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Authors

Roll, Richard
Schwartz, Eduardo S
Subrahmanyam, Avanidhar

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by

Richard Roll, Eduardo Schwartz, and Avanidhar Subrahmanyam

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Since arbitrage involves trading, it is potentially impeded by market frictions and costs. We study whether stock market liquidity is related to the efficacy of arbitrage. Specifically, we examine the joint time-series of the NYSE Composite index futures basis and aggregate liquidity on the NYSE for a relatively long time-period, over 3000 trading days, and find that the basis and liquidity are jointly determined. Contemporaneous innovations in the absolute basis and in bid-ask spreads are positively correlated. There is also evidence of two-way Granger causality between short-term absolute bases and effective spreads. Impulse response functions indicate that shocks to the absolute basis are significantly informative in predicting spreads.

Contacts

	Roll	**Schwartz**	**Subrahmanyam**
Voice:	1-310-825-6118	1-310-825-2873	1-310-825-5355
Fax:	1-310-206-8404	1-310-206-5455	1-310-206-5455
E-mail:	Rroll@anderson.ucla.edu	Eschwart@anderson.ucla.edu	Asubrahm@anderson.ucla.edu
Address:	Anderson School UCLA Los Angeles, CA 90095-1481	Anderson School UCLA Los Angeles, CA 90095-1481	Anderson School UCLA Los Angeles, CA 90095-1481

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Liquidity and Arbitrage

Abstract

Since arbitrage involves trading, it is potentially impeded by market frictions and costs. We study whether stock market liquidity is related to the efficacy of arbitrage. Specifically, we examine the joint time-series of the NYSE Composite index futures basis and aggregate liquidity on the NYSE for a relatively long time-period, over 3000 trading days, and find that the basis and liquidity are jointly determined. Contemporaneous innovations in the absolute basis and in bid-ask spreads are positively correlated. There is also evidence of two-way Granger causality between short-term absolute bases and effective spreads. Impulse response functions indicate that shocks to the absolute basis are significantly informative in predicting spreads.

Liquidity and Arbitrage

Introduction

A fundamental building block of modern financial theory is the law of one price. Indeed, several well-known pricing relations in finance depend on the notion that two traded or synthesized instruments with the same future cash flows should trade at the same market value. In accepting such a relation as true, the financial modeler typically presumes that arbitrage forces cause the relation to hold.

Arbitrage forces, however, are not akin to Newton's laws of motion but are governed by human agents. Thus, the efficacy of arbitrage (and the extent to which no-arbitrage relations hold) should depend on the frictions associated with transacting. Examples of such frictions include liquidity measures such as the bid-ask spread, short sale restrictions, and circuit breakers. Indeed, the influence of market imperfections on security pricing is receiving increasing prominence (see, for example, the AFA presidential address of Stoll, 2000).

Liquidity has attracted a lot of attention from traders, regulators, exchange officials and academics. It can be defined as the ability to buy or sell large quantities of an asset quickly and at low cost. Recent financial crises suggest that, at times, market conditions can be severe and liquidity can decline or even disappear.[1]

[1] "One after another, LTCM's partners, calling in from Tokyo and London, reported that their markets had dried up. There were no buyers, no sellers. It was all but impossible to maneuver out of large trading bets." - *Wall Street Journal*, November 16, 1998.

Many microstructure studies are devoted to the measurement of and time variation in liquidity. The study of liquidity is often justified to a broader finance audience by arguing that liquidity shocks affect asset prices. Amihud and Mendelson (1986) and Jacoby, Fowler, and Gottesman (2000) provide theoretical arguments and empirical evidence to show how liquidity impacts stock returns in the cross-section, and Jones (2001) as well as Amihud (2002) show that liquidity predicts expected returns. Pastor and Stambaugh (2003) find that expected stock returns are cross-sectionally related to liquidity risk.

In this paper we focus on another potential reason that liquidity is relevant for financial theory. Specifically, we examine whether liquidity is related to the efficacy of arbitrage activity.

Our empirical investigation involves the joint time-series of the basis associated with the NYSE Composite index futures contract and aggregate liquidity on the NYSE. We use a relatively long time-period of over 3000 trading days (1988-2002). While there have been many studies of index arbitrage and of the relation between index futures and the underlying cash index,[2] to the best of our knowledge, the dependence of arbitrage efficacy on frictions such as liquidity has not been investigated over an extensive time-period.[3]

[2] See, for example, Modest and Sundaresan (1983), Kawaller, Koch, and Koch (1987), Chung (1991), Subrahmanyam (1991), Bessembinder and Seguin (1992), Chan (1992), Miller, Muthuswamy, and Whaley (1994), Yadav and Pope (1994), Neal (1996), Barclay, Hendershott, and Jones (2003), and Wang (2003).

[3] In an interesting paper, Bakshi, Cao, and Chen (2000) consider the frequency with which index option prices violate theoretical comparative statics. They do not, however, relate the liquidity of the cash market to these violations. Such liquidity is key to arbitraging an index derivative as the cash instrument is a basket of many stocks. Bakshi, Cao, and Chen use data that span about three months; in contrast, we consider the relation between the futures-cash basis and aggregate stock liquidity using about fifteen years of data.

We perform our analysis in two stages. First, we separately adjust the raw time-series of liquidity and the absolute cash-futures basis to account for deterministic time-trends and calendar regularities. Then, we apply vector autoregression to the adjusted series to uncover the dynamic interplay between liquidity and the efficacy of arbitrage.

Section I below describes the data. Section II presents our analysis of the joint time-series of the basis and liquidity. Section III concludes and suggests further investigations.

I. The Data

A. Computation of the Basis

Let F be the current futures price, S be the stock market index, r the riskfree rate for lending and borrowing over the remaining life of the contract, t the time to expiration, and δ the dividend yield over the contract's lifetime. The absolute value of the relative index futures basis (henceforth, termed the "absolute basis") can be defined as

$$\frac{\left|Fe^{-(r-\delta)t} - S\right|}{S}.$$

In a frictionless world, the above quantity should be precisely zero. However, it is well known from earlier work (e.g., MacKinlay and Ramaswamy, 1988, Brennan and Schwartz, 1991) that the basis exhibits considerable time-series variation. Our goal is to examine the behavior of the absolute basis over a relatively long time-period and to relate it to a specific friction, namely, stock market illiquidity.

The basis is estimated as follows: F is the closing futures price on the NYSE composite index futures contract, while S is the closing value of the NYSE composite index. The rate r is measured by the continuously compounded yield on a Treasury Bill maturing as close to the future's expiration date as possible, and the yield is extrapolated from the last day of the bill's life to the end of the contract. The dividend yield δ is measured by the (continuously compounded) annual yield on the S&P500 index (obtained from CRSP) updated in January of every year.[4]

The NYSE Composite Index futures contract expires on the third Friday in four months, March, June, September, or December. Each contract starts trading a year prior to its maturity. As in MacKinlay and Ramaswamy (1988), we construct four continuous time-series of the basis from the first trading day in January 1988 to the last trading day in December 2002 by rolling over on contract expiration days into a new contract.[5]

We have available four different time-series of the bases, obtained by starting with a contract with X months to maturity and rolling over every third Friday of March, June, September and December into successive contracts with the same original time to maturity, where X=3, 6, 9, or 12.[6] In this study, we focus on the first three series, because the fourth is inactively traded (with an average volume of about two contracts per day) and we presume that this last series would not be subject to intense arbitrage forces because of its lack of liquidity. Henceforth, we refer to the three series as the three-month, six-month, and nine-month bases, respectively.

[4] There inevitably is some error associated with the measurement of both the risk-free rate and the dividend yield. However, the errors should be small (in particular, the dividend yield on a basket such as the NYSE Composite Index should not fluctuate much from day to day). Further, it seems unlikely that any measurement error would be related to liquidity, our variable of interest. Nonetheless, to mitigate possible measurement error problems, we control for the measured values of the above variables in adjustment regressions prior to conducting vector autoregressions.

[5] All contracts officially expire on Friday. However, within the futures dataset (obtained from www.normanshistoricaldata.com), in some cases there was no trading on Friday and we define Thursday as the last day of trading.

[6] Two days, December 20, 1996 and December 23, 1996, are deleted from the sample because there are no data available for the contract on those days, and we wish to keep the time-series of all bases synchronized.

Figure 1 plots the three bases over the sample period. The basis in each case appears fairly stationary though the nine-month basis appears to exhibit more and greater deviations from zero than the other two.

B. Stock Liquidity Data

An aggregate NYSE liquidity index on a daily basis for the period 1988-2002 is constructed as follows:

- To be included, a stock must be present at the beginning and at the end of the year in both the CRSP and the intraday databases.
- If the firm changes exchanges from Nasdaq to NYSE during the year (no firms switched from the NYSE to the Nasdaq during the sample period), it is dropped from the sample for that year.
- Because their trading characteristics might differ from ordinary equities, certificates, ADRs, shares of beneficial interest, units, companies incorporated outside the U.S., Americus Trust components, closed-end funds, preferred stocks, and REITs are expunged.
- To avoid the undue influence of high-priced stocks, which have inordinately large spreads, if the price at any month-end during the year is greater than $999, the stock is deleted from the sample for the year.

Intraday data are purged for one of the following reasons: trades out of sequence, trades recorded before the open or after the closing time, and trades with special settlement conditions (because they might be subject to distinct liquidity considerations). A preliminary investigation reveals that auto-quotes (passive quotes by secondary market dealers) have been eliminated in the ISSM database but not in TAQ. This causes the quoted spread to be artificially inflated in TAQ. Since there is no reliable way to filter out auto-quotes in TAQ, only BBO (best bid or offer)-eligible primary market (NYSE) quotes are used. Quotes established before the opening of the market or after the close are discarded. Negative bid-ask spread quotations, transaction prices, and quoted depths are discarded. Each bid-ask quote included in the sample is matched to a transaction; specifically, following Lee and Ready (1991) any quote less than five seconds prior to the trade is ignored and the first one at least five seconds prior to the trade is retained.

For each stock the following variables are calculated:

Quoted spread: the difference between the asked and the bid quote.

Effective spread: twice the absolute distance between the transaction price and the mid-point of the prevailing quote.

An initial scanning of the intraday data reveals a number of anomalous records that appear to be keypunching errors. Filters are applied to the transaction data by deleting records that satisfy the following conditions:[7]

1. Quoted spread>$5
2. Effective spread / Quoted spread > 4.0
3. Proportional effective spread / Proportional quoted spread > 4.0
4. Quoted spread/Mid-point of bid-ask quote >0.4

[7] The proportional spreads in condition 3 are obtained by dividing the unscaled spreads by the mid-point of the prevailing bid-ask quote.

These filters removed less than 0.02% of all stock transaction records. The above variables are averaged across the day to obtain stock liquidity measures for each day. To avoid excessive variation in the sample size, stocks are required to have traded for a minimum of 100 days in a year to be included in the sample for that year. Days for which stock return data were not available from CRSP are dropped from the sample. The dates of October 25, 1989 and September 4, 1991 are also dropped from the sample because of transparent reporting errors. Specifically, on the former date, there are no data at all, and on the latter, ISSM recorded only quotes, but no transactions. In addition, data are not available for the period from September 11 to September 14, 2001 (neither for the stock liquidity measures nor for the basis) because of market closures and disruption following the terror attacks in New York City.

The daily spread measures are averaged, value-weighted, across stocks (with weights proportional to market capitalizations at the end of the previous year) to obtain the aggregate market liquidity measures, i.e., the quoted spread and the effective spread that are used in this study.

Figure 2 plots aggregate quoted and effective spreads. They exhibit a downward trend throughout the sample period. In addition, there are two significant drops corresponding to the reduction in minimum tick size, first to sixteenths and then to cents. Because of the non-stationary nature of these liquidity series, in the next section we adjust these series to account for deterministic time trends, calendar regularities, and discrete reductions in the minimum tick size prior to using the adjusted series in vector autoregressions involving the basis variables.

Since our goal is to consider deviations from the no-arbitrage relation without regard to direction, the *absolute* basis is our object of enquiry. Table 1 presents summary statistics for the absolute basis and liquidity variables. In unreported t-tests, we find that all of the means are statistically significant at the 5% level or less. However, the mean value of the absolute basis across the three series is only about 0.3%, which suggests that, on average, arbitrage activities are quite effective. Table 1 reveals that the absolute basis increases with horizon, suggesting that arbitrageurs are more active in short-term contracts. The average value of the quoted spread is about sixteen cents and the quoted spread is larger than the effective spread, indicating that many transactions take place within the spread.

II. The Evidence.

Our goal is to explore intertemporal associations between market liquidity and the extent to which futures and cash markets diverge from the no-arbitrage relation. The basic notion is that liquidity shocks and information flows cause temporary deviations of the basis from zero (cf., Gorton and Pennacchi, 1993, Kumar and Seppi, 1994, and Subrahmanyam, 1991), and arbitrageurs step in to exploit these temporary deviations. We test for the dual possibilities that arbitrageurs both affect and are affected by liquidity. Since our interest lies in exploration at the daily horizon, rather than using the measures of Amihud (2002) and Pastor and Stambaugh (2003), which are available only over aggregated intervals such as a month, we use daily bid-ask spread measures obtained from transactions data.

It is known from prior research (Chordia, Roll, and Subrahmanyam, 2001) that spreads exhibit time-trends and calendar regularities. It seems plausible that the absolute basis could also exhibit such phenomena. For example, the increased risk of holding positions over the weekend could cause higher absolute bases towards the end of the week. To avoid the possible spurious correlation induced by such calendar regularities and time trends, our analysis proceeds in two stages. In the first stage, the raw time-series are adjusted for deterministic variations. In the second stage, innovations (residuals) from the adjustment regressions are related with a vector autoregression (VAR). The possibility of bivariate causality motivates the VAR approach. When the stock market is illiquid, arbitrageurs have difficulty closing the gap between the two markets. In the reverse direction, a large inventory imbalance caused by arbitrage trades could result in illiquidity.

A. Adjustment Regressions

The following adjustment variables are used for the basis: (i) A Friday dummy to account for increased costs of holding positions through the weekend, (ii) 11 month of the year dummies for January through November, (iii) a dummy for days prior to major holidays; Thanksgiving, Christmas Eve, and July 4, (iv) for the three-month basis, four dummies to account for the four days prior to expiration to account for any maturity-related effects (v) a time trend and the square of the time trend to remove any long-term trends[8], and (vi) the difference between the risk-free rate and the dividend yield to account for any errors in the measurement of these quantities. For spreads, explanatory variables include day-of-the-week, month-of-the-year, and holiday dummies, linear and quadratic time trends, and two dummies to account for the post sixteenth period (i.e., the period following the shift in minimum tick size from 1/8 to 1/16 on June 24, 1997) and the post-decimalization period (following January 29, 2002.)

Tables 2 and 3 present adjustment regressions for the absolute basis and spreads, respectively. The absolute basis tends to be lower in late summer and exhibits strong maturity-date related effects. The maturity effect is presumably due to traders unwinding large positions just prior to expiration. The difference between the risk-free rate and dividend yield is positive and significant for the two longer contracts but negative for the three-month contract. We exploit this variable to correct for the impact of measurement error in the borrowing/lending rate of arbitrageurs as well as in the dividend yield over the life of the contract.

[8] The time trends are orthogonalized; i.e., the linear time index t goes from –1 at the beginning of the sample to 1 at the end of the sample, whereas the quadratic term equals $(3\,t^2-1)/2$.

Note that the coefficients for month-of–the-year regularities in the basis adjustment regressions (Table 2) are usually more significant and larger in absolute magnitudes for the longer-term bases. Indeed, the August coefficient for the nine-month basis is about double that for the three-month basis, and is also more significant. The explanatory power of the regression is also larger for the longer-term bases. This observation makes intuitive sense. Since longer-term contracts are less liquid,[9] one might expect more deviations of the basis from zero for such contracts.

Table 3 reveals that during the sample period, spreads are lower on Mondays and Tuesdays and are also lower in the summer months. The effect of early fall and the end of the year on spreads is large; in absolute terms, the coefficient for the quoted spread in September (December) is about six (five) times as large as the coefficient in April. This phenomenon is intriguing and warrants further analysis in future research.

Spreads appear to be highest in January, the base case. This may be due to imbalances created by reallocations to speculative stocks by institutional money managers after window-dressing in December. Positive order flow from individual investors' buying activity after year-end cash inflows such as bonuses may also contribute to the "January" effect in spreads. There are significant reductions in spreads after decimalization and the shift to sixteenths, which is not surprising. In addition, spreads exhibit a strong downward trend over time. The behavior of both the quoted and effective series is very similar, and the explanatory power of the spread regressions is high.

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[9] The average daily trading volumes for the three-, six- and nine-month bases are respectively 3037, 455, and 14 contracts. These numbers are all statistically different from each other at the 5% level.

The augmented Dickey-Fuller test strongly rejects the existence of a unit root for all five adjusted time-series (three for the absolute basis, and two for spreads) at p-values less than 0.001. Hence, there is no evidence that the adjusted series are non-stationary.

Table 4 presents the correlation matrix for the three adjusted absolute bases (denoted ABAS3, ABAS6, and ABAS9) together with the two adjusted series of the quoted and effective spreads (QSPR and ESPR). All of the correlations are statistically significant at the 5% level. We find that the three absolute bases series are highly correlated. In addition, the correlations of the bases with the two spread measures are all positive, suggesting commonality between absolute bases and spreads. The correlations of the absolute bases with effective spreads are higher than those with quoted spreads, suggesting that the effective spread, which accounts for transactions executing within the quotes, has a stronger link with the absolute basis.

Another way to examine the illiquidity-absolute basis relation is simply to correlate the average absolute basis over a contract's lifetime with the average spread over its lifetime. The correlations between average effective spreads and the average absolute bases for the three- six- and nine-month absolute bases are 0.293, 0.290, and 0.211. The corresponding correlations for the quoted spreads are 0.253, 0.260, and 0.201. All six correlations are significant at the 5% level. This is independent support for commonality between the absolute basis and stock market illiquidity.

As a pre-amble to the analysis of whether shocks to the absolute basis have a lasting effect on liquidity and vice versa, in Table 5 we present the coefficients from OLS regressions of the absolute bases on one lag of the spreads and vice versa. The results indicate that the coefficient of the lagged effective spread is highly significant in explaining all three absolute bases. The lagged quoted spread, however, is significant only for the three-month absolute basis.

In the reverse regression of spreads on lags of the absolute bases, we find that lags of all three absolute bases are significant in explaining the current effective spread, where lags of the two shorter-term absolute bases are significant in explaining the quoted spread. In totality, the results of Tables 4 and 5 point to the notion that the absolute bases and liquidity are jointly determined, and that effective spreads (which account for trades executing within the posted quotes) bear a stronger relation to the absolute bases. We now turn to a vector autoregression that allows for a richer dynamic structure between the bases and liquidity measures.

B. Vector Autoregressions

The input data for VAR estimation are the adjusted series (i.e., the residuals) from the first-stage regressions described in the previous subsection. We choose the number of lags based on the Akaike and Schwarz information criteria. When these two criteria indicate different lag lengths, we choose the lesser lag length for the sake of parsimony. Typically, the slopes of the information criteria (as a function of lag length) are quite flat for longer lags, so the choice of shorter lag lengths is further justified. We run six bivariate VARs, pairing each of the three absolute basis measures (three-, six-, and nine-months) with the two liquidity measures (quoted and effective spreads). Our criteria imply a lag length of six days for all VARs.

Panel A of Table 6 reports the correlation matrix of the innovations (residuals) from the estimation of the six VARs. The cross-correlations between innovations in spreads and the absolute bases are all positive and their magnitudes range from about 0.14 to 0.07. They are also statistically significant at the 5% level in each case. The cross-correlations are higher for the effective spreads, again suggesting that these spreads are more relevant for arbitrageurs. Overall, this panel represents further evidence that illiquidity is positively related to deviations from the no-arbitrage relation.

Table 6 also reports pairwise Granger-causality tests. For the null hypothesis that variable i does not Granger-cause variable j, we test whether the lag coefficients of i are jointly zero when j is the dependent variable in the VAR. In Panel B of Table 6, the cell associated with the i'th row variable and the j'th column variable shows the Chi-square statistic associated with this test.

We find that the three-month absolute basis Granger-causes quoted and effective spreads. Further, reverse causality running from both spread measures to the absolute basis is found for the nine-month basis measure. The quoted spread Granger-causes the six-month absolute basis whereas this basis measure Granger-causes the effective spread. The overall conclusion that emerges from Panel B of Table 6 is that liquidity concerns are particularly relevant for arbitrageurs in longer-term, relatively less-active contracts. Further, absolute basis innovations for shorter-term contracts have a stronger effect on liquidity, likely because arbitrage activities are more intense in such contracts.[10] Note that the Granger causality results are based on analyses of coefficients from a single equation and do not account for joint dynamics implied by the VAR system. A clearer picture can potentially emerge by the use of impulse response functions (IRFs), which we now present.

[10] Mitchell, Pulvino, and Stafford (2002), Lamont and Thaler (2003), Hou and Moskowitz (2004), and Sadka and Scherbina (2004) suggest that frictions affect the efficacy of arbitrage. We explore this as well as the reverse: viz., that exploitation of arbitrage opportunities can impact an endogenous market friction, liquidity.

An IRF traces the impact of a one-time, unit standard deviation, positive shock to one variable (henceforth, termed simply a ”shock”’ or “innovation” for expositional convenience) on the current and future values of the endogenous variables. Since the innovations are correlated (as shown in Panel A of Table 6), they need to be orthogonalized, so we use the inverse of the Cholesky decomposition of the residual covariance matrix to orthogonalize the impulses. Results from the IRFs are generally sensitive to the specific ordering of the endogenous variables.[11] However, conclusions about our IRFs turn out to be insensitive to the ordering, and also robust to the computation of generalized impulse responses (Pesaran and Shin, 1988). In the following results, the absolute basis variables are always placed first in the ordering.

Figures 3 through 8 illustrate the responses of the illiquidity and basis measures to a unit standard deviation shock in a particular variable traced forward over a period of five days. In these figures day 1 gives the contemporaneous impact and days 2-6 plot the effect from +1 to +5 days. Monte Carlo two-standard-error bands (based on 1000 replications) are provided to gauge the statistical significance of the responses. The responses generally decay over time, confirming the unit root tests that indicate stationarity.

It can be seen that shocks to a variable are informative in predicting future values of that same variable in every instance. This confirms that both spreads and absolute bases are persistent. With regard to cross-effects, the IRFs are largely consistent with the Granger causality results. We find that innovations to the three-month absolute basis have a lasting and significantly positive effect on both spread measures (Figures 3 and 4.) This strongly suggests that the activities of arbitrageurs affect liquidity in the stock market. If the basis widens on a particular

[11] VAR coefficient estimates (and, hence, the Granger causality tests) are unaffected by the ordering of variables.

day, arbitrage forces on subsequent days cause an excess of buy or sell orders, which strains liquidity. While the impulse responses of spreads to absolute bases is significant on the second day for the six- and nine-month absolute bases as well, this response is most persistent for the three-month basis, again suggesting more intense arbitrage activity in short-term contracts.[12]

Impulse responses of the three-month and six-month absolute bases to spreads are generally positive, but not significant. Thus, even though the spread Granger-causes the shorter-term absolute bases, after accounting for the joint dynamics, including the persistence of the absolute basis and liquidity variables, shocks to spreads are statistically uninformative in forecasting these bases. Note, however, that for the nine-month case, the spread is indeed informative in predicting shocks to the absolute basis at the second period. This confirms the results from Table 6 that liquidity affects the future activities of arbitrageurs relatively more in the less liquid, longer-term contracts. While there are differences in the impulse responses of spreads and absolute bases to each other, note from Table 4 and Panel A of Table 6 that *contemporaneous* innovations in all the basis measures and in illiquidity are positively and significantly correlated, suggesting joint determination of the absolute bases and liquidity.

We note that fluctuations in financial market liquidity can arise either from demand-related shifts (e.g., shocks to investors' trading needs) or supply-related ones (e.g., shifts in borrowing terms which affect market makers' access to capital). While a detailed analysis of the causes of daily

[12] Order flow provides indirect evidence that arbitrage activity follows days when the absolute basis widens. Harford and Kaul (2004) document a significant index-related component in order flow. We do not include order flow in our VAR system because of possible multicollinearity between absolute order flow and the absolute basis. However, using an (imperfect) index of order flow based on Chordia, Roll, and Subrahmanyam (2002), univariate correlations between net buying activity and the lagged (signed) three- six- and nine-month bases are 0.069, 0.067 and 0.065, respectively, and are all statistically significant at the 1% level. This is evidence of buying activity in the cash market following days when the futures component of the basis is high relative to the cash component, which is consistent with arbitrageurs reacting to the size of the basis.

liquidity fluctuations warrants a separate paper, previous literature suggests that the most important candidates for common determinants of liquidity and the absolute basis are return volatility as well as the signed return (see, for example, Benston and Hagerman, 1974, MacKinlay and Ramaswamy, 1988, and Hasbrouck, 1991). As such, it is of interest to consider the extent to which our results are driven by return and volatility dynamics.

To address this issue, we perform a robustness check by including six lags of return and index volatility in the VAR system.[13] While we do not present the estimates from this exercise for brevity, and because the central goal of our paper is simply to analyze the bivariate relation between the efficacy of arbitrage and measures of liquidity, we find that our conclusions on commonality between the absolute basis and liquidity are largely unaltered even after accounting for volatility and returns. Specifically, the conclusions on all three aspects of the VAR, namely, innovation correlations, Granger causality results, and the impulse response functions, remain unchanged. Thus, volatility and returns alone do not capture the common dynamics of the absolute basis and liquidity. Our findings are consistent with the notion that changes in liquidity-related factors other than returns and volatility, such as shifts in market makers' ability to supply liquidity, and shifts in investors' liquidity needs, play an important role in the effectiveness of arbitrage activity.

With respect to the economic significance of the IRFs, a one-standard deviation shock from the three-month absolute basis impacts effective spreads and aggregates to an <u>annualized</u> extra trading cost of $2.7 million for a daily round-trip trade of one million shares in the basket of

[13] Index return and index volatility are obtained respectively as the residual and absolute value of the residual from a regression of the index return on the past twelve lagged returns and four dummies for days of the week (see Schwert, 1990, Jones, Kaul, and Lipson, 1994, and Chan and Fong, 2000).

NYSE-listed common stocks. Further, in the case of the nine-month basis measure, for an average stock price of $40 and a trade of one million shares, a one standard deviation shock to spreads results in an annualized cumulative divergence of $1 million between the futures and its cash value.[14] Thus, the economic relevance of both effects appears to be material.

[14]This calculation of economic significance is based on the six-day cumulative impulse response of one variable to a one-standard deviation shock in the other. In the first case (the response of liquidity to the absolute basis), taking the total incremental trading cost per million shares traded and multiplying by the number of trading days in an year (250) yields the dollar amount we report. In the second case, the cumulative impulse response over six days is multiplied by the dollar value of the trade, and then by 250, the approximate number of trading days in an year, to arrive at the reported number.

III. Conclusions

The assumption that no-arbitrage relations hold is taken virtually for granted in theoretical finance. However arbitrage is a costly human activity, and as such is impeded by market frictions. We investigate whether aggregate stock market illiquidity affects the efficacy of arbitrage.

We use three basis series consisting of contracts maturing within three, six- and nine-months, and two measures of liquidity, quoted and effective spreads, in our analysis. We find that the dynamics of the absolute futures-cash basis and liquidity are jointly determined. Contemporaneous innovations to the absolute basis and spreads are positively correlated. In addition, there is evidence of two-way Granger causality between the three-month absolute basis and stock liquidity, as measured by effective spreads. Impulse response analyses indicate that shocks to absolute bases are significantly informative in predicting stock spreads. Further, shocks to spreads are more informative in predicting shifts in the nine-month absolute basis than the shorter-term bases, suggesting that liquidity affects arbitrageurs more in the relatively less actively traded longer-term futures contracts.

Overall, our analysis indicates that the efficacy of arbitrage affects and is influenced by the liquidity of the stock market. If the degree to which markets satisfy the law of one price is taken to be a measure of the market's efficiency, then our results indicate that policy measures to improve a market's liquidity can have an important role in moving markets towards an efficient outcome.

In the reverse direction, deviations from the law of one price on a given day can help to predict market liquidity on subsequent days, since the actions of arbitrageurs materially affect liquidity. Hence, the absolute basis provides information to both retail and institutional investors that can be valuable in the forecasting and control of trading costs.

An extension of our analysis might consider the basis-liquidity relation for other markets such as fixed-income and foreign currency. It would also seem desirable to study the basis prior to news announcements (when the market might be particularly illiquid because of asymmetric information stemming from news leakage). Finally, it may be worthwhile to consider technological innovations in financial markets (such as electronic communication networks.) How such developments affect no-arbitrage relations by way of the liquidity channel would appear to be an interesting area for further research.

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Table 1 – Summary Statistics for Deviations from Arbitrage and Liquidity Measures

The summary statistics below pertain to the estimated absolute basis (in percent relative to the cash index value) for the three-month, six month, and nine-month NYSE Composite index futures contracts, and the (in dollars) for NYSE value-weighted quoted and effective spreads; daily data for 1988-2002 inclusive.

	Mean	Median	Standard Deviation
3-month absolute basis	0.186%	0.142%	0.173%
6-month absolute basis	0.298%	0.259%	0.224%
9-month absolute basis	0.425%	0.359%	0.307%
Quoted spread	\$0.164	\$0.175	\$0.053
Effective Spread	\$0.110	\$0.121	\$0.036

Table 2 –Absolute Basis Adjustment

In these OLS regressions, the dependent variable is the absolute basis for a NYSE Composite index futures contract. The time-period is 1988-2002 inclusive and the observations are daily. Dummy variables are included for Friday and for months of the year. RemTrm is the number of days until contract expiration, with 1 representing the last trading day. Holiday-1 is a dummy for the trading day prior to Thanksgiving, Christmas, or July 4. Time and Time**2 are orthogonalized linear and quadratic time trends. Rf-DYld is the difference between the estimated risk-free rate and the dividend yield. All coefficients are multiplied by 100.

	Maximum Maturity of Futures Contract (Months)					
	Three		Six		Nine	
Variable	Coefficient	T	Coefficient	T	Coefficient	T
Friday	0.007	1.02	-0.008	-0.94	-0.009	-0.77
January	-0.039	-2.88	-0.132	-7.95	-0.227	-10.35
February	-0.064	-4.65	-0.155	-9.18	-0.244	-10.92
March	-0.048	-3.63	-0.145	-8.86	-0.247	-11.42
April	-0.017	-1.28	-0.088	-5.29	-0.169	-7.71
May	-0.061	-4.56	-0.129	-7.83	-0.204	-9.37
June	-0.021	-1.57	-0.074	-4.51	-0.120	-5.50
July	-0.029	-2.17	-0.045	-2.74	-0.049	-2.26
August	-0.067	-5.03	-0.094	-5.79	-0.137	-6.34
September	-0.014	-1.08	-0.027	-1.60	-0.034	-1.52
October	0.019	1.45	0.034	2.08	0.051	2.35
November	-0.053	-3.96	-0.047	-2.81	-0.022	-1.02
RemTrm=1	-0.018	-1.17				
RemTrm=2	-0.031	-6.62				
RemTrm=3	0.051	8.51				
RemTrm=4	1.946	10.05				
Holiday-1	-0.098	-4.44	-0.008	-0.40	-0.014	-0.54
Time	-0.065	-2.97	-0.056	-9.60	-0.055	-7.10
Time**2	-0.076	-3.46	0.026	3.47	-0.022	-2.26
Rf-DYld	-0.059	-2.66	4.239	17.59	7.276	22.76
Intercept	0.182	17.27	0.281	21.89	0.376	21.98
Adjusted R^2	0.081		0.147		0.206	

Table 3 – Spread Adjustment

In these OLS regressions, the dependent variables are the daily NYSE value-weighted quoted and effective spreads from 1988-2002, daily observations. Dummy variables are included for days of the week and months of the year. Holiday-1 denotes the trading day prior to Thanksgiving, Christmas, or July 4. Post16 and postdeci denote periods following a reduction in the minimum tick size to 1/16 (on June 24, 1997) and a further reduction to \$.01 (on January 29, 2002). Time and time**2 are orthogonalized linear and quadratic time trends. All coefficients are multiplied by 100.

	Quoted spread		Effective spread	
Variable	Coefficient	T	Coefficient	T
Monday	-0.176	-3.16	-0.064	-1.76
Tuesday	-0.186	-3.41	-0.097	-2.70
Wednesday	-0.116	-2.13	-0.080	-2.24
Thursday	-0.072	-1.32	-0.071	-1.99
February	-0.031	-0.36	0.046	0.82
March	-0.223	-2.67	-0.119	-2.17
April	-0.168	-1.98	-0.007	-0.12
May	-0.678	-8.05	-0.294	-5.30
June	-0.906	-10.78	-0.407	-7.37
July	-0.840	-9.94	-0.422	-7.59
August	-0.898	-10.78	-0.423	-7.72
September	-1.086	-12.70	-0.578	-10.27
October	-0.583	-6.98	-0.225	-4.11
November	-0.931	-10.94	-0.535	-9.57
December	-0.850	-10.03	-0.464	-8.33
Holiday-1	0.055	0.56	0.026	0.41
post16th	-2.574	-29.32	-2.181	-37.77
postdeci	-6.459	-64.80	-4.441	-67.73
time	-4.427	-50.32	-2.835	-48.98
(time**2)	0.172	2.43	0.633	13.56
Intercept	18.85	239.2	12.68	244.6
Adjusted R^2	0.960		0.963	

Table 4 – Correlation Matrix for Adjusted Absolute Bases and Liquidity Measures

Correlations are presented between each measure of the adjusted absolute basis (ABASx, where x represents the basis horizon in months) and adjusted quoted and effective spreads (QSPR and ESPR respectively). The adjusted series are residuals from the regressions Tables 2 and 3.

	ABAS3	**ABAS6**	**ABAS9**	**QSPR**
ABAS6	0.725			
ABAS9	0.491	0.878		
QSPR	0.113	0.046	0.022	
ESPR	0.193	0.173	0.119	0.621

Table 5 – OLS Regressions

Regression results are presented for each measure of the adjusted absolute basis (ABASx, where x represents the basis horizon in months) on one lag of adjusted quoted and effective spreads (QSPR and ESPR respectively), and vice versa. The adjusted series are residuals from the regressions Tables 2 and 3. In Panel A, the coefficients are multiplied by 100.

Panel A: Absolute bases as dependent variables

	Independent variable			
	Lag(QSPR)		Lag(ESPR)	
	Coefficient	T	Coefficient	T
ABAS3	0.764	3.01	2.379	6.18
ABAS6	0.263	0.83	3.770	7.85
ABAS9	0.014	0.03	3.342	5.24

Panel B: Liquidity measures as dependent variables

	Independent variable					
	Lag(ABAS3)		Lag(ABAS6)		Lag(ABAS9)	
	Coefficient	T	Coefficient	T	Coefficient	T
QSPR	0.752	7.29	0.232	2.80	0.033	0.52
ESPR	0.843	7.60	0.587	10.93	0.237	5.78

Table 6 - Vector Autoregressions

Vector autoregressions pair each measure of the adjusted absolute basis (ABASx, where x represents the basis horizon in months) with adjusted quoted and effective spreads (QSPR and ESPR respectively). The adjusted series are residuals from the regressions Tables 2 and 3. The table presents correlations in VAR innovations and chi-square statistics and p-values (in parentheses) of pairwise Granger Causality tests between the endogenous variables. Numbers in boldface denote statistical significance at the 5% level.

Panel A: Correlations between VAR innovations

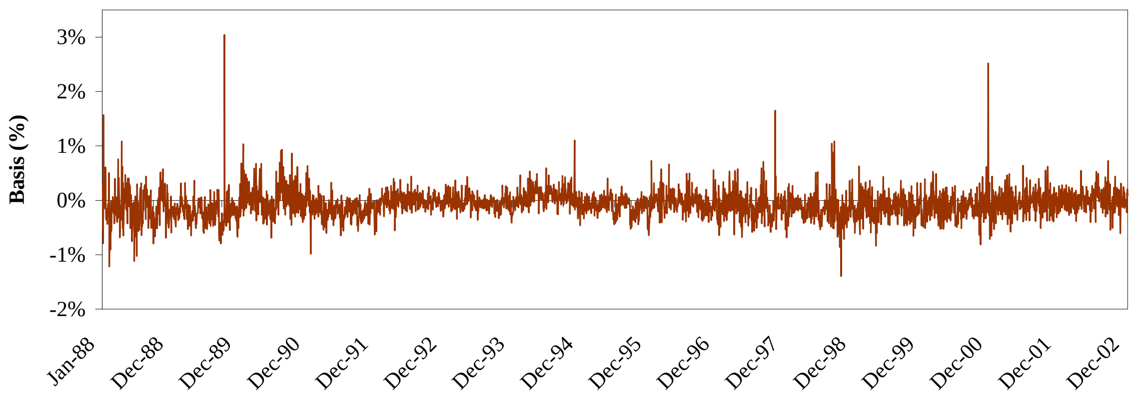
	ABAS3	ABAS6	ABAS9
QSPR	0.111	0.068	0.075
ESPR	0.138	0.080	0.106

Panel B: Granger Causality Tests
Null hypothesis: Row variable does not Granger-cause column variable

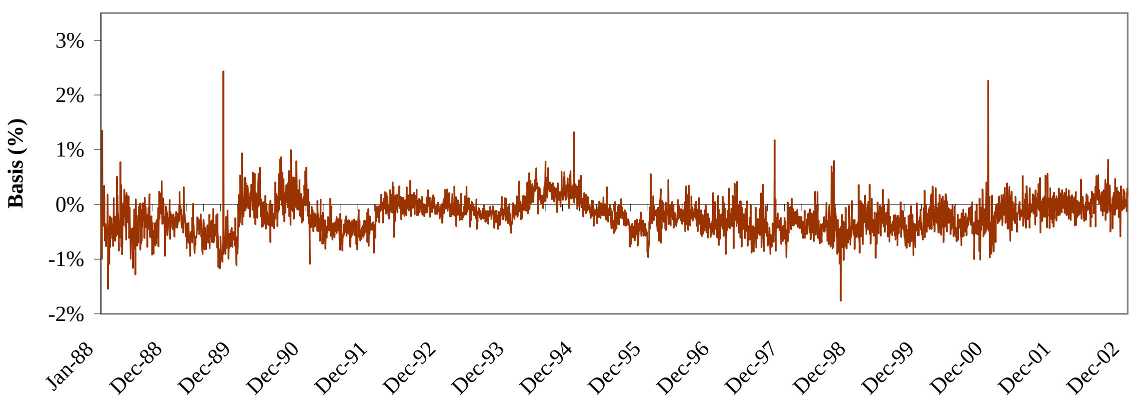
	ABAS3	ABAS6	ABAS9	QSPR	ESPR
ABAS3	-	-	-	32.94 (0.00)	45.55 (0.00)
ABAS6	-	-	-	9.892 (0.13)	17.62 (0.01)
ABAS9	-	-	-	8.68 (0.19)	9.10 (0.17)
QSPR	2.944 (0.82)	3.378 (0.76)	17.32 (0.01)	-	-
ESPR	15.74 (0.02)	5.292 (0.51)	28.05 (0.00)	-	-

Figure 1. NYSE Composite Index Futures Bases Over Time

Three-Month Basis



Six-Month Basis



Nine-Month Basis

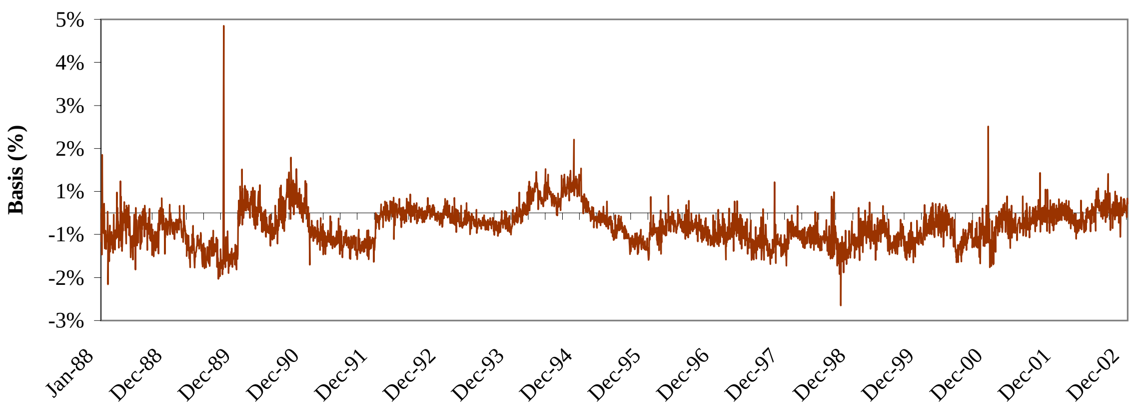


Figure 2. Liquidity Measures Over Time

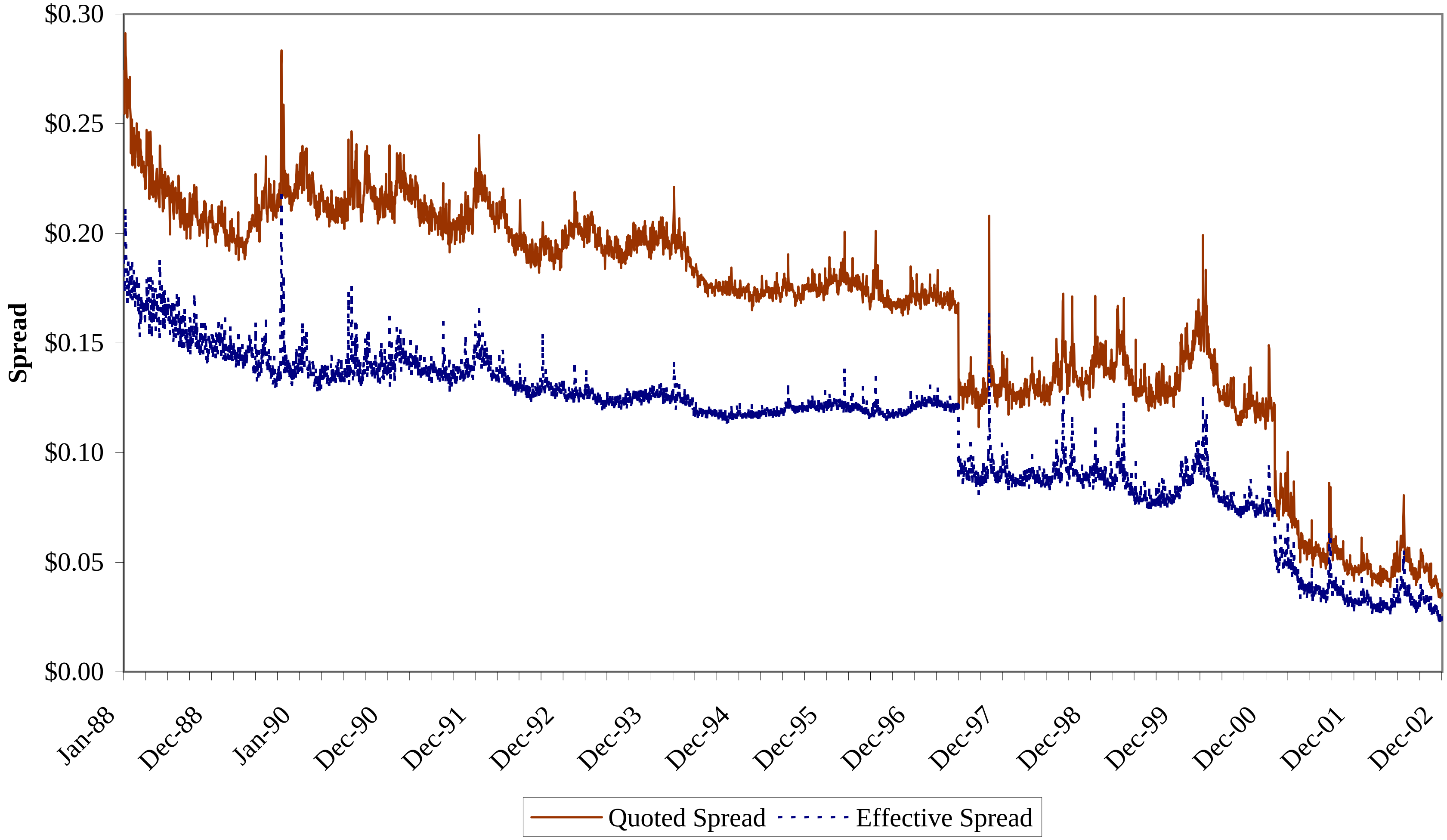


Figure 3 - Impulse response functions for the bivariate VAR with the adjusted three-month basis (ABAS3) and the adjusted quoted spread (QSPR) (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)

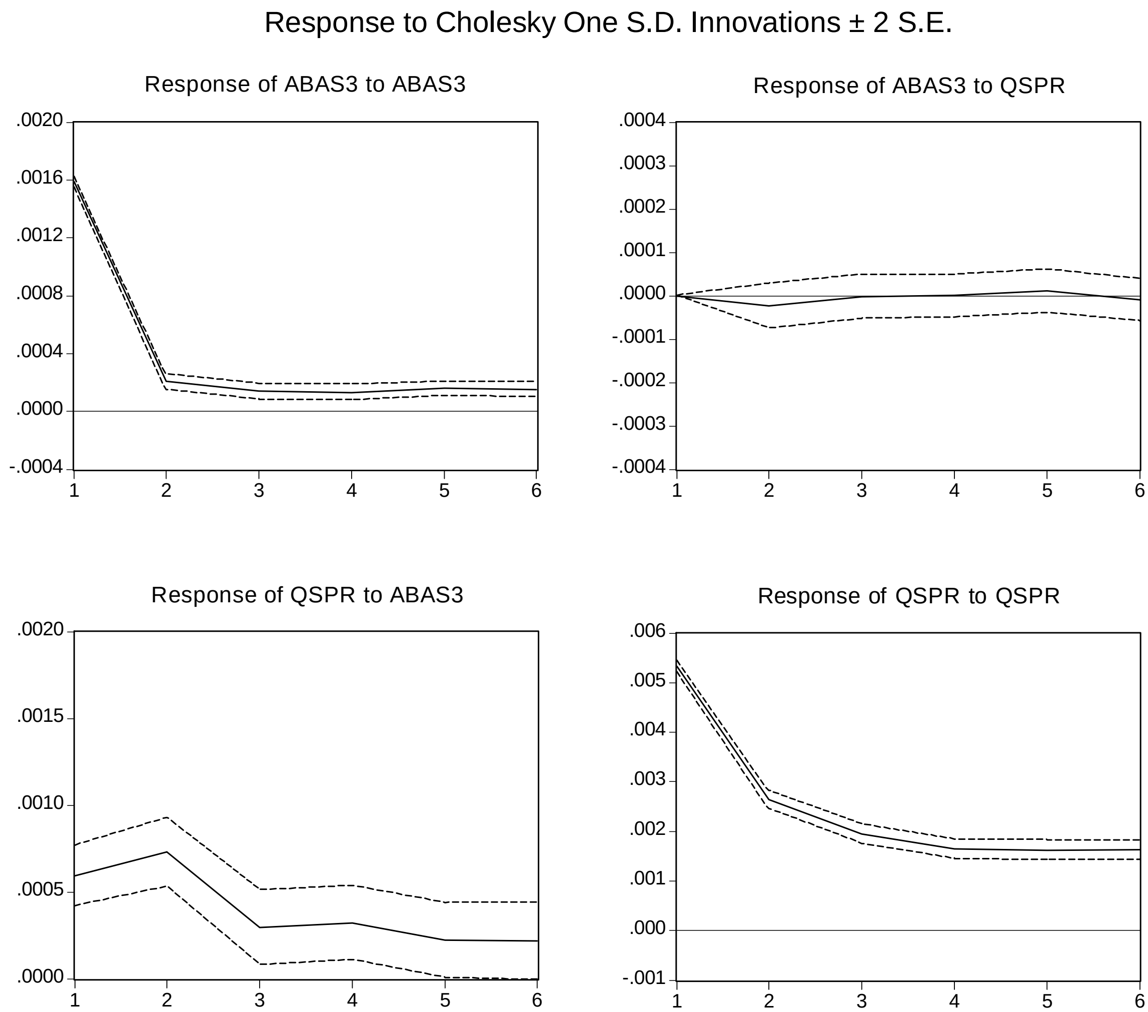


Figure 4 - Impulse response functions for the bivariate VAR with the adjusted three-month basis (ABAS3) and the adjusted effective spread (ESPR) (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)

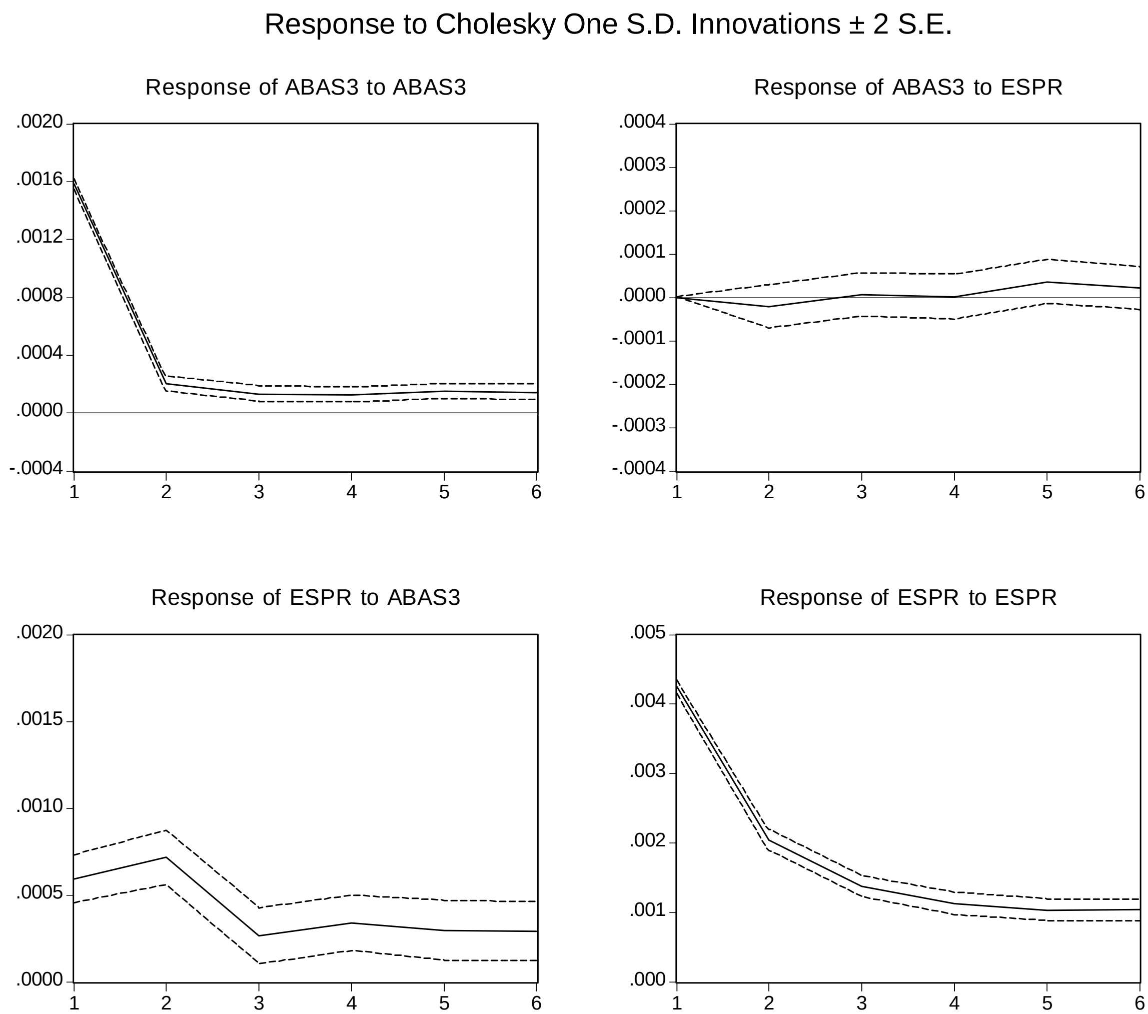


Figure 5 - Impulse response functions for the bivariate VAR with the six-month basis (ABAS6) and the quoted spread (QSPR (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)

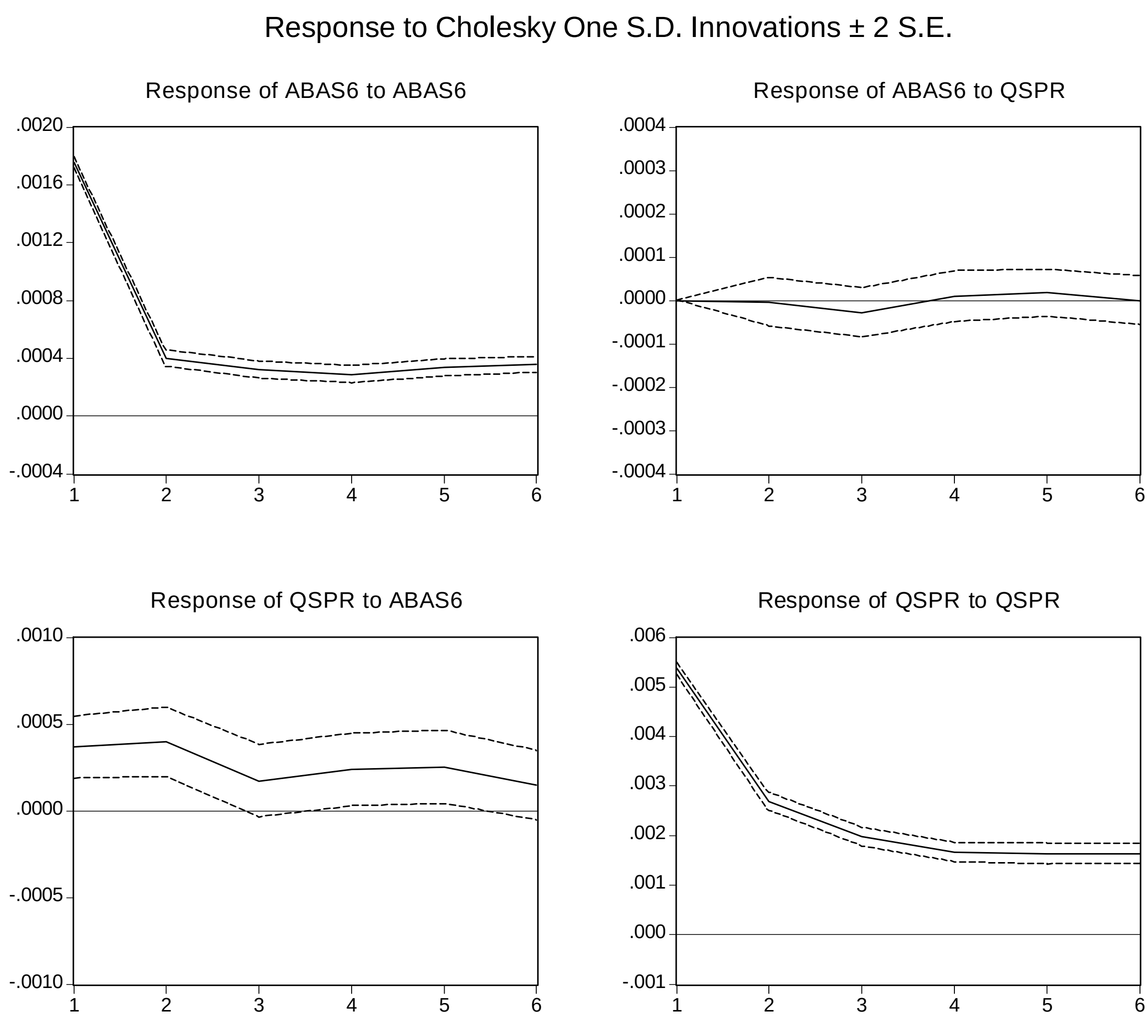


Figure 6 - Impulse response functions for the bivariate VAR with the adjusted six-month basis (ABAS6) and the adjusted effective spread (ESPR) (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)

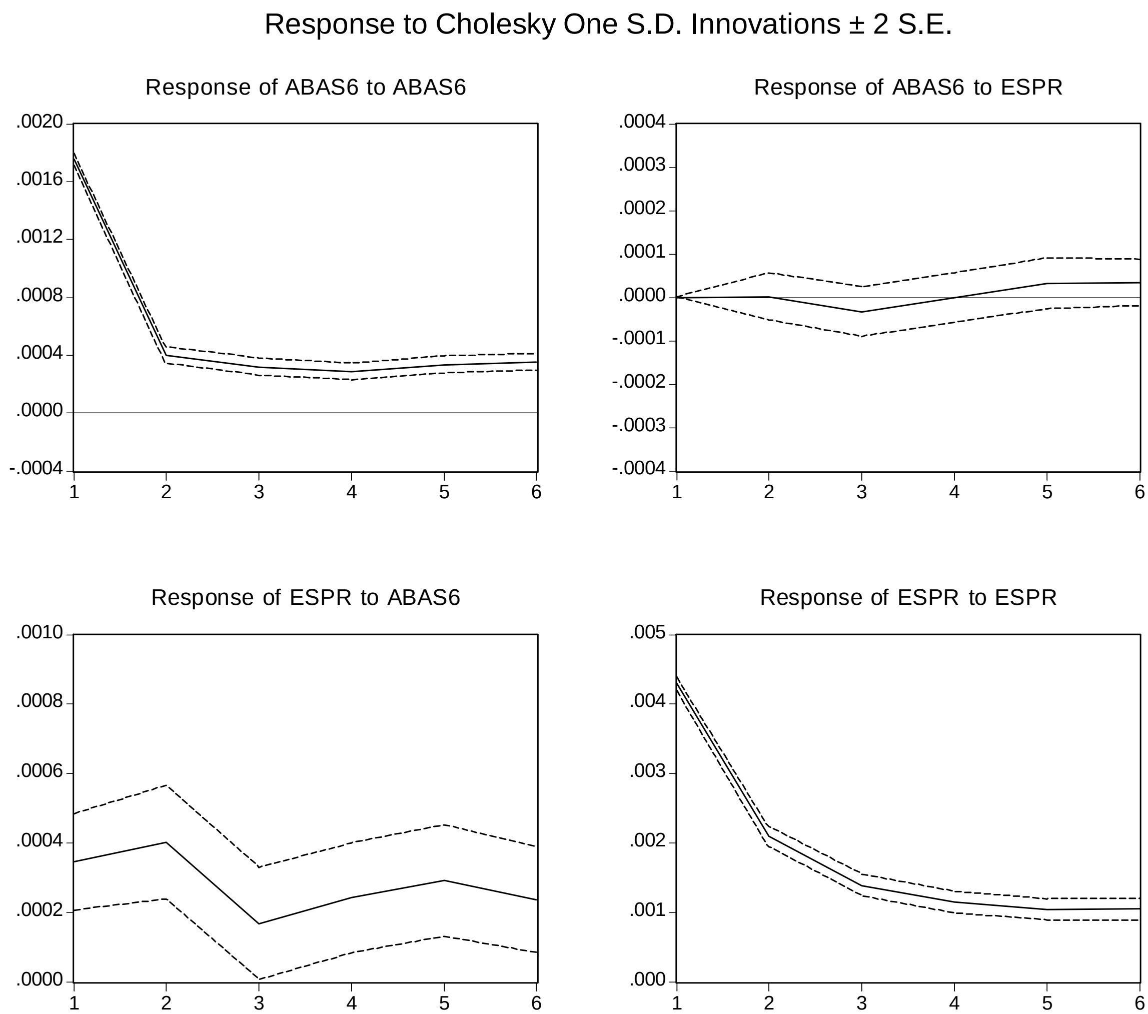


Figure 7 - Impulse response functions for the bivariate VAR with the adjusted nine-month basis (ABAS9) and the adjusted quoted spread (QSPR) (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)

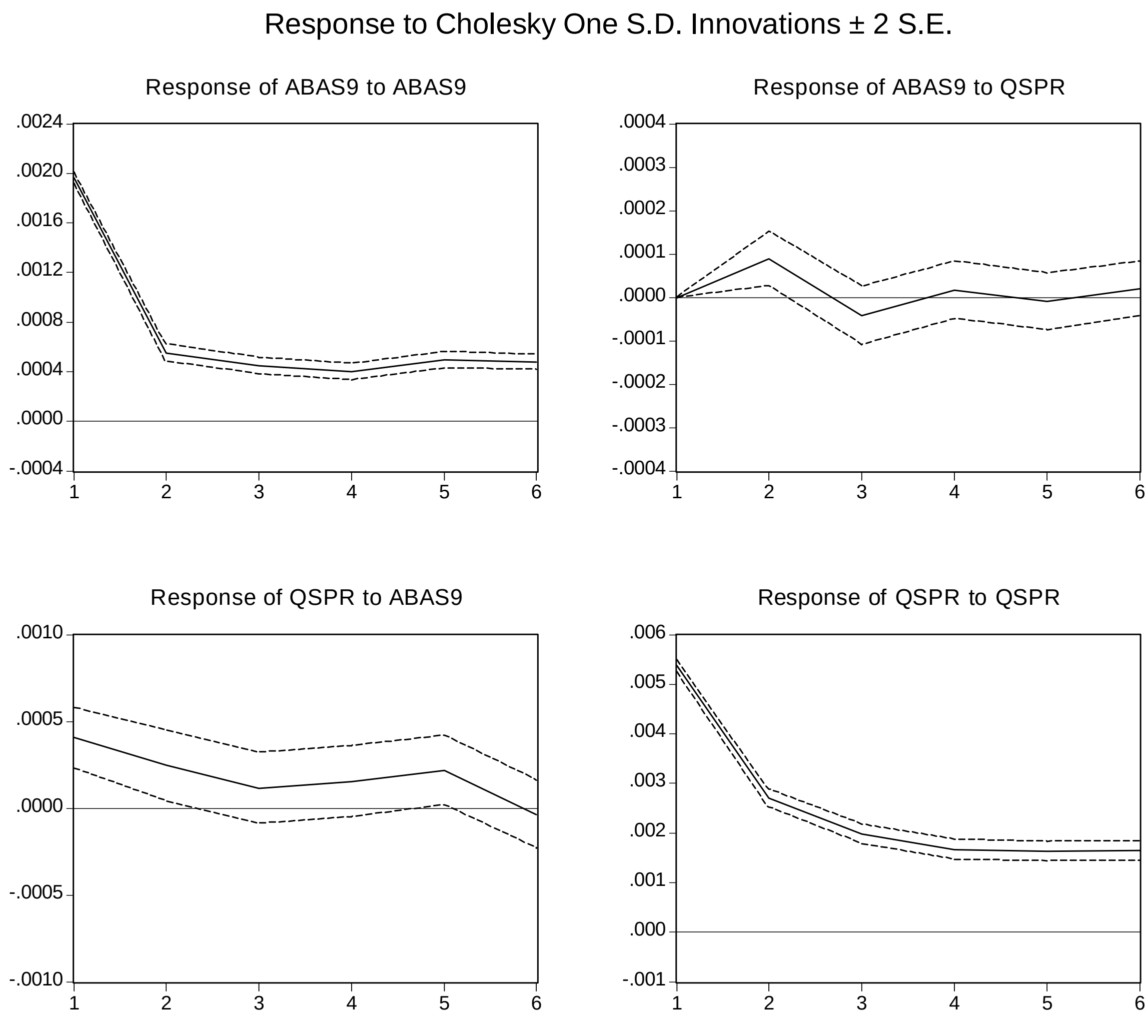


Figure 8 - Impulse response functions for the bivariate VAR with the adjusted nine-month basis (ABAS9) and the adjusted effective spread (ESPR) (responses on the vertical axis, time-period lags on the horizontal axis, starting with the contemporaneous period)