3}] \tag{4.10}$$

$$\phi_{1i}(t) = \frac{\tilde{f}_i}{M_i} - \frac{\tilde{f}_{i-1}}{M_{i-1}} \tag{4.11}$$

For $i = 1$.

$$\ddot{\epsilon}_1 + \alpha[(\frac{q_1 + q_4}{1 + q_3} + \lambda)\dot{\epsilon}_1 + \frac{\lambda(q_1 + q_4)}{1 + q_3}\epsilon_1] = (\alpha - 1)\ddot{x}_l + \frac{\tilde{f}_1}{M_1} \tag{4.12}$$

Define

$$||\frac{\tilde{f}}{M}(0)||_\infty \quad := \quad \sup_i \quad |\frac{\tilde{f}_i}{M_i}(0)| \tag{4.13}$$

Proposition **3** : Robustness to mass and tire drag variation

If

1. the conditions in Proposition 1 hold
2. $||\frac{\tilde{f}}{M}(0)||_\infty, ||\epsilon(0)||_\infty, ||\dot{\epsilon}(0)||_\infty$ exist

then $\sup_i \; ||\epsilon_i||_\infty$ is bounded (i.e the spacing errors are uniformly bounded in time and vehicle index),

Proof From equations 4.9 - 4.12. it follows that

$$||\epsilon_1||_\infty \leq \frac{1 + q_3}{\lambda(q_1 + q_4)}[(\alpha - 1)||\ddot{x}_l||_\infty + ||\frac{\tilde{f}}{M}||_\infty + (\beta_1||\epsilon(0)||_\infty + ||\dot{\epsilon}(0)||_\infty)] + \frac{\beta_1}{\beta_2}||\epsilon(0)||_\infty$$

where β_1, β_2 are the minimum and maximum absolute values of the roots of $\Delta(s) \quad \forall \ \alpha \in [\beta_l, \beta_h]$. For all $i \geq 2$,

$$||\epsilon_i||_\infty \leq \frac{q_1}{q_1 + q_4}||\epsilon_{i-1}||_\infty + \frac{2(1+q_3)}{\lambda(q_1+q_4)}||\frac{\tilde{f}}{M} \tag{4.14}$$

where C is the constant associated with the initial conditions. Clearly, $\sup_i \ ||\epsilon_i||_\infty$ is bounded.

Although the spacing errors are uniformly bounded in the presence of uncertainty in the rolling resistance moment, steady state spacing errors do exist. One may to avoid the problem of steady state errors is to incorporate integral action in the definition of sliding surface given by equation 4.2.

If there is any mismatch in the aerodynamic drag coefficient. individual vehicle stability cannot be guaranteed and hence. uniform boundedness of spacing errors or string stability cannot be assured. Parameter adaptation is required to improve the robustness of the control algorithm.

4.2 Direct Adaptive Control Algorithm

We assume that the lead vehicle makes a bounded velocity. acceleration and jerk maneuver. In the presence of parametric uncertainty, the control effort is given by:

$$u_i = \hat{c}_i \dot{x}_i^2 + \hat{f}_i + \hat{M}_i u_{isl} \tag{4.15}$$

Hence,

$$\dot{S}_{1i} + \lambda S_{1i} = \frac{1+q_3}{M_i}[\tilde{M}_i u_{isl} + \tilde{c}_i \dot{x}_i^2 + \tilde{f}_i] \tag{4.16}$$

Define a Lyapunov function candidate

$$V_i \ = \ \frac{M_i}{1+q_3}\frac{S_{1i}^2}{2} + \gamma_1\frac{\tilde{M}_i^2}{2} + \gamma_2\frac{\tilde{c}_i^2}{2} + \gamma_3\frac{\tilde{f}_i^2}{2} \tag{4.17}$$

Choose the adaptation laws as follows

$$\dot{\hat{M}}_i = -\frac{1}{\gamma_1} S_{1i} u_{isl} \quad (4.18)$$

$$\dot{\hat{c}}_i = -\frac{1}{\gamma_2} S_{1i} \dot{x}_i^2 \quad (4.19)$$

$$\dot{\hat{f}}_i = -\frac{1}{\gamma_3} S_{1i} \quad (4.20)$$

4.3 Analysis for uniform boundedness of spacing errors and parameter convergence

With the choice of adaptation laws and control in the previous section, we obtain

$$\dot{V}_i = -\frac{\lambda M_i}{1+q_3} S_{1i}^2 \quad \leq 0 \quad (4.21)$$

$$\ddot{x}_i = \frac{\hat{M}_i u_{isl} + \tilde{c}_i \dot{x}_i^2 + \tilde{f}_i}{M_i} \quad (4.22)$$

From equations 4.17 and 4.21, it follows that $S_{1i} \in L_\infty \cap L_2$; $\tilde{M}_i, \tilde{c}_i, \tilde{f}_i \in L_\infty$.

4.3.1 Uniform boundedness of spacing errors

Proposition 4 : Effectiveness of Parameter Adaptation
For all bounded

1. $||\tilde{M}(0)||_\infty, ||\tilde{c}(0)||_\infty, ||\tilde{f}(0)||_\infty, ||\epsilon(0)||_\infty, ||\dot{\epsilon}(0)||_\infty, ||\dot{w}(0)||_\infty, ||w(0)||_\infty$ and

2. $\frac{d}{dt}\ddot{x}_l(t)$.

the control law given by equation 4.15 together with the adaptation laws given by equations 4.18 - 4.20 guarantee that

1. $\sup_i \; ||\tilde{M}_i||_\infty, \sup_i \; ||\tilde{c}_i||_\infty, \sup_i \; ||\tilde{f}_i||_\infty$ are bounded. In other words, all the parameter estimation errors are bounded uniformly in time and vehicle index.

2. $\sup_i \ ||\epsilon_i||_\infty, \sup_i \ ||\dot{\epsilon}_i||_\infty, \sup_i \ ||w_i||_\infty, \sup_i \ ||\dot{w}_i||_\infty$ are bounded. This implies that all the spacing and velocity errors are uniformly bounded in time and vehicle index.

3. ϵ_i, $\dot{\epsilon}_i \to 0$ asymptotically

4. If, in addition, $\ddot{x}_l(t) \in L^\infty$, then $\ddot{\epsilon}_i \to 0$ asymptotically.

Proof

1. By hypothesis, $||V(0)||_\infty$ exists. Since $V_i(t)$ is decreasing, $\sup_i \ ||V_i||_\infty \leq ||V(0)||_\infty$ Therefore, $\sup_i \ ||S_{1i}||_\infty, \sup_i \ ||\tilde{M}_i||_\infty, \sup_i \ ||\tilde{c}_i||_\infty, \sup_i \ ||\tilde{f}_i||_\infty$ are bounded. Let $\sup_i \ ||S_{1i}||_\infty \leq K_s$, where K_s is some positive constant.

2.

$$(1+q_3)\dot{\epsilon}_1 + (q_1+q_4)\epsilon_1 = S_{11} \tag{4.23}$$

$$(1+q_3)\dot{\epsilon}_i + (q_1+q_4)\epsilon_i = \dot{\epsilon}_{i-1} + q_1\epsilon_{i-1} + S_{1i} - S_{1i-1} \tag{4.24}$$

It follows that

$$||\epsilon_1||_\infty \leq \frac{K_s + (1+q_3)||\epsilon(0)||_\infty}{q_1+q_4} \tag{4.25}$$

$$||\dot{\epsilon}_1||_\infty \leq \frac{2}{1+q_3}[K_s + (1+q_3)||\epsilon(0)||_\infty] \tag{4.26}$$

$$||\epsilon_i||_\infty \leq \frac{q_1}{q_1+q_4}||\epsilon_{i-1}||_\infty + \frac{2K_s + (2+q_3)||\epsilon(0)||_\infty}{q_1+q_4} \tag{4.27}$$

Hence, $\sup_i \ ||\epsilon_i||_\infty \leq K_z$ where K_z is a positive constant. The proof relies on $q_4 \neq 0$, i.e., that relative position of the lead vehicle is available to every controlled vehicle.

Observe that $\sup_i \ ||S_{1i}||_2 < \infty$. From Fact 2 of the previous chapter, we know that if the impulse response, $h(t)$, of a transfer funciton, $H(s)$, is always positive.

then all the input/output inducecl norms of the transfer function are equal. As a consequence.

$$||\epsilon_i||_2 \leq \frac{q_1}{q_1+q_4}||\epsilon_{i-1}||_2 + \frac{||S_{1i}-S_{1i-1}||_2}{q_1+q_4} + K_2$$

where K_2 bounds the 2-norm of exponentially decaying response due to initial conditions. Since S_{11} E L_2, it follows. from equation 4.23. that' ϵ_1 E L_2. Since $\sup_i \; ||S_{1i}||_2 < \infty$. it implies that $\sup_i \; ||\epsilon_i||_2 < \infty$.

Rewriting equation 4.24,

$$\dot{\epsilon}_i = \frac{1}{1+q_3}\dot{\epsilon}_{i-1} + \frac{q_1\epsilon_{i-1} - (q_1+q_4)\epsilon_i + S_{1i} - S_{1i-1}}{1+q_3} \tag{4.28}$$

$$\Rightarrow ||\dot{\epsilon}_i||_\infty \leq \frac{1}{1+q_3}||\dot{\epsilon}_{i-1}||_\infty + \frac{(2q_1+q_4)K_z + 2K_s}{1+q_3} \tag{4.29}$$

Clearly, $\sup_i \; ||\dot{\epsilon}_i||_\infty$ is bounded. By a similar argument, $\sup_i \; ||\dot{\epsilon}_i||_2 < \infty$

Since $\sup_i \; \{||\epsilon_i||_\infty, ||\epsilon_i||_2, ||\dot{\epsilon}_i||_\infty < \infty\}$, by Barbalat's Lemma, $\epsilon_i(t) \longrightarrow 0$.

3. If $||\tilde{M}_i||_\infty, ||\tilde{c}_i||_\infty, ||\tilde{f}_i||_\infty, ||S_i(0)||_\infty$ are sufficiently small so that for some $q < 1$, $\frac{\hat{M}_i}{M_i(1+q_3)} \leq q$, then $u_{isl}, \ddot{x}_i$ are uniformly bounded in time and vehicle index.

$$S_{1i} = \dot{\epsilon}_i + q_1\epsilon_i + q_3 w_i + q_4\dot{w}_i \tag{4.30}$$

where

$$\dot{w}_i(t) = \dot{x}_i - \dot{x}_l \tag{4.31}$$

$$w_i(t) = x_i - x_l + \sum_1^i L_j \tag{4.32}$$

$$S_{1i} - S_{11} = (1+q_3)\dot{w}_i + (q_1+q_4)w_i - \dot{w}_{i-1} - q_1\dot{w}_{i-1} \tag{4.33}$$

By an argument seen earlier for ϵ_i, we conclude that $\sup_i\{||w_i||_\infty, ||\dot{w}_i||_\infty, ||w_i||_2\} < \infty$ and that, by Barbalat's Lemma, $w_i \longrightarrow 0$ for all i.

$$u_{isl} = \frac{1}{1+q_3}[\frac{\hat{M}_i}{M_i}u_{(i-1)sl} + \frac{\tilde{c}_i\dot{x}_i^2 + \tilde{f}_i}{M_i} - q_1\dot{\epsilon}_i - q_4(v_i - v_l) - \lambda S_{1i}] \tag{4.34}$$

By hypothesis. $\frac{\hat{M}_i}{M_i(1+q_3)} < q < 1$. Since all the other terms on the Right Hand Side are uniformly hounded, it follows that u_{isl} is bounded uniformly in time and vehicle index. From equation 4.22. $\ddot{x}_i$ is also bounded uniformly in time and vehicle index. It also follows that, $\ddot{\epsilon}_i$ is uniformly bounded in time and vehicle index.

From equation 4.16. it follows that $\dot{S}_{1i} \in L_\infty$. Hence, by Barbalat's Lemma. $S_{1i} \rightarrow 0$ asymptotically. Since $\sup_i \{||\dot{\epsilon}_i||_\infty, ||\dot{\epsilon}_i||_2, ||\ddot{\epsilon}_i||_\infty\} < \infty$ .it follows, from Barbalat's Lemma. that $\dot{\epsilon}_i(t) \rightarrow 0$.

4. $\dot{S}_{1i}$ is uniformly continuous if $\frac{d}{dt}\ddot{x}_l, \ddot{x}_l, \dot{x}_l$ are bounded and continuous

From equations 4.4, 4.22. we have

$$\dot{u}_{isl} = \frac{1}{1+q_3}[\frac{d}{dt}\ddot{x}_{i-1} + q_3\frac{d}{dt}\ddot{x}_l - q_1\ddot{\epsilon}_i - q_4(\ddot{x}_i - \ddot{x}_l) - \lambda\dot{S}_i] \tag{4.35}$$

$$\frac{d}{dt}\ddot{x}_i = \frac{\dot{\hat{M}}_i u_{isl} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + \hat{M}_i\dot{u}_{isl} + 2\tilde{c}_i\dot{x}_i\ddot{x}_i}{M_i} \tag{4.36}$$

Substituting equation 4.35 into equation 4.36

$$\frac{d}{dt}\ddot{x}_i = \frac{\hat{M}_i}{(1+q_3)M_i}\frac{d}{dt}\ddot{x}_{i-1} + \frac{\hat{M}_i(q_3\frac{d}{dt}\ddot{x}_l - q_1\ddot{\epsilon}_i - q_4(\ddot{x}_i - \ddot{x}_l) - \lambda\dot{S}_i)}{(1+q_3)M_i} +$$

$$\frac{\dot{\hat{M}}_i u_{isl} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + 2\tilde{c}_i\dot{x}_i\ddot{x}_i}{M_i}$$

Since $\frac{\hat{M}_i}{(1+q_3)M_i} < q < 1$. and all the terms on the Right Hand Side of the above equation are uniformly bounded. it follows that $\ddot{x}_i$, and hence. $\dot{u}_{isl}$ are uniformly hounded. Since

$$\ddot{S}_{1i} = -\lambda\dot{S}_{1i} + \frac{1+q_3}{M_i}[\dot{\hat{M}}_i u_{isl} + \dot{\hat{c}}_i\dot{x}_i^2 + \dot{\hat{f}}_i + \tilde{M}_i\dot{u}_{isl} + 2\tilde{c}_i\dot{x}_i\ddot{x}_i] \tag{4.37}$$

it follows that $\ddot{S}_{1i}$ is uniformly bounded. Since S_{1i} and the Right Hand Side of equation 4.16 is continuous. it follows that $\dot{S}_{1i}$ is continuous, and hence. $\dot{S}_{1i}$ is uniformly continuous.

Since $\lim_{t\to\infty} \ |\int_0^t \dot{S}_{1i}(\tau)d\tau| \ = |S_{1i}(0)| \ \leq ||S_1||_\infty$ and $\dot{S}_{1i}$ is uniformly continuous, it follows, by Barbalat's Lemma, that $\dot{S}_{1i} \to 0$ asymptotically. Therefore, by equation 4.2, $\ddot{\epsilon}_i \to 0$ asymptotically.

4.3.2 Parametric Convergence

Since $\dot{S}_{1i} + \lambda S_{1i} \to 0$ asymptotically, $\tilde{M}_i u_{isl} + \tilde{c}_i \dot{x}_i^2 + \tilde{f}_i \to 0$ asymptotically. For parametric convergence, Persistence of Excitation condition has to be satisfied. Let $W_i = [u_{isl} \quad \dot{x}_i^2 \quad 1]^T$. Then, there should exist three positive constants. $\delta, \gamma_1, \gamma_2$ such that

$$\gamma_2 I \quad \geq \quad \int_t^{t+\delta} W_i W_i^T d\tau \quad \geq \quad \gamma_1 I \quad \forall \ t \geq 0 \tag{1.38}$$

Since $\epsilon_i, \dot{\epsilon}_i, \ddot{\epsilon}_i \to 0$ asymptotically, $u_{isl} \approx \ddot{x}_l, \dot{x}_i^2 \approx \dot{x}_l^2$. If $\dot{x}_l = A_0 + A_1 \sin(wt)$ where $A_0 > A_1 > 0$, choosing $\delta = \frac{12\pi}{w}$,

$$\int_t^{t+\delta} W_i W_i^T d\tau \approx \begin{pmatrix} 6\pi A_0^2 w & 0 & 0 \\ 0 & (A_0^2 + \frac{A_1^2}{2})^2 + \frac{A_1^4}{8} + 2A_0^2 A_1^2)\frac{12\pi}{w} & (A_0^2 + \frac{A_1^2}{2})\frac{12\pi}{w} \\ 0 & (A_0^2 + \frac{A_1^2}{2})\frac{12\pi}{w} & \frac{12\pi}{w} \end{pmatrix}$$

The above matrix is positive definite $\forall \ A_1 > 0$. Hence, parameter convergence is ensured.

4.4 Simulation Results

Simulations are performed for a 3-vehicle platoon. The plant, in all the simulations considers the effects of slip between the tire and the ground, slip across the torque converter, manifold air dynamics. and the lag in the brake torque. all of which are neglected in developing the controller. In all the simulations. all the vehicles in the platoon start' with zero initial position and velocity errors. The lead vehicle in the platoon makes the following acceleration maneuver:

$$a_l(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ -1.2\sin(\frac{2\pi(t-5)}{10}) & 5 \leq t \leq 5 + 10N \\ 0 & t \geq 3 + 10N \end{cases}$$

where N is a positive integer. The following gains are used for simulating the 5-vehicle platoon: $q_1 = 1$. $q_3 = 1$, $q_4 = 0.5$, $X = 1.0$, $\gamma_1 = 28.0$. $\gamma_2 = 0.008$ ancl $\gamma_3 = 30.0$. In all the figures that follow. the number "i" on the figure represents the plot for the i-th following vehicle. Figure 4.1 illustrates the effect of uncertainty in mass of the vehicle on the platoon performance. α. shown in the figure. denotes the ratio of the estimated mass used in the controller and the true mass. As expected, the peak spacing errors decrease geometrically at a ratio of $\frac{2}{3}$ with vehicle index. The spacing errors go to zero asymptotically. Figure 4.2 depicts uniform boundedness of spacing errors in the presence of uncertainty in the mass and rolling resistance. The controller's mass estimate is 20 % less than the actual estimate and the rolling resistance estimate is 0.0 Nm. We have non-zero steady state spacing errors in this case. Since all the vehicles are identical (including the estimates), $\phi_{1i}(t) \equiv 0 \;\; \forall \;\; i \geq 2$ and hence. by equation 4.9 the steady state spacing errors decrease with vehicle index. Figure 4.3 describes how uncertainty in aerodynamic drag coefficient affects the performance of the platoon. For this simulation, the controller has no knowledge of the aerodynamic drag coefficient and the rolling resistance moment. i.e $\hat{c}_i = 0, f_i = 0$. As in the previous case, the estimate of mass is 20 % less than its true value. The maximum spacing errors and the steady state spacing errors are higher than before clue to the aclditional uncertaintv. Figure 4.4 demonstrates the effectiveness of the adaptive controller. It can he seen that the errors go to zero asymptotically and the maximum spacing errors are significantly smaller compared to the non-adaptive case. Also. the peak spacing errors decrease monotonically with vehicle index. Figures 4.4, 4.5 show how the parameters behave during adaptation. Parameters do not converge, but oscillate in the neighborhood of their true d u e s . This is clue to the fact that the controller model neglects four states associated with torque converter, manifold air dynamics, slip between the tire and the wheels and the lag in delivering the desired brake torque. [h]

The robustness of adaptive control algorithms and parameter convergence to singularly perturbed actuator dynamics is a difficult, problem to be resolved. In this dissertation. no attempt has been made to investigate this problem.

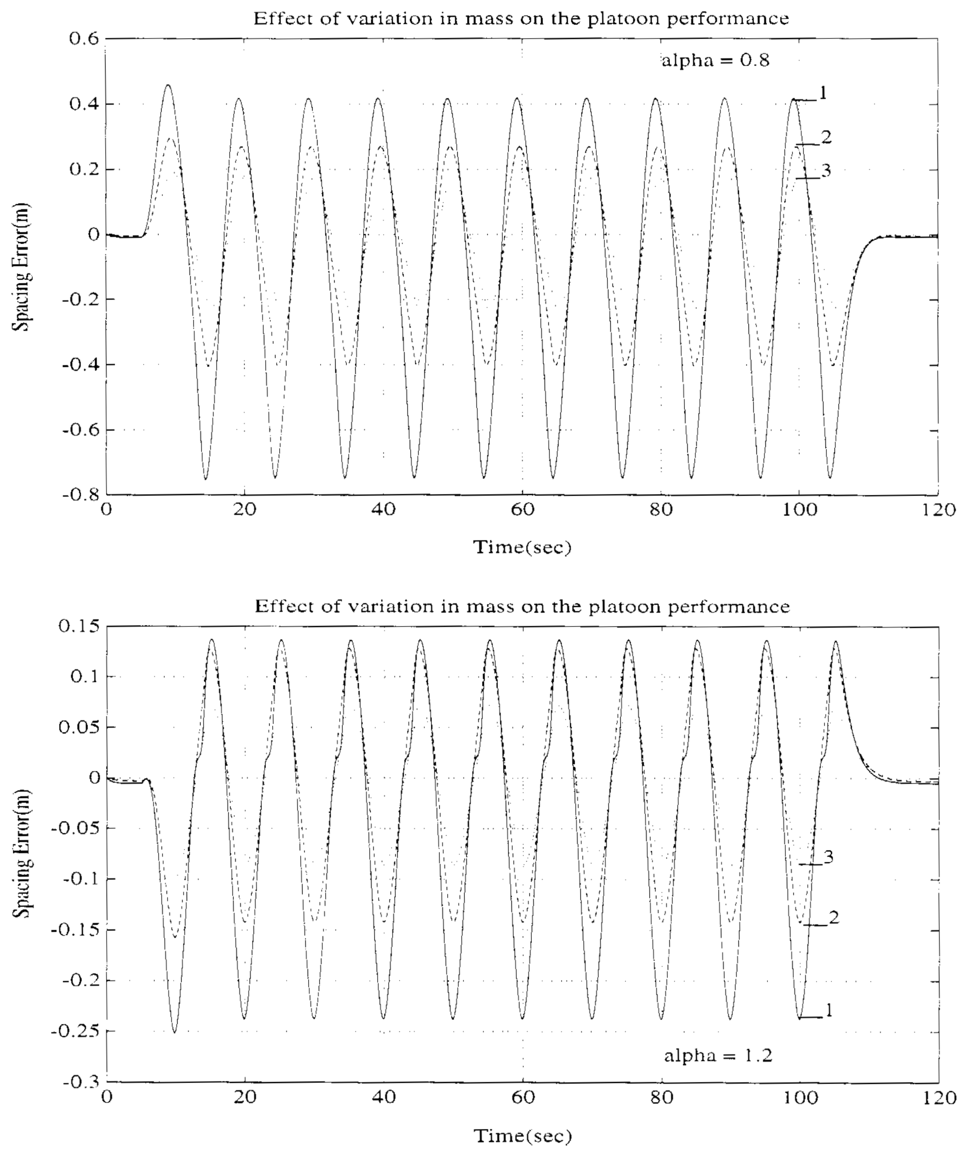


Figure 4.1: Effect' of uncertainty in mass on the platoon performance

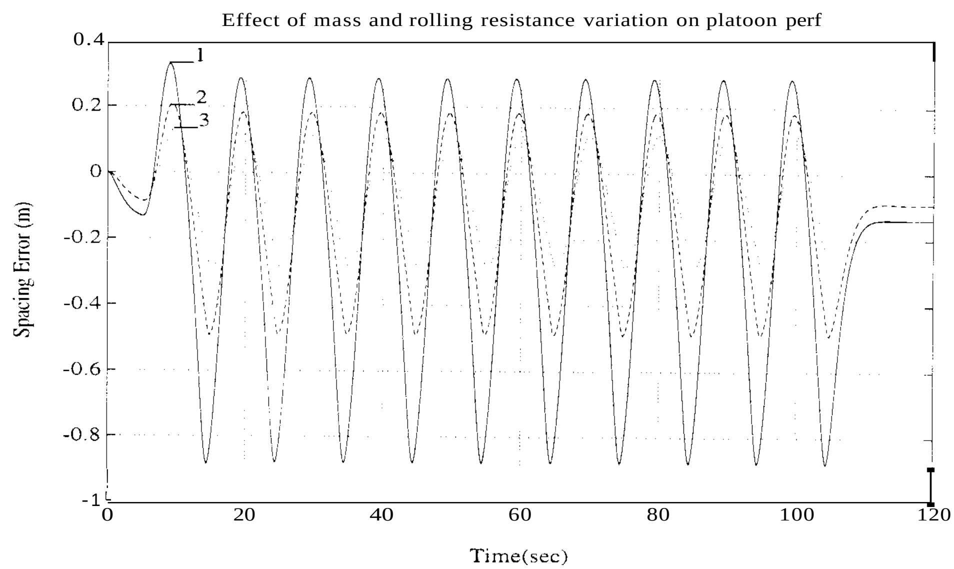


Figure 4.2: Effect of uncertainty in rolling resistance and mass on the platoon performance

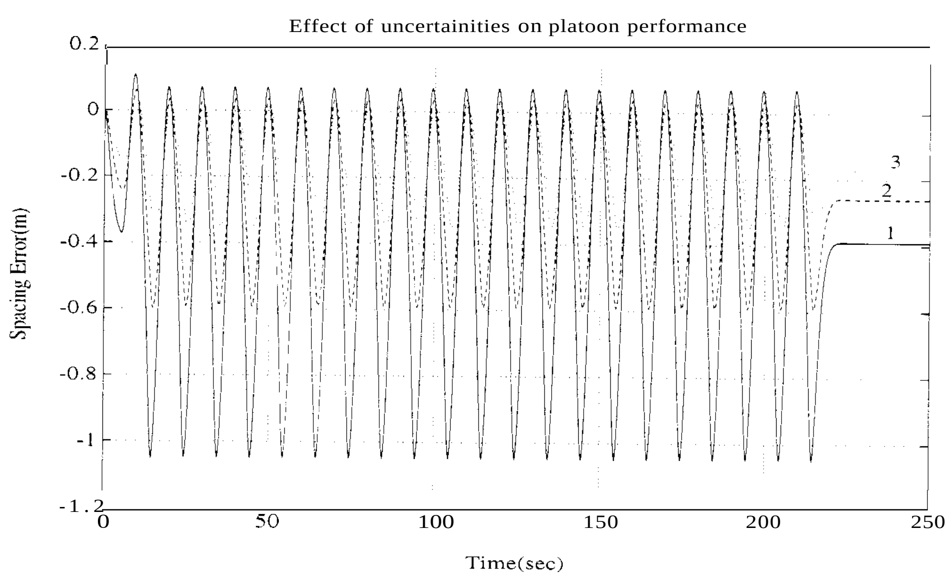


Figure **4.3:** Effect of uncertainty in parameters on the platoon performance

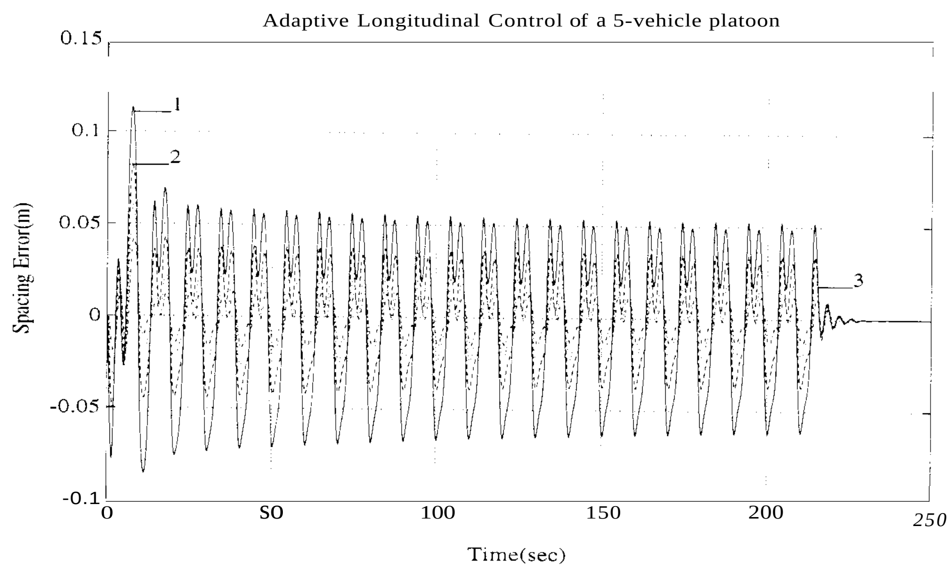
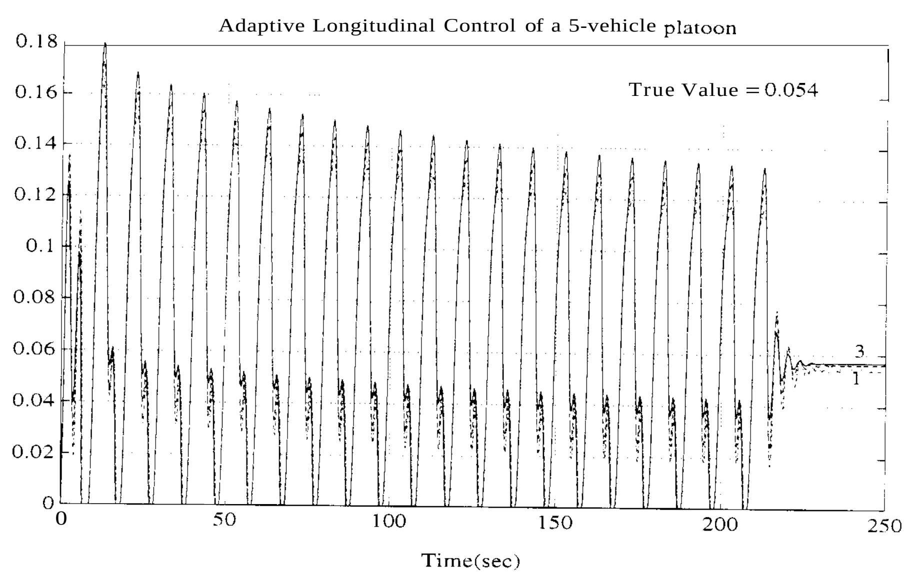


Figure **4.4:** Platoon performance with adaptation



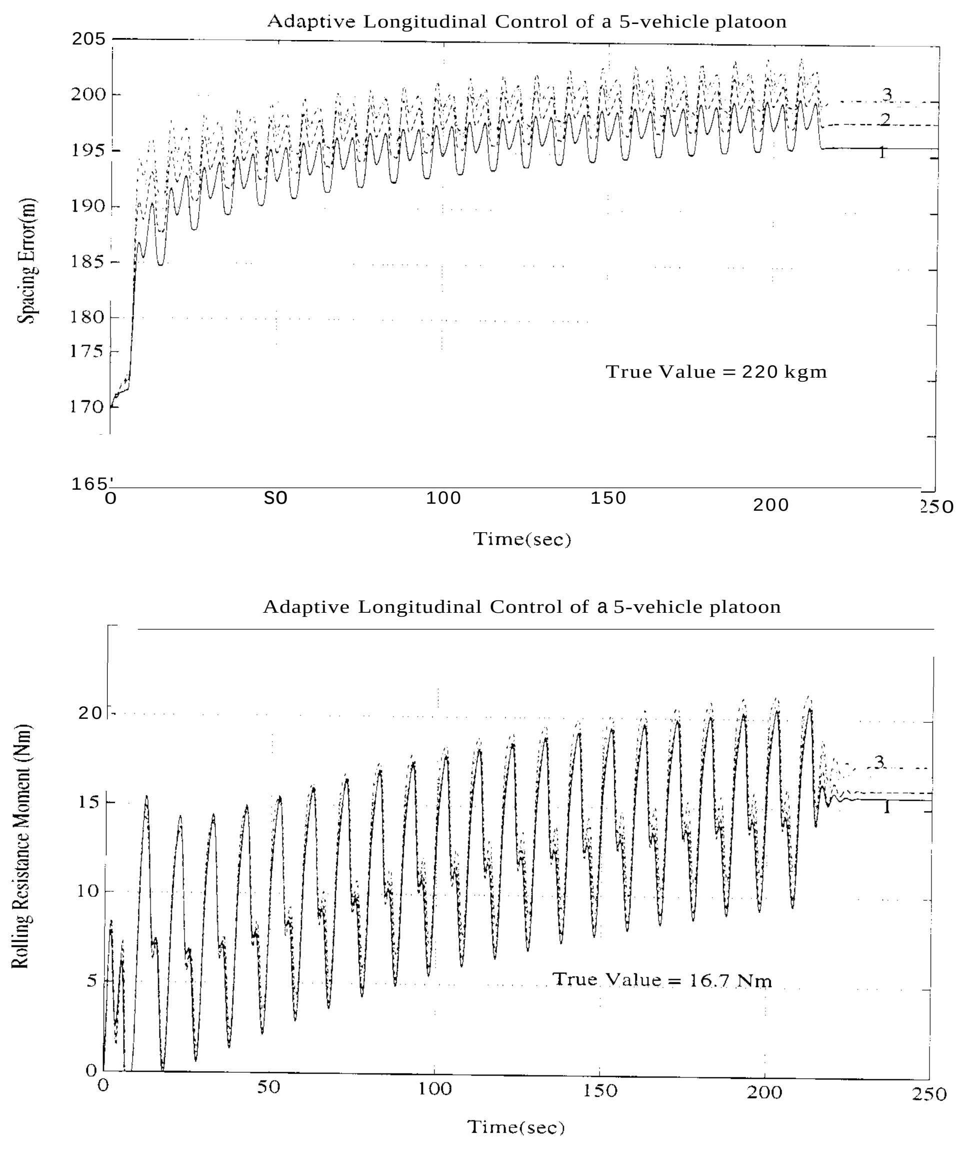


Figure 4.5: Behavior of parameters during adaptation

Chapter 5

String Stability of Interconnected systems

Earlier research on interconnected systems focussed on vehicle following applications [26], [22], [13], [35], [19], control of distributed systems, (e.g. regulation of seismic cables, vibration control in beams etc..) [10], [29], signal processing [5], power systems. Loosely speaking, string stability of an interconnected system implies uniform boundedness of the state of all the systems. For example. in automated vehicle following applications. tracking (spacing) errors should not amplify upstream from vehicle to vehicle for safety. Similarly. deflection at any point in a beam or a rod should remain bounded at all times. Spatial discretization and control of such distributed systems have relevance to the problem of string stability for interconnected systems. Although a precise definition of string stability was not coined. Kuo and Melzer [26], Levine and Athans [22] were seeking optimal control solutions to the automated vehicle following problem. Chu defined string stability in the contest of vehicle following [19]. In [5], Chang introduces a stronger version of stability for interconnected systems: namely, "γ -stability" for infinite interconnection of linear digital processors. Intuitively, "γ-stability" ensures that the state of all the systems decays to zero exponentially in time and system index. In this chapter. we generalize the concept of string stability to a class of interconnected systems and seek sufficient conditions to guarantee their string stability. We also examine their robustness to structural and singular perturbations.

This chapter is organized as follows. In section 5.1, we define string stability and asymptotic string stability, we present "small-gain" conditions that guarantee string stability for a class of interconnected systems and we demonstrate that exponential string stability is preserved under small structural perturbations. In section 3.2. we prove that every esponentially string stable interconnected system is string stable in the presence of small singular perturbations. In section 5.3. me discuss direct adaptive control of such interconnected systems.

5.1 String Stability

We use the following notations: $||f_i(\cdot)||_\infty$ or simply $||f_i||_\infty$ denotes $\sup_{t\geq 0} \ |f_i(t)|$ and $||f_i(0)||_\infty$ denotes $\sup_i \ |f_i(0)|$. For all $p < \infty$, $||f_i(\cdot)||_p$ or $||f_i||_p$ denotes $(\int_0^\infty |f_i(t)|^p dt)^{\frac{1}{p}}$ and $||f_i(0)||_p$ denotes $(\sum_1^\infty |f_i(0)|^p)^{\frac{1}{p}}$.

Consider the following interconnected system:

$$\dot{x}_i = f(x_i, x_{i-1}, \dots, x_{i-r+1}) \tag{5.1}$$

where $i \in \mathcal{N}$, $x_{i-j} \equiv 0 \ \forall \ i \leq j$, $x \in \mathcal{R}^n$, $f : \underbrace{\mathcal{R}^n \times \cdots \times \mathcal{R}^n}_{rtimes} \rightarrow \mathcal{R}^n$ and $f(0, \cdots, 0) = 0$.

Definition 1: The origin $x_i = 0$, i E $\mathcal{N}$ of the interconnected system 5.1 is string stable. if given any E > 0, there exists a $\delta > 0$ such that $||x_i(0)||_\infty < \delta \Rightarrow \sup_i \ ||x_i(\cdot)||_\infty < \epsilon$.

Definition 2: The origin $x_i = 0$, i E $\mathcal{N}$ of the interconnected system 3.1 is asymptotically (exponentially) string stable. if it is string stable and $x_i(t) \longrightarrow 0$ asymptotically (exponentially) for all i E $\mathcal{N}$. forall $||x_i(0)||_\infty$

X more general definition of string stability is :

Definition 3 : l_p string stability The origin $x_i = 0$, i E $\mathcal{N}$ of the interconnected system 5.1 is l_p string stable if for all $\epsilon > 0$. there exists a δ_i such that

$$||x_i(0)||_p < 1 \iff \sup_t ||x_i(t)||_p < \epsilon$$

where

$$||x_i(t)||_p = (\sum_1^\infty |x_i(t)|^p)^{\frac{1}{p}}$$

Definition 1 of string stability can be restated as l_∞ string stability of Definition 3. Henceforth, we will deal with string stability according to definition 1. The following theorem proves. under some 'weak coupling" conditions, that any countably infinite interconnection of exponentially stable nonlinear systems is string stable. Clearly, a string of uncoupled exponentially stable systems is asymptotically string stable. Intuitively. any interconnection of exponentially stable systems is string stable, if the interconnections are sufficiently weak.

Theorem 1 : Weak Coupling Theorem for String Stability
If the following conditions are satisfied:

- f is globally Lipschitz in its arguments. i.e

$$|f(y_1, \ldots, y_r) - f(z_1, \ldots, z_r)| \leq l_1|y_1 - z_1| + \cdots + l_r|y_r - z_r| \tag{5.2}$$

- the origin of $\dot{x} = f(x, 0, \ldots, 0)$ is globally exponentially stable.

Then. for sufficiently small $l_i, i = 2, \ldots, r$, the interconnected system is globally exponentially string stable.

Proof Since the origin of $\dot{x} = f(x, 0, \ldots, 0)$ is exponentially stable. by Converse Lyapunov Theorem. there exists a Lyapunov function $V(x)$ such that

$$\alpha_l||x||^2 \leq V(x) \leq \alpha_h||x||^2 \tag{3.3}$$

$$\frac{\partial V}{\partial x} f(x, 0, \ldots, 0) \leq -\alpha_1||x||^2 \tag{5.4}$$

$$||\frac{\partial V}{\partial x}|| \leq \alpha_3||x|| \tag{5.5}$$

For the sake of convenience. we denote $V(x_i)$ by V_i. Then.

$$\dot{V}_i = \frac{\partial V_i}{\partial x_i} f(x_i, x_{i-1}, \ldots, x_{i-r+1})$$

$$= \frac{\partial V_i}{\partial x_i} f(x_i, 0, \ldots, 0) + \frac{\partial V_i}{\partial x_i}[f(x_i, x_{i-1}, \ldots, x_{i-r+1}) - f(x_i, 0, \ldots, 0)]$$

$$\leq -\alpha_1 ||x_i||^2 + \alpha_3 ||x_i|| (\sum_{j=2}^{r} l_j ||x_{i-j+1}||)$$

Using the inequality that $xy \leq \frac{x^2+y^2}{2}$, the above equation results in

$$\dot{V}_i \leq -\frac{(\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j)}{\alpha_h} V_i + \frac{\alpha_3}{2\alpha_l} \sum_{j=2}^{r} l_j V_{i-j+1} \tag{3.6}$$

If $\sum_{j=2}^{r} l_j$ is sufficiently small such that $\sum_{j=2}^{r} l_j < \frac{2\alpha_l \alpha_1}{\alpha_3(\alpha_l + \alpha_h)}$, then $\frac{\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j}{\alpha_h} > \frac{\alpha_3}{2\alpha_l}\sum_{j=2}^{r} l_j > 0$. Consequently,

$$||V_i(\cdot)||_\infty \leq \frac{\alpha_h}{\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j} \frac{\alpha_3}{2\alpha_l} \sum_{j=2}^{r} l_j ||V_{i-j+1}(\cdot)||_\infty + V_i(0) \tag{5.7}$$

Since $\sum_{j=2}^{r} l_j$ is assumed small enough such that $\frac{\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j}{\alpha_h} > \frac{\alpha_3}{2\alpha_l}\sum_{j=2}^{r} l_j$, it follows that the roots of the polynomial $P_r(z) = z^{r-1} - \frac{\alpha_3 \alpha_h}{\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j} \sum_{j=2}^{r} l_j z^{r-j-1}$ lie within the unit circle. Therefore. from the BIBO property of stable LTI systems, there exists a positive constant. K, such that' $||V_i(\cdot)||_\infty \leq K \sup_i \; V_i(0)$. where K depends on $\alpha_l, \alpha_h, \alpha_1, \alpha_3$ and $\sum_{j=2}^{r} l_j$. Given any $\epsilon > 0$, choose $\delta = \sqrt{\frac{\alpha_l}{K\alpha_h}}\epsilon$. Clearly. if $||x_i(0)||_\infty < \delta$, $\sup_i \; ||x_i(\cdot)||_\infty < \epsilon$.

Let $d > 1$. Define $V(d^{-1}, t) = \sum_{j=1}^{\infty} V_i(t) d^{-i}$. Clearly, $V(d^{-1}, t)$ is defined whenever the weak coupling conditions are satisfied and whenever $||x_i(0)||_\infty$ exists.

$$\dot{V} = \sum_{j=1}^{\infty} \dot{V}_i(t) d^{-i} \leq -V d^{-(r-1)} P_r(d) \frac{\alpha_1 - \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j}{\alpha_h}$$

Clearly. $P_r(d) > 0$ whenever $d > 1 > \rho(P_r(z))$, the spectral radius of the polynomial $P_r(z)$. $V \to 0$ exponentially and hence, $V_i(t), x_i(t) \to 0$ exponentially.

From the definition of string stability, it is clear that the string stability of an interconnected system guarantees the stability of every subsystem. Under some stronger coupling condition, $\alpha_1 > \frac{\alpha_3}{2}\sum_{j=2}^{r} l_j$, any finite interconnections of 5.1 is asymptotically string stable. In the vehicle following applications. although the number of vehicles in every platoon (electronically interconnected system of vehicles) will be finite, it is necessary that the stability of the platoon be independent of the size of the platoon to prevent the saturation of the input actuators. Xnother interesting feature about the string stability of an interconnection of exponentially stable systems is that it is preserved under small structural perturbations. Consider

$\dot{x}_i = f(x_i, \ldots, x_{i-r+1}) + \epsilon f_p(x_i, \ldots, x_{i-r+1})$

Assume $f_p(0, \ldots, 0) = 0$ and $||f_p(p_1, \ldots, p_r) - f_p(q_1, \ldots, q_r)|| \leq \sum_{j=1}^{r} l_{fj}||p_j - q_j||$. From Theorem 1, the interconnection of the perturbed systems is string stable if

$$(\alpha_1 - \alpha_3\epsilon l_{f1}) - \sum_{j=2}^{r} \frac{\alpha_3(l_j + \epsilon l_{fj})}{2} > \frac{\alpha_h}{\alpha_l}\sum_{j=2}^{r} \frac{\alpha_3(l_j + \epsilon l_{fj})}{2}$$

This condition is satisfied when

$$\epsilon < \frac{\alpha_1 - \frac{\alpha_3(\alpha_l+\alpha_h)}{2\alpha_l}\sum_{j=2}^{r} l_j}{\alpha_3 l_{f1} + \frac{\alpha_3(\alpha_l+\alpha_h)}{2\alpha_l}\sum_{j=2}^{r} l_{fj}}$$

This concludes the proof that string stability is robust to small structural perturbations. It is desirable that the string stability property be preserved in the presence of parasitic actuator dynamics. In the next section, we present the conditions which guarantee string stability of the origin of the interconnected system in the presence of such parasitic actuator dynamics.

5.2 String Stability Of Singularly Perturbed Interconnected Systems

Before proceeding to study the string stability of the interconnected system. we present a result on the stability of a singularly perturbed system from [21].

Theorem 2 (Robustness of Exponentially Stable Nonlinear Systems to Singular Perturbations): Consider the autonomous singularly perturbed system

$$\dot{x} = f_1(x, z) \qquad (5.8)$$

$$\epsilon\dot{z} = g_1(x, z) \qquad (5.9)$$

where $x \in \mathcal{R}^n, z \in \mathcal{R}^m$ and assume that the origin is an isolated equilibrium point and the functions f_1 and g_1 are locally Lipschitz in an open connected set that contains the origin. Let $z = h_1(x)$ be an isolated root of $0 = g(x, z)$, such that $h_1(0) = 0$. Let $y = z - h_1(x)$. If the following conditions are satisfied

- The reduced system is exponentially stable, i.e there exists positive constants $\alpha_l, \alpha_h, \alpha_1, \alpha_3$ and a Lyapunov function $V(x)$ such that

$$\alpha_1 ||x||^2 \leq V(x) \leq \alpha_h ||x||^2$$

$$\frac{\partial V}{\partial x} f_1(x, h_1(x)) \leq -\alpha_1 ||x||^2$$

$$||\frac{\partial V}{\partial x}|| \leq \alpha_3 ||x||$$

- The boundary layer system is exponentially stable. uniformly for frozen x. i.e there esists positive constants $\beta_l, \beta_h, \alpha_2, \alpha_4$ and a Lyapunov function $W(x, y)$ such that

$$\beta_l ||y||^2 \leq W(x, y) \leq \beta_h ||y||^2$$

$$\frac{\partial W}{\partial y} g(x, y + h_1(x)) \leq -\alpha_2 ||y||^2$$

$$||\frac{\partial W}{\partial (x, y)}|| \leq \alpha_4 ||x \quad y||$$

- There exist positive constants. β_2 and γ such that

$$[\frac{\partial W}{\partial x} - \frac{\partial W}{\partial y} \frac{\partial h_1}{\partial x}] f_1(x, y + h_1(x)) \leq \beta_2 ||x|| ||y|| + \gamma ||y||^2$$

Let $\epsilon^* = \frac{\alpha_1 \alpha_2}{\alpha_1 \gamma + \beta_1 \beta_2}$. Then the origin of the singularly perturbed system is exponentially stable for $0 < \epsilon < \epsilon^*$.

Proof See Theorem 2.1 and Corollary 2.2 of [21]

Intuitively. origin of the perturbed interconnected system will be string stable if origin of every perturbed subsystem is stable and the origin of the **reduced" interconnected system is string stable. This observation leads us to the following theorem.

Consider the following perturbed interconnected system:

$$\dot{x}_i = f(x_i, z_i, x_{i-1}, \ldots, x_{i-r+1}) \quad (5.10)$$

$$\epsilon \dot{z}_i = g(x_i, z_i) \qquad i \in \mathcal{N} \quad (5.11)$$

where $f : \mathcal{R}^n \times \mathcal{R}^m \times \underbrace{\mathcal{R}^n \times \cdots \times \mathcal{R}^n}_{(r-1) times} \rightarrow \mathcal{R}^n$,
$g : \mathcal{R}^n \times \mathcal{R}^m \rightarrow \mathcal{R}^m$. Let $f(0, \ldots, 0) = 0; g(0, 0) = 0$ and let $z_i = h(x_i, \ldots, x_{i-r+1})$ be an isolated root of $0 = g(x_i, z_i)$. Let $y_i = z_i - h(x_i)$ and let $h(0) = 0$. and f, g, h be sufficiently smooth Lipschitz functions.

Theorem 3 (Robustness of Exponentially stable Interconnected Systems to Singular Perturbations):

If the following conditions are satisfied:

1. Let there exist a Lyapunov function. $V(x_i)$, such that

$$\alpha_l ||x_i||^2 \leq V(x_i) \leq \alpha_h ||x_i||^2$$

$$\frac{\partial V}{\partial x_i} f(x_i, h(x_i), x_{i-1}, \ldots, x_{i-r+1}) \leq -\alpha_1 ||x_i||^2 + \sum_{j=2}^{r} \alpha_{1j} ||x_{i-j+1}||^2$$

with $\alpha_{1j} > 0$ and $\alpha_1 > \frac{\alpha_h}{\alpha_l} \sum_{j=2}^{r} \alpha_{1j}$.

$$||\frac{\partial V}{\partial x_i}|| \leq \alpha_3 ||x_i||$$

These conditions imply the string stability of the interconnected of reduced(unperturbed) systems.

2. There exists a Lyapunov function $W(x_i, y_i)$ such that

$$\beta_l ||y_i||^2 \leq W(x_i, y_i) \leq \beta_h ||y_i||^2$$

$$\frac{\partial W}{\partial y_i} g(x_i, y_i + h(x_i)) \leq -\alpha_2 ||y_i||^2$$

$$(\frac{\partial W}{\partial x_i} - \frac{\partial W}{\partial y_i}\frac{\partial h}{\partial x_i})f(x_i, y_i + h(x_i), \ldots, x_{i-r+1}) \leq \beta_2||x_i||||y_i|| + \gamma||y_i||^2 + \sum_{j=2}^{r}\gamma_j||x_{i-j+1}||^2$$

with $\gamma_j > 0$. This condition implies the exponential stability of the singularly perturbed individual systems.

Then. the singularly perturbed interconnected system is string stable.

Proof: If $\sum_{j=2}^{r}\alpha_{1j} \neq 0$. let $k = \min\{\frac{\sum_{j=2}^{r}\alpha_{1j}}{\sum_{j=2}^{r}\gamma_j}, \frac{\alpha_h}{\beta_h}\}$. Otherwise, let $k = \min\{\frac{\alpha_1}{\sum_{j=2}^{r}\gamma_j}, \frac{\alpha_h}{\beta_h}\}$. Define $\nu(x_i, y_i) = \frac{1}{2}(V(x_i) + kW(x_i, y_i))$. Using shorthand notation ν_i for $\nu(x_i, y_i)$, V_i for $V(x_i)$ and W_i for $W(x_i, y_i)$, there exists a β_1 such that

$$\frac{\partial V_i}{\partial x_i}[f(x_i, y_i + h(x_i), \ldots, x_{i-r+1}) - f(x_i, h(x_i), \ldots, x_{i-r+1})] \leq \beta_1||x_i||||y_i||$$

$$\frac{\alpha_l||x_i||^2 + k\beta_l||y_i||^2}{2} \leq \nu_i \leq \frac{\alpha_h||x_i||^2 + k\beta_h||y_i||^2}{2}$$

$$\dot{\nu}_i = \frac{1}{2}[\dot{V}_i + k\dot{W}_i]$$

$$= \frac{1}{2}[-\alpha_1||x_i||^2 + \beta_1||x_i||||y_i|| + \sum_{j=2}^{r}\alpha_{1j}||x_{i-j+1}||^2]$$

$$+\frac{k}{2}[-\frac{\alpha_2}{\epsilon}||y_i||^2 + \beta_2||x_i||y_i|| + \gamma||y_i||^2 + \sum_{j=2}^{r}\gamma_j||x_{i-j+1}||^2]$$

$$\leq -\lambda(\epsilon)(||x_i||^2 + ||y_i||^2) + \sum_{j=2}^{r}\frac{\alpha_{1j} + k\gamma_j}{2}||x_{i-j+1}||^2$$

where

$$\lambda(\epsilon) = \frac{4\alpha_1 k(\alpha_2 - \epsilon\gamma) - \epsilon(\beta_1 + \beta_2 k)^2}{4(\epsilon\alpha_1 + k(\alpha_2 - \epsilon\gamma) + \sqrt{(\epsilon\alpha_1 - k(\alpha_2 - \epsilon\gamma))^2 + \epsilon^2(\beta_1 + \beta_2 k)^2})}$$

Since $\lambda(\epsilon)$ is a continuous function of ϵ, define

$$F(\epsilon) = \frac{2\lambda(\epsilon)}{\alpha_h} - \sum_{j=2}^{r}(\alpha_{1j} + k\gamma_j)$$

Since λ: $\sum_{j=2}^{r} \gamma_j < \sum_{j=2}^{r} \alpha_{1j}$, from Assumption 1, it follows that $F(0) > 0$ and $F(\frac{4\alpha_1\alpha_2 k}{4\alpha_1\gamma k+(\beta_1+\beta_2 k)^2}) < 0$. By intermediate value theorem, there esists ϵ_d such that $0 < \epsilon_d < \frac{4\alpha_1\alpha_2 k}{4\alpha_1\gamma k+(\beta_1+\beta_2 k)^2}$ such that $\forall\ 0 < \epsilon < \epsilon_d$, $F(\epsilon) > 0$. Therefore,

$$\dot{\nu}_i \leq -\lambda(\epsilon)(||x_i||^2 + ||y_i||^2) + \sum_{j=2}^{r}(\alpha_{1j} + k\gamma_j)\frac{||x_{i-j+1}||^2}{2} \leq -\frac{2\lambda(\epsilon)}{\alpha_h}\nu_i + \sum_{j=2}^{r}(\alpha_{1j} + k\gamma_j)\nu_{i-j+1}$$

By an argument, similar to that in Theorem 1, there esists a constant $K > 0$, such that $||\nu_i(\cdot)||_\infty \leq K||\nu_i(0)||_\infty$. This proves that the interconnection of singularly perturbed systems is string stable $\forall\ 0 < \epsilon < \epsilon_d$.

It also follows, by an argument similar to that in Theorem 1, that $\nu_i \to 0$ exponentially.

The above theorem justifies the use of control based on reduced(unperturbed) system model.

5.3 Adaptive Control of Interconnected Systems

Consider the following open loop interconnected system :

$$\dot{\xi}_i = f_o(\xi_i, \xi_{i-1}, \ldots, \xi_{i-r+1}) + g(\xi_i)u_i$$

where $\xi_i \in \mathcal{R}^{p+q+1}$ where p, q are positive integers. As assumed earlier, $\xi_j \equiv 0$ for all $j \leq 0$. f_o, g are smooth vector fields $u_i \in \mathcal{R}$ and $i \in \mathcal{N}$. The output of the i-th subsytem is $h_i = h(\xi_i)$ with $h_i \in \mathcal{R}$. The objective is to find a control such that the states of the closed loop interconnected system are always bounded and go to the origin, $\xi_i = 0$, $i \in \mathcal{N}$, asymptotically.

The following assumptions are used for obtaining the control effort and analyzing the closed loop behavior of the interconnected system:

- There esists a global diffeomorphism $z_i = \phi_1(\xi_i), y_i = \phi_2(\xi_i)$ with $z_i \in \mathcal{R}^{p+1}$, $y_i \in \mathcal{R}^q$ and

$$\dot{z}_i^{(1)} = z_i^{(2)}$$

$$\dot{z}_i^{(2)} = z_i^{(3)}$$

$$\dot{z}_i^{(p)} = z_i^{(p+1)}$$

$$\dot{z}_i^{(p+1)} = \theta_f^T W_f(\xi_i, \xi_{i-1}, \ldots, \xi_{i-r+1}) + \theta_g^T W_g(\xi_i) u_i$$

$$\dot{y}_i = \eta(z_i, y_i)$$

where $z_i^{(j)}$ is the j-th component of the vector z_i and $z_i^{(1)} = h_i$. The above condition implies the adaptive linearizability of the open loop system with a strict relative degree equal to $p + 1$. For details on adaptive linearizability. see Sastry and Isidori. [34]. The vector fields, f_o, g are implicitly assumed to be linearly parametrizable in the constant but uncertain parameters θ_f and θ_g. Estimates of these uncertain constant parameters will, henceforth. be represented by θ_f and 8, respectively. Similarly: the parameter estimation errors are given by $\tilde{\theta}_f$ and $\tilde{\theta}_g$. y_i represents the state that will be rendered unobservable by a input-output' linearizing control. In other words. the dynamics of y_i represents the internal dynamics of the i-th system.

- (The origin of) $\dot{y}_i = \eta(0, y_i)$ is globally exponentially stable. This assumption states that the zero dynamics of every system in the open loop interconnected system is esponentially stable. This assumption is required to establish that y_i is bounded uniformly in i when z_i is uniformly bounded in i. A more general form of internal dynamics that arises in such interconnected systems is of the following form

$$\dot{y}_i = \eta(z_i, y_i, z_{i-1}, y_{i-1}, \ldots, z_{i-r+1}, y_{i-r+1})$$

In order to analyze the closed loop interconnected system with such an internal dynamics, additional weak coupling conditions similar to those in Theorem 1 (on the magnitude of the Lipschitz constants associated with y arguments of η) have to be imposed to conclude that y_i is uniformly bounded in i if z_i are uniformly bounded in i. In order to keep the analysis simple. me will. however. not use this general form of internal dynamics.

- Every system avails the information of its state and the information of the states of "r" systems preceding it. This assumption is necessary to generate a feedback linearizing law which guarantees that the states of all the systems are bounded uniformly in i. The above assumption enables us to define S_i such that $S_i = 0$ describes the desired closed loop (string stable) dynamics. For example, one could define

$$S_i = z_i^{(p)} + \delta_1 z_i^{(p-1)} + \ldots + \delta_p z_i^{(1)} - \delta_{p+1} z_{i-1}^{(1)}$$

where $s^p + \delta_1 s^{p-1} + \ldots + \delta_p$ is a Hurwitz polynomial with real roots. On the surface. $S_i = 0$. $||z_i||_\infty < ||z_{i-1}||_\infty$ if δ_{p+1} is sufficiently small. In matrix form. S_i can be defined compactly as

$$\dot{x}_i = f_d(x_i, \ldots, x_{i-r+1}) + b_\lambda S_i$$

where $b_\lambda = [0, \ldots, 0.1]^T$ and $x_i = [z_i^{(1)} \ldots, z_i^{(p)}]^T$. f_d is a smooth vector field and it satisfies the weak coupling conditions described in Theorem 1 so that the dynamics on the surface $S_i = 0$ is string stable. Algebraically, S_i should be understood as

$$S_i = z_i^{(p)} + \psi_d(x_i, x_{i-1}, \ldots, x_{i-r+1})$$

Here ψ_d is a smooth scalar function.

The control input u_i should be chosen to drive S_i to the surface $S_i = 0$. In order to obtain the control effort. differentiate S_i

$$\dot{S}_i = z_i^{(p+1)} + \dot{\psi}_d(x_i, \ldots, x_{i-r+1}) = \theta_f^T W_f(\xi_i, \ldots, \xi_{i-r+1}) + \theta_g^T W_g(\xi_i) u_i + \dot{\psi}_d(x_i, \ldots, x_{i-r+1})$$

Choose u_i such that

$$\hat{\theta}_f^T W_f(\xi_i, \ldots, \xi_{i-r+1}) + \hat{\theta}_g^T W_g(\xi_i) u_i + \dot{\psi}_d(x_i, \ldots, x_{i-r+1}) = -\lambda S_i$$

Obtaining control effort requires inversion of $\theta_g W_g$ which may be singular. If it is known that $|\theta_g W_g| > C$ where C is a generic positive constant, projection algorithms could be employed to counter this problem.

As seen earlier. the closed loop dynamics of any adaptively linearizable non-linear systems with a coupling (interconnecting) control law can be cast in the following form :

$$\dot{x}_i = f_d(x_i, x_{i-1}, \ldots, x_{i-r+1}) + b_\lambda S_i$$

$$\dot{S}_i = -\lambda S_i + \tilde{\theta}_i^T W(x_i, y_i, S_i, x_{i-1}, \ldots, x_{i-r+1})$$

$$\dot{y}_i = \phi(x_i. y_i, S_i) \tag{5.12}$$

where $b_\lambda = [0 \cdots 0 \;\; 1]^T, \tilde{\theta}_i = \hat{\theta}_i - \theta_i$ where $\hat{\theta}_i$ is the estimate of the parameter and 8; is the actual (constant) value of the parameter. From the first equation. $S_i = 0$ describes the desired closed loop dynamics. The second equation describes the dynamics of S_i and the third equation indicates the behavior of the internal dynamics associated with this system. Any adaptively linearizable nonlinear system with a coupling (interconnecting) control law yields this form of equations. In order to analyze the effect of parameter adaptation, we assume the following :

1. There exists a Lyapunov function $V(x_i)$ (for convenience. V_i), such that

$$\alpha_l ||x_i||^2 \leq V_i \leq \alpha_i ||x_i||^2$$

$$\frac{\partial V_i}{\partial x_i} f_d(x_i, x_{i-1}, \ldots. x_{i-r+1}) \leq -l_1 ||x_i||^2 + \sum_{j=2}^{r} l_j ||x_{i-j+1}||^2$$

$$||\frac{\partial V_i}{\partial x_i}|| \leq \alpha_1 ||x_i||$$

with $l_1 > \frac{\alpha_h}{\alpha_l} \sum_{j=2}^{r} l_j$.

2. There esists a Lyapunov function $W_z(y_i)$ (for convenience. W_i) . such that

$$\beta_l||y_i||^2 \le W_i \le \beta_h||y_i||^2$$

$$\frac{\partial W_i}{\partial y_i}\phi(x_i, S_i, y_i) \le -\alpha_2||y_i||^2 + \alpha_3||y_i|||S_i| + \alpha_4||y_i||||x_i||$$

$$||\frac{\partial W_i}{\partial y_i}|| \le \alpha_5||y_i||$$

We assume the exponentially stable behavior of the zero dynamics. Assumptions 1 and 2 enable the string stability of the interconnected system in the absence of parameter mismatch.

3. $W(x_i, y_i, S_i, x_{i-1}, \ldots x_{i-r+1})$ is bounded for all its bounded arguments.

Theorem **4** (Effectiveness of Parameter Adaptation **for** Interconnected **sys-**tems): Under these conditions, the following parameter adaptation law

$$\dot{\hat{\theta}}_i = -\Gamma W(x_i, \ldots, x_{i-r+1})S_i, \quad \Gamma > 0$$

guarantees that for all bounded $||x_i(0)||_\infty, ||S_i(0)||_\infty, ||\tilde{\theta}_i(0)||_\infty$,

- $\sup_i \ ||x_i(\cdot)||_\infty, \sup_i \ ||S_i(\cdot)||_\infty, \sup_i \ ||\tilde{\theta}_i(\cdot)||_\infty$ are bounded.
- $x_i(t), S_i(t) \to 0$ asymptotically for all i.

Proof Let $V_{ai} = S_i^2 + \tilde{\theta}_i^T\Gamma^{-1}\tilde{\theta}_i$. Using the adaptation law.

$$\dot{V}_{ai} = -2\lambda S_i^2$$

$$\Rightarrow V_{ai}(t) \le V_{ai}(0) \le S_i^2(0) + \frac{||\tilde{\theta}_i(0)||^2}{\lambda_{min}(\Gamma)}$$

$$\sup_i \ ||S_i(\cdot)||_\infty \le \sqrt{||S_i(0)||_\infty^2 + \frac{||\tilde{\theta}_i(0)||_\infty^2}{\lambda_{min}(\Gamma)}}$$

Similarly.

$$\sup_i \quad ||\tilde{\theta}_i(\cdot)||_\infty \leq \sqrt{\lambda_{max}(\Gamma)}\sqrt{||S_i(0)||_\infty^2 + \frac{||\tilde{\theta}_i(0)||_\infty^2}{\lambda_{min}(\Gamma)}}$$

and

$$\sup_i \int_0^\infty S_i^2 dt = \sup_i \quad ||S_i(\cdot)||_2^2 \leq \frac{V_{ai}(0)}{2\lambda}$$

$$\leq \frac{S_i^2(0) + \frac{||\tilde{\theta}_i(0)||_\infty^2}{\lambda_{min}(\Gamma)}}{2\lambda}$$

Calculating $\dot{V}_i$ along the trajectories of x_i,

$$\dot{V}_i = \frac{\partial V_i}{\partial x_i}[f_d(x_i, \ldots, x_{i-r+1}) + b_\lambda S_i]$$

$$\leq -l_1||x_i||^2 + \sum_{j=2}^{r} l_j||x_{i-j+1}||^2 + \alpha_1||x_i|||S_i|$$

Since $\sup_i \quad ||S_i||_\infty \leq K$ where $K := \sqrt{||S_i(0)||_\infty^2 + \frac{||\tilde{\theta}_i(0)||_\infty^2}{\lambda_{min}(\Gamma)}}$.

$$\dot{V}_i \leq -l_1||x_i||^2 + \sum_{j=2}^{r} l_j||x_{i-j+1}||^2 + \alpha_1 K||x_i||$$

Define $e_i = \sqrt{V_i}$. Then

$$\dot{e}_i \leq -\frac{l_1}{2\alpha_h}e_i + \sum_2^r \frac{l_j}{2\alpha_l}e_{i-j+1} + \frac{\alpha_1}{2\alpha_l}|S_i|$$

$$||e_i||_p \leq \frac{\alpha_h}{l_1}\sum_2^r \frac{l_j}{\alpha_l}||e_{i-j+1}||_p + \frac{\alpha_h\alpha_1}{l_1\sqrt{\alpha_l}}||S_i||_p$$

where $p = 2, \infty$. Since $l_1 > \frac{\alpha_h}{\alpha_l}\sum_2^r l_j$, $||e_i||_p \leq M||S_i||_p$ where $M > 0$ is a constant. Since $\sup_i \quad \{||S_i||_\infty, ||S_i||_2\} < \max\{K, \sqrt{\frac{V_{ai}(0)}{2\lambda}}\} < \infty$, it follows that $\sup_i \quad \{||e_i||_\infty, ||e_i||_2\} < K_1$ for some positive K_1. This implies that $\sup_i \quad \{||x_i||_\infty, ||x_i||_2\} < \frac{K_1}{\sqrt{\alpha_l}}$. By Assumption 2 (that the zero dynamics of every individual system is minimum phase), $\sup_i \quad ||y_i(\cdot)||_\infty$ exists. By Assumption 3, $W(x_i, S_i, y_i, x_{i-1}, \ldots, x_{i-r+1})$ is bounded. Therefore, $\dot{S}_i \in L_\infty$. Consequently, by Barbalat's Lemma, $S_i \to 0$

Observe that $\sup_i ||\dot{e}_i||_\infty$ is bounded, since

$$\dot{e}_i \leq -\frac{l_1}{2\alpha_h}e_i + \sum_2^r \frac{l_j}{2\alpha_l}e_{i-j+1} + \frac{\alpha_1}{2\alpha_l}|S_i|$$

Since $\sup_i \quad sup_i\{||e_i||_\infty, ||e_i||_2\}$ are bounded, by Barbalat's Lemma, $e_i \to 0$. Therefore, $V_i, x_i \to 0$.

Remarks:

1. In Assumption1, $S_i \equiv 0$ yields the desired "string stable" dynamics

2. Designing decentralized adaptive controllers for interconnected systems can be done in two steps :

   (a) Identify the desired closed loop (string stable) dynamics. Design a controller to achieve the desired closed loop dynamics in the absence of parametric uncertainty.

   (b) Use a gradient adaptation law to update the parameters.

3. The dynamics of the sliding surface is usually given by

   $$\dot{S}_i = -\lambda sign(S_i) + \tilde{\theta}_i W(x_i, y_i, S_i, x_{i-1}, \ldots, x_{i-r+1})$$

   then the following adaptation law should be used

   $$\dot{\hat{\theta}}_i = -\Gamma W(x_i, \ldots, x_{i-r+1}) sign(S_i), \quad \Gamma > 0$$

   to conclude that $\sup_i ||x_i||_\infty, \sup_i ||S_i||_\infty, \sup_i ||\tilde{\theta}_i||_\infty$ are bounded and that $x_i(t), S_i(t) \to 0$ asymptotically for all i.

   The proof of the above remark is similar to the proof of Theorem 4.

4. The dynamics of the interconnected system considered in this chapter represents the dynamics of spacing errors of member vehicles in the platoon as described in earlier chapters.

Chapter 6

Conclusions and Future Research

In this dissertation. we studied the stability of a string of high speed. densely packed vehicles under automatic control in a unified approach. Vehicles are dynamically coupled in the string through their dynamics and the feedback control laws. Information structure dictates the spacing policy (constant spacing, constant headway time or constant safety factor) and the string stabilizing feedback control law.

In chapter 2, we developed a simplified vehicle model and partially validate the model.

In chapter *3*, we investigated string stability of various schemes in constant, spacing and variable spacing policies. Lack of reference vehicle information limits the stability performance of the platoon (in terms of the maximum attenuation of maximum spacing errors that can be achieved upstream from vehicle to vehicle). We showed. by analysis and simulation (and hopefully. experimentation) that lead vehicle relative position information helps attenuate the maximum spacing error geometrically with vehicle index in a platoon. It enhances robustness to the parasitic actuator lags and is necessary to guarantee stability during adaptation.

In chapter 4, we investigated the effect of parametric uncertainty in the string stability performance of the platoon. We presented a on-line implementable direct adaptive control lam that guarantees boundedness of the spacing errors of all vehicles at all time. Parametric convergence can be guaranteed for frequency rich lead vehicle maneuvers.

In chapter 3. we extended the concept of string stability to nonlinear systems. Analysis, however. is restricted to "look ahead" models. We have also demonstrated the robustness of string stability to structural and singular perturbations. We then developed a gradient adaptive algorithm to update uncertain parameters to guarantee boundedness of the state of all systems at all time.

The desired platoon size is constrained by several factors. Among the important ones are:

1. Capacity: For smaller platoon sizes: capacity increases linearly with the platoon size. For larger platoon sizes, capacity is almost a constant.

2. Ramp Length requirements: On-ramp length required increases atleast linearly for large platoon sizes.

3. Communication Protocols/Hardware: If we were to use token ring architecture. vehicle broadcast information update time increases linearly with platoon size. degrading the platoon performance. Increase in control sampling time also degrades performance.

4. String Stability: Depending on the information available. desired platoon size is limited by string stability and ride comfort requirements.

The goal of a control engineer: therefore. is to design control algorithms that do not restrict the platoon size at all. The control algorithms designed in sections 3.3 and **4.2** do not limit the platoon size if the sampling time is sufficiently small and there is no delay in broadcasting the lead vehicle information. Current efforts in evaluating masimum control sampling time for ensuring string stability (analytically and numerically) can be found in [31]. Sampled data control using control algorithm in section 3.3.4 couples the spacing error of every controlled vehicle with the spacing errors of all the preceding vehicles. However. the effect, of the i-th preceding vehicle wanes geometrically with i. Efforts are underway in evaluating the effect of communication delays/protocols on the stability of the vehicle string [56].

Optimal tuning of gains to maximize rider comfort while still ensuring string stability is an open issue. The structure of the controller given in section 3.3 does not satisy the conditions given in [19] and therefore. the corresponding results are inapplicable.

Performance of a platoon in degraded weather conditions is another important area which requires considerable attention. This dissertation only deals with performance of platoon under nominal conditions. The "No-slip" condition assumed in chapter 2 in developing the controller model is no longer valid when a vehicle is moving on a wet road. The deterioration in the platoon performance is proportional to the slip. A Considerable amount of slip causes instability- in the vehicle and the platoon.

The effect of slip across the torque converters at low vehicle speeds on the string stability of the platoon should also be investigated.

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