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Authors

Sweeney, Richard J.

Warga, Arthur D.

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AND DISCRIMINATING BETWEEN THE CAPM AND APT

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Richard J. Sweeney *

and

Arthur D. Warga * **

* Claremont Graduate School

and

** University of California, Los Angeles

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**ESTIMATING THE RISK PREMIUM ON THE MARKET,
AND DISCRIMINATING BETWEEN THE CAPM AND APT**

by

Richard J. Sweeney*

and

Arthur D. Warga**

*Claremont McKenna College and Claremont Graduate School

**Claremont Graduate School and University of California,
Los Angeles

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**ESTIMATING THE RISK PREMIUM ON THE MARKET,
AND DISCRIMINATING BETWEEN THE CAPM AND APT**

Section 1. Introduction

In estimating empirical versions of either the CAPM or the APT with previous methodologies, it is not possible to obtain estimates of the ex ante risk premium on the market (Black, Jensen and Scholes (1972); Fama and McBeth (1973); Sweeney and Warga (1983)). This paper shows that if the CAPM or APT considered includes the market plus other factors, and if the non-market factors are not orthogonal to the market, then an explicit point estimate of the premium on the market can be obtained. It is important to note at the outset that the consideration of models developed from linear generating relationships where one of the factors is the market actually covers many models of interest. For example, generalized versions of the CAPM, such as Merton's (1973) intertemporal model, include the market as one of the factors. The technique used in this paper relies on comparing estimates of the risk premia on non-market factors both when these factors are purged and not purged of the market's influence.

It turns out that obtaining estimates in these two ways is also important for discriminating between an APT

in which all factors are priced versus a CAPM with a multi-factor generating relationship in which only the market factor is priced. In a CAPM, as long as the non-market factors are correlated with the market, these non-market factors will have measured APT risk premia; these risk premia, however, simply reflect the correlation of the unpriced non-market factors with the market. Indeed, the expected value of such a measured premium ($P\Delta_j$) due to the j^{th} factor is equal to the expected premium on the market ($ER_M - ER_Z$) times the slope coefficient of the j^{th} factor regressed on the market (γ_j), or $P\Delta_j = (ER_M - ER_Z)\gamma_j$.

Alternatively, suppose the APT is true and the j^{th} factor is priced on its own and not just through correlation with the market. Then, we can obtain estimates of the premium ($P\Delta'_j$) on the j^{th} factor and the premium ($P\Delta''_j$) on the purged (of the market's influence) j^{th} factor. As will be shown below, the two premia are related by $P\Delta'_j = P\Delta''_j + \gamma_j(ER_M - ER_Z)$; thus an estimate of the premium on the market, $ER_M - ER_Z$, is obtainable.

Elton, Gruber and Rentzler (EGR, 1983) discuss a real asset pricing model, where the inflation-adjusted return on asset i is generated by two factors, the real return on the market and actual inflation. Despite its two-factor generating structure, pricing in the model will be as given in a conventional single-factor CAPM if actors value each

asset according to the covariance of its real return with the market's real return, relative to the market's variance. EGR show that this CAPM pricing implies that inflation can also be priced. They do not make clear, however, that pricing of inflation occurs in their CAPM just in case the market's real return and inflation are correlated. Nor do they derive the exact premium on inflation, as we do below and discussed briefly above. Our results are more general than EGR's in that we derive them for second factors other than inflation, and indeed for any number of non-market factors. Further, we show that if the CAPM is true, the ex ante premium on any factor j is $\gamma_j (ER_M - ER_Z)$.

This paper is organized as follows. Section 2 shows how the betas in the generating relationship affect pricing, or the expected return on asset i , ER_i . A key distinction is whether non-market factors are correlated with the market. When they are orthogonal, ER_i depends in the CAPM on the market beta and no other beta; in the APT, ER_i may, but need not, depend on non-market betas. When the market and the other factors are correlated, the beta in the CAPM on which pricing depends, or the pricing beta, is a composite of the market and all the other betas in the generating function; the CAPM pricing and generating betas on the market are different when non-market factors

are correlated with the market. The non-market generating betas enter the pricing beta with weights equal to the slope coefficient γ_j defined above. In the APT, the influence of non-market factors on ER_i depends both on $\gamma_j (ER_M - ER_Z)$ and some (possibly) non-zero risk premium.

Section 3 shows how estimates of the non-market risk premia can be used either to discriminate between the CAPM and APT, or to provide estimates of the premium on the market. Section 4 presents estimates based on empirical work in Sweeney and Warga (1983). They found a significant second factor, changes in long-term government bond yields, in pricing of electric utilities stocks. This interest-rate factor is empirically correlated with the market. However, if it is hypothesized that the true model is a CAPM and that the estimated premium on the second factor is due to its correlation with the market, the implied risk premium on the market is 48%. This seems implausibly high, so we reject the single factor CAPM. In the APT framework, the implied estimate of the risk premium on the market is 26%. This is somewhat higher than expected; it can be taken as casting some doubt on the APT specification used, or alternatively as suggesting that the risk premium on the market is somewhat higher than the 10% range that some have found intuitively plausible. Ibbotson and Sinquefeld (1977) estimated a

risk premium of 9.2% per year using 51 years of monthly data on returns on the market and T-bills; Merton (1980) derived a range of estimates of 8.2% to 12.04% for a period over which the average excess return on the New York Stock Exchange was 8.15% per year. We take our estimates as lending some weight to the upper end of the range that has been considered plausible.

Section 2. The Role of Generating Betas in Pricing

Suppose that returns are generated by a two-factor model (this is trivially generalized to k factors),

$$(1) \quad R_i = E(R_i) + \beta_{iM}(R_M - ER_M) + \beta_{i\Delta}(\Delta - E(\Delta)) + \varepsilon_i$$

where R_i is the nominal^{1/} return on asset i ($i = 1, \dots, n$), $E(\cdot)$ is the expectation operator, R_M the nominal return on the market, Δ the second factor, β_{iM} and $\beta_{i\Delta}$ are asset i 's generating betas on the two factors, and ε is a random term, $E(\varepsilon) = 0$, $\text{Cov}(\varepsilon, R_M) = \text{Cov}(\varepsilon, \Delta) = 0$.

Pricing in the CAPM

Issues of pricing involve ER_i but not the form of (1). In particular it is well-known that it is perfectly consistent with the CAPM for there to be multiple generating factors and betas.

The CAPM implication is

$$(2) \quad ER_i = ER_Z + [\text{cov}(R_i, R_M) / \text{var}(R_M)] (ER_M - ER_Z),$$

where cov and var are the covariance and variance operators, and R_Z is the return on the minimum-variance portfolio orthogonal to R_M (Black (1972)). Suppose that R_M and Δ are related by

$$(3) \quad \Delta = \delta + \gamma R_M + e, \quad Ee = 0, \quad \text{cov}(R_M, e) = 0.$$

Then, a straightforward application of the effect on a model coefficient when a variable has been omitted yields

$$(4) \quad \text{cov}(R_M, R_i) / \text{var}(R_M) = \beta_{iM} + \beta_{i\Delta} \gamma = \beta_{iM}^*$$

and (2) becomes

$$(5) \quad ER_i = ER_Z \beta_{iM} (ER_M - ER_Z) + \beta_{i\Delta} \gamma (ER_M - ER_Z).$$

Hence, looking cross sectionally, an increase in $\beta_{i\Delta}$ raises ER_i by $\gamma(ER_M - ER_Z)$. There is the seeming paradox, then, that Δ is priced in the CAPM, even though only the market is priced in the CAPM. The paradox is resolved by noting that the only reason Δ is priced is because of its

correlation with R_M ; an increase in $\beta_{i\Delta}$ makes R_i more sensitive to that component of Δ that is correlated with R_M , hence raises $\text{cov}(R_i, R_M)$ and hence raises ER_i .

For a different light on pricing, use (1), (3) and (4) to write

$$\begin{aligned} (6) \quad R_i &= ER_i + (\beta_{iM} + \beta_{i\Delta}\gamma)(R_M - ER_M) + \beta_{i\Delta}e + \varepsilon_i \\ &= ER_i + \beta_{iM}^*(R_M - ER_M) + \varepsilon'_i. \end{aligned}$$

where

$$\beta_{iM}^* = \beta_{iM} + \beta_{i\Delta}\gamma,$$

$$\varepsilon'_i = \beta_{i\Delta}e + \varepsilon_i,$$

$$\text{cov}(R_M, \varepsilon'_i) = 0.$$

Thus, even when R_M and Δ are correlated, there is always a reformulation of the generating relationship that expresses R_i as depending only on the single factor R_M and an error term ε'_i that is orthogonal to R_M .^{2/} The slope coefficient in this relationship (6) is β_{iM}^* , the beta that is used in pricing,

$$(7) \quad ER_i = ER_Z + \beta_{iM}^*(ER_M - ER_Z).$$

Thus, in the CAPM we can distinguish between generating betas and pricing betas. The generating betas on non-market factors enter the pricing beta only to the extent of the fortuitous correlation as measured by γ .

Pricing in the APT.

Assuming that Ross's (1976) pricing approximations hold exactly, the APT implies from the generating relationship (1) that

$$(8) \quad ER_i = a_0 + a_1 \beta_{iM} + a_2 \beta_{i\Delta} \quad i = 1, n$$

where the a 's are constants. From Ross (1976), we know $a_0 = ER_Z$ where R_Z is now the return on a portfolio orthogonal to both R_M and Δ . Further,

$$a_1 = ER_M - ER_Z.$$

Finally, call the premium on the second factor $P_{\Delta'}$, so

$$a_2 = P_{\Delta'}.$$

The APT makes virtually no prediction about $P_{\Delta'}$. $P_{\Delta'}$ may have any sign. Indeed, people may not care about Δ , so $P_{\Delta'}$ may be zero.

The CAPM may be thought of as a nested version of the APT, where the CAPM restricts a_2 in (8) to

$$(9) \quad a_2 = \gamma(ER_M - ER_Z).$$

This discussion suggests two issues. First, if a non-market factor seems to be priced, how can one decide whether this is in an APT or a multi-factor CAPM. As seen, this question comes to, how can you decide whether (9) holds or not? Secondly, (9) suggests that an estimate of $(ER_M - ER_Z)$, the premium on the market, can be obtained since both a_2 and γ can be estimated. Section 3 discusses these questions, in order, and Section 4 presents some estimates.

Section 3. Discriminating Between the CAPM and APT;
Estimating the Premium on the Market

Two means of discriminating suggest themselves. First, based on (9), one could obtain consistent estimates of a_2 and γ , and then ask whether

$$(10) \quad a_2 \approx \hat{\gamma}(ER_M - ER_Z).$$

The problem is, of course, that the market premium $ER_M - ER_Z$ is not known. However, if $a_2/\hat{\gamma}$ is negative, or is

positive but implies an annual percentage return greater than say 30% or less than 1%, then the CAPM restriction in (9) seems implausible. Clearly, one can find a point estimate of $(ER_M - ER_Z)$ as

$$(ER_M - ER_Z) = \frac{\hat{a}_2}{\hat{\gamma}} = \frac{\hat{P}_\Delta'}{\hat{\gamma}}.$$

Secondly, based on (6) one could estimate the purged version of the model

$$(11) R_i = ER_i + \beta_{iM}^* (R_M - ER_M) + \beta_{i\Delta} e + \epsilon_i.$$

The APT associated with (11) implies

$$a_0 = ER_Z, \quad a_1 = ER_M - ER_Z, \quad a_2 = P_\Delta'',$$

where P_Δ'' is the premium on the purged second factor, while the CAPM implies

$$a_0 = ER_Z, \quad a_1 = ER_M - ER_Z, \quad a_2 = 0.$$

Hence, discrimination between the CAPM and APT is possible by testing whether $\hat{a}_2 \neq 0$.

Insight is gained by considering purged and unpurged versions of the model, (11) and (1), which give

$$(12) \quad ER_i = ER_z + (ER_M - ER_z) \beta_{iM}^* + P\Delta'' \beta_{i\Delta}$$

$$(13) \quad ER_i = ER_z + (ER_M - ER_z) \beta_{iM} + P\Delta' \beta_{i\Delta}$$

respectively. Since purging Δ of the influence of R_M does not effect ER_i , ER_z or $(ER_M - ER_z)$, combining (12) and (13) gives

$$(14) \quad (ER_M - ER_z) (\beta_{iM}^* - \beta_{iM}) = (P\Delta' - P\Delta'') \beta_{i\Delta};$$

using

$$(15) \quad \beta_{iM}^* = \beta_{iM} + \beta_{i\Delta} \gamma$$

with (14) gives

$$(16) \quad P\Delta'' = P\Delta' - \gamma (ER_M - ER_z).$$

(16) says that the premium on the purged factor equals the premium on the unpurged factor and an adjustment to reflect the extent to which the unpurged factor reflects correlation with the market and hence bears the market

premium. Since the CAPM implies $P\Delta'' = 0$, again discrimination between the CAPM and APT comes to whether $P\Delta'$ simply is "close to" $\gamma(ER_M - ER_Z)$.

Alternatively, from (16) an estimate of the premium on the market in the APT can be formed as

$$(17) \quad (ER_M - ER_Z) = \frac{\hat{P}\Delta' - \hat{P}\Delta''}{\hat{\gamma}}$$

As an obvious generalization, if there are k non-market factors, k estimates of the market premium can be found as

$$(ER_M - ER_Z)_j = \frac{P\Delta'_j - P\Delta''_j}{\hat{\gamma}_j} \quad j = 1, \dots, k$$

Section 4. Estimates of the Risk Premium on the Market;

Tests of the APT v. CAPM.

Sweeney and Warga (1983) estimate a monthly two-factor model, where nominal returns on the i^{th} asset (R_i) depend on the nominal return on the market (R_M) and changes in an index of yields on long-term government bonds (DFY \equiv first difference of FYGT20 from the NBER Database). When DFY is estimated by OLS as a function of R_M , $\hat{\gamma}$ is highly significant for each period save the 60 months of 1960-64, as Table 1 shows. Consequently, estimates of the premium on DFY were obtained with DFY unpurged ($P\Delta'$) and purged ($P\Delta''$). In both cases, full-

information maximum likelihood estimates were obtained using LSQ in TSP (see Hall and Hall (1982)). Table 2 gives the estimates of $P_{\Delta'}$ and $P_{\Delta''}$ for three different groups of twenty-five electric utilities.

Sweeney and Warga (1983) report that most of the individual New York Stock Exchange firms with significant betas on DFY were in the electric and gas utility SIC listings (firms with two-digit primary SIC code 49). Consequently, they examined in detail 75 gas and electric utilities from industry 49. They split these alphabetically into 3 portfolios, A-C, with the first twenty-five firms in A, the second twenty-five in B, and the last twenty-five in C. Cross section-time series results for the individual betas and the cross-equation parameters were then found. Since virtually all of the $\hat{\beta}_{i\Delta}$ were negative, a negative value of $\hat{P}_{\Delta'}$ (or $\hat{P}_{\Delta''}$) was expected in order to give a positive effect of interest-rate risk ($\beta_{i\Delta}P_{\Delta'}$ or $\beta_{i\Delta}P_{\Delta''}$) on ER_i . In Table 2, the unpurged $\hat{P}_{\Delta'}$ seem on average to be significant, particularly for the three longer periods. However, the purged $\hat{P}_{\Delta''}$ are less significant, though mostly with the same negative sign as $\hat{P}_{\Delta'}$. One might be tempted to argue that the second factor is not priced in the APT sense but only in the CAPM sense of the factor being correlated with the market. Deriving the implied market premia, and

associated tests, helps discriminate between the CAPM and APT.

The CAPM.

Table 3 gives the implicit estimates of the return on the market, under the assumption that the CAPM is true and that $\hat{P}\Delta'$ arises from correlation of DFY and R_M . For example, portfolio A for the entire twenty year-period 1960-79 (240 months) had an estimate of $P\Delta'$ of $.5988 \times 10^{-4}$. From Table 1, $\hat{\gamma} = -.11817 \times 10^{-2}$. Thus, for A, $\hat{P}\Delta'/\hat{\gamma} = 5.006 \times 10^{-2}$ in decimal form,^{3/} or as a percentage/month is 5.066. This implies, when multiplied by 12, an estimated risk premium of 60.29% per year. The lowest for portfolio B, is 33.7% per year, and the average for the three portfolios is 48.14%.

Similar estimates for the ten-year periods 1960-69, 1970-79, and the five year periods 1960-64, 1965-1969, 1970-1974, and 1975-1979 are also reported in Table 3. This is done with the actual estimate of $\hat{\gamma}$ in the periods, and alternatively with the $\hat{\gamma}$ for the whole period. Thus, for A for 1960-69, the estimate of the premium on the market is 44.02% using that period's $\hat{\gamma}$, but only 23.87% using the overall period's.

On an intuitive basis, the implicit estimates of $(ER_M - ER_z)$ appear far too large on average. Both indirect and

intuitive estimates of the market premium tend to place it at around 10% per year. Thus, it appears that the hypothesis that the measured $\hat{P}\Delta'$ is due to correlation of DFY with R_M , with the true model a CAPM, is unsustainable.

More formal tests can be done using the statistic P which is conditional on a maintained hypothesis about the value of $(ER_M - ER_Z)$, where $P = \hat{P}\Delta' - (ER_M - ER_Z)\hat{\gamma}$. Suppose we hypothesize that $(ER_M - ER_Z) = .1$ per year (that is, 10% per year in percentage terms) or .008333 per month. Then, for portfolio A for 1960-79, from Table 2 $\hat{P}\Delta' = -.5988 \times 10^{-4}$ and from Table 1, $\hat{\gamma}$ for 1960-79 is $-.118 \times 10^{-2}$, giving $P = (-.5988 \times 10^{-4}) - .008333 (-.118 \times 10^{-2}) = (-.5988 \times 10^{-4}) + (.8333 \times .118 \times 10^{-4}) = -.5005 \times 10^{-4}$. If the CAPM holds

$$E(\hat{P}) = E(\hat{P}\Delta') - (ER_M - ER_Z)E(\hat{\gamma}) = 0,$$

since the CAPM implies

$$E(\hat{P}\Delta') = (ER_M - ER_Z)E(\hat{\gamma}).$$

Further,

$$\text{Var}(P) = \sigma_{\hat{P}\Delta'}^2 + (ER_M - ER_Z)^2 \sigma_{\hat{\gamma}}^2 - 2(ER_M - ER_Z) \text{cov}(\hat{P}\Delta', \hat{\gamma}).$$

$\hat{P}\Delta'$ and $\hat{\gamma}$ are positively correlated.^{4/} For A for 1960-79, $P = .5005 \times 10^{-4}$ and the standard error of P is $.2815 \times 10^{-4}$.^{5/} Thus, the t-statistic on P is 1.75. Some t-statistics are less significant, a few others more significant.^{6/} However, it seems likely one will want to reject one or both of the two joint hypotheses: (a) the annualized risk premium is $(ER_M - ER_Z) = .1$, and (b) the CAPM^{7/} is true.

The APT.

Table 4 presents estimates of the premium on the market under the assumption that the APT is valid and $\hat{P}\Delta'$ arises partly from the correlation of R_M and DFY, but also partly from an underlying premium $P\Delta''$. A consistent estimate of $P\Delta''$ is obtained by first purging DFY of the influence of R_M through OLS, obtaining the residuals \hat{e} , and then rerunning the LSQ equations with \hat{e} substituted for DFY. Table 4 then uses the difference of the two estimates of $P\Delta$, $\hat{P}\Delta' - \hat{P}\Delta''$, divided by the same $\hat{\gamma}$ as in Table 3, to estimate the premium on the market. Clearly, this estimate will be smaller (larger) than in Table 3 if $\hat{P}\Delta''$ is negative (positive); in nearly three quarters of the cases, $\hat{P}\Delta''$ is negative. In any case where $\hat{P}\Delta'' < 0$, the APT estimate of $(ER_M - ER_Z)$ is smaller and hence more reasonable than the CAPM's; but since the CAPM implies $E\hat{P}\Delta''$

= 0, the APT's $(ER_M - ER_Z)$ should fluctuate randomly around the CAPM's if the CAPM is valid. Note that $P^{\Delta''} < 0$ is required if $\beta_{i\Delta} P^{\Delta''}$ is to be positive so that with the average negative $\beta_{i\Delta}$ for electric utilities, a positive return is paid for bearing their interest-rate risk.

For A in 1960-79, $\hat{P}^{\Delta'} = -.5988 \times 10^{-4}$ and $\hat{\gamma} = -.1182 \times 10^{-2}$, as before, and $\hat{P}^{\Delta''} = -.3293 \times 10^{-4}$. Thus, $(\hat{P}^{\Delta'} - \hat{P}^{\Delta''})/\hat{\gamma} = 2.28 \times 10^{-2}$ per month, or in percentage form 2.28/month and 27.36% per year. The average estimate for A-C is 26.29% per year. While this is quite large relative to the intuitive estimate of a premium of 10%, it is substantially smaller than the estimate of 48.14% per year under the CAPM.

As above we can form a statistic PP conditional on a maintained value of $(ER_M - ER_Z)$ where

$$PP = \hat{P}^{\Delta'} - \hat{P}^{\Delta''} - \hat{\gamma}(ER_M - ER_Z),$$

with $(ER_M - ER_Z)$ taken as a decimal per month. If the APT is true, $E(PP) = 0$. Assuming an annualized risk premium of .1, $(ER_M - ER_Z) = .008333/\text{month}$, and for A for 1960-79, $PP = (-.5988 + .3293) \times 10^{-4} + .008333 \times .1182 \times 10^{-2} = -.171 \times 10^{-4}$.

$$\begin{aligned} \text{Var}(PP) &= \sigma^2_{\hat{P}^{\Delta'}} + \sigma^2_{\hat{P}^{\Delta''}} + (ER_M - ER_Z)^2 \sigma^2_{\hat{\gamma}} \\ &\quad - 2\text{cov}(\hat{P}^{\Delta'}, \hat{P}^{\Delta''}) - 2(ER_M - ER_Z)\text{cov}(\hat{P}^{\Delta'} - \hat{P}^{\Delta''}, \hat{\gamma}). \end{aligned}$$

Estimates of the second cov() show it is near zero and can be neglected in calculating estimates of var PP. We have been unable to find estimates of the first cov(\cdot), but can present a range of estimates for $\hat{\sigma}_{PP}$.^{8/} If the correlation of \hat{P}_{Δ}' and \hat{P}_{Δ}'' is unity, the t-statistic on PP is -3.27 and clearly a market premium of 10% per year is implausible; raising $(ER_M - ER_Z)$ to 15% per year gives a t-statistic of -2.21, and raising the premium to 20% gives a t-statistic of -1.24. Thus, the $(ER_M - ER_Z)$ of 27.36% per year for A in 1960-79 is evidently imprecisely enough estimated that it is not significantly different from 20% and is hardly different from 15%. However, with the correlation between $\Delta\hat{P}'$ and $\Delta\hat{P}''$ taken as zero, a 10% premium on the market gives a t-statistic of -.46, while assuming a correlation of .5 gives a t-statistic of -.65. All told, the APT estimated by Sweeney and Warga (1983), seems consistent with a reasonable range for the premium on the market.

Section 5. Conclusions

This paper examined some important caveats for deciding on the pricing of factors in asset models based on linear generating relationships. While the results are generalizable to any APT, nominal or real, with any number of factors, we focused discussion on simple nominal two-factor models where one of the factors is the market

portfolio. Such models are, aside from their importance as existing equilibrium based models containing the market (e.g. Merton (1973)), of ongoing concern to many authors due to issues involved in finding evidence at odds with a simple CAPM (see Elton, Gruber, and Rentzler (1982)). As we demonstrate, market based models also allow point estimates of the market premium to be calculated.

Methodologies for estimating asset pricing models have been unable to provide estimates of the ex ante risk premium on the market. Further, when estimating multi-factor models, it is not easy to decide if priced factors other than the market are priced simply because of the factors' correlation with the market, as the CAPM would imply, or are still priced when account is taken of such correlation, as would be consistent with the APT. This paper discusses an approach that allows discrimination between the CAPM and APT. It also provides a spot estimate of the premium on the market under either the CAPM or APT; the estimate can then be tested for significance.

The analytics are illustrated with an example that seeks to establish the pricing of a factor believed to be proxying for unanticipated changes in expected inflation. The second factor appears to be priced. While we make no claim that the generating relationship, using the market

portfolio and the inflation factor, is completely specified, the results appear to constitute evidence at odds with the simpler versions of the CAPM. We are strengthened in this belief because of the apparent sensitivity to the inflation factor by only the regulated segment of the market; the results thus do not seem to be due simply to sampling variability. Further, there seems little reason to believe a more complete specification would eliminate the pricing of this second factor.

FOOTNOTES

¹The discussion is in terms of nominal returns solely for convenience, and all analytical points hold mutatis mutandis for real returns models.

²There is no claim to originality of this result.

³ $\hat{P}_{\Delta'}/\hat{\gamma}$ is a consistent estimator of P_{Δ}/γ when maximum likelihood estimates are employed as is the case here.

⁴The positive correlation between $\hat{P}_{\Delta'}$ and $\hat{\gamma}$ is evident when one views $\hat{P}_{\Delta'}$ as being from an iterative scheme of cross section regressions on OLS β 's for R_m and DFY (see Gibbons (1982) for a discussion of this). An increase in $\hat{\gamma}$ will result in a smaller market beta and larger DFY beta (i.e., as the DFY beta is negative this means both coefficients will move toward zero). The premium on DFY will grow (in magnitude) to offset this.

⁵Even though $\text{cov}(\hat{P}_{\Delta'}, \hat{\gamma}) > 0$ as footnote 4 argues, it turns out that $2(ER_M - ER_Z)\text{cov}(\hat{P}_{\Delta'}, \hat{\gamma})$ can be neglected in finding Var P. For portfolio A for the period 1960-79,

$$\text{Var} P \approx \sigma_{\hat{P}_{\Delta'}}^2 + (ER_M - ER_Z)^2 \sigma_{\hat{\gamma}}^2$$

can be calculated as follows. From Table 2, $\hat{P}_{\Delta'} = -.5988 \times 10^{-4}$ and has t-statistic of 2.1, so $\sigma_{\hat{P}_{\Delta'}} = (.5988 \times 10^{-4})/2.1 = .2851 \times 10^{-4}$, and $\sigma_{\hat{P}_{\Delta'}}^2 = 8.128 \times 10^{-2} \times 10^{-8}$. On a monthly basis, a 10% annual premium on the market is in decimal form .008333, and $(ER_M - ER_Z)^2 = 6.9444 \times 10^{-5}$. From Table 1, $\hat{\gamma} = -.11817 \times 10^{-2}$ and its t-statistic is 6.4, so $\sigma_{\hat{\gamma}} = (.11817 \times 10^{-2})/6.4 = 1.8464 \times 10^{-4}$, and $\sigma_{\hat{\gamma}}^2$

= 3.4092×10^{-8} . Hence, $\text{Var } P \approx$

$$10^{-8} \{(8.128 \times 10^{-2}) + (6.9444 \times 10^{-5})(3.4092)\}$$

and

$$\begin{aligned} \hat{\sigma}_P &= 10^{-4} \{(8.128) \times 10^{-2} + 23.675 \times 10^{-5}\}^{\frac{1}{2}} \\ &= 10^4 \{.08128 + .00023675\}^{\frac{1}{2}} \\ &= 10^{-4} \{.08152\}^{\frac{1}{2}} \\ &= 10^{-4} \times .2855. \end{aligned}$$

Including an estimate of $2(ER_M - ER_Z)\text{cov}(\hat{P}_\Delta', \hat{\gamma})$ does not much affect results. An estimate of the $\text{cov}(\cdot)$ is $.14339 \times 10^{-9}$. Multiplying by $2 \times .008333$ gives $.0023898 \times 10^{-9}$ which reduces the estimate of $\text{Var } P$ from $.8151 \times 10^{-9}$ to $.81217 \times 10^{-9}$ and reduces $\hat{\sigma}_P$ to $.28508 \times 10^{-4}$ from $.2855 \times 10^{-4}$. The new t-statistic is raised marginally to $-.5005/.2855 = -1.756$. The $\hat{c}\hat{o}v$ is found by estimating the equations for A as before but including the equation $DFY = \delta + \gamma R_M$.

⁶For groups B and C for the overall period, the P-statistics are $-.2335 \times 10^{-4}$ and $-.3930 \times 10^{-4}$, respectively, with t-statistics of 1.27 and 1.92. For 1970-79, groups A, B and C have the P and t values of $-.6901 \times 10^{-4}(1.93)$, $-.2581 \times 10^{-4}(1.09)$, $-.6035 \times 10^{-4}(2.24)$, while for 1960-69, the groups have $-.1366 \times 10^{-4}(1.16)$, $-.1048 \times 10^{-4}(.98)$, $-.1786 \times 10^{-4}(1.10)$. Clearly, group B and the earlier period provide the weakest results.

⁷As in all empirical applications of the CAPM, the actual hypothesis being tested is the ex ante mean-variance efficiency of the actual market proxy being employed (see Roll (1977)).

⁸Consider first the results when the correlation between \hat{P}_Δ' and \hat{P}_Δ'' is unity. Then,

$$\sigma_{\hat{P}_\Delta'} + \sigma_{\hat{P}_\Delta''}^2 - 2 \text{cov}(\hat{P}_\Delta', \hat{P}_\Delta'') = (\sigma_{\hat{P}_\Delta'} - \sigma_{\hat{P}_\Delta''})^2$$

With $\text{cov}((\hat{P}_\Delta' - \hat{P}_\Delta''), \hat{\gamma}) = 0$,

$$\text{Var}(PP) = (\sigma_{\hat{P}_\Delta'} - \sigma_{\hat{P}_\Delta''})^2 + (ER_M - ER_Z)^2 \sigma_{\hat{\gamma}}^2.$$

Footnote 5 showed $\hat{\sigma}_{P\Delta'} = .2851 \times 10^{-4}$; similarly, since $\hat{P}\Delta'' = -.3292 \times 10^{-4}$ and its t-statistic is 1.4 in Table 2, $\hat{\sigma}_{P\Delta''} = .2351 \times 10^{-4}$. Thus, $(\hat{\sigma}_{P\Delta'} - \hat{\sigma}_{P\Delta''})^2 = \{(.2851 - .2351) \times 10^{-4}\}^2 = (.05)^2 \times 10^{-8} = .0025 \times 10^{-8}$. Footnote 5 showed that with an annual premium on the market of 10%, $(E R_M - E R_Z)^2 \hat{\sigma}_Y^2 = .0002344 \times 10^{-8}$. Hence, $\text{Var}(PP) = 10^{-8} \{ .0025 + .0002344 \}$ and

$$\begin{aligned}\hat{\sigma}_{PP} &= 10^{-4} \{ .0027344 \}^{\frac{1}{2}} \\ &= 10^{-4} \times .05229.\end{aligned}$$

Since the text showed $PP = -.171 \times 10^{-4}$, the t-statistic is $-.1711 / .05229 = -3.27$.

If the premium on the market is 20% per year, in monthly decimal form $(E R_M - E R_Z) = .016667$, $(E R_M - E R_Z)^2 = .00027778$, $(E R_M - E R_Z)^2 \hat{\sigma}_Y^2 = .00027778 = .0009376$, and

$$\begin{aligned}\hat{\sigma}_{PP} &= 10^{-4} \{ .0025 + .0009376 \}^{\frac{1}{2}} \\ &= 10^{-4} \{ .003438 \}^{\frac{1}{2}} \\ &= 10^{-4} \times .0586\end{aligned}$$

PP is now

$$\begin{aligned} &(-.5988 + .3293) \times 10^{-4} + .016667 \times .1182 \times 10^{-2} \\ &= -.2695 \times 10^{-4} + .197 \times 10^{-4} \\ &= -.0725 \times 10^{-4}.\end{aligned}$$

Thus, the t-statistic is $-.0725 / .0586 = -1.24$.

If the market premium is 15% per year, $(E R_M - E R_Z) = .0125$, $(E R_M - E R_Z)^2 = .00015625$, and $(E R_M - E R_Z)^2 \hat{\sigma}_Y^2 = .0005274$, and

$$\begin{aligned}\text{Var}(PP) &= 10^{-4} \{ .0025 + .0005274 \}^{\frac{1}{2}} \\ &= 10^{-4} \{ .0030274 \}^{\frac{1}{2}} \\ &= 10^{-4} \times .05502.\end{aligned}$$

PP is now

$$\begin{aligned} &-.2695 \times 10^{-4} + .0125 \times .1182 \times 10^{-2} \\ &= -.2695 \times 10^{-4} + .14775 \times 10^{-4} \\ &= -.10^{-4} \times .12175.\end{aligned}$$

Thus, the t-statistic is $-.12175 / .05502 = -2.21$.

Alternatively, suppose $\text{cov}(\Delta P', \Delta P'') = 0$. Then, with an annual risk premium of .1,

$$\begin{aligned}\text{Var}(PP) &= \hat{\sigma}_{\Delta P'}^2 + \hat{\sigma}_{\Delta P''}^2 + (E(R_M) - E(R_Z))^2 \hat{\sigma}_Y^2 \\ &= (.0813 \times 10^{-8}) + (.0553 \times 10^{-8}) \\ &\quad + (.0002344 \times 10^{-8}) \\ &= .1368 \times 10^{-8},\end{aligned}$$

$$\hat{\sigma}_{PP} = .3699 \times 10^{-4}.$$

For $PP = -.171 \times 10^{-4}$, the t-statistic is $-.171/.3699 = -.46$.

Finally, if the correlation between $\Delta P'$ and $\Delta P''$ is .5, then $2\text{cov}(\Delta P', \Delta P'') = 2 \times .5 \times .2851 \times 10^{-4} \times .2351 \times 10^{-4} = .0670 \times 10^{-8}$, $\text{Var}(PP) = .1368 \times 10^{-8} - .0670 \times 10^{-8} = .0698 \times 10^{-8}$, and $\hat{\sigma}_{PP} = .2642 \times 10^{-4}$; this gives a t-statistic of $-.171/.2642 = -.65$.

TABLE 1*

$$DFY = \delta + \gamma R_m + e$$

	$\hat{\gamma}$	t-stat	Standard Error
1960-79	-.11817	-6.4	1.846x10 ⁻⁴
1960-69	-.06407	-3.12	2.054x10 ⁻⁴
1970-79	-.1485	-5.2	2.856x10 ⁻⁴
1960-64	-.0102	-.55	1.855x10 ⁻⁴
1965-69	-.11496	-3.3	3.484x10 ⁻⁴
1970-74	-.1541	-3.83	4.024x10 ⁻⁴
1975-79	-.1577	-3.75	3.087x10 ⁻⁴

*DFY = first difference of FYGT20 from the NBER database. R_m is the value weighted return on the market (including dividends) from the CRSP monthly returns file. All figures for $\hat{\gamma}$ are multiplied by 100.

TABLE 2*

Estimates of $P\Delta'$ and $P\Delta''$ (t-statistics in parentheses)

	A	B	C
1960-79	$P\Delta'$ -.5988(2.1)	-.3320(1.8)	-.4915(2.4)
	$P\Delta''$ -.3292(1.4)	-.1099(.65)	-.2063(1.2)
1960-69	$P\Delta'$ -.2351(2.0)	-.2033(1.9)	-.2771(1.7)
	$P\Delta''$ -.0245(.22)	.005(.05)	-.0466(.32)
1970-79	$P\Delta'$ -.7946(2.2)	-.3566(1.5)	-.7020(2.6)
	$P\Delta''$ -.4853(1.6)	-.0900(.4)	-.3585(1.6)
1960-64	$P\Delta'$ -.0571(.91)	-.081(1.2)	NC
	$P\Delta''$ -.0818(1.3)	-.1046(1.5)	NC
1965-69	$P\Delta'$ -.343(2.1)	-.2654(2.3)	-.535(3.1)
	$P\Delta''$.1335(.86)	.2129(1.8)	-.024(.2)
1970-74	$P\Delta'$ -.3488(1.3)	-.4894(2.3)	-.7071(2.3)
	$P\Delta''$ -.2800(1.2)	-.3438(1.6)	-.5152(1.9)
1975-79	$P\Delta'$ -.7837(3.3)	-.761(2.7)	.0205(0.1)
	$P\Delta''$ -.1677(.87)	-.1239(.54)	-.3924(2.1)

*Full information maximum-likelihood estimates obtained from the LSQ routine in TSP. $P\Delta'$ is for the unpurged series DFY, and $P\Delta''$ for the purged series DFY. Estimates have been multiplied by 104. NC denotes non-convergence of the algorithm.

TABLE 3*

CAPM IMPLIED PREMIA

$$\hat{P}\Delta' / \hat{\gamma} = (ER_M - ER_Z)$$

(figures are in percentage per year)

	A	B	C	Average
1969-79	60.79/ -	33.71/ -	49.9/ -	48.14/ -
1960-69	44.02/ 23.87	38.06/ 20.64	51.88/ 28.14	44.65/ 24.22
1970-79	64.21/ 80.65	28.82/ 36.20	56.72/ 71.24	49.92/ 62.70
1960-64	67.18/ 5.80	95.6/ 8.25	NC	81.4/ 7.0
1965-69	-35.8/ 34.8	27.77/ 26.94	55.8/ 54.3	39.8/ 38.7
1970-74	27.16/ 35.41	38.11/ 49.68	55.06/ 71.79	40.11/ 52.29
1975-79	59.63/ 79.56	57.9/ 77.3	NC	58.8/ 78.4

*Top figure uses $\hat{\gamma}$ calculated in the same period as $P\Delta'$. Bottom figure uses $\hat{\gamma}$ calculated from 1960-1979.

NC denotes non-convergence of the algorithm.

TABLE 4*

$$\frac{\hat{P}\Delta' - \hat{P}\Delta''}{\hat{\gamma}} = (ER_M - ER_Z)$$

	A	B	C	Average
1960-79	27.36/-	22.55/-	28.96/-	26.29/-
1960-69	39.43/21.38	39.0/21.15	43.08/23.36	40.5/22.0
1970-79	25.00/31.40	21.54/27.05	27.72/34.82	24.75/31.09
1960-64	-29/-2.5	-27.8/-2.4	NC	-28.4/-2.45
1965-69	49.7/48.4	49.91/48.56	53.3/51.9	51.0/49.6
1970-74	5.36/6.98	11.34/14.78	14.94/19.48	10.55/13.75
1975-79	46.87/62.54	48.5/64.7	NC	47.7/63.6

*Figures are in percent per year. Top figure uses $\hat{\gamma}$ calculated in the same period as $\hat{P}\Delta' - \hat{P}\Delta''$. Bottom figure uses $\hat{\gamma}$ calculated from 1960-79.

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