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Contracting with Externalities

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Contracting with Externalities

Abstract

The paper studies inefficiencies arising in contracting between one principal and N agents when the utility of each agent depends on all agents' trades with the principal. When the principal commits to a set of publicly observable bilateral contract offers, the arising inefficiency is due entirely to the externalities imposed on non-signers. In contrast, when the principal's offers are privately observed, the distortion is due to the externalities given agents' equilibrium trades. Comparison of the two externalities determines the relative efficiency of the two contracting regimes. In both cases, we show that when N is large, each agent can be treated as non-pivotal, provided that appropriate continuity assumptions are satisfied.

We also study the case in which the principal can condition each agent's trade on other agents' messages. We characterize the set of such mechanisms in which each agent's participation is voluntary. When the principal can commit to any such mechanism, she implements the first-best outcome, while threatening each deviator with the harshest possible punishment. However, in the presence of noise that goes to zero slower than N goes to infinity, in the limit we obtain a (generally inefficient) outcome in which each agent feels non-pivotal.

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1 Introduction

In many economic situations, bilateral contracting imposes externalities on third parties. Here are some examples of such externalities that have received attention in the economic literature:

- A shareholder tendering his shares to a corporate raider has a positive externality on other shareholders (Grossman-Hart [1980]).
- A creditor exchanging debt for equity in a distressed firm has a positive externality on other creditors of the firm (Gertner-Scharfstein [1991]).
- A buyer of a VCR has a positive "network" externality on owners of compatible VCRs (Katz-Shapiro [1986b]).
- A party which accepts its offer in multi-party bargaining has a positive externality on other parties left at the bargaining table (Cai [1996a,b]).
- A merger of competing firms has a positive externality on other firms in the same market (Mackay [1984]).
- A private contributor to a public good has a positive externality on other consumers of the good (Bergstrom-Blume-Varian [1986]).
- A buyer signing an exclusive dealing contract which hinders competition imposes a negative externality on other buyers (Rasmusen-Ramseyer-Wiley [1991]).
- A producer purchasing an intermediate input from a manufacturer imposes a negative externality on competing producers (Hart-Tirole [1990], Katz-Shapiro [1986a]).
- A principal designing an incentive scheme in a common agency situation imposes an externality on other principals dealing with the same agent (Bernheim-Whinston [1986], Pauly [1974]).

In all these instances, contracting externalities have been shown to yield inefficiencies. However, the connections among existing models, and the general nature of arising inefficiencies, have not been well understood.

This paper develops and studies a general model of contracting with externalities which unifies the examples listed above, and many more. In the model, outlined in Section 2, one party (the principal) can make contract offers to N other parties (agents). The utility of each agent depends on the vector of all agents' trades with the principal. In Section 3, I describe a large number of existing models which can all be seen as applications of the general model.

Section 4 focuses on the case in which the principal commits to a set of bilateral contract offers. In this setting, inefficiency arises because of the principal's incentive to extract rents from the agents by reducing their reservation utilities, i.e., the utilities they would obtain by refusing to sign. Therefore, even though in equilibrium all agents may sign contracts with the principal, distortions are due entirely to the externalities on *non-signers*.

In most applications, the most interesting question is how the principal's rent-extraction motive affects the total trade (the sum of all agents' trades). To obtain an unambiguous answer to this question, I impose three assumptions, which are satisfied in almost all applications. First, I restrict the principal's profit to depend only on the total trade, and restrict each agent's utility to depend only on his own trade and on the total trade. Second, I make an assumption that ensures that the total surplus depends only on the total trade. Third, I make an assumption on the agents' trade domains. I show that under these assumptions, the total trade is socially insufficient or excessive depending on whether the externalities on non-signers externalities are positive or negative.

In Section 5, I study a contracting game in which the principal's offers are privately observed. Since the principal is now unable to commit to compensate agents for the externalities imposed on them in equilibrium, it is these externalities that determine the direction of distortion. In particular, under the same assumptions as in the previous section, I find the total trade is insufficient or excessive depending on whether the externalities on *signers* in equilibrium are positive or negative.

Using the techniques of monotone comparative statics (Milgrom-Shannon [1994], Edlin-Shannon [forth.]), I am able to establish these results in considerable generality, thus generalizing and systematizing the results in the specific applications listed above. Under additional assumptions, I am also able to compare (in Section 5) the outcomes under the two contracting regimes. The comparison hinges on the relative sizes of the externalities on signers and those on non-signers.

In both Section 4 and Section 5, I also study an asymptotic setting in which the number N of agents goes to infinity. Is it justified to assume that each agent thinks of himself as non-pivotal (i.e. not affecting the total trade by his decision) when N is large? While this assumption is sometimes made to simplify analysis (see e.g. Katz-Shapiro [1986b], Gertner-Scharfstein [1991]), it has generated much controversy in the context of takeovers (see e.g. Grossman-Hart [1980], Bagnoli-Lipman [1988]). In the settings of both public offers (Section 4) and private offers (Section 5), I show that under appropriate continuity assumptions, the equilibrium correspondence is upper hemi-continuous at $N = \infty$. That is, as $N \to \infty$, every converging sequence of equilibrium outcomes converges to a non-pivotal outcome. Therefore, the raider's ability to make stockholders pivotal for any N is due to the assumed discontinuity of the takeover process. At the same time, continuity assumptions do not guarantee that the equilibrium correspondence is lower hemi-continuous at $N = \infty$, i.e., some non-pivotal equilibria may not be approximated by equilibria with a large finite N.

In Section 6, I allow the principal to commit to a mechanism which makes her trade with every agent contingent on all agents' messages. I show that if the principal is restricted to choose from a family of such mechanisms in which agents' participation constraints bind, distortions will again arise due to the principal's rent-extraction incentive, and thus will be entirely determined by the externality on non-signers. At the same time, the principal's fully optimal mechanism involves offering the agents a first best trade profile, while threatening any deviator with the worst possible punishment. In this way, the principal maximizes total surplus and minimizes agents' reservation utilities at the same time. However, this solution requires making each agent pivotal, which seems unrealistic when the number of agents is large and not precisely known by the principal. To formalize this intuition, I consider a setting in which each acceptance message has a probability ε of being lost in the mail. I show that when $N \to \infty$ and $\varepsilon_N \to 0$ in such a way that $N\varepsilon_N \to \infty$, asymptotically a non-pivotal outcome obtains.

2 The Model

Consider a model in which one party, "the principal," can contract with N other parties, "the agents" (the parties will have various interpretations in examples below).[1] The principal's "trade" with each agent i is denoted by $x_i \in \mathfrak{X}_i$, where $\mathfrak{X}_i$ is a compact subset of the set $\Re_+$ of non-negative real numbers, with $0 \in \mathfrak{X}_i$. We assume that all parties' utilities are quasilinear in money, and let $t_i \in \Re$ denote the monetary transfer from agent i to the principal. The default ("no trade") point for each agent i is $t_i = x_i = 0$. Let the vector $x = (x_1, \ldots x_N) \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N$ denote the agents' trade profile.

Externalities among agents arise because each agent's utility depends not only on his own trade x_i, but also on other agents' trades. Specifically, the parties' payoffs are

$$\begin{aligned} \text{Agent } i\text{'s payoff} &= u_i(x) - t_i, \text{ for } i = 1, \ldots N, & (1) \\ \text{The principal's payoff} &= f(x) + \sum_i t_i. & (2) \end{aligned}$$

In some examples described in the next section, the principal will be "selling" x_i to agent i, in which case both $u_i(x)$ and $(-f(x))$ (the principal's cost of producing x) are increasing in x_i, and we can expect the price t_i paid by agent i to be positive. In other examples, the principal will be "buying" x_i from agent i, in which case both $(-u_i(x))$ and $f(x)$ (the principal's net benefit of x) are increasing in x_i, and we can expect t_i to be negative. This distinction between buying and selling will prove immaterial for our results. What *will* prove important is whether agents' utilities are increasing or decreasing in other agents' trades, i.e., whether externalities are positive or negative.

For future reference, let $\mathfrak{M}^* \subset \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N$ denote the set of trade profiles that maximize the total surplus of the $N + 1$ parties:

$$\mathfrak{M}^* = \underset{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N}{\arg\max} \; f(x) + \sum_i u_i(x). \quad (3)$$

It should be noted that this definition does not take into account the welfare of parties who do not participate in contracting but may be affected by its

[1] Lacking better unifying terminology, we use these terms to reflect this paper's focus on games in which the "principal" makes contract offers to "agents". No "agency relationship" is implied.

outcome. These include, for example, final consumers in vertical contracting applications (Hart-Tirole [1990], Katz-Shapiro [1986a]), the firm's employees in takeovers (Grossman-Hart [1990]) and debt workouts (Gertner-Scharfstein [1991]), etc. In each specific application, the contracting outcome should be examined from the viewpoint of aggregate social welfare. However, in this paper it will be convenient to use the surplus-maximizing outcomes as a benchmark against which contracting outcomes are compared. Thus, the word "inefficiency" will denote the contracting parties' failure to maximize their joint surplus.

Some of our results will require additional assumptions on the parties' payoffs and trade domains. Here we state all the assumptions that will later prove useful. In the next section, we will point out which of them hold in specific applications.

Condition A: $u_i(x) = U_i(x_i, X)$ for all i, and $f(x) = F(X)$, where $X = \sum_j x_j$.

In words, each agent's utility depends only on his own trade and on the sum of all agents' trades, and the principal's utility only depends on the sum of all agents' trades.

Condition L: Condition A is satisfied, and $U_i(x_i, X) = x_i\alpha(X) + \beta_i(X)$ for all i.

In addition to Condition A, this assumption requires that each agent i's utility be linear in x_i, and the coefficient with x_i be the same for all agents. The usefulness of this condition will stem from the fact that it ensures that the total surplus can be written as a function of the total trade X only:

$$W(X) = F(X) + X\alpha(X) + \sum_i \beta_i(X).$$

Condition D: Either $\mathfrak{X}_i = [0, \overline{x}_i]$ or $\mathfrak{X}_i = \{kz : k = 0, 1, ...\overline{k}_i\}$ for all i, for some $z > 0$.

This assumption says that all trades are measured in the same increments, which could be either infinitesimal or finite.[2]

[2]Given that the sets $\mathfrak{X}_i$ are compact, an alternative way to state this condition is that there exists z (interpreted as a unit of trade) such that $\sup\{y \in \mathfrak{X}_i : y < x\} = x - z$ for all i and all $x \in \mathfrak{X}_i \backslash \{0\}$.

Condition S: $\mathfrak{X}_i = \{0,1\}$ for all i. Furthermore, for any trade profile $x \in \{0,1\}^N$ and any permutation π of $\{1,\ldots,N\}$, letting x_π denote the permuted trade profile $(x_{\pi(1)},\ldots,x_{\pi(N)})$, we have $f(x_\pi) = f(x)$, and $u_i(x_\pi) = u_{\pi(i)}(x)$ for all i.

The second part of this assumption states that all agents are identical, in the sense that their payoffs are symmetric with respect to permutations of agents. Observe that Condition S implies all the other Conditions. Indeed, given the imposed symmetry among agents, the principal's profit $f(x)$ will be completely determined by the total number $X = \sum_j x_j$ of agents who have $x_j = 1$. Similarly, by considering all the permutations π that hold i fixed, we can see that agent i's utility can be written as $U_i(x_i, X)$. Therefore, Condition S implies Condition A. The symmetry imposed by Condition S also implies that the function $U_i(\cdot,\cdot)$ is the same for all agents i. Condition L then also holds, since the fact that $x_i \in \{0,1\}$ implies the linearity of $U(x_i, X)$ in x_i. Condition D also trivially follows from Condition S.

3 Applications

Application 1: Vertical Contracting (Hart-Tirole [1990], O'Brien-Schaffer [1992], McAfee-Schwartz [1994], Rey-Tirole [1996]). The principal is a supplier of an intermediate good to N agents (downstream firms), who produce substitute goods. $x_i \geq 0$ is firm i's purchase of the intermediate good, and t_i is its payment to the supplier. Due to the ensuing downstream competition, each firm i's utility, $u_i(x_i, x_{-i})$, is decreasing in other firms' purchases x_{-i}. The literature assumes that a downstream firm cannot produce without using the principal's input, and therefore $u_i(0, x_{-i}) \equiv 0$.

Hart-Tirole [1990] and Rey-Tirole [1996] study a specific model of downstream competition which further restricts the firms' payoffs. Namely, they assume that the downstream firms produce a homogeneous final good, using a technology that transforms a unit of the intermediate good into a unit of the final good at a cost c. After purchasing their inputs, the firms play a standard Bertrand-Edgeworth game of downstream price competition with capacity constraints. Assuming that the firms utilize all purchased inputs in equilibrium,[3] and letting $P(X)$ denote the inverse demand

[3] See Tirole [1988, ch. 5] for more detail.

function for the final good, the parties' payoffs satisfy Condition A with $U_i(x_i, X) = [P(X) - c]x_i$. Observe that this model also satisfies Conditions L and D.

Application 2: Vertical Contracting with an Inferior Substitute. (Katz-Shapiro [1986a], Kamien et al. [1992]). As in the previous example, N downstream firms (agents) use the input supplied by the principal to produce the final good, but now each downstream firm also has access to an inferior technology which does not use the principal's input. Therefore, unlike in the previous application, $u_i(0, x_{-i})$ can now be positive and can depend on x_{-i} (the importance of this difference will be shown in Section 4). As in the previous application, due to the downstream competition, $u_i(x_i, x_{-i})$ is decreasing in x_{-i}.[4]

The two referenced papers assume that the intermediate good is a fixed input: $x_i \in \{0, 1\}$ (specifically, they interpret the "input" as a licence to use the principal's patent). They also assume that the downstream firms are identical. These assumptions imply Condition S, and therefore all the other Conditions.

Application 3: Exclusive Dealing (Rasmusen-Ramseyer-Wiley [1991], Segal-Whinston [forth.]). The principal is an incumbent monopolist who offers exclusive dealing contracts to N identical buyers (agents). The contract obliges the buyer not to purchase from a rival seller. Let $x_i = \{0, 1\}$ indicate whether buyer i signs such a contract, and $(-t_i)$ be the compensation paid to the buyer by the incumbent. After observing the number X of signers, a potential entrant decides whether to enter. Due to the entrant's economies of scale, the probability of entry $\rho(X)$ is a non-increasing function of X. In the case of no entry, in the second stage the incumbent makes the monopoly profit π^m on each buyer by charging him the monopoly price p^m. In the case of entry, the entrant and incumbent compete for the "open" buyers (those who have not signed in the first stage), and the incumbent, whose marginal cost is higher than the entrant's, makes no profit on these buyers. The incumbent can still charge p^m to the buyers who have signed exclusives, and earn π^m on each of them.

Since the buyers are assumed to be identical, the model satisfies Condition S, and consequently all the other Conditions. Specifically, the incumbent's

[4]Kamien et al. derive this from explicit models of Cournot and Bertrand type models of downstream competition.

net profit can be written as

$$F(X) = [\rho(X)N + (1-\rho(X))X]\,\pi^m.$$

Normalizing each buyer's surplus under price p^m to zero, and letting Δ denote his surplus under the competitive price, his utility can be written as

$$U_i(x_i, X) = (1-x_i)\rho(X)\Delta.$$

Since $U_i(x_i, X)$ is non-increasing in X, by signing an exclusive contract, each buyer imposes a negative externality on other buyers.

Application 4: Selling a Nuclear Weapon (Jehiel-Moldovanu [1996,1997], Jehiel at al. [1996]) The principal has only one unit of an indivisible good (a nuclear weapon). Let $x_i \in \{0,1\}$ indicate whether agent i obtains this weapon, and t_i be the agent's payment to the principal. We can model the principal's inability to sell more than one weapon by assuming that $f(x) = -C$ whenever $\sum_j x_j \geq 2$, with C very large. Letting α_{ij} denote the utility of agent i if agent j gets the weapon, we can write $u_i(x) = \sum_j \alpha_{ij} x_j$. Observe that this model in general does not satisfy Condition A, since agent i's utility may depend on which of the other agents obtains the weapon. Therefore, out of the Conditions described in Section 2, only Condition D is satisfied.

Application 5: Common Insurance. (Pauly [1974]) The principal is a risk-averse individual who contracts with N risk-neutral insurance firms (agents). There are two possible outcomes, $y \in \{0,1\}$. The individual suffers a monetary loss $a > 0$ when $y = 1$ (the "accident" state), and suffers no loss when $y = 0$ (the "no accident" state). The individual's insurance contract with each firm i specifies a premium $(-t_i)$ and a payment $x_i \geq 0$ the individual receives when $y = 1$.

Externalities among insurance firms are due to the individual's moral hazard. For example, suppose that the individual can choose the probability ρ of accident, at a private cost $c(\rho)$. If the individual's risk preferences satisfy Constant Absolute Risk Aversion, then her certainty equivalent can be written as

$$v(\rho, X) - c(\rho) + \sum_i t_i, \text{ with } v(\rho, X) = -\frac{1}{r}\log\left[1-\rho+\rho e^{-r(X-a)}\right],$$

where $X = \sum_i x_i$, and $r > 0$ is the individual's coefficient of absolute risk aversion. Then the parties' expected payoffs satisfy Condition A, with

$$\begin{aligned} F(X) &= v(\rho^*(X)) - c(\rho^*(X)), \\ U_i(x_i, X) &= -\rho^*(X)x_i, \end{aligned}$$

where $\rho^*(X)$ is the individual's optimal choice of ρ. It is easy to see that $\rho^*(X)$ is non-decreasing in X.[5] Therefore, by increasing the individual's insurance and thereby raising the probability of accident, each company imposes a negative externality on other companies. Observe that the model satisfies Conditions L and D.[6]

Application 6: Common Agency. (Bernheim-Whinston [1986]) Consider the model of the previous application, with two modifications. First, the individual herself suffers no monetary loss from either outcome, i.e., $a = 0$. Second, each firm i now receives a benefit b_i when $y = 1$ (and the benefit from $y = 0$ is normalized to zero). As in the previous application, the individual's contract with each firm i specifies a lump-sum payment t_i to the individual and a "bonus" payment x_i for the "good" outcome $y = 1$. Unlike in the previous application, however, the motivation for contracting is not to insure the individual, but to make her choose an action which increases the probability of an outcome that is desirable for the firms.

The parties' expected utilities satisfy Condition A, with

$$\begin{aligned} F(X) &= v(\rho^*(X)) - c(\rho^*(X)), \\ U_i(x_i, X) &= \rho^*(X)\left[b_i - x_i\right]. \end{aligned}$$

Assume that $b_i \geq 0$ for all i, and restricting attention to contracts with $x_i \in [0, b_i]$ (higher bonuses are not likely to arise in equilibrium). Then by increasing the total bonus and thereby raising the probability of $y = 1$, each firm has a positive externality on other firms. Observe that the model satisfies Conditions L and D.[7]

[5] By observing that the individual's utility is supermodular in (ρ, X), and applying Topkis' Monotonicity Theorem (see e.g. Milgrom and Shannon [1994]).

[6] In the model of Pauly, the individual's risk preferences need not satisfy CARA, and therefore her behavior may exhibit wealth effects (in particular, her optimal choice of ρ may depend on $\sum_i t_i$). To simplify analysis, we abstract from wealth effects in this paper.

[7] The model of Bernheim-Whinston is substantially more general than our simple model

Application 7: Takeovers. (Grossman-Hart [1980], Bagnoli-Lipman [1988], Holmstrom-Nalebuff [1992]). The principal is a corporate raider, who makes a tender offer to N shareholders (agents). Let x_i denotes the number of shares tendered by shareholder i, and $(-t_i)$ denote the raider's payment to this shareholder. Let $v(X)$ denote the expected value of the firm's shares as a function of X, the total number of shares tendered. Finally, let $c(X)$ denote the raider's "transaction cost" of acquiring X shares (it could also be negative, reflecting the raider's private benefit from controlling X shares). Then the parties' payoffs satisfy Condition A, with

$$\begin{aligned} F(X) &= Xv(X) - c(X), \\ U_i(x_i, X) &= (\overline{x}_i - x_i)v(X), \end{aligned}$$

where $\overline{x}_i$ is shareholder i's endowment of shares. It is usually assumed that the raider is more efficient than the incumbent management, and therefore $v(X)$ is non-decreasing in X. In this case, a shareholder who tenders his shares has a positive externality on other shareholders.[8] The model satisfies Condition L, and it satisfies Condition D provided that either shares are infinitely divisible or all indivisible shares have the same value.

Application 8: Debt Workouts. (Gertner-Scharfstein [1991]) The principal is a collective of shareholders of a financially distressed firm, and the agents are the firm's creditors (of equal seniority). The shareholders offer the creditors a debt-equity swap. Let $\bar{x}_i$ denote the face value of debt initially held by creditor i, and suppose that the swap results in the creditor forgiving an amount $x_i \in [0, \bar{x}_i]$ of debt in exchange for an equity stake s_i in the firm. Let $F(X)$ denote the expected value of the firm's equity, and $d(X)$ denote the expected value of a \$1 face value of debt, as functions of the

in several important respects. First, the individual's risk preferences may exhibit wealth effects (see the previous footnote). Second, they allow for more than two possible outcomes, which requires considering contracts of more than two dimensions. (For example, Holmstrom-Milgrom [1988] and Dixit [1995] study common agency models with a separate dimension of agent's performance for each principal.) Third, the firms' preferences over the two outcomes may diverge, e.g., we may have $b_i < 0$ for some firms, in which case contracting may have negative, as well as positive, externalities.

[8] It is also possible that an increase in X could reduce the firm's public value $v(X)$. For example, the raider could use her control over the firm to divert some of its value to her private benefit. In that case, which is considered, e.g., by Bagnoli-Lipman, a tendering shareholder imposes a negative externality on other shareholders.

total amount $X = \sum_i x_i$ of debt tendered. The principal's payoff can then be written as $(1 - \sum_i s_i) F(X)$, and each agent i's payoff can be written as $(\bar{x}_i - x_i) d(X) + s_i F(X)$. If the offers are made publicly, then each agent i will be able to calculate the monetary value $t_i = s_i F(X)$ of the shares he is offered. In this notation, the parties' payoffs can be written in the standard form. Observe that the model satisfies Conditions A, L, and D.

We can expect $d(X)$ to be increasing in X, for two reasons: (i) when the firm's debt is lower, it will repay a greater portion of it in any state of the world, and (ii) when the firm's debt is lower, the firm is less likely to engage in extracting surplus from creditors, e.g. by undertaking risky investments, or triggering inefficient bankruptcy. Thus, a creditor who accepts his offer has a positive externality on other creditors.[9]

Application 9: Merger for Monopoly. (Mackay [1984]). The principal makes acquisition offers to N competing firms (agents). Let $x_i \in \{0, 1\}$ indicate whether firm i accepts the offer, and let $(-t_i)$ be the principal's payment to this firm. If the firms are identical, then the model satisfies Condition S, and therefore all the other Conditions. Since a firm that sells out does not make any profit in the market, we have $U_i(1, X) \equiv 0$.

Since the market's concentration is increasing in X, it is natural to expect the profit $U_i(0, X)$ of a firm which did not sell out to be increasing in X. Therefore, a firm which sells out has a positive externality on other firms.

Application 10: Network Externalities. (Katz-Shapiro [1986b]) The principal is a seller of a good for which each agent (buyer) has a unit demand. $x_i \in \{0, 1\}$ is buyer i's purchase of the good, and t_i is his payment to the seller. Since all buyers are assumed to be identical, Condition S is satisfied, and therefore all the other Conditions. Because of a "network externality", each buyer's valuation of the good is an increasing function $\alpha(X)$ of the total number X of units sold,[10] thus we have $U_i(x_i, X) = x_i \alpha(X)$. A buyer who

[9] Gertner-Scharfstein also consider exchanges of debt for cash or senior debt. These exchanges cannot be captured by our simple model, since *both* of the goods exchanged involve externalities on other creditors. For example, a creditor has a positive externality on other creditors by forgiving his debt, but he imposes a negative externality on them by accepting senior debt in exchange. Gertner-Scharfstein find that the net external effect of such exchanges may be negative.

[10] Karni-Levin [1994] assume a similar "network externality" for restaurant dinners: a diner's valuation for a restaurant meal is assumed to be increasing in the total number of diners at that restaurant.

buys the good therefore has a positive externality on other buyers owning this good.

Application 11: Bargaining Externalities. (Cai [1996a,b]) Consider a two-period bargaining game in which the principal needs the agreement of all N agents to realize a surplus $S > 0$. All players have the same discount factor $\delta \in (0, 1)$. In the first period, the principal makes an offer to each agent i. Let $x_i \in \{0, 1\}$ indicate whether agent i accepts his offer, and $(-t_i) \geq 0$ denote the principal's payment to the agent. If some agents reject their offers, in the second period all such agents cooperatively split the surplus S with the principal. Specifically, suppose that the principal receives a share $\rho \in (0, 1)$ of the surplus, and the rest is equally shared by the agents who have not signed in the first period.

Since all agents are identical, the model satisfies Condition S, and therefore all the other Conditions. Specifically, the parties' payoffs can be written as

$$F(X) = \begin{cases} S & \text{if } \; X = N, \\ \delta\rho S & \text{if } \; X < N, \end{cases} \quad \text{and } U(x_i, X) = \begin{cases} 0 & \text{if } \; x_i = 1, \\ \delta\frac{1-\rho}{N-X}S & \text{if } \; x_i = 0. \end{cases}$$

Since $U(0, X)$ is increasing in X, this stylized model captures the intuition of Cai that by accepting an offer in the first period, an agent has a positive externality on the agents remaining at the bargaining table.[11]

Application 12: Pure Public Goods. (Bergstrom-Blume-Varian [1986]) The principal is a provider of a public good, who can contract with N consumers of the good (agents). Let $x_i \geq 0$ be the amount of the public good "contributed by agent i", and let t_i be agent i's payment to the provider. Then the parties' utilities satisfy Condition A, with $U_i(x_i, X) = v_i(X)$ being consumer i's benefit from X units of the public good. We assume that $v_i(X)$ is non-decreasing in X, i.e., each consumer's contribution to the public good has a positive externality on other consumers. Observe that the model satisfies Conditions L and D.

Application 13: Pure Public Bads. (Neeman [1997]) The model is the same as in the previous application, except that now $U_i(x_i, X) = v_i(X)$ is non-

[11] Cai analyzes an infinite-horizon bargaining game in which agents are approached by the principal sequentially, and inefficient delay occurs in some (but not all) equilibria. It will follow from the analysis of the next section that the unique equilibrium of my model involves inefficient delay when δ is close enough to 1.

increasing in X. Neeman offers several examples of such public bads. One example is vote trading: a party (the principal) may be able to implement a policy which hurts voters, by buying up their votes.[12] Another example is "yellow dog" employment contracts, which require workers not to join labor unions. The assumption here is that workers' wages are increasing in the union's bargaining strength, which is in turn increasing in the number of union workers. In both cases, individual agents care only about the total trade X, and not about their individual trades x_i per se. The model satisfies Conditions L and D.[13]

All the applications, and the Conditions they satisfy, are listed in Table 1. Observe that all of the above applications satisfy Conditions A, L, and D, with the exception of Application 1 (Vertical Contracting) in the case when the final goods are differentiated, and Application 4 (Nuclear Weapons). (Many of the applications actually satisfy the stronger Condition S.) Observe, however, that even with these conditions imposed, our model cannot be reduced to the pure public good/bad settings described in the last two applications. Indeed, in our general model, contracting has both a public and a private component. Agent i's benefit from the public component X in general depends on his private trade x_i, and, conversely, his willingness to pay for x_i in general depends on X.

4 Bilateral Contracting with Public Offers

In this section, we analyze the following two-stage game: In the first stage, the principal commits to a set $\{(x_i, t_i)\}_{i=1}^N$ of publicly observable bilateral contract offers to all agents. In the second stage, all agents simultaneously decide whether to accept or reject their respective offers. We analyze the principal's preferred Subgame-Perfect Nash Equilibrium (SPNE) of the game.

[12]In particular, this situation would arise in Takeovers (Application 7) in the absence of a "one share-one vote" rule, i.e., if shareholders could sell their voting rights to an inefficient raider while keeping their profit shares.

[13]Neeman also claims, erroneously, that Exclusive Dealing (as described in Application 3) offers another example of a pure public bad. The claim is incorrect, since each buyer i's valuation for competition (and therefore for X) depends on whether he has signed an exclusive (i.e., x_i). Specifically, competition does not benefit buyers who have signed exclusives, since they are charged the monopoly price in any case.

(In Appendix B, we study conditions under which the game may have other SPNE, and analyze the properties of such equilibria.)

4.1 Inefficiency Results

Since the principal can always offer $(x_i, t_i) = (0, 0)$, without loss of generality we can restrict attention to equilibria in which every agent accepts his offer. It will be a second stage Nash equilibrium for all agents to accept if and only if the following participation constraint is satisfied for every agent i:

$$u_i(x) - t_i \geq u_i(0, x_{-i}) \tag{4}$$

It is clear that in the principal's preferred SPNE all agents' participation constraints must bind (otherwise, the principal could increase her profit by raising t_i for some agent i, while still keeping all participation constraints satisfied). Substituting the resulting transfers in the principal's objective function, we can write her problem as

$$\max_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N} f(x) + \sum_i u_i(x) - \sum_i u_i(0, x_{-i}). \tag{5}$$

Let $\mathfrak{M} \subset \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N$ denote the set of solutions to this program.

The principal's objective function differs from that of the first-best program (3) by its last term. This term is the sum of the agents' reservation utilities, i.e., utilities they would obtain by rejecting their offer, provided that everyone else accepts. If each agent's reservation utility does not depend on other agents' trades (in which case we will say that there is no externality on signers), then the profit-maximization program (5) is equivalent to the first-best program (3), and we obtain

Proposition 1 *If $u_i(0, x_{-i})$ does not depend on $x_{-i} \in \mathfrak{X}_{-i}$ for all i, then $\mathfrak{M} = \mathfrak{M}^*$.*

Intuitively, when externalities on non-signers are absent, and the principal can commit to compensate the agents who contract with her for the externalities imposed on them, externalities do not result in distortions. Externalities on non-signers are absent in Applications 1 (Vertical Contracting), 5 (Common Insurance), and 10 (Network Externalities), and the principal's commitment to a set of public offers yields first-best outcomes in these applications.[14]

On the other hand, when contracting affects the agents' reservation utilities, inefficiency arises because of the principal's incentive to reduce the agents' rents. Intuitively, this implies that the contracting outcome x will be distorted to reduce the sum of agents' reservation utilities, $r(x) = \sum_i u_i(0, x_{-i})$. To formalize this intuition, we use the monotone comparative statics techniques described by Milgrom-Shannon [1994], which utilize the concept of the strong set order:

Definition 1 *For two sets A, B, we say that $A \leq B$ in the strong set order if whenever $a \in A$, $b \in B$, and $a \geq b$, we must also have $a \in B$ and $b \in A$.*

In this paper we will only be concerned with cases in which A and B are subsets of $\Re$, in which case $A \leq B$ in the strong set order if and only if $A \backslash B$ lies below $A \cap B$, which in turn lies below $B \backslash A$. Armed with this concept, we can formulate the following result:

Proposition 2 *$r(\mathfrak{M}) \leq r(\mathfrak{M}^*)$ in the strong set order.*

Proof. Using the "aggregation method" of Milgrom-Shannon [1994], define

$$w(\overline{r}) = \max \left\{ f(x) + \sum_i u_i(x) : x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N, r(x) = \overline{r} \right\}.$$

- the maximum value of total surplus which is consistent with a given sum $\overline{r}$ of agents' reservation utilities. Consider the following parametrized program:

$$\max_{\overline{r}} w(\overline{r}) - z\overline{r},$$

where $z = 0$ corresponds to surplus-maximization (program (3)) and $z = 1$ to profit-maximization (program (5)), and let $r^*(z)$ denote the set of solutions

[14]In the context of Application 11 (Pure Public Good), if the public good were excludable, there would also be no externality on non-signers. In the context of Application 5 (Common Agency), Bernheim-Whinston assume that a firm that does not sign a contract is not affected by the individual's actions, in which case there would also be no externality on non-signers.

to this program. Since the objective function is supermodular in $(\bar{r}, z)$, by Topkis' Monotonicity Theorem, $r(\mathfrak{M}^*) = r^*(0) \geq r^*(1) = r(\mathfrak{M})$. ■

The next question of interest is how the principal's rent extraction motivation affects the contracting outcome x. Since the distortion is caused by the externalities imposed on non-signers, intuition suggests that the answer will hinge on whether these externalities are positive or negative:

Definition 2 *We have positive [negative] externalities on non-signers if* $u_i(0, x_{-i})$ *is non-decreasing [non-increasing] in* $x_{-i} \in \mathfrak{X}_{-i}$ *for all* i.

Thus, Applications 2 (Vertical Contracting with an Inferior Substitute), 3 (Exclusive Dealing), and 13 (Pure Public Bads) described in Section 3 exhibit negative externalities on non-signers, while Applications 6 (Common Agency), 7 (Takeovers), 8 (Debt Workouts), 9 (Merger for Monopoly), 11 (Bargaining), and 12 (Pure Public Goods) exhibit positive externalities on non-signers. In Application 4 (Nuclear Weapon), externalities could be either positive or negative. These facts are summarized in Table 1.

Intuition suggests that with positive externalities on non-signers we will have too little trade from the social viewpoint, and with negative externalities we will have too much. Unfortunately, this intuition is not correct in general. For example, with positive externalities, the principal has an incentive to distort each agent i's trade x_i downward *given* x_{-i}, the vector of other agents' trades. But since we can also expect x_{-i} to be different from its first-best value, we cannot conclude that the principal's choice of x_i will be lower than its first-best level.

In almost all applications of the model, Condition A is satisfied (see Table 1), and the main question of interest is whether the total trade X at the principal's profit-maximizing outcome is socially too high or too low. It turns out, however, that Condition A alone does not guarantee a definitive answer to this question. Here is an example which satisfies Condition A and exhibits positive externalities on non-signers, but in which the principal distorts the total trade X upward:

Example 1 *Let* $N = 2$, $\mathfrak{X}_1 = \mathfrak{X}_2 = \Re_+$, *suppose that Condition A holds, and let* $U_1(x_1, X) = x_1\left[1 - X^{10}\right] + X$, $u_2(x_2, X) = 3X$, *and* $F(X) = \begin{cases} 0 \text{ when } X \leq 1, \\ -100X \text{ when } X > 1. \end{cases}$

Consider first the first-best program (3), which can now be written as

$$\max_{\substack{x_1, x_2 \geq 0 \\ x_1 + x_2 = X}} F(X) + x_1 \left[1 - X^{10}\right] + 4X.$$

First, observe that given the shape of $F(\cdot)$, we can restrict attention to $X \leq 1$. Second, observe that it is optimal to allocate any given $X \leq 1$ between the agents by setting $x_1 = X$, $x_2 = 0$. Therefore, the optimal X will solve $\max_{X \in [0,1]} W(X) = X\left[1 - X^{10}\right] + 4X$. Since $W'(1) = -10 + 4 < 0$, any solution X^* of this program must satisfy $X^* < 1$.

As for the profit-maximization program (5), it is now written as

$$\max_{\substack{x_1, x_2 \geq 0 \\ x_1 + x_2 = X}} F(X) + x_1 \left[1 - X^{10}\right] + 4X - x_2 - 3x_1 = F(X) + x_1 \left[1 - X^{10}\right] - 2x_1 + 3X.$$

Again, given the shape of $F(X)$, we can restrict attention to $X \leq 1$. Observe that now the optimal way to allocate any X between the agents is by setting $x_1 = 0$, $x_2 = X$. Then the program can be written as $\max_{X \in [0,1]} 3X$, and its solution is $X = 1$. Therefore, the profit-maximizing total trade exceeds the first-best total trade, despite the fact that we have positive externalities on non-signers.

The key feature of the example is that the socially optimal allocation of a total trade $X \leq 1$ between agents involves trading with agent 1 only, while the principal's profit-maximizing allocation involves trading with agent 2 only. Intuitively, the interaction between the principal's choice of X and her choice of allocation of a given X among the agents prevents us from making a definitive comparison with the first-best.

In order to eliminate this interaction, we assume Conditions L and D, which are satisfied in all economic applications which satisfy Condition A (see Table 1). Observe that under Condition L we have positive [negative] externalities on non-signers if and only if all functions $\beta_i(\cdot)$ are non-decreasing [non-increasing]. Condition L ensures that the allocation of a given X among agents is irrelevant in the first-best problem, and the set $M^* = \{\sum_i x_i : x \in \mathfrak{M}^*\}$ of surplus-maximizing total trades can be defined as

$$M^* = \arg\max_{X \in \sum_i \mathfrak{X}_i} W(X) = \arg\max_{X \in \sum_i \mathfrak{X}_i} F(X) + X\alpha(X) + \sum_i \beta_i(X) \qquad (6)$$

(where $\sum_i \mathfrak{X}_i = \{\sum_i x_i : x_i \in \mathfrak{X}_i \text{ for all } i \in N\}$).

As for the principal's profit-maximization program (5), under Condition L it becomes

$$\max_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N} F(\sum_j x_j) + \left[\sum_j x_j\right] \alpha(\sum_j x_j) + \sum_i \beta_i(\sum_j x_j) - \sum_i \beta_i(\sum_{j \neq i} x_j).$$

Using the "aggregation method" of Milgrom-Shannon [1994], define

$$R(X) = \min_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N} \left\{ \sum_i \beta_i(\sum_{j \neq i} x_j) : \sum_j x_j = X \right\}$$

- the minimum sum of agents' reservation utilities which is consistent with the total trade X.[15] Then, letting $M = \{\sum_i x_i : x \in \mathfrak{M}\}$ denote the set of the principal's profit-maximizing total trades, the above program implies that

$$M = \arg\max_{X \in \sum_i \mathfrak{X}_i} F(X) + X\alpha(X) + \sum_i \beta_i(X) - R(X). \tag{7}$$

In order to compare M and M^*, we first establish the following lemma:

Lemma 1 *If Condition D holds and all functions $\beta_i(\cdot)$ are non-decreasing [non-increasing], then $R(X)$ is non-decreasing [non-increasing] over those values of X for which it is defined.*

Proof. First, let all $\beta_i(\cdot)$ be non-decreasing. Take any $X', X \in \sum_i \mathfrak{X}_i$, with $X' \leq X$. Suppose that $R(X) = \sum_i \beta_i(\sum_{j \neq i} x_j)$, with $\sum_i x_i = X$. Under condition D, there exist feasible $x'_j \leq x_j$ such that $\sum_i x'_i = X'$. This implies that $R(X') \leq \sum_i \beta_i(\sum_{j \neq i} x'_j) \leq R(X)$ whenever $R(X')$ is defined.

Now, let all $\beta_i(\cdot)$ be instead non-increasing. Take any $X', X \in \sum_i \mathfrak{X}_i$, with $X' \geq X$. Suppose that $R(X) = \sum_i \beta_i(\sum_{j \neq i} x_j)$, with $\sum_i x_i = X$. Under condition D, there exist feasible $x'_j \geq x_j$ such that $\sum_i x'_i = X'$. This implies that $R(X') \leq \sum_i \beta_i(\sum_{j \neq i} x'_j) \leq R(X)$ whenever $R(X')$ is defined. ■

The Lemma implies

[15] The minimum may not exist when the functions $\beta_i(\cdot)$ are not continuous. When the minimum does not exist for a certain total trade X, we will say that $R(X)$ is not defined. Note that such total trade cannot arise at a profit-maximizing outcome.

Proposition 3 *If Conditions L,D hold, then with positive [negative] externalities on non-signers, $M \leq M^*$ [$M \geq M^*$] in the strong set order.*

Proof. Consider the parametrized program

$$\max_{X \in \sum_i \mathfrak{X}_i} F(X) + X\alpha(X) + \sum_i \beta_i(X) - zR(X),$$

where $z = 0$ corresponds to the first-best program (6), and $z = 1$ corresponds to the profit-maximization program (7). Using Lemma 1, we see that with positive [negative] externalities on non-signers, the objective function is supermodular in $(-X, z)$ [in (X, z)]. Topkis' Monotonicity Theorem implies the result. ■

This Proposition neatly summarizes many existing contracting inefficiency results. For instances of positive externalities on non-signers, the Proposition predicts that an individual's effort in a common agency situation may be lower than its second-best level (Application 6), that takeovers and debt-equity swaps are less likely to occur than is socially optimal (Applications 7,8), that merger for monopoly may not occur even though it maximizes producers' profits (Application 9), that multiparty bargaining may exhibit inefficient delays (Application 11), and that a pure public good may be privately underprovided (Application 12). For instances of negative externalities on non-signers, the Proposition predicts that socially inefficient exclusion may occur (Application 3), that an intermediate good manufacturer may sell more than the vertical profit-maximizing quantity when an inferior substitute is available (Application 2), and that a pure public bad may be overprovided (Application 13).[16] When externalities on non-signers are absent (Applications 1,5,10), they can be thought of as both "positive" and "negative" at the same time. In this case, the proposition implies that the set of contracting outcomes coincides with the set of first-best outcomes.

As one application, this result demonstrates the fundamental difference between excludable and non-excludable public goods (in the context of Application 12). The possibility of exclusion eliminates the positive externality on

[16] While suggestive of distortions, all of our comparisons in this section are weak, i.e., they do not rule out the possibility that contracting outcomes are socially efficient. Numerous examples of strict inefficiency can be found in the papers referenced in Section 4. In subsection 4.3, we will provide sufficient conditions for contracting outcomes to be inefficient when N is large enough.

non-contributors and results in efficient provision (despite the remaining externality on contributors), while without exclusion the positive externality on non-contributors results in underprovision. To my knowledge, this is the first formal demonstration of the role of excludability for the efficient provision of a public good in a symmetric-information setting (in a private-information setting, a similar point has been made by Maskin [1994]).[17]

Our analysis assumes that all agents accept in the second stage whenever it is a Nash equilibrium for them to do so. It turns out, however, that even when "all accept" is a Nash equilibrium, there may exist other Nash equilibria in which some or all agents reject their offers, and that these Nash equilibria may sometimes be preferred by all agents. In Appendix B, we study conditions under which this happens, and characterize the contracting outcome when the agents always coordinate on a coalition-proof equilibrium. We find that with some additional assumptions, the results of this section are preserved.

4.2 Effect of Contracting on Agents' Utilities

In the absence of externalities, the agents, who have no bargaining power, would receive no contracting surplus. With externalities, we have the following result:

Proposition 4 *With positive [negative] externalities on non-signers, a ban on contracting weakly reduces [raises] each agent's utility.*

Proof. Since in equilibrium each agent i's participation constraint binds, his utility equals $u_i(0, x_{-i})$, which, with positive [negative] externalities, is weakly smaller [greater] than $u_i(0, 0)$, his utility if contracting were banned. ■

The case of negative externalities is particularly interesting: in equilibrium, each agent would be better off if all of them rejected their offers, yet all of them accept. A ban on contracting would raise all agents' utilities, and may even raise the total surplus.[18]

[17]A caveat: when the public good's provider cannot commit to a set of contract offers, inefficiency would arise due to the externality on signers, as will be shown in Section 5.

[18]Neeman [1997] independently argues, in the context of Pure Public Bad (Application 13) which exhibits negative externalities, that a restriction on the freedom of contract may be beneficial.

4.3 Asymptotics

When the number N of agents is very large and Condition A holds, it seems natural to expect that each individual agent will take the total trade X as given in making his decision, i.e., he will think of himself as *non-pivotal.* Many papers have indeed made this assumption, including Gertner-Scharfstein [1991] in the context of Debt Workouts (Application 8), Katz-Shapiro [1986b] in the context of Network Externalities (Application 10), and Grossman-Hart [1980] in the context of Takeovers (Application 7). In the last context, however, the assumption has been subject to much controversy, which we outline here.

Most of the literature on takeovers focuses on the case in which the value of the firm as a function of the proportion X of equity tendered is given by $v(X) = \begin{cases} \overline{v} \text{ when } X \geq 0.5, \\ \underline{v} < \overline{v} \text{ otherwise.} \end{cases}$ If the raider takes over (i.e., $X \geq 0.5$), using (7), her profit can be written as

$$\Delta v \cdot \sum_{i \in N:\, X - x_i < 0.5} \overline{x}_i - c(X),$$

where $\Delta v = \overline{v} - \underline{v}$ is the firm's value increase due to the takeover, and $c(X)$ is the raider's "transaction cost". This expression demonstrates that *the raider appropriates only the appreciation in the holdings of those shareholders who are pivotal for the takeover.* Therefore, whenever some shareholders are not pivotal, the raider receives less than 100% of the firm's value improvement, and in the presence of positive takeover costs she will not implement some socially efficient takeovers. Moreover, Grossman-Hart [1980] argue that when the number N of shareholders is large, each of them will think of himself as non-pivotal. Therefore, the raider will not be able to appropriate any of the firm's value improvement, and will not bid in the presence of any positive bidding cost.

In response to Grossman-Hart, Bagnoli-Lipman [1988] and Holmstrom-Nalebuff [1992] pointed out that for any finite N, the raider can make each tendering shareholder pivotal by setting $X = 0.5$. This allows the raider to capture a significant fraction of the firm's value improvement Δv. For example, consider the case in which each shareholder i holds one indivisible share: $\mathfrak{X}_i = \{0, 1/N\}$, and assume for simplicity that N is even. In this case, $X = 0.5$ corresponds to having exactly $N/2$ shareholders tender. Since each of the tendering shareholders is pivotal, the raider appropriates 50% of the firm's value improvement. But the raider can do even better when individual shareholdings are divisible. For example, by setting $X = 0.5$ and $x_i = \overline{x}_i/2 > 0$ for all i, the raider would make *each* shareholder pivotal, thus capturing 100% of the firm's value improvement! Observe that the raider can implement this outcome simply by offering the price $\underline{v} - \Delta v$ for each share tendered. Given this offer, there exists an equilibrium in which each shareholder tenders exactly 1/2 of his holding. Indeed, even though the bid is *below* the firm's current value, each shareholder will tender, since he knows that the takeover, for which he is pivotal, will raise the value of the shares he keeps by exactly the same amount as he loses on the shares he sells. Thus, the raider will appropriate 100% of the firm's value improvement, and a takeover will take place whenever it is socially efficient, regardless of N.[19] This is very different from the non-pivotal outcome described by Grossman-Hart.

In order to study conditions under which the agents feel non-pivotal when N is large, we introduce the following "asymptotic setting": Take one big agent, whose payoff (under Condition A) is $U(x, X) - t$, and split his trade among N small identical agents, so that $\sum_{i=1}^{N} x_i = x$ and $\sum_{i=1}^{N} = t$. Suppose that the payoff of each of these small agents is $U^N(x_i, X) - t_i$. We want to require that the total payoff of the small agents always equals to the big agent's payoff: $\sum_i U^N(x_i, X) - \sum_i t_i. = U(x, X) - t$. For this to hold, we need $U^N(x_i, X)$ (and consequently $U(x, X)$) to satisfy Condition L. Thus, we can write $U(x, X) = x\alpha(X) + \beta(X)$, and $U^N(x_i, X) = x_i\alpha(X) + \frac{1}{N}\beta(X)$. We take the latter utility function to be each agent's utility in the asymptotic setting with N agents.

It remains to define the trade domain for each small agent. To reflect the fact that each small agent i is a $1/N$th replica of the big agent, we assume that his trade domain is $\mathfrak{X}_i = \mathfrak{X}/N$. The total trade, therefore, lies in the set $\sum_i \mathfrak{X}_i = {}^N\mathfrak{X}/N$, where we use the notation ${}^J\mathfrak{X} = \underbrace{\mathfrak{X} + \ldots + \mathfrak{X}}_{J \text{ times}}$. Asymptotically,

[19] Bagnoli-Lipman only study indivisible shareholdings, and Holmstrom-Nalebuff do not consider bids below the firm's current value. For these reasons, in both setups, the raider cannot appropriate more than 50% of the firm's value improvement. Both papers study symmetric mixed-strategy equilibria, in which the raider generally does worse than that. However, in the "focal" symmetric equilibrium studied by Holmstrom-Nalebuff, randomization disappears as shares become infinitely divisible, and the raider comes close to capturing 50% of the improvement.

we will allow all total trades from the convex hull of $\mathfrak{X}$, which we denote by $\overline{\mathfrak{X}}$. Observe that ${}^N\mathfrak{X}/N \subset \overline{\mathfrak{X}}$ for all N.

Now we are prepared to define non-pivotal outcomes. When each agent i takes the total trade X as given, his participation constraint can be written as $x_i\alpha(X) - t_i \geq 0$. Substituting all the binding participation constraints in the principal's objective function, we find that her profit is given by $F(X) + X\alpha(X)$. Conveniently, it is only a function of the total trade X, and not of its allocation among agents. The set of non-pivotal outcomes can now be defined as

$$M_0 = \arg\max_{X \in \overline{\mathfrak{X}}} \pi_0(X) = F(X) + X\alpha(X).$$

Since the set $\overline{\mathfrak{X}}$ is compact, a sufficient condition for M_0 to be non-empty is for $\pi_0(\cdot)$ to be upper semi-continuous.[20] This condition is more often met in applications than the continuity of $\pi_0(\cdot)$. For example, in the context of takeovers (Application 7), $\pi_0(X) = -c(X)$. One often mentioned case is one in which the raider's bidding cost is a fixed cost, i.e., $\pi_0(X) = \begin{cases} -c < 0 \text{ when } X > 0, \\ 0 \text{ when } X = 0. \end{cases}$ While this function is not continuous, it is upper semi-continuous, and the set of non-pivotal outcomes is $M_0 = \{0\}$. In words, due to the positive cost of making a bid, the raider does not attempt a takeover when she expects all shareholders to feel non-pivotal.

Let M_N denote the set of solutions to the principal's profit-maximization program with N agents. Rewriting the program (7) for our asymptotic setting, we obtain

$$\begin{aligned} M_N &= \arg\max_{X \in {}^N\mathfrak{X}/N} \pi_N(X), \text{ where} \\ \pi_N(X) &= F(X) + X\alpha(X) + \beta(X) - R_N(X) = \pi_0(X) + \beta(X) - R_N(X), \\ \text{with } R_N(X) &= \min\left\{\frac{1}{N}\sum_{i=1}^{N}\beta(\sum_{j\neq i} x_j) : x_j \in \mathfrak{X}/N \text{ for all } j,\ \sum_j x_j = X\right\} \end{aligned}$$

Since $\mathfrak{X}$ is compact, ${}^N\mathfrak{X}/N$ is also compact, and non-emptiness of M_N would be guaranteed by upper semi-continuity of $\pi_0(\cdot)$ together with con-

[20] A function $g(\cdot)$ is upper semi-continuous at x_0 if for any $\varepsilon > 0$ there exists a neighborhood of x_0 in which $g(x) < g(x_0) + \varepsilon$. Any continuous function is, of course, upper semi-continuous. More generally, if we take a continuous function and increase its value at any one point, we obtain an upper semi-continuous function.

tinuity of $\beta(\cdot)$ (observe that the latter implies that $R_N(\cdot)$ is continuous as well).

Observe that unless $\mathfrak{X}$ is a closed interval, the domain ${}^N\mathfrak{X}/N$ of the above program is a proper subset of the domain $\overline{\mathfrak{X}}$ of the non-pivotal program. For example, when $\mathfrak{X} = \{0, 1\}$, ${}^N\mathfrak{X}/N$ contains only N points, while $\overline{\mathfrak{X}} = [0, 1]$. The set $\cup_{N=1}^{\infty} {}^N\mathfrak{X}/N$ in this case coincides with the set of all rational numbers in $[0, 1]$. Of course, if $\pi_0(\cdot)$ is maximized at an irrational point at which it is discontinuous, we cannot expect convergence to the solution of the non-pivotal program. Such pathological cases are ruled out by assuming that the domain $\mathfrak{X}$ satisfies the following property:

Definition 3 *The domain $\mathfrak{X}$ is asymptotically adequate for a function $g(\cdot)$ if $\lim_{N\to\infty} \sup g({}^N\mathfrak{X}/N) = \sup g(\overline{\mathfrak{X}})$.*

Observe that $\mathfrak{X}$ is asymptotically adequate for $g(\cdot)$ when $\mathfrak{X}$ it is a closed interval (in which case ${}^N\mathfrak{X}/N = \overline{\mathfrak{X}} = \mathfrak{X}$ for all N), or when $g(\cdot)$ is continuous from a side. We are going to require that $\mathfrak{X}$ is asymptotically adequate for $\pi_0(\cdot)$.

In order to analyze convergence of M_N to M_0, we need to define a notion of distance between two sets. For any two sets $A, B \subset \Re$, define $d(A, B) = \sup_{a\in A} \inf_{b\in B} |a - b|$ - a measure of how far A extends beyond B. For example, $d(A, B) = 0$ whenever $A \subset B$.[21] For future reference, we also define $d_+(A, B) = \sup_{a\in A} \inf_{b\in B}(a-b)_+$ - a measure of how far A extends *above* B. Now we are prepared to formulate the following result:

Proposition 5 *Suppose that the domain $\mathfrak{X}$ is asymptotically adequate for $\pi_0(\cdot)$, and $\beta(\cdot)$ is continuous on $\overline{\mathfrak{X}}$. Then (i) $\sup \pi_N({}^N\mathfrak{X}/N) \to \sup \pi_0(\overline{\mathfrak{X}})$. If, in addition, $\pi_0(\cdot)$ is upper semi-continuous on $\overline{\mathfrak{X}}$, then (ii) $d(M_N, M_0) \to 0$, and (iii) $d(W(M_N), W(M_0)) \to 0$.*

Proof. See Appendix A. ∎

While the proofs of this and other asymptotic results are given in Appendix A, here we briefly outline the logic of the proof. Part (i) of the proposition follows immediately from the continuity of $\beta(\cdot)$. If the functions $\pi_N(\cdot)$ were known to be continuous, part (ii) would follow from part (i) by

[21] Observe that $d(X, Y)$ is not symmetric. The (symmetric) Hausdorff distance between X and Y can be defined as $\max\{d(X, Y), d(Y, X)\}$.

Berge's "maximum theorem" (see e.g. Walker [1979]). Since I was unable to find a version of the theorem stated for upper semi-continuous functions, I essentially had to adapt the theorem's proof for this case. Part (iii) follows from parts (i) and (ii) in a straightforward way.

If we define $M_\infty = M_0$, then Proposition 5(ii) can be interpreted as saying that when $\beta(\cdot)$ is continuous, the equilibrium correspondence M_N is upper hemi-continuous at $N = \infty$. This result offers a resolution to the "Grossman-Hart Paradox": in practice, a takeover is not a discrete event. For example, a raider who acquires 49% of the firm will probably be able to implement most of the value improvement that she would implement after acquiring 51% of the firm. Thus, the firm's value, $v(X)$, should be a continuous function of the proportion X of shares tendered. As Proposition 5 establishes, this continuity will make a small shareholder feel non-pivotal for a large N.[22] (Another solution to the paradox is to introduce some noise into the model, as in Subsection 6.7 below.)

It is worth noting that the equilibrium correspondence M_N is not necessarily lower hemi-continuous, as the following example demonstrates:

Example 2 *Consider the setting of Takeovers (Application 7), with* $\mathfrak{X} = \{0, 1\}$*, with the firm's value given by* $\beta(X) = v(X) = 1 - (1 - X)^2$*, and with no bidding costs, thus* $\pi_0(X) \equiv 0$*. In this model, we have* $M_0 = \overline{\mathfrak{X}} = [0, 1]$*. In words, when agents feel non-pivotal, the raider makes zero profits no matter how many shares she buys, and she is indifferent among all possibilities.*

On the other hand, $\pi_N(X) = X\left[\beta(X) - \beta(X - \frac{1}{N})\right] = \frac{X}{2N}\left(1 - X + \frac{1}{2N}\right)$*, which is maximized at* $M_N = \{\frac{1}{2} + \frac{1}{4N}\}$*. The maximizer converges to* $\frac{1}{2}$*, and* $W(M_N) \to \beta(\frac{1}{2}) = \frac{3}{4}$*.*

Thus, we get a unique prediction for any finite N, but the correspondence "explodes" at $N = \infty$. The example demonstrates that some non-pivotal outcomes may not be good approximations for true outcomes.

Finally, we can use our convergence result to demonstrate that inefficiency is in some sense "generic" when N is large.[23] For this purpose, we first

[22] Neeman [1997], in the context of Pure Public Bads (Application 13), independently observes the role of continuity in ensuring that agents feel non-pivotal when $N \to \infty$.

[23] If trade domains $\mathfrak{X}_i$ are intervals, then using the strong monotone comparative static technique of Edlin-Shannon [forth.], inefficiency can be shown to be generic for any $N \geq 2$. For discrete domains, however, we may need N to be large enough.

compare the non-pivotal outcomes to first-best outcomes when externalities on non-signers are strictly positive or negative:

Proposition 6 *Suppose that $\pi_0(\cdot)$ and $\beta(\cdot)$ are differentiable on $\overline{\mathfrak{X}}$, and $\beta'(X) > 0$ [< 0] for all $X \in \overline{\mathfrak{X}}$. Define $M_0^* = \arg\max_{X \in \overline{\mathfrak{X}}} W(X)$. Then for any $X^* \in M_0^* \cap int\overline{\mathfrak{X}}$ and any $X_0 \in M_0$, $X < X^*$, [$X > X^*$].*

Proof. Consider the parametrized program

$$\max_{X \in \overline{\mathfrak{X}}} \pi_0(X) + z\beta(X),$$

where $z = 0$ corresponds to the non-pivotal program and $z = 1$ corresponds to the first-best program. Under the assumptions, when $\beta'(X) > 0$ [< 0], the objective function has increasing marginal returns in (X, z) [$(-X, z)$], as defined in Edlin-Shannon [forth.] The result follows by their Strict Monotonicity Theorem. ■

Using Proposition 5(ii), this strict inefficiency result can be extended to the asymptotic settings with N large enough:

Corollary 1 *Suppose that $\pi_0(\cdot)$ and $\beta(\cdot)$ are differentiable on $\overline{\mathfrak{X}}$, and $\beta'(X) > 0$ [< 0] for all $X \in \overline{\mathfrak{X}}$. For any N, define $M_N^* = \arg\max_{X \in {}^N\mathfrak{X}/N} W(X)$. Then when N is large enough, for any $X^* \in M_N^* \cap int\overline{\mathfrak{X}}$, and any $X \in M_N$, we have $X_N < X^*$, [$X_N > X^*$].*

Proof. The assumptions imply that $W(\cdot)$ is continuous, which in turn implies that $d(M_N^*, M_0^*) \to 0$ as $N \to \infty$. Since the assumptions of Proposition 5 hold, we also know that $d(M_N, M_0) \to 0$ as $N \to \infty$. Together with the previous proposition, this implies the result. ■

5 Bilateral Contracting with Private Offers

Some of the papers referenced in Section 3 study contracting games in which the principal does not have as much commitment power as assumed in the previous section. Thus, in the context of Network Externalities (Application 10), Katz-Shapiro [1986b] study a game in which the principal approaches two different groups of agents in two periods, and cannot commit to the second period's price in the first period. In the context of Vertical Contracting (Application 1), Hart-Tirole [1990], McAfee-Schwartz [1994], O'Brien-Shaffer [1992], and Rey-Tirole [1996] consider a game in which the principal makes offers to all agents simultaneously, but each agent only observes his own offer. In the context of Common Agency (Application 6), Bernheim-Whinston [1986] study a game in which it is the agents (firms) who make simultaneous offers to the principal (a risk-averse individual). All these papers make assumptions that rule out externalities on non-signers, thus, as shown in the previous section, if the principal could commit to a set of publicly observable bilateral offers, she would implement the first-best outcome. However, when the principal is unable to commit to compensate signers for the externality imposed on them, inefficiency arises.

In this section, we analyze this inefficiency in our general setting, in the framework of the game often studied in the Vertical Contracting setting (Application 1).[24] The game consists of two stages: in the first stage, the principal makes each agent i an offer (x_i, t_i), which is privately observed by the agent. In the second stage, the agents simultaneously decide whether to accept or reject.[25,26]

Each agent's acceptance decision in this game depends on his beliefs about offers extended to other agents. Since in a Perfect Bayesian Equilibrium we can assign arbitrary beliefs following a probability-zero deviation, this gives rise to enormous multiplicity of equilibria. To make a more precise prediction, I follow the papers in the Vertical Contracting literature in restricting the agents to hold so-called "passive beliefs": even after observing an unexpected

[24]While we think that our qualitative results would generalize for other games in which the principal does not have full commitment power, a comprehensive study of such games is left to future research.

[25]The papers on vertical contracting referenced above actually analyze a more complicated game, in which in the first stage the principal offers a tariff $t_i(x_i)$ (sometimes restricted to be a two-part tariff), and in the second stage each agent i chooses his trade x_i. However, if agent i chooses x_i without observing other agents' tariffs (McAfee-Schwartz call this game the *ex post unobservability* game), this game can be seen to produce the same outcome as our (simpler) game.

[26]In modeling debt-equity swaps (Application 8), we assumed that each creditor is able to calculate the expected value of the equity he is offered. This assumption is not legitimate when the creditor does not observe the offers extended to other creditors. Therefore, the analysis of this section will not be valid for this application.

offer from the principal, they continue to believe that other agents face their equilibrium offers.

5.1 Inefficiency Results

As in the previous section, since the principal can always offer $(x_i, t_i) = (0, 0)$, without loss of generality we can restrict attention to equilibria in which all agents accept their offers. If the equilibrium trade profile is $\widehat{x} = (\widehat{x}_1, \ldots \widehat{x}_N)$ and agent i holds passive beliefs, he will accept an offer of (x_i, t_i) if and only if $u_i(x_i, \widehat{x}_{-i}) - t_i \geq u_i(0, \widehat{x}_{-i})$. The equilibrium trade profile $\widehat{x}$ should maximize the principal's profit subject to these participation constraints:

$$\begin{aligned} \widehat{x} &\in \arg\max_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N,\ t \in \Re^N} f(x) + \sum_i t_i \\ \text{s.t. } u_i(x_i, \widehat{x}_{-i}) - t_i &\geq u_i(0, \widehat{x}_{-i}) \text{ for all } i \in N. \end{aligned}$$

Since an equilibrium outcome must satisfy the same participation constraints as those in the commitment program (5), the principal's profit cannot exceed that in the commitment case. Moreover, the principal is likely to suffer from his lack of commitment, because of the additional requirement that $\widehat{x}$ should be his best-response to the agents' beliefs.[27]

All participation constraints in the above program clearly bind, since otherwise the principal could profitably deviate by raising t_i for some agent i. Using this, and the fact that the principal takes $u_i(0, \widehat{x}_{-i})$ as given, the above program can be rewritten as

$$\widehat{x} \in \arg\max_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N} f(x) + \sum_i u_i(x_i, \widehat{x}_{-i}). \tag{8}$$

Let $\mathfrak{E}$ denote the set of trades $\widehat{x}$ which satisfy this condition. The condition implies (but is stronger than) what McAfee-Schwartz [1994] call "pairwise proofness", which says that for each agent i,

$$\widehat{x}_i \in \arg\max_{x_i \in \mathfrak{X}_i} f(x_i, \widehat{x}_{-i}) + u_i(x_i, \widehat{x}_{-i}).$$

[27] By the same logic, a player always prefers being a Stackelberg leader to moving simultaneously with other players.

Indeed, "pairwise proofness" only ensures that the principal does not find it profitable to deviate by changing his offer to a single agent, and does not check the profitability of multi-agent deviations:

Example 3 *Consider the takeover model of Example 2, except that the raider has a positive fixed bidding cost:* $c(X) = \begin{cases} c, & X > 0, \\ 0, & X = 0. \end{cases}$ *Whenever*

$$c > (1/N)\,\beta(1/N) = (2 - 1/N)\,/N^2,$$

$\widehat{x}_1 = \ldots = \widehat{x}_N = 0$ *constitutes a pairwise equilibrium, since*

$$F(1/N) + U_i(1/N, 1/N) = 1/N \cdot \beta(1/N) - c < 0 = F(0) + U_i(0,0).$$

At the same time, whenever $c < \beta(1) = 1$, *the point* $\widehat{x}$ *does not satisfy the true equilibrium condition (8), since the principal can profitably deviate by offering* $x_1 = \ldots x_N = 1/N$:

$$F(1) + \sum_i U_i(1/N, 1/N) = \beta(1) - c = 1 - c > 0.$$

In order to ensure the existence of an equilibrium, we need to impose certain additional assumptions, which are discussed in Appendix C. All of this section's results will be vacuous (but formally correct) when the equilibrium set $\mathfrak{E}$ is empty.

Intuition suggests that since the principal can no longer commit to properly compensate the signing agents for the externalities imposed on them, distortion will be due to these externalities, rather than due to externalities on non-signers. Indeed, it turns out that when externalities on signers are absent at a first-best trade profile, private contracting produces efficient outcomes, regardless of any externalities that might exist at other trade profiles:

Proposition 7 *If there exists* $x^* \in \mathfrak{M}^*$ *such that* $u_i(x_i^*, x_{-i})$ *does not depend on* $x_{-i} \in \mathfrak{X}_{-i}$ *for all* i, *then* $\mathfrak{E} \subset \mathfrak{M}^*$.

Proof. For any $\widehat{x} \in \mathfrak{E}$, the equilibrium condition (8) implies that

$$f(\widehat{x}) + \sum_i u_i(\widehat{x}) \geq f(x^*) + \sum_i u_i(x_i^*, \widehat{x}_{-i}) = f(x^*) + \sum_i u_i(x^*),$$

where the equality follows from the assumption that $u_i(x_i^*, x_{-i})$ does not depend on x_{-i} for all i. Therefore, $\widehat{x} \in \mathfrak{M}^*$. ■

In Applications 5 (Takeovers), 9 (Merger for Monopoly), and 11 (Bargaining), externalities are absent on agents who "sell out", i.e., trade the maximum amount. At the same time, it is often assumed in these contexts that trading the maximum amount maximizes total surplus. Under these assumptions, Proposition 7 establishes that the only candidate equilibrium outcomes are first-best outcomes.

Example 4 *Consider again the takeover model of Example 2, with no bidding costs. Then the unique first-best outcome has all shareholders selling to the raider, i.e.,* $x_i^* = 1/N$ *for all* i*. But at this trade profile there are no externalities on signers:* $U_i(1/N, X) = 0$ *for all* X*. Therefore, according to Proposition 7,* x^* *is the only candidate equilibrium outcome. It is easy to see that it is indeed an equilibrium outcome.*[28] *On the other hand, as shown in Example 2, we get inefficiency with public offers:* $X = \frac{1}{2} + \frac{1}{4N} < 1$*. (For a general comparison of efficiency with private offers and public offers, see Subsection 5.3.)*

As another application of Proposition 7, consider Application 6 (Common Agency) with a risk-neutral individual. In this case there exists a first-best allocation in which all firms "selling out" to the individual, i.e., $x_i^* = b_i$ for all i, and in this allocation there are no externalities on the firms. Proposition 7 then establishes that every equilibrium outcome in this case is a first-best outcome, which parallels Theorem 2 of Bernheim-Whinston [1986].

On the other hand, when externalities on signers are always present, they will distort the contracting outcome. To formalize this intuition, consider the following definition:

Definition 4 *We have globally positive [globally negative] externalities if for each agent* i*,* $u_i(x_i, x_{-i})$ *is non-decreasing [non-increasing] in* $x_{-i} \in \mathfrak{X}_{-i}$ *for all* $x_i \in \mathfrak{X}_i$*.*

[28] Note, however, that in the presence of a bidding cost $c > [\beta(1) - \beta(1/N)]\, N = 1/N$, the described outcome is not an equilibrium outcome, hence no equilibrium exists. See Appendix C for sufficient conditions for existence of equilibrium.

It is easy to see that we have globally negative externalities in Applications 1-5 and 13, and globally positive externalities in Applications 6-12 (see Table 1).

We are tempted to conjecture that under Condition A, with globally positive [globally negative] externalities, the total equilibrium trade $\widehat{X} = \sum_i \widehat{x}_i$ will be weakly lower [higher] than socially optimal, but this conjecture runs into the same problem as in the case of public offers. For example, with positive externalities it is easy to see that in equilibrium each agent i's trade $\widehat{x}_i$ will be too low *given* $\widehat{x}_{-i}$, but this generally does not imply that $\widehat{x}_i$ will be lower than its first–best level. To obtain a definitive comparison, we again assume Conditions D and L. Then, letting E denote the set of equilibrium total trades X (and recalling that M^* is the set of surplus-maximizing outcomes defined in (6)), we obtain the following result:

Proposition 8 *If Conditions D, L hold, then with globally positive [negative] externalities, $E \cup M^* \leq M^*$ [$E \cup M^* \geq M^*$] in the strong set order.*

Proof. Consider the case of globally negative externalities. Suppose that $X^* \in M^*$ and $\widehat{X} \in E \cup M^*$, and that $(\widehat{x}_1, ..., \widehat{x}_N) \in \mathfrak{E}$, with $\sum_i \widehat{x}_i = \widehat{X} \leq X^*$. Since we trivially have $X^* \in E \cup M^*$, to establish the strong set order comparison we only need to prove that $\widehat{X} \in M^*$.

Under condition D, we can choose $X^* = \sum_i x_i^*$ with $x_i^* \in \mathfrak{X}_i$ for all i so that $x_i^* \geq \widehat{x}_i$ for all i, and consequently $X_{-i}^* = \sum_{j \neq i} x_j^* \geq \widehat{X}_{-i} = \sum_{j \neq i} \widehat{x}_j$ for all i. Now we can write

$$\begin{aligned} W(\widehat{X}) &= F(\widehat{X}) + \sum_i U_i(\widehat{x}_i, \widehat{X}) \geq F(X^*) + \sum_i U_i(x_i^*, x_i^* + \widehat{X}_{-i}) \geq \\ &\geq F(X^*) + \sum_i U_i(x_i^*, X^*) = W(X^*). \end{aligned}$$

The first inequality obtains by the equilibrium condition (8). The second inequality follows from the fact that $X_{-i}^* \geq \widehat{X}_{-i}$ for all i and externalities are globally negative. For the last equality, use Condition L. Therefore, we must also have $\widehat{X} \in M^*$. This implies the result for globally negative externalities. The proof for globally positive externalities is analogous. ■

The Proposition compares the sets E and M^* in a somewhat weaker way than would the strong set order. For example, with globally positive externalities, the statement means that the set $E \backslash M^*$ lies below M^*, but allows some elements of $E \cap M^*$ to lie above $M^* \backslash E$. But when $|M^*| = 1$, the statement says that $\sup E \leq M^*$, which is a pretty strong result.

Finally, it is easy to see that Section 4's Proposition 4 on the effect of contracting on agents' utilities also obtains in this setting, since in equilibrium the agents' participation constraints bind:

Proposition 9 *With positive [negative] externalities on non-signers, a ban on contracting with privately observed offers weakly reduces [raises] each agent's utility.*

Note that unlike all the other results of this section, this result depends on the sign of externalities on *non-signers.*

5.2 Asymptotics

As in the case of public offers, when the number N of agents is very large and Condition A holds, it seems natural to expect that each individual agent will think of himself as *non-pivotal*, i.e., he will take the total trade X as given in making his decision. In that case, under Condition L, agent i accepts his offer if and only if $t_i \leq x_i \alpha(\widehat{X})$, where $\widehat{X}$ is his expectation of total trade. Since in equilibrium all participation constraints bind, the principal solves

$$\max_{X \in \overline{\mathfrak{X}}} \pi_0(X|\widehat{X}) = F(X) + X\alpha(\widehat{X}).$$

$\widehat{X} \in \overline{\mathfrak{X}}$ is a non-pivotal equilibrium total trade if and only if $\widehat{X} \in \arg\max_{X \in \overline{\mathfrak{X}}} \pi_0(X|\widehat{X})$. Let E_0 denote the set of such non-pivotal equilibrium trades. But will the contracting outcomes with N trades converge to the outcomes from E_0 as $N \to \infty$?

Consider, for example, the setting of Vertical Contracting (Application 1) with private offers. If the supplier takes her total sales X as given in deciding how much to sell to each downstream firm, this is equivalent to taking the downstream price $P(X)$ as given. Therefore, even though the supplier is a monopolist, the non-pivotal outcome is a competitive (price-taking) outcome. Rey-Tirole [1996] indeed show that in a very specific setting of linear demand and constant marginal costs, the equilibrium outcome converges to the competitive outcome as $N \to \infty$. But how general is this convergence result?

As another example, consider the setting of Common Insurance (Application 5). In this setting, if an insurance company takes an individual's total purchase of insurance (and therefore the chosen probability of accident) as given, it will insure him fully. Therefore, in the non-pivotal outcome, the agent will be fully insured, and will choose the least-cost level of care. Pauly [1974] indeed argues that this outcome will obtain in the "competitive" equilibrium. But will this outcome be approximated in a setting in which the number N of insurers is very large but finite?

In this subsection, we present a result which provides sufficient conditions for convergence to the non-pivotal outcome. Letting E^N denote the set of equilibrium total trades in the asymptotic setting with N agents introduced in Subsection 4.3, we have

Proposition 10 *Suppose that $\alpha(\cdot)$ and $\beta(\cdot)$ are continuous functions, $f(\cdot)$ is upper semi-continuous, and domain $\mathfrak{X}$ is asymptotically adequate for the function $\pi_0(\cdot|X_0)$ for all $X_0 \in \overline{\mathfrak{X}}$. Then (i) $d(\pi_0(E_N), \pi_0(E_0)) \to 0$, (ii) $d(E_N, E_0) \to 0$, and (iii) $d(W(E_N), W(E_0)) \to 0$ as $N \to \infty$.*

Proof. See Appendix A. ■

In Vertical Contracting (Application 1), the proposition's assumptions are satisfied whenever the inverse demand function $P(\cdot)$ is continuous and the principal's cost function is lower semi-continuous (this allows, for example, for a positive fixed cost). In such cases the price-taking outcome obtains in the limit. In Common Insurance (Application 5), the proposition's assumptions are satisfied whenever the probability of accident $\rho(X)$ is a continuous function of the amount X of insurance.[29] In such cases, the individual will fully insure herself in the limit.[30]

[29]This rules out cases in which the insuree only has a finite number of actions to choose from. Whether convergence to full insurance outcomes obtains in such cases is an open question.

[30]Pauly [1974] observes that in some cases the individual strictly prefers not to purchase any insurance, rather than fully insuring herself. However, it is easy to see that not purchasing any insurance cannot be an equilibrium outcome. Therefore, in such cases no equilibrium exists. For sufficient conditions for the existence of equilibrium, see Appendix C.

5.3 Comparison to the Case of Public Offers

As we have seen, with public offers the distortion stems from the externalities on non-signers, while with private offers it stems from the externalities on signers. This suggests that in order to compare the outcomes in the two cases, we need to compare the two externalities. For this purpose, consider the following definition:

Definition 5 *We have increasing [decreasing] externalities if for each agent i and all $x_i \in \mathfrak{X}_i$ the difference $u_i(x_i, x_{-i}) - u_i(0, x_{-i})$ is non-decreasing [non-increasing] in $x_{-i} \in \mathfrak{X}_{-i}$.*[31]

The definition can be interpreted in two ways:

1. With increasing [decreasing] externalities, agent i is more [less] willing to accept his offer when he expects more other agents to accept. This interpretation is used in Appendix B to study the possibility of multiple equilibria.

2. The definition can be restated as

   $$u_i(x_i, x'_{-i}) - u_i(x_i, x_{-i}) \geq \ [\leq]\ u_i(0, x'_{-i}) - u_i(0, x_{-i}) \text{ whenever } x'_{-i} \geq x_{-i},$$

   i.e., with increasing [decreasing] externalities, the externality imposed on agent i by increasing x_{-i} is more [less] positive when he signs than when he does not. This interpretation is used in this subsection.

It is easy to verify that we have increasing externalities in Applications 10 (Network Externalities) and 3 (Exclusive Dealing), and we have decreasing externalities in Applications 1 (Vertical Contracting), 5 (Common Insurance), 9 (Merger for Monopoly), and 11 (Bargaining). We also have decreasing externalities in Application 7 (Takeovers) as long as we restrict attention to the case in which $\mathfrak{X}_i = \{0, \overline{x}_i\}$ for all i.[32] In Applications 12 and 13 (Pure Public Good and Bad), externalities are increasing when the functions $v_i(\cdot)$

[31]This is implied by, but weaker than, the requirement that the function $u_i(x_i, x_{-i})$ satisfy increasing differences in (x_i, x_{-i}) [in $(x_i, -x_{-i})$] (Milgrom-Shannon [1994]).

[32]This has been informally noted by Holmstrom-Nalebuff [1992]: "... Everyone wants to be negatively correlated with the majority. When the majority tenders, everyone wants to hold out, while when the majority holds out, everyone wants to tender."

are convex, and decreasing when they are concave. In Application 2 (Vertical Contracting with a Substitute), externalities may be either increasing or decreasing (both cases are analyzed in Katz-Shapiro [1986a]).

Intuition suggests that with increasing externalities, the principal's incentive to trade too much will be lower, or her incentive to trade too little will be higher, with private offers than with public offers, and conversely with decreasing externalities. Unfortunately, just as with similar intuitions above, this conjecture is not always true because of the interaction of the principal's choice of X and her choice of allocating a given X among agents. Moreover, unlike with intuitions above, even Conditions L and D do not guarantee a definitive comparison. The problem is that while Conditions L ensures that the allocation of a given X is irrelevant for total surplus, it may still be relevant in the principal's program both with public offers and with private offers. For this reason, we are only able to establish a definitive comparison under the stronger Condition S, which ensures that the allocation of a given X among agents is always irrelevant:

Proposition 11 *When Condition S holds and externalities are increasing [decreasing], we have* $E \cup M \leq [\geq] M$.

Proof. Under Condition S, the set M of profit-maximizing outcomes with public offers solves

$$M = \arg\max_{X \in \{0,\ldots,N\}} F(X) + X\left[U(1,X) - U(0,X-1)\right]$$

Consider the case of increasing externalities. Suppose that $X \in M$ and $\widehat{X} \in E$, and $X \leq \widehat{X}$. Since we obviously have $X \in E \cup M$, to establish the strong set order comparison we only need to prove that $\widehat{X} \in M$. Since the agents' participation constraints bind in the private-offer equilibrium $\widehat{X}$, each agent i with $x_i = 1$ pays $t_i = U(1,\widehat{X}) - U(0,\widehat{X}-1)$. Consider a deviation from the equilibrium in which the principal offers $x_i = t_i = 0$ to $\widehat{X} - X$ agents who previously had $x_i = 1$. Since the deviation must be unprofitable, we have

$$\begin{aligned} F(\widehat{X}) + \widehat{X}\left[U(1,\widehat{X}) - U(0,\widehat{X}-1)\right] &\geq F(X) + X\left[U(1,\widehat{X}) - U(0,\widehat{X}-1)\right] \geq \\ &\geq F(X) + X\left[U(1,X) - U(0,X-1)\right], \end{aligned}$$

where the second inequality follows from the property of increasing externalities. Therefore, we must also have $\widehat{X} \in M$. The proof for decreasing externalities is similar, except that we should consider the principal's deviation to offer $x_i = 0$ to $\widehat{X} - X$ agents who previously had $x_i = 1$. ■

Another way to compare the outcomes with private and public offers is to focus on the situations in which N is large. Observe that in the corresponding non-pivotal programs under condition L, the allocation of a given X does not matter. Thus, we should be able to compare the sets M_0 and E_0 of the corresponding non-pivotal outcomes. Then we can use asymptotic convergence results to compare the outcomes in the asymptotic setting with N large enough. Observe that the proper notion of increasing [decreasing] externalities in the non-pivotal setting in which each agent takes the total trade as given is to have the function $\alpha(\cdot)$ non-decreasing [non-increasing]. The sign of $\alpha'(\cdot)$ for all the examples of Section 3 is given in Table 1. Note that in all cases of increasing externalities we have $\alpha'(\cdot) > 0$, and in all cases of decreasing externalities we have $\alpha'(\cdot) < 0$. In the settings of Pure Public Good and Bad (Application 12,13), we may have increasing or decreasing externalities for any finite N, but not asymptotically, since $\alpha'(\cdot) = 0$.

Using the technique of strictly monotone comparative statics developed by Edlin-Shannon [forth.], we obtain the following result:

Proposition 12 *Suppose that $F(\cdot)$ and $\alpha(\cdot)$ are differentiable on $\overline{\mathfrak{X}}$, and that $\alpha'(X) > 0$ [< 0] for all $X \in \overline{\mathfrak{X}}$. Then for any $X_0 \in M_0 \cap int\overline{\mathfrak{X}}$ and any $\widehat{X} \in E_0$, $\widehat{X} < X_0$, [$\widehat{X} > X_0$].*

Proof. Consider the case in which $\alpha'(X) > 0$ for all $X \in \overline{\mathfrak{X}}$. First, observe that if we had $\widehat{X} > X_0$, then, using the fact that $\widehat{X} \in E_0$, we could write

$$\pi_0(\widehat{X}) = \pi_0(\widehat{X}|\widehat{X}) \geq \pi_0(X_0|\widehat{X}) = F(X_0)+X_0\alpha(\widehat{X}) > F(X_0)+X_0\alpha(X_0) = \pi_0(X_0),$$

which would contradict the assumption that $X_0 \in M_0$. Therefore, we must have $\widehat{X} \leq X_0$. To see that the inequality is strict, observe that since $X_0 \in M_0 \cap int\overline{\mathfrak{X}}$, the following first-order condition must be satisfied:

$$\left.\frac{\partial \pi_0(X)}{\partial X}\right|_{X=X_0} = F'(X_0) + \alpha(X_0) + X_0\alpha'(X_0) = 0.$$

Since $\alpha'(X_0) > 0$, this implies that

$$\left.\frac{\partial \pi_0(X|X_0)}{\partial X}\right|_{X=X_0} = F'(X_0) + \alpha(X_0) < 0,$$

therefore $X_0 \notin E_0$, and consequently $X_0 \neq \widehat{X}$. Therefore, we must have $\widehat{X} < X_0$. The proof for the case in which $\alpha'(\cdot) < 0$ is similar. ■

Using Propositions 5(ii) and 10(ii), this comparison is easily extended to the asymptotic setting with N large enough:

Corollary 2 *Suppose that $F(\cdot)$ and $\alpha(\cdot)$ are differentiable on $\overline{\mathfrak{X}}$, that $\beta(\cdot)$ is continuous on $\overline{\mathfrak{X}}$, and that $\alpha'(X) > 0$ [< 0] for all $X \in \overline{\mathfrak{X}}$. Then when N is large enough, for any $X_0 \in M_N \cap int\overline{\mathfrak{X}}$ and any $\widehat{X} \in E_N$, we have $\widehat{X} < X_0$, [$\widehat{X} > X_0$].*

What do these comparisons imply about the relative efficiency of outcomes with private and public offers? The implications are unambiguous when the total surplus $W(X)$ is a quasiconcave function of X. Under this assumption, which is reasonable in all applications listed in Section 3, the closer is X to its first-best value, the greater is the total surplus:

Proposition 13 *Suppose that Conditions L,D hold, we have globally positive [globally negative] externalities, $W(\cdot)$ is quasiconcave, $X \in M$ and $\widehat{X} \in E$. Then we have $W(X) \geq W(\widehat{X})$ if and only if $X \geq \widehat{X}$ [$X \leq \widehat{X}$].*

Proof. Consider the case of globally positive externalities. In this case, Propositions 3 and 8 imply that there exists $X^* \in M^*$ such that $X, \widehat{X} \leq X^*$. If $X \geq \widehat{X}$, then $X \in [\widehat{X}, X^*]$, and quasiconcavity of $W(\cdot)$ implies that $W(X) \geq \min\left\{W(\widehat{X}), W(X^*)\right\} = W(\widehat{X})$. Conversely, if $X \leq \widehat{X}$, then $\widehat{X} \in [X, X^*]$, and quasiconcavity of $W(\cdot)$ implies that $W(\widehat{X}) \geq \min\{W(X), W(X^*)\} = W(X)$. The proof for the case of globally negative externalities is analogous. ■

Putting this result together with the results of Proposition 11 and Corollary 2, we see that when the externalities on signers have the same sign, but a greater absolute value, than those on non-signers, then (under Condition S) contracting with private offers is more distortionary than contracting with private offers. This includes cases in which externalities are globally positive and increasing, such as Application 10 (Network Externalities), and cases in which externalities are globally negative and decreasing, such as Application 1 (Vertical Contracting) and 5 (Common Insurance). In these three settings, externalities on signers are absent altogether, and inefficiencies can be ascribed to the principal's inability to commit. This point has been made in the three respective literatures.

At the same time, to my knowledge, it has not been observed before that the principal's inability to commit can *raise* the total surplus of the contracting parties. This happens when the externalities on signers have the same sign, but a smaller absolute value, than those on non-signers. This includes cases in which externalities are globally positive and decreasing, such as Applications 7 (Takeovers), 9 (Merger for Monopoly), and 11 (Bargaining), and cases in which externalities are globally negative and increasing, such as Applications 3 (Exclusive Dealing) and 9 (Common Agency). This observation gives rise to novel policy implications. In the contexts of Exclusive Dealing, Common Agency, and Takeovers, total welfare is increased when the principal is not legally allowed to commit herself.[33,34]

6 General Commitment Mechanisms

In this section, I assume that the principal can commit to a mechanism in which one agent's trade can be made contingent on other agents' messages. In the context of Vertical Contracting with a substitute (Application 2), Katz-Shapiro [1986a] and Kamien et al. [1992] study a first-price auction in which the seller commits to sell $\bar{X}$ units of the good to the highest bidders.

[33]This conclusion has an imperfect analogy in the "Coase conjecture", which says that a durable good monopolist who is unable to commit to a high price will sell a more efficient quantity (see e.g. Tirole [1988]). In the same way, a raider who is unable to commit will buy more shares from shareholders.

Observe also that if the durable good monopolist needs to incur a sunk cost before producing, her inability to commit may prevent her from recouping this cost. If this happens, she will not produce at all, which hurts consumers. In the same way, a raider's inability to commit may prevent her from recouping the sunk cost of a takeover. Taking such a cost into account would qualify our policy recommendation.

[34]In the setting of Merger for Monopoly, which also exhibits globally positive and decreasing externalities, the policy implication is different if we take the welfare of consumers into account. The acquirer's commitment, this reduces the market's concentration, which benefits consumers.

Unlike with bilateral contracting, whether buyer i obtains the good now depends on other buyers' bids. In the context of Takeovers (Application 7), Bagnoli-Lipman [1988] study conditional bids, in which the raider commits to purchase exactly $\bar{X}$ shares at a certain price if at least $\bar{X}$ shares are tendered by stockholders, and to buy no shares otherwise. Unlike with bilateral contracting, the number of shares sold by shareholder i now depends on other stockholders' tenders.

We begin this section with characterizing all the feasible mechanisms in which agents' participation is voluntary. We then study principal's problem of choosing from a family of such mechanisms. We find that all the results of Section 4 generalize for this case; however, in the particular case in which the family includes all conceivable mechanisms, the first-best is always attained. This happens because the principal can now maximize surplus and at the same time extract the maximum rent from the agents, by threatening a deviator with the harshest possible punishment. However, in the presence of noise which goes to zero slower than the number N of agents goes to infinity, the principal will no longer be able to make each agent pivotal, and in the limit we obtain the non-pivotal outcome described in Subsection 4.3 (which, as demonstrated in that subsection, is usually inefficient).

6.1 A Characterization

A general mechanism (game form) can be described as $\Gamma = \langle S_1, \ldots, S_N, g(\cdot)\rangle$, where S_i is agent i's message space, and $g : S_1 \times \ldots \times S_N \to \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N \times \Re^N$ is the outcome function describing all agents' trades and monetary transfers as functions of the messages sent. We will write $g(s_1, \ldots s_N) = (x(s_1, \ldots s_N), t(s_1, \ldots s_N))$, where $x(s_1, \ldots s_N)$ and $t(s_1, \ldots s_N)$ are the trade and transfer profiles prescribed for the message profile $(s_1, \ldots s_N)$. We assume that the principal can commit to the mechanism, and can induce the agents to play any given Nash equilibrium of the mechanism. The concept of implementation can then be defined as follows:

Definition 6 *A mechanism $\Gamma = \langle S_1, \ldots, S_N, g(\cdot)\rangle$ implements an allocation $(\bar{x}, \bar{t}) \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N \times \Re^N$ if there exists a Nash equilibrium s of Γ such that $g(s) = (\bar{x}, \bar{t})$.*

In addition to the standard incentive-compatibility requirement that the agents play a Nash equilibrium of the message game, we will reflect the fact that agents' participation in the mechanism is voluntary by endowing each agent i with a special "reject" message $s_i = 0$, which guarantees him the bundle $(x_i, t_i) = (0, 0)$.

Definition 7 *A mechanism* $\Gamma = \langle S_1, \ldots, S_N, (x(\cdot), t(\cdot))\rangle$ *is voluntary if* $0 \in S_i$ *for all* i, *and* $s_i = 0 \Rightarrow x_i(s) = t_i(s) = 0,$.

Definition 8 *An allocation is implementable if it can be implemented by a voluntary mechanism.*

A special class of voluntary mechanisms consists of mechanisms in which each agent can only send two possible messages, "reject" ($s_i = 0$), and "accept" ($s_i = 1$), and in equilibrium all agents accept. We will call such mechanisms "direct":

Definition 9 *A voluntary mechanism* $\Gamma = \langle S_1, \ldots, S_N, g(\cdot)\rangle$ *is a direct mechanism if* $S_i = \{0, 1\}$ *for all* i, *and* $(1, \ldots, 1)$ *is a Nash equilibrium of the mechanism.*

Every play $s \in \{0,1\}^N$ in a direct mechanism can be represented by the corresponding "acceptance set" $A(s) = \{i \in N : s_i = 1\}$. Thus, the outcome function of a direct mechanism can be described by functions $\mathbf{x} : 2^N \to \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N$ and $\mathbf{t} : 2^N \to \Re^N$, so that $g(s) = (x(A(s)), t(A(s)))$.[35] In words, $x(A)$ is the trade profile prescribed when the set of accepting agents is A, and $t(A)$ is the transfer profile in this situation. Since a direct mechanism must be voluntary, we must have $x_i(A) = t_i(A) = 0$ for $i \notin A$.

By definition, a direct mechanism $(\mathbf{x}, \mathbf{t})$ implements an outcome $(\overline{x}, \overline{t})$ if and only if (i) $(x(N), t(N)) = (\overline{x}, \overline{t})$, and (ii) $A = N$ is a Nash equilibrium play. The latter requirement means that each agent weakly prefers to accept if he expects all other agents to accept, which gives rise to the following participation constraints:

$$u_i(x(N)) - t_i(N) \geq u_i(0, x_{-i}(N \backslash i)) \text{ for all } i \in N. \tag{9}$$

Since agents do not have any private information to report, in the spirit of the revelation principle it can be seen that an arbitrary mechanism can be replaced with a direct mechanism which implements the same allocation:

[35] We will use boldface type to denote functions on 2^N, e.g. $\mathbf{x}$, and normal type to denote their values, e.g. $x(\{1, 2\})$.

Proposition 14 *An allocation $(\overline{x},\overline{t}) \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N \times \Re^N$ is implementable if and only if it can be implemented by a direct mechanism.*

Proof. Suppose that a voluntary mechanism $\Gamma = \langle S_1, \ldots, S_N, (x(\cdot), t(\cdot))\rangle$ implements $(\overline{x},\overline{t})$. By definition, this means that there exists a message profile $\overline{s} = (\overline{s}_1, \ldots, \overline{s}_N) \in S_1 \times \ldots \times S_N$ such that (i) $(x(\overline{s}), t(\overline{s})) = (\overline{x}, \overline{t})$, and (ii) $\overline{s}$ is a Nash equilibrium of Γ, i.e.,

$$u_i(\overline{x}(\overline{s})) - t_i(\overline{s}) \geq u_i(x(s_i', \overline{s}_{-i})) - t_i(s_i', \overline{s}_{-i}) \text{ for all } i \in N, \text{ and all } s_i' \in S_i.$$

For any $A \subset N$, and any $i \in N$, define $\widetilde{s}_i(A) = \begin{cases} 0, & i \notin A, \\ \overline{s}_i, & i \in A. \end{cases}$ Consider a direct mechanism $(\widetilde{\mathbf{x}},\widetilde{\mathbf{t}})$ given by $\widetilde{x}(A) = x(\widetilde{s}(A))$ and $\widetilde{t}(A) = x(\widetilde{t}(A))$. Then we know that $\left(\widetilde{x}(N), \widetilde{t}(N)\right) = (x(\widetilde{s}(N)), t(\widetilde{s}(N))) = (x(\overline{s}), t(\overline{s})) = \left(\overline{x}, \overline{t}\right)$. Also, the above inequalities imply that $(\widetilde{\mathbf{x}},\widetilde{\mathbf{t}})$ satisfies the participation constraints (9). Therefore, the direct mechanism $(\widetilde{\mathbf{x}},\widetilde{\mathbf{t}})$ implements the allocation $(\overline{x}, \overline{t})$. ■

Just like the revelation principle in the standard implementation setting, this is a very powerful result, which will allow us to restrict attention to direct mechanisms.[36] The difference from the standard implementation setup lies in the fact that even though the agents have no private information to report, we need to allow the agents to send messages in order to give them an option not to participate in the mechanism.

Using the proposition, we can restrict our analysis to the study of direct mechanisms. We will allow the principal to choose from a family of such mechanisms. Participation constraints (9) demonstrate that as long as we are interested in Nash implementation and do not worry about the existence of unwanted equilibria, only equilibrium transfers $t(N)$, and only trades $x(A)$ for $|A| \geq N - 1$ are relevant.

In addition, we impose another restriction on the mechanisms we consider:

[36] Just like the standard revelation principle, our result does not guarantee that the direct mechanism will not have "unwanted" Nash equilibria, in which case "indirect" mechanisms may be useful for ruling out such equlibria. For example, Segal-Whinston [forth.] find that in the setting of Exclusive Dealing (Application 3), the incumbent firm can exclude at zero cost if it can induce the buyers to play its preferred Nash equilibrium, and at a positive cost if the buyers coordinate on their preferred Nash equilibrium. Segal-Whinston find that in the latter case, exclusion can be attained at a lower cost when the agents play a sequential acceptance game, rather than the corresponding direct mechanism (which is a simultaneous acceptance game).

Definition 10 *A direct mechanism* $(\mathbf{x}, \mathbf{t})$ *is binding if all agents' participation constraints (9) bind.*

One justification for focusing on binding mechanisms is that if the principal is able to charge fixed participation fees, she will optimally make all agents' participation constraints bind. Moreover, as will be seen in examples below, even when the principal is not allowed to charge participation fees, she may optimally adjust some other parameters of the mechanism (such as linear prices) to make all agents' participation constraints bind.

6.2 Examples

Bilateral Contracting with Public offers (studied in Section 4): A set $\left\{(\hat{x}_i, \hat{t}_i)\right\}_{i=1}^N$ of bilateral contract offers is equivalent to a direct mechanism $(\mathbf{x}, \mathbf{t})$ which has $(x_i(A), t_i(A)) = \begin{cases} (\hat{x}_i, \hat{t}_i) \text{ if } i \in A, \\ 0 \text{ otherwise.} \end{cases}$ Since the principal is free to adjust $\hat{t}_i$, she will optimally make all agents' participation constraints bind.

Auctions (Katz-Shapiro [1986a], Kamien et al. [1992]): For definiteness, suppose that the principal *sells* x_i to the agents. The principal commits to sell a quantity $\hat{X} \in \sum_i \mathfrak{X}_i$ via an auction. (To ensure that $\hat{X}$ can always be allocated among participating agents, assume that condition D holds.) While Katz-Shapiro study an auction in which the $\hat{X}$ highest bidders receive exactly one unit of the good each, it will be clear below that their results generalize to a much wider class of auctions.

Taking into account that the principal cannot sell the target quantity unless sufficiently many agents participate, we find that the corresponding direct mechanism $(\mathbf{x}, \mathbf{t})$ has $\sum_i x_i(A) = \min\{\hat{X}, \sum_{i \in A} \max \mathfrak{X}_i\}$ for all $A \subset N$. Since only acceptance sets A with $|A| \geq N - 1$ are relevant for Nash implementation, whenever $\hat{X} \leq \sum_i \mathfrak{X}_i - \max_i \mathfrak{X}_i$, we have $\sum_i x_i(A) = \hat{X}$ for such acceptance sets. In this case each agent will take the total trade $\hat{X}$ as given when making his acceptance decision.

But under Condition L, an agent who takes the total trade $\hat{X}$ as given is willing to pay exactly $\alpha(\hat{X})$ for each unit of the good. Therefore, in a wide class of auctions, all trades will take place at this price. This implies, in particular, that all agents' participation constraints (9) will bind, even when the principal cannot charge a participation fee.

Conditional bids (Bagnoli-Lipman [1988]): For definiteness, suppose that the principal *buys* x_i from the agents. Suppose that the principal sets a price p at which all agents can tender their goods. If fewer than $\hat{X}$ units are tendered, no trade takes place. If more than $\hat{X}$ units are tendered, then the principal purchases only $\hat{X}$ units, using some rationing device. Suppose that in equilibrium exactly $\hat{X}$ units are tendered, and let $\hat{x}_i$ denote the equilibrium tender from agent i. Then the corresponding direct mechanism $(\mathbf{x}, \mathbf{t})$ is given by $(x_i(A), t_i(A)) = \begin{cases} (\hat{x}_i, p\bar{x}_i) \text{ if } A = N, \\ 0 \text{ otherwise.} \end{cases}$

If the principal is constrained to offer a linear price p, in order to have all participation constraints (9) binding, we would need to have $p\bar{x}_i = u_i(\hat{x}_i, \hat{X}) - u_i(0,0)$ for all agents i. In the symmetric case in which Condition L is satisfied, this implies that

$$p\bar{x}_i = \alpha(\hat{X})\hat{x}_i + \beta(\hat{X}) - \beta(0) \text{ for all agents } i.$$

This can hold if either $\beta(\hat{X}) = \beta(0)$, or the equilibrium trade $\hat{x}_i$ is the same across agents. The latter property is satisfied in takeover bids studied by Bagnoli-Lipman, in which the raider offers to buy 100% of the shares, and stockholders are homogeneous (and therefore, $\hat{x}_i = \max \mathfrak{X}_i$ is the same across agents).[37]

The examples demonstrate that restricting attention to binding mechanisms is reasonable in many, but not all, contexts.

6.3 Inefficiency Results

In a binding direct mechanism $(\mathbf{x}, \mathbf{t})$, the equilibrium transfers $t(N)$ can be obtained from the trade component $\mathbf{x}$ of the mechanism using the binding participation constraints (9). Since all other transfers are irrelevant for Nash implementation, our mechanisms will be fully described by their trade components. Let $\mathfrak{D}$ denote the set of all binding direct mechanisms:

$$\mathfrak{D} = \{\mathbf{x} \in (\mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N)^{2^N} : x_i(A) = 0 \text{ whenever } i \notin A\}.$$

[37] Bagnoli-Lipman also argue that even if the stockholders were heterogeneous, the raider would be able to extract their surplus by tailoring her offers appropriately (presumably, using fixed fees), and thus participation constraints (9) would still bind.

We assume that the principal is restricted to choose from a subset ("family") $\mathfrak{F} \subset \mathfrak{D}$ of such mechanisms. (The case in which $\mathfrak{F} = \mathfrak{D}$ will be studied in more detail in Subsection 6.5 below.) The set $\mathfrak{M}^*_{\mathfrak{F}}$ of mechanisms that maximize total surplus within the family $\mathfrak{F}$ can be defined as[38]

$$\mathfrak{M}^*_{\mathfrak{F}} = \arg\max_{\mathbf{x}\in\mathfrak{F}} f(x(N)) + \sum_i u_i(x(N)), \tag{10}$$

On the other hand, if the principal chooses a binding mechanism from $\mathfrak{F}$ to maximize her profit, she solves the following program:

$$\max_{\mathbf{x}\in\mathfrak{F}} f(x(N)) + \sum_i u_i(x(N)) - \sum_i u_i(0, x_{-i}(N\backslash i)). \tag{11}$$

Let $\mathfrak{M}_{\mathfrak{F}}$ denote the set of solutions to this program. It is clear now that when there are no externalities on non-signers, the two problems coincide, and we have

Proposition 15 *If $u_i(0, x_{-i})$ does not depend on $x_{-i} \in \mathfrak{X}_{-i}$ for all i, then $\mathfrak{M}_{\mathfrak{F}} = \mathfrak{M}^*_{\mathfrak{F}}$.*

When externalities on non-signers are present, on the other hand, the principal will in general distort her choice of mechanism to extract surplus from the agents. To see this, define $r(\mathbf{x}) = \sum_i u_i(0, x_{-i}(N\backslash i))$.[39] Then we have the following result:

Proposition 16 *For any family $\mathfrak{F}$ of binding direct mechanisms, $r(\mathfrak{M}_{\mathfrak{F}}) \leq r(\mathfrak{M}^*_{\mathfrak{F}})$.*

Proof. Using the "aggregation method" of Milgrom-Shannon, define

$$w(\overline{r}) = \max\left\{ f(x(N)) + \sum_i u_i(x(N)) : \mathbf{x} \in \mathfrak{F},\ r(\mathbf{x}) = \overline{r} \right\}.$$

Consider the parametrized program $\max_{\overline{r}\in\Re} w(\overline{r}) - z\overline{r}$, where $z = 0$ corresponds to surplus-maximization and $z = 1$ to profit-maximization, and

[38]Observe that when $\mathfrak{F}(N) = \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N$, i.e. all possible trade profiles can arise in equilibrium of mechanisms from family $\mathfrak{F}$, then we have $\mathfrak{M}^*_{\mathfrak{F}} = \mathfrak{M}^*$, as defined in Section 2. For the sake of generality, however, we do not restrict attention to this case.

[39]The definition of this function is changed from that in Section 4.

let $r^*(z)$ denote the set of solutions to this program. Since the objective function is supermodular in (v, z), by Topkis' Monotonicity Theorem, $r(\mathfrak{M}^*_{\mathfrak{F}}) = r^*(0) \geq r^*(1) = r(\mathfrak{M}_{\mathfrak{F}})$. ■

In order to see the direction of distortion in the total trade X, we need to assume Condition L, just as we did in Section 4. Under this condition, the objective functions in both the surplus-maximization program (10) and the profit-maximization program (11) can be written as functions of total trades, $X(A) = \sum_{i \in J} x_i(A)$, for $A \subset N$. For any family $\mathfrak{F}$ of mechanisms, let $\Sigma\mathfrak{F} = \{\sum_i \mathbf{x}_i : \mathbf{x} \in \mathfrak{F}\}$ denote the "aggregate representation" of family $\mathfrak{F}$. Then $\Sigma\mathfrak{M}^*_{\mathfrak{F}}(N)$ will represent the set of equilibrium total trades in surplus-maximizing mechanisms, and $\Sigma\mathfrak{M}_{\mathfrak{F}}(N)$ will represent the set of equilibrium total trades in profit-maximizing mechanisms.[40] Observe that $\Sigma\mathfrak{M}^*_{\mathfrak{F}}(N) = M^*$ (as defined in program (6) in Section 4) whenever $\Sigma\mathfrak{F}(N) = \sum_i \mathfrak{X}_i$, i.e., all possible total trades can be implemented as equilibrium total trades in family $\mathfrak{F}$.

To determine the direction of distortion, we will also require the family $\mathfrak{F}$ to have the following property:

Definition 11 *For any $\bar{X} \in \sum_i \mathfrak{X}_i$, define $\Sigma\mathfrak{F}|\bar{X} = \{\mathbf{X} \in \Sigma\mathfrak{F} : X(N) = \bar{X}\}$. We will say that the mechanism family $\mathfrak{F}$ is ascending if for any $\bar{X}, \bar{Y} \in \sum_i \mathfrak{X}_i$ such that $\bar{X} \leq \bar{Y}$, we have*

(i) for any $\mathbf{X} \in \Sigma\mathfrak{F}|\overline{X}$ there exists $\mathbf{Y} \in \Sigma\mathfrak{F}|\overline{Y}$ such that $\mathbf{X} \leq \mathbf{Y}$, and

(ii) for any $\mathbf{Y} \in \Sigma\mathfrak{F}|\overline{Y}$ there exists $\mathbf{X} \in \Sigma\mathfrak{F}|\overline{X}$ such that $\mathbf{X} \leq \mathbf{Y}$.

In words, a family $\mathfrak{F}$ is ascending if there is always a way to increase or decrease the equilibrium total trade $X(N)$ within the family while increasing (or decreasing) the out-of-equilibrium total trades $X(A)$ for all acceptance sets A at the same time.

To give some examples, the family of bilateral contracting mechanisms studied in Section 4 and the family of auctions described above are both ascending whenever Condition D on trade domains is satisfied. The family of conditional bids is also ascending, since we always have $X(N \backslash i) = 0$. We can think of the ascendance property as the appropriate generalization of Condition D to general mechanism families. This property allows us to compare the firm's profit-maximizing outcomes to the first-best outcomes:

[40]We use the notation $\Sigma\mathfrak{F}(A) = \{X(A) : \mathbf{X} \in \Sigma\mathfrak{F}\}$.

Proposition 17 *Suppose that Condition L holds, and $\mathfrak{F}$ is an ascending family of binding direct mechanisms. Then with positive [negative] externalities on non-signers, we have $\Sigma\mathfrak{M}_{\mathfrak{F}}(N) \leq \Sigma\mathfrak{M}^*_{\mathfrak{F}}(N)$ [$\geq \Sigma\mathfrak{M}^*_{\mathfrak{F}}(N)$] in the strong set order.*

Proof. When $\mathfrak{F}$ is ascending, the function $R(\bar{X}) = \min_{\mathbf{X}\in\Sigma\mathfrak{F}|\bar{X}} \sum_i \beta_i(X(N\backslash i))$ can be shown to be non-decreasing [non-increasing in $\overline{X}$] with positive [negative] externalities on non-signers. Consider the parametrized program

$$\max_{\bar{X}\in\Sigma\mathfrak{F}(N)} W(\bar{X}) - zR(\bar{X}),$$

where $z = 0$ corresponds to the first-best program (10), and $z = 1$ corresponds to the principal's profit maximization program (11). With positive [negative] externalities on non-signers, the objective function is supermodular in $(-\bar{X}, z)$ [in $(\bar{X}, z)$] Using Topkis' Monotonicity Theorem, we obtain the result. ■

This proposition generalizes Proposition 3 of Section 4. It demonstrates that the relation between the direction of distortion and the sign of externality on signers is not specific to bilateral contracting; rather, it is holds quite generally when the principal can commit to a mechanism from a family of binding mechanisms. For example, when the principal uses auctions (as in Katz-Shapiro [1986a], Kamien et al. [1992]), the arising distortion will be of the same sign as when the principal uses bilateral contracts.[41]

Section 4's Proposition 4 on the effect of contracting on agents' utilities also generalizes in a straightforward way:

Proposition 18 *With positive [negative] externalities on non-signers, a ban on contracting weakly reduces [raises] each agent's utility.*

Proof. Since each agent i's participation constraint binds, his utility equals $u_i(0, x_{-i}(N\backslash i))$, which, with positive [negative] externalities, is weakly smaller [greater] than $u_i(0,0)$, his utility if contracting were banned. ■

[41]For example, in the context of Vertical Contracting with a Substitute (Application 2), Kamien et al. [1992] find that if the principal's input (patent) provides a sufficiently large improvement so that the downstream firms who do not purchase it are driven out of the market, the principal's optimal auction yields an efficient outcome. Our result demonstrates that in this case, due to the absence of externality on signers, the principal's choice from *any* ascending family of mechanisms yields an efficient outcome.

6.4 Asymptotics

We can also generalize the asymptotic result of Subsection 3.3 under appropriate assumptions. Consider a sequence $\{\mathfrak{F}_N\}_{N=1}^{\infty}$ of mechanism families in asymptotic settings with N agents (as defined in Subsection 3.3).

Definition 12 *A sequence $\{\mathfrak{F}_N\}_{N=1}^{\infty}$ of mechanism families in the asymptotic settings with N agents is asymptotically adequate if*

$$\sup_{\mathbf{X}\in\Sigma\mathfrak{F}_N} \pi_0\left(X(N)\right) \to \sup \pi_0\left(\overline{\mathfrak{X}}\right) \text{ as } N \to \infty.$$

(This definition generalizes the asymptotic adequacy of domain defined in Subsection 3.3)

Definition 13 *A sequence $\{\mathfrak{F}_N\}_{N=1}^{\infty}$ of mechanism families in the asymptotic settings with N agents is asymptotically continuous if*

$$\sup_{X\in\Sigma\mathfrak{F}_N,\ i\in N} |X(N\backslash i) - X(N)| \to 0 \text{ as } N \to \infty.$$

Intuitively, a sequence of mechanism families is asymptotically continuous if a single agent asymptotically has a negligible effect on the total trade. The bilateral contracting and auction examples described in Subsection 6.2 satisfy this property, while conditional bids described in that subsection do not.

Letting $M_{\mathfrak{F}_N} = \Sigma\mathfrak{M}_{\mathfrak{F}_N}(N)$ denote the set of equilibrium total trades in the asymptotic setting with N agents, and recalling that M_0 is the set of non-pivotal total trades as defined in Section 4, we have the following result:

Proposition 19 *Suppose that the sequence $\mathfrak{F}_N$ of families is asymptotically adequate and continuous, and $\beta(\cdot)$ is continuous on $\overline{\mathfrak{X}}$. Then (i) $\sup \pi_N({}^N\mathfrak{X}/N) \to \sup \pi_0(\overline{\mathfrak{X}})$. If, in addition, $\pi_0(\cdot)$ is upper semi-continuous on $\overline{\mathfrak{X}}$, then (ii) $d\left(M_{\mathfrak{F}_N}, M_0\right) \to 0$, and (iii) $d\left(W(M_{\mathfrak{F}_N}), W(M_0)\right) \to 0$.*

The proof of this result is virtually identical to that of Proposition 5.

This result establishes that when $\beta(\cdot)$ is continuous and N is very large, the outcome of contracting does not depend on which mechanisms the principal can use, as long as they are asymptotically adequate and asymptotically continuous. For example, since both auctions and bilateral contracts have this property, the advantage of the former over the latter which was demonstrated by Katz-Shapiro [1986a] and Kamien et al. [1992] disappears when $N \to \infty$. On the other hand, conditional bids are not asymptotically continuous and are not covered by this proposition. In the next section, we will see that using such discontinuous mechanisms, the principal may be able to do much better than in the non-pivotal program even when N is large.

6.5 Fully Optimal Mechanisms

Here we study at the principal's optimal choice when all mechanisms are allowed, i.e. $\mathfrak{F} = \mathfrak{D}$:

Proposition 20 $\mathfrak{M}_{\mathfrak{D}} = \mathfrak{M}_{\mathfrak{D}}(N) \times \prod_{i=1}^{N} \mathfrak{M}_{\mathfrak{D}}(N\backslash i) \times \prod_{A \subset N, |A| < N-1} \mathfrak{M}_{\mathfrak{D}}(A)$, *where*

$$\begin{aligned} \mathfrak{M}_{\mathfrak{D}}(N) &= \mathfrak{M}^* \text{ (as defined in Section 4)}, \\ \mathfrak{M}_{\mathfrak{D}}(N\backslash i) &= \arg\min_{x_{-i} \in \mathfrak{X}_{-i}} u_i(0, x_{-i}) \text{ for all } i \in N, \text{ and} \\ \mathfrak{M}_{\mathfrak{D}}(A) &= \mathfrak{D}(A) \text{ for all } A \subset N \text{ with } |A| < N-1. \end{aligned}$$

Proof. Follows from the additive separability of the objective function in the profit-maximization program (11) in $x(N)$ and $x(N\backslash i)$ for all i, and the fact that $\mathfrak{D} = \prod_{A \subset N} \mathfrak{D}(A)$. ■

Corollary 3 *With positive [negative] externalities,*

$$\mathfrak{M}_{\mathfrak{D}} \supset \{\mathbf{x} \in \mathfrak{D} : x(N) \in \mathfrak{M}^*,\ x_j(N\backslash i) = 0 \ [= \max \mathfrak{X}_j] \text{ whenever } j \neq i\}.$$

These results can be interpreted as follows. If the principal can choose any mechanism from $\mathfrak{D}$, then she maximizes her profit by implementing a surplus-maximizing trade profile $x(N)$ in equilibrium, while at the same time minimizing the agents' rents by choosing the harshest punishment $x(N\backslash i)$ following each agent i's deviation (and the trades following multilateral deviations do not matter, since we are only concerned with Nash equilibrium implementation).[42] With positive externalities, the harshest punishment for

[42]One might wonder whether these optimal mechanisms may have other Nash equilibria which are preferred by all agents. In fact, it is easy to ensure that "all accept" is a unique

agent i is $x_j(N\backslash i) = 0$ for all j. This punishment is implemented by an offer that requires unanimous acceptance: if at least one agent rejects, no trade takes place. (A mechanism of this kind has been suggested by Bagnoli-Lipman [1988] in the context of Takeovers (Application 7).) With negative externalities, the harshest punishment is $x_j(N\backslash i) = \max \mathfrak{X}_j$ for all $j \neq i$. In words, the deviator is punished by implementing the maximum possible trades with all other agents. (Mechanisms of this kind have been suggested by Katz-Shapiro [1986a] and Kamien et al. [1992] in the context of Vertical Contracting with a Substitute (Application 2).[43]) In both cases, the separation between rent extraction and surplus maximization results in efficiency.

Observe that this result does not contradict the previous results of this section. Indeed, all the inefficiency results of Subsection 6.3 are weak: they determine the direction of distortion, but do not rule out the possibility that an efficient outcome arises. The asymptotic result of Proposition 19 is stronger: it establishes that for a large N we obtain a non-pivotal outcome, which is likely to be inefficient (see Subsection 4.3 for sufficient conditions for this to occur). However, this result is only true for asymptotically continuous mechanisms, and the fully optimal mechanism derived above is clearly not asymptotically continuous: it requires triggering the harshest possible punishment following any individual agent's deviation.[44]

6.6 Noisy Asymptotics

The discontinuity of the fully optimal mechanism derived above seems unrealistic in environments in which N is large and the principal faces some uncertainty about the number of accepting agents. To formalize this intuition, consider an asymptotic setting in which the principal cannot foresee

Nash equilibrium. Indeed, since the out-of equilibrium payments $t_i(A)$ for $A \neq N$ do not affect the principal's equilibrium profits, she can promise these payments to be large enough so that each agent strictly prefers to accept if he knows that at least one other agent rejects. This would make acceptance a weakly dominant strategy. Furthermore, by slightly raising the equilibrium payments $t_i(N)$, the principal could ensure that acceptance is a strictly dominant strategy.

[43]In the context of Nuclear Weapons (Application 4), a similar mechanism has been suggested by Jehiel et al. [1996].

[44]Note in passing that with negative externalities, each agent is pivotal in the sense opposite to that of bilateral contracting (Section 4): his rejection *increases* the total trade X!

the number of accepting agents precisely. Namely, suppose that with some probability $\varepsilon_N > 0$ any given agent is unable to respond to the principal's offer (e.g. because he has passed away, he has not seen the offer, or his acceptance message is lost in the mail). Suppose also that these events are independent across agents. If an agent's response has not been received by the principal, she will not be able to trade with that agent. Since the principal cannot precisely predict the exact number of responders, it becomes significantly harder to make agents pivotal.

Let $A \subset N$ denote the random set of agents who are able to respond to the principal's offer. As before, without loss of generality we can restrict attention to direct mechanisms, in which all agents who are able to respond accept in equilibrium. For these strategies to constitute a Nash equilibrium, each agent must prefer accepting to not responding when he knows that others accept whenever they can, i.e., the following participation constraints must hold:

$$E_{A\subset N}\left[u_i(x(A)) - t_i(A)|\ i \in A\right] \geq E_{A\subset N}\left[u_i(0, x_{-i}(A\backslash i))|\ i \in A\right] \text{ for all } i \in N.$$

Note that unlike in the case of certainty studied before, all acceptance sets $A \subset N$ will be observed in equilibrium with a positive probability, and thus the values of $x_i(A)$ and $t_i(A)$ for all these sets A are relevant for agent i's acceptance decision. The principal will optimally choose payments $t_i(A)$ to make all agents' participation constraints bind, from which we can obtain each agent i's expected payment to the principal, $E_{A\subset N}\left[t_i(A)|\ i \in A\right]$.[45] Substituting in the principal's objective function, we can write her expected profit as

$$E_{A\subset N}\left[f(x(A)) + \sum_{i\in A} t_i(A)\right] = E_{A\subset N}\left[f(x(A)) + \sum_{i\in A}\left(u_i(x(A)) - u_i(0, x_{-i}(A\backslash i))\right)\right].$$

We will study the asymptotic setting defined in Subsection 4.3, which satisfies Condition L. In this setting, the principal's expected profit can be written as a function of the mechanism's aggregate representation $\mathbf{X} = \sum_i \mathbf{x}_i$ only:

$$\pi_N(\mathbf{X}) = E_{A\subset N}\left[F(X(A)) + X(A)\alpha(X(A)) + \frac{1}{N}\sum_{i\in A}\left[\beta(X(A)) - \beta(X(A\backslash i))\right]\right].$$

[45]Observe that the principal is indifferent about the choice of payments $t_i(A)$ for different acceptance sets A, as long as each agent i's participation constraint binds. She could achieve this, for example, by charging each accepting agent i a fixed fee t_i.

Letting $\mathfrak{D}_N$ denote the set of all direct mechanisms in the asymptotic setting with N agents, the set $\mathbf{M}_N$ of the aggregate representations of the principal's profit-maximizing mechanisms can be defined as

$$\mathbf{M}_N = \arg\max_{\mathbf{X}\in\Sigma\mathfrak{D}_N} \pi_N(\mathbf{X})$$

Since the acceptance set A is now random, we can think of $\mathbf{X}$ as a random variable, and of $\mathbf{M}_N$ as a random set. We are therefore going to use the concept of convergence in probability to establish the convergence of $\mathbf{M}_N$ to the set M_0 of non-pivotal total trades defined in Subsection 4.2.

Our result will require the domain $\mathfrak{X}$ to satisfy a property which is slightly stronger than the condition introduced in Subsection 4.3:

Definition 14 *The domain $\mathfrak{X}$ is strongly asymptotically adequate if for some $\gamma < 1$, $\lim_{N\to\infty} \sup \pi_0({}^{\lceil\gamma N\rceil}\mathfrak{X}/N) = \sup \pi_0(\overline{\mathfrak{X}})$.*

This is a stronger property than simple asymptotic adequacy, since ${}^{\lceil\gamma N\rceil}\mathfrak{X}/N \subset {}^{N}\mathfrak{X}/N \subset \overline{\mathfrak{X}}$, and therefore

$$\sup \pi_0({}^{\lceil\gamma N\rceil}\mathfrak{X}/N) \le \sup \pi_0({}^{N}\mathfrak{X}/N) \le \sup \pi_0(\overline{\mathfrak{X}}).$$

We need this strengthened assumption to rule out situations in which profit maximization requires us to implement the maximum possible total trade $\max \mathfrak{X}$, and this cannot be approximated due to noise. The property is satisfied, for example, when $M_0 \neq \{\max \mathfrak{X}\}$, and either $\pi_0(\cdot)$ is continuous from a side, or $\mathfrak{X}$ is a closed interval (in which case ${}^{N}\mathfrak{X}/N = \overline{\mathfrak{X}} = \mathfrak{X}$ for all N).

Finally, perhaps the most important condition for our asymptotic result is that noise goes to zero slower than N goes to infinity. Intuitively, it is hard to make an agent pivotal if the probability that any given number of signers is realized goes to zero as $N \to \infty$. To ensure this, we are going to assume that the variance in the number of signers, which equals $N\varepsilon_N(1-\varepsilon_N)$, goes to infinity. Then we obtain the following result:

Proposition 21 *Suppose that Condition L holds, the domain is strongly asymptotically adequate, and $\beta(\cdot)$ and $\pi_0(\cdot)$ are bounded on $\overline{\mathfrak{X}}$. Suppose also that $N\varepsilon_N \to \infty$ and $\varepsilon_N \to 0$ as $N \to \infty$. Then (i) $\sup \pi_N(\mathfrak{D}_N) \to \sup \pi_0(\overline{\mathfrak{X}})$. If, in addition, $\pi_0(X)$ is upper semi-continuous, then (ii) $d(\mathbf{M}_N, M_0) \xrightarrow{p} 0$. If, in* *addition, $\beta(\cdot)$ is upper semi-continuous*[46]*, then (iii)* $d_+(W(\mathbf{M}_N), W(M_0) \xrightarrow{p} 0$. *If, moreover, $\beta(\cdot)$ is continuous, then (iv)* $d(W(\mathbf{M}_N), W(M_0)) \xrightarrow{p} 0$.

Proof. See Appendix A. ■

Note that for bounded random variables, convergence in probability implies convergence of expectations. Therefore, parts (ii) and (iv) of the theorem imply that the expected values of the total trade and total surplus converge to the sets M_0 and $W(M_0)$ respectively: $d(E[\mathbf{M}_N], M_0) \to 0$, and $d(E[W(\mathbf{M}_N)], W(M_0)) \to 0$.

To obtain some intuition for the result, consider the principal's incentives to choose total trades $X(A)$ for various acceptance sets $A \subset N$. First observe that the choice of $X(N)$ affects only the total surplus in the case $A = N$ (all agents accept), and does not affect any agent's reservation utility. Therefore, in any profit-maximizing mechanism, the principal will choose $X(N)$ to maximize total surplus (i.e., $X(N) \in M_N^*$). But what is the probability of $A = N$? If N is large and ε_N is small, this probability can be approximated by

$$(1-\varepsilon_N)^N = \left[(1-\varepsilon_N)^{-1/\varepsilon_N}\right]^{-N\varepsilon_N} \approx e^{-N\varepsilon_N}.$$

Therefore, if $N\varepsilon_N$ could be bounded regardless of N, then with a positive probability, all agents would accept and the first-best outcome would obtain. On the other hand, if $N\varepsilon_N \to \infty$, as assumed in Proposition 21, the probability that all agents accept goes to zero as $N \to \infty$ (moreover, it can be seen that the maximum probability of any given number of responders goes to zero). The principal's optimal choice of $X(A)$ for $A \neq N$ is determined by two considerations: first, it affects the total surplus when A is the set of agents whose messages are received, and second, it affects the reservation utility of each agent $i \in N \backslash A$ when the set of agents who are able to respond is $A \cup i$. Thus, unlike in the case of perfect certainty, profit maximization and rent extraction can no longer be separated. As a result of the optimal trade-off between the two motives, the principal asymptotically chooses a non-pivotal outcome.

Observe that in contrast to the asymptotic results of Proposition 19, parts (i) and (ii) of Proposition 21 do not require the function $\beta(\cdot)$ to be continuous. Intuitively, the presence of noise smooths all discontinuities, whether

[46]Which is satisfied in all applications described in Section 3.

they arise in the mechanism or come with the function $\beta(\cdot)$. Therefore, our analysis supports and extends the Grossman-Hart conjecture on takeovers: regardless of the mechanisms the raider can use (e.g. conditional offers considered by Bagnoli-Lipman [1988], restricted offers, etc.), and regardless of how the firm's value depends on the raider's final share, with a large number of shareholders and in the presence of noise an inefficient non-pivotal outcome will obtain.

Proposition 21 bears some resemblance to the asymptotic inefficiency results of Rob [1989] and Mailath-Postlewaite [1990] on the one hand, and that of Levine-Pesendorfer [1995] on the other. However, my model is substantially different from either setup, as I argue below.

Rob and Mailath-Postlewaite study a setting of Pure Public Good (Application 12), in which "noise" is present in the form of agents' private information about their willingness to pay for the public good. When N is large, agents will asymptotically feel non-pivotal and will not contribute to the public good. My setup is more general in that it incorporates situations in which contracting has a private, as well as a public, component. While my agents will asymptotically take the amount of the public good X as given, they will still trade the private good with the principal.

Levine-Pesendorfer show that noisy observation ensures that agents in some two-stage games become non-pivotal as their number goes to infinity. This result is applied, in particular, to resolve the Grossman-Hart paradox. My setup is more general in that rather than considering a specific game, I show that if the principal can choose *any* mechanism, asymptotically the same non-pivotal outcome obtains. For example, the Grossman-Hart conjecture is established not only for non-discriminating single-price offers, as in Levine-Pesendorfer, but for arbitrary mechanisms. A second difference is that the result of Levine-Pesendorfer is based on "disappearance of information": observation noise asymptotically prevents the principal from observing any single agent's deviation. In my setting, on the other hand, the principal asymptotically perfectly observes any given agent's deviation, and can design a mechanism which makes a single agent (or a finite group of agents) pivotal. What Proposition 21 shows is that asymptotically the principal will maximize her profits by choosing a mechanism in which the vast majority of agents are non-pivotal.

7 Conclusion

In recent years, economic theorists have focused on private information as a source of all "transaction costs". Our analysis identifies and studies a new kind of "transaction cost", which arises in environments with a large number of agents. Our last result, in particular, demonstrates that in such environments, the presence of private information is not necessary to produce contracting inefficiencies: any exogenous source of noise (such as a probability of messages being lost in the mail) would asymptotically give rise to the same non-pivotal outcome. This gives a theoretical foundation for the study of inefficiencies in the various instances of contracting with externalities.

Since our analysis identifies a failure of the Coase theorem, it should have implications for the optimal design of property rights. As one application, we can now better understand the role of the right to exclude others from using an asset, which Hart-Moore [1990] consider to be the most important of property rights. Consider first an asset in "common property", which many parties can use at the same time, and from which no party can exclude others. Due to the well-known "tragedy of the commons", such an asset is likely to be used inefficiently. Moreover, our analysis suggests that contracting will not necessary yield efficient outcomes in this situation. On the other hand, suppose that one party (the principal) has the right to exclude others from using the asset. According to our analysis, if the principal has commitment power, an efficient outcome will arise.[47] This happens because giving the principal the right to exclude others eliminates externalities on non-signers, even though in equilibrium all parties may work with the asset and impose externalities on each other.[48]

[47]This result obtains regardless of who is given the property right over the asset. In particular, the principal may be an outsider, in which case $f(X) \equiv 0$. However, if the parties can make asset-specific investments, Hart-Moore [1990] in deomstrate that it may be optimal to give the property right to the party whose investment is the most important.

[48]In a different but related model, Jehiel-Moldovanu [1996b] study various allocations of property rights which involve negative externalities on non-signers. They find that the same inefficient outcome obtains regardless of the initial allocation of property rights.

A Proofs of Asymptotic Results

The following three lemmas will be useful in some of the proofs:

Lemma 2 (Triangle Inequality) *For any three sets A, B, C, $d_{[+]}(A, C) \leq d_{[+]}(A, B) + d_{[+]}(B, C)$.*

The Lemma easily follows from the triangle inequality: $(a-c)_{[+]} \leq (a-b)_{[+]} + (b-c)_{[+]}$.

Lemma 3 *If $d(M^N, M_0) \to 0$, M_0 is compact, and $\beta(\cdot)$ is upper semi-continuous, then $d_+(\beta(M_N), \beta(M_0)) \to 0$. If, moreover, $\beta(\cdot)$ is continuous, then $d(\beta(M_N), \beta(M_0)) \to 0$.*

Proof. For the first statement: Suppose not. Then there exists a subsequence such that $d_+(\beta(X_N), \beta(M_0)) \geq \sigma \geq 0$ for some $X_N \in M_N$, for all N. Since $\overline{\mathfrak{X}}$ is compact, we can choose this sequence to be converging: $X_N \to \widehat{X} \in \overline{\mathfrak{X}}$. Since $\beta(\cdot)$ is upper semi-continuous, we have $d_+(\beta(\widehat{X}), \beta(M_0)) \geq \sigma$. On the other hand, since $d(X_N, M_0) \to 0$, we must have $d(\widehat{X}, M_0) = \lim_{N\to\infty} d(X_N, M_0) = 0$, which together with compactness of M_0 implies $\widehat{X} \in M_0$, and consequently $d(\beta(\widehat{X}), \beta(M_0)) = 0$ - a contradiction.

The proof of the second statement is obtained by replacing d_+ with d and u.s.c. with continuity. ■

Lemma 4 *If π_0 is u.s.c., $M_0 = \arg\max_{X\in\overline{\mathfrak{X}}} \pi_0(X)$, and $d(\pi_0(M_N), \pi_0(M_0)) \to 0$, then $d(M_N, M_0) \to 0$.*

Proof. Suppose not. Then we could find a subsequence $X_N \in M_N$ such that $d(X_N, M_0) \geq \delta > 0$ for all N. Since $\overline{\mathfrak{X}}$ is compact, we can choose this sequence to be converging: $X_N \to \widehat{X}$, with $d(\widehat{X}, M_0) \geq \delta > 0$. On the other hand, by assumption, $\pi_0(X_N) \to \pi_0(M_0)$, and u.s.c. of $\pi_0(\cdot)$ implies $\pi_0(\widehat{X}) \geq \pi_0(M_0)$, and therefore $\widehat{X} \in M_0$ - a contradiction. ■

Proof of Proposition 5: Since $\beta(\cdot)$ is continuous, it is also uniformly continuous, which implies

$$\sup_{X\in^N\mathfrak{X}/N} |\pi_N(X) - \pi_0(X)| = \sup_{X\in^N\mathfrak{X}} |R_N(X) - \beta(X)| \to 0 \text{ as } N \to \infty. \quad (12)$$

Therefore,

$$\left|\sup \pi_N({}^N\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \leq$$
$$\leq \left|\sup \pi_N({}^N\mathfrak{X}/N) - \sup \pi_0({}^N\mathfrak{X}/N)\right| + \left|\sup \pi_0({}^N\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \leq$$
$$\leq \sup_{X\in {}^N\mathfrak{X}/N} \left|\pi_N(X) - \pi_0(X)\right| + \left|\sup \pi_0({}^N\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right|,$$

and now (12) and asymptotic adequacy of $\mathfrak{X}$ imply (i).

Now, using the triangle inequality, we can write

$$\begin{aligned} d(\pi_0(M_N), \pi_0(M_0)) &\leq d(\pi_0(M_N), \pi_N(M_N)) + d(\pi_N(M_N), \pi_0(M_0)) \leq \\ &\leq \sup_{X\in {}^N\mathfrak{X}/N} |\pi_N(X) - \pi_0(X)| + |\sup \pi_N({}^N\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})|. \end{aligned}$$

Using (12) and (i), we see that $d(\pi_0(M_N), \pi_0(M_0)) \to 0$, which together with Lemma 4 implies (ii).

Finally, note that $W(X) = \pi_0(X) + \beta(X)$, and therefore

$$d(W(M_N), W(M_0)) \leq d(\pi_0(M_N), \pi_0(M_0)) + d(\beta(M_N), \beta(M_0)).$$

The first term has been proven to go to zero. The second term goes to zero by Lemma 3, since $\beta(\cdot)$ is continuous (ii) holds. Thus, we obtain (iii). ■

Proof of Proposition 10: Let $\mathfrak{E}_N$ denote the set of equilibrium trade vectors from $(\mathfrak{X}/N)^N$. Observe that $E^N = \{\sum_{i=1}^N x_i : x \in \mathfrak{E}^N\}$. Define

$$\begin{aligned} \pi_N(X|\widehat{x}) &= f(X) + \psi(X|\widehat{x}), \text{ where} \\ \psi_N(X|\widehat{x}) &= \sup_{x\in(\mathfrak{X}/N)^N, \sum_i x_i = X} \sum_i \left[x_i \alpha(x_i + \sum_{j\neq i} \widehat{x}_j) + \frac{1}{N}\beta(x_i + \sum_{j\neq i} \widehat{x}_j) - \frac{1}{N}\beta(\sum_{j\neq i} \widehat{x}_j) \right]. \end{aligned}$$

Take any sequence $x^N \in \mathfrak{E}_N$ such that $X^N = \sum_i x_i^N \to X_0$. Since $\alpha(\cdot)$ and $\beta(\cdot)$ are continuous on the compact set $\overline{\mathfrak{X}}$, they must also be uniformly continuous, which implies that

$$\sup_{X\in {}^N\mathfrak{X}/N} \left|\pi_N(X|x^N) - \pi_0(X|X_0)\right| = \sup_{X\in {}^N\mathfrak{X}/N} \left|\psi_N(X|x^N) - X\alpha(X_0)\right| \leq$$

$$\begin{aligned} &\leq \sup_{x_i, x_i^N \in \mathfrak{X}/N} \left(X \cdot \left|\alpha(x_i + X^N - x_i^N) - \alpha(X_0)\right| + \left|\beta(x_i + X^N - x_i^N) - \beta(X^N - x_i^N)\right|\right) \to \\ &\to 0 \text{ as } N \to \infty. \end{aligned} \quad (13)$$

Since $\pi_N(X^N|x^N) = \sup \pi_N(^N\mathfrak{X}/N|x^N)$, we can write

$$\begin{aligned}
&\left|\pi_N(X^N|x^N) - \sup \pi_0(\overline{\mathfrak{X}}|X_0)\right| \leq \\
&\leq \left|\sup \pi_N(^N\mathfrak{X}/N|x^N) - \sup \pi_0(^N\mathfrak{X}|X_0)\right| + \left|\sup \pi_0(^N\mathfrak{X}|X_0) - \sup \pi_0(\overline{\mathfrak{X}}|X_0)\right| \leq \\
&\leq \sup_{X\in {}^N\mathfrak{X}/N} \left|\pi_N(X|x^N) - \pi_0(X|X_0)\right| + \left|\sup \pi_0(^N\mathfrak{X}|X_0) - \sup \pi_0(\overline{\mathfrak{X}}|X_0)\right|
\end{aligned}$$

As $N \to \infty$, the first term goes to zero by (13), and the second term goes to zero by asymptotic adequacy of domain $\mathfrak{X}$ for the function $\pi_0(\cdot|X_0)$. Therefore,

$$\lim_{N\to\infty} \pi_N(X^N|x^N) = \sup \pi_0(\overline{\mathfrak{X}}|X_0).$$

On the other hand, using Lemma 2, we can write

$$\begin{aligned}
&d_+\left(\pi_N(X^N|x^N), \pi_0(X_0|X_0)\right) \leq \\
&\leq d_+\left(\pi_N(X^N|x^N), \pi_0(X^N|X_0)\right) + d_+\left(\pi_0(X^N|X_0), \pi_0(X_0|X_0)\right) \leq \\
&\leq \sup_{X\in {}^N\mathfrak{X}/N} \left|\pi_N(X|x^N) - \pi_0(X|X_0)\right| + d_+\left(\pi_0(X^N|X_0), \pi_0(X_0|X_0)\right).
\end{aligned}$$

As $N \to \infty$, the first term goes to zero by (13), and the second term goes to zero by upper semi-continuity of $\pi_0(\cdot|X_0)$ (which is implied by upper semi-continuity of $f(\cdot)$). Putting together with the last result, we see that $\sup \pi_0(\overline{\mathfrak{X}}|X_0) = \lim_{N\to\infty} \pi_N(X^N|x^N) \leq \pi_0(X_0|X_0)$, which implies that $X_0 \in E_0$. Since this is true for any converging sequence $X^N \in E_N$, and the set $\overline{\mathfrak{X}}$ is compact, this establishes (ii).

Now we can also see that

$$\lim_{N\to\infty} \pi_N(X^N|x^N) = \sup \pi_0(\overline{\mathfrak{X}}|X_0) = \pi_0(X_0|X_0) = \pi_0(X_0) \in \pi_0(E_0). \quad (14)$$

Since this is true for any converging sequence $X^N \in E_N$, and the set $\overline{\mathfrak{X}}$ is compact, this establishes (i).

For the welfare result, since $W(X) = \pi_0(X|X) + \beta(X)$, we can write

$$\begin{aligned}
\left|W(X^N) - W(X_0)\right| &\leq \left|\pi_0(X^N|X^N) - \pi_0(X_0|X_0)\right| + \left|\beta(X^N) - \beta(X_0)\right| \leq \\
&\leq \left|\pi_0(X^N|X^N) - \pi_0(X^N|X_0)\right| + \left|\pi_0(X^N|X_0) - \pi_N(X^N|x^N)\right| + \\
&\quad + \left|\pi_N(X^N|x^N) - \pi_0(X_0|X_0)\right| + \left|\beta(X^N) - \beta(X_0)\right| \\
&\leq X^N\left|\alpha(X^N) - \alpha(X_0)\right| + \sup_{X\in {}^N\mathfrak{X}/N} \left|\pi_N(X|x^N) - \pi_0(X|X_0)\right| + \\
&\quad + \left|\pi_N(X^N|x^N) - \pi_0(X_0|X_0)\right| + \left|\beta(X^N) - \beta(X_0)\right|
\end{aligned}$$

The second term goes to zero by (13), the third term goes to zero by (14). Continuity of $\alpha(\cdot)$ and $\beta(\cdot)$ implies that the first and the last term go to zero. Therefore, $\lim_{N\to\infty} W(X^N) = W(X_0) \in W(E_0)$. Since this is true for any converging subsequence $X^N \in E_N$, and the set $\overline{\mathfrak{X}}$ is compact, this establishes (iii). ■

Proof of Proposition 21: The principal's expected profit can be rewritten as

$$\pi_N(\mathbf{X}) = E_A\pi_0(X(A)) + E_A\left[\sum_{i\in A}\frac{1}{N}[\beta(X(A)) - \beta(X(A\backslash i))]\right].$$

The first term is the profit in a non-pivotal mechanism. The second term can be rearranged as

$$\begin{aligned}
&\textstyle\sum_{A\subset N}\sum_{i\in A} p_N^A \frac{1}{N}[\beta(X(A)) - \beta(X(A\backslash i))] = \\
&= \textstyle\sum_{A\subset N} A p_N^A \frac{1}{N}\beta(X(A)) - \sum_{A\subset N}(N-A)p_N^{A+1}\frac{1}{N}\beta(X(A)) = \\
&= \textstyle\sum_{A\subset N} p_N^A \frac{1}{N}[A - (N-A)\frac{1-\varepsilon}{\varepsilon}]\beta(X(A)) = \\
&= E_A\left[\frac{A-N(1-\varepsilon)}{N\varepsilon}\beta(X(A))\right],
\end{aligned}$$

where $p_N^A = \varepsilon^{N-A}(1-\varepsilon)^A$ - probability of a given acceptance set $A \subset N$. (With a slight abuse of notation, we sometimes let A and N stand for the sizes of the respective sets.)

This term can be bounded using Jensen's inequality:[49]

$$\begin{aligned}
\left|E_A\left[\frac{A-N(1-\varepsilon)}{N\varepsilon}\beta(X(A))\right]\right| &\leq \overline{\beta}E_A\left|\frac{A-N(1-\varepsilon)}{N\varepsilon}\right| \leq \overline{\beta}\sqrt{E_A\left(\frac{A-N(1-\varepsilon)}{N\varepsilon}\right)^2} = \\
&= \overline{\beta}\sqrt{Var\left(\frac{A}{N\varepsilon}\right)} = \overline{\beta}\sqrt{\frac{N\varepsilon(1-\varepsilon)}{N^2\varepsilon^2}} = \overline{\beta}\sqrt{\frac{1-\varepsilon}{N\varepsilon}},
\end{aligned}$$

where $\overline{\beta} = \sup|\beta(\mathfrak{X})|$. Therefore,

$$\begin{aligned}
\left|\sup\pi_N(\Sigma\mathfrak{D}_N) - \sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N} E_A\pi_0(X(A))\right| &\leq \sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N}|\pi_N(\mathbf{X}) - E_A\pi_0(X(A))| \leq \\
&\leq \overline{\beta}\sqrt{\frac{1-\varepsilon_N}{N\varepsilon_N}} \to 0 \text{ as } N\to\infty. \quad (15)
\end{aligned}$$

[49] I am grateful to Jim Powell for suggesting this bound.

Now we can write

$$\left|\sup \pi_N(\Sigma\mathfrak{D}_N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \leq \left|\sup \pi_N(\Sigma\mathfrak{D}_N) - \sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N} E_A\pi_0(X(A))\right| + \left|\sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N} E_A\pi_0(X(A)) - \sup \pi_0(\overline{\mathfrak{X}})\right|. \quad (16)$$

The first term has been shown to go to zero, so it remains to show that the second term goes to zero as well. For this purpose, observe that we have ${}^{\lceil\gamma N\rceil}\mathfrak{X}/N \subset {}^{A}\mathfrak{X}/N \subset \overline{\mathfrak{X}}$ when $A > \gamma N$, and therefore we have the following double inequality

$$\sup \pi_0(\overline{\mathfrak{X}}) \geq \sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N} E_A\pi_0(X(A)) = E_A \sup \pi_0({}^{A}\mathfrak{X}/N) \geq \quad (17)$$
$$\geq \sup \pi_0({}^{\lceil\gamma N\rceil}\mathfrak{X}/N) - \Pr\{A \leq \gamma N\} \cdot 2\sup\left|\pi_0(\overline{\mathfrak{X}})\right|.$$

As $N \to \infty$, the first term in the last expression goes to $\sup \pi_0(\overline{\mathfrak{X}})$ by strong asymptotic adequacy of the domain. To bound the second term, we use Chebyshev's inequality, which says that

$$\Pr\left\{\left|\frac{A - N(1-\varepsilon)}{N\varepsilon}\right| \geq \delta\right\} \leq \frac{Var(A)}{\delta^2 (N\varepsilon)^2} = \frac{1-\varepsilon}{\delta^2 N\varepsilon} \text{ for any } \delta > 0. \quad (18)$$

The inequality implies that

$$\Pr\{A \leq \gamma N\} \leq \Pr\left\{\left|\frac{A - N(1-\varepsilon)}{N\varepsilon}\right| \geq \frac{1-\varepsilon-\gamma}{\varepsilon}\right\} \leq \frac{(1-\varepsilon)\,\varepsilon_N^2}{(1-\varepsilon-\gamma)^2 N\varepsilon_N} \to 0 \text{ as } N \to \infty, \quad (19)$$

since by assumption $\varepsilon_N \to 0$ and $N\varepsilon_N \to \infty$. Therefore, from the double inequality (17) we see that $\sup_{\mathbf{X}\in\Sigma\mathfrak{D}_N} E_A\pi_0(X(A)) \to \sup \pi_0(\overline{\mathfrak{X}})$ as $N \to \infty$. Now we can use (16) and (15) to obtain (i).

To show (ii), rewrite the principal's problem as

$$\max_X E_A\left[\pi_0(X(A)) + \frac{A - N(1-\varepsilon)}{N\varepsilon}\beta(X(A))\right].$$

The expectation can be maximized statewise, i.e. for every A,

$$M_N(A) = \arg\max_{X\in{}^{A}\mathfrak{X}/N} \pi_N^A(X) = \pi_0(X) + \frac{A - N(1-\varepsilon)}{N\varepsilon}\beta(X).$$

Using Chebyshev's inequality (18) and the assumption that $N\varepsilon_N \to \infty$, we see that $\sup_{X\in\overline{\mathfrak{X}}} \left|\pi_N^A(X) - \pi_0(X)\right| \xrightarrow{p} 0$.

Now, using the triangle inequality, when $A > \gamma N$, we can write

$$\begin{aligned} d(\pi_0(M_N(A)), \pi_0(M_0)) &\leq d(\pi_0(M_N(A)), \pi_N^A(M_N(A))) + d(\pi_N^A(M_N(A)), \pi_0(M_0)) \leq \\ &\leq \sup_{X\in\overline{\mathfrak{X}}} \left|\pi_N^A(X) - \pi_0(X)\right| + \left|\sup \pi_N^A({}^A\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \leq \\ &\leq 2\sup_{X\in\overline{\mathfrak{X}}} \left|\pi_N^A(X) - \pi_0(X)\right| + \left|\sup \pi_0({}^A\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \leq \\ &\leq 2\sup_{X\in\overline{\mathfrak{X}}} \left|\pi_N^A(X) - \pi_0(X)\right| + \left|\sup \pi_0({}^{\lceil\gamma N\rceil}\mathfrak{X}/N) - \sup \pi_0(\overline{\mathfrak{X}})\right| \end{aligned}$$

The first term has been shown to go to zero in probability, the second term goes to zero by strong asymptotic adequacy of domain, and $\Pr\{A > \gamma N\} \to 1$ by (19). Therefore,

$$d(\pi_0(\mathbf{M}_N), \pi_0(M_0)) \xrightarrow{p} 0 \tag{20}$$

Since $\pi_0(\cdot)$ is u.s.c., Lemma 4 now implies (ii).

For the welfare results, write

$$W(X(A)) = \pi_0(X(A)) + \beta(X(A))$$

and therefore

$$d_+(W(\mathbf{M}_N), W(M_0)) \leq d_+(\pi_0(\mathbf{M}_N), \pi_0(M_0)) + d_+(\beta(\mathbf{M}_N), \beta(M_0))$$

The first term is zero. When $\beta(\cdot)$ is u.s.c. and (ii), the second term goes to zero in probability by Lemma 3. Thus we obtain (iii).

Finally, we can write

$$d(W(\mathbf{M}_N), W(M_0)) \leq d(\pi_0(\mathbf{M}_N), \pi_0(M_0)) + d(\beta(\mathbf{M}_N), \beta(M_0))$$

When $\beta(\cdot)$ is continuous and (ii), Lemma 3 implies that the second term goes to zero in probability. Using in addition (20), we obtain (iv). ■

B Multiple Equilibria and Coalition-Proofness

In the main body of the paper we assume that all agents accept their offers whenever it is a Nash equilibrium for them to do so. But can there be other Nash equilibria in which some or all agents reject? In this appendix we show that the answer depends on whether externalities are increasing or decreasing, as defined in Subsection 4.3. We find that with decreasing externalities, multiplicity of equilibria is not a serious a problem for the principal, while with increasing externalities, it is. In the latter case, we study the contracting outcome under the assumption that all agents coordinate on a coalition-proof equilibrium. We find that agents' coordination reduces contracting distortions, but does not eliminate them completely.

Before proceeding, we need to take care of the following technical problem. Suppose that we want to constrain the principal to makes offers such that in the 2nd stage, "all accept" is a *unique* Nash equilibrium, or that it is a *coalition-proof* Nash equilibrium. In either case, it is easy to see that the set S of offer profiles $\{(x_i, t_i)\}_{i=1}^N$ which have this property may not be closed. Hence, the principal's profit-maximizing point in S may not exist.[50]

We sidestep the problem by introducing the following concept. We will say that an offer profile $(x, t) \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N \times \Re^N$ is *nearly* in S (belongs to "near S") if for any $\varepsilon > 0$ there exists $t' \in \Re^N$ such that $\|t' - t\| < \varepsilon$ and $(x, t') \in S$. When the sets $\mathfrak{X}_i$ are finite, "near S" is the closure of S, and standard assumptions ensure that a profit-maximizing point within the set exists.[51] One way to approach this point is by restricting the principal to offer payments in the multiples of a small $\delta > 0$, which corresponds to the feasible set $S_\delta = \{(x, t) \in S : t_i = z_i\delta, \text{ with } z_i \in Z, \text{ for all } i\}$. Since the set S_δ is closed, it is easy to ensure that a profit-maximizing point within this set exists. Using Berge's "maximum theorem" (see e.g. Walker [1979]), we can see that if the set "near S" is closed and the principal's objective function is

[50]The problem does not arise when the equilibrium concept is Nash, since under standard continuity assumptions, Berge's "maximum theorem" implies that the graph of the Nash equilibrium correspondence (the parameters in our case being (x, t)) is closed (see e.g. Walker [1979]). The theorem cannot be applied to unique Nash equilibrium and coalition-proof equilibrium correspondences.

[51]For general compact sets $\mathfrak{X}_i$, the sets "near S" corresponding to unique Nash equilibrium and coalition-proof Nash equilibrium can also be shown to be closed, provided that the functions $u_i(\cdot, \cdot)$ are continuous.

continuous, as $\delta \to 0$, any limit of a sequence of profit-maximizing points from S_δ will maximize profit within "near S". This argument offers a justification for using "near S" as the principal's feasible set.

B.1 Decreasing Externalities

With decreasing externalities, an agent finds it more profitable to accept his offer when fewer other agents accept. Therefore, whenever it is a Nash equilibrium for all agents to accept, accepting must be a (weakly) dominant strategy. Thus, multiplicity of equilibria does not constitute a serious problem: by slightly reducing agents' payments, the principal can make acceptance a strictly dominant strategy, thus ensuring that "all accept" is the unique equilibrium:

Proposition 22 *With decreasing externalities, whenever "all accept" is a Nash equilibrium of the continuation game following $\{(x_i, t_i)\}_{i=1}^N$, it is nearly a unique Nash equilibrium.*

Proof. Using participation constraints (4) and the property of decreasing externalities, for any $\varepsilon > 0$, for any agent $i \in N$ and for any set $A \subset N \backslash i$, we have

$$u_i(x_i, x_A, 0) - u_k(0, x_i, 0) - t_i + \varepsilon \geq u_i(x_i, x_{-i}) - u_k(0, x_{-i}) - t_i + \varepsilon \geq \varepsilon > 0.$$

This establishes that for an offer profile $\{(x_i, t_i - \varepsilon)\}_{i=1}^N$ with an arbitrarily small $\varepsilon > 0$, acceptance is a strictly dominant strategy, and therefore "all accept" is a unique Nash equilibrium. This implies the statement. ■

B.2 Increasing Externalities

With increasing externalities, an agent is finds it less profitable to accept his offer when he expects fewer other agents to accept, and the multiple equilibrium problem is more serious. For example, since all agents' participation constraints bind at a solution to the principal's profit-maximization program, "all reject" will also be a 2nd stage Nash equilibrium for this offer profile. With positive externalities, Proposition 4 implies that the "all accept" equilibrium is preferred by all the agents. In this case, it may be reasonable for the principal to expect the agents to coordinate on their preferred Nash equilibrium, as Katz-Shapiro [1986b] assume in the context of Network Externalities (Application 10). With negative externalities, however, Proposition 4 implies that the "all reject" Nash equilibrium will be Pareto preferred by the agents to the "all accept" equilibrium!

Katz-Shapiro [1986a] and Segal-Whinston [forth.] encountered this problem in the context of Applications 2 (Vertical Contracting with a Substitute) and 3 (Exclusive Dealing) respectively. While Katz-Shapiro assume that the principal is able to induce the agents to play a Pareto inferior equilibrium, Segal-Whinston instead assume that the agents always choose a coalition-proof equilibrium. Here we will make the same assumption, and analyze the principal's problem in the case of increasing negative externalities.

Our analysis is greatly simplified by observing that under the natural strategy ordering "accept" > "reject", the property of increasing externalities implies that the 2nd stage acceptance game is supermodular (as defined in Milgrom-Roberts [1990]), and the property of negative externalities implies that each player's payoff is non-increasing in other players' strategies. Milgrom-Roberts [1996] demonstrate that in such games, a large class of coalitional equilibrium refinements produces the same prediction. Specifically, they define the concept of a coalition-proof equilibrium (CPE) with communication structure Σ, which, for different choices of "admissible" communication structures, spans a large number of coalitional refinements, including the concepts of "coalition-proof Nash equilibria" (Bernheim-Peleg-Whinston [1987]) and "semistrong" equilibria (Kaplan [1992]). For this equilibrium concept, we obtain the following result:

Proposition 23 *Choose any admissible communication structure Σ. With increasing negative externalities, "All accept" is nearly a CPE with of the continuation game following an offer profile $\{(x_i, t_i)\}_{i=1}^N$ if and only if*

$$\min_{D \subset N, D \neq \emptyset} \max_{k \in D} \left\{ u_k(x_k, x_{N \setminus D}, 0) - t_k - u_k(0, x_{N \setminus D}, 0) \right\} \geq 0, \tag{21}$$

Proof. Since the acceptance game is a supermodular game with each player's payoff non-decreasing in others' strategies, Theorem 2(3) of Milgrom-Roberts [1996] establishes that, for any admissible communication structure Σ, the unique CPE of the game is the lowest Nash equilibrium (which always exists). Therefore, "all accept" is the unique CPE if and only if it is the unique Nash equilibrium of the game.

When the inequality holds, by offering $\{(x_i, t_i - \varepsilon)\}_{i=1}^N$ with an arbitrarily small $\varepsilon > 0$, the principal can ensure that no acceptance play $N \backslash D \neq N$ is a Nash equilibrium, hence "all accept" is the unique CPE by Milgrom-Roberts [1996].

Conversely, when the inequality is violated, it is also violated for any offer profile (x, t') with t' close enough to t. Then for any such profile there will exist a play $N \backslash D \neq N$ in which all rejecting agents strictly prefer to reject. While this play is not necessarily a Nash equilibrium itself, we could then find a Nash equilibrium play $A \subset N \backslash D \neq N$. Hence, "all accept" is not the lowest Nash equilibrium of the game, and by Milgrom-Roberts [1996], it is not a CPE. ■

Intuitively, the constraint (21) requires that for every deviating coalition $D \neq \emptyset$, there exists a "defector" $k \in D$ who (weakly) prefers to accept. (Observe that if we restrict attention to $|D| = 1$, we obtain the Nash equilibrium participation constraints (4).) Now we can study the principal's problem of maximizing profit subject to (21). Since a general analysis of this program seems quite difficult, we restrict attention to cases satisfying Condition S. For this case, define the set of "trading" agents $N_1 = \{i \in N : x_i = 1\}$. Since agents are symmetric, without loss of generality we can assume that $N_1 = \{1, \ldots, X\}$, where X is the total trade.

Since an agent i who is offered $x_i = 0$ will accept if and only if $t_i \geq 0$ regardless of what he expects other agents to do, the principal will optimally set $t_i = 0$ for such agents. The constraint (21) will then be automatically satisfied for all D which contain some agents with $x_i = 0$. Therefore, in verifying the constraint, we can restrict attention to $D \subset N_1$.

Let us start with $D = N_1$. The constraint (21) says that there exists an agent $k \in D$ who weakly prefers to accept in this situation. In the symmetric case, we can assume without loss of generality that $k = 1$, i.e., $U(1,1) - t_1 \geq U(0,0)$. But since we have increasing externalities, this agent will also want to accept for any $D \subset N_1$. Thus, we can now restrict attention to deviating coalitions $D \subset N_1 \backslash \{1\}$. In particular, for $D = N_1 \backslash \{1\}$, we must have an agent $k \in D$ who weakly prefers to accept in this situation. In the symmetric case, we can assume without loss of generality that $k = 2$, i.e., $U(1,2) - t_2 \geq U(0,1)$. But since we have increasing externalities, agent 2 will then also want to accept for any $D \subset N_1 \backslash \{1\}$. Thus, we can now restrict attention to deviating coalitions $D \subset N_1 \backslash \{1, 2\}$. Continuing this argument, we see that up to a renumbering of agents, the constraint in program (21) reduces to

$$\begin{aligned} t_k &\leq 0 \text{ for all } k \in N \backslash N_1, \\ U(1,k) - t_k &\geq U(0,k-1) \text{ for all } k \in N_1. \end{aligned}$$

At the principal's profit-maximizing point, all these constraints will be binding. Expressing t_k from the binding constraints and adding up over all k, we obtain

$$\sum_{k \in N} t_k = \sum_{k \in N_1} t_k = \sum_{k=1}^{X} \left[U(1,k) - U(0,k-1) \right].$$

Therefore, the principal's program can be written as

$$\max_X F(X) + \sum_{k=1}^{X} \left[U(1,k) - U(0,k-1) \right].$$

Let M^{CP} denote the set of solutions of this program.

To compare this set to the set M^* of first-best total trades and the set M of trades that occur in the principal's preferred NE, consider the following intuition. Start with a 2nd stage CPE in which the constraint (21) binds, and the set of trading agents is $N_1 \subset N$, with $|N_1| = X - 1$. Suppose that the principal contemplates deviating to a CPE with total trade X. It is easy to see that the principal will optimally do making a new agent $i \notin N_1$ an offer at a price of $t_i = U(1,X) - U(0,X-1)$, and holding her offers to other agents fixed. Indeed, this will insure that all other agents accept even if they expect the new agent to reject, hence "all accept" remains a CPE. This deviation will impose externalities on agents other than i, both those who trade and those who do not trade. Therefore, when externalities are globally negative, the principal will have a socially excessive incentive to increase X. At the same time, when externalities are increasing, the principal will have a smaller incentive to do it than he would have in program (5), in which only externalities on non-signers matter. Formalizing this intuition using Topkis' Monotonicity Theorem, we obtain the following result:

Proposition 24 *When Condition S holds and we have increasing and globally negative externalities, $M^* \leq M^{CP} \leq M$ in the strong set order.*

Thus, with increasing and globally negative externalities, coordination among agents reduces contracting distortions, but does not fully eliminate them. This generalizes the finding of Segal-Whinston [forth.] that in the setting of Exclusive Dealing (Application 3), inefficient exclusion always occurs when the buyers can be induced to play the incumbent firm's preferred equilibrium, but occurs only for some parameters when the buyers coordinate on a coalition-proof equilibrium.

With private offers, we could also allow agents to coordinate on a coalition-proof equilibrium. As should be clear from the above analysis, with decreasing externalities this would not affect the equilibrium set. With increasing externalities, on the other hand, an equilibrium is unlikely to exist, for the same reasons as outlined in Appendix C.

C Existence of Equilibria with Private Offers

A set of sufficient conditions for existence of equilibrium with private offers is provided by the following proposition:

Proposition 25 *When $\mathfrak{X}_i$ is an interval for all i, and the function*

$$g(x, \widehat{x}) = f(x) + \sum_i u_i(x_i, \widehat{x}_{-i})$$

is continuous in $(x, \widehat{x})$ and quasi-concave in x, we have $E \neq \emptyset$.

Proof. A trade profile $\widehat{x}$ satisfies the equilibrium condition (8) if and only if $\widehat{x} \in B(\widehat{x}) = \arg\max_{x \in \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_N} g(x, \widehat{x})$. Under the assumptions, the correspondence $B(x)$ satisfies the conditions of Kakutani's fixed-point theorem, therefore a solution $\widehat{x}$ exists. ■

A sufficient condition for quasi-concavity of $g(x, \widehat{x})$ in x is for $f(x)$ to be concave and for $u_i(x_i, \widehat{x}_{-i})$ to be concave in x_i for all i and all x_{-i}. In the context of Vertical Contracting (Example 1), this means that the supplier's cost function is convex and the revenue functions are quasiconcave, which generalizes the setting analyzed by Hart-Tirole [1990] and Rey-Tirole [1996].

In an asymptotic setting with a large N, supposing that the second derivatives of $F(\cdot)$, $\alpha(\cdot)$, $\beta(\cdot)$ exist and are bounded on $\overline{\mathfrak{X}}$, we can write

$$g_N(x,\widehat{x}) = F\left(\sum_i x_i\right) + \sum_i \left[x_i\alpha\left(x_i + \sum_{j\neq i}\widehat{x}_i\right) + \frac{1}{N}\beta\left(x_i + \sum_{j\neq i}\widehat{x}_i\right)_i\right].$$

$$\begin{aligned}
\frac{\partial g_N(x,\widehat{x})}{\partial x_k} &= F'\left(\sum_i x_i\right) + \alpha\left(x_i + \sum_{j\neq i}\widehat{x}_i\right) + O(\frac{1}{N}) \\
\frac{\partial^2 g_N(x,\widehat{x})}{\partial x_k^2} &= F''\left(\sum_i x_i\right) + 2\alpha'\left(x_i + \sum_{j\neq i}\widehat{x}_i\right) + O(\frac{1}{N}), \\
\frac{\partial^2 g_N(x,\widehat{x})}{\partial x_k \partial x_l} &= F''\left(\sum_i x_i\right) \text{ for } k \neq l.
\end{aligned}$$

Thus, having $F(\cdot)$ concave and $\alpha'(X) < 0$ for all X is sufficient to satisfy the conditions of Proposition 25 for N large enough. On the other hand, when $\alpha'(X) > 0$ for all X, the conditions of Proposition 25 are violated when N is large enough. To see this, observe that in this case the 3×3 leading principal minor of the bordered Hessian of $g_N(\cdot,\widehat{x})$ at the point $x_i = \widehat{x}_i = \overline{X}/N$ for all i equals to

$$-4\left[F'\left(\overline{X}\right) + \alpha\left(\overline{X}\right)\right]^2 \cdot \alpha'\left(\overline{X}\right) + O(\frac{1}{N}) > 0 \text{ for } N \text{ large enough.}$$

Therefore, in this case the function $g_N(\cdot,\widehat{x})$ is not quasiconcave.

Since $\alpha'(\cdot) < 0$ is the asymptotic version of decreasing externalities, this suggests that the existence of equilibrium is intimately tied to decreasing externalities. While we do not have a general result to this effect, we have a result for a special case:

Proposition 26 *Suppose that Condition S holds. Then*

(i) When we have decreasing externalities and $F(\cdot)$ concave, we have $E \neq \emptyset$.

(ii) When $U(1, X+1) - U(0, X)$ is strictly increasing in X (i.e., we have strictly increasing externalities), any equilibrium $\widehat{x} \in E$ must have $\widehat{x}_1 = \ldots = \widehat{x}_N$.

Proof. Define

$$\Delta(X)=\begin{cases}[F(X+1)+U(1,X+1)]-[F(X)+U(0,X)] \text{ when } X\in\{0,\ldots,N-1\}, \\ 0 \text{ when } X<0 \text{ or } X>N-1.\end{cases}$$

Note that $\Delta(X)$ can be interpreted in two ways:

- As the principal's gain from trading with one more agent when she is expected to trade with X agents, and agents hold passive beliefs.
- As the principal's loss from trading with one fewer agent when she is expected to trade with $X+1$ agents, and agents hold passive beliefs.

It is now clear that $X\in\{0,\ldots,N\}$ is a "pairwise equilibrium" (as defined in Subsection 5.1) if and only if $\Delta(X)\leq 0$ and $\Delta(X-1)\geq 0$. It is easy to see that the point $\widehat{X}=\min\{X\in\{0,\ldots,N\}:\Delta(X)\leq 0\}$ satisfies this condition, and exists (indeed, the set always includes N, and is therefore non-empty, and it is finite). Therefore, a pairwise equilibrium $\widehat{X}$ must exist.

To see that $\widehat{X}$ must be a true equilibrium, consider the general multi-agent deviation, in which the principal trades with k new agents and gives up trade with l old agents. The principal's gain from this deviation can be written as

$$F(\widehat{X}+k-l)-F(\widehat{X})+k\left[U(1,\widehat{X}+1)-U(0,\widehat{X})\right]-l\left[U(1,\widehat{X})-U(0,\widehat{X}-1)\right].$$

When $k\geq l$, this gain can be rewritten as

$$F(\widehat{X}+k-l)-F(\widehat{X})+(k-l)\left[U(1,\widehat{X}+1)-U(0,\widehat{X})\right]+$$
$$+l\left[\left(U(1,\widehat{X}+1)-U(0,\widehat{X})\right)-\left(U(1,\widehat{X})-U(0,\widehat{X}-1)\right)\right].$$

When $F(\cdot)$ is concave, the first term can be bounded from above by $(k-l)\,\Delta(\widehat{X})\leq 0$. When we have decreasing externalities, the second term must also be non-positive. Thus, the deviation is unprofitable. Since a similar argument works when $k<l$, $\widehat{X}$ must be a true equilibrium, which establishes (i).

To see (ii), suppose in negation that we have $\widehat{x}\in E$ with $\widehat{x}_k=1$ and $\widehat{x}_l=0$. Consider the principal's deviation in which she trades with one new agent and gives up trade with one old agent. Letting $\widehat{X} = \sum_i \widehat{x}_i = 0$, the principal's gain can be written as

$$\left[U(1, \widehat{X}+1) - U(0, \widehat{X})\right] - \left[U(1, \widehat{X}) - U(0, \widehat{X}-1)\right].$$

With strictly increasing externalities, the principal's gain must be strictly positive, which contradicts the hypothesis that $\widehat{x} \in E$. ■

While this result does not rule out the possibility that private-offers equilibria exist with increasing externalities, it does reinforce our intuition that such existence is, in some sense, less likely than with decreasing externalities. One way to ensure existence is to allow the agents to hold arbitrary, and not just passive, beliefs. Segal-Whinston [forth] examine the set of such equilibria in the context of Application 3 (Exclusive Dealing), which exhibits increasing externalities.

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Settings:	Conds.* A,L,D	Cond. S	Extern. on non-signers	Global Extern.	Increase in Externality	α' (Asympt. Increase)
1. Vertical Contracting	sometimes	no	0	-	-	-
2. V. C. with a Subst.	yes	yes	-	-	+/-	+/-
3. Exclusive Dealing	yes	yes	-	-	+	+
4. Nuclear Weapons	no	no	+/-			
5. Common Insurance	yes	no	0	-	-	-
6. Common Agency	yes	no	+	+	+/-	-
7. Takeovers	yes	sometimes	+	+	-	-
8. Debt Workouts	yes	sometimes	+	+	-	-
9. Merger for Monopoly	yes	yes	+	+	-	-
10. Network Extern.	yes	yes	0	+	+	+
11. Bargaining Extern.	yes	yes	+	+	-	-
12. Pure Public Goods	yes	sometimes	+	+	+/-	0
13. Pure Public Bads	yes	sometimes	-	-	+/-	0

Table 1: Applications

* In these two columns, "yes" means that the respective condition(s) are always satisfied in the literature, "sometimes" means that they are satisfied only in some models in the literature, and "no" means that they are not satisfied and they are not reasonable in the specific context.

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