Monetary General Equilibrium with Transaction Costs

Ross M. Starr

University of California, San Diego
June 28, 2002

Abstract

Commodity money arises endogenously in a general equilibrium model with convex transaction cost technology and with separate budget constraints for each transaction. Transaction costs imply differing bid and ask (selling and buying) prices. The most liquid good --- with the smallest proportionate bid/ask spread --- becomes commodity money. General equilibrium may not be Pareto efficient, but if zero-transaction-cost money is available then the equilibrium allocation is Pareto efficient. Fiat money is an intrinsically worthless instrument. Its positive price comes from acceptability in paying taxes, and its use as a medium of exchange is based on low transaction cost.

Keywords: commodity money, fiat money, general equilibrium, transaction cost

JEL classification: C62, E40, E42

Monetary General Equilibrium with Transaction Costs*

Ross M. Starr

University of California, San Diego
June 28, 2002

Abstract

Commodity money arises endogenously in a general equilibrium model with convex transaction cost technology and with separate budget constraints for each transaction. Transaction costs imply differing bid and ask (selling and buying) prices. The most liquid good --- with the smallest proportionate bid/ask spread --- becomes commodity money. General equilibrium may not be Pareto efficient, but if zero-transaction-cost money is available then the equilibrium allocation is Pareto efficient. Fiat money is an intrinsically worthless instrument. Its positive price comes from acceptability in paying taxes, and its use as a medium of exchange is based on low transaction cost.

"[An] important and difficult question...[is] not answered by the approach taken here: the integration of money in the theory of value..."

------ Gerard Debreu, *Theory of Value* (1959)

I. Menger's "Origin of Money" in General Equilibrium

Why do economies use money? is one of the classic issues in the foundations of economic theory, with contributions extending from Smith's *Wealth of Nations*, to the present. *Money*, like *written language* and the *wheel*, is one of the fundamental discoveries of civilization. As an explanation for the use of money, it is not sufficient to say that barter is awkward, requiring a double coincidence of wants. That argument explains why monetary trade is superior to barter exchange. To explain the use of money, monetary trade should be the outcome of market equilibrium based on primitive assumptions that do not directly posit its use as a medium of exchange. The analysis should explain how

* This paper has benefited from the comments of Walter P. Heller, Mukul Majumdar, and Joel Sobel. It is a pleasure to acknowledge colleagues' remarks in seminars at: University of California, San Diego (2001), Far West Regional Economic Theory Meeting (San Diego, 2002), NBER General Equilibrium Theory Meeting (Minneapolis, 2002), North American Summer Meeting of the Econometric Society (Los Angeles, 2002), Stanford Institute for Theoretical Economics - SITE (Palo Alto, 2002), and the European Meeting of the Econometric Society (Venice, 2002). Remaining errors are the author's responsibility.

rational optimizing agents individually choose in market equilibrium to use a medium of exchange as part of a market clearing allocation guided by prices.

Despite the evident superiority of monetary trade over barter, there is a counterintuitive --- superficially irrational --- quality to monetary exchange. Monetary trade involves one party to a transaction giving up something desirable (labor, his production, a previous acquisition) for something useless (a fiduciary token or a commonly traded commodity for which he has no immediate use) in the hope of advantageously retrading this latest acquisition. A fundamental issue in monetary theory is to articulate the elementary economic conditions that allow this paradox to be sustained as an individually rational market equilibrium.

Over a century ago, Carl Menger (1892), issued that challenge to monetary theory and he proposed a solution based on differing liquidity ('saleability') of commodities:

> It is obvious ... that a commodity should be given up by its owner ...for another more useful to him. But that every[one] ... should be ready to exchange his goods for little metal disks apparently useless as such...or for documents representing [them]...is...mysterious...
>
> why...is...economic man ...ready to accept a certain kind of commodity, *even if he does not need it,* ... in exchange for all the goods he has brought to market[?]
>
> [Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution...
>
> when any one has brought goods not highly saleable to market, the idea uppermost in his mind is to exchange them, not only for such as he happens to be in need of, but...for other goods...more saleable than his own...By...a mediate exchange, he gains the prospect of accomplishing his purpose more surely and economically than if he had confined himself to direct exchange...Men have been led...without convention, without legal compulsion,...to exchange...their wares...for other goods...more saleable...which ...have ...become generally acceptable media of exchange.

The theory of money Menger proposes here is strongly based on price theory. In Menger's view, not only does a price system provide prices for goods and services; liquidity ('saleability' in Menger's terminology) is priced as well. The price of liquidity is the bid/ask spread, "the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution..." A market equilibrium then includes not only a choice of supply and demand; it includes a choice of means of payment. Each buyer and seller chooses the goods that he will both accept in exchange for his sales, and that he will use to pay for his purchases. He chooses to carry the most liquid ('saleable') goods as carriers of value between his selling and buying transactions. The most liquid goods, those with the narrowest bid/ask spreads, become media of exchange. This happens not because of fiat, legal tender rules, a social contract, or expectations of future acceptability; it is a result of the price-guided general equilibrium. Menger argues that the price system includes much more information than is conventionally supposed. Not only do prices convey relative scarcity; they also convey information about liquidity. The price system conveys information not only of what to buy and sell, but how best to carry value from one transaction to the next. Acceptability in trade is positive bid price. Liquidity is clearly and explicitly priced in the spread between buying and selling prices. The price system includes enough information to decide what each agent wants and how he wishes to pay for it. The price system tells traders what good(s) is(are) commodity 'money' simply by conveying prices and the spread between buying and selling prices.

It is well known that the classic Arrow-Debreu model, Debreu (1959), cannot support the presence of money. The full set of futures markets eliminates any need for a store of value. More fundamentally, the single budget constraint facing each household means that there is no function for a medium of exchange to carry purchasing power between transactions. Debreu (1959), p. 36, notes "the integration of money in the theory of value" is an "important and difficult question ... not answered by the approach taken here." Hahn (1971) and Starrett (1973) argue that the use of a sequence economy with a time sequence of budget constraints generates a demand for a store of value function of money. This paper uses the general equilibrium with transactions cost structure of Foley (1970), Hahn (1971) and Starrett (1973) to formalize Menger's (1892) argument, and to introduce commodity and fiat money in an Arrow-Debreu general equilibrium model. This paper's model posits two additions to the Arrow-Debreu model: segmented markets and transaction costs. Segmented markets imply a multiplicity of budget constraints and a demand for a carrier of purchasing power among them; transaction costs create a spread between public buying (ask) and public selling (bid) prices in equilibrium. Then the lowest transaction cost goods become the natural commodity moneys. Fiat money is introduced as an intrinsically worthless government-issued instrument. Acceptability in payment of taxes gives fiat money positive value and low transaction cost makes it the common medium of exchange.

The concept of a segmented market here is merely the separate budget balance requirement fulfilled at each distinct transaction. Each market segment merely represents a separate transaction: buying food from the supermarket, selling labor to an employer, buying cleaning services from the laundry, ... Each of these is a separate segment since a separate budget constraint is fulfilled in each one. Markets are segmented (with households fulfilling a separate budget constraint in each segment) reflecting the many distinct transactions a typical household undertakes, generating a demand for a carrier of value between transactions (media of exchange). The most liquid asset, the instrument that provides liquidity at lowest cost, will be chosen as the common medium of exchange, the carrier of value between market segments. Thus, the choice of a (possibly unique) 'money' is the outcome of optimizing behavior of economic agents in market equilibrium. Menger's views are fulfilled. The most liquid goods are chosen as the media of exchange. This treatment presents a general equilibrium model alternative to the random matching sequential trade model[1], e.g. Kiyotaki and Wright (1989), and an alternative to a cash-in-advance model (which typically assumes the existence of valued fiat money), e.g. Lucas and Stokey (1987).

Allocative efficiency is defined subject to initial endowment and transaction technology; reallocation is necessarily a resource using activity. Nevertheless, general equilibrium need not be Pareto efficient (see Hahn(1971), Starrett(1973), Ostroy and Starr(1990)). However, if there is a transaction-costless common medium of exchange

[1] To distinguish the static finite horizon general equilibrium model of money here from the dynamic infinite horizon random matching model of money, consider the relatively rare case of a double coincidence of wants (the grocery store clerk who needs food, the auto worker who wants a car). The random matching model concludes that double coincidence leads to a barter trade between the well matched traders. The present general equilibrium model with transaction costs concludes that the pattern of trade depends on transaction costs. Transaction costs may well be lower with monetary trade, and double coincidence will be resolved with trade in money. Casual empiricism suggests that the latter conclusion is the more realistic. Auto workers pay for their cars in money, not in kind; grocery employees pay for their food in money, not in kind.

then a general equilibrium will be Pareto efficient (Starrett(1973), Theorem 3 and Corollary 1 below).

To prove existence of a general equilibrium in a segmented market with transaction cost this paper combines two available treatments. Foley (1970) provides a demonstration of existence of general equilibrium with bid and ask prices and transaction costs in a single unified market. Arrow and Hahn (1971, chapter 6) demonstrates the existence of general equilibrium with externalities. The composite household model below then expands the commodity space and the population of households. Each commodity is treated as distinct depending on which market segment it trades in. Each household is treated as being many distinct counterparts depending on which market segment it trades in. The counterparts are then combined by formalizing an external effect (in the form of a common consumption and common maximand) among them. The general equilibrium of the composite household model with externalities is then a general equilibrium of the original segmented market economy. The equilibrium concept used is a quasi-equilibrium (compensated equilibrium) rather than a competitive equilibrium. This reflects two technical problems: the complexity of moving budgets among constituent elements of the composite households; the difficulty of assuring ample real income in the presence of a bid/ask spread.

II. Segmented Market Model

Hahn (1971) and Starrett (1973) introduce a sequence economy model with a budget constraint at each of a succession of dates. The present model is agnostic on the time structure, positing a multiplicity of budget constraints, each separately to be fulfilled. The sequence economy models should be a special case of the segmented market model. The segmented market model with its multiple budget constraints is intended to represent the requirement that at each of a variety of transactions, agents are required to pay for their purchases. Each household faces many budget constraints, not just one.

Households: There is a finite set of households, H; the typical element is denoted $i \in H$ or $h \in H$. In an interpretation taken from Foley(1970), households are not modeled as directly incurring transaction costs themselves. Rather they face the bid/ask spread presented by the market and see the transaction costs built into retail prices. Thus a household's decision whether to expend gasoline and time in going shopping is idealized as taking place in a separate firm. An alternative treatment attributing the transaction cost decision to the household itself is in Kurz (1974).

Goods: There is a finite list of goods, n = 1,...,N. As in Debreu (1959), the list of commodities is subject to interpretation. They may simply all be spot goods. Alternatively, some may be spot and others for future delivery. Under uncertainty, they may be defined as well by the state of the world in which they are deliverable.

Firms: There is a finite set of firms F, with the typical element $j \in F$. Firms can perform conventional production activities, buying inputs and selling outputs. Following Foley (1970), one of the principal activities of a firm is to undertake transactions. Changing the ownership of a commodity --- buying from a seller and selling to a distinct buyer, buying at the bid (wholesale) price and selling at the ask (retail) price --- is treated as a production activity. It is resource using; the transactions take place at differing prices; the firm undertakes it to make a profit on the difference between buying and selling prices. Hence, the model here interprets the actions of wholesalers, retailers, brokers --- any business whose specialty is making a market --- as a special case of production activity.

The structure of markets described below leads to an awkward oversimplification on the structure of firms as market makers and intermediaries. So long as a firm is active only on a single market, the firm's profit as its maximand is well defined. A firm with actions on several markets does not have a well-defined concept of profit due to the differing prices across markets. This leads to the unsatisfactory usage that a typical firm is active on only one market. Hahn (1971) treated this difficulty in precisely the same fashion. Hence the arbitrage functions we might expect a firm to undertake are here left to households.

Markets: In specifying the structure of markets the present paper breaks with most of the general equilibrium theory, and follows most closely Hahn (1971) and Starrett (1973). The essential elements here are the structure of budget constraints and the evaluation of firm profits. A market in this paper is the locus of transactions. Each household is expected to fulfill a budget constraint on each market separately. Because of the difficulty of assessing discount rates across markets, each firm is supposed to optimize its profits on a single market and confine its transactions to that market. There is a finite set of markets M, each denoted $k \in M$. This construct is used to represent the notion that budgets balance, that payment is made for purchases, at each of many separate transactions. Thus a multiplicity of budget constraints, one at each transaction, replaces the single budget constraint of the Arrow-Debreu model. Hahn's (1971) construction of this model emphasized the notion of a sequence economy; that budgets balance at each date. The present treatment is intended to be more agnostic and more general; budgets balance at each transaction.

Prices: Inasmuch as transactions are a resource using activity there will be a spread between selling and buying prices. Thus, on any single market, there are two prices for each good. The vector $p^{kS} \in \mathbb{R}^N_+$, represents the vector of selling (bid, wholesale) prices on market k ('selling' as viewed by the public). Similarly the vector $p^{kB} \in \mathbb{R}^N_+$, represents the vector of buying (ask, retail) prices on market k ('buying' as viewed by the public). We'll specialize the price space to a simplex directly. Prices are not quoted in monetary terms; they are pure numbers. The vector of retail margins, the spread between buying and selling prices on market k, is represented by $\pi^k = p^{kB} - p^{kS}$. Note that typically $p^{kB} \geq p^{kS}$ (coordinatewise), and $\pi^k \geq 0$.

Though each firm is active on only one market k, a typical household can transact on a variety of markets. The household must take account of all prevailing prices and price ratios in order to choose the best markets on which to transact. The space of prices facing a household includes a bid price and an ask price for each good in each market. The resulting array of prices lies in $\mathbb{P} \subset \mathbb{R}^{2\#MN}_+$. $\mathbb{P}$ is the unit simplex in $\mathbb{R}^{2\#MN}_+$, the space of possible price vectors, with typical element $p = (p^{1B}, p^{1S}; \ldots; p^{\#MB}, p^{\#MS})$, alternatively denoted $(p^{kB}, p^{kS})|_{k\in M} \in \mathbb{P}$.

$x^i \in \mathbb{R}^{2\#MN}$, denotes i's full transaction plan, $x^{ikB} \in \mathbb{R}^N_+$, $x^{ikS} \in \mathbb{R}^N_-$; note that x^{ikS} is a (negative) vector of sales.

$X^i \subseteq \mathbb{R}^N$, i's possible net trade space

$u^i(x): X^i \rightarrow \mathbb{R}$, i's utility function.

$Y^j \subseteq \mathbb{R}^{2N}$, j's technology set. j is typically active on only one of the segmented markets, $k \in M$. $F(k) \subset F$ is the set of firms j active on market k. The typical element of Y^j is (y^j, y^{jB}). y^j is j's net transaction; y^{jB} is the portion of j's transaction undertaken at the higher retail (ask) prices, p^{kB}. The value of this production plan is $p^{kS}y^j + \pi^k y^{jB}$.

$0 \leq \theta^{ij} \leq 1$, i's share of firm j .

The segmented market model embodies the concept that a typical household will make many separate transactions, with retailers, service providers, an employer, and so forth. In each of these transactions a budget constraint prevails. At prices prevailing in each transaction, budgets must balance; each party delivers value to the other equal to that he receives. Since there is a multiplicity of separate budget constraints, the market is said to be segmented. This concept is formalized as the multiplicity of markets $k \in M$, where M is the set of available distinct markets. In addition, there are transaction costs in each market creating differing bid and ask prices. The notion of transactions as a resource using activity is embodied in firms with a production technology transforming goods between purchased (from the public, 'wholesale' at bid prices) to sold (to the public, 'retail' at ask prices). Finally, transaction costs may differ across markets, so prevailing bid and ask prices may differ as well. Thus with N commodities and #M market segments there are 2#MN prevailing bid and ask prices. The reason for investigating this construct is to derive the monetary structure that it generates. The multiplicity of budget constraints implies a demand for a carrier of value between transactions. Based on prevailing bid and ask prices, a typical household might then decide to sell good 1 on one segmented market, acquiring there good 2, which it will then take to a second segmented market to trade for its desired purchase, good 3. Trade may occur in this fashion because prevailing transaction costs make it prohibitive to trade good 1 directly for good 3. In this example good 2 acts as a carrier of value from one market segment to another; it becomes a commodity money. The household decision-making that leads the household to choose good 2 to act in this fashion is based on the household's endowment, preferences, and prevailing prices. Menger (1892) argued that the choice of a commodity money will be based on asset liquidity, reflected in the bid/ask spread. Theorem 2 below confirms this viewpoint.

Household i's actions are $x^i \in \mathbb{R}^{2\#MN}$. $x^i = (x^{i1B}, x^{i1S}, ..., x^{ikB}, x^{ikS}, ..., x^{i\#MB}, x^{i\#MS})$. That is, x^i lists i's N-dimensional buying (a nonnegative vector x^{ikB}) and selling (a nonpositive vector x^{ikS}) actions on each of the segmented markets $k \in M$. i's net trade can then be characterized as $\sigma(x^i) = \sum_{k \in M} (x^{ikB}+x^{ikS})$. $\sigma(x^i)$ is i's net trade aggregated over the markets $k \in M$.

We characterize prevailing prices as $p=(p^{kB}, p^{kS})|_{k\in M} \in \mathbb{P}$ and firms' profit maximizing actions at these prices as (y^j, y^{jB}). Then for $x^{oi} \in \mathbb{R}^{2\#MN}$, we say that $x^{oi} \in D^i(p)$, if x^{oi} maximizes $u^i(\sigma(x^i))$ subject to,

$$p^{kB} \cdot x^{ikB} + p^{kS} \cdot x^{ikS} \le \sum_{j \in F(k)} \theta^{ij}[p^{kS} y^j + \pi^k y^{jB}] \text{ , for each } k \in M. \tag{1}$$

Prices $(p^{*kB}, p^{*kS})|_{k\in M} \in \mathbb{P}$, household plans $x^{*i} \in \mathbb{R}^{2\#MN}$, and firm plans $(y^{*j}, y^{*jB}) \in Y^j$ are said to constitute a quasi-equilibrium if

(y^{*j}, y^{*jB}) maximizes $p^{kS} \cdot y^j + \pi^k \cdot y^{jB}$ subject to $(y^j, y^{jB}) \in Y^j$, for each $j \in F(k)$, each $k \in M$,

$x^{*i} \in \mathbb{R}^{2\#MN}$ maximizes $u^i(\sigma(x^i))$ subject to

$$p^{*kB} \cdot x^{ikB} + p^{*kS} \cdot x^{ikS} \le \sum_{j \in F(k)} \theta^{ij}[p^{*kS} y^{*j} + \pi^{*k} y^{*jB}] \text{ , for each } k \in M \tag{2}$$

<u>or if</u>

x^{*i} minimizes $p^{*kB} \cdot x^{*kB} + p^{*kS} \cdot x^{*kS}$ subject to $u^i(\sigma(x)) \ge u^i(\sigma(x^{*i}))$ for each $k \in M$, and

$$\sum_{i \in H} (x^{*ikB} + x^{*ikS}) - \sum_{j \in F} y^{*jk} \le 0 \text{, co-ordinatewise, for each } k \in M, \tag{3}$$

$$\text{and } \sum_{i \in H} x^{*ikB} - \sum_{j \in F(k)} y^{*jB} \le 0 \text{, co-ordinatewise, for each } k \in M. \tag{4}$$

<u>III. Media of Exchange</u>

Let $p \in \mathbb{P}$, $x^i \in D^i(p)$. i's net trade consists of $\sum_{k \in M} (x^{ikB} + x^{ikS})$. But i's gross trades may be much larger than his net trades. The gross trades (as positive values) are

$\sum_{k\in M} x^{ikB} - \sum_{k\in M} x^{ikS}$. How do gross and net trades differ? In rare cases, a double coincidence of wants in each market, they may not differ at all. If i sells in each market k goods of which he has a net supply and buys goods there that he eventually consumes, gross trades equal net trades. More generally, however, i may deliver most of his excess supplies on one market k' and acquire his consumption on a variety of other markets k'', k''', ... In that case he will accept goods in k' in payment for his supplies that he will subsequently trade away on k'', k''', ..., in exchange for his planned consumption. Those goods temporarily held between markets are acting as media of exchange, commodity money.

Pricing for good n on market k is said to be arbitrage free if the following inequality is fulfilled:

$$\min_{\substack{m=1,2,\dots,N\\ m\neq n\\ k'\in M}} \left[\frac{p_n^{kB}}{p_m^{kS}} \Big/ \frac{p_n^{k'S}}{p_m^{k'B}} \right] > 1 \ . \tag{5}$$

Note that the expression above is a strict inequality. The property of being arbitrage free means that a round trip purchase and sale of good n in markets k and k' is (strictly) unprofitable. A general equilibrium will have the property that pricing is arbitrage free.[2]

For $z\in \mathbb{R}^N$, let $[z]_+$ be the N-vector of nonnegative elements in z (zero's in place of other elements), and let $[z]_-$ be the N-vector of nonpositive elements in z (zero's in place of other elements). Let the notation $\{x^{ikB}\}$ and $\{x^{ikS}\}$ denote the vectors x^{ikB} and x^{ikS} where 0 has been substituted for each co-ordinate that is not arbitrage free. The expression

$$e^i(x^i) \equiv [\sum_{k\in M}\{x^{ikB}\}] - [\sum_{k\in M}(\{x^{ikB}\}+\{x^{ikS}\})]_+ - [\sum_{k\in M}\{x^{ikS}\}] + [\sum_{k\in M}(\{x^{ikB}\}+\{x^{ikS}\})]_- \tag{6}$$

represents i's flow of media of exchange. $e^i(x^i)$ is gross purchases minus net purchases, (algebraically) minus (negative) gross sales plus net sales, ignoring arbitrage flows. $e^i(x^i)$

[2] A general equilibrium is consistent with net flow of a good between markets undertaken by households (at zero-profit) where the good may fulfill a medium of exchange function for the households but fill a demand in the receiving market and absorb a supply in the sending market. Thus, if good n is abundant in market k and scarce in k', a seller of good m in k who is a buyer of m' in k' may find it convenient to take payment at k in n and spend n in k' to buy m'.

is the flow of goods in excess of those minimally required physically to implement i's net trade.

IV. Pareto Efficiency

Any reallocation may require incurring transaction costs. The presence of transaction costs and of the wedge between buying and selling prices are not in themselves indications of inefficient allocation. In this literature (Hahn (1971), Starrett (1973)) efficiency is defined relative to initial endowment and transaction technology; necessary transaction costs incurred in moving from endowment to a preferred allocation are a technical necessity and not inefficient. But transaction costs incurred merely in fulfilling budget constraints, condition (B), are regarded as wasted resources. These are the transaction costs incurred in implementing media of exchange, $e^i(x^i)$. An additional related source of inefficient allocation is preferable reallocations (net of technically necessary transaction costs) discouraged by the prospect of transaction costs incurred in fulfilling budget constraints.[3]

V. Assumptions

The following assumptions are familiar in conventional general equilibrium models and correspond essentially to those of Foley (1970). H.1 to H.4 apply to the households of the economy. P.1 to P.4 correspond to the production sector of the economy, including the transactions process as a resource using activity. These are sufficient to develop a model including an equilibrium with a commodity money in Theorems 1, 2 and 3 below. An additional family of assumptions on taxation and fiat money issue, M.1 through M.8, is developed later in the paper to characterize a fiat money equilibrium in Theorems 4 and 5.

H.1 $X^i \subset \mathbb{R}^N$. X^i has a lower bound.

H.2 X^i is closed and convex; $0 \in X^i$.

H.3 u^i: $X^i \to \mathbb{R}$ is continuous, quasi-concave.

[3] In conversation with Nobuhiro Kiyotaki, he argued that the notion of efficiency above is too restrictive. In the view he expressed, as I understand it, budget balance (1) is a technical necessity just as much as is a transaction technology, so the notion of Pareto efficiency should be subject to endowment, transaction technology, and budget balance.

H.4 $x' >> x^o$ implies $u^i(x') > u^i(x^o)$

P.1 $0 \in Y^j$, $Y^j \subset \mathbb{R}^{2N}$.

P.2 There is no $(y^j, y^{jB}) \in Y^j$ so that $(y^j, y^{jB}) > 0$.

P.3 Y^j is a convex cone.

P.4 $Y_k = \sum_{j \in F(k)} Y^j$, is closed for each $k \in M$.

VI. Model of Composite Households with Consumption Externalities

We seek to establish the existence of a general (quasi-) equilibrium in the segmented market model. Rather than prove this directly, we take the approach of restating the model in a way that treats the model as a special case of Foley (1970) with externalities in consumption. That model's sufficient conditions are then adequate to ensure existence of equilibrium in the segmented market model. The strategy of proof is to expand the dimension of the commodity space by a factor of #M, the number of distinct market segments. That is, there are #MN formally distinct goods. Identical goods in distinct segments are then treated as different goods, with distinct prices, transacted by different firms, and consumed by formally distinct households. In the original segmented market model, each household is active in each of the #M segmented markets. We now restate this as each household i having #M distinct counterparts, ik, $k \in M$, active on market segment k. The #M households are linked in their preferences by an external effect. For each i, and each of the formally distinct households ik' and ik" (for k', k"$\in$ M, k' $\neq$ k"), ik" 's consumption plans enter as an external effect in ik' 's utility, as though those consumptions were ik' 's own. Hence we can represent the #M-segment complex of purchase plans of the typical household i in the original segmented market model, as #M distinct purchase plans of #M distinct households linked by an external effect in the composite household model. There are then #H#M formally distinct households, each one with preferences linked by an external effect to #M-1 counterparts. Since each household takes fully into account the consumptions of his #M-1 counterparts, and since they share a common utility function, optimization for the complex of #M distinct households ik, $k \in M$, in the composite household model is equivalent to that of household i in the segmented market model. We demonstrate the existence of equilibrium in the composite household model and then note that the conditions for equilibrium there are precisely equivalent to those of the segmented market model. Hence the segmented market model has a general equilibrium.

In the composite household economy we consider a revised population of households, $HM = \{ik| i\in H, k\in M\}$. For each $ik \in HM$, ik's buying and selling plans are restricted to segment k. $x^{ik} \in \mathbb{R}^{2\#MN}$ with all co-ordinates for markets other than k set identically equal to 0. $x^{ik}=(0, 0, ..., 0, x^{ikB}, x^{ikS}, 0, ..., 0, 0)$. Conversely, let $x^{i-k} \in \mathbb{R}^{2\#MN}$ denote the 2#MN-dimensional vector of external effects on ik coming from its counterparts active on markets $k'\neq k$. $x^{i-k} \equiv (x^{i1B}, x^{i1S}, ..., x^{ik-1B}, x^{ik-1S}, 0, 0, x^{ik+1B}, x^{ik+1S}, ..., x^{i\#MB}, x^{i\#MS})$ setting at 0 the k-indexed co-ordinates. That is, each household $i\in H$ of the original segmented market model appears in the composite household model as #M distinct households, one for each segmented market $k\in M$. The separate households are related by an external effect --- each of the separate households appreciates fully the consumption decisions of its counterparts.

Household ik's utility function is characterized by strong external effects. ik's preferences are those of i in the segmented market model, applied to ik's net trades plus those of ik', $k'\neq k$. That is,

$$u^{ik}(x^{ik}; x^{i-k}) \equiv u^{i}(x^{ikB}+x^{ikS} +\sigma(x^{i-k})) \qquad (7)$$

where x^{i-k} is treated parametrically. The constraint set on ik's transactions then is X^{ik} defined as

$$X^{ik}(x^{i-k}) \equiv \{ x^{ik} \in \mathbb{R}^{2\#MN} \mid x^{ik}=(0, 0, ..., 0, x^{ikB}, x^{ikS}, 0, ..., 0, 0), \\ (x^{ikB}+x^{ikS} +\sigma(x^{i-k}))\in X^{i}\} \ . \qquad (8)$$

ik's income, to be spent on market k, comes from sales of goods on k, x^{ikS}, and from ik's share of profits of firms active on k. We take ik's shares of firms $j \in F(k)$ to be identical to i's, θ^{ij} for $j\in F(k)$. Thus ik's budget constraint is

$$p^{kB}\cdot x^{ikB} + p^{kS}\cdot x^{ikS}\leq \sum_{j\in F(k)} \theta^{ij}[p^{kS}y^{j} + \pi^{k}y^{jB}] \ . \qquad (9)$$

Prices $(p^{*kB}, p^{*kS})|_{k\in M}\in \mathbb{P}$, household plans $x^{*ik} \in \mathbb{R}^{2\#MN}$, firm actions $(y^{*j}, y^{*jB})\in Y^{j}$ are said to constitute a quasi-equilibrium in the composite household model if

(y^{*j}, y^{*jB}) maximizes $p^{*kS}y^j + \pi^{*k}y^{jB}$ subject to $(y^j, y^{jB}) \in Y^j$, for each $j \in F(k)$, each $k \in M$, and for each $ik \in HM$

x^{*ik} maximizes $u^{ik}(x^{ik}; x^{*i-k})$ on $X^{ik}(x^{*i-k})$ subject to

$$p^{*kB} \cdot x^{ikB} + p^{*kS} \cdot x^{ikS} \leq \sum_{j \in F(k)} \theta^{ij}[p^{*kS}y^{*j} + \pi^{*k}y^{*jB}] \tag{10}$$

<u>or if</u>

x^{*ik} minimizes $p^{*kB} \cdot x^{kB} + p^{*kS} \cdot x^{kS}$ subject to $u^{ik}(x^k; x^{*i-k}) \geq u^i(\sigma(x^{*i}))$ for each $k \in M$, and

$$\sum_{i \in H} (x^{*ikB} + x^{*ikS}) - \sum_{j \in F(k)} y^{*j} \leq 0, \text{ co-ordinatewise,} \tag{11}$$

for each $k \in M$, and

$$\text{and } \sum_{i \in H} (x^{*ikB}) - \sum_{j \in F(k)} y^{*jB} \leq 0, \text{ co-ordinatewise, for each } k \in M. \tag{12}$$

<u>VII. Results</u>

Lemma 1 (Existence of a quasi-equilibrium in the composite household economy): Assume H.1-H.4, P.1-P.4. Then the composite household economy has a quasi-equilibrium with prices $(p^{*kB}, p^{*kS})|_{k \in M} \in \mathbb{P}$.

Proof: See the Appendix. Foley (1970), Theorem 4.1. Arrow and Hahn (1971)'s discussion on p. 135 of an economy with continuous external effects demonstrates the existence of a quasi-equilibrium (compensated equilibrium). QED

Theorem 1 (Existence of a quasi-equilibrium): Assume H.1-H.4, P.1-P.4. Then the segmented market economy has a quasi-equilibrium with prices $(p^{*kB}, p^{*kS})|_{k \in M} \in \mathbb{P}$.

Proof: Apply Lemma 1 to the composite household economy. For each $i \in H$, the first order conditions to maximize $u^{ik}(\bullet)$ subject to budget constraint in $k \in M$ in the composite household model are identical to those of maximizing $u^i(\bullet)$ subject to budget constraint in the segmented market model. QED

Theorem 2 (Demand for media of exchange): Let $p \in \mathbb{P}$ be a quasi-equilibrium price vector, $x^i \in D^i(p)$ and consider $e^i(x^i)$. Designate some n*=1,...,N and all other n≠n*, n=1,...,N. Let $p^{kB}_{n*}>0$ and $p^{kS}_{n*}>0$, for all $k \in M$. Further, let

$$\frac{\boldsymbol{p}^{k}_{n*}}{p^{kB}_{n*}} = \frac{p^{kB}_{n*} - p^{kS}_{n*}}{p^{kB}_{n*}} < \frac{p^{kB}_{n} - p^{kS}_{n}}{p^{kB}_{n}} = \frac{\boldsymbol{p}^{k}_{n}}{p^{kB}_{n}} \tag{13}$$

and $$\frac{\boldsymbol{p}^{k}_{n*}}{p^{kS}_{n*}} = \frac{p^{kB}_{n*} - p^{kS}_{n*}}{p^{kS}_{n*}} < \frac{p^{kB}_{n} - p^{kS}_{n}}{p^{kS}_{n}} = \frac{\boldsymbol{p}^{k}_{n}}{p^{kS}_{n}} \tag{14}$$

for all $k \in M$. That is, on all markets, both on the buying and selling side, suppose good n* has the narrowest proportionate bid/ask spread of any good. Then the only nonull co-ordinates of $e^i(x^i)$ are in good n*.

Proof: Suppose $e^i_n(x^i)>0$ for $n \neq n^*$. Then there is an alternative x'^i fulfilling (1) with utility higher than x^i (x'^i using more n* as medium of exchange, less n). QED

Example 1 (Inefficient allocation in equilibrium): Pareto inefficient allocation is possible in equilibrium of the segmented market economy. A special case of the segmented market model is a sequence economy, Hahn (1971). See the examples of inefficiency in a sequence economy in Starrett (1973) or Ostroy and Starr (1990).

Theorem 3 (Efficiency of allocation with a transaction-costless medium of exchange): Let $p^* \in \mathbb{P}$ be a quasi-equilibrium price vector and $x^{*i} \in D^i(p^*)$ be the corresponding equilibrium trading plans. Let $p^{*kB} \cdot x^{*ikB} > 0$ for some $i \in H$, some $k \in M$. Let (y^{*j}, y^{*jS}) be the corresponding firm plans. Let there be good n* so that $p^{*kB}_{n*} = p^{*kS}_{n*} > 0$ for all $k \in M$. Then the allocation x^{*i} is Pareto efficient.

Proof: For all i,k,n so that $x^{*ikB}_n \neq 0$ we have $[p^{*kB}_n / p^{*kS}_{n*}] \leq [p^{*k'B}_n / p^{*k'S}_{n*}]$ for $k' \neq k$. For all i,k,n so that $x^{*ikS}_n \neq 0$ we have $[p^{*kS}_n / p^{*kB}_{n*}] \geq [p^{*k'S}_n / p^{*k'B}_{n*}]$ for $k' \neq k$. These prices display what Hahn (1971) called the 'Debreu property.'

Denote $r^B_n = [p^{*kB}_n / p^{*kS}_{n*}]$ for k so that $x^{*ikB}_n \neq 0$. Denote $r^S_n = [p^{*kS}_n / p^{*kB}_{n*}]$ for k so that $x^{*ikS}_n \neq 0$. Then (r^B, r^S) is a 2N-dimensional price vector supporting the allocation x^{*i}, (y^{*j}, y^{*jS}). The allocation is Pareto efficient by the First Fundamental

Theorem of Welfare Economics (for a compensated or quasi-equilibrium where at least one household has positive income, Arrow and Hahn, Theorem 5.3). QED

VIII. Monetary and Financial Structure

Theorems 1, 2, and 3 above develop the model of commodity money equilibrium. Theorem 1 merely states that the assumptions are sufficient to generate existence of a quasi-equilibrium. Theorem 2 embodies Menger's (1892) argument that commodity money is based on liquidity. Theorem 2 proposes that there be a single good n* with narrowest proportional bid/ask spread at prevailing prices. Then n* will be the unique medium of exchange. Clearly Theorem 2 poses a simplified case --- there could be several goods tied for narrowest bid/ask spread or goods with a narrow bid/ask spread could vary across markets (in which case there would be no common medium of exchange). Nevertheless, the underlying principle is clear. Liquidity is priced in the bid/ask spread and the most liquid good(s) will be the medium(a) of exchange.

When a household engages in trade, its sales from endowment or its income from business may be concentrated on one market k' but its purchases for consumption may center on another market k". The two values, sales (plus profits) and purchases, must balance on each market separately, (B). Hence the household uses a carrier of value, commodity money $e^i(x^i)$, to shift purchasing power among markets. It seeks to do so in the most advantageous fashion possible, losing as little purchasing power as possible in the process. That is how the household forms its optimizing choice of $e^i(x^i)$. For arbitrary $p \in \mathbb{P}$, there is no simple general characterization of $e^i(x^i)$. But Theorem 2 describes the most interesting special case. Suppose --- at prevailing prices --- there is a natural money, n*, a good with such low transaction costs that the prevailing bid/ask spread makes it the least costly way to move purchasing power across all markets. Then n* is the only good that will be used as $e^i(x^i)$. All transactions will either be for directly useful trades --- delivering supplies, fulfilling demands --- or they will be in n* acting as a medium of exchange.

Jevons (1875) reminds us that money (particularly a commodity money) should possess the following properties: 1. Utility and Value, 2. Portability, 3. Indestructibility, 4. Homogeneity, 5. Divisibility, 6. Stability of value, 7. Cognizibility. How do these notions stack up in the current model? Following Theorems 1 and 2, suppose we have $p^* \in \mathbb{P}$, a segmented market equilibrium price vector, with n* the low transaction-cost instrument in all markets $k \in M$. What can we expect of n* at p* with regard to Jevons's specifications?

1. Utility and Value . As stated in Theorem 2, the bid and ask price of n* must be positive in all markets $k \in M$. For n* to act as a medium of exchange, it must have a positive price. In order to maintain a positive bid price, n* must have a desirable use, so the model is fully in agreement with Jevons. Clearly modern fiat money does not directly fulfill this property, so some explanation of its positive value is required. That discussion appears below.

2. Portability, 3. Indestructibility. The resource requirements for dealing in n* , including its transportation requirements and any n* used up through deterioration in the process of trade, are included in the transaction/production technology Y^j. The choice of n* as the low bid/ask spread instrument in equilibrium reflects a low transaction cost. The resources required for dealing in and transporting n*, including shrinkage in the quantity of n*, should be small and of low value. The notion of portability is embodied in Theorem 2 through the transaction technology and its attendant costs.

5. Divisibility. Since the commodity space and transaction technologies are assumed to be convex, divisibility is a matter of assumption and definition in this model.

4. Homogeneity, 7. Cognizibility. A lack of uniformity in quantity or quality of a good will increase the resources required to trade it, since resources will need to be expended in verifying and quantifying the good (see Banerjee and Maskin (1996)). Hence the notions of the resource cost and resource saving associated with homogeneity and cognizibility are embodied in the transaction technology and in Theorem 2.

6. Stability of value. This issue is trickier to treat; it involves the time and uncertainty structure of the model. Assume a model with a full set of time-dated, state-of-world labeled goods. In an Arrow-Debreu model we'd call this a full set of futures markets under uncertainty. That nomenclature doesn't precisely fit here, since the markets may be available but with high transaction costs so that they are inactive in equilibrium, Hahn (1971). A typical inactive market will have posted bid and ask prices for each future/contingent commodity. But bid/ask spreads may be wide enough so that no trading takes place, so that the risk of price variation is not hedged because hedging itself is too expensive. As events unfold, the price (or more precisely, the array of bid and ask prices) of a commodity may vary as events move down an event tree. The implication of Theorem 2 is that to the extent that 'stability of value' leads to market liquidity (narrow bid/ask spread) then stability is important to the designation of the common medium of exchange, n*. The link between stability and liquidity is that risk aversion discounts the value of the more unpredictable contingent commodities without reducing their transaction costs, hence resulting in illiquidity.

Example 1 and Theorem 3 emphasize the notion of Pareto efficiency. The example (the restatement that the examples of Starrett (1973) and of Ostroy and Starr (1990) still apply) demonstrates that in a segmented market with transaction cost equilibrium need not be Pareto efficient. However, if there is a medium of exchange that operates with zero transaction cost ('money'), then common general equilibrium prices can be re-established and the allocation is Pareto efficient by the usual First Fundamental Theorem of Welfare Economics.

<u>Banks, Financial Intermediation</u>

Assume a model with a full set of time-dated, state-of-world labeled goods. Banking (and more complex financial intermediation) functions are just another market making function: buying and selling dated goods (particularly those of the common medium of exchange, n*, if it exists). Borrowing, or accepting deposits, consists of buying current goods, including n*. Repaying deposits consists of disbursing corresponding future goods, including future n*. Lending consists of selling current goods, including n* while buying future goods. A bank arranging future repayment of a loan is buying future dated goods, including n*. The present model assumes convexity of transaction technologies, so the intermediation function simply falls on the firms best suited to perform it, those with the lowest transaction costs in the spot and futures markets. However, the scale economy and law of large numbers arguments common in partial equilibrium descriptions of banking cannot be fully accommodated here, since they involve a nonconvexity in the transaction technology.

IX. Fiat Money and Government

The outlines of how to incorporate fiat money in this finite horizon static general equilibrium model can be briefly sketched. There are two issues to be addressed: How does an intrinsically worthless instrument become positively valued in equilibrium? How does this instrument become the common medium of exchange? The answer to the first question is "taxes." The answer to the second is "low transaction costs." Introduce a government in the model with the powers to issue fiat money and to collect taxes. Fiat money is intrinsically worthless; it enters no one's utility function. But the government is uniquely capable of issuing it and of declaring it acceptable in payment of taxes. Adam Smith(1776) notes "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money…" (v. I, book II, ch. 2). Abba Lerner(1947) comments

> The modern state can make anything it chooses generally acceptable as money and thus establish its value quite apart from any connection, even of the most formal kind, with gold or with backing of any kind. It is true that a simple declaration that such and such is money will not do, even if backed by the most convincing constitutional evidence of the state's absolute sovereignty. But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done. Everyone who has obligations to the state will be willing to accept the pieces of paper with which he can settle the obligations, and all other people will be willing to accept these pieces of paper because they know that the taxpayers, etc., will be willing to accept them in turn.

Taxation --- and fiat money's guaranteed value in payment of taxes --- explains the positive equilibrium value of fiat money[4]. Assume the fiat money also to have very low transaction costs. Then the conclusion follows. Fiat money becomes the common medium of exchange[5]. For an example of how this logic works see Starr (2001). To formalize these views we extend the model above by modeling government and fiat money.

Define taxes, fiat money and government in the following way. Government, denoted G, is formalized as another household with distinctive properties for its achievable net trade set, X^G. Tax receipt certificates are good N-1. Every household

[4] See also Dubey and Geanakoplos (2001), Li and Wright (1998) and Starr (1974).

[5] A more complex argument involves a scale economy, ruled out by the present paper's convexity assumption. If there is a scale economy in transaction costs and if government is a large economic agent, then government transactions in fiat money ensure sufficient scale to result in low transaction costs. Hence fiat money becomes the unique common medium of exchange.

$h \in H$, has a tax quota $\theta^h > 0$, so that there is positive marginal utility from acquiring additional x^h_{N-1} up to the level θ^h . No firm $j \in F$ can produce N-1 and no household in H can achieve a net disbursement of N-1 (that is, no household is endowed with N-1). Government, denoted G, is the unique source of N and is endowed with N-1 (more formally, X^G admits the possibility of a net disbursement of N-1, tax receipts), but not so much that it becomes a drug on the market. Good N will be treated as fiat money. We assume that no household gets positive utility from good N. Government G, declares its willingness to accept N (which nobody wants) in exchange for N-1 (which everybody wants). This amounts merely to defining u^G, G's utility function, with strictly positive marginal utility for N and for N-1. We formalize these notions as[6]

M.1 For each $h \in H$, and each $x^h \in X^i$, each n=1,2,...,N-2, there is $\theta^h > 0$ so that if $x^h_{N-1} \leq \theta^h$, then $+\infty > \dfrac{\frac{\partial u^h(x^h)}{\partial x^h_{N-1}}}{\frac{\partial u^h(x^h)}{\partial x^h_n}} >> 0$.

M.2 $X^G \equiv \mathbb{R}^N_+ - \{(0,0, ..., \Sigma_{h \in H} \theta^h , \Sigma_{h \in H} \theta^h)\}$. For all $h \in H$, all $x \in X^h$, $x_{N-1} \geq 0$.

M.3 For all $x \in X^G$, $\dfrac{\partial u^G(x^G)}{\partial x^G_{N-1}} = \dfrac{\partial u^G(x^G)}{\partial x^G_N} > 0$.

M.4 For all $h \in H$, $\dfrac{\partial u^h(x^h)}{\partial x^h_N} = 0$.

M.5 For all $h \in H$, all $x \in X^h$, x_{N-1}, $x_N \geq 0$.

M.6 For all $j \in F$, all $(y^j, y^{jB}) \in Y^j$, $y^j_{N-1} = y^j_N = 0$.

M.7 For all $j \in F$, let $(y'^j, y'^{jB}) \in Y^j$, let $y'^j = y''^j$, $y'^{jB}_n = y''^{jB}_n$ for n=1,2,...,N-1. Let $y''^{jB}_N \geq y'^{jB}_N$. Then $(y''^j, y''^{jB}) \in Y^j$. [M.7 creates an exception to P.2; trade in money is not resource using.]

M.8 Let $(y'^j, y'^{jB}) \in Y^j$, let $y'^j = y''^j$, $y'^{jB}_n = y''^{jB}_n$ for n=1,2,...,N-2. Let $y''^{jB}_{N-1} \geq y'^{jB}_{N-1}$, $y''^{jB}_N \geq y'^{jB}_N$. Then for at least one $j \in F$, $(y''^j, y''^{jB}) \in Y^j$. [M.8 creates an exception to P.2. For at least one $j \in F$, trade in money and taxes is not resource using. Under M.6 note that money and taxes are pure exchange goods; they are not produced.]

[6] The partial derivatives representing marginal utilities in the assumptions below are assumed to exist everywhere in the (relative) interior of X^G. The assumptions can be restated without differentiability, but the notion of marginal utility is clearly useful here.

Assumptions M.1 through M.8 define the notions of fiat money and taxation. M.1 says that households try to arrange their affairs to pay their taxes and that the marginal rate of substitution of tax payment for other goods is bounded away from zero when taxes have not fully been paid. Since fiat money is acceptable in payment of taxes, M.1 guarantees a finite price level in terms of N. That is, M.1 puts a floor on the value of fiat money. M.2, M.5 and M.6 say that government, G, is the unique source of money, good N, and of tax receipt certificates, good N-1. M.3 says that government, G, is willing to accept money, good N, one for one, in exchange for tax receipt certificates, N-1. M.4 says there is no utility to money, good N, for any household; only government G behaves as though money, N, is desirable. M.7 says that money, good N, carries low transaction costs. M.8 says that there is at least one market where both money and tax receipts, goods N-1 and N, carry low transaction costs.

Theorem 4 (Existence of a fiat money quasi-equilibrium): Assume H.1-H.4, P.1-P.4, M.1-M.8. Then the economy has a quasi-equilibrium with prices $(p^{*kB}, p^{*kS})|_{k\in M} \in \mathbb{P}$. Further, $p^{*kB}_N, p^{*kS}_N > 0$, for all $k \in M$.

Proof: We cannot directly apply Theorem 1 because of free transactions in N-1 and N under M.7 and M.8, violating P.2. Let $K^q \subset \mathbb{R}^{2N}$ be a cube centered at the origin of side q = 1, 2, 3, ... Consider the truncated economy characterized by firms with production technologies $Y^j \cap K^q$. Apply the Theorem of Debreu(1962) to the composite household economy with truncated technology. Let the (truncated) economy's quasi-equilibrium prices and firm actions be $(p^{qkB}, p^{qkS})|_{k\in M}$, $(y^{qj}, y^{qjB}) \in Y^j$. Under M.6, $y^{qj}_{N-1}=y^{qj}_N=0$ for all j. Firm j's retail actions in N-1 and N, y^{qjB}_{N-1}, y^{qjB}_N, may increase without bound as q becomes large. This can occur in equilibrium of the q^{th} truncated economy as q becomes large only in the case of wash sales of N-1 and N, which have no effect on household utility (since they wash) or on firm inputs or profits (by M.7 and M.8). Real equilibrium activity, in goods 1, 2, 3, ..., N-2, is necessarily bounded by P.1 - P.4. Then household and firm actions are set-valued and there is also bounded equilibrium household and firm action. Take a convergent subsequence in prices and actions. Its limit is market equilibrium prices and household and firm actions, $(p^{*kB}, p^{*kS})|_{k\in M} \in \mathbb{P}$, $(y^{*j}, y^{*jB}) \in Y^j$, x^{*i}. $p^{*kB}_{N-1}, p^{*kS}_{N-1} > 0$ by M.1, M.7, and M.8.

Then p^{*kB}_N, $p^{*kS}_N>0$ by arbitrage under M.3. Thus there is a monetary equilibrium for the composite household economy. Then apply the same argument as in the proof of Theorem 1: equilibrium prices of the composite household economy are equilibrium prices of the segmented market economy. QED

Theorem 5 (Demand for fiat money): Assume H.1-H.4, P.1-P.4, M.1-M.8. Let $(p^{*kB}, p^{*kS})|_{k\in M} \in \mathbb{P}$ be quasi-equilibrium prices with p^{*kB}_N, $p^{*kS}_N>0$. Let $x^i \in D^i(p^*)$ and consider $e^i(x^i)$. Then

$$\frac{p^{*k}_N}{p^{*kB}_N} = \frac{p^{*kB}_N - p^{*kS}_N}{p^{*kB}_N} \leq \frac{p^{*kB}_n - p^{*kS}_n}{p^{*kB}_n} = \frac{p^{*k}_n}{p^{*kB}_n} \tag{15}$$

and $$\frac{p^{*k}_N}{p^{*kS}_N} = \frac{p^{*kB}_N - p^{*kS}_N}{p^{*kS}_N} \leq \frac{p^{*kB}_n - p^{*kS}_n}{p^{*kS}_n} = \frac{p^{*k}_n}{p^{*kS}_n} \tag{16}$$

for all $k \in M$, n=1,2,...,N-1. That is, on all markets, both on the buying and selling side, good N has the narrowest proportionate bid/ask spread of any good. Then there is $e^i(x^i)$ so that the only nonnull co-ordinates of $e^i(x^i)$ are in N.

Proof: Apply M.7, Theorem 4 (p^{*kB}_N, $p^{*kS}_N>0$) and Theorem 2. QED

Corollary 1 (to Theorems 3, 4 and 5; Pareto efficiency of Fiat Money Equilibrium): Assume H.1-H.4, P.1-P.4, M.1-M.8. Consider the allocation of the segmented market economy in quasi-equilibrium with prices $(p^{*kB}, p^{*kS})|_{k\in M} \in \mathbb{P}$, p^{*kB}_N, $p^{*kS}_N>0$. Let $x^{*i} \in D^i(p^*)$, $i\in H$, be equilibrium trading plans. Let $p^{*kB}\cdot x^{*ikB}>0$ for some $i\in H$, some $k\in M$. Then $p^{*kB}_N = p^{*kS}_N$ for all $k\in M$, and the equilibrium allocation is Pareto efficient.

Proof: Theorem 4 tells us that there is a quasi-equilibrium. By M.7 and marginal cost pricing we have $p^{*kB}_N = p^{*kS}_N$ and these prices are positive by Theorem 5. Then Theorem 3 implies a Pareto efficient allocation. QED

In Theorem 4 positivity of the price of fiat money, that is p^{*kB}_N, $p^{*kS}_N>0$ for each k, comes from the fiat money structure developed in M.1 through M.8. Good N is desirable since it is desired by G on the same basis as N-1 (M.3) and all households want

N-1 (M.1). These statements hold net of transaction costs since these are small (M.7, M.8). Further the scarcity value of N and N-1 is ensured by limitations on supply (M.2, M.5, M.6). Theorem 5 merely restates Theorem 2 for fiat money. Since fiat money is a low transaction cost instrument, it is priced with the lowest bid/ask spread of any good. Then it will be the universal medium of exchange.

X. Conclusion

A modified Arrow-Debreu general equilibrium model can support monetary trade as a consequence of the equilibrium, not as an assumption, using two constructs in addition to the Arrow-Debreu model. Transaction costs and distinct budget constraints representing budget balance in each of a number of transactions constitute sufficient additional structure. Then commodity money flows are endogenously determined as part of the equilibrium actions of firms and households. In an economy with segmented markets and the resultant multiple budget constraints, goods act as carriers of value between transactions. A price system formalizes the transaction cost structure in bid and ask prices. Liquidity is priced; its price is the bid/ask spread, and the price system provides a direct incentive to concentrate the medium of exchange function on goods with the narrowest bid/ask spread. This results in the commodity money structure of a segmented market equilibrium (Theorems 1 and 2). The monetary equilibrium fulfills Menger (1892)'s research agenda. Positive value of fiat money issued by government can be supported by fiat money's acceptability in payment of taxes (Theorem 4). Fiat money as the common medium of exchange derives from its low transaction cost (Theorem 5). Commodity or fiat money with a zero transaction cost leads to Pareto efficient allocation in equilibrium (Theorem 3 and Corollary 1; see also Starrett (1973)).

Appendix: Foley's Transaction Cost Model and Arrow and Hahn's Treatment of External Effects

Foley (1970) noted the formal equivalence of the existence of a quasi-equilibrium in Debreu (1962) to its existence in a model with a doubled dimension of the commodity space with a convex transaction technology. Arrow and Hahn (1971) noted that the results demonstrating existence of quasi-equilibrium (compensated equilibrium) could be generalized to a model including continuous external effects among households. The combined result below notes that the same logic means that the Foley (1970) result holds in the presence of continuous external effects among households. All of these results apply in a single unified market. Now consider #M separate markets where each household and each firm is allowed to be active in only one market and each household has ownership shares only of the firms in its separate market. The adapted result below extends the combined result to this separated market model: continuous external effects are consistent with a quasi-equilibrium (compensated equilibrium) in the separated market model. Equilibrium of the composite household economy is a special case of this result.

Published Results

(Foley(1970)) Let #M=1, and assume H.1-H.4, P.1-P.4, with no external effects. Then there is a quasi-equilibrium $(p^{*kB}, p^{*kS}) \in \mathbb{R}^{2N}_{+}$.

(Arrow and Hahn (1971)) Let #M=1, and assume H.1-H.4, P.1-P.4, with continuous external effects (each household's utility function is continuous in the consumptions of other households). Let transaction costs be nil. Then there is a compensated equilibrium (quasi-equilibrium) $p^* \in \mathbb{R}^{N}_{+}$.

Combined Result

Assume H.1-H.4, P.1-P.4, with continuous external effects (each household's utility function is continuous in the consumptions of other households). Let #M=1. Then there is a quasi-equilibrium $(p^{*kB}, p^{*kS}) \in \mathbb{R}^{2N}_{+}$.

Adapted Result

Assume H.1-H.4, P.1-P.4, with continuous external effects. Let #M>1 with each firm and household active on only one $k \in M$. Then there is a quasi-equilibrium $(p^{*kB}, p^{*kS})|_{k \in M} \in \mathbb{R}^{2\#MN}_{+}$.

<u>References</u>

Arrow, K. J. and F. H. Hahn (1971), *General Competitive Analysis*, San Francisco: Holden-Day.

Banerjee, A. and E. Maskin (1996), "A Walrasian Theory of Money and Barter," *Quarterly Journal of Economics*, v. CXI, n. 4, November, pp. 955-1005.

Debreu, G. (1959), *Theory of Value,* New Haven: Yale University Press

Debreu, G. (1962), "New Concepts and Techniques for Equilibrium Analysis," *International Economic Review*, v.3, pp. 257-273 (September).

Dubey, P. and J. Geanakoplos (2001), "Inside and Outside Fiat Money, Gains to Trade and IS-LM," Cowles Foundation Discussion Paper no. 1257R, Yale University, June.

Foley, D. K. (1970), "Economic Equilibrium with Costly Marketing," *Journal of Economic Theory*, v. 2, n. 3, pp. 276 - 291. Reprinted in R. Starr, ed., *General Equilibrium Models of Monetary Economies*, San Diego: Academic Press, 1989.

Hahn, F. H. (1971), "Equilibrium with Transaction Costs," *Econometrica*, v. 39, n. 3, pp. 417 - 439. Reprinted in R. Starr, ed., *General Equilibrium Models of Monetary Economies*, San Diego: Academic Press, 1989.

Jevons, W. S. (1875), *Money and the Mechanism of Exchange,* London: Macmillan.

Kiyotaki, N. and R. Wright (1989), "On Money as a Medium of Exchange," *Journal of Political Economy*, v. 97, pp. 927 - 954.

Kurz, M. (1974), "Equilibrium in a Finite Sequence of Markets with Transaction Cost," *Econometrica*, v. 42, n.1, pp. 1 -20. Reprinted in R. Starr, ed., *General Equilibrium Models of Monetary Economies*, San Diego: Academic Press, 1989.

Li, Y., and R. Wright (1998), "Government Transaction Policy, Media of Exchange, and Prices," *Journal of Economic Theory*, v.81, pp. 290-313.

Lucas, R. E. and N. Stokey (1987), "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, v. 55, n. 3 (May), pp. 491-513.

Menger, C. (1892), "On the Origin of Money," *Economic Journal*, v. II, pp. 239-255. Translated by Caroline A. Foley. Reprinted in R. Starr, ed., *General Equilibrium Models of Monetary Economies*, San Diego: Academic Press, 1989.

Ostroy, J. and R. Starr (1990), "The Transactions Role of Money," in B. Friedman and F. Hahn, eds., *Handbook of Monetary Economics*, New York: Elsevier North Holland.

Starr, R. M. (1974), "The Price of Money in a Pure Exchange Monetary Economy with Taxation," *Econometrica*, v. 42, pp. 45-54.

Starr, R. (2001) "Why Is There Money? Endogenous Derivation of 'Money' as the Most Liquid Asset: A Class of Examples," UCSD Economics Dept. Working Paper no. 2000-25R, October.

Starrett, D. A. (1973), "Inefficiency and the Demand for 'Money' in a Sequence Economy," *Review of Economic Studies*, v. XL, n.4, pp. 437 - 448.