Title
Neutrino gravitational redshift and the electron fraction above nascent neutron stars

Journal
Nuclear Physics A, 606(1-2)

ISSN
0375-9474

Authors
Fuller, George M
Qian, Yong-Zhong

Publication Date
1996-08-01

DOI
10.1016/0375-9474(96)00268-0

Peer reviewed

NUCLEAR PHYSICS A

ELSEVIER Nuclear Physics A 606 (1996) 167–172

Neutrino gravitational redshift and the electron fraction above nascent neutron stars

George M. Fuller [a], Yong-Zhong Qian [b]

[a] *Department of Physics, University of California, San Diego, La Jolla, California 92093-0319, USA*
[b] *Physics Department, 161-33, California Institute of Technology, Pasadena, California 91125, USA*

Received 29 July 1996

Abstract

Neutrinos emitted from near the surface of the hot proto-neutron star produced by a supernova explosion may be subject to significant gravitational redshift at late times. Electron antineutrinos ($\bar{\nu}_e$) decouple deeper in the gravitational potential well of the neutron star than do the electron neutrinos (ν_e), so that the $\bar{\nu}_e$ experience a larger redshift effect than do the ν_e. We show how this differential redshift can increase the electron fraction Y_e in the neutrino-heated ejecta from the neutron star. Any r-process nucleosynthesis originating in the neutrino-heated ejecta would require a low Y_e, implying that the differential redshift effect cannot be too large. In turn, this effect may allow nucleosynthesis to probe the nuclear equation of state parameters which set the neutron star radius and surface density scale height at times of order $t_{\mathrm{pb}} \approx 10\,\mathrm{s}$ to $25\,\mathrm{s}$ after core bounce.

In this paper we examine the effects of gravitational redshift on the energy spectra of neutrinos and antineutrinos emitted from the surface regions of the hot neutron star remnants of Type II and Type Ib supernova explosions. These considerations allow us to find a serendipitous link between the neutron-to-proton ratio required for r-process nucleosynthesis in the neutrino-heated supernova ejecta and the size and structure of the neutron star at times of order one or several neutrino diffusion timescales after core bounce.

This link is significant, because it may be in just this time frame ($t_{\mathrm{pb}} \approx 10\,\mathrm{s}$ to $25\,\mathrm{s}$, where t_{pb} stands for time *post core bounce*) that enough deleptonization of the proto-neutron star has occurred to trigger an exotic phase transition to quark (strange) matter or, perhaps more likely, a kaon condensate [1]. Kaon condensation, for example, would have a fairly dramatic effect on the evolution of the deleptonizing neutron star. As Gerry Brown and his co-workers have elucidated [2], such a phase transition almost

PII S0375-9474(96)00268-0

inevitably leads to the formation of a black hole. Whether or not there is an exotic phase transition, as the neutron star deleptonizes its radius would shrink and we might expect general relativistic effects near the star's surface to become more pronounced.

Interestingly, calculations indicate that the neutrino-heated ejecta originating from the vicinity of the neutron star surface at these late times is a leading candidate for the site of r-process nucleosynthesis [3,4]. In these calculations neutrinos not only provide the requisite energy to eject this material [5], but the competition [6] between electron neutrino and antineutrino captures on neutrons and protons, respectively,

$$\nu_e + \mathrm{n} \to \mathrm{p} + e^-, \tag{1a}$$

$$\bar{\nu}_e + \mathrm{p} \to \mathrm{n} + e^+, \tag{1b}$$

sets the neutron-to-proton ratio n/p= $1/Y_e - 1$, where Y_e is the electron fraction, or the net number of electrons per baryon. Integration of the rate equations [6] corresponding to the reactions in Eqs. (1a) and (1b) shows that,

$$Y_e \approx \left[1 + \lambda_{\bar{\nu}_e p}(r)/\lambda_{\nu_e n}(r)\right]^{-1}, \tag{2a}$$

where $\lambda_{\bar{\nu}_e p}(r)$ and $\lambda_{\nu_e n}(r)$ correspond to the rates of the reactions in Eqs. (1b) and (1a), respectively, as evaluated at radius r. We can approximate this expression as,

$$Y_e \approx \left[1 + \left(L_{\bar{\nu}_e}\langle E_{\bar{\nu}_e}\rangle\right)/\left(L_{\nu_e}\langle E_{\nu_e}\rangle\right)\right]^{-1}, \tag{2b}$$

where $\langle E_{\bar{\nu}_e}\rangle$ and $\langle E_{\nu_e}\rangle$ represent the average $\bar{\nu}_e$ and ν_e energies, respectively, while $L_{\bar{\nu}_e}$ and L_{ν_e} are the $\bar{\nu}_e$ and ν_e luminosities, respectively. Eq. (2b) follows from Eq. (2a) on noting that, for example, the cross section for the process in Eq. (1a) scales like the square of the neutrino energy, and that the rate for this process is given by the neutrino number flux times the appropriate cross section with, in turn, the number flux given by the ratio of the neutrino luminosity and the appropriate neutrino average energy.

In the absence of neutrino flavor transformations and extreme gravitational redshift effects, the average local energies of the various neutrino species above the neutrino sphere obey the hierarchy:

$$\langle E_{\nu_{\mu(\tau)}}\rangle \approx \langle E_{\bar{\nu}_{\mu(\tau)}}\rangle > \langle E_{\bar{\nu}_e}\rangle > \langle E_{\nu_e}\rangle. \tag{3}$$

This energy hierarchy reflects the fact that the neutron star is mostly neutrons: the ν_e have a higher opacity contribution from the charged current process in Eq. (1a) than the $\bar{\nu}_e$ acquire from the process in Eq. (1b). As a result, the $\bar{\nu}_e$ tend to decouple deeper in the star where it is hotter, and the ν_e tend to decouple further out where it is relatively cooler. Since the muon and tau neutrinos and antineutrinos lack any charged current opacity contribution, they decouple deepest and thus have larger average energies than either of the electron-type neutrino species. Typical average energies at $t_{\mathrm{pb}} \sim 10\,\mathrm{s}$ are [6]: $\langle E_{\nu_{\mu(\tau)}}\rangle \approx \langle E_{\bar{\nu}_{\mu(\tau)}}\rangle \approx 25\,\mathrm{MeV}$; $\langle E_{\bar{\nu}_e}\rangle \approx 16\,\mathrm{MeV}$; and $\langle E_{\nu_e}\rangle \approx 11\,\mathrm{MeV}$. At these times, numerical calculations suggest that the luminosities of all six neutrino species are nearly the same, so that by Eq. (2b) it is clear that $Y_e < 0.5$ and neutron-rich conditions will obtain so long as the average energy hierarchy is as in Eq. (3).

Neutron-rich conditions, $Y_e < 0.5$, are *necessary* for r-process nucleosynthesis to take place in the neutrino-heated ejecta generated in the post-core-bounce supernova environment. However, current models of the r-process based on this environment, though promising, are also problematic. The Y_e, expansion timescale, and entropy conditions predicted in these models are at best barely adequate to give the neutron-to-seed ratio required for the production of nuclei like platinum. It is very clear that these models would not be viable if Y_e were to be *increased* significantly over the predictions of Eqs. (2a) and (2b) with the average neutrino energies as above. Though the neutrino energy spectra are themselves rather uncertain as a result of, for example, transport calculation deficiencies, one might hope that the fractional *difference* between $\langle E_{\bar{\nu}_e}\rangle$ and $\langle E_{\nu_e}\rangle$, which has the most leverage on Y_e, could be obtained somewhat more reliably. Numerical models show that the fractional difference between $\langle E_{\bar{\nu}_e}\rangle$ and $\langle E_{\nu_e}\rangle$ increases with time, as would be expected with continuing deleptonization and the concomitant lessening of the charged current opacity contribution [Eq. (1a)] for the $\bar{\nu}_e$. It might be tempting to argue that the r-process in the neutrino-heated ejecta comes from very late times post-core-bounce, when this electron antineutrino/neutrino fractional average energy difference is large and, therefore, Y_e in the ejecta is small.

But this may be dangerous, as invoking very late times and significant deleptonization raises the possibility that gravitational redshift effects can become important, especially if there is a phase transition to an exotic equation of state. At this epoch we expect the radius of the neutron star to change only slowly with time, essentially on a neutrino diffusion timescale. Therefore, to gauge the effects of redshift on the neutrino and antineutrino energy spectra it is adequate to approximate the spacetime as static. We will also neglect rotation and magnetic fields, and assume that spherical symmetry is a good approximation. With these approximations, we can describe adequately the region outside the neutron star as the vacuum Schwarzschild geometry with metric components $g_{\alpha\beta}$ defined through the line element, $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$,

$$ds^2 = -e^{\{2\Phi(r)\}}dt^2 + e^{\{2\Lambda(r)\}}dr^2 + r^2 d\Omega^2, \tag{4}$$

where $d\Omega^2 = (d\theta^2 + \sin^2\theta d\phi^2)$, and $\{x^\alpha\} \to (t, r, \theta, \phi)$ are the Schwarzschild coordinates. Here $\Phi(r)$ and $\Lambda(r)$ are the usual radial coordinate-dependent Schwarzschild metric functions [7]. In vacuum the metric function in the time-time component of the metric is given by $\exp\{2\Phi(r)\} = 1 - r_s/r$, where the Schwarzschild radius is $r_s = 2M \approx 4.134\,\mathrm{km}\,(M/1.4\,M_\odot)$, with M the gravitational mass of the neutron star, and where we have set $G = c = 1$.

In any static spacetime, the timelike *covariant* component of the four-momemtum $p_0 = g_{0\beta}p^\beta$ [where $\mathbf{p} \Rightarrow \{p^\beta\} \to (p^0, p^1, p^2, p^3)$] of a freely falling particle is a constant of the motion for covering coordinates in which the metric functions are time independent, *e.g.* the Schwarzschild coordinates. Neutrinos emitted from a "neutrino sphere" near the neutron star's surface can be taken to be freely streaming, and thus following geodesics, in the region well above the surface. The energy $E^*(r)$ measured by a locally inertial observer (with four velocity $\mathbf{u}$) at some point at radius r along a neutrino trajectory will be $E^*(r) = -\mathbf{p}\cdot\mathbf{u}$.

If we take this observer to be momentarily at rest at position r, and evaluate the frame invariant inner product in the Schwarzschild coordinates [so that the four velocity has components $\mathbf{u} \to (\exp\{-\Phi\}, 0, 0, 0)$], then the locally measured energy will be,

$$E^*(r) = E\exp\{-\Phi(r)\}, \tag{5a}$$

where E is a conserved quantity defined in terms of the time-like covariant momentum component in the Schwarzschild coordinates as $E \equiv -p_0$. With this definition, E is interpreted as the energy of the neutrino as measured at infinity. In terms of the neutrino production (emission) energy at the neutrino sphere, $E_{\rm em}$, as measured by a locally inertial observer at rest, we can write,

$$E = E_{\rm em}\exp\{\Phi(r_{\rm em})\}, \tag{5b}$$

where $r_{\rm em}$ is the radius at which the neutrino is produced (the neutrino sphere). The redshift at Schwarzschild radial coordinate r is then $z_r = E_{\rm em}/E^*(r) - 1$, so that $1 + z_r = \exp\{\Phi(r) - \Phi(r_{\rm em})\}$, and the redshift at radial infinity is $z \equiv z_\infty = (1 - r_s/r_{\rm em})^{-1/2} - 1$.

At Schwarzschild radial coordinate r, the electron fraction is determined as in Eqs. (2a) and (2b) by the *local* values of the luminosities and average energies of the $\bar{\nu}_e$ and ν_e. However, since these neutrino species have differing production/emission radii (*i.e.*, their neutrino spheres have different values of Schwarzschild radial coordinate), they are subject to different gravitational redshift effects. If we define the ν_e neutrino sphere to be at $r^{\rm sp}_{\nu_e}$ and the $\bar{\nu}_e$ neutrino sphere to be at $r^{\rm sp}_{\bar{\nu}_e}$, then Eq. (2b) can be recast as:

$$Y_e = 1/\{1 + R_{n/p}\}, \tag{6a}$$

$$R_{n/p} \equiv R^0_{n/p} \cdot \Gamma, \tag{6b}$$

$$R^0_{n/p} \approx \left[\frac{L^{\rm sp}_{\bar{\nu}_e}\langle E^{\rm sp}_{\bar{\nu}_e}\rangle}{L^{\rm sp}_{\nu_e}\langle E^{\rm sp}_{\nu_e}\rangle}\right]. \tag{6c}$$

In these equations, $\langle E^{\rm sp}_{\bar{\nu}_e}\rangle$ and $L^{\rm sp}_{\bar{\nu}_e}$ are the average $\bar{\nu}_e$ energy and luminosity as measured by a locally inertial observer at rest at the $\bar{\nu}_e$ neutrino sphere; and similarly for the quantities which characterize the ν_e energy and luminosity at the ν_e neutrino sphere. We are making the approximation that the (anti)neutrino energy spectrum does not evolve significantly with increasing radius above the (anti)neutrino sphere as a result of emission, absorption and scattering processes. The quantity $R_{n/p}$ is the local neutron-to-proton ratio, and as is evident from Eq. (2a), this can be approximated as $R_{n/p} \approx \lambda_{\bar{\nu}_e p}(r)/\lambda_{\nu_e n}(r)$, where the rates are understood to be evaluated from the neutrino and antineutrino energy spectra extant at Schwarzschild radial coordinate r. In Eq. (6b), $R^0_{n/p}$ can be interpreted as the neutron-to-proton ratio [corresponding to the electron fraction $Y^0_e = 1/(1 + R^0_{n/p})$] in the absence of gravitational redshift effects. Here the effects of the gravitational field are represented by the term Γ which can be written as,

$$\Gamma \equiv \exp\left\{3\left[\Phi\left(r^{\rm sp}_{\bar{\nu}_e}\right) - \Phi\left(r^{\rm sp}_{\nu_e}\right)\right]\right\}, \tag{7a}$$

$$\Gamma = \left(\frac{1 - r_s/r^{\rm sp}_{\bar{\nu}_e}}{1 - r_s/r^{\rm sp}_{\nu_e}}\right)^{3/2}, \tag{7b}$$

where Eq. (7b) obtains in vacuum. Note that Γ contains three factors of the redshift term which appears in Eqs. (5a) and (5b). This is because the product of neutrino luminosity and average energy has contained in it two "energy" factors and one inverse time interval (*i.e.*, frequency), which redshifts in the same manner as energy.

Let us define the difference of the neutrino and antineutrino sphere positions to be $\delta r \equiv r^{\rm sp}_{\nu_e} - r^{\rm sp}_{\bar{\nu}_e}$. If we now assume that $\delta r << r^{\rm sp}_{\bar{\nu}_e}$, then we can estimate that,

$$R_{n/p} \approx R^0_{n/p} \cdot \left(1 + \frac{3}{2}\gamma \frac{\delta r}{r^{\rm sp}_{\bar{\nu}_e}}\right)^{-1}, \tag{8a}$$

where the redshift amplification factor is defined to be,

$$\gamma \equiv 1/\left(r^{\rm sp}_{\bar{\nu}_e}/r_s - 1\right). \tag{8b}$$

For example, if $\delta r/r^{\rm sp}_{\bar{\nu}_e} \approx 0.1$ and the predicted electron fraction in the absence of gravitational redshift effects is $Y^0_e \approx 0.4$, then we must have $r_s/r^{\rm sp}_{\bar{\nu}_e} < 0.76$ if we are to have more neutrons than protons, $Y_e < 0.5$ when gravitational redshift effects are taken into account. In fact, the differential redshift effect is even greater when the threshold effects are taken into account in the evaluation of the rates which enter into $R_{n/p} \approx \lambda_{\bar{\nu}_e p}(r)/\lambda_{\nu_e n}(r)$. Since the neutron is heavier than the proton by $\Delta_{np} \approx 1.293\,\mathrm{MeV}$, the process in Eq. (1b) has a threshold, and any reduction in the average energies of the neutrinos and antineutrinos has a disproportionate effect on reducing $\lambda_{\bar{\nu}_e p}(r)$. A semi-numerical calculation including these threshold effects shows that for $L^{\rm sp}_{\bar{\nu}_e}/L^{\rm sp}_{\nu_e} \approx 1.2$, and $\epsilon^{\rm sp}_{\bar{\nu}_e} \equiv \langle (E^{\rm sp}_{\bar{\nu}_e})^2\rangle/\langle E^{\rm sp}_{\bar{\nu}_e}\rangle \approx 25\,\mathrm{MeV}$ and $\epsilon^{\rm sp}_{\nu_e} \equiv \langle (E^{\rm sp}_{\nu_e})^2\rangle/\langle E^{\rm sp}_{\nu_e}\rangle \approx 10\,\mathrm{MeV}$ (which imply that $Y^0_e \approx 0.32$), and $\delta r/r^{\rm sp}_{\bar{\nu}_e} \approx 0.1$, we must have $r_s/r^{\rm sp}_{\bar{\nu}_e} < 0.58$ if we are to guarantee that the electron fraction is $Y_e < 0.4$. The same parameters, but now assuming that $\delta r/r^{\rm sp}_{\bar{\nu}_e} \approx 0.01$, would require $r_s/r^{\rm sp}_{\bar{\nu}_e} < 0.75$ if we demand that $Y_e < 0.4$. A value of $Y_e \approx 0.4$ may actually be a conservative limit on the maximum allowable electron fraction for the r-process.

We see that *plausible* values of the neutron star mass (or r_s), radius ($\approx r^{\rm sp}_{\bar{\nu}_e}$) and density scale height near the surface (which determines δr) could produce enough differential gravitational redshift to *preclude* neutrino-heated supernova ejecta as the site of the r-process. If eventually it is argued convincingly that this site *does* give rise to the r-process, then differential neutrino redshift could allow the nucleosynthesis abundance yield to become an interesting probe of the nuclear equation of state parameters that set $r^{\rm sp}_{\bar{\nu}_e}$ and δr. Clearly, the differential redshift effects become more dramatic as γ becomes larger and the star becomes more relativistic. In turn, we know from the work of Gerry Brown and others that a phase transition to a soft kaon condensed environment (or interacting strange quark matter) will cause the star to become more relativistic. One might argue that, for a particularly soft equation of state, δr might approach zero, and the neutrinos and antineutrinos could then decouple from the same radius with (possibly) the same energy spectrum, thus ensuring that there will be no differential redshift effect. However, this particular case will also preclude r-process nucleosynthesis, as nearly identical energy spectra for the $\bar{\nu}_e$ and the ν_e will produce $Y_e > 0.5$ as a result of the energy threshold for the process in Eq. (1b). In any case, the effects of an exotic state of matter on the neutrino and antineutrino energy spectra call for further examination.

Acknowledgements

This work was supported by NSF Grant PHY95-03384 and NASA Grant NAG5-3062 at UCSD. Y.-Z. Qian was supported by the D. W. Morrisroe Fellowship at Caltech.

References

[1] G. E. Brown, K. Kubodera, M. Rho, and V. Thorsson, Phys. Lett. B 291 (1992) 355.
[2] G. E. Brown and H. A. Bethe, Astrophys. J. 423 (1994) 659.
[3] B. S. Meyer, W. M. Howard, G. J. Mathews, S. E. Woosley, and R. D. Hoffman, Astrophys. J. 399 (1992) 656.
[4] S. E. Woosley, J. R. Wilson, G. J. Mathews, R. D. Hoffman, and B. S. Meyer, Astrophys. J. 433 (1994) 229.
[5] Y.-Z. Qian and S. E. Woosley, Astrophys. J. 471 (1996) in press.
[6] Y.-Z. Qian, G. M. Fuller, G. J. Mathews, R. W. Mayle, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. 71 (1993) 1965.
[7] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, San Francisco, W. H. Freeman Press (1973).