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NATURAL RESOURCES, GOODS, BADS AND ALTERNATIVE INSTITUTIONAL FRAMEWORKS

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Problems of determining optimal intertemporal use patterns for exhaustible and/or renewable natural resources can and have been analyzed in a number of settings. Much of this analysis began with the seminal work of Hotelling [20]. This work has been extended by Gordon [16], Scott [31], Crutchfield and Zellner [9], Turvey [41], Smith [35], Quirk and Smith [26] in the context of fisheries; by Herbindahl [19], Gordon [17], Scott [32], Cummings and Burt [10] in the context of mines; by Davidson [13] in the context of petroleum; by Brown and McGuire [5] and Burt [6] in the context of groundwater; by Plourde [27], Vousdan [42], Dasgupta and Heal [12], Solow [36,37], and Weinstein and Zeckhauser [44] in the context of exhaustible natural resources; by Anderson [1], and Stiglitz [38] in the context or macroeconomic growth and exhaustible resources; by Vousden [43] in the context of international trade and exhaustible resources; and by Smith [34], Burt and Cummings [7] and Rausser [28] in the context of resource production along with the rate of capital investment and associated technical progress.

In Solow's neoclassical framework, the continuous advances in technology, along with substitution among different inputs allow the economic resource base to expand. The stock of resources itself is not limited in either size of composition, with the result that per capita well being is in principal capable of continuous improvement. Solow's view of these issues is nicely summarized by a statement appearing in his Ely lecture [37, p. 11]:

"If it is easy to substitute other factors for natural resources, then there is in principal no 'problem'. The world can in effect get along without natural resources, so exhaustion is just an event, not a castastrophy."

The above observations and many of the conclusions derived from the recent conceptual work on natural resources is based on conventional neoclassical formulations with the names of the inputs altered. Many of these conceptual treatments restrict natural resources to only those goods which exchange on primary commodity markets and thus neglect the role of environmental common property resources. In an empirical context, this is not surprising since market transactions on the utilization of common property resources are nonobservable. For this reason the whole range of common property resources has been omitted from our system of accounts. Nevertheless, to adequately examine important policy issues, conceptual formulations must explicitly admit natural resources as original endowments which comprise the basic life support system.

For analytical purposes, many common property resources must be analyzed as though they were depleteable, or renewable but with maximum capacities. As d'Arge and Kogiku [11, p. 68] point out,

> "If extractive resources are finite in magnitude and can for all practical purposes be exhausted, then optimal environmental management involves a 'conjunctive use' type of allocation problem where one must consider rates of extraction and rates of waste generation. Thus, the 'pure' mining problem must be coupled with the 'pure' pollution problem and questions like these become relevant which should be run out of first, air to breathe or fossil fuels to pollute the air we breathe?"

In the above context, the purpose of this paper is to extend and modify the neoclassical paradigm to adequately account for the unique features of natural resources. These unique features relate to the common property nature of resources, pollution and associated externalities, environmental degradation, technological progress, and the possibility of maximum resource capacities. This extension of the conventional neoclassical paradigm can be most easily accomplished in the context of joint product goods and bads of productive inputs. These products are significant in the context of natural resources and admit a number of interesting conceptual issues. These conceptual issues should be brought front and center in the examination of alternative institutional frameworks and policies.

To be sure, once a natural resource has been extracted it is typically used as an input in the production of final goods. Most existing conceptual formulations either neglect this fact or reflect it by treating the natural resource as a factor (usually the only factor) in the production of some desired output. However, most natural resources have the characteristics of a joint input which leads not only to the production of desired outputs, but as well undesired byproducts. The latter products result in externalities which may be characterized as "bads". For example, once fossil fuels are extracted and employed in the production process they produce desired outputs along with smoke. Similar sorts of goods and bads result from the employment of minerals, water, and in some instances even fish. This joint input characteristic of natural resources results in multiple products in the final goods sector, some subset of which leads to degradation of environmental resources. This degradation process imposes an external cost to society in terms of the finite capacity of environmental resources and thus should be explicitly recognized. Unfortunately, these characteristics of the joint input properties of natural resources seem to have escaped existing paradigms.

Society's attempt to internalize the above external cost on the final goods sector has led to the emergence of a new industry, the pollution abatement industry. This industry also employs natural resources as factors in the production of emission and waste byproduct control devices. As yet, in this industry, little in the way of technical progress has been achieved (Cohen, et al. [8]). Nevertheless, this industry cannot be treated in isolation or as a predetermined component in the analysis of the interrelationships between the natural resource, final goods and environmental sectors.

A conceptual framework must be sufficiently general to encompass most natural resources, problems of stock and flow dynamics for both renewable and exhaustable resources, social control or regulation, public investment, and the importance of property rights arising under various institutional arrangements. These characteristics along with those previously noted should be explicitly incorporated in the conceptual formulation in order to adequately evaluate alternative resource user or royalty charges, technological or abatement subsidies, and pollution taxes. Some of the relevant policy questions include: are both taxes and subsidies needed? Under what conditions are the taxes and/or subsidies unique? What is the relationship between resource user charges and technological advancement? If pollution emissions cannot be adequately measured, does this influence the user charge and/or abatement subsidy level? For a public agency setting user charges, pollution taxes, technological, and/or abatement subsidies are the limits imposed by endogenous budgets? Under what conditions on endogenous policy budgets are only second best solutions possible.

2. Model

The economy envisaged consists of three principal sectors. In the first sector (N) production and investment take place to exploit available stocks of natural resources. The second and third sectors pertain to the production of final goods along with bads (Q) and pollution abatement goods (C), respectively.

2.1. Natural Resources Sector. The amount of resource stocks available during period t will be denoted by X_t. Associated with X_t is a rate of resource utilization, production, or extraction at time t, u_t. This rate is constrained by some function of X_t, the level of capital stock Y_t employed to exploit the natural resource, and the level of knowledge stocks or accumulated technical progress (W_t^u). Specifically,

$$(2.1) \qquad u_t \leq X^u(X_t, Y_t, W_t^u)$$

where $H^u(\cdot)$ is concave. The relationships $H^u(\cdot)$ is such that larger resource, capital, and knowledge stocks allow larger rates of production, i.e., $\partial H^u/\partial X_t$, $\partial H^u/\partial Y_t$, $\partial H^u/\partial W_t^u > 0$. As the stock of knowledge W_t^u is increased, the efficiency with which say minerals are extracted, gas is produced, or groundwater is utilized, is improved.

Once u_t is produced it may be allocated to either the Q or C sectors.[1] Define u_t^q as the amount of natural resource allocated to the Q sector and u_t^c as the amount of natural resource allocated to the C sector. The sum of these two amounts is constrained by u_t, i.e.,

$$(2.2) \qquad u_t^q + u_t^c \leq u_t$$

To complete the representation for this sector, the generation of the stocks X_t, Y_t, and W_t^u must be specified. These stocks are presumed, at beginning of any period t+1, to be obtained from

(2.3) $$X_{t+1} \leq X_t + G(u_t, X_t)$$

(2.4) $$X_{t+1} \leq Y_t + D^u(u_t, v_t, Y_t)$$

(2.5) $$W_{t+1}^u \leq W_t^u + L^u(u_t, W_t^u)$$

where each of the functions $G(\cdot)$, $D^u(\cdot)$, and $L^u(\cdot)$ are concave. The variable v_t denotes gross investment to the extraction capital stock.

The function $G(\cdot)$ obviously represents the net additions to natural resource stock during period t; its assumed properties are $\partial G/\partial X_t \leq 0$ and $\partial G/\partial X_t \geq 0$. For the case of exhaustible natural resources and no new discoveries $\partial G/\partial X_t = 0$, while for renewable natural resources $\partial G/\partial X_t > 0$. The depreciation function $D^u(\cdot)$ measures the net changes in extraction of capital stocks. This relationship is a nonincreasing function of capital stocks and an increasing function of gross investment, i.e., $\partial D^u/\partial v_t > 0$, and $\partial D^u/\partial Y_t \leq 0$. The presence of the additional variable u_t in $D^u(\cdot)$ reflects the consumption of capital resulting from current production, i.e., $\partial D^u/\partial u_t \leq 0$. Finally, the evolvement of technical progress (2.5) associated with allowable resource production (2.1) is stated in terms of the net learning function $L^u(u_t, W_t^u)$. This function is a generalization of the usual progress function found in the learning by doing literature (Oi[24], Rosen [30]). More specifically, the learning function includes as its arguments not only the rate of production but also the stock of knowledge. It is assumed that $\partial L^u/\partial W_t^u \leq 0$ and that learning is a positive function of resource production, i.e., $\partial L^u/\partial u_t > 0$.

2.2 Final Goods Sector. The quantity, u_t^q, produced in the N sector is employed as an input in the production of salable outputs. The q sector employs this input along with labor (r_t^q) to produce not only final consumption goods (q_t) but also the investment goods (v_t) purchased by the N sector. This transformation process will be represented by

$$v_t + q_t \leq H^q(u_t^q, r_t^q, c_t) \tag{2.6}$$

where c_t denotes gross investment in pollution abatement capital. The latter variable is incorporated in $H^q(\cdot)$ to reflect the difficult adjustments in production of salable outputs resulting from the introduction of new abatement capital. Following Lucas [23] and Treadway [40], the effect of this investment on the production of salable outputs is negative, i.e., $\partial H^q/\partial c_t < 0$. As usual, of course, the inputs have positive effects, i.e., $\partial H^q/\partial u_t^q > 0$, $\partial H^q/\partial r_t^q > 0$.

The utilization of u_t^q in this sector also results in production of by-products which have adverse effects on environmental quality. In other words, u_t^q has the characteristics of a joint input which, in part, is responsible for the production of goods q_t, v_t and, in part, responsible for the production of bads. The latter products degradate the environment, a process which is represented by

$$Z_{t+1} \geq Z_t + H^z(u_t^q, K_t, W_t^z, Z_t), \tag{2.7}$$

where

$$K_{t+1} \leq K_t + D^c(c_t, K_t) \tag{2.8}$$

and

$$W_{t+1}^z \leq W_t^z + L^z(c_t, W_t^z) . \tag{2.9}$$

The functions $H^z(\cdot)$, $D^c(\cdot)$ and $L^z(\cdot)$ are presumed concave with properties $\partial H^z/\partial u_t^q > 0$, $\partial H^z/\partial K_t < 0$, $\partial H^z/\partial W_t^z < 0$, $\partial H^z/\partial Z_t \leq 0$, $\partial D^c/\partial c_t > 0$, $\partial D^c/\partial K_t < 0$, $\partial L^z/\partial c_t > 0$, and $\partial L^z/\partial W_t^z \leq 0$.

The function $H^z(\cdot)$ represents the net flow of pollution from the final goods sector; the first three arguments contribute to the rate of externality output while the fourth argument reflects the rate of environmental assimilation. This relationship depends upon climatic conditions, geography, and chemical reactions. As noted in Tietenberg [39], the relationship should summarize the principal phases of pollution dispersion, namely transport, dillution, depletion, and reaction.[2] This function in conjunction with (2.6) implies a production structure for the final goods sector in which the transformation between any salable output and the pollution externality is represented by a single point. That is, given fixed amounts of all inputs, the amounts of salable and externality outputs cannot be varied.[3]

Equation (2.8) describes the stock of pollution abatement capital available to the final goods sector. The depreciation function $D^c(\cdot)$ has the same basic characteristics as $D^u(\cdot)$ in the resource sector. In the case of knowledge accumulation (2.9), learning is presumed to result from the investment and operation of emission control devices (c_t). This physical learning process can be transferred into cost reductions of controlling emissions due to increased investments in pollution abatement capital.

2.3 Pollution Abatement Goods Sector. To simplify the analysis, the amount of natural resource employed in this sector u_t^c will be presumed to contribute only to the production of c_t but not the degradation of environmental quality.[4] This specification involves little loss in generality and its relaxation overly complicates the analysis. In any event, this sector will be characterized by

$$c_t \leq H^c(u_t^c, r_t^c, W_t^c) \tag{2.10}$$

where

$$W_{t+1}^c \leq W_t^c + L^c(c_t, W_t^c) \tag{2.11}$$

and both functions $H^c(\cdot)$ and $L^c(\cdot)$ are assumed concave. The bound on the production of c_t has the usual properties of a production function, i.e., $\partial H^c/\partial u_t^c > 0$, $\partial H^c/\partial r_t^c > 0$, and $\partial H^c/\partial W_t^c > 0$. Equation (2.11) has the basic properties of equations (2.5) and (2.9), i.e., the learning is a positive function of c_t ($\partial L^c/\partial c_t > 0$) and a nonincreasing function of previously accumulated knowledge ($\partial L^c/\partial W_t^c \leq 0$).

The presence of W_t^c in $H^c(\cdot)$ implies that as learning experience associated with manufacturing of pollution abatement equipment increases, the allowable rate of producing c_t is augmented. Hence, the principal distinction between the processes (2.9) and (2.11) is that the former pertains to learning resulting from the operation while the latter pertains to the manufacture of abatement equipment.

2.4 Population Component. Labor employed in both the Q and C sectors is constrained by the working population in each period t. This population (r_t) will be treated exogenously. In other words,

$$r_t \geq r_t^q + r_t^c \tag{2.12}$$

2.5 Additional Constraints. Any optimal program in the above economy must, of course, satisfy equations (2.1) through (2.12). These equations represent constraints to any societal optimization problem where the stock values X_t, Y_t, Z_t, K_t, W_t^c, W_t^z, and W_t^u may be viewed as state variables, and the flow values u_t^q, u_t^c, u_t, v_t, r_t^q, r_t^c, q_t, and c_t may be viewed as control or decision variables. In addition to these constraints, all control and state variables are restricted to nonnegative values. That is,

$$u_t^q,\ u_t^c,\ u_t,\ v_t,\ r_t^q,\ r_t^c,\ q_t,\ c_t \geq 0 \qquad (2.13)$$

$$X_t,\ Y_t,\ Z_t,\ K_t,\ W_t^c,\ W_t^z,\ W_t^u \geq 0$$

for all t. Furthermore, the selection of the optimal path is constrained by initial conditions on each of the state variables, i.e.

$$\hat{X}_o = \hat{X},\ Y_o = \hat{Y},\ Z_o = \hat{Z},\ K_o = \hat{K},\ W_o^c = \hat{W}^c,\ W_o^z = \hat{W}^z,\ W_o^u = \hat{W}^u\ . \qquad (2.14)$$

2.6 Societal Criterion Function. To complete the specification of the (centralized) optimization problem, we require a measure of societal welfare by which alternative time paths of the state and control variables may be evaluated. Such a function is presumed to exist for a planning horizon of length T. In general form, this function is also assumed stationary, additive, and concave over each period of the specified planning horizon. It is designated as $U(q_t, Z_t)$ for each period t to reflect the value society places on the consumption of goods (q_t) and bads (Z_t). The three sectors N, Q, and C emerge to facilitate these societal values. A centralized organization is presumed to exist which desires to maximize the present value of societal values over the interval [0, T]. For the case in which the planning horizon is finite, i.e., $T < \infty$, a concave value function, $\Psi(X_T, Y_T, Y_T, Z_T, W_T^c, W_T^u, W_T^z)$, associated with the terminal levels of the stock variables will be specified so as to establish some continuity with future periods beyond T.

2.7 Necessary Centralized Conditions. Summarizing the above discussion more formally, the postulated centralized optimization problem may be stated as

$$(2.15) \qquad \text{Max} \left[\sum_{t=o}^{T-1} \beta^t U(q_t, Z_t) + \beta^T \Psi(X_T, Y_T, Z_T, W_T^c, W_T^u, W_T^z) \right]$$

subject to (2.1) through (2.14) where the variables of optimization are u_t^q, u_t^c, u_t, v_t, r_t^q, q_t, c_t, $t = 0, 1, \ldots, T - 1$ and X_T, Y_t, Z_t, K_t, W_t^c, W_t^z, W_t^u, $t = 1, \ldots T$; T is assumed finite; Max denotes the maximum operator; and $\beta = 1/(1 + i)$, i being the appropriate positive discount rate.

Assuming the usual limiting and continuity properties for the functions (2.1) through (2.15), the above optimization problem can be treated within a Kuhn-Tucker analytical framework. For this framework, the relevant Lagrangian function ($\mathcal{L}$) and necessary conditions are represented in the Appendix.[5] As these relationships indicate the decision process corresponds to a Markovian dependence structure, i.e., given levels of the stock variables, the optimal levels of control variables in period are independent of the control variables in the previous periods, say $j < t$. The stock or state variables completely summarize the influence of all previous decisions upon current and future optimal actions. These centralized conditions are examined and given economic interpretations in the following section.

3. Economic Interretations and the Steady-State

The model outlined in Section 2 is a description of a fairly complex economy in which there are stocks of two capital goods, learning stocks, a resource stock and a pollution stock. The framework incorporates the relation between choices of control variables (such as resource extraction) and the impact of these control variables on future stock levels. Within this framework, the central planner must choose the level of the control variables at each t, subject to the appropriate constraints on these activities, in order to maximize the present discounted value of utility, plus some terminal value function. This allocational decision is greatly complicated by the intertemporal affects of any such decision; for example, a change in resource extraction will affect the future stock of the resource, as well as the learning stock and the extraction capital stock. Similarly, an increased utilization of the resource in the Q sector ceteris paribus will affect the pollution stock and the stock of pollution abatement equipment (along with the learning stocks associated with the production of C), in addition to altering the current output of consumer goods. For an optimal program, the planner must, at the margin, balance these costs.

As a practical matter, the solution of the optimizing equations, and a characterization of the optimal path (see Appendix) is an extremely complex task, particularly for the case of a finite planning horizon. Intuitively, an optimal program must meet several criteria:

(i) It must equate the marginal benefit of extra consumption with the increased costs due to higher pollution levels. The tradeoff between consumption and pollution occurs in two ways: a) divert resources from the Q sector to the C sector, thereby decreasing future pollution, but also decreasing consumption; b) by, for given output of C, varying the use of natural resources and labor in the consumption sector. Since only resource use in the Q sector creates pollution, it is possible to increase output of Q, given C, at the expense of higher pollution levels by allocating more of the resource to the Q sector. This implies that, for an optimal program, the MRTS will differ between sectors.

(ii) In addition, an optimal program must concern itself with the intertemporal production of consumption and pollution.[6] Because the utility function is assumed concave, it is desirable to "smooth" out the flow of consumption as well as adverse flows of pollution; thus resource extraction decisions and investments in durable goods must attempt to achieve this intertemporal balance. In particular, this implies that resource extraction, especially during early stages of the plan, may occur at less than maximal rates (particularly for a nonrenewable resource) in order to transfer this resource intertemporally. However, the "excess capacity" in the extraction sector does not imply that no capital investment is needed since later periods will generally be characterized by operating at full extraction capacity. Thus, both current abstention from resource utilization and investment in extraction capital can be used to transfer the resource intertemporally, and thereby obtain a more even flow of consumption.

Furthermore, since pollution is treated as a stock variable, similar intertemporal considerations hold with respect to creation of new pollution flows (through resource utilization in Q) and to investment in abatement equipment. Unfortunately, little of a specific nature can be said about the optimal path, since this solution will depend upon the functional forms and the terminal value function.

The case of an infinite planning horizon proves more tractable and permits insight into the fundamental nature of the optimal solution. If the resource is nonrenewable, an infinite planning horizon is impractical unless either: (i) the resource is not essential in the production of commodities, or (ii) resource-augmenting technical progress occurs in the final goods sector. The former case has been analyzed by Vousden, while the latter case has been studied by Anderson. Thus, in the analysis that follows we restrict attention to the case of a renewable resource.

Assuming the resource is renewable, then a steady-state solution is achievable if: a) the learning stocks are bounded, and b) if pollution stocks can be held at some finite level. The former condition can be fulfilled by the assumption of knowledge decay: $\partial L^i/\partial W^i < 0$, $\partial^2 L^i/\partial (W^i)^2 \leq 0$, whereas the latter condition can be met (assuming new pollution emissions can never be reduced to zero if $U^q > 0$) if there is some decay of the pollution stock: $(\partial H^z/\partial Z) < 0$. Ecologically, this decay reflects the ability of the environmental system to purify itself.

Thus, assuming renewability of the resource, a steady-state solution can exist.[7] However, as in the case of the neoclassical one sector growth model, alternative steady-states exist corresponding to different rates of resource extraction (unless $\partial G/\partial X \leq 0$), and to different levels of K, Z, q, etc. It is the task of the planner to choose steady-state, among the set of all possible steady-state paths, which maximizes social welfare.[8]

From the model outlined in Section 2, there are 12 equations and 15 unknowns (in the steady-state all equations hold with equality; thus the extraction sector will operate at capacity - otherwise the marginal product of Y will be zero). Hence, there are three degrees of freedom in determining the steady-state path. The additional information needed to determine the optimal steady-state path is, of course, obtained from optimizing the Lagrangean function and solving for the steady-state values. Given this specification, we are free to choose the level of three control (control or stock) variables in determining the optimal steady-state solution. For example, a choice of u uniquely determines the stationary values of X, W^u, Y and hence v. Similarly, a choice of c determines K, W^z, and W^c. Finally, the choice of u^q determines r^q, (and hence q), u^c, r^c. and Z thereby completing the description of the steady-state solution.

Intuitively, to resolve the three degrees of freedom for the optimal steady-state three efficiency can be inferred by treating the model as though it were a two-commodity, two-capital good model. In such a model an optimum steady-state allocation would be characterized by equating the net marginal product on each capital good to the discount rate, and by equating the marginal rate of transformation between commodities to the marginal rate of substitution. But these three conditions are precisely those needed in our model. For example, a choice of u uniquely determines the steady-state levels of the resource stock (X) and of the extraction capital (Y). Thus, Y (or X) can be viewed as one capital good. Similarly, abatement capital (K) represents the second capital good, and a choice of the steady-state level of K uniquely determines c, and the learning stocks W^c, W^z. In the optimum steady-state, the return on each of these capital goods must be equal to the discount rate.[9]

Finally for given (Y, X, W^u, v) and K (C, W^c, W^z), it is still possible to alter the output of consumer goods (q) and the pollution stock (Z) through reallocating the natural resource and labor across sectors. Thus, the third efficiency condition states that the marginal rate of transformation between (the steady-state levels of) q and Z must equal the marginal rate of substitution between these commodities.

Formally, the three efficiency conditions obtained from the optimization procedure are:

$$(3.1) \qquad 1 - \frac{\frac{\partial H^u}{\partial X^u} \cdot \frac{\partial G}{\partial u}}{\left(i - \frac{\partial G}{\partial X^u}\right)} - \frac{\frac{\partial H^u}{\partial W^u} \cdot \frac{\partial L^u}{\partial u}}{\left(i - \frac{\partial L^u}{\partial W^u}\right)} - \frac{\frac{\partial H^u}{\partial Y^u} \cdot \frac{\partial D^u}{\partial u}}{\left(i - \frac{\partial D^u}{\partial X^u}\right)} - \frac{\frac{\partial H^u}{\partial Y^u} \cdot \frac{\partial D^u}{\partial v} \cdot \frac{\partial H^q}{\partial r^q} \cdot \frac{\partial H^c}{\partial u^c}}{\left(\frac{\partial H^c}{\partial r^c}\right)\left(i - \frac{\partial D^u}{\partial Y^u}\right)} = 0$$

$$(3.2) \qquad 1 - \frac{\frac{\partial H^c}{\partial W^c} \cdot \frac{\partial L^c}{\partial c}}{\left(i - \frac{\partial L^c}{\partial W^c}\right)} - \frac{\frac{\partial H^c}{\partial r^c} \cdot \frac{\partial H^q}{\partial c}}{\frac{\partial H^q}{\partial r^q}}$$

$$+ \frac{\left(\frac{\partial H^c}{\partial r^c}\right)}{\left(\frac{\partial H^z}{\partial u^q}\right)}\left[\frac{\frac{\partial H^q}{\partial u^q}}{\frac{\partial H^q}{\partial r^q}} - \frac{\frac{\partial H^c}{\partial u^c}}{\frac{\partial H^c}{\partial r^c}}\right]\left[\frac{\frac{\partial H^z}{\partial K} \cdot \frac{\partial D^c}{\partial c}}{\left(i - \frac{\partial D^c}{\partial K}\right)} + \frac{\frac{\partial H^z}{\partial W^q} \cdot \frac{\partial L^z}{\partial c}}{\left(i - \frac{\partial L^z}{\partial W^z}\right)}\right] = 0$$

$$(3.3) \qquad \frac{\partial U}{\partial q}\left[\frac{(\partial H^q/\partial u^q)}{(\partial H^q/\partial r^q)} - \frac{(\partial H^c/\partial u^c)}{(\partial H^c/\partial r^c)}\right] = \frac{-\left(\frac{\partial U}{\partial Z}\right)}{\left(i - \frac{\partial H^z}{\partial Z}\right)}\left(\frac{\partial H^z/\partial u^q}{\partial H^q/\partial r^q}\right) > 0$$

Note that, as in the case of standard growth models, the utility function does not appear in (3.1) or (3.2); these equations reflect the condition that the net rate of return on capital must equal the discount rate. On the other hand, (3.3) reflects the equality between the MRT and MRS of consumption for pollution. Furthermore, note that because of the pollution associated with resource utilization in the Q sector, the marginal rate of technical substitution of the inputs is not equalized across sectors.

It can readily be verified that (3.3) does indeed reflect the optimal choice of the consumption, pollution, point on the transformation frontier. For example, assume resource extraction (u, and hence X, Y, W^u) is fixed, as is the level of production of c (and thus W^c, W^z, and K). Consider whether any reallocation of resources between sectors can raise discounted welfare:

(3.4) $$du^q + du^c = 0; \; dr^q + dr^c = 0$$

(3.5) $$dc = \left(\frac{\partial H^c}{\partial U^c}\right) du^c + \frac{\partial H^c}{\partial r^c} \cdot dr^c = 0 .$$

Assume there is an increase in u^q in period 1; this increases pollution emissions and raises pollution stocks in period 2. Further, assume du_2^q is chosen so that pollution stocks return to their original level in period 3. Assuming u^q also returns to its original level in period 3, the only changes in consumption and pollution occur in periods 1 and 2. Thus, let:

(3.6) $$du_c^q = \varepsilon ; \quad dZ_2 = \left(\frac{\partial H^z}{\partial U^q} \cdot \varepsilon\right)$$

(3.7) $$dZ_3 = dZ_2 \left[1 + \frac{\partial H^z}{\partial Z}\right] + \frac{\partial H^z}{\partial u^q} \cdot du_2^q = 0 \implies du_2^q = \varepsilon \left(1 + \frac{\partial H^z}{\partial Z}\right)$$

Utilizing (3.4) and (3.5) we find:

$$\text{(3.8)} \qquad dq_1 = \left[\frac{\partial H^q}{\partial u^q} - \frac{\partial H^q}{\partial r^q} \cdot \left(\frac{\partial H^c/\partial u^c}{\partial H^c/\partial r^c}\right)\right] \varepsilon$$

$$\text{(3.9)} \qquad dq_2 = -\left[\frac{\partial H^q}{\partial u^q} - \frac{\partial H^q}{\partial r^q}\left(\frac{\partial H^c/\partial u^c}{\partial H^c/\partial r^c}\right)\right] \varepsilon \cdot \left(1 + \frac{\partial H^z}{\partial Z}\right)$$

The change in welfare is:

$$\text{(3.10)} \qquad dU = \frac{\partial U}{\partial q_1} \quad dq_1 + \frac{\frac{\partial U}{\partial q_2} \cdot dq_2}{(1 + i)} + \frac{\frac{\partial U}{\partial Z} \cdot dZ_2}{(1 + i)}$$

Substituting (3.6), (3.8), and (3.9) in (3.10), and setting dU = 0, we obtain:

$$\text{(3.11)} \qquad dU = \varepsilon \left(\frac{\partial H^q}{\partial r^q}\right)\left(\frac{i - \frac{\partial H^z}{\partial Z}}{1 + i}\right)\left[\frac{\partial U}{\partial q}\left(\frac{\partial H^q/\partial u^q}{\partial H^q/\partial r^q} - \frac{\partial H^c/\partial u^c}{\partial H^c/\partial r^c}\right) + \frac{\frac{\partial U}{\partial Z} \cdot \frac{\partial H^z}{\partial u^q}}{\left(i - \frac{\partial H^z}{\partial Z}\right)\frac{\partial H^q}{\partial r^q}}\right] = 0 ,$$

which is precisely (3.3).

In a similar fashion it can be shown that (3.1) and (3.2) reflect the result that the net rate of return on capital equals the discount rate. For example, (3.2) can be derived by assuming there is a slight increase in investment in abatement capital in period o, ($dc_o = \varepsilon$), followed by no further investment. Choosing input combinations such that the change in pollution is zero in each period, (3.2) then reflects the condition that the present discounted value of the change in consumption must be zero at an optimum, i.e., that the net rate of return on capital equals the rate of time preference. Not only is the productivity of capital crucial in determining the optimum, but the learning associated with the investment and the loss in production associated with installation of the equipment also influences the actual steady-state values.

Similarly, (3.1) states that the net rate of return on extraction capital must equal the discount rate. This can be shown by assuming there is an initial increase in investment in extraction capital in period 1 (and hence a decrease in consumption), and by calculating the change in the present discounted value of consumption due to such a change (for constant stocks of pollution). At an optimum, the change in the discounted value of the stream of consumption must be zero, as is indicated by (3.1). As for abatement equipment, the return to extraction capital depends not only on the marginal physical product of capital but also on the change in the resource stock and learning stock due to changes in research extraction, as well as on the depreciation rates. By analogy to neoclassical growth models, if we assume there is no learning, no depreciation due to extraction (just exponential decay), and no depletion of the resource due to extraction (so that $u = H^u(Y)$), then (3.1) reduces to:

$$\text{(3.1')} \qquad \frac{\partial H^u}{\partial Y^u}\left(\frac{\partial D^u}{\partial v}\right)\left(\frac{\partial H^q}{\partial r^q}\right)\left(\frac{\partial H^c/\partial u^c}{\partial H^c/\partial r^c}\right) = \left(i - \frac{\partial D^u}{\partial Y^u}\right)$$

But (3.1') is precisely the modified golden rule of the neoclassical growth models; the LHS of (3.1') represents the marginal product of investment in extraction capital (assuming $du^q = 0$, so that $dZ = 0$). Thus, both equations (3.1) and (3.2) are simply "disguised" versions of the modified golden rule that determine optimal levels of capital stocks.

In summary, for the rather complex model of Section 2, the efficiency conditions used to determine the optimum steady-state path are similar to the usual results of growth theory. However, because of the learning associated with several activities, these rules must be modified to incorporate the indirect benefits associated with resource extraction or investment in abatement equipment. Furthermore, the fact that resource utilization in the Q sector causes pollution must also be recognized, leading to a disparity in the MRTS of inputs between sectors, and hence to the need for some form of government intervention to support this solution. In Section 4 we discuss how prices can be used to sustain the optimal solution.

4. Alternative Institutional Frameworks

In the previous section, a centralized planning economy was analyzed. To admit private control and decentralized actions, in this section we examine four alternative institutional structures. For each of these structures, it is shown how the steady-state allocation described in Section 3 can be supported by means of a price system. The first structure assumes the production of the resource is controlled by a central authority, but that the Q and C sectors are decentralized and operate under competitive conditions. For the second structure, the resource sector is also decentralized and again we show how prices can be employed to support the steady-state solution. In the third structure, endogenous governmental budgets are examined while in the fourth structure we recognize that it is difficult, if not impossible, to accurately measure environmental quality (Z). Throughout the analysis for each of these four structures, at the steady state $r_t = r^*$, i.e., zero population growth will be presumed.

Hence, it makes no difference, from an efficiency standpoint, whether we operate with a total or per capita societal criterion function.

4.1 Centralized N and Decentralized Q and C Sectors. For this structure, the number of firms in the final goods sector and in the capital abatement sector are specified as fixed and identical. Furthermore, we assume that firms behave as price-takers and face the following prices which they assume are static over their planning horizon:

p^z - price per unit on gross emissions of pollution;

p^{ui} - price of the resource to sector i; i = C, Q;

p^{ri} - price of labor to sector i; i = C, Q;

p^c - price paid to producer of pollution abatement equipment;

p^k - purchase price to users of pollution abatement equipment; and

p^q - price of the final good.[10]

Let the number of (identical) firms in C be n^c, and let the number of (identical) firms in sector Q be n^q.[11] Further, assume that the production function for each firm in C is:

$$c^i \leq F^c(c^{ci}, r^{ci}, W^c); \; i = 1, \ldots, n^c; \tag{4.1}$$

and that W^c (and W^z) are treated as pure externalities by each firm in C. Then the profit-maximizing conditions for each of these firms are given by the usual static conditions:[12]

$$p^c \frac{\partial F^c(\;)}{\partial u^{ci}} \leq p^{uc}; \qquad u^{ci}[p^{uc} - p^c \frac{\partial F^c}{\partial u^{ci}}] = 0 \tag{4.2}$$

$$p^c \frac{\partial F^c}{\partial r^{ci}} \leq p^{rc}; \qquad r^{ci}[p^{rc} - p^c \frac{\partial F^c}{\partial r^{ci}}] = 0 \tag{4.3}$$

The profit-maximizing conditions for the final goods sector are more complicated since these conditions are inherently dynamic. Assuming firms maximize the present discounted value of profits over (an infinite) planning horizon, that the discount factor, β, is the same as society's, and that firms have static expectations with respect to prices, we have:

$$(4.4) \qquad \Pi_i = \sum_{t=0}^{\infty} \Big\{ \beta^t \, [p^q F^q(u_t^{qi}, r_t^{qi}, c_t^i) - p^z G^z(u_t^{qi}, K_t^i, W_t^z) - p^{rq} r_t^{qi} - p^{uq} u_t^{qi} - p^k c_t^i] + \beta^{t+1} \lambda_{t+1} [K_t^i + J(c_t^i, K_t^i) - K_{t+1}^i] \Big\}$$

where $F^q(u_t^{qi}, r_t^{qi}, c_t^i)$ is the production function of firm i and $G^z(\cdot)$ is the gross emissions of firm i. Furthermore, we assume W^z is external to each firm, and that gross emissions of pollution can be separated from the decay of the existing stock of pollution.[13]

The optimizing conditions, assuming the controls u^{qi}, r^{qi}, c^i are used at positive levels and that the stock variable K^i is positive for all i, may be aggregated across firms to obtain[14]

$$(4.5) \qquad p^q \frac{\partial H^q}{\partial u^q} - p^z \frac{\partial H^z}{\partial u^q} = p^{uq}$$

$$(4.6) \qquad p^q \frac{\partial H^q}{\partial r^q} = p^{rq}$$

$$(4.7) \qquad \frac{p^z \left|\frac{\partial H^z}{\partial K}\right| \frac{\partial D^c}{\partial c}}{(i - \frac{\partial D^c}{\partial K})} - p^q \left|\frac{\partial H^q}{\partial c}\right| = p^k$$

Given the behavior of firms, as summarized in (4.2), (4.3), (4.5), (4.6) and (4.7), the task of the central planner is to choose a price vector p* (as well as the output and investment decisions for the resource sector) that will induce firms to produce and utilize factors at their optimal steady-state level.

It is well-known that if the prices are equated to the shadow prices of the corresponding control (or stock) variables, then this system of prices will support the optimal steady-state solution (given the appropriate initial conditions on K^i). Specifically, let:[15]

(4.8) $$p^{rq} = p^{rc} = \bar{\alpha}^r$$

(4.9) $$p^q = \bar{\phi}^q$$

(4.10) $$p^{uq} = p^{uc} = \bar{\gamma}$$

(4.11) $$p^c = \bar{\phi}^c$$

(4.12) $$p^z = \beta\bar{\phi}^z$$

(4.13) $$p^k = \beta\bar{\eta}^k \frac{\partial D^c}{\partial c} - \bar{\phi}^q \left|\frac{\partial H^q}{\partial c}\right|$$

Note that $p^z = \beta\bar{\eta}^k$ because gross emissions in t affect the pollution stock in (t + 1). In addition, since p^k is the user price for new abatement equipment (c), the price reflects the lag in installation of that equipment and the fact that installation reduces current output. Finally, the subsidy to be applied to abatement equipment due to learning (W^c, W^z) associated with production of c is S*:

(4.14) $$S^* = p^c - p^k = \bar{\phi}^c - \beta\bar{\eta}^k \frac{\partial D^c}{\partial c} + \phi^q \left|\frac{\partial H^q}{\partial c}\right| = \beta\bar{\omega}^z \left|\frac{\partial W^z}{\partial c}\right| + \beta\bar{\omega}^c \frac{\partial W^c}{\partial c} ,$$

as determined from (A.9) of the Appendix.

Given the prices (4.8) - (4.13), if the profit-maximizing conditions (4.2), (4.3), (4.5) and (4.6) are satisfied, then it follows that conditions (A.2), (A.3), (A.7) and (A.8) of the Appendix are also satisfied. Therefore, if the appropriate production and investment decisions are made for the resource sector, and given that markets clear, this price system will support the optimal solution.

However, it is apparent that this price system is not unique (given the choice of numeraire). For example, a change in p^c will not affect resource allocation, provided that p^{uc} and p^{rc} are altered accordingly. Similarly, a change in p^z will not affect the profit-maximizing conditions if p^{uq} and p^k are suitably adjusted. Thus, there are two degrees of freedom in specifying the supporting price system.

Specifically, let:

(4.15) $$p^q = \overline{\phi}^q$$

(4.16) $$p^c = \overline{\phi}^c + d_1$$

(4.17) $$p^z = \beta\overline{\phi}^z + d_2$$

and choose the remaining prices as follows:

(4.18) $$p^{rc} = \overline{\alpha}^r + d_1 \left.\frac{\partial H^c}{\partial r^c}\right)^*$$

(4.19) $$p^{uc} = \overline{\gamma} + d_1 \left.\frac{\partial H^c}{\partial u^c}\right)^*$$

(4.20) $$p^{rq} = \overline{\alpha}^r$$

(4.21) $$p^{uq} = \bar{\gamma} - d_2 \left.\frac{\partial H^z}{\partial u^q}\right)^*$$

(4.22) $$p^k = \beta\bar{\eta}^k \left.\frac{\partial D^c}{\partial c}\right)^* - \bar{\phi}^q \left|\frac{\partial H^q}{\partial c}\right|^* + \frac{d_2 \left|\frac{\partial H^z}{\partial K}\right|^* \left(\frac{\partial D^c}{\partial c}\right)^*}{\left(i - \frac{\partial D^c}{\partial K}\right)^*}$$

Hence, the subsidy on new capital becomes:

(4.23) $$S = S^* + d_1 - \left[\frac{d_2 \left(\left|\frac{\partial H^z}{\partial K}\right|^*\right)\left(\frac{\partial D^c}{\partial c}\right)^*}{\left(i - \frac{\partial D^c}{\partial K}\right)^*}\right]$$

where S* is defined in (4.14), and * in (4.18) - (4.23) indicates as before that the partial derivatives are evaluated at their steady-state levels. Give prices (4.15) - (4.23), it is apparent that the profit-maximizing conditions are satisfied at the steady-state solution. Thus, the planner has two degrees of freedom in choosing prices.

To provide further elaboration, suppose the planner wishes to reduce the subsidy $(p^c - p^k)$ on new pollution-abatement capital, while maintaining the same steady-state solution. This can be accomplished by raising the price charged on new pollution emissions $(d_2 > 0)$; this increase in p^z leads to altered demand for resources in the final good sector. In order to achieve the same resource allocation pattern, the user price of new abatement equipment must be increased, while the use of the resource in the final good sector must be subsidized. By choosing these price variations as in (4.21) and (4.22), he will leave the demands for each factor unchanged.[16]

4.2 Decentralized N Sector. For this structure we demonstrate how prices can be used to support decentralization of decisions in the resource sector. In our analysis, the behavior of competitive firms in the other two sectors will remain unaltered.

The competitive solution for the resource sector is complicated by the fact that the firms share a common property resource. Let the number of firms, n^u, be fixed and assume that each firm has the following (identical) production function:

$$u_t^i \leq F^{ui}(X_t^u, Y_t^{ui}, W_t^u); \; i = 1, \ldots, n^u \tag{4.24}$$

In (4.24) u_t^i is the resource output of each firm, Y_t^{ui} is the extraction capital of each firm, W_t^u is the common pool of knowledge and X_t^u is the common property resource. The intertemporal profit-maximizing behavior of firms depends upon their perceptions as to how their own output decisions (u_t^i) affect the stock of learning (W_t^u) and the stock of the common property resource (X_t^u). If firms perceive that their decisions affect these stocks and realize that the decisions of other firms also affects these stocks, then an optimal decision (for each firm) requires expectations on the plans of other firms in the industry.

To simplify, following Quirk and Smith [26] we assume that the number of firms (n^u) is sufficiently large, so that each firm assumes its decisions have a negligible impact on X_t^u and W_t^u. Furthermore, to be consistent with the steady-state solution, we assume firms expect these stocks to remain constant through time. Then the only decision to be made by each firm is how much to invest (v_t^i) in each time period. Let:

p^o - price paid to producer for resource; and

p^I - price paid by producer for new investment.

These prices are assumed (by firms) to remain constant through time. For each firm we have:

(4.25) $$Y_{t+1} \leq Y_t + M^i\,(u_t^i,\ v_t^{\ i},\ Y_t^{\ i})$$

and thus discounted profits for the ith firm are given by:

(4.26) $$\Pi^i = \sum_{t=0}^{\infty} \beta^t [p^o u_t^i - p^I v_t^{\ i} + \lambda_t\ (F^{ui}(X_t^u,\ Y_t^{\ i},\ W_t^u) - u_t^i)]$$
$$+ \beta^{t+1}\theta_{t+1}[Y_t^{\ i} + M^i(u_t^i,\ v_t^{\ i},\ Y_t^{\ i}) - Y_{t+1}^{\ i}]$$

Optimizing with respect to u^i, v^{ui}, and Y^{ui}, and solving for the steady-state solution we obtain:

(4.27) $$p^o = \frac{p^I \left[\dfrac{\dfrac{\partial F^{ui}}{\partial Y^{\ i}}\ \dfrac{\partial M^i}{\partial u^i}}{(i - \dfrac{\partial M^i}{\partial Y^{\ i}})} \right]}{\dfrac{\dfrac{\partial F^{ui}}{\partial Y^{\ i}}\ \dfrac{\partial M^i}{\partial v^{\ i}}}{(i - \dfrac{\partial M^i}{\partial Y^{\ i}})}}$$

If we let:

(4.28) $$p^o = \alpha_1\ \overline{Y};\ p^I = \alpha_2\ \overline{\phi}^q,$$

and aggregate across firms,[17] it follows that

$$(4.29) \qquad \alpha_1 \bar{\gamma} = \alpha_2 \bar{\phi}^q \left[1 + \frac{\frac{\partial H^u}{\partial Y} \left|\frac{\partial D^u}{\partial u}\right|}{(i - \partial D^u/\partial Y)} \right] \Bigg/ \left[\frac{\frac{\partial H^u}{\partial Y} \frac{\partial D^u}{\partial v}}{(i - \frac{\partial D^u}{\partial Y})} \right]$$

Finally, define R* as

$$(4.30) \qquad R^* = \frac{\left[1 + \frac{\frac{\partial H^u}{\partial Y} \frac{\partial D^u}{\partial u}}{(i - \frac{\partial D^u}{\partial Y})} + \frac{\frac{\partial H^u}{\partial X} \frac{\partial G}{\partial u}}{(i - \frac{\partial G}{\partial X})} - \frac{\frac{\partial H^u}{\partial W^u} \frac{\partial L^u}{\partial u}}{(i - \frac{\partial L^u}{\partial w^u})} \right]^*}{\left[1 + \frac{\frac{\partial H^u}{\partial Y} \left|\frac{\partial D^u}{\partial u}\right|}{(i - \partial D^u/\partial Y^u)} \right]^*}$$

where again the * indicates the expressions are evaluated at their steady-state values. The planner should choose α_1 (given α_2) in order to insure the optimal rate of resource extraction. Given the stationary values of $\bar{\phi}^q$ and $\bar{\gamma}$

$$(4.31) \qquad \bar{\phi}^q = \frac{\partial U}{\partial q},$$

$$(4.32) \qquad \bar{\gamma} = \frac{\partial U}{\partial q} \frac{\partial H^q}{\partial u^q} - \frac{\left|\frac{\partial U}{\partial Z}\right| \frac{\partial H^z}{\partial U^q}}{(i - \frac{\partial H^z}{\partial Z})}$$

the relationshiup between α_1 and α_2 may be represented as:

$$(4.33) \qquad \alpha_1 = (\alpha_2/R^*)$$

Thus, choose:

$$(4.34) \qquad p^I = \alpha_2 \bar{\phi}^q, \text{ and}$$

$$(4.35) \qquad p^o = \alpha_2 (\bar{\gamma}/R^*)$$

In particular, for $\alpha_2 = 1$,

$$(4.36) \qquad p^o = \frac{\overline{\gamma}}{R^*} = \overline{\gamma}\,(1-\tau); \qquad \tau = \left(\frac{R^* - 1}{R^*}\right),$$

where τ is the royalty rate the government should charge for the natural resource. If increased extraction "depletes" the resource stock faster than it increases the stock of knowledge, i.e., $R^* > 1$ then there should be royalty charged for use of the common property resource.

Thus, a price vector, as determined by (4.15) - (4.22) and (4.34) - (4.35) will lead the competitive system to support the optimal steady-state solution. If $d_1 = d_2 = 0$, $\alpha_2 = 1$, then a subsidy is necessary to producers of C and a tax is needed on the extraction of the common property resource. However, d_1, d_2 and α_2 can be varied, thereby necessitating differential taxes or subsidies on the factors of production (depending upon the sector in which they are used).

4.3 Decentralization and Agency Budget Constraints. The decentralized price solution described in 4.1 and 4.2 presupposes that profits to each sector are non-negative. Moreover, we have seen that when shadow prices are used as the supporting prices, the government must levy a tax on new pollution emissions and must subsidize new pollution abatement equipment. If the planning agency has complete taxing power, then it can raise sufficient revenue to implement this plan.[18] However, if the revenue of the agency is limited to the fees paid for pollution emissions, it does not necessarily follow that this optimal solution can be reached.

For example, suppose that the pricing system defined by equations of 4.2 is implemented (with $d_1 = 0$).[19] Moroever, assume that the resource is sold by the resource sector at the price $p^u = \bar{\gamma}$; then profits in sector Q are:

$$(4.37) \qquad \Pi^q = \bar{\phi}^q H^q - \bar{\alpha}^r r^q - [\bar{\gamma} - d^2 \frac{\partial H^z}{\partial u^q}] u^q - (\beta\bar{\phi}^z + d_2) H^z - p_c^k$$

$$= \bar{\phi}^q H^q - \bar{\alpha}^r r^q - \bar{\gamma} u^q - \beta\bar{\phi}^z H^z - (\beta\bar{\eta}^k \frac{\partial D^c}{\partial c} - \bar{\phi}^q |\frac{\partial H^q}{\partial c}|) c$$

$$- d_2 [H^z - \frac{\partial H^z}{\partial u^q} u^q + \frac{|\frac{\partial H^z}{\partial K}| \quad (\frac{\partial D^c}{\partial c})}{(i - \partial D^c/\partial K)} c]$$

As noted earlier, d_2 can be varied without affecting the resource allocation pattern. However, values of $d_2 > 0$ (< 0), which represent "overpricing" ("underpricing") pollution emissions necessitate offsetting variations in p^k and p^{uq}.

Assuming the only source of revenue for the pollution agency is the revenue from pollution emissions and that it must finance the subsidy on new capital abatement equipment and a fraction, α, of the subsidy (or receive a fraction α of the net tax) on the resource allocated to q, $u^q(\bar{\gamma} - p^{uq})$ we have for this agency's budget:

$$(4.38) \qquad B = (\beta\bar{\phi}^z + d_2)H^z - (p^c - p^k)c - \alpha(\bar{\gamma} - p^{uq})u^q = \beta\phi^z H^z -$$

$$(\phi^c - \beta\bar{\eta}^k(\frac{\partial D^c}{\partial c}) + \phi^q \, |\frac{\partial H^q}{\partial c}|) \; c + d_2[H^z - \alpha \frac{\partial H^z}{\partial u^q} u^q + \frac{\frac{\partial H^z}{\partial K}|\frac{\partial D^c}{\partial c}}{(i - \frac{\partial D}{\partial K})} c]$$

In order to support the optimal solution there must exist some d_2 such that $B \geq 0$ and $\Pi^q \geq 0$.

Note that $\alpha < 1$ implies that some of the revenue from the resource sector can be taxed away (for $d_2 > 0$) since the agency does not pay for the entire resource subsidy to q. On the other hand, if $\alpha = 1$, then no financial transfer from the resource sector is possible. Thus, for $\alpha = 1$, the only affect of altering d_2 is to transfer "money" between the Q sector and the planning agency, without altering the sum $(B + \Pi^q)$. Letting $\alpha = 1$, we have, from (4.37) and (4.38):

$$(4.39) \qquad B + \Pi^q = [\bar{\phi}^q H^q - \bar{\alpha}^r r^q - \bar{\gamma} u^q - \bar{\phi}^c c],$$

Substituting the steady-state values for the dual variables, (4.39) reduces to:

$$(4.40) \qquad B + \Pi^q = \bar{\phi}^q \left\{ H^q - r^q \frac{\partial H^q}{\partial r^q} - \left[\left(\frac{\partial H^c/\partial u^c}{\partial H^c/\partial r^c} \right) \frac{\partial H^q}{\partial r^q} \right] \left[u^q + \frac{u^c}{\frac{\partial H^c}{\partial u^c} \frac{u^c}{H^c}} \right] \right\}$$

where the variables and partial derivatives in (4.40) are evaluated at their steady-state levels. The sign of (4.40) depends upon the properties of the production functions, as well as the utility function. For example, if we assume $H^q(u^q, r^q, c)$ is homogeneous of degree one in u^q and r^q, and if we define θ:[20]

$$(4.41) \qquad \theta = \frac{\frac{\partial H^c}{\partial u^c}}{\frac{\partial H^c}{\partial r^c}} \frac{\partial H^q/\partial r^q}{\partial H^q/\partial u^q} < 1,$$

(4.42) $$B + \Pi^q = \phi^q \frac{\partial H^q}{\partial u^q} \left[(1 - \Theta)u^q - \frac{\Theta \, u^c}{\left(\frac{\partial H^c}{\partial u^c} \frac{u^c}{H^c}\right)} \right]$$

A priori, it does not appear possible to ascertain the sign of (4.42). If $(B + \Pi^q) \geq 0$, then the first best solution can be obtained by proper choice of d_2. For example, if at $d_2 = 0$, $\Pi^q > 0$, and $B < 0$, then increasing d_2 will transfer the surplus to the planning agency.[21] Similar results hold for $\Pi^q < 0$ at $d_2 = 0$.

On the other hand, if $\Pi^q + B < 0$, then the first best solution cannot be reached unless new sources of revenue are made available to the planning agency. If the budget constraint is binding, then we are in a second best world, and the problem must be resolved in that context.[22]

4.4 Optimal Policies When Pollution Taxes are Not Feasible. The preceding discussion has assumed that it is possible to measure and tax new pollution emissions. However, as a practical matter this may not be possible. Large transaction costs are usually associated with attempts to measure pollution emissions of each firm (Rausser and Howitt [29]). Furthermore, there may be substantial measurement problems associated with this tax. Thus, considerations of this type could involve setting $p^z = 0$.

From Section 4.2 we see that this implies that the following prices are needed to support the optimal solution

(4.43) $$p^z = 0 \Rightarrow d_2 = -\beta\bar{\phi}^z$$

(4.44) $$p^{uq} = \bar{\gamma} + (\beta\phi^z \frac{\partial H^z}{\partial u^q}) = \phi^q \frac{\partial H^q}{\partial u^q}$$

(4.45) $$p^k = \beta\left[\bar{\eta}^k - \frac{\phi^z|\frac{\partial H^z}{\partial K}|}{(i - \frac{\partial D^c}{\partial K})}\right]\frac{\partial D^c}{\partial c} - \phi^q|\frac{\partial H^q}{\partial c}| = -\bar{\phi}^q \frac{\partial H^q}{\partial c}$$

Two issues immediately arise regarding this solution. First, the restriction that $p^z = 0$ entails placing a tax $(\beta\phi^z\partial H^z/\partial u^q)$ on the resource used in sector Q. The levels of this tax corresponds to the damage caused by the resource. This result corresponds to the customary way of dealing with pollution caused by a particular resource.

However, the resource tax alone will not yield an optimal solution since it provides no incentive for firms to employ pollution abatement equipment. In order to provide this incentive, and since this equipment yields no revenue (and doesn't reduce operating costs), the firm must be paid an amount $[-\bar{\phi}^q|\frac{\partial H^q}{\partial c}|]$ for each machine purchased and installed. This payment just equals the value of output foregone as a result of installation of the last unit of abatement equipment.

While theoretically this solution yields the optimum solution (provided $\Pi^q \geq 0$) there are obvious practical difficulties. In particular, enforcement activities must be introduced to insure that the machines are actually used. From the firm's viewpoint, it would be optimal to take the machines (and receive the subsidy), but not actually install them. Therefore, penalties would be needed, as well as enforcement personnel, in order to guarantee that the equipment is actually used.

In addition, if firms are not identical (in their location or production of pollution), then this policy may not lead to an optimum distribution of abatement equipment across firms. Thus, while theoretically the optimum solution may be obtainable under the restriction $p^z = 0$, the practical difficulties of enforcing this optimum should not be minimized.

Furthermore, if there is an agency budget constraint as described in 4.3, the restriction $p^z = 0$ removes the degree of freedom the agency has in attempting to balance its budget. While this restriction will not alter the sum $(B + \Pi^q)$, if resource taxes are paid to the agency it does eliminate its power to alter the distribution of B and Π^q. From (4.37) with $d_2 = -\beta\bar{\phi}^z$:

$$\Pi^q = \bar{\phi}^q[H^q - \frac{\partial H^q}{\partial r^q} r^q - \frac{\partial H^q}{\partial u^q} u^q + |\frac{\partial H^q}{\partial c}|c] \tag{4.46}$$

If H^q ( ) is homogenous of degree one in u^q and r^q, then $\Pi^q > 0$; if it is homogenous of degree one in u^q, r^q and c, $\Pi^q = 0$. For the agency budget, we find, using (4.40) and (4.41) (and assuming H^q ( ) is homogenous of degree one in u^q and r^q):

$$B = \bar{\phi}^q[\frac{\partial H^q}{\partial u^q}] \quad [(1 - \theta)u^q - \frac{\theta \; u^c}{\frac{\partial H^c}{\partial u^c} \; \frac{u^c}{H^c}} - |\frac{\partial H^q}{\partial c}|c] \tag{4.47}$$

The sign of (4.47) cannot be determined without further information on the functional forms. Thus, the additional restriction $p^z = 0$ may cause violation of the agency budget constraint and make the first best solution unobtainable.

5. Conclusions

We have argued that most natural resources have the characteristics of a joint input which result not only in the production of desirable but, as well, undesirable outputs. This joint input feature of natural resources leads to multiple products in the final goods sector, some subset of which results in the degradation of environmental resources. In turn, the societal external costs associated with this degradation process has led to the emergence of a new industry, the pollution abatement industry. Hence, any viable representation of natural resource problems requires the explicit recognition of at least three sectors: the extractive or natural resource sector (N), the pollution abatement sector (C), and the final goods sector (Q). One interesting feature of the C sector is that, given its relatively infant state, technical progress in both the manufacture and use of abatement equipment become important elements of any realistic formulation. Moreover, the introduction of the sector implies that the tradeoff between consumption and pollution can occur in two ways: (i) resources may be diverted from the Q to the C sector in order to produce pollution abatement equipment that will reduce pollution stocks; or (ii) since pollution is associated with the use of natural resources in the Q sector, for given levels of capital good production, the consumption - pollution mix can be altered through factor reallocation between the sectors.

Among other results, the formulation advanced in this paper for the case of a centralized economy leads to the following conclusions: (a) the marginal rates of technical substitution for the Q and C sectors should not be equated; (b) three degrees of freedom are available in determining the optimal steady-state solution; (c) along with the stationary state solution for the dual variables, three efficiency conditions must be satisfied to achieve the optimal steady-state, where the first pertains to selecting a point on the transformation frontier between pollution and consumption, the second refers to the tangency between the social indifference curve and the transformation frontier, and the third is concerned with the proper rate of resource extraction, which is in many ways equivalent to determining the optimal savings rate in a neoclassical growth model; and (d) under the special circumstances of no learning in the resource sector, no depreciation due to resource extraction, no depletion of resources, and no pollution we obtain precisely the modified golden rule.

The basic formulation was also examined in the context of four alternative institutional structures. For each of these structures, we demonstrated how the optimal steady-state can be supported by means of a price system. In particular, for the first structure in which the Q and C sectors are decentralized and operate under competitive conditions, a tax on pollution emitted and a subsidy on new abatement equipment due to learning are both needed to support the optimal steady-state. Furthermore, the levels of this tax and subsidy are not unique; two degrees of freedom are available in determining this supporting price system. For the second structure, the previous structure is augmented by decentralizing decisions in the resource sector. Under this situation, in addition to the previous elements of the price support system, a user charge must be imposed upon resource extraction which takes into account the effect of current resource production on both the resource and knowledge stocks. For example, only if increased extraction "depletes" the resource stock at a faster rate than it increases the stock of knowledge will a fee be imposed upon users of the common property resource. The third structure investigates endogenous agency budgets presuming the revenues received by the agency are limited to the fees paid for pollution emissions. Under this specification, it no longer follows that the optimal solution can be reached. The conditions which must be met to reach this solution are found to depend critically upon the various functional forms.

Perhaps the most operational result is associated with the frequent impracticality of measuring pollution emissions. For this structure, the tax on pollution must be discarded and a tax on the resource input employed in the final goods sector is required. This resource input tax simply recognizes that the joint nature of resource use in the Q sector is responsible for environmental damages. However, the imposition of this tax alone will not yield an optimal solution since it provides no incentive for firms to employ pollution abatement equipment. Thus, a subsidy for each device purchased and installed by firms in the Q sector must be offered to provide this incentive. In this setting, of course, once endogenous agency budgets are admitted the restriction that the tax on pollution be zero removes the degree of freedom the agency has in attempting to balance its budget.

A comparison of the above institutional structure to the conventional Pigouvian tax approach requires the explicit introduction of uncertainty. This is particularly evident given the present lack of information on pollution dispersion orpcesses and the need to measure statistically (via experimental design procedures) the level of externality emissions. On the basis of transactions cost and expected benefits, the relevant issue is under what conditions of uncertainty or information precision with respect to the generation and dispersion of pollution is it optimal to select the resource input tax, abatement subsidy structure or the conventional pollution tax combined with the stochastic externality measurement institutional structure. We are in the process of investigating this issue by (i) introducing realistic stochastic components into the resource input tax, abatement subsidy institutional structure advanced in Section 4.4 of this paper and (ii) comparing the associated transaction costs and expected benefits for this structure with those for the pollution tax, stochastic externality measurement structure. Preliminary results indicate that for most natural resources, under present states of information, the input tax, abatement subsidy approach is preferred. However, it is also reasonably clear that active attempts to collect more precise information on the generation and dispersion of pollution resulting from natural resource use can lead to an optimal timing problem for switching from the input tax, abatement subsidy to the pollution tax, stochastic externality measurement institutional structure.

APPENDIX

The Lagrangian function for the maximization problem discussed in Section 2 is:

$$
\begin{aligned}
\text{(A.1)} \quad \mathcal{L} = \sum_{t=0}^{T-1} & (\beta^t U(q_t, Z_t) + \beta^{t+1}\lambda_{t+1}[X_t + G(u_t, X_t) - X_{t+1}] \\
& + \beta^{t+1}\eta^u_{t+1}[Y_t + D^u(u_t, v_t, Y_t) - Y_{t+1}] + \beta^t\phi^u_t[H^u(X_t, Y_t, W^u_t) - u_t] \\
& + \beta^{t+1}\omega^u_{t+1}[W^u_t + L^u(u_t, W^u_t) - W^u_{t+1}] + \beta^t\gamma_t[u_t + u^q_t - u^c_t] \\
& + \beta^t\phi^c_t \, [H^c(u^c_t, r^c_t, W^c_t) - c_t] + \beta^{t+1}\omega^c_{t+1} \, [W^c_t + L^c(c_t, W^c_t) - W^c_{t+1}] \\
& + \beta^{t+1}\phi^z_{t+1} \, [Z_{t+1} - Z_t - H^z(u^q_t, K_t, W^z_t, Z_t)] + \beta^t\phi^q_t \, [H^q(u^q_t, r^q_t, c_t) \\
& - v_t - q_t] + \beta^{t+1}\eta^k_{t+1} \, [K_t + D^c(c_t, K_t) - K_{t+1}] \\
& + \beta^{t+1}\omega^z_{t+1} \, [W^z_t + L^z(c_t, W^z_t) - W^z_{t+1}] + \beta^t\alpha^r_t \, [r_t - r^q_t - r^c_t] \\
& + \beta^T\psi(X_T, Y_T, Z_T, W^c_T, W^u_T, W^z_T) \; .
\end{aligned}
$$

Given the values of the stock variables at $t = 0$, we wish to maximize $\mathcal{L}(\ldots)$ with respect to the control and state variables. For example, optimizing with respect to the control variable u^q_t yields the following (for $t=0, 1, \ldots, t-1$):

$$
\frac{\partial \mathcal{L}}{\partial u^q_t} = \beta^t[\phi^q_t \frac{\partial H^q}{\partial u^q_t} - \gamma_t - \beta\phi^z_{t+1} \frac{\partial H^z}{\partial u^q_t}] \leq 0
$$

This equation can be solved only if a solution can be obtained for the shadow value ϕ^z_{t+1} . A solution for this value can be derived by differentiating the Lagrangian expression with respect to the pollution stock. In particular, if pollution stocks are at positive levels, we can use the resulting recursive relationship to solve for the dual variable ϕ_t^z . Specifically, from

$$\frac{\partial \mathscr{L}}{\partial Z_t} = \beta^t \, [\beta\phi_{t+1}^z \, (1 + \frac{\partial H^z}{\partial Z_t}) \; - \; \phi_t^z + \frac{\partial U}{\partial Z_t}] \; \leq \; 0$$

we obtain for positive levels of Z_t,

$$\phi_t^z = \sum_{j=t}^{T-1} \beta^{j-t} \frac{\partial U_j}{\partial Z_j} \frac{\partial Z_j}{\partial Z_t} + \beta^{T-t} \frac{\partial \psi_T}{\partial Z_T} \frac{\partial Z_T}{\partial Z_t} \quad .$$

Substituting this result back into the necessary condition on the control variable u_t^q we obtain the following necessary condition:

$$\text{(A.2)} \qquad \phi_t^q \frac{\partial H^q}{\partial U_t^q} - \gamma_t \leq \left[\sum_{j=t+1}^{T-1} \beta^{j-t} \frac{\partial U_j}{\partial Z_j} \frac{\partial Z_j}{\partial Z_{t+1}} + \beta^{T-t} \frac{\partial \psi_T}{\partial Z_T} \frac{\partial Z_T}{\partial Z_{t+1}} \right] \frac{\partial H_t^z}{\partial u_t^q} \; .$$

Assuming that each stock or state variable is positive over all periods of the planning horizon, the necessary conditions for the remaining control variables may be derived in a similar fashion. For the planning horizon, $t = 0, \ldots, t-1$, this result for u_t^c is:

$$\text{(A.3)} \qquad \phi_t^c \frac{\partial H^c}{\partial u_t^c} \leq \gamma_t \quad ;$$

for u_t

$$\text{(A.4)}\qquad \gamma_t + \left[\sum_{j=t+1}^{T-1} \beta^{j-t} \phi_j^u \frac{\partial H_j^u}{\partial W_j^u} \frac{\partial W_j^u}{\partial W_{t+1}^u} + \beta^{T-1} \frac{\partial \Psi}{\partial W_T^u} \frac{\partial W_T^u}{\partial W_{t+1}^u} \right] \left(\frac{\partial L_t^u}{\partial u_t} \right)$$

$$- \phi_t \leq \left[\sum_{j=t+1}^{T-1} \beta^{j-t} \phi_j^u \frac{\partial H_j^u}{\partial X_j} \frac{\partial X_j}{\partial X_{T+1}} + \beta^{T-t} \frac{\partial \Psi}{\partial X_T} \frac{\partial X_T}{\partial X_{t+1}} \right] \left(- \frac{\partial G_t}{\partial u_t} \right)$$

$$+ \sum_{j=t+1}^{T-1} \left[\beta^{j-t} \phi_j^u \frac{\partial H_j^u}{\partial Y_j} \frac{\partial Y_j}{\partial Y_{t+1}} + \beta^{T-t} \frac{\partial \Psi}{\partial Y_T} \frac{\partial Y_T}{\partial Y_{t+1}} \right] \left(- \frac{\partial D_t^u}{\partial u_t} \right) ;$$

for v_t

$$\text{(A.5)}\qquad \left[\sum_{j=t+1}^{T-1} \beta^{j-t} \phi_j^u \frac{\partial H_j^u}{\partial Y_j} \frac{\partial Y_j}{\partial Y_{T+1}} + T-1 \frac{\partial \Psi}{\partial Y_T} \frac{\partial Y_T}{\partial Y_{t+1}} \right] \left(\frac{\partial D_t^u}{\partial v_t} \right) \leq \phi_t^q ;$$

for r_t^q

$$\text{(A.6)}\qquad \phi_t^q \frac{\partial H^q}{\partial r_t^q} \leq \alpha_t^r ;$$

for r_t^c

$$\text{(A.7)}\qquad \phi_t^c \frac{\partial H_t^c}{\partial r_t^c} \leq \alpha_t^r ;$$

for q_t

$$\text{(A.8)}\qquad \frac{\partial U_t}{\partial q_t} \leq \phi_t^q ;$$

and finally for c_t

$$
\begin{aligned}
\text{(A.9)} \quad & \left[\sum_{j=t+1}^{T-1} \beta^{j-t} \left(\sum_{i=j+1}^{T-1} \beta^{i-j} \frac{\partial U_i}{\partial Z_i} \frac{\partial Z_i}{\partial Z_{j+1}} + \beta^{T-j} \frac{\partial \Psi}{\partial Z_T} \frac{\partial Z_T}{\partial Z_{j+1}} \right) \cdot \right. \\
& \left. \frac{\partial H_j^z}{\partial K_j} \frac{\partial K_j}{\partial K_{t+1}} + \beta^{T-t} \frac{\partial \Psi}{\partial K_T} \frac{\partial K_T}{\partial K_{t+1}} \right] \left(\frac{\partial D_t^c}{\partial c_t} \right) + \left[\sum_{j=t}^{T-1} \beta^{j-t} \right. \\
& \left(\sum_{i=j+1}^{T-1} \beta^{i-j} \frac{\partial U_i}{\partial Z_i} \frac{\partial Z_i}{\partial Z_{j+1}} + \beta^{T-j} \frac{\partial \Psi}{\partial Z_T} \frac{\partial Z_T}{\partial Z_{j+1}} \right) \cdot \frac{\partial H_j^z}{\partial W_j^z} \frac{\partial W_j^z}{\partial W_{t+1}^z} + \\
& \left. \beta^{T-t} \frac{\partial \Psi}{\partial W_T^z} \frac{\partial W_T^z}{\partial W_{t+1}^z} \right] \left(\frac{\partial L_t^z}{\partial c_t} \right) + \left[\sum_{j=t}^{T-1} \beta^{j-t} \phi_j^c \frac{\partial H_j^c}{\partial W_j^c} \frac{\partial W_j^c}{\partial W_{t+1}^c} + \right. \\
& \left. \beta^{T-t} \frac{\partial \Psi}{\partial W_T^c} \frac{\partial W_T^c}{\partial W_{t+1}^c} \right] \left(\frac{\partial L_t^c}{\partial c_t} \right) + \phi_t^q \frac{\partial H^q}{c_t} \le \phi_t^c \ .
\end{aligned}
$$

If the strict inequality holds for any of these equations, the corresponding decision variable is 0; however, if the decision variable is positive, the corresponding condition is a strict equality.

FOOTNOTES

*The senior author wishes to thank George Tolley for a number of useful insights related to the framework advanced in this paper.

1. In the initial specification of this sector, storage of "free" resource reserves was included as a specific activity. Since the current model contains sufficient richness, however, uncertainty and seasonality in production are not admitted. Without such influences, the above-ground storage of resources is undesirable; it entails unnecessary construction of storage facilities and reduces the rate of resource renewal at the stationary state.

2. For similar specifications for the dynamics of the pollution process, see Baumol [2], Baumol and Oates [3], and Rausser and Howitt [29].

3. This specification generalizes the usual fixed proportion model of externalities, i.e., once the level of saleable output is set, the externality output is automatically determined no matter what the rate of input use. It is also a more appropriate specification than the multiproduct formulation involving a single relationship. Such a joint product specification, found in most intermediate economic texts, is not generally applicable to the case of externalities. It implies that, given amounts of all inputs, more saleable output can be produced by altering the amount of externality output. This is clearly incorrect; the externality output can only be varied by changing the joint natural resource input (e.g., type of fuel used) or the amount of fixed or other variable inputs (Whitcomb [45]).

4. One possible justification for this specification could be based upon the relative locations of the Q and C sectors. If the output q is perishable it seems reasonable that the Q sector would be located close to population centers and thus the use of the natural resource by this sector would result in environmental damages. In contrast, since c is durable the C sector could be located sufficiently far from population centers to result in little if any environmental damages.

5. In addition to the concavity assumptions on the functions (2.1)-(2.12) and (2.15) another restriction is needed for the Kuhn-Tucker conditions to imply sufficiency as well as necessity. This restriction is that the Lagrangian multipliers or dual variables be positive which may imply free disposal. Of course, free disposal of any of the specified stocks may be extremely restrictive for some situations. Note that Gale [15] has proved necessary and sufficient conditions for the existence of the Lagrangian multipliers in the context of dynamic framework advanced in this paper.
6. Since pollution stocks effect utility, the trade-off between consumption and pollution must also be viewed as an intertemporal problem.
7. The existence of the steady state may be obtained by appealing the dynamic programming approach of Bellman [4]. Assuming (i) the utility specification is not explicitly a function of time; (ii) all state variables are confined to a bounded region; (iii) the difference equations generating the state variables are consistent with this bounded region; and the (iv) utility function is uniformly bounded for all values ot its arguments belonging to the constraint set and the bounded region for the state variables, the value of the criteria function in (2.15) at its maximum point, stated in terms of initial states, converges as $T \to \infty$ to a functional equation which has a unique solution, (Bellman [4, p. 121]). Moreover, under these assumptions along with the specified concavity and continuity assumptions, either a steady state must exist or resource stocks become zero in a finite period. If we append to these assumptions, the Inada conditions on all production functions, viz.

$$\lim_{u \to o} \frac{\partial H}{\partial u} = \infty$$

then all stock and flow variables will be utilized at positive levels in the steady state.

8. The necessary conditions appearing in the Appendix can be shown to hold in the limiting case where $T \to \infty$. Under the assumptions in footnote 7, Inada conditions on all production functions and Bellman's theorem, it is clear that we can approximate the necessary conditions for an infinite planning horizon to any desired degree of accuracy by setting T at arbitrarily large levels. This result obtains when the terms involving summations in (A.2)-(A.9) ultimately converge to zero. Such convergence follows from $|1 + \partial G/\partial X| < 1/\beta$, $|1 + \partial D^u/\partial Y| < 1/\beta$, $|1 + \partial L^u/\partial W^u| < 1/\beta$, $|1 + \partial H^z/\partial Z| < 1/\beta$, $|1 + \partial D^c/\partial K| < 1/\beta$, $|1 + \partial L^z/\partial W^z| < 1/\beta$, and $|1 + \partial L^c/\partial W^c| < 1/\beta$. These conditions are indirectly assured by the assumptions appearing in footnote 7 and the specifications imposed on (2.15).

9. Alternatively, these two conditions can be viewed as: (i) obtaining the production possibility frontier between Z and q and (ii) choosing a proper intertemporal allocation of resources.

10. Obviously, these prices can only be determined up to some scalar multiple; the choice of a numeraire good remains.

11. For simplicity, we define total population to be one.

12. Since firms are assumed identical and face identical prices, we have: $u^{ci} = u^{cj}$, $r^{ci} = r^{cj}$; and thus:

(i) $c = n^c[F_c(u^{ci}, r^{ci}, W^c)] = n^c F_c(\frac{u^c}{N^c}, \frac{r^c}{n^c}, W^c)$

(ii) $H^c(u^c, r^c, W^c) = n^c F_c(\frac{u^c}{n^c}, \frac{r^c}{n^c}, W^c)$,

and

(iii) $\frac{\partial H^c}{\partial x} = \frac{\partial F_c}{\partial x^i}$, $x = u^c, r^c$.

13. Thus, we assume from an aggregative viewpoint:

$$H^z(u^q, K, W^z, Z) = \hat{H}^z(u^q, K, W^z) + \bar{H}^z(Z)$$

14. Since all firms are identical:

$$H^q(u^q, r^q, c) = n^q F^q(u^{qi}, r^{qi}, c^i) = n^q F^q(\frac{u^q}{n^q}, \frac{r^q}{n^q}, \frac{c}{n^q}),$$

and

$$\frac{\partial H^q}{\partial x} = \frac{\partial F^q}{\partial x^i}, \qquad x = u^q, r^q, c$$

Similarly, from footnote 13;

$$H^z(u^q, K, W^z, Z) = \hat{H}^z(u^q, K, W^z) + \bar{H}^z(Z) = n^q G^z(\frac{u^q K}{n^q n^q}, W^z) + \bar{H}^z(Z)$$

and

$$\frac{\partial H^z}{\partial x} = \frac{\partial \hat{H}^z}{\partial x} = \frac{\partial G^z}{\partial x^i}, \qquad x = u^q, K.$$

Furthermore, since $K_{t+1}{}^i = K_t{}^i = J(c_t{}^i, K_t{}^i)$, $K_{t+1} = K_t + D^c(c_t, K_t)$, and $K_t = n^q K_t{}^i$, then

$$D^c(c_t, K_t) = n^q J(\frac{c}{n^q}, \frac{K}{n^q}) \text{ and } \frac{\partial D^c}{\partial x^i} = \frac{\partial J}{\partial x^i}; \quad x = C, K.$$

15. Naturally, all prices can be scaled by the same constant value without altering the solution.

16. Obviously, the profit-maximizing conditions apply onto to the interior solution. Moreover, we must also be sure profits are non-negative at the optimal level. Variations in d_2 will affect the profits of the final goods sector.

17. Given n^u, $u = n^u u^i$ implies $H^u(X^u, Y^u, W^u) = n^u F^{ui}(X^u, Y^{ui}, W^u)$. Since all firms are identical:

$$\frac{\partial H^u}{\partial Y} = \frac{\partial F^{ui}}{\partial Y^i} \text{ for } Y = n^u Y^i$$

Similarly,

$$Y_{t+1} = Y_t + D^u(u_t, v_t, Y_t)$$

and

$$Y_{t+1}^{\ i} = Y_t^{\ i} + M^i(u^i, v^i, Y^i)$$

implies

$$D^u = n^u M^i[\frac{u}{n^u}, \frac{v}{n^u}, \frac{Y}{n^u}]$$

assuming extraction is allocated efficiently across firms, as it would be under competitive conditions. Therefore:

$$\frac{\partial D^u}{\partial x} = \frac{\partial M^i}{\partial x^i}, \text{ where } x = u, v, Y.$$

18. Since the return to labor is a pure rent in this model an income tax (or lump sum tax) can be imposed on labor's wages without altering the resource allocation. Moreover, the rents earned from the resource stock could be used to augment the agency's budget. Thus, with these revenues and taxing powers, the optimal solution can be achieved.

19. If $H^c(u^c, r^c, W^c)$ is homogeneous of degree one in u^c and r^c, then profits in sector C are zero, and nothing is altered by varying d_1. Thus, we let $d_1 = 0$.

20. θ is the ratio of the MRS between sectors, and thus $\theta < 1$ because of the pollution associated with u^q.

21. This assumes $H^z - \frac{\partial H^z}{\partial u^q} u^q + \frac{|\frac{\partial H^z}{\partial K}| (\frac{\partial D^c}{\partial c})c}{(1 - \partial D^c/\partial K)} > 0$ at

the steady-state solution. If the sign is reversed, then increasing d_2 raises Π^q and lowers B; if it equals zero, d_2 has no effect on the distribution of revenue and alternative taxing schemes must be found. Note that if $H^z(u^q, K, W^z, Z)$ is homogeneous of degree one in u^q and K, and if $(\partial D^c/\partial c) \equiv 1$, $(\frac{\partial D^c}{\partial K}) \equiv \delta$, so $c = \delta K$ in the steady-state, then:

$$H^z - \frac{\partial H^z}{\partial u^q} u^q + \left|\frac{\partial H^z}{\partial K}\right| \frac{(\delta K)}{(1+\delta)} = \frac{K}{1+\delta} \left|\frac{\partial H^z}{\partial K}\right| K < 0$$

22. If $(B + \Pi^q) < 0$, then the optimal solution is not attainable unless alternative sources of funds are available to the Agency. If the constraint is tight, then we must proceed with a Second Best Solution and it is well-known that, in this case, the other optimality conditions may be violated (Lipsey and Lancaster, [17]). Under these circumstances, the Agency must choose a set of prices p^c, p^k, p^z (let $p^q \equiv 1$) and a tax τ on the resource used in the final goods sector such that the profit-maximizing conditions for the firms are fulfilled (as defined by equations (4.2), (4.3), and (4.5) - (4.7) and such that the following two constraints are not violated:

(i) $\Pi^q = H^q - p^r r^q - (p^u + \tau) u^q - p^z H^z - p^k c \geq 0$

(ii) $B = p^z H^z - (p^c - p^k) c + \tau u^q \geq 0.$

Note that we assume p^r, the price of labor and p^u, the price of the resource are beyond the Agency's control. If this were not the case, then the optimal solution is always attainable through an (implicit) tax on labor or the resource.

Also, we assume that technology is such that $\Pi^c \geq 0$ whenever the first order conditions for profit-maximization are satisfied in sector C. The equilibrium values of p^r and p^u should be determined by their respective dual variables, but they must be taken as exogenous to the Agency.

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