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A General Equilibrium Approach

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Predatory Behavior in Vertical Market Structures:
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Introduction

Predatory selling has been evaluated and assessed by antitrust regulators, the courts, and the economics profession.[1] Recently the spotlight has turned to alleged predatory buying.[2] The criteria for determining in output markets whether monopolists or oligopolists are engaged in predatory actions has been debated and various criteria have been expressed both by courts and professional economists. In the case of monopsonists or oligopolists as buyers in input markets, many have argued that the same criteria used to evaluate predatory selling should also hold for predatory buying.[3]

The economic literature has focused the evaluation of predatory conduct on the trade-off between a predator's short run losses and the benefits that might be achieved after its prey is harmed (Telser 1977, Joskow and Klevorick 1979, Easterbrook 1981, Elzinga and Mills 1989 and 1994, McGee 1980, Milgrom and Roberts 1982, Scherer 1976, Williamson 1977). The short run losses suffered by the predator are viewed as an investment incurred that is designed to discipline or eliminate its rivals. This investment is presumed to be motivated by monopoly or monopsony rent seeking. Accordingly, in this two-stage view, the rents or benefits accruing to

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[1] Areeda and Turner (1975), Areeda and Hovenkamp (Supp. 1993), *Brooke Group v. Brown and Williamson Tobacco Cor.,*(1993), Burns (1986), Easterbrook (1981), Elzinga and Mills (1989), Elzinga and Mills (1994), Joskow and Klevorick (1979), *Matsushita Elec. Indus. C. v. Zenith Radio Corp.* (1986), McGee (1980), Milgrom and Roberts (1982), Scherer (1976), *William Inglis and Sons Baking Co. V. ITT Continental Baking Co* (9th Cir. 1982), and Williamson (1977).

[2] Blair and Harrison (1993), Carstensen (2004), Kirkwood (2005), Noll (2005), Salop (2005), Zerbe (2005), and *Weyerhaeuser Company v. Ross-Simmons Hardwood Lumber Co, Inc.*, No.05-381 U.S. (9th Cir. 2006); *Khan v. State Oil Co.,*93 F.3d 1358, 1361 (7th Cir. 1996), 522 U.S. 3 (1997); *Todd v. Exxon Corp.,* 275 F.3d 191, 202 (2nd Cir. 2001); *United States v. Syufy Enters.,* 903 F2d 659, 663 n.4 (9th Cir. 1990); *Houser v.Fox Theaters Mgmt. Corp.,* 854 F.2d 1225, 1228 and 1231 (3rd Cir. 1988); *Betaseed, Inc. v. U and I Inc.,* 681 F.2d 1203, 1221 (9th Cir. 1982).

[3] *Weyerhaeuser Company v. Ross-Simmons Hardwood Lumber Co, Inc.*, No.05-381 U.S. (9th Cir. 2006).

predatory actions can only be rationalized during some recoupment period as clearly stated by

Elzinga and Mills (1994, p. 560):

> In simplest terms, conventional predation occurs in two stages. In the first stage the predator prices at nonrenumerative levels to drive rivals or an entrant from the market or to coerce rivals to cede price leadership to the predator. In the second stage the predator flexes its monopolistic muscles by charging supracompetitive prices and recouping the losses sustained during the initial stage.

The lens used by the courts and much of the antitrust literature on predatory selling

versus predatory selling, however, is based on partial equilibrium methodology. The purpose of

this paper is to determine whether partial equilibrium methodology is robust and can be relied

upon in assessments of predatory monopoly or monopsony conduct. The focus is on related

markets and the role they play in general equilibrium analysis of such conduct. Does the

existence of substitutable versus complimentary products materially change the results implied

by a partial equilibrium analysis?

Given the complexity of a general versus partial equilibrium framework, we isolate the

impact of related markets in a temporally aggregated analysis. In other words, our results are

developed from a static model rather than a two-stage model where the firm with market power

first drives out its competitors and then exercises greater market power than previously in an

open-ended subsequent recoupment stage. While much of the relevant legal literature and court

opinions consider only a two-stage framework as an explanation for overbuying, most such

analyses fail to consider the anticompetitive barriers to reversibility that would be required

during recoupment, versus the re-entry that would otherwise occur following predation.. In

contrast, we show that such conduct is profitably sustainable under certain conditions on a

continual basis (or, by implication, with temporal aggregation under reversibility) using a static

framework where general equilibrium adjustments are considered. Further, we suggest that such

models offer a practical explanation for the substantive impacts of overbuying because two-stage

models do not explain why firms do not re-enter markets just as easily as they leave unless other anticompetitive factors are present.

In the two-stage framework, if a competing firm's best use of its resources is to produce a particular product under competitive pricing but finds switching to production of an alternative to be optimal when a predatory buyer drives up its input price, then its optimal action is to return to its first best use of resources as soon as the predatory behavior is reversed. Thus, unless this competitive readjustment is artificially prevented, such as by buying up fixed production resources, two-stage predatory behavior cannot be optimal. Thus, proving two-stage predatory behavior should require identification of an artificial barrier to other firms' re-entry or return to previous production levels in the recoupment period. Alternatively, the conditions outlined in this paper would be required for a temporal aggregation of the two-stage problem presuming, of course, that predatory behavior is optimal for any firm.

Fundamentally, we suggest that understanding of the general equilibrium outcomes of the single-stage static model, which implicitly assumes reversibility, is needed before a full understanding of two-stage possibilities can be achieved. In this paper, we present such a static general equilibrium framework. After specifying the general equilibrium model and the competitive equilibrium benchmark, we turn to the general case where a single firm or colluding group of firms has potential market power in both their input and output markets. We develop a number of results that turn on characteristics of technologies of competing industries and the characteristics of input supplies and output demands including the degree of substitutability or complementarity. In these cases we find specific conditions where overbuying can occur profitably. Interestingly, however, profitable overbuying in this model can occur on a continuing basis so that a predatory period may not be evidenced by losses such as are used as a prerequisite for predatory behavior by the courts. Further, we find that a mirror image of this behavior in

terms of overselling is not possible. Finally we present the case of *naked overbuying* as a means of exercising market power.

Equilibrium Analysis of Economic Welfare

To assess the consequences of possible predatory conduct, we use the approach advanced by Just, Hueth, and Schmitz (2004, pp. 355-361) for comparison of welfare effects where equilibrium adjustments occur across many markets as well as many types of consumers and producers. This approach permits an analysis of indirect equilibrium adjustments that determine the implications of monopolistic behavior in markets that are interdependent with other markets. Such a framework can explain seemingly extreme monopoly behavior including overbuying even in static models where recoupment periods are not necessary. Before developing specific results for the market structure considered in this paper, we summarize the underlying equilibrium measurement of welfare.

Assumption 1. Suppose each of *J* utility-maximizing consumers has exogenous income m_j, and is endowed with a nonnegative *N*-vector of resources r_j, has monotonically increasing, quasiconcave, and twice differentiable utility $U_j(c_j)$, where c_j is a corresponding nonnegative *N*-vector of consumption quantities, the budget constraint is $p(c_j - r_j) = m_j$, and p is a corresponding *N*-vector of prices faced by all consumers and firms in equilibrium.

Assumption 2. Suppose each of *K* firms maximizes profit pq_k given an implicit multivariate production function $f_k(q_k) = 0$ where q_k is an *N*-vector of netputs ($q_{kn} > 0$ for outputs and $q_{kn} < 0$ for inputs) where each scalar function in f_k is monotonically increasing, concave, and twice differentiable in the netput vector, other than for those netputs that have identically zero marginal effects in individual equations (allowing each production process to use a subset of all goods as inputs producing a different subset of all goods as outputs).

Proposition 1. *Under Assumptions 1 and 2, the aggregate equilibrium welfare effect (sum of compensating or equivalent variations in the case of compensated demands evaluated at ex ante or ex post utility, respectively) of moving from competitive pricing to distorting use of market power in a single market n is given by*

$$\Delta W = \int_{p_n^s(0)}^{p_n^s(\delta)} q_n^s(p_n) dp_n - \int_{p_n^d(0)}^{p_n^d(\delta)} q_n^d(p_n) dp_n$$

where $q_n^s(\cdot)$ is the aggregate equilibrium quantity supplied of good n, and $q_n^d(\cdot)$ is the aggregate equilibrium quantity demanded of good n, $\delta = p_n^d(\delta) - p_n^s(\delta)$ is the effective price distortion introduced in market n, and $p_n^s(\delta)$ and $p_n^d(\delta)$ represent the respective marginal cost and marginal benefit of good n considering all equilibrium adjustments in other markets in response to changes in δ.

Proof: See Just, Hueth, and Schmitz (2004, pp. 355-361).

Proposition 1 allows an account of equilibrium adjustments that occur throughout an economy in response to the distortion in a single market. Further, the welfare effects (compensating or equivalent variation) of a change in δ can be measured for individual groups of producers using standard estimates of profit functions and for individual groups of consumers using standard estimates of expenditure or indirect utility functions by evaluation at the initial and subsequent equilibrium price vectors.[4] If other markets are distorted, then this result can be modified accordingly (Just, Hueth, and Schmitz 2004, pp. 361-365) but, in effect, only the case of a single distortion is needed for results in this paper.

The graphical implications of Proposition 1 are presented in Figure 1. With no distortion, equilibrium in market *n* is described by the intersection of ordinary supply, $\overline{q}_n^s(p_n, \tilde{p}(0))$, and ordinary demand, $\overline{q}_n^d(p_n, \tilde{p}(0))$, where $\tilde{p}(0)$ denotes conditioning on all other equilibrium

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[4] In the case of indirect utility functions, the welfare effects are not measured by the change in the function. Rather, compensating variation, CV, is defined by $V(\boldsymbol{p^1}, m_j^1 - \text{CV}) = V(\boldsymbol{p^0}, m_j^0)$ and equivalent variation, EV, is defined by $V(\boldsymbol{p^1}, m_j^1) = V(\boldsymbol{p^0}, m_j^0 + \text{EV})$ where V is the indirect utility function and superscripts 0 and 1 represent initial and subsequent equilibrium conditions.

prices throughout the economy under no distortions, i.e., when $\delta = 0$.[5] If the distortion $\delta = \delta_0$ is introduced in market n, then after equilibrium adjustments throughout the economy, ordinary supply shifts to $\bar{q}_n^s(p_n, \tilde{p}(\delta_0))$ and ordinary demand shifts to $\bar{q}_n^d(p_n, \tilde{p}(\delta_0))$, which are conditioned on prices throughout the economy with a specific distortion, $\delta = \delta_0$, in market n. The effective general equilibrium supply and demand relationships that implicitly include equilibrium adjustments throughout the economy in response to changes in the distortion δ are $q_n^s(p_n^s(\delta))$ and $q_n^d(p_n^d(\delta))$, respectively.

With monopoly pricing in market n, $p_n^d(\delta)$ represents the equilibrium market n price, and $\delta = p_n^d(\delta) - p_n^s(\delta)$ represents the difference in price and general equilibrium marginal revenue, *EMR*. This marginal revenue is not the marginal revenue associated with either the ordinary demand relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopoly problem, it is the marginal revenue associated with the general equilibrium demand, $q_n^d(p_n^d(\delta))$, that describes how price responds with equilibrium adjustments throughout the economy in response to changes in the market n distortion. In this case, $q_n^s(p_n^s(\delta))$ represents how marginal cost varies with equilibrium adjustments in other markets, so marginal cost is equated to *EMR* at $q_n^d(p_n^d(\delta_0))$.

With monopsony, $p_n^s(\delta)$ represents the equilibrium market n price and $\delta = p_n^d(\delta) - p_n^s(\delta)$ represents the difference in the general equilibrium marginal outlay, *EMO*, and price. This marginal outlay is not the marginal outlay associated with either the ordinary supply relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopsony problem, it is the marginal outlay associated with the general equilibrium supply, $q_n^s(p_n^s(\delta))$, that describes how price responds with equilibrium adjustment throughout the economy to changes in the market n distortion. In this case, $q_n^d(p_n^d(\delta))$ represents

how marginal revenue varies with equilibrium adjustments in other markets, so marginal revenue is equated to *EMO* at $q_n^s(p_n^s(\delta_0))$.

The application of this result to the related market structure of this paper is illustrated simplistically for the case of perfect substitutes in demand for final products and perfect substitutes in supply of inputs in Figure 2. Suppose in Figure 2(a) that output demand jointly facing two products or industries is $p(y + z)$ where y and z are the quantities sold of each of the products. Suppose also that both products or industries use the same input in production and thus the input supply jointly facing the two industries is $w(x_y + x_z)$ where x_y and x_z are the respective quantities of the input used by the two industries (both the input and output market are represented on the same diagram, assuming for the graphical analysis that the production process transforms the input unit-for-unit into outputs). If the y industry consists of a single firm whereas industry z is a competitive industry, we have the dominant-firm-competitive-fringe structure as a special case of Figure 2. Figure 2(b) represents the competitive response of production activity in the z industry as a function of the difference in the input and output price. Specifically, supply at the origin of Figure 2(b), shown in reverse, is the point at which the corresponding difference in prices in (a) is just high enough that the z industry would start to produce. Suppose with increasing marginal cost for the z industry that at output price p^0 and input price w^0 the z industry uses input quantity x_z^0. The corresponding excess demand, *ED*, and excess supply, *ES*, to the y industry are shown in Figure 2(a). Note that the two vertical dotted lines in Figure 1(a) sum to the vertical dotted line in Figure 2(b).

To maximize profits, the y industry can use the input supply and excess supply relationships directly from Figure 2(a) as shown in Figure 2(c). For comparability, the output demand and excess demand relationships from Figure 2(a) must be transformed into input price equivalents by inversely applying the production technology of the y industry for purposes of determining how much to produce. That is, where $y = y(x_y)$ is the production function of the y

industry and $x_y = y^{-1}(y)$ is the associated inverse function, the equivalent input demand D^* in Figure 2(c) is found by substituting the demand relationship in Figure 2(a) into $y^{-1}(\cdot)$. The equivalent excess demand, ED^*, in Figure 2(c) is found similarly. Then the y industry maximizes profit by equating the general equilibrium marginal revenue, MR^*, associated with ED^*, and the general equilibrium marginal outlay, MO, associated with the excess supply.

The core insights in this paper arise because the production technologies for the two industries may not be similar and may not be unit-for-unit technologies. In contrast to the traditional monopoly-monopsony result where market quantities are restricted to increase profits, equilibrium adjustments can cause displacement of the z industry by the y industry in the case of overbuying. Moreover, these results are modified when the outputs are not perfect substitutes in demand or the inputs are not purchased from the same market but in related markets.

The Model

Based on the general economy model, we are now in a position to evaluate a related market structure that exists within the general economy. To abstract from the complications where compensating variation does not coincide with equivalent variation (nor with consumer surplus), consumer demand will be presumed to originate from a representative consumer, and that prices of all goods, other than two related goods of interest, are set by competitive conditions elsewhere in the economy. As a result, expenditures on other goods can be treated as a composite commodity, n, which we call the numeraire. More concretely, suppose that demand is generated by maximization of a representative consumer utility that is quasilinear in the numeraire, $u(y,z)+n,$ where y and z are non-negative consumption quantities of the two goods of interest

and standard assumptions imply $u_y > 0$, $u_z > 0$, $u_{yy} < 0$, $u_{zz} < 0$, and $u_{yy}u_{zz} - u_{yz}^2 \geq 0$ where subscripts of u denote differentiation.[6]

Suppose the consumer's budget constraint is $p_y y + p_z z + n = m$ where p_y and p_z are prices of the respective goods and m is income. Substituting the budget constraint, the consumer's utility maximization problem becomes $\max_{y,z} u(y,z) + m - p_y y - p_z z$. The resulting first-order conditions yield the consumer demands in implicit form,

(1) $p_y = u_y(y,z)$

(2) $p_z = u_z(y,z)$.

Downward sloping demands follow from the concavity conditions, $u_{yy} < 0$ and $u_{zz} < 0$. The two goods are complements (substitutes) in demand if $u_{yz} > (<) \, 0$.

Suppose the two goods, y and z, each has one major input. For simplicity and clarity, suppose the quantities of any other inputs are fixed. Thus, the respective production technologies can be represented by

(3) $y = y(x_y)$

(4) $z = z(x_z)$

where x_y and x_z represent the respective input quantities and standard assumptions imply $y' > 0$, $y'' < 0$, $z' > 0$, and $z'' < 0$, where primes denote differentiation.

Suppose the inputs are related in supply so that the industries or products compete for inputs as well as sales of total output. To represent the related nature of supply, suppose the

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[6] While the weaker assumption of quasi-concavity can be assumed for consumer problems, we use the more restrictive assumption that $u_{yy}u_{zz} - u_{yz}^2 \geq 0$ to attain symmetry of the mathematical analysis, which saves space and enhances intuition..

respective inputs are manufactured by a third competitive industry with cost function $c(x_y, x_z)$. [7]

Thus, input supplies in implicit form follow

(5) $w_y = c_y(x_y, x_z)$

(6) $w_z = c_z(x_y, x_z)$

where y and z subscripts of c represent differentiation with respect to x_y and x_z, respectively, and

standard assumptions imply $c_y > 0$, $c_z > 0$, $c_{yy} > 0$, $c_{zz} > 0$, and $c_{yy}c_{zz} - c_{yz}^2 \geq 0$, where

$c_{yz} > (<) \, 0$ if x_y and x_z are substitutes (complements) in supply. [8] For convenience, we also define

$x_z = \hat{c}(w_y, x_z)$ as the inverse function associated with $w_y = c_y(x_y, x_z)$, which implies

$\hat{c}_w \equiv 1/c_{yz} > (<) \, 0$ and $\hat{c}_x \equiv -c_{yy}/c_{yz} < (>) \, 0$ if x_y and x_z are substitutes (complements).

Suppose that the z industry always operates competitively as if composed of many firms.

The profit of the z industry is $\pi_z = p_z \cdot z(x_z) - wx_z$. The first-order condition for profit

maximization requires

(7) $w_z = p_z z'(x_z)$.

The second-order condition for a maximum is satisfied because $z'' < 0$ and prices are regarded

as uninfluenced by the firm's actions.

Finally, suppose behavior of the y industry is given by

(8) $\max_{x_y} \pi_y = p_y \cdot y(x_y) - w_y x_y$.

Equations (1)-(7) are sufficient to determine the general equilibrium supply and demand

relationships facing the y industry. A variety of cases emerge depending on market structure and

the potential use of market power by the y industry.

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[7] This industry may represent a hypothetical firm formed by aggregating the behavior of many producers under competitive conditions.

[8] For the special case where $c_{yy}c_{zz} - c_{yz}^2 = 0$, which is not normally admitted in standard convexity conditions, we introduce a concept of perfect substitutes in supply where, in effect, $c(x_y, x_z)$ becomes $c(x_y + x_z)$ and $c(\cdot)$ is a convex univariate function.

Competitive Behavior

If the y industry is composed of many firms that do not collude, then the first-order condition for (8) requires

(9) $\quad w_y = p_y y'.$

As for the z industry, the second-order condition is satisfied because $y'' < 0$ and prices are regarded as uninfluenced by firm actions. This yields the case where $\delta = 0$ in Figure 1.

Focusing on the y industry for given x_y, the system composed of (1)-(7) can be reduced to a two equation system that describes the general equilibrium input supply and output demand facing the y industry, viz.,

(10) $\quad p_y = u_y(y(x_y), z(\hat{c}(w_y, x_y)))$

(11) $\quad c_z(x_y, \hat{c}(w_y, x_y)) = u_z(y(x_y), z(\hat{c}(w_y, x_y)))z'(\hat{c}(w_y, x_y)).$

Equations (10) and (11) define implicitly the general equilibrium supply and demand relationships for the y industry. Because (10) and (11) are not in explicit form, comparative static methods can be used to determine

(12) $\quad \dfrac{dp_y}{dx_y} = u_{yy}y' + \dfrac{u_{yz}z'}{c_{yz}}\left[\dfrac{dw_y}{dx_y} - c_{yy}\right]$

(13) $\quad \dfrac{dw_y}{dx_y} = c_{yy} + \dfrac{(c_{yz} - u_{yz}y'z')c_{yz}}{\pi_{zz}}$

where throughout this paper we define for notational simplicity $\pi_{zz} \equiv u_{zz}z'^2 + u_z z'' - c_{zz} < 0,$ which is the marginal effect of x_z on the first-order condition of the z industry given demand for z and supply of x_z. The relationships in (12) and (13) implicitly define the input and output prices for the y industry as a function of its input level x_y, or equivalently in terms of its output level, $y = y(x_y)$.

The relationship in (12) is a core effect in this paper that measures the effect of an increase in the purchased quantity of the y industry's input on the y industry's output demand through its indirect effect transmitted through the z industry markets. If more of the y industry's input is purchased, then its input price is bid up, the supply of a competing input produced for the z industry (which is a substitute output for input suppliers) is reduced, the production activity of the z industry is then reduced, and the reduction in z output causes the demand for y to increase (decrease) if y and z are substitutes (complements) in demand. This effect can be compared to the direct effect on the price of the y industry's input in maximizing profit if the y industry consists of a single firm with market power. However, with competitive behavior by the y industry, condition (9) together with (10) and (11) defines the competitive equilibrium output price $p_y = \overline{p}_y$, input price $w_y = \overline{w}_y$, and input quantity $x_y = \overline{x}_y$, where other equilibrium quantities and prices follow from $\overline{y} = y(\overline{x}_y)$, $\overline{x}_z = \hat{c}(\overline{w}_y, \overline{x}_y)$, $\overline{z} = z(\overline{x}_z)$, $\overline{w}_z = c_z(\overline{x}_y, \overline{x}_z)$, and $\overline{p}_z = u_z(\overline{y}, \overline{z})$.

The results of our analysis depend critically on whether ordinary demands and supply are conditioned on prices or quantities in related markets. Intuitively, a price-dependent ordinary demand (or supply) is more elastic than its quantity-dependent counterpart because it allows the consumer (or supplier) to readily shift to other markets in response to a price change. Furthermore, price-dependent ordinary demand (or supply) is more elastic than its general equilibrium counterpart because it assumes, in effect, that prices in other markets are not affected by those shifts. A quantity-dependent ordinary demand (or supply) is less elastic than its general equilibrium counterpart because it does not consider the quantity adjustments in other markets.

Which specification provides the right standard of comparison depends on the empirical facts. In particular, this specification issue depends on how relevant business managers assess output demand and input supply, or how economists, lawyers, and courts estimate demand and supply relationships when investigating the use of market power. Such empirical facts have been

originally used by Okun (1975) and Dornbusch (1976) to divide the general economy into two types of markets: contract or sticky price markets and flex-price markets. The former markets adjust slowly to shocks while the latter markets adjust quickly. Typically, a prevalence of contract-type markets is found among upstream industries while downstream transactions, especially at the retail or consumer level, are dominated by flex-price markets.

Accordingly, in the case of consumer market demand relationships, ordinary demands are typically estimated as conditioned on prices in other markets. Popular second-order flexible demand system specifications as well as typical ad hoc specifications used for estimation by economists are dependent on the prices of competing products. Also, typical business practices conduct retail reconnaissance to determine competitors' pricing in assessing final good market opportunities. Additionally, price data on final goods markets are readily available and typically abundant, in many cases from public sources, to facilitate conducting price-dependent analysis.

In the case of supply relationships, economists have also developed second-order flexible supply system specifications that depend on prices of competing products. However, such analyses are typically infeasible because price data on intermediate goods are often unavailable and even impossible to collect for less competitive markets where the use of market power is an issue. Furthermore, such models may not apply in the short run when transactions are limited by contractual relationships. Many of these transactions take place by contract where prices from suppliers are closely guarded proprietary secrets. On the other hand, trade organizations often publish some form of quantity data and some market information services offer such data commercially. In some cases, import-export records or other public regulatory agencies reveal quantity data. Moreover, while lawyers and expert witnesses typically have access to the proprietary data of their clients and opponents in legal proceedings, access to proprietary data of indirectly related industries usually cannot be made available under court jurisdiction. In these circumstances, estimation or assessment of price-dependent ordinary supply is infeasible. Thus,

conventional partial equilibrium analysis is often feasible only for the quantity-dependent ordinary supply specification.

The pervasiveness of supply contracts in primary and intermediate goods markets (as compared with final goods markets) suggests that quantity dependence may be more appropriate for short-run input supply analysis because contracts take time to change, particularly when changes benefiting both parties cannot be identified. Even in absence of contracting, the threat of competitive retaliation can cause input markets to function as if quantity-restricting supply contracts were present. For example, business executives often hold the view that they will not invite retaliation as long as they do not invade the territorial market of another firm or as long as their share of some measure of market activity does not exceed a given level. Finally, producers of intermediate goods typically have large investments in plant and equipment that have fixed capacities that limit short-term quantity adjustments. As a result, assessments of ordinary supply in intermediate markets are more likely to be conditioned on market quantities than on market prices.

In contrast, final goods purchasing involves little contracting except for financing consumer durables. Consumers are typically free to adjust quantities instantaneously in response to minor price changes. Thus, final goods prices are more likely regarded as the key determinants of final good demands, which is another reason why typical estimates of consumer demands are specified as price dependent. Regardless, in the results that we present we analyze both the cases of price as well as quantity dependent ordinary demands as well as ordinary supplies.

Market Power in Both Input and Output Markets

For the purpose of deriving the core results, we introduce the following definition, which facilitates a shorthand notation representing the strength of substitution in input markets versus complementarity in output markets, upon which many results depend.

Definition. *Define* $S = c_{yz} - u_{yz} y' z'$ *as the measure of input substitution relative to output complementarity where complementary is represented by the additive inverse of substitution as measured by the cross derivative of consumer utility or input industry cost. If* $S > (<) 0$ *then inputs are more (less) substitutes in supply than outputs are complements in demand (which also includes the case where outputs are substitutes in demand), or inputs are less (more) complements in supply than outputs are substitutes in demand. Similarly, if* $S < (>) 0$ *then outputs are more (less) complements in demand than inputs are substitutes in supply (which also includes the case where inputs are complements in supply), or outputs are less (more) substitutes in demand than inputs are complements in supply. All cases where* $S > (<) 0$ *will be described as having input substitution greater (less) than output complementarity. If both inputs and outputs are substitutes (complements), then* $S > (<) 0$.

The intuition of this definition follows from noting that c_{yz} is the cross derivative of the cost function of the supplying industry with respect to the two input quantities, while $u_{yz} y' z'$ is the cross derivative of consumer utility with respect to the two input quantities after substituting the production technologies, $u(y, z) = u(y(x_y), z(x_z))$. For simplicity, the relationship of inputs will always refer to input supply and the relationship of outputs will always refer to output demand.

Again, for given x_y the system composed of (1)-(7) can be reduced to the two equation system in (10) and (11), which generates (12) and (13), for which further manipulation reveals

(12') $\quad \dfrac{dp_y}{dx_y} = u_{yy} y' + \dfrac{u_{yz} z' S}{\pi_{zz}} = p_{yy} y' > (<) 0 \text{ as } u_{yz} S < (>) -u_{yy} \pi_{zz} y' / z'$

(13') $\quad \dfrac{dw_y}{dx_y} = c_{yy} + \dfrac{c_{yz} S}{\pi_{zz}} = s_{yy} > (<) 0 \text{ as } c_{yz} S < (>) -c_{yy} \pi_{zz}.$

An interesting aspect of these results is that the general equilibrium demand is not necessarily more or less elastic than the ordinary demand. From (12'), p_{yy} differs from u_{yy} by

$$\text{(14)} \quad p_{yy} - u_{yy} = \frac{u_{yz} z' S}{\pi_{zz} y'} > (<) \ 0 \text{ as } u_{yz} S < (>) \ 0.$$

Proposition 2 (Quantity Conditioning). *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium demand relationship facing the concentrated industry is less (more) elastic than the ordinary demand conditioned on quantity in the related output market if outputs are complements and input substitution is greater (less) than output complementarity, or outputs are substitutes and input substitution is less (greater) than output complementarity. In particular, the general equilibrium demand relationship is more elastic than the quantity-dependent ordinary demand if either both inputs and outputs are substitutes or both are complements.*

Intuitively, when inputs are substitutes, monopsonizing the x_y input market by reducing purchases causes an increase in supply of inputs to the z industry and thus an increase in z industry output, which, if y and z are complements, causes an increase in demand for y that permits further exploitation by the y industry in its output market. Thus, the general equilibrium demand is less elastic than where these adjustments are ignored. Conversely, when inputs are complements, monopsonizing the x_y input market by reducing purchases causes a reduction in supply of inputs to the z industry and thus a decrease in z industry output, which, if y and z are substitutes, causes an increase in demand for y that permits further exploitation. Thus, the general equilibrium demand is less elastic than where these adjustments are ignored just as in the case where inputs are substitutes and outputs are complements.

Alternatively, the elasticity of the general equilibrium demand can be compared to the elasticity of the more common ordinary demand conditioned on price in the related market, viz

$$\text{(15)} \quad p_{yy} - p_{yy}^* = \frac{u_{yz}^2 y' \pi_{zz} + u_{zz} u_{yz} z' S}{\pi_{zz} u_{zz} y'} > (<) \ 0 \text{ as } u_{yz} S < (>) \ -\frac{u_{yz}^2 \pi_{zz} y'}{u_{zz} z'}$$

$$\text{or equivalently as } c_{yz} u_{yz} < (>) \ (u_{yy} - p_{yy}^*)(c_{zz} - u_z z'')(y'/z').$$

where p_{yy} is the slope of the typical price-dependent ordinary demand for y.

Proposition 2' (Price Conditioning). *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium demand relationship facing the concentrated industry is less elastic than the price-conditioned ordinary demand in every case where it is less elastic than the quantity-conditioned ordinary demand, and is more likely to be so as (i) the difference in the quantity-conditioned and price-conditioned demand elasticities is greater, (ii) the marginal productivity of the competitive industry is smaller compared to the concentrated industry, (iii) the marginal productivity of the competitive industry is more rapidly diminishing, and (iv) the competitive sector's marginal input cost is more rapidly increasing. In particular, the general equilibrium demand relationship is less elastic than the price-dependent ordinary demand if either inputs are substitutes while outputs are complements or outputs are substitutes while inputs are complements.*

The intuition of additional conditions in this proposition is as follows. The indirect price effects through the *z* industry of a reduction in purchasing of x_y in the case of input substitutes tends to cause a larger increase in the price of x_z when the marginal cost of x_z is increasing more rapidly (for a given effect of x_y on that marginal cost). Further, the increase in the price of x_z tends to be translated into a larger increase in the price of *z* if the marginal productivity in the *z* industry is diminishing more rapidly. Also, the transmission of effects of changing x_y through the z industry tends to be relatively greater than through the *y* industry as the marginal productivity in the z industry is relatively greater than the marginal productivity in the *y* industry. Finally, as the difference in quantity-conditioned and price-conditioned demand elasticities given by $u_{yy} - p_{yy}^* = u_{yz}^2/u_{zz}$ is greater, the cross-price effects on the *y* market arising from the *z* industry are greater making the general equilibrium demand less elastic. Similar reasoning applies to the case where inputs are complements and outputs are substitutes except that the *z* industry declines.

A further interesting and peculiar nature of the equilibrium relationship in (12') is that the general equilibrium demand facing the y industry is not necessarily downward sloping. In fact, comparing to (14) as π_{zz} approaches zero, the condition for $p_{yy} > 0$ becomes the same as for $p_{yy} > u_{yy}$. Recalling that $\pi_{zz} \equiv u_{zz}z'^2 + u_z z'' - c_{zz}$, this is the case where the technologies that produce z and x_z approximate linearity ($z'' \approx 0$, $c_{zz} \approx 0$) and consumer demand for z approximates linearity ($u_{zz} \approx 0$).

Proposition 3. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and production and demand in the competitive industry are approximately linear, the general equilibrium demand becomes upward sloping if outputs are substitutes and input substitution is greater than output complementarity, or outputs are complements and input substitution is less than output complementarity.*

While upward sloping demands are generally counterintuitive according to accepted economic wisdom, the possibility exists with general equilibrium adjustment when the effects of adjustment are transmitted more effectively through the competitive industry than the concentrated industry. Consider the case where the y industry increases production and input use. Intuitively, when inputs are substitutes, increasing input purchases causes a reduction in supply of inputs to the z industry and thus a reduction in z industry output, which, if y and z are substitutes, causes an increase in demand for y. If this transmission of effects through the z industry is sufficiently effective, e.g., because marginal productivity in the y industry is relatively low, then this upward pressure on the demand for y can be greater than the downward pressure on p_y caused by the increase in y output. If so, then the general equilibrium demand for y is upward sloping.

To examine plausibility of the conditions in Proposition 3 and later results, we will consider extreme but plausible cases of substitution and complementarity. For perfect substitutes in demand, we consider the specific case where the utility function takes the form $u(y,z) = u(y +$

z). Thus, demand in implicit form satisfies $p_y \equiv p_z \equiv p = u'(y + z)$ in which case

$u_y \equiv u_z \equiv u' > 0$ and $u_{yy} \equiv u_{zz} \equiv u_{yz} \equiv u'' < 0$. In this case, $u_{yz} < 0$ and the assumption

$u_{yy}u_{zz} - u_{yz}^2 \geq 0$ is satisfied with strict equality. With perfect substitutes in demand, both

industries effectively sell into the same market. Conversely, we define perfect complements in

demand as the case where $u_{yz} > 0$ and $u_{yy}u_{zz} - u_{yz}^2 \geq 0$ is satisfied with strict equality. For

simplicity in this case we assume $u_{yy} \equiv u_{zz} \equiv -u_{yz} \equiv u'' < 0$. While various other definitions of

perfect complements are used in standard consumer theory, this case is sufficient to demonstrate

plausibility of certain possibilities, and this terminology simplifies subsequent discussion.[9]

Nevertheless, more general results are also indicated parenthetically for cases where $u_{yy} \neq u_{zz}$.

Similarly, let the case where x_y and x_z are perfect substitutes in supply be defined by the

case where $c(x_y, x_z) = c(x_y + x_z)$. With perfect substitutes in supply, both industries effectively use

the same input in their respective production processes. Thus, supply of the input in implicit

form becomes $w_y \equiv w_z \equiv w = c'(x_y + x_z)$ in which case $c_y \equiv c_z \equiv c' > 0$, $c_{yy} \equiv c_{zz} \equiv c_{yz} \equiv c'' > 0$,

$\hat{c}_w \equiv 1/c''$, and $\hat{c}_x \equiv -1$. This is the extreme case of positive c_{yz} where the assumption

$c_{yy}c_{zz} - c_{yz}^2 \geq 0$ is satisfied with strict equality. Similarly, we define as perfect complements in

supply the case where c_{yz} is negative and $c_{yy}c_{zz} - c_{yz}^2 \geq 0$ is satisfied with strict equality. For

simplicity in stating results, we assume in this case that $c_y \equiv c_z \equiv c' > 0$ and

$c_{yy} \equiv c_{zz} \equiv -c_{yz} \equiv c'' > 0$.[10] But more general results are also indicated parenthetically for cases

where $c_{yy} \neq c_{zz}$.

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[9] A typical approach defines perfect complements as the case where consumption must occur in fixed-proportions, in which case derivatives of the utility function are discontinuous (or more generally where a unique combination of goods is consumed at each indifference level). We use a weaker definition of perfect complements that maintains continuity of second-order derivatives of the utility function and limits the degree of complementarity by concavity of the utility function, which permits deriving the cases of complements and substitutes simultaneously. However, identical qualitative results can be derived for separate analyses of the more typical case of perfect complements involving fixed-proportions consumption.

[10] Various definitions of perfect complements are used in the production literature as well. A typical approach is to represent perfect complements by fixed-proportions production, in which case the cost function has discontinuous derivatives. Another approach considers production of a composite good, say x, such that the production of x_y and x_z are each monotonically increasing functions of x. Here we maintain continuity of second-order derivatives of the cost function by limiting complementarity by convexity of the cost function so the results for complements and

To see that each of the conditions of Proposition 3 are plausible, suppose that the z industry technology is linear and both inputs and outputs are perfect complements or both are perfect substitutes. Then the condition in (12') can be expressed as

$$(u_{yy}u_{zz} - u_{yz}^2)y'z'^2 + u_{yy}y'(u_z z'' - c_{zz}) + c_{yz}u_{yz}z' = u''c''(z' - y') < (>) \ 0 \text{ as } z' > (<) \ y'.$$

While the first left-hand term is generally non-negative, it vanishes with perfect substitutes or perfect complements in demand. The third left-hand term is negative if both inputs and outputs are complements or both are substitutes and dominates the second term in the case where both inputs and outputs are perfect substitutes or perfect complements (or more generally where $c_{zz}u_{yy} \geq c_{yz}u_{yz}$) if $z'' = 0$ and $z' > y'$. In this case, from (12'),

$p_{yy} = (z'/y' - 1)u''c''/(u''z'^2 - c'') > 0$ if $z' > y'$. Thus, the general equilibrium demand is upward sloping if the marginal productivity in the z industry is higher than in the y industry. With these results, Proposition 3 can be restated.

Proposition 3'. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, and the competitive industry technology is approximately linear, the general equilibrium demand becomes upward sloping if both inputs and outputs are sufficiently strong substitutes or both are sufficiently strong complements, and marginal productivity in the competitive industry is sufficiently higher than in the concentrated industry.*

Similarly, the general equilibrium supply is not necessarily more or less elastic than the quantity-dependent ordinary supply. From (13'), s_{yy}, differs from c_{yy} by

$$(16) \quad s_{yy} - c_{yy} = \frac{c_{yz}S}{\pi_{zz}} > (<) \ 0 \text{ as } c_{yz}S < (>) \ 0.$$

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substitutes can be derived simultaneously. However, identical qualitative results can be derived from a separate analysis of fixed-proportions production of inputs as long as conflicting fixed proportions are not imposed on consumption.

Proposition 4. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium supply relationship facing the concentrated industry is more (less) elastic than the ordinary supply conditioned on quantity in the related output market if inputs are substitutes and input substitution is greater (less) than output complementarity, or inputs are complements and input substitution is less (greater) than output complementarity. In particular, the general equilibrium supply relationship is more elastic than the quantity-dependent ordinary supply if either both inputs and outputs are substitutes or both are complements.*

Intuitively, when outputs are substitutes, monopolizing the *y* output market by reducing the quantity sold causes in increase in demand for the output of the *z* industry and thus an increase in z industry output and input use, which, if x_z and x_y are substitutes, causes a reduction in supply of x_y that reduces the benefit of monopsonistic exploitation by the *y* industry in its input market. Thus, the general equilibrium supply is more elastic than where these adjustments are ignored. Conversely, when inputs are complements, monopolizing the *y* output market by reducing the quantity sold causes a reduction in demand for the output of the z industry and thus a decrease in z industry output and input use, which, if x_z and x_y are complements, causes a reduction in supply of x_y that also reduces the benefit of monopsonistic exploitation by the *y* industry in its input market. Thus, the general equilibrium supply is more elastic than where these adjustments are ignored in this case as well.

Alternatively, the elasticity of the general equilibrium supply can be compared to the elasticity of the price-dependent ordinary supply, viz

$$s_{yy} - s_{yy}^* = \frac{c_{yz}S}{\pi_{zz}} + \frac{c_{yz}^2}{c_{zz}} > (<) \ 0 \text{ as } c_{yz}S < (>) - \frac{c_{yz}^2\pi_{zz}}{c_{zz}}$$

or equivalently as $c_{yz}u_{yz} > (<) \ (c_{yy} - s_{yy}^*)(u_{zz}z'^2 + u_z z'')/(y'z').$

where s_{yy}^* represents the slope of the price-dependent ordinary supply that is conditioned on w_z.

Proposition 4' (Price Conditioning). *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium supply relationship facing the concentrated industry is less elastic than the price-conditioned ordinary supply in every case where it is less elastic than the quantity-conditioned ordinary supply, and is more likely to be so as (i) the difference in the quantity-conditioned and price-conditioned supply elasticities is greater, (ii) the marginal productivity of the concentrated industry is relatively smaller, (iii) the marginal productivity of the competitive industry is more rapidly diminishing, and (iv) consumers have more rapidly diminishing marginal utility of the competitive good. In particular, the general equilibrium supply relationship is less elastic than the price-dependent ordinary demand if either inputs are substitutes while outputs are complements, or outputs are substitutes while inputs are complements.*

The intuition of the additional conditions in Proposition 4' is as follows. The indirect price effects on the z industry price of a reduction the quantity of y sold tend to be greater when consumers have more rapidly diminishing marginal utility in the competitive good. Further, if the marginal productivity in the z industry is more rapidly diminishing, then the increase in z industry activity widens the margin between input and output prices. Again, the transmission of effects through the y industry tend to be relatively less as marginal productivity in the y industry is relatively less, and this is particularly true relative to marginal productivity in the z industry when $z'' = 0$. Finally, as the difference in quantity-conditioned and price-conditioned supply elasticities given by $c_{yy} - s^*_{yy} = c^2_{yz}/c_{zz}$ is greater, the cross-price effects on the x_y market arising from the z industry are greater making the general equilibrium supply less elastic. Similar reasoning applies to the case where outputs are complements and inputs are substitutes except z industry activity declines.

Further, the general equilibrium supply facing the y industry in (13') is not necessarily upward sloping. In fact, comparing to (15), as π_{zz} approaches zero, the condition for $s_{yy} < 0$

becomes the same as for $s_{yy} < c_{yy}$. This is the case where the technologies that produce z and x_z are nearly linear $(z'' \approx 0, c_{zz} \approx 0)$ and consumer demand for z is nearly linear $(u_{zz} \approx 0)$.

Proposition 5. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, and production and demand in the competitive industry are approximately linear, the general equilibrium supply becomes downward sloping if inputs are complements and input substitution is greater than output complementarity, or inputs are substitutes and input substitution is less than output complementarity.*

While downward sloping supplies are also generally counterintuitive according to accepted economic wisdom, this possibility also exists with general equilibrium adjustment when the effects of adjustment are transmitted more effectively through the competitive industry than the concentrated industry. Consider the case where the y industry increases production and input use. Intuitively, when outputs are substitutes, increasing the output quantity causes a reduction in demand for the output of the z industry and thus a reduction in z industry input use, which, if x_y and x_z are substitutes, causes an increase in supply of x_y. If this transmission of effects through the z industry is sufficiently effective, then this upward pressure on the supply of x_y can be greater than the downward pressure on w_y caused by the increase in the quantity of input use by the y industry. If so, then the general equilibrium supply of x_y is downward sloping. In the case of indirect effects from output markets to input markets, a low marginal productivity causes the effects of a given output market change to be more dramatic in the input market, and therefore a low marginal productivity in the z industry relative to the y industry makes the indirect effects through the z sector more likely to dominate the direct effects of increasing production and input use in the y industry,

To see that the conditions of Proposition 5 are plausible, suppose the z technology is linear $(z'' = 0)$ and either inputs are prefect substitutes while outputs are perfect complements, or

inputs are perfect complements while outputs are perfect substitutes. Then the condition in (13')
can be expressed as

$$c_{yy}u_{zz}z'^2 - c_{yz}u_{yz}y'z' - (c_{yy}c_{zz} - c_{yz}^2) = c''u''z'(z' - y') < (>) \ 0 \ \text{as} \ z' > (<) \ y'.$$

The last left-hand term vanishes with perfect substitutes or perfect complements in supply while
the first left-hand term is negative. The second left-hand term including its sign is positive if both
inputs and outputs are substitutes or both are complements. Further, the second term dominates
the first if both inputs and outputs are perfect substitutes or perfect complements (or more
generally if $c_{yy}u_{zz} \geq c_{yz}u_{yz}$) and $y' > z'$. In this case, from (13'),
$s_{yy} = c''u''z'(z' - y')/(u''z'^2 - c'') < 0$ if $z' < y'$. Thus, the general equilibrium supply is downward
sloping if the marginal productivity in the y industry is higher than in the z industry. With these
results, Proposition 5 can be restated.

Proposition 5'. *With the market structure in (1)-(8) where the concentrated industry has market
power in both its input and output markets and the competitive industry technology is
approximately linear, the general equilibrium supply becomes downward sloping if both inputs
and outputs are sufficiently strong substitutes or both are sufficiently strong complements and
marginal productivity in the competitive industry is sufficiently lower than in the concentrated
industry.*

Propositions 3' and 5' suggest that negative sloping general equilibrium supply cannot
occur simultaneously with positively sloping general equilibrium demand because the conditions
on marginal productivity comparisons between the two industries are mutually exclusive even
with extreme cases of substitution and complementarity. Adding concavity in the z technology
only makes the conditions more stringent. More generally, the slopes of the general equilibrium
output demand and input supply can be compared using (12') and (13') to show that supply
always cuts demand from below regardless of unconventional slopes. To do this, either the slope

of the output demand must be converted by multiplying by y' for comparability because the marginal output price effect of a marginal change in input use pertains to y' units of the output at the margin, or the slope of the input supply must be converted equivalently by dividing by y'. The former reveals

$$(17) \quad \left[\frac{dw_y}{dx_y} - \frac{dp_y}{dx_y}\right]\pi_{zz} = (c_{yy} - u_{yy})u_z z'' - (c_{yy} - u_{yy}y')(c_{zz} - u_{zz}z'^2) + (c_{yz} - u_{yz}z')(c_{yz} - u_{yz}y'z').$$

To see that this expression is positive, suppose consumer utility is expressed as a function of the inputs, $u^*(x_y, x_z) \equiv u(y(x_y), z(x_z))$. Then $u^*(x_y, x_z)$ must be concave in x_y and x_z because $u(y, z)$, $y(x_y)$, and $z(x_z)$ are each concave. Further, $c(x_y, x_z) - u^*(x_y, x_z)$ must then be convex, which implies

$$(18) \quad \begin{aligned} C &= (c_{yy} - u^*_{yy})(c_{zz} - u^*_{zz}) - (c_{yz} - u^*_{yz})^2 \\ &= (c_{yy} - u_{yy}y' + u_y y'')(c_{zz} - u_{zz}z' + u_z z'') - (c_{yz} - u_{yz}y'z')^2 > 0. \end{aligned}$$

Comparing (17) and (18) thus proves

$$\left[\frac{dw_y}{dx_y} - \frac{dp_y}{dx_y}y'\right]\pi_{zz} = C - u_y y''(c_{zz} - u_{zz}z' + u_z z'') > C > 0.$$

Proposition 6. *With the market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets then the general equilibrium supply relationship facing the concentrated industry after transformation by its production technology always intersects the general equilibrium demand relationship from below.*

Proposition 6 is worded in terms of the general equilibrium relationships as a function of the output y, rather than the input x_y. The result is proven above in terms of the input but holds for output as well because the production transformation is monotonic. With Proposition 6, analyzing the sign of δ is sufficient to determine whether the equilibrium input use of the concentrated sector is larger or smaller than in the competitive equilibrium. Because the input quantity and output quantity of the concentrated industry have a monotonic relationship, both

will be above the competitive level if either is, and both will be below the competitive level if either is. But by Proposition 6, the conditions for overbuying and overselling are mutually exclusive.

To consider the net effect of the results above, the first-order condition for maximizing y industry profit, (8) is

(9') $\qquad w_y = \dfrac{dp_y}{dx_y} y + p_y y' - \dfrac{dw_y}{dx_y} x_y.$

Equations (12') and (13') imply, in the notation of Figure 1, that

(19)
$$\delta = -\dfrac{dp_y}{dx_y} y + \dfrac{dw_y}{dx_y} x_y = -\left[u_{yy} y' + \dfrac{u_{yz} z' S}{\pi_{zz}} \right] y + \left[c_{yy} + \dfrac{c_{yz} S}{\pi_{zz}} \right] x_y$$
$$= c_{yy} x_y - u_{yy} y y' + (c_{yz} x_y - u_{yz} y z')(S / \pi_{zz}).$$

Because either dp_y/dx_y can be positive or dw_y/dx_y can be negative, the result in (19) raises the question of whether the distortion δ can be negative. If $\delta > 0$, as in the cases of either monopoly or monopsony alone, then the y industry reduces its production to exercise market power most profitably. However, if $\delta < 0$, then the y industry finds expanding production and input use beyond the competitive equilibrium increases profit. If this occurs because the general equilibrium demand is upward sloping as in Propositions 3 and 3', then the firm with market power in both its input and output markets finds bidding up the price of its input, by buying more than in the competitive equilibrium, increases its demand sufficiently that the increase its revenue from monopoly pricing more than offsets the cost of buying its input (and more of it) at a higher input price. Thus, if $\delta < 0$ and $z' > y'$, then overbuying of the input occurs under the conditions of Propositions 3 and 3', which is motivated by the increased ability to exploit market power in the output market.

On the other hand, if $\delta < 0$ occurs because the general equilibrium supply is downward sloping as in Propositions 5 and 5', then the firm with market power in both its input and output

markets finds bidding down the price of its output and selling more than in the competitive equilibrium to increase its input supply sufficiently that the reduction in its cost with monopsony pricing more than offsets the loss of revenue from selling its output at lower prices. Thus, if $\delta < 0$ and $z' < y'$, then overselling of the output would occur under the conditions of Propositions 5 and 5', as motivated by the increased ability to exploit market power in the input market.

To clarify the outcomes that are possible under (19), we consider special cases involving either perfect substitutes or perfect complements in input supply and output demand. With perfect substitutes, both industries use the same input and effectively sell into the same output market using different technologies for production. If the z technology is linear and y' is represented as $y' = z'e$ where both inputs and outputs are perfect substitutes or both are perfect complements, then

$$\delta = c''x_y - u''yz'e + \frac{(c''x_y - u''yz')(c'' - u''z'^2e)}{u''z'^2 + u'z'' - c''} = 0 \text{ if } z'' = 0 \text{ and } e = 1.$$

Differentiating δ with respect to e to determine the sign of δ by whether $z' > (<) y'$ or, equivalently, by whether $e < (>) 1$ when $z'' = 0$ obtains

$$\frac{\partial \delta}{\partial e} = \frac{c''u''z'(y - x_y z')}{\pi_{zz}} > (<) 0 \text{ as } y/x_y - z' > (<) 0.$$

Thus, if $z'' = 0$, then $\delta > (<) 0$ as $(y' - z')(y/x_y - z') > (<) 0$, which implies that overbuying occurs if $z' > y'$ and $z' < y/x_y$, while overselling occurs if $z' < y'$ and $z' > y/x_y$. If marginal productivity in the z sector is diminishing $(z'' < 0)$, then the denominator of the latter right-hand term in (19) is increased (negatively) in magnitude so the strength of the latter term that generates the possibility of overbuying or overselling $(\delta < 0)$ is reduced. Thus, the conditions leading to overbuying or overselling become more stringent.

Proposition 7. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and either both inputs and outputs are perfect substitutes or both are perfect complements with those of a competitive sector, overbuying of the*

input relative to the competitive equilibrium is profitably sustainable if the marginal productivity of the competitive sector is both greater than marginal productivity of the concentrated industry and less than the average productivity of the concentrated industry, and the competitive sector has a sufficiently linear technology. Relaxing linearity of the technology of the competitive sector further restricts the conditions for overbuying.

The intuition of Propositions 2, 3 and 3' suggests why the case of overbuying occurs only when either both inputs and outputs are substitutes or both are complements. If a firm with market power in both its input and output markets bids up the price of its input by overbuying in the case of substitutes in supply, then the supply of inputs to the competitive sector contracts and accordingly the supply of the competitive sector output declines. This can enhance output market conditions for the concentrated industry only when the outputs are substitutes so that reduced output supply and higher output price in the competitive sector increases demand for the concentrated industry. If outputs were complements when inputs are substitutes, then the reduced output of the competitive sector would drive up price for the competitive output causing demand for the concentrated industry to contract so that no benefits could be gained by overbuying the input.

Next consider the potential for overselling. Interestingly, the condition for overselling under linearity of production in the competitive sector is not symmetric with the case of overbuying.

Proposition 8. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and either both inputs and outputs are perfect substitutes or both are perfect complements with those of a competitive sector, overselling of the output relative to the competitive equilibrium is profitably sustainable if the marginal productivity of the competitive sector is both less than marginal productivity of the concentrated industry and greater than the average productivity of the concentrated industry, and the*

competitive sector has a sufficiently linear technology. Relaxing linearity of the technology of the competitive sector further restricts the conditions for overbuying.

While Proposition 8 suggests the case for overselling the output is a mirror image of the case of overbuying, further analysis reveals that this is not the case. Positive profit requires $p_y y - w_y x_y > 0$ or, equivalently, $y/x_y > w_y/p_y$. However, the first-order condition in (9') implies that $w_y / p_y = y' - (\delta / p_y)$. Combining these two conditions implies $y/x_y > y' - (\delta / p_y)$. Because overselling requires $\delta < 0$, positive profit requires $y/x_y > y' - (\delta / p_y) > y'$, which is contrary to the conditions of Proposition 8 where $z' < y'$ and $z' > y / x_y$ jointly imply $y / x_y < z' < y'$. Thus, neither the case of perfect substitutes or perfect complements where $u_{yy} u_{zz} - u_{yz}^2 = 0$ is sufficient to generate overselling. In contrast, a similar analysis guarantees the marginal productivity condition of Proposition 7, which requires $y/x_y < y'$ when $\delta < 0$.

Proposition 9. *With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, sustained overselling cannot occur profitably as can the case of overbuying.*

As for previous cases, the second-order conditions for Propositions 7 and 8 involve complicating third derivatives for the utility and cost functions as well as the z technology. Again, some local conditions are clearly possible where the second-order condition fails, but distinct conditions exist where the second-order condition holds for each of the special cases of Propositions 7 and 8, including cases where δ is positive and δ is negative. If third derivatives of the utility and cost functions vanish and the z production technology is linear, then the second-order condition is

(20)
$$p_{yy} y' + p_y y'' - c_{yy} + \frac{(c_{yy} c_{zz} - c_{yz}^2) + (u_{yy} u_{zz} - u_{yz}^2)(y'^2 + yy'')z'^2}{\pi_{zz}}$$
$$- \frac{c_{zz} u_{yy}(y'^2 + yy'') + c_{yy} u_{zz} z'^2 - c_{yz} u_{yz} z'(2y' + x_y y'')}{\pi_{zz}} < 0$$

The first four left-hand terms are clearly negative if the y production technology is not too sharply downward bending, $y'^2 + yy'' > 0$. To evaluate the last term, without loss of generality, suppose similar to the approach used above for (19) that (20) is evaluated at arbitrary values of y, z, x_y, and x_z and that the units of measurement for y and z are chosen so that $u_{yy} = u_{zz}$ and the units of measurement for x_y and x_z are chosen so that $c_{yy} = c_{zz}$ at these arbitrary values. Then the numerator of the last term can be written as

(21) $\quad c_{yy}u_{yy}(y'-z')^2 + (2y'z' + yy'')(c_{yy}u_{yy} - c_{yz}u_{yz}) + c_{yz}u_{yz}y''(y - x_y z')$

where the first term is clearly negative and the second term is negative if the y production technology is not too sharply downward bending, in this case $2y'z' + yy'' > 0$, because $c_{yy}u_{yy} < c_{yz}u_{yz}$ follows from $c_{yy}c_{zz} - c_{yz}^2 > 0$ and $u_{yy}u_{zz} - u_{yz}^2 > 0$. Thus, (20) is negative as long as the last term of (21) does not dominate all other terms in (20). Two practical conditions make the last term small. First, as the y technology approaches linearity, the latter term vanishes. Second, the latter term vanishes as the average productivity of x_y in y approaches the marginal productivity of x_z in z. Thus, practical cases satisfying the second-order condition can possibly hold for all qualitative combinations of u_{yz} and c_{yz}. In particular, when third-order terms and concavity of z production are unimportant, the second-order condition holds for all cases where both inputs and outputs are substitutes or both are complements if the average productivity of x_y in y is less than the marginal productivity of x_z in z, which it must be in the case of overbuying in Proposition 7 but cannot be in the case of overselling in Proposition 8.

Naked Overbuying as a Means of Exercising Market Power

Another form of predatory behavior that can be examined in a general equilibrium framework is naked overbuying where the firm with market power buys amounts either of its own input or that of its competitor that are simply discarded. To analyze this case, we consider only buying amounts of the competitors input, which is equivalent to buying additional amounts

of its own input in the case of perfect substitutes, and is a more efficient way to influence the

market in the case of less-than-perfect substitutes. In this case, equation (6) is replaced by

$$(6^*) \qquad w_z = c_z(x_y, x_z + x_0)$$

where x_0 is the amount of the competitors' input bought and discarded by the firm with market

power. For this case, the system composed of (1)-(5), (6^*), and (7) can be solved for

$$(10^*) \qquad p_y = u_y(y(x_y), z(\hat{c}(w_y, x_y) - x_0))$$

$$(11^*) \qquad c_z(x_y, \hat{c}(w_y, x_y)) = u_z(y(x_y), z(\hat{c}(w_y, x_y) - x_0))z'(\hat{c}(w_y, x_y) - x_0),$$

which define the general equilibrium supply and demand.

Comparative static analysis of (10^*) and (11^*) yields

$$(12^*) \qquad \frac{dp_y}{dx_0} = \frac{c_{zz}u_{zz}z' - (u_{zz}z'^2 + u_z z'')(u_{zz} - u_{yz})z'}{\pi_{zz}}$$

$$(13^*) \qquad \frac{dw_y}{dx_0} = \frac{(u_{zz}z'^2 + u_z z'')c_{yz}}{\pi_{zz}} > (<)\ 0 \text{ as } c_{yz} > (<)\ 0$$

as well as the same results in (12') and (13'). Further, writing (6^*) as $w_z = c_z(x_y, \hat{c}(w_y, x_y))$ yields

$$(22) \qquad \frac{dw_z}{dx_y} = \frac{c_{yz}(u_{zz}z'^2 + u_z z'') - c_{zz}u_{yz}y'z'}{\pi_{zz}} > (<)\ 0 \text{ as } c_{yz} > (<)\ 0 \text{ and } u_{yz} > (<)\ 0$$

$$(23) \qquad \frac{dw_z}{dx_0} = \frac{u_{zz}z'^2 + u_z z''}{\pi_{zz}}c_{zz} > 0.$$

Proposition 10. *With the market structure in (1)-(5), (6^*), and (7)-(8) where the concentrated*

industry has market power in both its input and output market, naked overbuying of the related

industry's input unambiguously causes the related industry's input price to increase while it

causes the industry's own input price to increase (decrease) if inputs are substitutes

(complements). Demand for the concentrated industry increases if (i) outputs are complements

or (ii) outputs are perfect substitutes and the marginal cost of producing the competitive

industry's input is increasing.

To verify the latter claim of Proposition 10, note that the latter numerator term of (12^*) vanishes under perfect substitutes $(u_{zz} = u_{yz} = u'')$, but is positive (excluding the minus sign that is offset by negativity of the denominator) if $(u_{zz} - u_{yz} < 0)$. While this may appear to include all possible output relationships, some cases of near-perfect substitutes can have $u_{zz} - u_{yz} > 0$ without violating concavity conditions if u_{yy} is large relative to u_{zz}. On the other hand, the former numerator term is negative (vanishes) when the marginal cost of producing the competitive industry's input is increasing (constant), which together with the denominator contributes to non-negativity of dp_y/dx_0.

The firm with market power evaluating naked overbuying solves the profit maximization problem given by

$$(8^*) \qquad \max_{x_y, x_0 \geq 0} \pi_y = p_y y - w_y x_y - w_z x_0$$

using (12'), (13'), (12^*), (13^*), (22), and (23). The first-order condition for x_y again leads to (9') where (16) applies if $x_0 = 0$, while the first-order condition for x_0 yields

$$
(24) \qquad
\begin{aligned}
&\frac{dp_y}{dx_0} y - \frac{dw_y}{dx_0} x_y - \frac{dw_z}{dx_0} x_0 - w_z \\
&= \frac{c_{zz} u_{zz} yz' - (u_{zz} z'^2 + u_z z'')(u_{zz} - u_{yz}) yz'}{\pi_{zz}} - \frac{(u_{zz} z'^2 + u_z z'')(c_{yz} x_y + c_{zz} x_0)}{\pi_{zz}} - w_z.
\end{aligned}
$$

Because the signs of terms in (12') and (13') are unaffected by the addition of x_0 to the problem at $x_0 = 0$, the firm with market power is better off with naked overbuying if and only if the first-order condition for x_0 is positive when evaluated at $x_0 = 0$ where x_y solves the profit maximization problem at $x_0 = 0$ (assuming second-order conditions hold). If this first-order condition is negative at this point, then the results without x_0 in the problem apply because the firm would choose $x_0 = 0$ at the boundary condition.

The result in (24) is qualitatively ambiguous. The first right-hand term is clearly positive if outputs are complements and the second right-hand term evaluated at $x_0 = 0$ is clearly positive

if inputs are complements. Further, the left-hand term is also positive when outputs are perfect substitutes. The second right-hand term can be positive or negative but, evaluated at $x_0 = 0$, is negative (positive) if inputs are substitutes (complements). Of course, the third right-hand term is negative and can dominate if the related industry's input price is sufficiently high.

Proposition 11. *With the market structure in (1)-(5), (6^*), and (7)-(8) where the concentrated industry has market power in both its input and output market, naked overbuying of the related industry's input is profitably sustainable if inputs are complements, outputs are complements or perfect substitutes, and the related input industry's input price is sufficiently low.*

The case where both inputs and outputs are complements is the case where the concentrated industry overbuys the input because the beneficial effects on its output market dominate the increased cost of input purchases. The intuition of the major case of Proposition 11 is similar but the concentrated industry is better off because it does not have to use the increased purchase of inputs to relax the monopoly-restricted size of its output market. On the other hand, if inputs are complements and outputs are substitutes then buying the competitive sector's input and discarding it both increases the supply of the concentrated industry's input and, because of indirect effects though discouraging z industry activity, increases the concentrated industry's demand. These effects tend to improve the concentrated industry's ability to exploit both its input and output markets. By comparison, if inputs are substitutes then buying the competing sector's input and discarding it not only raises the input price of the competing sector but also the input price of the concentrated sector. In this case, the output market effect of causing a contraction in z industry activity must be greater to make such action profitable.

Conclusion

This paper has developed a framework to evaluate static explanations for predatory overbuying in input markets and predatory overselling in output markets. The intent is to fully

understand predatory behavior that is profitably sustainable. Much can be learned from the comparative static analysis before developing the two-stage predatory formulation where optimality depends on a second-stage recoupment period (at least in the case with related industries).[11]

While the literature on predatory behavior has drawn a distinction between raising rivals' costs and predatory overbuying that causes contraction of a related industry, our results show that optimal behavior can involve a combination of the two.[12] In the case of substitutes in a static model, raising rivals' costs is the means by which contraction of the related industry is achieved. Given the existence of a related competitive industry, a firm with market power in both its input and output markets can be attracted either to overbuy its input as a means of raising rivals' costs so as to take advantage of opportunities to exploit monopoly power in an expanded output market. Interestingly, this can be attractive even though a similar explanation for overselling is not applicable. That is, overbuying can be profitable sustainable whereas overselling appears to require a two-stage explanation with irreversibility. Nevertheless, our results show that (i) predatory buying in input markets will not necessarily lead to short-run costs above prices because the output market is exploited to increase output prices relatively more, and that (ii) a second-stage recoupment period after driving competitors from the market is not necessary to make this behavior profitable.

Moreover, such action may result in raising prices to consumers, which not only causes loss in overall economic efficiency (Carlton, 2007), but also loss in consumer welfare in particular (thus satisfying the narrower legal definition of efficiency emphasized by Salop 2005).

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[11] The conceptual results of this paper apply for various time horizons. Any substantive difference in a two-stage model will depend on having costs of expansion and contraction that differ from one another or that differ between industries. If the costs of expansion and contraction follow standard cost curves over longer time periods and are reversible as in classical theory of short-, intermediate-, and long-run cost curves, then the model of this paper is applicable and two-stage issues are inapplicable. So understanding of how two-stage results differ from classical theory depends on understanding how marginal costs of expansion differ from marginal costs of contraction.

[12] Of course, we recognize that much of the literature on predatory overbuying is based on the presumption that overbuying causes firms to exit, as in a two-stage case of recoupment.

But this loss in consumer welfare may occur either through higher prices for the primary consumer good (in cases of overbuying where $dp_y/dx_y > 0$), or by causing a relatively higher price for a related consumer good (in cases of overbuying where $dp_z/dx_y > 0$).

A further set of results in this paper applies to the case of complements. Overbuying can reduce costs to a related industry in the case of complements, and thus increase the ability to exploit an output market if the related output is also a complement. The general equilibrium model reveals that the case where both inputs and outputs are complements is virtually identical in effect as the case where both are substitutes. While the case of complements is less common in reality, it seems that any competitive standard should treat the cases symmetrically.

The framework of our analysis allows standard estimates of supply, demand, and production technologies to be used to determine the resulting behavior and its deviation from competitive standards. The general equilibrium model is the basis for determining, by standard measures of welfare economics, whether overbuying leads to consumer harm and thus violates the rule of reason under the Sherman Act. In particular, results show in a static model of perpetual predatory overbuying that the purpose of overbuying and consequent raising of rivals' costs is to more heavily exploit the output market, which necessarily harms consumers. This can happen even if the market output of the subject good does not contract from competitive levels because greater market demand for the subject good is achieved by influencing the related output market through predatory buying of its input.

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Zerbe, Richard O., Jr., "Monopsony and the Ross-Simmons Case: A Comment on Salop and Kirkwood," Vol. 72. Issue 2 (2005):717-726.

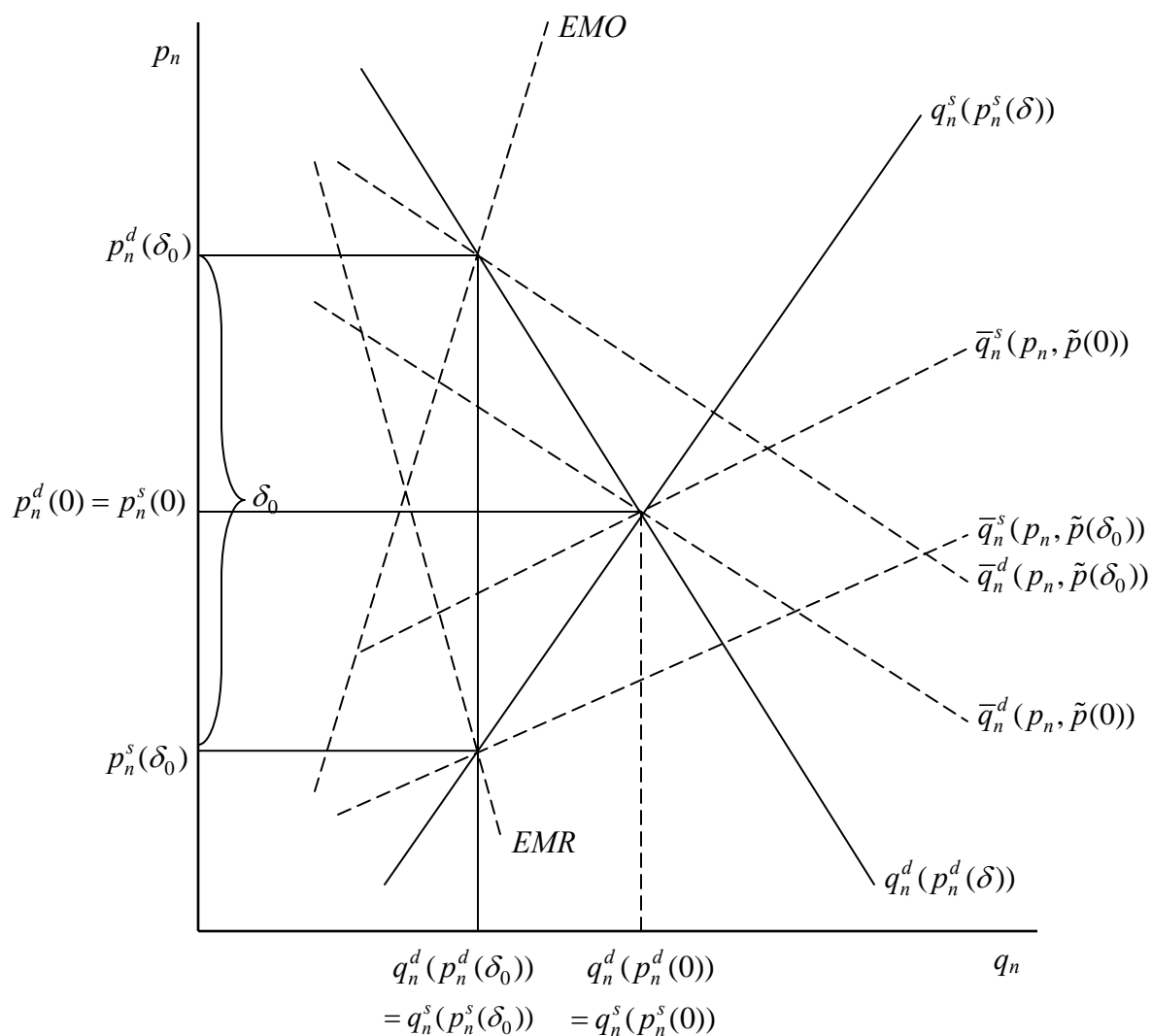**Figure 1. Equilibrium Measurement of the Welfare Effects of Monopolization.**

Figure 2. Use of Market Power by One Industry with Parallel Vertical Structures.
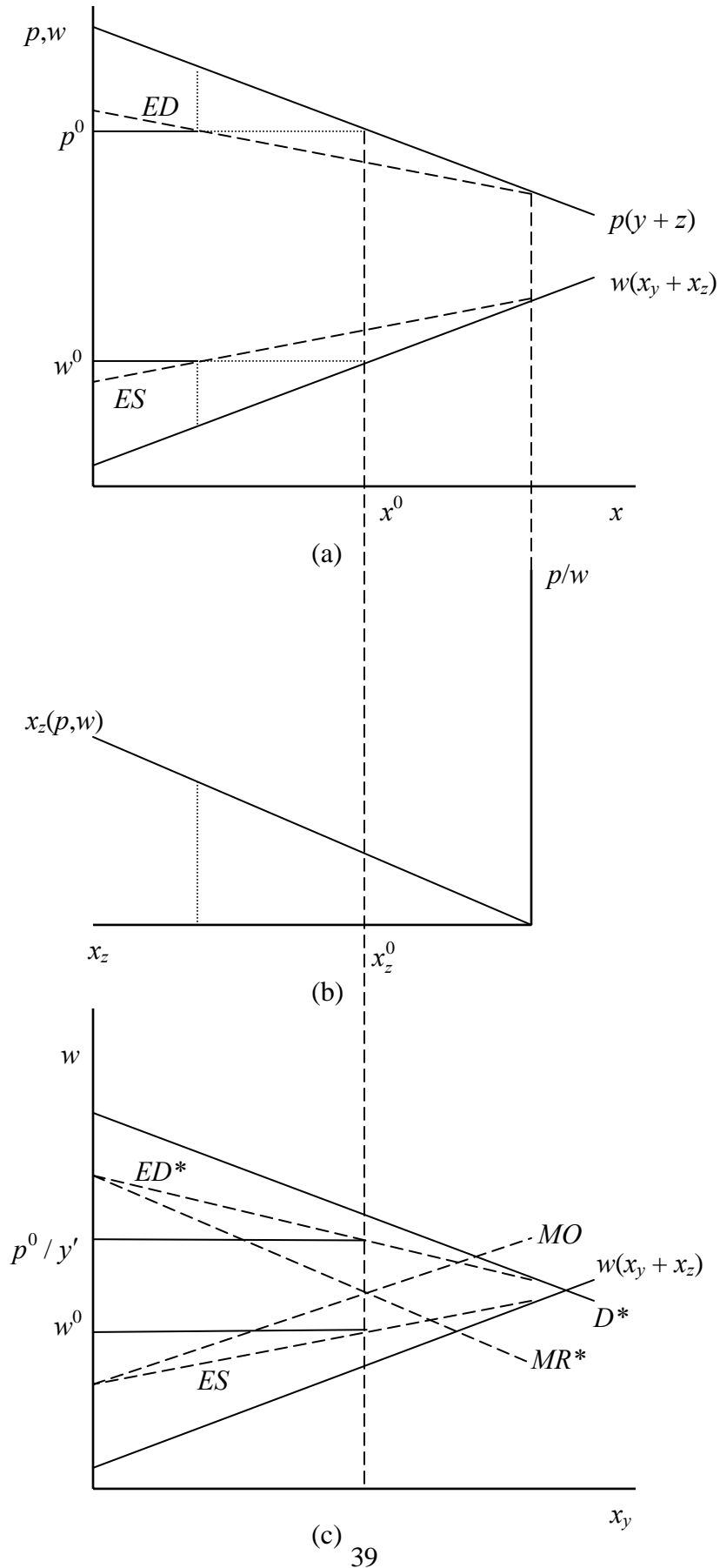

(a)

(b)

(c)

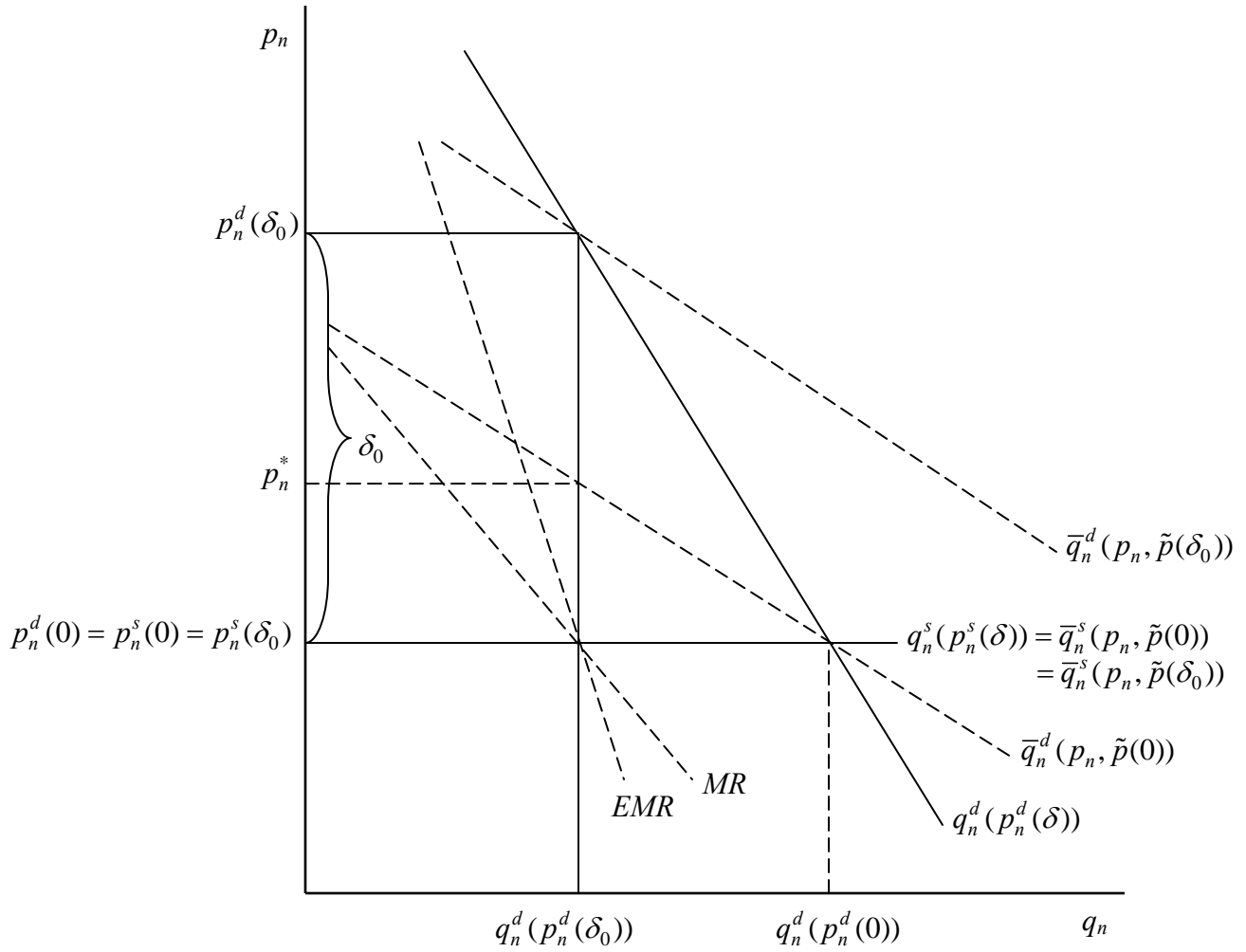**Figure 3. Equilibrium Welfare Effects of Monopoly with Vertically Parallel Structure.**

Figure 4. Equilibrium Welfare Effects of Monopsony with Vertically Parallel Structure.