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AND DECENTRALIZATION

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July 1980

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Epsilon-Rational Consistent Equilibria and Decentralization *

by

Steven M. Goldman and Kenneth M. Kletzer

1. Introduction

For a large class of strictly rational economic models, it is possible to describe equilibria wherein each agent is assigned a decision rule and, under suitable assumptions regarding that agent's conjectures about the decision rules of others, has no incentive to depart from that assignment. Yet the specification of this decision rule may require more than mere market information and rationality by the agent. Since the agents themselves have no guide to selecting from possibly many rational conjectures regarding the reactions of others, they have no means of inferring the "equilibrium" decision rule from their observations alone. It is in this sense that such equilibria cannot properly be termed "decentralized."

We propose, in this paper, to examine the problem of decentralized choice within the context of the problem of planning under intertemporally inconsistent preferences (Strotz (1956), Pollak (1968), Peleg and Yaari (1973), Goldman (1980)). After an example illustrating the possible failure of existence for decentralized solutions with both pure and mixed strategies, we will introduce a weaker concept of consistency requiring that agents have "plausible" beliefs regarding the possibility of later choices. This conjectural consistent equilibrium notion, though more general than the mixed strategy consistent equilibrium approach is still insufficient to guarantee a decentralized solution. It can, however be combined with the concept of "epsilon-rationality" (e.g. Radner (1979a,b)). Such epsilon-rational consistent equilibria will generally exist without the need for coordination among the agents.

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The allocations described by these two conjectural equilibrium concepts will be shown to be "close" when the behavior described by "epsilon-rationality" is nearly rational - i.e. epsilon is small. Thus, the centralized solutions may be viewed as an approximation to those resulting from this type of bounded rational behavior.

2. Pure and Mixed Strategies in Consistent Plan Equilibria

Suppose a finite sequence of agents is to divide one unit of a resource. Each agent selects that portion of the unconsumed resource for his own consumption and passes along the remainder to the next. The agents evaluate the outcome of this sequential game by preference orderings over the entire inter-agent consumption space -- i.e. an agent may "care" about the consumption levels of others. The consistent (or Perfect Nash) equilibrium may possibly be obtained through backward induction: the next to last agent can choose so as to maximize his own preference given the history of consumption prior to his own and the knowledge that the last will consume what remains. The previous agent (second from the last) can then infer the decision rule for the next to last and thereby choose his best level of consumption. The extension of this process backwards to the first agent would produce a consistent solution in which each agent has chosen optimally given the optimal responses of future agents.

Now as noted by Peleg and Yaari (1973), this procedure may fail when agents are limited to pure strategies. As will be shown below, it is simple to illustrate that the possibility of such failure is not removed by allowing for mixed strategies as well.

The difficulties arise in the following manner: Suppose that some agent has two equally desirable best levels of consumption. Then the agent preceeding him cannot know which will be chosen and this agent needn't be indifferent between the two outcomes! In the following example, almost all rules by which the indifferent agent would resolve his ambiguity leave the preceeding agent without a "best" choice.

Example 1:

Suppose that there are three agents. The feasible inter-agent consumption space is described by the face of the unit three-dimensional simplex ($c_1 + c_2 + c_3 = 1$) depicted in figure 1. Given a choice of \bar{c}_1 by the first agent, the feasible allocations are limited to a line such as aa' in figure 1 ($\bar{c}_1 + c_2 + c_3 = 1$). The second agent would choose the "best" point on this line from his perspective. The locus of such points for all possible choices by agent one, describes a reaction curve faced by the first agent. This is illustrated in figure 1 by the curve bb' with an indifference curve for the second agent shown as c .

The first agent's problem is then to select that level of c_1 associated with his best point on the reaction curve.

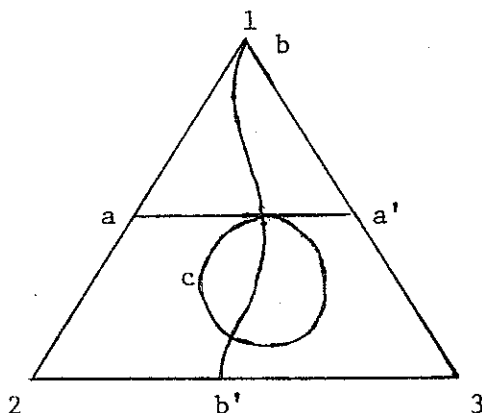


Figure 1

Thus far, there is no problem. The first agent is presented with a unique reaction to each of his possible choices. But suppose instead that the second agent's reaction curve had looked instead like $bb'b''^*$ in figure 2. That is, for some choice by the first agent, \bar{c}_1 , the second is indifferent between b' and b'' as indicated by the indifference curve c for the second agent. (Note: convexity could have been preserved in a four person example). Suppose further that the first agent's best choice is at b'' .

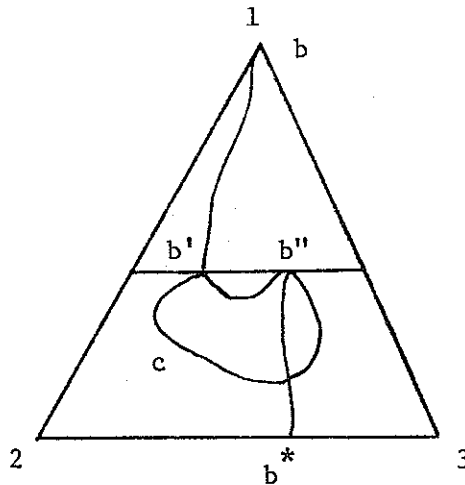


Figure 2

Now if the first agent believes that if he chooses \bar{c}_1 the second will choose at b'' there is no problem. However, if instead the first believes that the second will choose b' or some mixed strategy between b' and b'' , then a "best" solution for the first agent fails to exist. His "best" points lie arbitrarily close to b'' along $b''b^*$.

In a refinement to the work of Peleg and Yaari (1973) by Goldman (1980) it is proven that a perfect Nash equilibrium will always exist to such problems but that it will require the presumption (here) by

the first agent that when he selects \bar{c}_1 , the second will select at b'' .

But what is the first agent's justification for this presumption?

Let us now proceed to a formal statement of the existence problem.

Consider a storage economy with one unit of a single consumption good which is to be divided among a sequence of agents $t = 1, \dots, T$.

Letting c_s denote the consumption of an arbitrary agent s , we shall define the history of consumption prior to t by the vector

$$c^{t-1} = (c_1, \dots, c_s, \dots, c_{t-1}).$$

The feasible inter-agent consumption space is then described by the face of the T dimensional unit simplex, or

$$S = \{c^T \mid c^T \geq 0, \sum_{s=1}^T c_s = 1\}.$$

Given a history \bar{c}^{t-1} , the feasible choices for the t -th agent are described by those levels which do not more than exhaust the remaining stock, or

$$C_t(\bar{c}^{t-1}) = \{c_t \mid 0 \leq c_t \leq 1 - \sum_{s=1}^{t-1} \bar{c}_s\},$$

and the feasible final consumption vectors are limited to

$$S(\bar{c}^{t-1}) = \{c^T \mid c^T \in S, c^{t-1} = \bar{c}^{t-1}\}.$$

A1. Suppose now that each agent t has a bounded, continuous, real valued utility function $U^t(c^T)$ defined over S and, in the presence of uncertainty, receives the expected level of utility.

A2. A strategy for the t -th agent is a function which assigns to every feasible history a conditional distribution function over the

agent's feasible choices, or $F^t(c_t; c^{t-1})$ is a c.d.f. with support over $C_t(c^{t-1})$ and is defined for all feasible c^{t-1} .

A3. Given a vector of such strategies for all agents, the expected utility from a choice of \bar{c}_t by the t -th agent is given by:

$$V^t(\bar{c}^t) = \int_{c_{T-1} \in C_{T-1}(\bar{c}^t, c_{t+1}, \dots, c_{T-2})} \dots \int_{c_{t+1} \in C_{t+1}(\bar{c}^t)} U^t(\bar{c}^t, c_{t+1}, \dots, c_{T-1}, 1-c_s) dF^{t+1}(c_{t+1}; \bar{c}^t) \dots dF^T(c_T; (\bar{c}^t, c_{t+1}, \dots, c_{T-1})).$$

A4. A mixed strategy consistent equilibrium is described by a vector of strategies $F = (F^1(\cdot), \dots, F^T(\cdot))$ where, for each t , $F^t(\cdot)$ maximizes $V^t(\cdot)$ given c^{t-1} and $F^s(\cdot)$ for all $s > t$, and, $dF^t(\bar{c}_t; \bar{c}^{t-1}) = 0$ unless $V^t(\bar{c}^{t-1}, \bar{c}_t) \geq V^t(\bar{c}^{t-1}, c_t)$ for all $c_t \in C_t(\bar{c}^{t-1})$.

The equilibrium allocations are described as

$$E = \{c^T \mid \forall s, dF^s(c_s; c^{s-1}) \neq 0\}.$$

Since pure strategy equilibria exist in general for this problem (Goldman (1980)) and since pure strategies are also mixed ones, then clearly the set of mixed strategies isn't null. We shall argue below that by considering mixed strategies, we may increase the set of equilibria beyond those of pure ones alone.

But first, it is noteworthy that our definition of mixed strategy equilibrium requires that all agents prior to t have the same presumption regarding t 's strategy. Thus, the mixed strategy equilibria are no less in need of coordination than the pure ones. Further, as is clear from the example in figure 2, not every rational strategy by agent 2

will result in an equilibrium -- only the strategy assigning b'' when \bar{c}_1 is selected.

In the following example, we shall display a mixed strategy consistent equilibrium which is not a pure strategy one.

Example 2:

Consider a sequence of four agents. After the first agent has chosen some c_1 , the consumption space remaining for the other agents is typically depicted in figure 3 below -- the simplex $c_2 + c_3 + c_4 = 1 - \bar{c}_1$. Suppose now, that the indifference map for the third agent is as shown by the dotted line in figure 3. Then the reaction set faced by the second agent is described by the heavy border xyz . That is, if the second agent selects any positive level of consumption, the third will consume the remainder. If the second agent consumes nothing, then the third is indifferent as to his level of consumption.

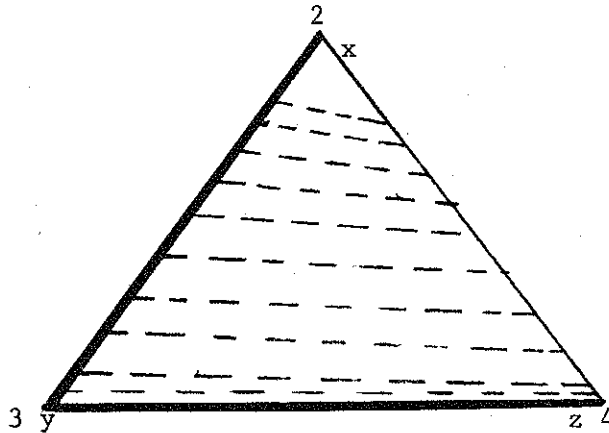


Figure 3

We shall first examine a possible mixed strategy equilibrium.

Now consider the strategy by the third agent as follows:

whenever c_2 is positive, consume whatever remains;

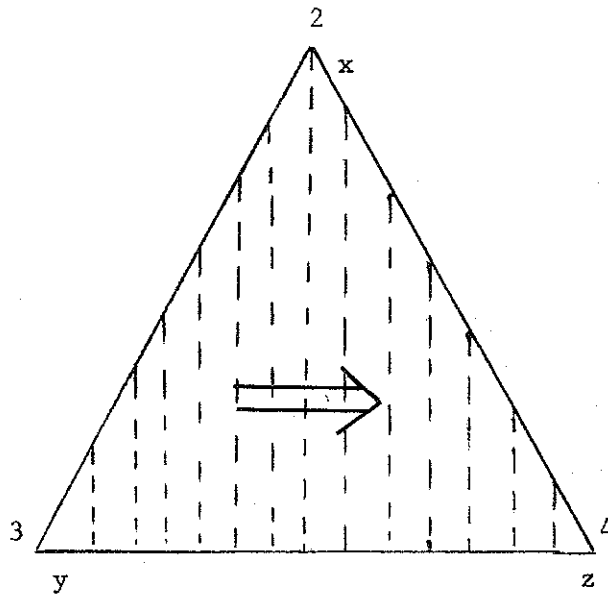


Figure 4

We have yet to address the problem of finding a decentralized solution for our problem. Indeed, so long as we require that all agents prior to t have the same perception of t 's strategy, we cannot escape the need for centralization since there is no way of inferring t 's strategy from strict rationality alone.

Let us now remove this restriction and suppose merely that each agent has a prior belief about the choices of later ones which are among those selections which are maximal for those agents. That is, an agent needn't be "correct" in his supposition about the strategy of a later agent; rather he expects choices which that agent "might" make.

This generalization, of course, will still include all of the mixed strategies described previously but adds still other possibilities. In fact, we shall demonstrate below that there will exist such conjectural consistent equilibria which are not mixed strategy consistent equilibria. The generalization of the concept of equilibrium is still not enough to provide for the existence of a decentralized equilibrium, however.

The example illustrated in figure 2 still suffices for this point -- let the first agent ascribe to the second a strategy of choosing b' and b'' with equal probability. The reaction set seen by the first agent remains open at b''.

Formally the definition of a conjectural consistent equilibrium (or C.C.E.) is given by the following:

Define

$$Z^T(c^{T-1}) = \{c_T | c_T = 1 - \sum_{s=1}^{T-1} c_s\} .$$

$$\begin{aligned} \text{and } F^T(c_T; c^{T-1}) &= 0 \text{ for } c_T < 1 - \sum_{s=1}^{T-1} c_s \\ &= 1 \text{ for } c_T = 1 - \sum_{s=1}^{T-1} c_s . \end{aligned}$$

$Z^t(c^{t-1})$ and $F^t(c_t; c^{t-1})$ are defined recursively by induction

$$\mathcal{F}^t(c^{t-1}) = \{F^t(c_t; c^{t-1}) | \int_{c_t \in C_t(c^{t-1})} \dots \int_{c_T \in C_T(c^{T-1})} U^t(c^{t-1}, c_t, c_{t+1}, \dots, c_T)\}$$

$dF^t(c_t; c^{t-1})dG^{t+1}(c_{t+1}; c^t) \dots dG^T(c_T; c^{T-1})$ is maximal

given some vector of c.d.f.'s G^{t+1}, \dots, G^T where G^s has

support over $Z^s(c^{s-1})$ for all s and F^t is a c.d.f.}

$$Z^t(c^{t-1}) = \{c_t | \exists F^t(\cdot) \in \mathcal{F}^t(c^{t-1}) \text{ s.t. } dF^t(c_t; c^{t-1}) \neq 0\} .$$

A4'. A conjectural consistent equilibrium is described by a set of strategies F^1, \dots, F^T where, for all t , and feasible c_t, c^{t-1}

$F^t(c_t; c^{t-1}) \in \mathcal{F}^t(c^{t-1})$. The equilibrium allocations are given by

$$E = \{c^T | \forall s, c_s \in Z^s(c^{s-1})\}.$$

In the following example, we shall show that a CCE equilibrium needn't be an MS equilibrium.

Example 3:

Let us consider a small modification to Example 2. Taking the preferences of the first and third agents as before, let us suppose that the second agent has a utility function given by $U^2(c^4) = c_2 + c_4$. Thus for any choice of c_1 by the first agent, the second would receive maximum satisfaction along the line xz .

Now, suppose the following conjectures and CCE equilibrium:

1. The first agent believes that the third will choose $1-c_1$ with probability one if $c_2 = 0$. (That is, along yz the third agent will choose at y).
2. The first agent believes that the second agent believes that the third agent will choose zero with probability one if $c_2 = 0$. (That is along yz the third agent will choose at z).

These beliefs are, of course contradictory in that the third agent cannot choose both ways, but it is not contradictory for the first agent to believe that the second will differ with him in guessing the actions of the third!

3. The first agent believes that the second will choose $c_2 = 0$ with probability one. This choice would be rational given the first agent's belief in the second's conjecture about the third. (That is, according to the first agent, the second would expect to receive $U^2(c^4) = c_4 = 1-c_1$.)

4. Therefore the first agent expects the final result to be $c_3 = 1 - c_1$,
 $c_2 = c_4 = 0$ and thus chooses c_1 to maximize $U^1(c^4) = c_1 + [1.9(1 - c_1)]^2$.
So $c_1 = 0$!

Alternatively, let's examine the MS equilibria.

1. If three's strategy puts any positive probability on $c_3 > 0$, then the second agent chooses $c_2 = 1 - c_1$, and the first agent maximizes his utility by choosing $c_1 = 1$.
2. If the third agent chooses $c_3 = 0$ with probability one, then a rational strategy for the second agent chooses $c_2 = 1$ with some probability, p and $c_2 = 0$ with probability $1 - p$. Either way the first agent receives utility equal to c_1 and his best choice is again $c_1 = 1$!

Therefore, a CCE equilibrium needn't be an MS equilibrium.

We shall now turn our attention to a weakening of the rationality assumption in order to exhibit the general existence of a consistent equilibrium.

3. On the Existence of Epsilon-Rational Conjectural Equilibria

The weakening from MS to CCE has relieved part of the decentralization problem at the expense of ascribing to each agent some expectation of the strategy of each later one. Yet, even with this introduction of "beliefs," there is still no assurance that an equilibrium will result. The difficulty here lies in the strictness of the rationality condition. It may happen, as indeed the first example illustrates, that an agent's best choice lies on the boundary of an open set. Now this particular type of equilibrium failure seems more the consequence of the mathematical convenience of continuity or divisibility, than of economic significance. If we believe that agents are satisfied to come close - within

epsilon - of their maxima, then we shall always be able to find equilibria. Below, we define an epsilon-rational conjectural consistent equilibrium (or ϵ -CCE) and prove its existence and decentralized nature for the consistent choice problem.

Define $F^t(c_t; c^{t-1}, \epsilon)$, $\mathcal{F}^t(c^{t-1}, \epsilon)$ and $Z^t(c^{t-1}, \epsilon)$ as follows:

$$Z^T(c^{T-1}, \epsilon) = \{c_T \mid c_T = 1 - \sum_{s=1}^{T-1} c_s\}$$

$$F^T(c_T; c^{T-1}, \epsilon) = \begin{cases} 0 & \text{for } c_T < 1 - \sum_{s=1}^{T-1} c_s \\ 1 & \text{for } c_T = 1 - \sum_{s=1}^{T-1} c_s \end{cases}$$

$$\mathcal{F}^T(c^{T-1}, \epsilon) = \{F^T(c_T; c^{T-1}, \epsilon)\}.$$

$$\mathcal{F}^t(c^{t-1}, \epsilon) = \{F^t(c_t; c^{t-1}, \epsilon) \mid V^t(\epsilon^t) = \int_{c_t \in C_t(c^{t-1})} \dots \int_{c_T \in C_T(c^{T-1})} U^t(c^{t-1}, c_t, c_{t+1}, \dots, c_T)\}$$

$dF^t(c_t; c^{t-1}, \epsilon) dG^{t+1}(c_{t+1}; c^t) \dots dG^T(c_T; c^{T-1})$ is within ϵ_t of its supremum given some vector of c.d.f.'s G^{t+1}, \dots, G^T where G^s has support over $Z^s(c^{s-1}, \epsilon)$ for all s and F^t is a c.d.f.}

$$Z^t(c^{t-1}, \epsilon) = \{c_t \mid \exists F^t(\cdot; c^{t-1}, \epsilon) \in \mathcal{F}^t(c^{t-1}, \epsilon), dF^t(c_t; c^{t-1}) \neq 0\}$$

A4" An epsilon-rational conjectural equilibrium is described by a vector of tolerances, $\epsilon = (\epsilon_1, \dots, \epsilon_T)$, and a vector of strategies $F = (F^1, \dots, F^T)$ where, for all t , $F^t(\cdot; c^{t-1}, \epsilon) \in \mathcal{F}^t(c^{t-1}, \epsilon)$.

The equilibrium allocations are given by $E(\epsilon) = \{c^T \mid \forall s, c_s \in Z^s(c^{s-1}, \epsilon)\}$

We note in passing that $E(0) = E$, that is, that the set of ϵ CCE for $\epsilon = 0$ is simply the set of CCE.

Loosely, each agent selects an action that results in an outcome within ϵ_t of the best he could hope to achieve given his subjective beliefs about how future agents will select from their satisfactory sets.

Theorem 1: Under A1, A2, A3 and A4" above, for any strictly positive vector of tolerances, an epsilon-rational conjectural equilibrium exists.

Proof: We shall proceed through inductive recursion to demonstrate that the support for members of each \mathcal{F}^t is not null.

The last agent simply consumes $c_T = 1 - \sum_{s=1}^{T-1} c_s$.

Suppose that for all $s > t$ and feasible c^{s-1} that the support for members of $\mathcal{F}^s(c^{s-1}, \epsilon)$ is non-null.

Therefore, since V^t is bounded, $\sup_{c_t \in C_t(c^{t-1})} V^t(c^{t-1}, c_t)$ exists

for all feasible c^{t-1} and feasible conjectures about the strategies of later agents. So $\{c'_t \mid \sup_{c'_t \in C_t(c^{t-1})} V^t(c^{t-1}, c'_t) - V^t(c^{t-1}, c_t) \leq \epsilon_t, c'_t \in S(c^{t-1})\}$

is non-empty for feasible strategies by the later agents. A feasible strategy for t is then a c.d.f. with this set for its support.

Q.E.D.

When the tolerance vector is zero, then the definition of an ϵ -CCE and CCE coincide. The proof of existence however fails since $\sup V^t$ needn't actually be achievable. Indeed, without permitting inexact maximization behavior, we have the same type of problems as with pure strategy equilibria. After all, the agent could conjecture

the pure strategies for later agents which would result in b' in the first example.

4. The Relationship Between Epsilon-Equilibria

We wish now to address the relationship between the sets of epsilon equilibria for different values of epsilon and, in particular, the relationship between the CCE and ϵ -CCE as epsilon converges to zero. The results will indicate that the CCE (which may need central coordination) may be viewed as approximations to the ϵ -CCE (which never require centralization) for small epsilon.

The procedure to be followed entails:

A description of the set of epsilon-rational conjectural consistent equilibrium allocations along the lines of Goldman (1980) for the pure strategy consistent equilibria. We shall draw heavily on these earlier results.

In order to view the CCE as approximations to the ϵ -CCE for small epsilon, we need to establish the continuity of $E(\epsilon)$ at $\epsilon = 0$. Then, every CCE is "close" to some ϵ -CCE and the limit of any convergent sequence of ϵ -CCE as $\epsilon \rightarrow 0$ is some CCE.

Define $X^T(c^T, \epsilon) = \{c^T\}$ where c^T is any allocation in S , and $Y^T(\epsilon) = S$.

Proceeding recursively,

let $X^t(c^t, \epsilon) = \{\bar{c}^T | \bar{c}^t = c^t, \exists \hat{c}^T \in X^{t+1}(\bar{c}^{t+1}, \epsilon), U^{t+1}(\hat{c}^T) + \epsilon_{t+1} \geq$

sup $\min U^{t+1}(\tilde{c}^T)\}$
 $c''_{t+1} \in C_{t+1}(\bar{c}^t) \quad \tilde{c}^T \in X^{t+1}(\bar{c}^t, c''_{t+1}, \epsilon)$

and

$$Y^t(\varepsilon) = \bigcup_{c^t \in S^t} X^t(c^t, \varepsilon) \quad \text{where } S^t = \{c^t \mid \sum_{s=1}^t c_s \leq 1\} .$$

It is quite straightforward to show that $Y^t(\varepsilon)$ is compact and non-empty for all t . The proof is virtually identical to the Lemma in Goldman (1980) p. 535, and is deleted here. Consequently, the X^t 's are non-empty.

We will now argue that $X^0(\varepsilon)$ is identical to the set of possible ε -CCE allocations, or $E(\varepsilon)$. The reasoning closely follows that of the Theorem in Goldman (1980) and is sketched below:

$X^{t+1}(c^{t+1}, \varepsilon)$ describes the outcomes which might occur after the first $t+1$ agents have selected c^{t+1} . The $t+1$ st agent can guarantee an outcome arbitrarily close in value (to him) to the sup min described in the definition of $X^t(c^t, \varepsilon)$. Thus he will most certainly not select any level of consumption which results in outcomes all of which are more than ε_{t+1} worse than the sup min!

Further, if some level of consumption c_{t+1}^* does result in possible outcomes better in value than the sup min minus ε_{t+1} , then by expecting the worst outcomes for other levels of his consumption and the best outcome for this level, c_{t+1}^* becomes within ε_{t+1} of the utility of his best choice under these expectations.

Therefore, these levels of c_{t+1} which may result in outcomes at least as good as the sup min minus ε_{t+1} are identical to the $Z^{t+1}(c^t, \varepsilon)$ previously described in the definition of ε -CCE. In this manner, $X^t(c^t, \varepsilon)$ becomes the possible outcomes given c^t .

Extending this procedure backwards $X^0(\varepsilon) = E(\varepsilon)$ for all $\varepsilon \geq 0$ and $E(0) = E$, the CCE allocations.

Theorem : $\lim_{\epsilon \rightarrow 0} E(\epsilon) = E$

Proof: (by induction)

1. $X^T(c^T, \epsilon) = X^T(c^T) \quad \forall \epsilon \geq 0, \forall c^T \text{ feasible.}$

2. Suppose

$$X^S(c^S, \epsilon) \geq X^S(c^S) \quad \forall \epsilon \geq 0, \forall c^S \text{ feasible, } s > t$$

$$\lim_{\epsilon \rightarrow 0} X^S(c^S, \epsilon) = X^S(c^S) \quad \forall c^S \text{ feasible, } s > t$$

Show A: $X^t(c^t, \epsilon) \geq X^t(c^t) \quad \forall \epsilon \geq 0, \forall c^t \text{ feasible}$

B: $\lim_{\epsilon \rightarrow 0} X^t(c^t, \epsilon) = X^t(c^t) \quad \forall c^t \text{ feasible}$

Recall the definition of $X^t(c^t, \epsilon)$

$$X^t(c^t, \epsilon) = \{ \hat{c}^T \mid \hat{c}^T = c^t, \hat{c}^T \in X^{t+1}(\bar{c}^{t+1}, \epsilon), U^{t+1}(\hat{c}^T) + \epsilon_{t+1} \geq$$

$$\sup_{c_{t+1}'' \in C_{t+1}(\bar{c}^t)} \min_{\tilde{c}^T \in X^{t+1}(\bar{c}^t, c_{t+1}'', \epsilon)} U^{t+1}(\tilde{c}^T), \bar{c}^T \in X^{t+1}(\bar{c}^{t+1}, \epsilon) \}$$

A. Since $\sup_{c_{t+1}'' \in C_{t+1}(\bar{c}^t)} \min_{\tilde{c}^T \in X^{t+1}(\bar{c}^t, c_{t+1}'', \epsilon)} U^{t+1}(\tilde{c}^T) \leq \sup_{c_{t+1}'' \in C_{t+1}(\bar{c}^t)} \min_{\tilde{c}^T \in X^{t+1}(\bar{c}^t, c_{t+1}'', \epsilon)} U^{t+1}(\tilde{c}^T)$

because $X^{t+1}(\bar{c}^t, c_{t+1}'', \epsilon) \supseteq X^{t+1}(\bar{c}^t, c_{t+1}'')$

then, for any \hat{c}^T where

$$U^{t+1}(\hat{c}^T) \geq \sup_{c_{t+1}'' \in C_{t+1}(\bar{c}^t)} \min_{\tilde{c}^T \in X^{t+1}(\bar{c}^t, c_{t+1}'', \epsilon)} U^{t+1}(\tilde{c}^T)$$

$$\Rightarrow U^{t+1}(c^T) + \varepsilon_{t+1} \geq \sup_{c''_{t+1} \in C_{t+1}(\bar{c}^t)} \min_{\tilde{c}^T \in X^{t+1}(\bar{c}^t, c''_{t+1}, \varepsilon)} U^{t+1}(\tilde{c}^T)$$

and therefore,

$$X^t(c^t, \varepsilon) \supseteq X^t(c^t) .$$

B. Consider a convergent sequence $\{\varepsilon^n\}$, where $\lim_{n \rightarrow \infty} \varepsilon^n = 0$.

Let $\{x^n\}$ be any sequence $x^n \in X^t(c^t, \varepsilon^n)$ where $\lim_{n \rightarrow \infty} x^n = x$. We must

establish that $x \in X^t(c^t)$.

By the definition of $X^t(c^t, \varepsilon)$ there exists a sequence $\{\hat{c}^n\}$, where $\hat{c}^{n,t+1} = x^{n,t+1}$, and

$$U^{t+1}(\hat{c}^n) + \varepsilon_{t+1} \geq \sup_{c''_{t+1} \in C_{t+1}(x^{n,t})} \min_{\tilde{c}^T \in X^{t+1}(x^{n,t}, c''_{t+1}, \varepsilon^n)} U^{t+1}(\tilde{c}^T)$$

$\{\hat{c}^n\}$ must have a convergent subsequence as $n \rightarrow \infty$, say, w.l.o.g.,

$$\lim_{n \rightarrow \infty} \hat{c}^n = c .$$

$$\begin{aligned} U^{t+1}(c) &= \lim_{n \rightarrow \infty} U^{t+1}(\hat{c}^n) = \lim_{n \rightarrow \infty} \sup_{c''_{t+1} \in C_{t+1}(x^{n,t})} \min_{\tilde{c}^T \in X^{t+1}(x^{n,t}, c''_{t+1}, \varepsilon^n)} U^{t+1}(\tilde{c}^T) \\ &= \sup_{c''_{t+1} \in C_{t+1}(x^t)} \min_{\tilde{c}^T \in X^{t+1}(x^t, c''_{t+1})} U^{t+1}(\tilde{c}^T) \end{aligned}$$

So, by this inequality and $c^{t+1} = x^{t+1}$; $x, c \in X^{t+1}(x^{t+1})$

and therefore $x \in X^t(x^t)$!

Extending backwards,

$$E(\varepsilon) = X^0(\varepsilon) \supseteq X^0 = E$$

and $\lim_{\varepsilon \rightarrow 0} E(\varepsilon) = E.$

Q.E.D.

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