

# UC Santa Barbara

## Departmental Working Papers

**Title**

Auction design for the allocation of emission permits

**Permalink**

<https://escholarship.org/uc/item/9pn661n6>

**Author**

Sunnevåg, Kjell J.

**Publication Date**

2001-06-11

# Auction design for the allocation of emission permits

Kjell J. Sunnevåg\*

*Foundation for Research in Economics and Business Administration (SNF), Bergen, Norway,  
and University of California, Santa Barbara, USA.*

UCSB, Department of Economics, Working Paper in Economics #

This version: June 11, 2001

*Abstract:* To the extent that emission permits have been allocated using market mechanisms, this has been done using a sealed-bid auction design, typically with discriminatory price. Emission permits, being a homogenous good with a fixed supply within a certain period, can be allocated using an auction design which can be considered as a “share auction”, where the bidders receive fractional shares of the item at a sale price that equates the demand and supply of “shares”. Since marketable emission permits is an exclusive right where signals from competing bidders can refine own bid estimates, the ascending multi-round auction format has been recommended over the sealed bid format by several authors for the allocation of emissions permits. Basically, two “competing” designs have been recommended, i.e. the standard ascending-clock auction and an ascending-clock implementation of Vickrey-pricing. The purpose of this paper is to evaluate pros and cons of these two formats, particularly in a setting where the authorities wish to allocate exclusive rights, e.g. emission permits, efficiently, and where the allocation of these exclusive rights has consequences for the level of production as well as market shares.

JEL No.: D44 (Auctions)

Keywords: auctions, multi-item auctions, emission permits.

---

\* Foundation for Research in Economics and Business Administration, Breiviksveien 40, NO-5045 Bergen, Norway. E-mail: [Kjell.Sunnevag@snf.no](mailto:Kjell.Sunnevag@snf.no). Financial support from the Norwegian Research Council research program OSIE gratefully acknowledged.

# Auction design for the allocation of emission permits

Kjell J. Sunnevåg

## I. Introduction

Typically, multiple units of a homogenous good have been allocated using sealed bid auctions, where the bidders simultaneously submit demand schedules. These are added to form the aggregate demand curve. The point at which the aggregate demand curve crosses the (vertical) supply curve determines the clearing price. The price paid for each item can either be the price bid or the clearing price. The former case is typically referred to as discriminatory or pay-your-bid pricing whereas the latter is referred to as uniform pricing. Bidder behavior will be different in the two formats. With the discriminatory price format it may be costly to bid significantly above the clearing price. Thus, the strategy will be to guess the clearing price and bid slightly above it. This might seem less important with uniform pricing since each winning bidder pays the clearing price. However, Ausubel and Cramton (1996) and Ausubel (1997) show that with uniform pricing there is a strong incentive for bid-shading.

Multi-unit homogenous goods can also be allocated using the open, ascending auction format. The ascending auction format has been recommended for allocating treasury securities and emissions permits (see e.g. Cramton and Kerr (1999) and Ausubel and Cramton (1998)).<sup>1</sup> As opposed to the sealed bid format, this format provides the bidders with information through the process of bidding. Moreover, the distinction between uniform and discriminatory pricing is much less important in an ascending auction than in sealed bid auctions (Cramton (1998)). The reason is that a bidder has little incentive to raise the bid much more than one bid increment above the clearing price. Unfortunately, Ausubel (1997) show that there still is an incentive for bid shading. He proposes an alternative ascending-clock auction that achieves full efficiency

---

<sup>1</sup> One notable example of practical experience with this format is the Norwegian import quota auctions which are conducted using an ascending auction, discriminatory price format.

under private values and affiliated signals. This alternative design is an ascending format implementation of Vickrey-pricing.

This paper intends to compare the outcome under the standard and alternative ascending auction format in a setting where there is some form of market power both in the product market as well as the market for permits. We will first study the outcome when the firm's bid assuming they have no market power. Thereafter, we will focus on possible outcomes when the firms try to exploit different strategies affecting the market-clearing price. The main result is that the alternative ascending auction makes it less costly to pursue strategies to raise the rivals' cost, thus offsetting the incentive to shade bids.

After some theoretical perspectives on multi-unit ascending auctions, the model is presented in Section III, first presenting results with sincere bidding, thereafter with strategic bidding. A short summary is presented in Section IV.

## **II. Background**

The auction formats used to sell many identical items can basically be divided in two: Sealed bid, one round auctions and open multi-round auctions. In sealed-bid auctions, the bidders simultaneously submit demand schedules. These are aggregated to form a demand curve, and the intersection between the demand curve and available supply distinguish winners from losers. Depending on whether the uniform or a discriminatory price format is used, the winners either pays the clearing price (highest losing or lowest winning) or what the actually bid. If there are several bidders at the market-clearing price, their requested volumes are prorated. This is a one-shot auction design, and losing bidders can not improve their bids. Thus, in order to get any of the items being auctioned, losing bidders must rely on the secondary market.

An important issue in discussions of auction design of for e.g. treasury securities and sulfur dioxide emissions has been whether to use a uniform or a discriminatory price format, the goal being to identify the format resulting in the lowest interest costs / highest revenue for the seller. It has been claimed that the uniform price format is superior to the discriminatory; the uniform-price format would more than offset the decline in revenue the seller receives as a discriminating monopolist due to increasing demand caused by more aggressive bidding and reduced bid participation costs.

For instance Wilson (1979) studies a *simultaneous* “share auction”, i.e. an auctions where each bidder offers a schedule specifying a price for each possible fraction of the item, and where the sale price is the one that equates the demand and supply of shares. He find that the uniform-price format leads to collusive outcomes, and that the seller’s revenue can be significantly lower than under a “unit-auction”. Also Back and Zender (1993) find that discriminatory auctions are likely to be more profitable for the seller if winning bidders pays the price they bid for each share since collusive strategies are self-enforcing in uniform-price divisible-good auctions. The “uniform-price fallacy” is also the theme of work by Ausubel and others (see e.g. Ausubel and Cramton (1996) and Ausubel (1997)), where it is demonstrated that a bidder who desires more than one unit in a uniform price auction has an incentive to shade the bid. In addition to the revenue effect, this shading also adversely affects the goal of allocating the objects to those who value them the most.

To a certain extent, the argument for the superiority of the uniform price format also seem to be based on Vickrey’s important result that bidding one’s own true value is a weakly-dominant strategy in the second-price auction of a single item (with independent private values). In a multi-item auction the second-price can be thought of as the market-clearing price. Nevertheless, Ausubel and Cramton (1996) point out that the notion that sincere bidding does not extend to a uniform-price auction where bidders desire multiple units is a result that actually can be found in the seminal work of Vickrey (1961). While the single-item Vickrey auction is well known, the multi-unit version of the auction design in the same 1961 article has received less attention. Here, submitted bids for the  $n$  objects are ranked in descending order, and the objects are awarded to the bidders with the  $n$  highest bids. The highest ranking winning bidder pay the amount of the highest rejected bid for the first object, the second highest rejected bid is paid for the second object and so forth. Consequently, each winner’s payment is independent of own bid.

This multi-unit auction design is efficient in private-value contexts (see e.g. Ausubel and Cramton (1996)). But as pointed out in Ausubel (1997), the reason for the lack of attention Vickrey’s multi-item design has received can (a) be its perceived complexity (b) that existence of common value components. Ausubel quotes Barry and Bulow (1993), which state that even economics Ph.D. students have trouble understanding the design, and that “...if people do not understand the payment rules of the auction then we do not have any confidence that the end result will be efficient.” With common value components in the objects auctioned, the sealed-

bid design of the auction restricts information available to the bidders, thus reducing efficiency. In addition, Klemperer (1999), point out that the Vickrey auction would be problematic for practical policy because an implication of the design is that high-valuers often are required to pay less than low-valuers.

As an alternative to the sealed bid format, multi-unit auctions can also be designed using an open format, either with ascending or descending price. The descending format corresponds to the Dutch auction for heterogeneous items: The auctioneer starts with a price that is gradually lowered until one of the bidders signals and announces the quantity she wishes to buy at that price. The auction proceeds like this until the whole available supply is sold or the price has come down to zero. The strength of this format is the same that applies to the standard Dutch auction format: The auction is less vulnerable to collusion since the collusive outcome is more difficult to enforce. However, the weakness is also the same, the amount of information available to bidders, which is necessary for the resulting allocation to be efficient, is limited.

A reliable process of price discovery is the primary advantage of the open ascending auction format over the sealed-bid format. In particular, as shown by Milgrom and Weber (1982), open auctions have important advantages over the sealed bid format in situations with positive correlation of parameter values ('affiliatedness'), i.e. bidders' valuations depend on information held by others.<sup>2</sup> The information revealed as the open auction proceeds raises the expected selling price and increases the probability for an efficient outcome. In multi-object auctions, each buyer's reservation value for an object may depend upon the other objects he obtains. A reliable process of price discovery refines bidder's valuation estimates, enabling them to bid more aggressively without fear of the winner's curse and to adjust the portfolio of objects in the course of the auction. Thus, in an ascending auction, both price and allocation are determined through a competitive auction process where each bidder in the next round has the opportunity to change losing bids into winning bids. In the end, those bidders valuing the auctioned objects most win.

---

<sup>2</sup> This result holds for risk-neutral buyers.

One approach to allocate multiple objects in an ascending auction is to invite *demand schedules* from bidders.<sup>3</sup> This is a method that can be considered as a multiple-round version of the sealed-bid auctions. All bids must be entered in the initial round and the total quantity requested by a bidder can only decrease. After each round the demand schedules are aggregated to form the demand schedule. The intersection between the demand curve and available supply defines the clearing price. Tentatively losing bidders, i.e. those with bids on the right side of the vertical supply curve, must improve their bids in the next round; otherwise the bidder is permanently rejected from further bidding. The auctioneer announces the minimum bid increment, i.e. how much the improved bid must exceed the clearing price. The minimum bid increment can be high in the early stages of the auction, and lowered as the auction proceeds.

The *ascending clock* auction is another approach to multiple-unit auctions. The clock indicates the current price. In each round, the bidders submit the quantity they are willing to buy at that price. If the total quantity bid exceeds the available supply, the clock is increased one step. Bidders can then decide if they want to reduce the quantity they want. This continues so long as there is excess demand. The objects are then allocated at the clearing price. According to Cramton and Kerr (1999), this design shares all the advantages of the ascending auction with demand schedules, but is easier to implement since a buyer only bids a single quantity in each round, rather than a schedule. In addition there are some other advantages: There is no possibility of undesirable bid signaling since only the total quantity bid is reported. Moreover, the process assures rapid convergence since the price is increased by one bid increment with each round.

As mentioned above, a critical distinction between the sealed-bid and the ascending auction format is that the latter provides the bidders with information through the process of bidding. However, information may not always serve the purpose of contributing to competition and optimal allocation. It may also be used by the bidders to enforce collusive outcomes. *Ex ante* asymmetries between bidders and weak competition may favor a sealed bid design. But as pointed out by Cramton (1998), in other cases, an ascending auction is likely to perform better in efficiency and revenue terms. Moreover, the information in an ascending auction can be tailored to limit collusion.

---

<sup>3</sup> The description of the ascending auction formats is based on Cramton (1998).

Nevertheless, motivated by potential bid shading problems in the standard formats, and the practical and conceptual problems associated with implementing the multi-item Vickrey format, Ausubel (1997) presents an alternative design with an ascending price. As in the standard ascending format bidders can present their demand at the current price in each round. The novelty of the design is that if a volume is “clinched”, i.e. it is mathematically impossible that the bidder not can get the “clinched” volume, this volume is allocated to the bidder at the current price. For instance, if the available quantity is 100, and bidder A bids for 120 units whereas bidder B bids for 80 at the current price, then 20 units is allocated to bidder A at the current price. The auction then proceeds until the total available volume is allocated. Ausubel (1997) show that this auction design gives incentives for sincere bidding and thus an efficient outcome since all the payoff-relevant events in the auction occur through clinching. Nevertheless, Cramton and Kerr (1999) find reason to state that in a “setting where market power is apt to be slight, the inefficiencies from a standard ascending-clock auction are likely to be insignificant” (page 8).

This paper intends to compare the incentives under the *standard* and what will hereafter be called the *alternative* ascending auction design. This will be done in a setting where the authorities face an oligopolistic industry whose emissions are controlled under a tradable emission permit regulation, where emissions permits are considered as an input with fixed, exogenously determined supply. All emission permits are auctioned.

We know from the theoretical literature related to this setting, that with perfect competition in product and permit markets, tradable pollution permits are a cost-effective means for reducing pollution. All sources will acquire the number of permits at which the marginal abatement costs equal the permit price, i.e. marginal abatement costs will be the same for all sources. Thus, total abatement costs for all sources are minimized, regardless of the initial allocation of the permits. Basically, the initial allocation of permits can be done through grandfathering rights to existing polluters or auctioning rights to the highest bidders.<sup>4</sup> However, even though permits market imperfections might not be an important concern when the number

---

<sup>4</sup> Although efficiency gains to some extent can be recovered through permit trades in secondary markets, Cramton and Kerr (1999) presents a long list of arguments why auctioning is superior: It allows reduced tax distortions, provides greater incentives for innovation, provides more flexibility in distribution of costs, and reduces the need for politically contentious arguments over the allocation of rents.



of regulated firms is large, it could be a serious problem in more localized permits markets where a small number of participants not guarantees competitive behavior.

This paper combines issues related to auction design with strategic behavior in the permits markets. Previous work related to the issue of strategic behavior in the permits markets include Malueg (1990) who show that efficient trading of permits does not necessarily yield the first-best allocation of resources when product markets are imperfectly competitive, even if marginal abatement costs are equated across sources. Efficiency requires that marginal production costs also be equated across manufacturers in the same market, with marginal cost equaling the output price. When there is market power in both product and the permits market, intuition suggests that there are strong incentives to cooperative in cost-manipulating strategies. When antitrust-policies not prevents monopolization of the product market, sale of all emission permits from one of the market participants to the other may be a profitable strategy for monopolizing the product market. Fehr (1993) derive the conditions under which a strategy of monopolization is profitable. Even when monopolization of the product market not is an option, leadership in the permits market may distort the allocation of abatement effort between firms as well as market shares and production level. Sartzetakis (1997) analyze the effect that positioning strategies in permits markets have on the degree of competition and social welfare. The analysis is based on the concept of raising rivals' cost strategies and he find that the ability to pursue such strategies might lessen competition in the product market substantially. However, efficiency may not be reduced if the leader expands its market share at the expense of a less efficient rival.

The model used in this paper is based on Sartzetakis (1997). The analysis extends this literature by explicitly focusing on the impact of the design of the institution used to allocate the homogenous good. The contribution to the auction literature lies in the study of incentives under two alternative auction designs for multi-unit homogeneous rights allocation in a setting where there is a link between rights acquired in the auction and the product market.

### **III. The model**

As mentioned above, the focus here is the allocation of a homogenous good within an oligopolistic industry whose emissions are controlled under a tradable emission permit regulation. We will model the allocation of permits and subsequent production as a two stage

process, with permits trading occurring in the first stage and production in the second. It is assumed that monopolization of the product market is not an option.

The inverse demand curves facing the firms' are given by  $p = a - bQ$ , where  $Q = q_1 + q_2$ . The firms face constant and equal marginal cost of production  $c$ . Tradable emission permits are considered as input with fixed supply  $\bar{E}$ , exogenously determined by the authorities. All permits are auctioned periodically, where each acquired permit allows the acquiring firm to emit one unit for the next period.  $E_i$ ,  $i = 1, 2$  is firm  $i$ 's share of the emission quota, such that  $\sum_i E_i = \bar{E}$ .

Each unit produced generates emissions at the proportional rate of  $\rho_i$ .<sup>5</sup> However, each firm can substitute away from permits by either engaging in abatement or reducing production, subject to  $\rho_i q_i - \alpha_i q_i = E_i$ , where  $\alpha_i$  is the abatement level and  $q_i$  firm  $i$ 's production level. Consequently, decisions in the product and permits markets are linked. The cost of abatement is assumed to be quadratic in both output  $q$  and abatement per unit of output  $\alpha$

$k(q_i, \alpha_i) = d\alpha_i q_i + e\alpha_i^2 q_i^2$ ,  $i = 1, 2$ , where  $d$  and  $e$  represent technological parameters ( $d, e \geq 0$ ).

If both firms are assumed to be price takers in the permits market, firm  $i$ 's profit maximization problem becomes:

$$\max_{q_i, \alpha_i} \Pi_i = pq_i - cq_i - k(q_i, \alpha_i) - \lambda E_i, \quad i = 1, 2,$$

where  $\lambda$  is the equilibrium price of permits. Optimization implies that each firm choose an abatement level so that  $\alpha_i = \frac{d - \lambda}{2q_i e}$ . At the equilibrium we have that the distribution of

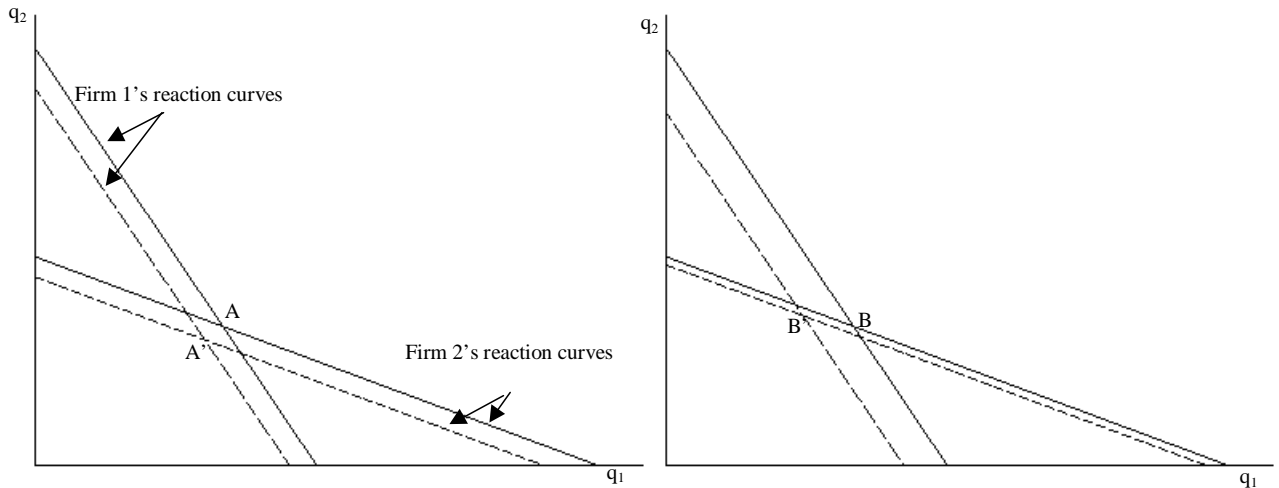
abatement effort yields minimization of compliance costs since marginal cost of abatement is equalized between firms. Moreover, the first order conditions yields firm  $i$ 's output reaction

$$\text{function } q_i = \frac{a - bq_j - c - \lambda \rho_i}{2b}, \quad i \neq j, \quad i, j = 1, 2.$$

---

<sup>5</sup> In this respect the model differs from Sartzetakis (1997). Here production technologies differ, while emission reduction technologies are similar between firms, whereas Sartzetakis assume that emission reduction technologies are different but emissions per unit of production is the same. Our assumption allows for more interesting interaction between permits acquired and resulting market shares.

The distribution of production in the unregulated and the regulated case for a) the case where emission generation is identical between the firms, and b) firm 2 generates more emissions per unit of production is presented in the figure below.



**Figure 1.** Distribution of production between two Cournot players in the unregulated case ( $\lambda=0$ ) and the case where  $\lambda>0$ . In the figure to the left we have that  $\rho_1=\rho_2$ , whereas in the figure to the right:  $\rho_1>\rho_2$ .

We see from the figure above that in both cases, as permit prices increase from zero, both firms' marginal costs increase and consequently their reaction functions are shifted inwards and total production is reduced. In the first case where production technologies are the same, we have a proportional reduction between the two firms (from A to A'), whereas in the second case we see a redistribution of production, where the less polluting firm (firm1) actually increase its production somewhat (from B to B') on behalf of the "dirty" firm.

Two alternative auction institutions can be used for the allocation of emission permits. Using the definitions in Ausubel (1997), we can define the two alternative auction formats as follows:

**Standard ascending-bid auction.** The auctioneer operates a continuously ascending clock. For each price,  $\lambda$ , each bidder  $i$  simultaneously indicates the quantity,  $E_i(\lambda)$ , she desires, where demands are required to be nonincreasing in price. When a price,  $\lambda^*$ , is reached such that

aggregate demand is less or equal to supply, the auctioneer concludes the auction. Each bidder  $i$  is then assigned the quantity  $E_i(\lambda^*)$ , and each bidder  $i$  is charged a unit price of  $\lambda^*$ .

**Alternative ascending-bid auction.** The auctioneer operates a continuously ascending clock. For each price,  $\lambda$ , each bidder  $i$  simultaneously indicates the quantity,  $E_i(\lambda)$ , she desires, where demands are required to be nonincreasing in price. At each price, a quantity  $E_i$  of available total supply  $\bar{E}$  is defined as being “clinched” by bidder  $i$  according to:

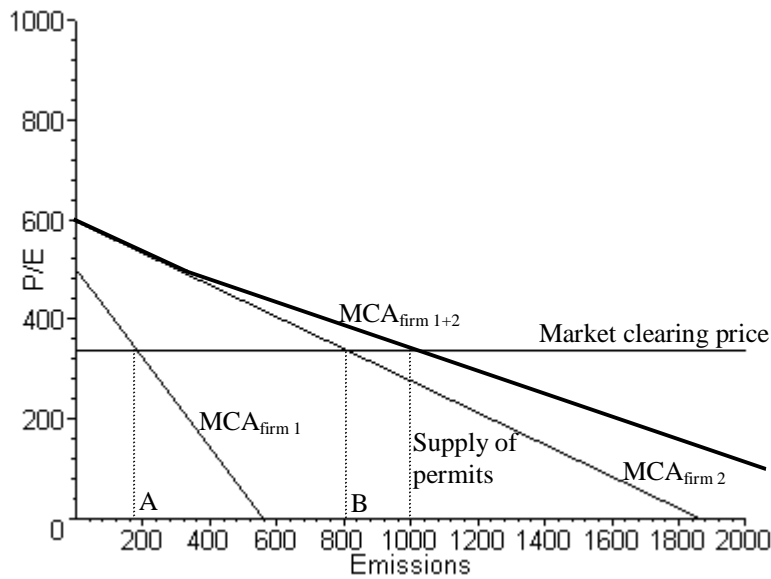
$$E_i(\lambda) = \max \left[ 0, \bar{E} - \sum_{j \neq i} E_j(\lambda) \right].$$

When a price,  $\lambda^*$ , is reached such that aggregate demand is

less or equal to supply, the auctioneer concludes the auction. Each bidder  $i$  is then assigned the clinched quantities, and is charged the standing price at which she “clinched” the respective quantities.

#### **A. Auction outcome with sincere bidding**

The Standard ascending-bid auction process and outcome with this design is presented in the figure below for a simulated case where we have one relatively ”clean” firm, i.e. firm 1 and one ”dirty” firm, firm 2 ( $\rho_1=0.2$ ,  $\rho_2=0.8$ ). Their respective marginal cost of abatement curves, thus demand for permits, are represented by  $MCA_{\text{firm 1}}$  and  $MCA_{\text{firm 2}}$ , respectively, whereas their aggregate demand is represented by the curve  $MCA_{\text{firm 1+2}}$ .



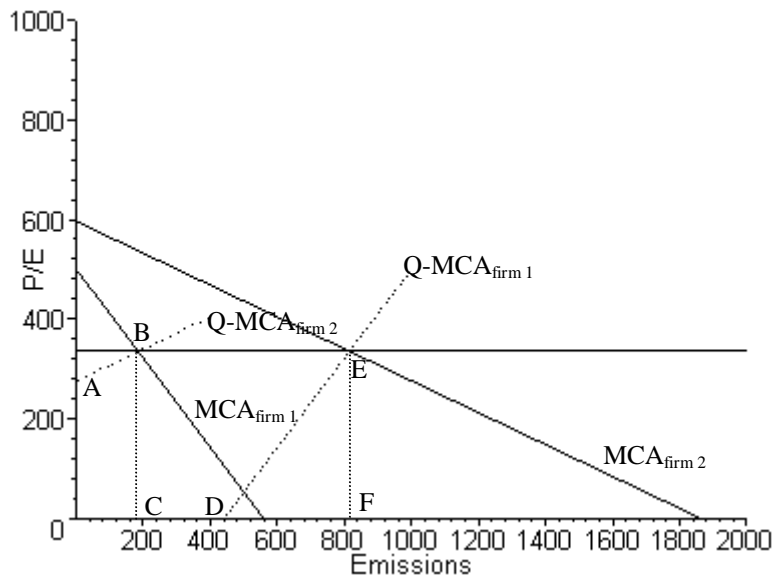
**Figure 2.** Distribution of available emission quota with the standard auction format, and sincere bidding.

In this example, the available supply of quotas is  $\bar{E} = 1000$  permits<sup>6</sup>, where each permit allows the firm to emit one unit. As the auctioneer raise the price, both firms will reduce their demand for permits until the market clears at the point where the aggregate demand curve intersects available supply. Firm 1 gets 185 permits, whereas Firm 2 get 815 units. Each firm's payment is determined by the market-clearing price multiplied by the acquired permits.

The auction process for the allocation of the available quota using the alternative design of the auction is depicted in the figure below.

---

<sup>6</sup> The other parameters are for the demand function:  $a=1500$ ,  $b=.2$ ; for the production cost:  $c=200$ ; for the abatement cost function:  $d=100$ ,  $e=.4$ ; and for the emissions per unit of production,  $\rho_1=.20$  and  $\rho_2=.80$ .



**Figure 3.** Distribution of available emission quota with the alternative auction format, and sincere bidding.

As the auction proceeds and the price ascends, we assume that each firm sincerely reports its marginal cost of abatement at the current price, i.e. its demand for emission permits. The permits allocated at the current price are a function of the other bidders' demand. The line DE represents the permits Firm 2 "clinch" as the price ascends. We see that when the price is zero, each firm's respective reported demand is 558 and 1858, respectively. Thus, it is mathematically impossible for Firm 2 not to get 442 permits. Consequently, 442 emission permits are allocated to Firm 2 at the price of zero. The price must rise to almost 300 before Firm 1 "clinch" any permits. However, the resulting allocation of the available quota is the same as with the previous method. The resulting market clearing price is also the same. Only the payment differs. Now firm 1's payment depends on Firm 2's demand. Firm 1 pays an amount represented by the area 0ABC, whereas Firm 2 pays an amount represented by the triangle DEF.

The resulting profit and allocation for varying parameter values is presented in the table below.

**Table 1.** Production, profits ( $\times 10^3$ ) and distribution of emissions quota.

Parameter values	Firm 1			Firm 2		
	$\Pi_1$	$q_1$	$E_1$	$\Pi_2$	$q_2$	$E_2$
	Unregulated, $\lambda=0$	939	2167	0	939	2167
Regulated, <b>standard</b> auction format						
$\rho_1=0.5, \rho_2=0.5, \lambda=425$	723	1812	500	723	1812	500
$\rho_1=0.2, \rho_2=0.8, \lambda=335$	1,177	2390	185	419	1386	815
$\rho_1=0.1, \rho_2=0.9, \lambda=287$	1,274	2502	16	388	1353	984
Regulated, <b>alternative</b> auction format						
$\rho_1=0.5, \rho_2=0.5, \lambda=425$	798	1812	500	798	1812	500
$\rho_1=0.2, \rho_2=0.8, \lambda=335$	1,182	2390	185	629	1386	815
$\rho_1=0.1, \rho_2=0.9, \lambda=287$	1,274	2502	16	624	1353	984

Both firms will be better off with the alternative auction format, since the payment for the permits are lower. To the consumers, auction format will not matter since produced quantities and thus market prices not change. The relative welfare effects depend on to which extent the payment for the permits reflect the externalities in production of the dirty good.

## **B. Strategic bidding**

So far we have assumed that both firms' bid their true marginal costs of abatement and demand for permits at each turn of the auctioneer's clock. This may not be the case, particularly not in this situation where two firms compete for a limited supply of emission permits. We know from Ausubel and Cramton (1996) that in the standard auction format, firms have a strong incentive to underreport their demand, thus reducing the realized price at the auction, suggesting that the alternative format should be used instead.

To investigate the incentives during the auction process in the same setting as we have used thus far, we assume that each firm has three possible strategies: i) report its demand sincerely; ii) report a demand lower than its true demand, thus inducing an earlier stop at a lower price; iii) report a higher demand than true demand thus acquiring more emission permits than it otherwise would have got, but at the cost of a higher equilibrium price for permits. We assume that each of the firms has no information with respect to what constitutes the other firm's true demand. It can only observe the actual bid at each time, not what kind of strategy the other firm is following. It can, however, decide on the basis of this information whether it is relatively "dirty" or "clean".

Assuming initially that we have two firms equally “dirty”, i.e.  $\rho_1=0.5$ ,  $\rho_2=0.5$ . The remaining parameter values are the same as before. The market clearing prices for emission permits under the different strategies is presented in the table below.

**Table 2.** Market clearing prices for emission permits,  $\rho_1=0.5$ ,  $\rho_2=0.5$ .

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	365	395	425
	Sincere	395	425	455
	Increase	425	455	485

The respective firms’ profit from following the different strategies using the standard auction format is presented in the table below.

**Table 3.** Expected outcome of different strategies, standard auction format,  $\rho_1=0.5$ ,  $\rho_2=0.5$ .

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	(744, 744)	(720, 747)	(700, 745)
	Sincere	(747, 720)	<b>(723, 723)</b>	(704, 721)
	Increase	(745, 700)	(721, 704)	(702, 702)

We see that if both firms follow a strategy of reducing demand relative to true demand, this will give both firms a higher profit than if both reported demand sincerely. However, this is not a Nash-equilibrium. For instance, if Firm 1 decide to cheat and report sincerely while the other underreports demand, it can gain market shares on behalf of the other firm, thus increasing profit relative to the (Reduce, Reduce) case. But if it tries to increase its demand relative to true demand, it will find this too costly relative to gained market shares, since the increased price for permits must be paid on all acquired permits, not only the marginal permits. Since the firms are symmetric, the Nash-equilibrium become to bid sincerely for both firms.

If we now focus on the alternative auction format, where we should expect sincere bidding to be the equilibrium strategy, yet another surprising result occur.

**Table 4.** Expected outcome of different strategies, alternative auction format,  $\rho_1=0.5$ ,  $\rho_2=0.5$ .

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	(819, 819)	(781, 838)	(748, 853)
	Sincere	(838, 781)	(798, 798)	(764, 812)
	Increase	(853, 748)	(812, 764)	<b>(777, 777)</b>



Now it becomes less expensive for the firms to pursue a strategy of increasing demand relative to true demand. This results from the payment function, which is now designed so that the increased price only is paid on marginal units of emissions permits. Still, (Reduce, Reduce) is better than (Sincere, Sincere), but since both firms find it optimal to try to increase their demand and market share on behalf of the other, we end up with (Increase, Increase) as the Nash-equilibrium.

Assuming now that we have one “dirty” firm and one relatively “clean” firm ( $\rho_1=0.2$ ,  $\rho_2=0.8$ ). The market clearing prices for permits is presented below.

**Table 5.** Market clearing prices for emission permits,  $\rho_1=0.2$ ,  $\rho_2=0.8$ .

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	287	311	335
	Sincere	311	335	358
	Increase	335	358	382

The following table summarizes the expected profit for the different strategies, for the two firms.

**Table 6.** Expected outcome of different strategies, standard auction format,  $\rho_1=0.2$ ,  $\rho_2=0.8$ .

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	1 143, 471	1 145, 453	1 151, 432
	Sincere	1 175, 435	1 177, 419	1 182, 398
	Increase	<b>1 198, 406</b>	1 199, 390	1 204, 371

Now we see that the “clean” firm, Firm 1, always will find it opportunistic to increase its reported demand relative to the true demand. Firm 1’s market share with sincere bidding is 63 per cent, whereas with strategic bidding this will increase to 65 per cent. The value of increasing its market share on behalf of the other exceeds the payment for the extra emission permits it acquires. Knowing that it is “dirty”, the best the Firm 2 can do is to reduce its reported demand below the true demand in order to stop the auction as early as possible. The corresponding results from the alternative auction format are presented below.

**Table 7.** Expected outcome of different strategies, alternative auction format,  $\rho_1=0.2$ ,  $\rho_2=0.8$

		FIRM 2		
		Reduce	Sincere	Increase
FIRM 1	Reduce	1 146, 673	1 147, 676	1 152, 675
	Sincere	1 182, 623	1 182, 629	1 186, 628
	Increase	1 211, 523	<b>1 210, 584</b>	1 213, 583

We see from the table above that it is still optimal for the “clean” firm to exploit the fact that it is relatively clean to the other. However, now Firm 2 finds it optimal to bid sincere, resulting in (Increase, Reduce) as the Nash-equilibrium.

#### **IV. Summary**

Typically, when governments allocate multiple units of homogenous goods, a standard sealed bid auction format is used. Bids are aggregated to form a demand curve. The cut off point, and the market-clearing price, is where this demand curve intersects available supply. The main part of the discussion on this auction format has revolved on the issue whether a discriminatory or uniform price format should be chosen. The ascending auction format for multiple items of a homogeneous good resolves this issue to some extent. In addition, this format has another advantage, it provides bidder with information that can reduce the potential for the winner’s curse problem and increase the probability of an efficient allocation of the goods. The increased information content this design provides has, however, a flip side: It can be used to establish and enforce collusive outcomes. Moreover, there is a strong incentive to effect price through bid shading. Ausubel (1997) presents an alternative auction mechanism that removes the incentives for bid shading.

In this paper, it is shown that in a setting where two companies compete in the product market as well as in the market for emission permits, and acquired permits has consequences for the production decision, it is not obvious that bid shading is the optimal strategy in the standard ascending auction format, neither that sincere bidding is the optimal strategy in the alternative ascending auction format. If the bidders are symmetric, it is shown that the alternative auction format makes it less costly to pursue strategies to increase market shares through acquiring emission permits and increasing the competitor's costs, leading to overbidding as the optimal strategy for both firms. Under the standard auction format, overbidding costs more than it tastes, leading to sincere bidding as the optimal strategy for both. If the bidders not are symmetric, it becomes optimal for the bidder in relatively low demand for the good in question to try to exploit this, and acquire market shares on behalf of the other.

## V. References

- Ausubel, L.M. (1997). "An Efficient Ascending-Bid Auction for Multiple Objects," Working Paper. University of Maryland, Department of Economics, June 25.
- Ausubel, L.M., and P. Cramton (1996). "Demand Reduction and Inefficiency in Multi-Unit Auctions," Working Paper. University of Maryland.
- Back, K., and J.F. Zender (1993). "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *Review of Financial Studies* 6 ( 4): 733-764.
- Barry, J.N., and J.I. Bulow (1993). "Designing the PCS Auction," Comment to the FCC on behalf of Bell Atlantic.
- Cramton, P. (1998). "Ascending Auctions," *European Economic Review* 42 ( 3-5): 745-756.
- Cramton, P., and S. Kerr (1999). "Tradeable Carbon Permit Auctions - How and why to auction not grandfather," Working Paper. University of Maryland, March 1999.
- Fehr, N.-H.M.v.d. (1993). "Tradeable Emission Rights and Strategic Interaction," *Environmental and Resource Economics* 3: 129-151.
- Klemperer, P. (1999). "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys* 13 ( 2): 227-286.
- Malueg, D.A. (1990). "Welfare Consequences of Emission Credit Trading Programs," *Journal of Environmental Economics and Management* 18: 66-77.
- Milgrom, P., and R. Weber (1982). "A Theory of Auctions and Competitive Bidding," *Econometrica* 50 ( 5): 1089-1122.
- Sartzetakis, E.S. (1997). "Raising Rivals' Costs Strategies via Emission Permits Markets," *Review of Industrial Organization* 12: 751-765.
- Sartzetakis, E.S. (1997). "Tradeable Emission Permits Regulations in the Presence of Imperfectly Competitive Product Markets: Welfare Implications," *Environmental and Resource Economics* 9: 65-81.
- Vickrey, W. (1961). "Counterspeculation, Auction, and Competitive Sealed Tenders," *Journal of Finance* 16 ( March): 8-37.
- Wilson, R. (1979). "Auctions of Shares," *Quarterly Journal of Economics* 93: 675-689.