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TRADES AND QUOTES: A BIVARIATE POINT PROCESS

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Trades and Quotes: A Bivariate Point Process*

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Recent empirical work has studied point processes of transactions in financial markets and observed clear time dependent patterns in these arrival times. However these studies do not examine the timing of quoted price changes. This paper formulates a bivariate point process to jointly analyze transaction and quote arrivals. In microstructure models, transactions may reveal private information which is then incorporated into new prices. This paper examines the speed of this information flow and the circumstances which govern it. One of the main conclusions are that conditional on past quote times, the impact of trade information is to make quote durations longer when there is more information flow rather than less. This is interpreted as evidence that limit order suppliers become more cautious in the presence of apparent informational trading.

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1. Introduction

Financial markets are designed to rapidly match buyers and sellers of assets at mutually agreeable prices. When this process is examined in detail, there are two types of events which occur. Traders buy and sell assets and market makers post quotes. Traders observe the posted prices and previous transactions to determine their strategies, and similarly, the market makers observe past transactions and prices to decide what quotes to post. Since transactions and quote revisions do not occur simultaneously, the times of each event presumably represent some optimization and potentially convey information. This paper analyses these two time scales and estimates a model that relates them.

The detailed records of financial transactions typically include two prices with different interpretations. A quote reflects one market participant's willingness to trade. It is firm only up to a given size and may be improved on, both in terms of price and/or quantity. It may well reflect limit orders that are known to the market maker but often not to other participants. Transaction prices are agreed prices between counter parties, however they may not reflect opportunities to trade. For example, a transaction which occurs at the ask price is likely to be between a buyer and the market maker or a limit order. This price is not available to a seller. Similarly, a transaction price for a small volume transaction is not generally available to a large transaction trader.

In analyzing the information content of a transaction, it is now common not only to examine the impact on prices of the direction of the trade, i.e. Hasbrouck (1996) and its many references, but also of the timing of the trade as in Easley and O'Hara (1992), Engle (1996), and Dufour and Engle (1997). The timing of the quote response however has not been examined. How long do market makers wait until they post new quotes? This is relevant for determining the speed of the price response to transactions and ultimately to the rapidity of information absorption and market clearing.

In this paper the arrival of trades and quotes is treated as a bivariate, dependent point process. The arrival of each type of event is influenced by the past history of both processes and also by information such as the bid ask spread, volume of transactions

and other predetermined variables. In the next section, the economic background is sketched, and then the statistical framework is presented. Section 3 presents the data and basic statistics while section 4 gives the results and discussion. Section 5 concludes.

2. Economic Motivation

Empirical findings of recent studies such as Engle (1996), Engle and Russell (1997) and Dufour and Engle (1997) are consistent with the predictions of the literature studying the *market microstructure* of financial markets (see (O'Hara 1995)).

In the early microstructure models time does not matter *per se*. In Kyle (1985) orders are batched together and cleared at predetermined points in time. Hence the arrival times of the individual orders are of no relevance to the market maker. The sequential trade framework suggested by Glosten and Milgrom (1985) have orders arriving according to some stochastic fashion independent of any time parameters. Thus the timing of trades is also irrelevant to this model. If, however, time can be correlated with any factor related to the asset price, then the rate of trade arrival convey information to the agents. And as the agents learn from watching the flow of trades the adjustment of prices to information will also depend on time (O'Hara 1995).

The notion of time was introduced into economic models by Diamond and Verrecchia (1987) and Easley and O'Hara (1992). Put very shortly the first model predicts that observing a low rate of trade arrival implies the presence of *bad news*. This result derives from short sell constraints. Easley and O'Hara (1992) introduce event uncertainty into the sequential trade framework. The uncertainty is whether informed traders received a signal about the value of the asset. Their model predicts that a low trading intensity means *no news*, because the informed traders only trade when they get a signal.

The empirical studies mentioned above seem to favour the Easley and O'Hara model. Dufour and Engle (1997) found that time durations are negatively correlated with the absolute value of the following quote revision, and that the spread is negatively correlated with lagged duration. Engle (1996) and Engle and Russell (1997) derived a

relationship between arrival rates and volatility. Engle (1996) modelled both the arrival times of transactions and characteristics of these events sometimes called marks. He modelled time according to the ACD model and the marks are modelled conditional on the times. Thus the estimated expected durations are included in the volatility equation of an ARCH-type model. It was found that longer durations were associated with lower volatilities, supporting Easley and O'Hara interpreted as no news reduces volatility.

In an asymmetric information framework, the market maker quotes bid and ask prices to offset the expected losses from trading with informed traders. Once a trade has occurred, the market maker can reevaluate his quotes. If the trade was a buy, then there is a slightly increased probability that the information possessed by a fraction of the traders is positive for the asset. He will increase both bid and ask prices at this time and possibly change the spread. The amount by which he moves the quotes depends on the information he has from trades thus far and the assessment of the fraction of traders who are informed. The higher the fraction, the greater the response to the trade.

A central question in market microstructure is how fast and how completely new information is incorporated into prices. A key but unnoticed part of this question is the timing of quote changes in response to transactions. The timing of the market maker's response is assumed to be immediate in models such as Glosten and Milgrom (1985). However, in Easley and O'Hara (1992), the calendar time between his revisions can change. If there is no information event, then trading will slow down and consequently the time between quote revisions will become longer. However there is still no delay from a trade to a quote revision. Only in the case where trades have no information, or where the discreteness of quotes is greater than the size of the desired revision, will there be a delay between transactions and revisions. Thus there is a prediction that in a market with fewer information traders and slower transaction rates, the time to revise quotes should be longer.

A deeper analysis of the timing of quote setting must be tied to the supply of limit orders. Since on the NYSE, the specialist participates in a relatively small number of transactions, his quotes reflect the tightness of the limit order book. If limit buy orders

are all above his asking quote, he will increase the quote to execute new market orders against the book. Similarly if there are limit buy orders within his spread, he may reduce the ask to again reflect the prices at which transactions can be executed. In this interpretation, quotes may change in the absence of transactions or other news simply because of changes in the order book. Similarly, transactions may not result in a quote change if the limit order book is unchanged.

3. Statistical Formulation

In transaction or quote datasets only one type of event can occur namely a trade or the post of a quote, respectively. When combining trade and quote data a more complicated situation arises. Now two types of events will be occurring as time passes and the associated marks may be different variables for the two types. There are very few general accounts on multivariate point processes in the literature. A comprehensive treatment was given by Cox and Lewis (1972) from whom we adopted some of the terminology and notation.

Denote by $t_1, \dots, t_{i-1}, t_i, \dots$ denote the sequence of clock times at which a transaction of a given asset occurred, and by $q_1, \dots, q_{i-1}, q_i, \dots$ denote the timing of bid/ask quote revisions for this particular asset. A general bivariate model for these processes could involve associating with the bivariate point process a counting process $N(s_1, s_2) = \{(N^t(s_1), N^q(s_2))\}$, where $N^t(s_1)$ and $N^q(s_2)$ are the number of trades and quotes in $(0, s_1]$ and $(0, s_2]$ respectively. Further a bivariate sequence of intervals $\{T_i, Q_j\}$ is defined. Here T_i is the time between the $(i - 1)$ 'th and the i 'th trade; and the $\{Q_j\}$ sequence is defined similarly. This specification might suggest constructing a bivariate model from $\{(T_i, Q_i)\}_{i=1}^N$. While this in some situations may be fruitful, it is not a useful general approach, because in the present case events in the two processes with a common serial number will be far apart in real time. This leaves the specification of the of dependence between pairs of T_i and Q_i very tricky. Below it is seen how our model circumvents this problem of asynchronous starting points of duration pairs.

3.1. The Model

Let $t_1, \dots, t_{i-1}, t_i, \dots$ be defined as above. Using these arrival times we define the sequence $t'_1, \dots, t'_{i-1}, t'_i, \dots$, with t'_i denoting the clock time of the first quote arriving after the transaction at t_{i-1} . Given these point processes two types of durations are defined. Both types start with a transaction occurring at time t_{i-1} . Hence, define by $X_i = t_i - t_{i-1}$ the forward recurrence time to the next trade, and by $Y_i = t'_i - t_{i-1}$ denote the forward recurrence time to the next quote. We call these random variables a *forward trade duration* and a *forward quote duration* respectively. Together X_i and Y_i constitute a bivariate duration process which eliminates the synchronization problem mentioned above. The transaction times are the forcing process, each initiating a waiting time for the next quote to occur.

It will often be the case for very frequently traded stocks that a new transaction occurs before the next quote, that is t'_i might be less than t_i . The transaction conveys information that is likely to change our beliefs about when the next quote will occur and especially our beliefs about what will happen at the next quote. This means that at time t_i our expectation of the arrival of the next quote will change even though the initiated quote spell was not completed. To embed this feature in the model, cases of $t'_i > t_i$ are treated as *censored* forward quote durations. This is done by defining

$$\tilde{Y}_i = \min(Y_i, X_i) = \tilde{t}_i - t_{i-1}, \quad \text{where } \tilde{t}_i = \min(t'_i, t_i)$$

We call \tilde{Y}_i the *observed forward quote duration*, and associate with it an indicator d_i taking the value 1 if \tilde{Y}_i was censored. Note that in this case we only know that the i 'th forward quote duration was longer than \tilde{Y}_i .

The statistical model can now be build by specifying the parameterization of the bivariate duration process given by $\{(X_i, \tilde{Y}_i)\}_{i=1}^N$. Assume that the i 'th observation has a joint density conditional on all relevant and available information as of time t_{i-1} . Modelling this distribution directly would be a very complex matter, but fortunately a simpler expression can easily be obtained. Without loss of generality, the joint density can be written as the product of the conditional density and the resulting marginal

density. Hence we write this as

$$p(x_i, \tilde{y}_i | \mathcal{H}_{i-1}; \boldsymbol{\omega}) = g(x_i | \mathcal{H}_{i-1}; \boldsymbol{\omega}_1) f(\tilde{y}_i | x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2) \quad (1)$$

and call $g(\cdot | \mathcal{H}_{i-1}; \boldsymbol{\omega}_1)$ the trade density and $f(\cdot | x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)$ the quote density.

Before we turn to the actual parameterization a few words about the model defined so far are required. The process for the forward trade duration is assumed to be of the *Autoregressive Conditional Duration* (ACD) type suggested by Engle and Russell (1997). However, the observed forward quote durations are censored and this must be modelled carefully. The process of censoring times are in fact the forward trade duration process, and hence the censoring times will be a dependent process. This is not a problem even though there will be clusters of short censoring time and periods with longer ones, and these these periods may correspond to similar periods in quote intensities. Cox and Oakes (1984) require that, conditionally on the values of any explanatory variables, the prognosis for any forward quote duration not terminated at the censoring time should not be affected if it is censored. This is clearly not true in the model we propose. But this is exactly the feature the model is constructed to deal with. The model explicitly states how our prognosis about the next occurrence of a quote should be altered in the case of an intervening trade. Hence the introduction of this type of censoring is a way of updating our beliefs about the expected forward quote duration. It is this feature that gives the effect equivalent of time-dependent covariates in a Cox regression for inter quote arrival times. Unlike the Cox and Oakes model and competing risk models we observe the censoring threshold for each observation, and can model it directly.

We now return to the parametric specification of the model. In specifying the trade density let $\psi_i(\mathcal{H}_{i-1}; \boldsymbol{\omega}_1) = E(X_i | \mathcal{H}_{i-1}; \boldsymbol{\omega}_1)$, then

$$g(x_i | \mathcal{H}_{i-1}; \boldsymbol{\omega}_1) = \psi_i(\mathcal{H}_{i-1}; \boldsymbol{\omega}_1)^{-1} \exp \left\{ \frac{x_i}{\psi_i(\mathcal{H}_{i-1}; \boldsymbol{\omega}_1)} \right\} \quad (2)$$

where the expected duration follows an exponential linked ACD-type model, given by

$$\psi_i(\mathcal{H}_{i-1}; \boldsymbol{\omega}_1) \equiv \psi_i = \exp \left\{ \alpha + \delta \ln(\psi_{i-1}) + \gamma \frac{x_{i-1}}{\psi_{i-1}} + \boldsymbol{\beta} \mathbf{Z}_{i-1} \right\} \quad (3)$$

with \mathbf{Z}_{i-1} being a vector of explanatory variables known at time t_{i-1} . Equation (3) will be referred to as the trade equation. Note that ψ^{-1} , the inverse of the expected duration, is the trading intensity. It gives the rate at which trades arrive to the market.

The quote density takes into account that some of the observations are censored. This is done in the usual way for models with censored observations. Thus we have

$$f(\tilde{y}_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2) = h_Y(\tilde{y}_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)^{1-d_i} S_Y(\tilde{y}_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)^{d_i} \quad (4)$$

where h_Y and S_Y are respectively the density function and the survivor function for the forward quote duration. $h_Y(\cdot|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)$ is the actual forward quote density and will be termed so. The density is similar to the density for the forward trade durations, except for the important feature that it is conditional on x_i . Let $\varphi_i(x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2) = E(Y_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)$, then

$$h_Y(\tilde{y}_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2) = \varphi_i(x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)^{-1} \exp\left\{\frac{\tilde{y}_i}{\varphi_i(x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)}\right\} \quad (5)$$

Again the expected duration follows an exponential ACD-type model, hence the quote equation is given by

$$\varphi_i(x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_1) \equiv \varphi_i = \exp\left\{\mu + \rho \ln(\varphi_{i-1}) + \delta_1 \frac{\tilde{y}_{i-1}}{\varphi_{i-1}} + \delta_2 \frac{\tilde{y}_{i-1}}{\varphi_{i-1}} d_{i-1} + \tau \frac{x_i}{\psi_i} + \boldsymbol{\eta} \mathbf{V}_{i-1}\right\} \quad (6)$$

where \mathbf{V}_{i-1} typically contains some of the variables of \mathbf{Z}_{i-1} . Here φ^{-1} is the quoting intensity, that is the rate at which the market maker post his quotes. Note that the inclusion of $\frac{\tilde{y}_{i-1}}{\varphi_{i-1}} d_{i-1}$ allows us to asses the impact of having some observations censored.

3.2. Estimation and Inference

Under the specification (1), the log likelihood can be expressed as:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\omega}; \mathbf{X}, \tilde{\mathbf{Y}}) &= \sum_{i=1}^N [\ln g(x_i|\mathcal{H}_{i-1}; \boldsymbol{\omega}_1) + \ln f(\tilde{y}_i|x_i, \mathcal{H}_{i-1}; \boldsymbol{\omega}_2)] \\ &= \sum_{i=1}^N l_i^g(\boldsymbol{\omega}_1) + \sum_{i=1}^N l_i^f(\boldsymbol{\omega}_2) \end{aligned} \quad (7)$$

which has to be maximized with respect to the parameters (ω_1, ω_2) . If we can assume that the observed forward quote durations are weakly exogenous for the parameters of interest, ω_1 in this case, then joint estimation is not required and the parameters can be estimated efficiently by maximizing the first term. Similarly, maximizing just the second term is justified if the forward trade durations are weakly exogenous for the parameters in ω_2 . We take the approach of first maximizing $\sum_{i=1}^N l_i^g(\omega_1)$ and then conditional on this $\sum_{i=1}^N l_i^f(\omega_2)$ is maximized. This is not equivalent to maximum likelihood as some constraints are ignored.

The estimation approach is semiparametric, in that we do not assume that the true densities of g and h are exponential as stated in (2) and (5)¹. The log likelihood function is called a quasi-likelihood function. This method only requires specifying the mean of the distribution. Then QML estimators can be obtained which are consistent for ω_1 and ω_2 and have a well defined asymptotic covariance matrix. QML methods were introduced into econometrics by White (1982) and the results that justify the present application are analogous to Bollerslev and Wooldridge (1992). The robust covariance matrix is calculated as:

$$\left[\sum_{i=1}^N \frac{\partial^2 l_i}{\partial \omega \cdot \partial \omega'}(\hat{\omega} \cdot) \right]^{-1} \left[\sum_{i=1}^N \frac{\partial l_i}{\partial \omega}(\hat{\omega} \cdot) \frac{\partial l_i}{\partial \omega}(\hat{\omega} \cdot)' \right] \left[\sum_{i=1}^N \frac{\partial^2 l_i}{\partial \omega \cdot \partial \omega'}(\hat{\omega} \cdot) \right]^{-1}. \quad (8)$$

It is important to note that it is only for the trade equation that the QMLE's of the parameters are consistent for the mean, as this part satisfies the requirement that the expected score must equal zero. For the quote equation these estimates are, in general, only consistent for the mean if the true duration density is exponential. Otherwise it is not true that the expected score is equal to zero. However if this assumption is violated the QMLE's will still be consistent in the sense of White, that is consistent for whatever they are estimating. It is easy to test for the fit of quote durations and misspecification may be resolved by using a more general distribution as suggested in Lunde (1997).

3.3. Residuals and Specification tests

The residual analysis assess the validity of the exponential duration distributions used in the the QML approach, and the amount of remaining autocorrelation not explained by the specified model. Hence under the null that the model is truly exponential and that the expected duration is correctly specified, the the residuals should be identical and independent unit exponential distributed.

Generally residuals with a unit exponential distribution is defined as follows. If T_i has survivor function $S(t|\mathcal{H}_{i-1}; \boldsymbol{\omega})$ then $S(T_i|\mathcal{H}_{i-1}; \boldsymbol{\omega})$ is uniformly distributed and $-\ln(S(t|\mathcal{H}_{i-1}; \boldsymbol{\omega}))$ has a unit exponential distribution. Thus for the trade part we define the residual to be

$$\begin{aligned}\xi_i &= -\ln(S(x_i|\mathcal{H}_{i-1}; \hat{\boldsymbol{\omega}}_1)) \\ &= x_i\psi_i(\mathcal{H}_{i-1}; \hat{\boldsymbol{\omega}}_1)^{-1}\end{aligned}\tag{9}$$

which is identical to the residual defined in Engle and Russell (1997). These residuals are often called Cox-Snell residuals as they derive from the general definition of residuals given by Cox and Snell (1968) ².

If the i th individual is censored, so too is the corresponding residual and thus in general we obtain a set of uncensored and a set of censored residuals which cannot be regarded on the same footing. We might therefore seek to modify the Cox-Snell residuals taking explicit account of the censoring. Suppose that \tilde{y}_i is censored. The Cox-Snell residual for this observation is then given by

$$r_i = \tilde{y}_i\varphi_i(\mathcal{H}_{i-1}; \hat{\boldsymbol{\omega}}_2)^{-1}$$

If the fitted model is correct, then the values r_i can be taken to have a unit exponential distribution. The cumulative hazard function of this distribution increases linearly with time, hence the greater the value of the duration, the greater the value of that residual. It hereby follows that the residual for this duration at the actual unobserved termination time will be greater than the residual evaluated at the observed censoring time. To account for this the Cox-Snell residual can be modified by adding a positive

constant, call the excess residual. Using the lack of memory property of the exponential distribution, we know that since r_i has a unit exponential distribution the excess residual will also have a unit exponential distribution. The expected value of the excess residual is therefore one. This suggests defining the residuals for the quote part to be

$$\varepsilon_i = \tilde{y}_i \varphi_i(\mathcal{H}_{i-1}; \hat{\omega}_2)^{-1} + d_i$$

A number of LM tests defined as given in Wooldridge (1994) section 4.6 are presented. The same type of comments on effect of the censoring in the quote part, as given in the end of the previous section applies here. We used the heteroskedasticity consistent covariance matrix (8) when computing these tests.

4. Data Description

The data are extracted from the The Trade and Quote (TAQ) database. The TAQ database is a collection of all trades and quotes in New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotation (Nasdaq) securities. We only consider trades on the NYSE. Schwartz (1993) and Hasbrouck, Sofianos, and Sosebee (1993) document NYSE trading and quoting procedures.

Among the fifty stocks with the highest capitalization value at December 13 1996 nine stocks were randomly selected. The names and some summary statistics are given in Table 1 and 2. The numbers of trades and quotes given in Table 1 are numbers left after anomalous data were filtered out. Further, trades reported within the same second were treated as one trade, with the volumes of the multiple trade aggregated. For the quotes we also filter out multiple occurrences of quotes. The sample period is the two months from August, 1997 to September 30, 1997, which gives a total of 42 trading days. All stocks in the period had more quotes than trades.

To construct the bivariate duration process of forward trade and quote durations we begin as outlined in section 2 by calculating the forward trade durations simply as the inter trade arrival times. The first duration every day is the duration from the second

to the third trade that day. This excludes the NYSE opening and the high volume of this from the analysis. Overnight durations were also omitted from the sample. Now for every transaction the prevailing quote is the most recent quote which occurred at least five seconds before the transaction. As on the NYSE floor posting new quotes is given priority over recording completed transactions, a quote revision will often precede the trade from which it was instigated. Hence to compute the forward quote durations we delay every quote time five seconds and then the forward quote duration is the time from the present trade to the next quote. Matching transactions with quotes in this way overcomes the concern over mis-timed recordings. This *five second* rule was suggested by Lee and Ready (1991). To build the observed forward quote durations every pair of forward trade and quote durations are compared. If the forward trade is longer than the forward quote duration, the observed forward quote duration is censored.

With the dependent process defined we need to specify the explanatory variables to be put into \mathbf{Z}_{i-1} and \mathbf{V}_{i-1} . There are surely a lot of possibilities. More lagged values of the dependent process, the time since the most recent quote, the spread, the volume etc. Of course it is preferable to include variable that have economic interpretations. In Table 4 we present and explain the computation of the explanatory variables associated with the parameters β and η of equations (3) and (6). It is important to note that variables are lagged with respect to the trade time. Hence the third lagged spread would be the prevailing spread three trades ago.

Trades may be classified into buys and sells using the technique developed by Lee and Ready (1991). Trades at prices above the midquote are associated with buys (initiated by a buyer) and are marked 1; trade below the midquote are called sells (initiated by a seller) and given the mark -1 . This variable is often referred to as a buy-sell trade indicator variable. The rationale for this classification is that trades originating from buyers are most likely to be executed at or near the ask, while sell orders trade at or near the bid. This scheme classifies all trades except those that occur at the midquote. We do not apply the *tick* rule; trades at the midquote are given a zero mark. Using these sign marks we compute the accumulated signed volume. This is calculated use

a moving window of ten trades. We include the absolute value of this variable as an explanatory variable. This variable is related to the depth measure *VNET* introduced by Engle and Lange (1997).

Typically the market exhibits high activity in the morning and before closure. Around lunch time the activity is mostly lower. To handle this time-of-the-day effect, $E[\cdot|t_{i-1}]$, for every second of the day was computed. This daily pattern was estimated by using the Splus routine called `smooth.spline`, which can be used for one-dimensional cubic spline smoothing as discussed in chapters 1, 2 & 3 of Green and Silverman (1994). The routine uses a basis of *B*-splines. The splines for trade-trade durations and quote-quote duration all have the characteristic inverse U-shaped form found in similar studies. The same methodology was used to filter the spread and volume variables. The estimated splines for volume and spread are shown in Figure 1. We only use this periodical filter for the trade equation. To stress this we mark all filtered variables with a check; \checkmark . In the quote equation we re-color the expected forward trade duration, that is we use

$$\hat{\psi}_i = \check{\psi}_i(\hat{\omega}_1)E[X_i|t_{i-1}]$$

as the expected trade duration. Further the time-of-the-day conditional expected duration is used as a explanatory variable and all variables are in deviations from their mean value.

The estimated functional forms are given as follows:

$$\begin{aligned} \ln(\check{\psi}_i) &= \alpha + \delta \ln(\check{\psi}_{i-1}) + \gamma \frac{\check{x}_{i-1}}{\check{\psi}_{i-1}} + \beta_1 \text{QQ.d}\check{u}r_{i-1} + \beta_2 \Delta \check{\text{Spr}}_{i-1} + \beta_3 \text{lev}.\check{\text{Spr}}_{i-1} \\ &+ \beta_4 \sqrt{\check{\text{vol}}_{i-1}} + \beta_5 \text{lev}.\sqrt{\check{\text{vol}}_{i-1}} + \beta_6 \text{Abs}(\text{s.v}\check{\text{ol}})_{i-1} \end{aligned}$$

and

$$\begin{aligned} \ln(\varphi_i) &= \mu + \rho \ln(\varphi_{i-1}) + \delta_1 \frac{\check{y}_{i-1}}{\varphi_{i-1}} + \delta_2 \frac{\check{y}_{i-1}}{\varphi_{i-1}} d_{i-1} + \tau \frac{x_i}{\psi_i} + \eta_1 \frac{x_{i-1}}{\psi_{i-1}} + \eta_2 \ln(\hat{\psi}_{i-1}) \\ &+ \eta_3 \text{QQ.dur}_{i-1} + \eta_4 \text{lev.QQ}_{i-1} + \eta_5 \Delta \text{Spr}_{i-1} + \eta_6 \text{lev.Spr}_{i-1} + \eta_7 \sqrt{\text{vol}}_{i-1} \\ &+ \eta_8 \text{lev}.\sqrt{\text{vol}}_{i-1} + \eta_9 \text{Abs}(\text{s.vol})_{i-1} + \eta_{10} \text{Back.Q}_{i-1} \end{aligned}$$

Table 3 gives some simple correlations of the dependent variable and the explanatory variables in the quote equations. We will return to these correlations later in the discussion.

5. Estimation and Results

Maximization of the log likelihood as outlined in section 2.2 was performed in C++ using the simplex method as found in Press, Teukolsky, Vetterling, and Flannary (1992). We formulate the discussion in term of intensities, as it is ultimately the dynamics of the trading- and quote reaction intensities about which we are concerned.

The estimates of the trade equation are presented in Table 5. We present the significant trade equation, stressing the signs of the coefficients

$$\begin{aligned} \ln(\check{\psi}_i) &= \bar{\alpha} + \overset{+}{\delta} \ln(\check{\psi}_{i-1}) + \overset{+}{\gamma} \frac{\check{x}_{i-1}}{\check{\psi}_{i-1}} + \overset{+}{\beta}_1 \text{QQ.d}\check{u}r_{i-1} + \bar{\beta}_2 \Delta \check{\text{Spr}}_{i-1} \\ &+ \overset{+}{\beta}_3 \text{lev.}\check{\text{Spr}}_{i-1} + \bar{\beta}_4 \sqrt{\check{\text{vol}}_{i-1}} + \bar{\beta}_5 \text{lev.}\sqrt{\check{\text{vol}}_{i-1}} \end{aligned}$$

The trading intensity show a very high degree of persistence, with δ bigger then 0.95 for most stocks. All stocks have γ , the coefficient on the surprise term positive and significant. Hence these two parameters are as expected from earlier studies.

The most conspicuous of the explanatory variables are the change in the spread. The coefficient, β_2 , is negative and highly significant for all stocks. Hence a rise in the spread leads to a rise in the trading intensity. The Easley and O'Hara model predicts this. They show that the spread rises because of the presence of informational traders. These agents only trade in response to a news event. Hence theory predicts that rising spreads follows because the market maker believes that some traders have advantageous information about the stocks. The trading intensity rises as these traders try to exploit this information.

The fact that the coefficient on the spread-level, β_3 , seems to be positive might be though of as a contradiction of the preceding results. We don't think this is the case. It is likely that by having reached a high spread-level the market maker has incorporated

his knowledge of adverse information in to his prices, and is thus insured against the informational traders. The value of this information will shrink and the trade intensity dampen.

The negative coefficients β_4 and β_5 on volume are perfectly consistent with the economic literature such as Easley and O'Hara (1987) where larger volume reveals informational traders. Informed traders do not miss an opportunity to trade so that trade durations are short and intensities are high.

Including the duration between previous adjacent quotes is a novelty in this type of model. Its coefficient, β_1 , is significant for all stocks except for MTC. The positive value of β_1 implies that shorter durations between quotes from the market maker increases the trading intensity.

The residuals are surprisingly well behaved. The autocorrelation is reduced considerably and it could be reduced further by introducing more lags of especially the variables on δ and γ . We did not pursue this as our main interest lies in the quote equation.

Some excess dispersion is still present as the standard deviation of the residuals exceed 1. To assess the significance of this excess dispersion we apply a simple test suggested by Engle and Russell (1997). The null of no excess dispersion is based on the statistics $\sqrt{N}(\frac{\hat{\sigma}_\xi^2 - 1}{\sigma_\nu})$, where σ_ν is the sample variance of $\hat{\xi}$, which should be 1 under the null hypothesis. σ_ν is the standard deviation of $(\xi - 1)^2$ which equals $\sqrt{8}$ under the null of a unit exponential distribution. This statistics has a limiting normal distribution under the null with a 5% critical value of 1.645. Performing this test for our samples reveals that excess dispersion is still left in the residuals. In Lunde (1997) a generalized gamma version of trade equation is estimated. This model is able to remove the excess dispersion completely.

The LM tests show the following: A) that the spline-filter has left no time-of-the-day effects in the residuals. B) It doesn't seem to be the case that interquote durations have more explanatory power than the amount supplied by $QQ.d\check{ur}_{i-1}$. C) Maybe the accumulated signed volume should be used instead of the absolute value of this. Overall

the trade equation is consistent with economic theory and quite stable across stocks. It reveals the importance of quotes, spreads, and volume in predicting transaction intensities.

Table 6 reports the estimates of the quote equation. Remember that this equation models the market makers speed of reaction. We just refer to this as the quoting intensity, as it is the intensity by which the market maker reacts to a trade event. This intensity is also persistent, but considerably less than the trading intensity. To assist the reading of Table 5 we reproduce the model stressing the signs of the coefficients which are found to be generally significant.

$$\begin{aligned} \ln(\varphi_i) = & \bar{\mu} + \overset{+}{\rho} \ln(\varphi_{i-1}) + \overset{+}{\delta_1} \frac{\tilde{y}_{i-1}}{\varphi_{i-1}} + \overset{+}{\tau} \frac{x_i}{\psi_i} + \overset{-}{\eta_1} \frac{x_{i-1}}{\psi_{i-1}} + \overset{-}{\eta_2} \ln(\widehat{\psi}_{i-1}) \\ & + \overset{+}{\eta_3} \text{QQ.dur}_{i-1} + \overset{+}{\eta_5} \Delta \text{Spr}_{i-1} + \overset{+}{\eta_7} \sqrt{\text{vol}_{i-1}} + \overset{+}{\eta_{10}} \text{Back.Q}_{i-1} \end{aligned} \quad (10)$$

We find that the lagged standardized observed forward quote duration enter the quote equation positively and highly significant. Note that δ_2 is insignificant which implies that censored durations enter in the same fashion as uncensored duration.

The remaining variables can be interpreted as showing whether quotes are revised with long or short durations when there is little information. Clearly the surprise in the current trade duration is highly significant and positive as expected. However the important quote intensity is conditional on only information known at the time of the last transaction. The expected quote duration therefore depends negatively on lagged trade duration surprises, positively on rising spreads, positively on the square root of volume, and positively on the time since the last quote. The first three are all indicators of informed trading. The last can be interpreted as follows: if it has been long since the last quote, then probably there is little news so the duration will be even longer. So overall, the results seem to say, conditional on past quote times, the impact of trade information is to make quote durations longer when there is more information flow, rather than less.

The diagnostics for the quote equation show that the autocorrelation has been removed. Some excess dispersion is left (we did not use the simple test for excess dispersion

due to the censoring) which will have to be removed using a more general duration distribution. The LM test for hourly dummies is only significant for BAC. Hence the intra-day seasonality seems to be removed by the seasonality of the right hand side variables.

We turn finally to an interpretation of the results in light of the economic explanations for delayed quote revisions. The a priori most plausible finding is that quotes are revised more slowly when transactions carry little information. The empirical results contradict this finding, which also is supported by the simple correlations for the spread variables in Table 3. Instead, one could hypothesize that it takes a longer time to calculate the correct quotes when information is flowing and consequently, quotes are delayed.

Another more convincing explanation may be deduced from an interpretation of the market makers use of the spread. Primarily the market maker sets the spread to insure himself against the risk of trading with informed agents. In the light of this it is likely that the market maker, when suspecting the presence of informational traders, sets such a high spread that he is unlikely to lose from trading. Until he has a clear indication of the nature of the hidden information, he will keep the spread high, only changing it when new information is learned. This strategy is available only to a monopolistic specialist. Although the exchange will discipline specialists who persist in setting high spreads, this fast market setting is unlikely to last very long. NYSE specialists however are not entirely free of competition since limit orders have priority over the specialist. Thus a substantial supply of limit orders will prevent the specialist from maintaining a wide spread. The ability of the specialist to maintain wide spreads in the face of informational traders, is directly a consequence of the hesitation of limit order suppliers to transact in this risky environment. So the empirical evidence suggests that in the face of informed traders, the specialist widens his spread and maintains it until new limit orders arrive, but these limit orders are delayed because of the concern for getting "picked off" by the informed traders.

6. Conclusion

In this paper the arrival of trades and quotes is treated as a bivariate, dependent point process. The arrival of each type of event is influenced by the past history of both processes and also by information such as the bid ask spread, volume of transactions and other predetermined variables or marks.

The trading intensities show a very high degree of persistence. A rise in the spread leads to a rise in the trading intensity. Easley and O'Hara (1992) predicts this. The trading intensity rises as informational traders try to exploit their advantage.

When having reached a high spread-level the market maker has incorporated his knowledge of adverse information into his prices, and is thus insured against the informational traders. The value of this information will shrink and the trade intensity dampen.

Including the duration between previous adjacent quotes is a novelty in this type of model. It was seen that shorter durations between quotes from the market maker increases the trading intensity.

Conditional on past quote times, the impact of trade information is to make quote durations longer when there is more information flow rather than less. This is interpreted as evidence that limit order suppliers become more cautious in the presence of apparent informational trading.

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NOTES

¹It is clearly possible to use a general density as the generalized gamma distribution as suggested by Lunde (1997). But we have no economic theory suggesting what the shapes of these densities should be. And it is not a question we are going to address in this paper.

²A very readable exposition of residuals for survival models may be found in Collett (1994)

APPENDIX A: Tables and figures.

Table 1

Selected NYSE stocks

Company	Symbol	Listed Shares	Market Value	#Trades	#Quotes
Procter & Gamble Company	PG	694	74,595	46933	59678
Disney (Walt) Company (The)	DIS	682	47,496	28391	39308
Federal National Mortgage Ass.	FNM	1129	42,053	24911	34536
General Motors Corporation	GM	757	42,182	32619	39016
Bank American Corporation	BAC	387	38,632	34765	47527
McDonald's Corporation	MCD	830	37,572	24721	27698
Monsanto Company	MTC	822	31,954	25325	30217
Schlumberger Limited	SLB	309	30,848	27788	40374

Table 1 shows the nine randomly selected stocks from the fifty leading NYSE stocks in market value, as of December 31, 1996. Shares and value are in millions.

Table 2

Summary Statistics

Comp.	Av. tr. dura.	Av. price	Av. size	Sells	Buys	Av. f.qu. dura.	Av.o.f.qu midqu.	Av. spread	Cens.
PG	20.73	129.39	1148.06	37.8%	49.9%	7.09	10.32	0.13	33.9%
DIS	34.27	78.72	1473.11	37.1%	44.5%	10.25	15.96	0.15	36.3%
FNM	38.93	45.53	2915.59	39.1%	42.1%	11.25	16.96	0.20	31.9%
GM	29.88	64.92	2459.70	36.6%	43.6%	10.75	18.62	0.15	39.9%
BAC	27.99	71.30	1840.85	39.0%	48.3%	7.71	11.20	0.17	33.4%
MCD	39.29	48.61	2623.67	43.8%	41.2%	12.76	21.45	0.19	34.3%
MTC	38.00	42.81	2950.54	35.4%	48.9%	11.43	17.43	0.27	35.7%
SLB	34.96	77.71	1658.58	42.6%	53.2%	9.71	14.20	0.18	33.0%

Table 2 gives summary statistics for the datasets. The fifth and the sixth column give the percentage of the trades marked as sells and buys. The spread is the percentage spread calculated as 100 times the log of the ask price minus by the log bid price. The seventh column gives the forward quote duration and the next column gives the observed (the censored) forward quote duration. The last column states the percentage of the observed forward quote duration that were censored.

Table 3**Simple correlations**

Company	lagged tr.dur	lagged volume	lagged spread	lagged dif. spread
PG	0.0760	0.0273	0.0533	0.0265
DIS	0.0650	0.0050	0.0188	0.0227
FNM	0.0479	-0.0115	0.0552	0.0262
GM	0.0471	0.0420	0.0488	0.0199
BAC	0.0844	0.0097	0.0479	0.0208
MCD	0.0448	-0.0280	0.0201	0.0193
MTC	0.0343	-0.0134	0.0340	0.0282
SLB	0.0695	0.0141	0.0391	0.0225

Table 3 gives some simple correlations of the observed forward quote durations with lagged trade durations, lagged volume, lagged spread and the change in the lagged spread.

Table 4**Description of explanatory variable**

Variable	Name	Description
$Z_{i-1}^{\beta_1}, V_{i-1}^{\eta_3}$	QQ.dur _{i-1}	Time between the most- and next most recent quotes
$Z_{i-1}^{\beta_2}, V_{i-1}^{\eta_5}$	Δ Spr _{i-1}	Change in spread between the most- and next most recent quotes
$Z_{i-1}^{\beta_3}, V_{i-1}^{\eta_6}$	lev.Spr _{i-1}	Mean of 10 lagged spreads
$Z_{i-1}^{\beta_4}, V_{i-1}^{\eta_7}$	$\sqrt{\text{vol}}_{i-1}$	Square root of the size of the previous trade
$Z_{i-1}^{\beta_5}, V_{i-1}^{\eta_8}$	lev. $\sqrt{\text{vol}}_{i-1}$	Mean of square root of the size of the 10 previous trades
$Z_{i-1}^{\beta_6}, V_{i-1}^{\eta_9}$	Abs(s.vol) _{i-1}	Abs. value of accumulated signed size of the 10 previous trades
$V_{i-1}^{\eta_1}$	$\frac{x_{i-1}}{\psi_{i-1}}$	lagged surprise trade duration
$V_{i-1}^{\eta_2}$	ψ_{i-1}	Expected forward trade duration
$V_{i-1}^{\eta_4}$	lev.QQ _{i-1}	Mean of 10 lagged QQ durations
$V_{i-1}^{\eta_{10}}$	Back.Q _{i-1}	Time since the most recent quote

This Table defines the explanatory variables included in the two lagged information sets, \mathbf{Z}_{i-1} and \mathbf{V}_{i-1} .

Table 5

Estimates for the trade equation

$$\ln(\check{\psi}_i) = \alpha + \delta \ln(\check{\psi}_{i-1}) + \gamma \frac{\check{x}_{i-1}}{\check{\psi}_{i-1}} + \beta_1 \text{QQ.d}\check{u}r_{i-1} + \beta_2 \Delta \check{S}\check{p}r_{i-1} + \beta_3 \text{lev.}\check{S}\check{p}r_{i-1} \\ + \beta_4 \sqrt{\text{vol}_{i-1}} + \beta_5 \text{lev.}\sqrt{\text{vol}_{i-1}} + \beta_6 \text{Abs(s.vol)}_{i-1}$$

Par.	PG	DIS	FNM	GM	BAC	MCD	MTC	SLB
α	-0.0265 (-7.94)	-0.0223 (-4.70)	-0.0728 (-4.38)	-0.0246 (-7.22)	-0.0604 (-4.4)	-0.0131 (-4.13)	-0.0128 (-4.94)	-0.0191 (-5.24)
δ	.9649 (137.4)	.9538 (57.9)	.3134 (4.85)	.9896 (385.4)	.8790 (20.9)	.9630 (57.6)	.9824 (267.7)	.9859 (228.5)
γ	.0407 (10.98)	.0387 (6.98)	.0351 (4.97)	.0246 (8.64)	.0569 (9.77)	.0325 (4.6)	.0312 (9.07)	.0254 (7.16)
β_1	.0081 (4.46)	.0114 (2.98)	.0943 (10.3)	.0032 (3.46)	.0315 (3.05)	.0055 (2.37)	.0013 (1.84)	.0050 (2.59)
β_2	-0.0575 (-4.77)	-0.3281 (-20.3)	-0.3246 (-18.4)	-0.2767 (-16.8)	-0.2798 (-19.5)	-0.2348 (-13.6)	-0.1791 (-12.5)	-0.1221 (-7.89)
β_3	.42e-4 (3.92)	.13e-4 (1.21)	.94e-3 (5.9)	-.87e-5 (-1.41)	.66e-4 (2.49)	-.16e-4 (-1.21)	.17e-4 (3.64)	-.17e-4 (-2.19)
β_4	-0.0266 (-7.14)	-0.0290 (-3.76)	-0.1110 (-9.74)	-0.0061 (-3.22)	-0.0396 (-5.28)	-0.0273 (-3.45)	-0.0215 (-5.84)	-0.0123 (-4.29)
β_5	-0.63e-4 (-3.79)	-.52e-4 (-2.06)	-0.0017 (-5.78)	.15e-4 (1.54)	-.12e-4 (-2.56)	.25e-4 (1.24)	-0.33e-4 (-3.77)	.27e-4 (1.90)
β_6	.13e-5 (0.73)	.28e-4 (2.52)	.58e-4 (1.52)	-.17e-5 (-1.43)	.48e-5 (1.47)	-.71e-5 (-1.89)	-.73e-5 (-2.27)	.32e-5 (1.47)
LB(\check{X})	2776.3	898.5	582.4	1427.3	1438.7	1009.3	2584.8	1419.7
LB(ξ)	31.2	7.7	204.2	32.7	33.4	26.1	24.3	53.4
E(ξ)	0.9997	0.9993	0.9995	0.9998	0.9997	0.9995	0.9998	0.9996
St(ξ)	1.0656	1.1556	1.0764	1.1115	1.1382	1.0316	1.1028	1.1444
Ex.dis.	10.38	19.98	8.85	15.03	19.47	3.57	12.16	18.25
LM test								
A)	2.31	3.68	3.51	2.28	4.68	3.14	2.07	1.96
B)	16.98	0.12	7.88	0.83	0.22	0.01	1.53	0.01
C)	0.43	0.66	7.56	2.45	10.28	0.22	10.03	0.60

Table 4 gives the estimates of the trade equation defined as above. Several diagnostics are given. Ex.dis. is the Engle and Russell test for excess dispersion. The LM test are: A) Four hourly dummies, 9:30-10:00, 10:00-11:00, 11:00-12:00 and 15:00-16:00. B) Mean of 10 lagged QQ durations. C) Accumulated 10 signed volume. Numbers in italic boldface are significant on a 99% level, number in normal font are significant on a 95% level. The numbers typed with very small types are insignificant.

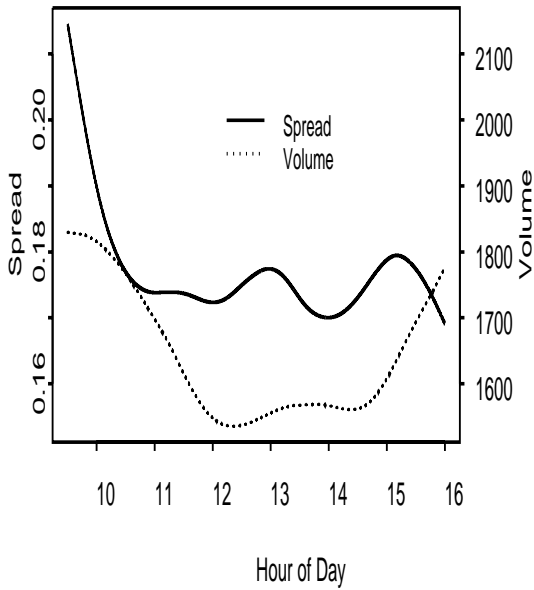
Table 6
Estimates for the quote equation

$$\begin{aligned} \ln(\varphi_i) = & \mu + \rho \ln(\varphi_{i-1}) + \delta_1 \frac{\tilde{y}_{i-1}}{\varphi_{i-1}} + \delta_2 \frac{\tilde{y}_{i-1}}{\varphi_{i-1}} d_{i-1} + \tau \frac{x_i}{\psi_i} + \eta_1 \frac{x_{i-1}}{\psi_{i-1}} + \eta_2 \ln(\widehat{\psi}_{i-1}) \\ & + \eta_3 \text{QQ.dur}_{i-1} + \eta_4 \text{lev.QQ}_{i-1} + \eta_5 \Delta \text{Spr}_{i-1} + \eta_6 \text{lev.Spr}_{i-1} + \eta_7 \sqrt{\text{vol}_{i-1}} \\ & + \eta_8 \text{lev.}\sqrt{\text{vol}_{i-1}} + \eta_9 \text{Abs(s.vol)}_{i-1} + \eta_{10} \text{Back.Q}_{i-1} \end{aligned}$$

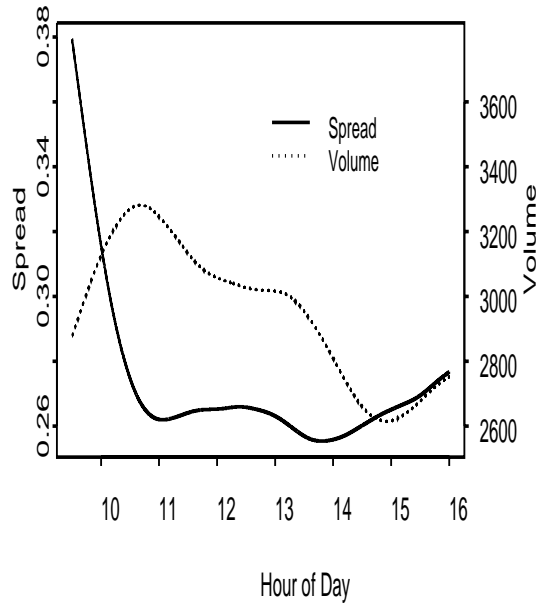
Par.	PG	DIS	FNM	GM	BAC	MCD	MTC	SLB
μ	-.1055 (-9.40)	-.0463 (-5.71)	-.0662 (-5.55)	-.0951 (-9.59)	-.0616 (-7.12)	-.1238 (-7.58)	.0189 (-1.1)	-.076 (-7.24)
ρ	.8787 (51.0)	.9324 (49.9)	.9094 (46.5)	.9141 (51.9)	.8922 (26.6)	.8501 (32.5)	.8680 (27.7)	.8831 (44.24)
δ_1	.0885 (8.62)	.0468 (4.33)	.0655 (6.27)	.0972 (8.25)	.0775 (3.94)	.0838 (7.94)	.0921 (6.64)	.067 (6.13)
δ_2	.0040 (0.40)	-.0112 (-1.15)	.0133 (1.22)	-.0123 (-1.23)	-.0010 (-0.08)	.0441 (3.86)	.0061 (0.48)	.0069 (.539)
τ	.2769 (17.0)	.2309 (7.51)	.2983 (6.23)	.1974 (6.87)	.3119 (11.3)	.4567 (8.65)	.3704 (7.74)	.293 (8.64)
η_1	-.2300 (-14.2)	-.2254 (-7.48)	-.3558 (-7.84)	-.1880 (-6.75)	-.2608 (-8.69)	-.4033 (-7.59)	-.304 (-5.78)	-.27 (-7.81)
η_2	-.0042 (-0.66)	-.76e-3 (-0.11)	-0.0219 (-1.78)	-.0280 (-5.25)	-.0106 (-1.63)	-.0451 (-2.71)	-.0487 (-3.98)	.022 (2.37)
η_3	.0130 (-4.35)	.0100 (3.13)	.0109 (2.11)	.0095 (3.03)	.0119 (3.10)	.0237 (3.82)	.0144 (2.15)	.012 (2.97)
η_4	-.56e-5 (-0.20)	0.91e-4 (2.29)	.83e-4 (1.51)	-.44e-4 (-0.94)	.94e-4 (1.06)	-.98e-4 (-0.82)	-.40e-3 (-3.07)	.008 (-1.64)
η_5	.2516 (16.8)	.1949 (8.81)	.4065 (11.0)	.2866 (11.6)	.1550 (9.39)	.3373 (9.18)	0.2729 (8.80)	.174 (7.66)
η_6	.35e-4 (1.65)	-.63e-4 (-2.05)	-.12e-4 (-0.29)	-.74e-5 (-0.18)	-.61e-4 (-1.71)	.67e-4 (0.72)	.45e-3 (3.08)	.005 (1.22)
η_7	.0490 (5.80)	.0133 (2.14)	.0314 (3.40)	.03705 (5.83)	.36e-3 (0.05)	.0347 (3.41)	-.0147 (-1.42)	.061 (6.08)
η_8	-.45e-4 (-0.93)	-.11e-4 (-0.32)	-.13e-3 (-1.12)	.10e-3 (1.40)	-.46e-4 (-0.44)	.41e-4 (0.44)	-.12e-3 (-2.35)	.036 (0.40)
η_9	-.18e-6 (-0.03)	-.33e-4 (-2.18)	.19e-4 (1.27)	-.37e-4 (-3.43)	-.80e-5 (-1.50)	.85e-5 (0.80)	-.11e-3 (3.05)	.57e-3 (0.05)
η_{10}	.0218 (5.02)	.0187 (4.47)	.0309 (4.209)	.0323 (6.36)	.0236 (3.46)	.0391 (4.39)	.0177 (2.69)	.0119 (2.21)
LB(\bar{Y})	6359.0	577.2	1654.8	4021.9	1113.6	2358.9	14183	1160.2
LB(ξ)	15.9	16.4	24.69	53.1	20.5	15.75	9.98	13.3
E(ε)	0.9998	0.9994	0.9996	0.9997	0.9996	0.9996	0.9998	0.9996
St(ε)	1.0473	1.0490	1.3036	0.9469	1.0799	1.3390	1.1298	1.1484
LM test	9.81	9.29	4.39	2.72	15.66	11.21	3.02	7.35

Table 5 gives the estimates of the quote equation defined as above. Several diagnostics are given. The LM test is: Four hourly dummies, 9:30-10:00, 10:00-11:00, 11:00-12:00 and 15:00-16:00. Numbers in italic boldface are significant on a 99% level, number in normal font are significant on a 95% level. The numbers typed with very small types are insignificant. T-statistics are in parentheses.

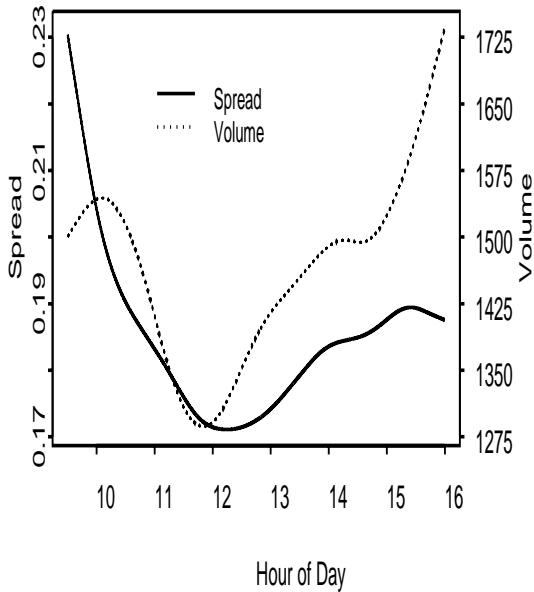
Schlumberger Limited



Monsanto Company



McDonald's Corporation



Bank American

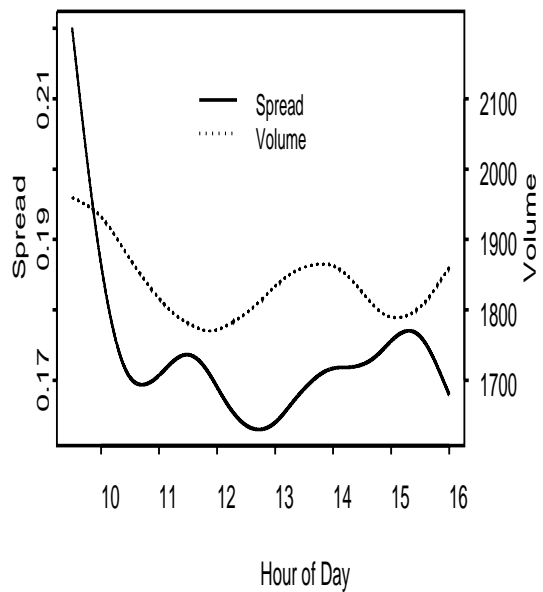
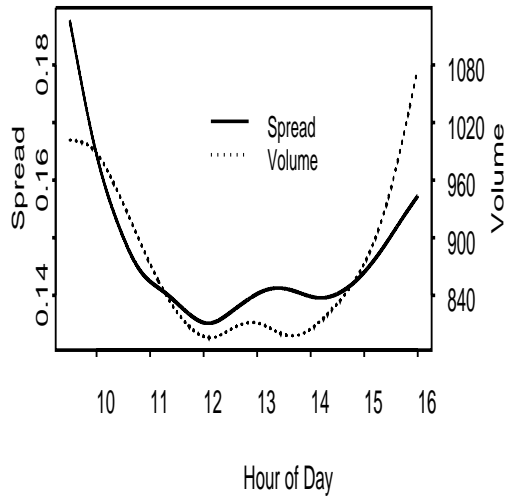


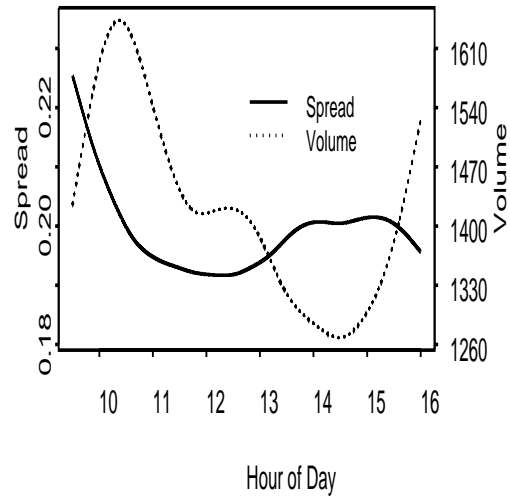
Figure 1a

Time-of-the-day splined mean of volume and spread

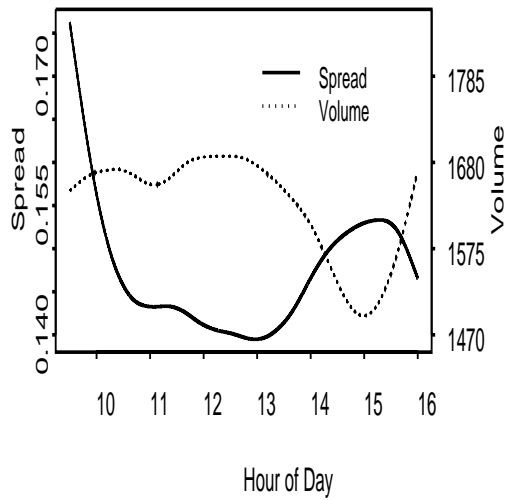
General Motors



Federal National Mortgage



Disney Company



Proctor & Gamble

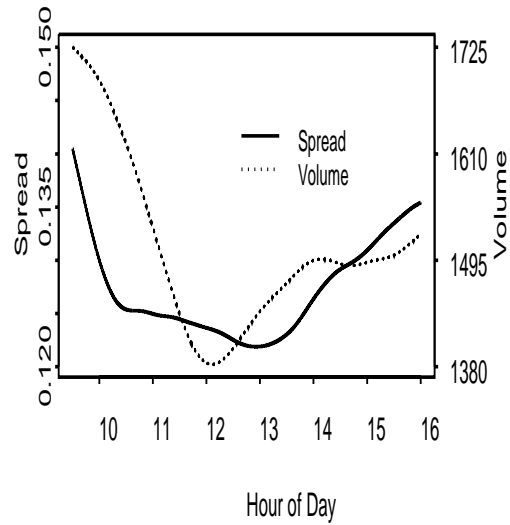


Figure 1b

Time-of-the-day splined mean of volume and spread