

Lawrence Berkeley National Laboratory

Recent Work

Title

String Solitons and Singularities of K^3

Permalink

<https://escholarship.org/uc/item/0041m83w>

Author

Ooguri, H.

Publication Date

1995-12-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

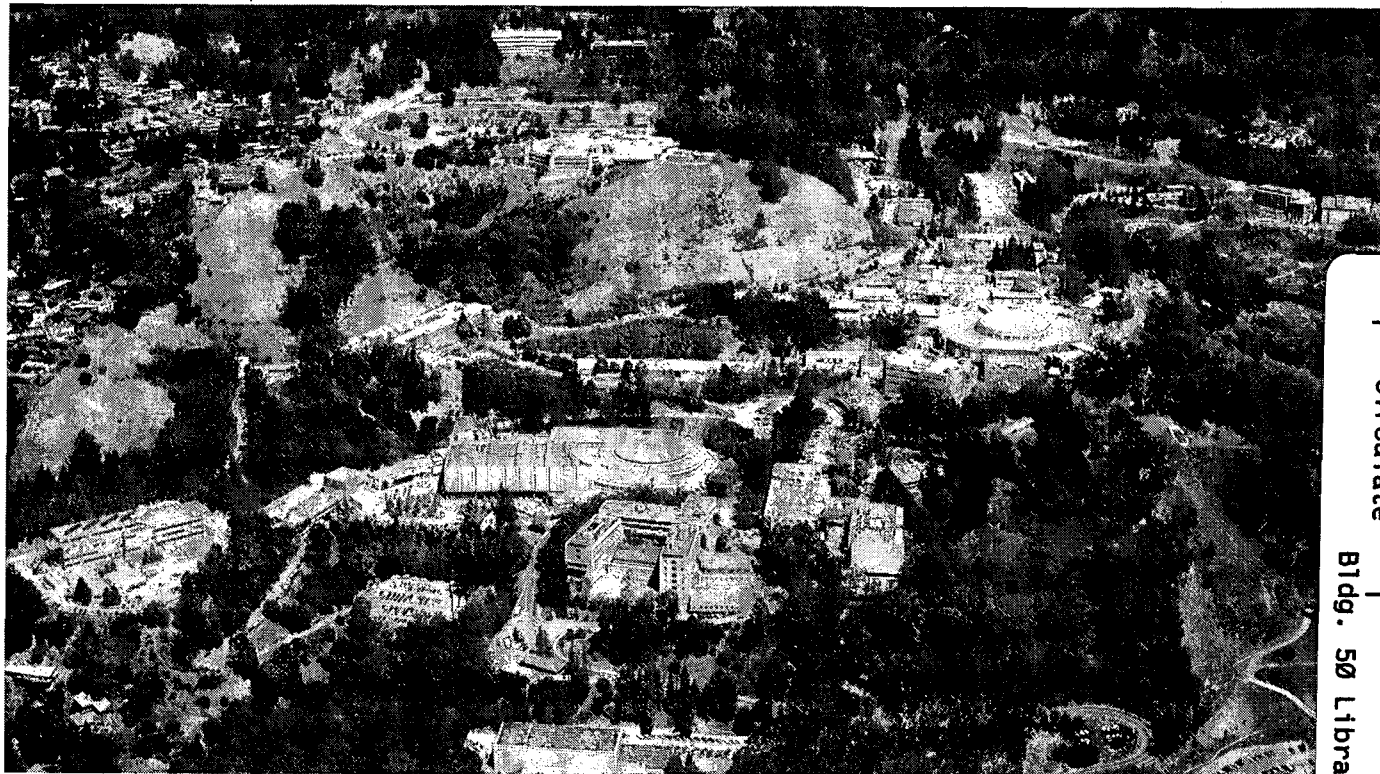
Physics Division

Presented at the Workshop "Frontiers in Quantum Field Theory,"
in honor of the 60th birthday of Professor Keiji Kikkawa,
Osaka, Japan, December 14-16, 1995, and to be published
in the Proceedings

String Solitons and Singularities of K_3

H. Ooguri

December 1995



REFERENCE COPY |
Does Not |
Circulate |
Copy 1
Bldg. 50 Library.
LBL-38591

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBL-38591
UC-416

String Solitons and Singularities of K_3

Hiroshi Ooguri

Department of Physics
University of California, Berkeley

and

Theory Group
Physics Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

December 1995

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and by the National Science Foundation under Grant PHY-90-21139.

String Solitons and Singularities of K_3

Hiroshi Ooguri

*366 Le Conte Hall, Department of Physics, University of California at Berkeley
Berkeley, CA 94720, USA.*

and

*Theory Group, Mail Stop 50A-5101, Lawrence Berkeley National Laboratory
Berkeley, CA 94720, USA.*

Talk presented at the Workshop "Frontiers in Quantum Field Theory" in honor of the 60th birthday of Prof. Keiji Kikkawa, Osaka, Japan, December 1995

It is my pleasure and honor to give a talk on occasion of Professor Kikkawa's 60th birthday. His various scientific achievements have been mentioned throughout the meeting, but I also want to add that he has been a great role model for generations of aspiring theoretical physicists. I myself frequented his group at Osaka during my graduate school years at Kyoto in order to discuss with him, and I am very grateful for his advice on scientific and other matters.

I have to come back to one of his scientific contributions, the discovery of the T -duality in string theory¹. Let me start with the T -duality in the context of the heterotic string. The heterotic string on $\mathbf{R}^9 \times S^1$ is invariant under transformation $r \rightarrow 2/r$ where r is the radius of the circle S^1 in the target space. (Once compactified on S^1 , the heterotic strings with $E_8 \times E_8$ and $SO(32)$ gauge groups are continuously connected, and we do not have to

¹This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

distinguish the two.) One can interpret ² this transformation, called the T -duality, as a remnant of non-abelian gauge symmetry which manifests itself at the self-dual radius $r = \sqrt{2}$ and which is spontaneously broken for $r \neq \sqrt{2}$. To understand this, one only have to remember that the string on S^1 carries separate momenta for the left and the right movers parameterized by a pair of integers (n, m) as $(p_L, p_R) = (n/r + rm/2, n/r - rm/2)$. In particular at $r = 2$, the integers $(n, m) = (1, -1)$ and $(-1, 1)$ give $(p_L, p_R) = (0, \pm\sqrt{2})$. Vertex operators corresponding to them are $e^{\pm i\sqrt{2}X_R}$, which together with $i\bar{\partial}X_R$ generate the affine $SU(2)$ symmetry on the worldsheet (the Frenkel-Kac construction). In the string theory, the worldsheet affine symmetry implies the target space gauge symmetry. Therefore the heterotic string acquires an extra $SU(2)$ gauge symmetry at the self-dual radius $r = \sqrt{2}$. Away from this radius, the $SU(2)$ is spontaneously broken to $U(1)$, which is the standard Kaluza-Klein gauge symmetry generated by $i\bar{\partial}X_R$. Therefore the $SU(2)/U(1)$ is non-linearly realized away from the self-dual point, and in fact one can show that it transforms r to $2/r$.

I should mention that this argument does not apply to the type II superstring theory since the would-be gauge bosons generated by $e^{\pm i\sqrt{2}X_R}$ are removed by the GSO projection. In fact the T -duality transforms the type IIA string into the type IIB string and vice versa ^{2,3}.

The story becomes more interesting if we combine this observation with the recent conjecture on dualities among string theories. It was suggested by Hull and Townsend ⁴ that the the heterotic string on $\mathbf{R}^6 \times T^4$ is equivalent to the type IIA string on $\mathbf{R}^6 \times K_3$. Under this duality transformation, the value of the string coupling constant λ is inverted to $1/\lambda$. This conjecture implies the following interesting prediction on non-perturbative effects in the type IIA string theory ⁵. As we saw in the above, the heterotic string gets an enhanced non-abelian gauge symmetry when the torus T^4 takes a special shape. This is the Frenkel-Kac mechanism on the worldsheet, and therefore is perturbative in λ . One can show that the type IIA string dual to this heterotic string lives on K_3 which has an orbifold-type singularity with vanishing 2-cycles. Since the string coupling constant is inverted, this enhancement of symmetry must take place non-perturbatively in the type IIA side. Strominger noted that the type IIA string has a BPS-saturated solitonic state and that such a state, if it wraps around the 2-cycle, may become massless as the cycle collapses to a point. He thus proposed that they could be candidates for the extra gauge bosons ⁶. (Actually he proposed this scenario in a slightly different situation, but it is equally applicable to this case.)

In my talk, I discussed two issues:

- (1) How the type IIA string on $\mathbf{R}^6 \times K_3$ behaves near the orbifold singularity

where the 2-cycle degenerates.

(2) How the enhancement of gauge symmetry occurs non-perturbatively along the line of Strominger's proposal.

Since the details have already appeared elsewhere^{7,8}, here I only describe main points of these discussions.

In [7], we have shown that the type IIA string in the neighborhood of the singularity on K_3 is equivalent to the type IIB string in a background of a soliton which is extended over 5 spacial dimensions. The equivalent can be established as follows. The geometry of K_3 in the neighborhood of the (resolved) orbifold singularity is modeled by one of the asymptotically locally Euclidean gravitational instantons. The asymptotic region of such a gravitational instanton has a geometry of S^3/Γ where Γ is a discrete subgroup of $SU(2)$ and it fits nicely in the neighborhood of the orbifold singularity. Let us consider the case of $\Gamma = Z_n$. The instanton can be constructed by considering 2-dimensional torus fibered over \mathbf{R}^2 such that the torus degenerates at n points, $x_1, \dots, x_n \in \mathbf{R}^2$. The total space is 4-dimensional and allows a self-dual metric. The metric is regular provided $x_i \neq x_j$ for $i \neq j$, and it becomes singular as these points approach to each other. If all the n points coincide, there will be $(n - 1)$ degenerating 2-cycles. This gives a model for the orbifold singularity on K_3 .

Now let us perform the T -duality on the torus. Before the T -duality, the special points x_i on \mathbf{R}^2 have codimension 2 on the 4-dimensional space because the torus over x_i is still of finite volume albeit degenerate. Since the T -duality on the torus interchanges its complex moduli and the Kähler moduli, the volume of the torus becomes zero at x_i after the T -duality. This means that the codimension of the singularity is now 4. If one looks more closely, one finds that this configuration is exactly that of the solitonic 5-brane solution by Horowitz and Strominger⁹. It is called 5-brane since the string actually lives in $(9 + 1)$ dimensions, so the codimension 4 means that the singularity is spread over $(5 + 1)$ dimensions. One may regard this as a trajectory of a 5-dimensional extended object, i.e. 5-brane. One of the important properties of this solitonic 5-brane state is that each of them carries a unit charge with respect to the rank-2 anti-symmetric tensor field $B_{\mu\nu}$ in the NS-NS sector of the string, which couples to the vertex operator $\partial X^\mu \bar{\partial} X^\nu$.

Thus we find that the type IIA string on in the neighborhood of the orbifold singularity of Z_n type is equivalent to the type IIB string with n solitonic 5-branes each of which carries a unit charge with respect to the NS-NS B -field. Here we have moved from type IIA to type IIB since the T -duality exchanges the two. As mentioned in the above, when all the points x_i coincide, K_3 generates $(n - 1)$ collapsing 2-cycles. Under the T -duality, this is mapped to

the n solitonic 5-branes approaching to each other.

This observation can be used to explain the non-perturbative enhancement of the gauge symmetry in the following way⁸. The type IIB string has two rank-2 anti-symmetric tensor fields, one comes from the NS-NS sector and the other from the R-R sector. The solitonic 5-brane state that appeared in the above carries the unit charge with respect to the NS-NS field, but no charge with respect to the R-R field. The type IIB string is supposed to have the $SL(2, Z)$ S -duality in 10 dimensions, as explained in the talk by J. Schwarz at this meeting. Under the S -duality, the charges with respect to the NS-NS and R-R fields transform as a doublet of $SL(2, Z)$. Thus the S -duality in 10 dimensions predicts that there is a solitonic 5-brane which carries a unit R-R charge and zero NS-NS charge.

Polchinski has shown that the Dirichlet brane carries the R-R charge and thus is a candidate for such a solitonic state¹⁰. The Dirichlet brane is characterized by the fact that an elementary closed string may disappear on the trajectory of the brane. If we exchange the space and the time coordinates on the worldsheet of the closed string disappearing on the brane, one finds that the Dirichlet brane introduces an open string whose end-point lies on the trajectory of the brane. If there is only one such brane, both ends of the open string should be on the same brane. The open string spectrum then contains the $U(1)$ gauge boson on the $(5 + 1)$ -dimensional trajectory of the 5-brane. If there are more than one branes, an open string may stretch between two branes. If the branes are apart, the stretched string is massive. However as the branes approach each other, the lowest energy states of the open string become massless. In the limit when all the n branes coalesce, the open string spectrum acquires the $U(n)$ gauge boson, with each brane carrying the Chan-Paton factor that can be attached to the end-points of the open string.

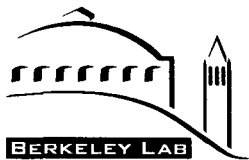
Since we have performed the T -duality and the S -duality to reach at this picture, it is useful to go back the process and unwind these duality transformations. We start with n coalescing Dirichlet 5-branes in the type IIB theory, with elementary open strings stretching between them to generate the $U(n)$ gauge symmetry. If we do the S -duality transformation backward, one finds Dirichlet strings stretching between the solitonic 5-branes of Horowitz and Strominger. We then perform the T -duality back to go from type IIB to type IIA. Under the T -duality, the Dirichlet strings stretching between the 5-branes is mapped to the Dirichlet 2-branes wrapping around the collapsing 2-cycles. Thus we have made a full circle and come back to the original proposal of Strominger. It is the Dirichlet 2-brane which generate the non-perturbative enhancement of gauge symmetry in the type IIA string on $\mathbf{R}^6 \times K_3$.

Acknowledgements

This work was supported in part by the National Science Foundation under grants PHY-9501018 and PHY-9514797 and in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

References

1. K. Kikkawa and M. Yamasai, *Phys. Lett.* 149B (1984) 357.
2. M. Dine, P. Huet and N. Seiberg, *Nucl. Phys.* B322 (1989) 301.
3. J. Dai, R. G. Leigh and J. Polchinski, *Mod. Phys. Lett.* A4 (1989) 2073.
4. C. M. Hull and P. K. Townsend, *Nucl. Phys.* B438 (1995) 109.
5. E. Witten, *Nucl. Phys.* B443 (1995) 85.
6. A. Strominger, *Nucl. Phys.* B451 (1995) 96.
7. H. Ooguri and C. Vafa, e-Print Archive: hep-th/9511164.
8. M. Bershadsky, C. Vafa and V. Sadov, e-Print Archive: hep-th/9510225.
9. G. Horowitz and A. Strominger, *Nucl. Phys.* B360 (1991) 197.
10. J. Polchinski, *Phys. Rev. Lett.* 75 (1995) 4724.



ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
TECHNICAL AND ELECTRONIC INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA | BERKELEY, CALIFORNIA 94720