Title
Energy-aware Trajectory Optimization of Connected and Automated Vehicle Platoons through a Signalized Intersection

Permalink
https://escholarship.org/uc/item/00d6591g

Authors
Han, Xiao, PhD
Ma, Rui, PhD
Zhang, H. Michael, PhD

Publication Date
2019-06-01
Energy-aware Trajectory Optimization of Connected and Automated Vehicle Platoons through a Signalized Intersection

Xiao Han, Postdoctoral Researcher, Department of Civil and Environmental Engineering, University of California, Davis
Rui Ma, Postdoctoral Researcher, Department of Civil and Environmental Engineering, University of California, Davis
H. Michael Zhang, Professor, Department of Civil and Environmental Engineering, University of California, Davis

June 2019
## Abstract

Traffic signals, while serving an important function to coordinate vehicle movements through intersections, also cause frequent stops and delays, particularly when they are not properly timed. Such stops and delays contribute to significant amount of fuel consumption and greenhouse gas emissions. The recent development of connected and automated vehicle (CAV) technology provides new opportunities to enable better control of vehicles and intersections, that in turn reduces fuel consumption and emissions. In this paper, we propose platoon-trajectory-optimization (PTO) to minimize the total fuel consumption of a CAV platoon through a signalized intersection. In this approach, all CAVs in one platoon are considered as a whole, that is, all other CAVs follow the trajectory of the leading one with a time delay and minimum safety gap, which is enabled by vehicle to vehicle communication. Moreover, the leading CAV in the platoon learns of the signal timing plan just after it enters the approach segment through vehicle to infrastructure communication. We compare our PTO control with the other two controls, in which the leading vehicle adopts the optimal trajectory (LTO) or drive with maximum speed (AT), respectively, and the other vehicles follow the leading vehicle with a simplified Gipps’ car-following model. Furthermore, we extend the controls into multiple platoons by considering the interactions between the two platoons. The numerical results demonstrate that PTO has better performance than LTO and AT, particularly when CAVs have enough space and travel time to smooth their trajectories. The reduction of travel time and fuel consumption can be as high as 40% and 30% on average, respectively, in the studied cases, which shows the great potential of CAV technology in reducing congestion and negative environmental impact of automobile transportation.

### Key Words

Connect vehicles, autonomous vehicles, traffic platooning, fuel consumption, vehicle trajectories, trajectory control
About the UC Institute of Transportation Studies

The University of California Institute of Transportation Studies (UC ITS) is a network of faculty, research and administrative staff, and students dedicated to advancing the state of the art in transportation engineering, planning, and policy for the people of California. Established by the Legislature in 1947, ITS has branches at UC Berkeley, UC Davis, UC Irvine, and UCLA.

Acknowledgements

This study was made possible through funding received by the University of California Institute of Transportation Studies from the State of California via the Public Transportation Account and the Road Repair and Accountability Act of 2017 (Senate Bill 1). The authors would like to thank the State of California for its support of university-based research, and especially for the funding received for this project. The authors would also like to thank CTECH: Center for Transportation, Environment and Community Health, a US DOT funded university center, for its support of this research.

Disclaimer

The contents of this report reflect the views of the author(s), who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the State of California in the interest of information exchange. The State of California assumes no liability for the contents or use thereof. Nor does the content necessarily reflect the official views or policies of the State of California. This report does not constitute a standard, specification, or regulation.
Energy-aware Trajectory Optimization of Connected and Automated Vehicle Platoons through a Signalized Intersection

UNIVERSITY OF CALIFORNIA INSTITUTE OF TRANSPORTATION STUDIES

June 2019

Xiao Han, Department of Civil and Environmental Engineering, University of California, Davis

Rui Ma, Postdoctoral Scholar, Department of Civil and Environmental Engineering, University of California, Davis

H. Michael Zhang, Professor, Department of Civil and Environmental Engineering, University of California, Davis
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Literature review</td>
<td>3</td>
</tr>
<tr>
<td>Optimal control of one CAV</td>
<td>4</td>
</tr>
<tr>
<td>Platoon optimization</td>
<td>7</td>
</tr>
<tr>
<td>The framework of PTO method</td>
<td>7</td>
</tr>
<tr>
<td>Two other methods for comparison</td>
<td>9</td>
</tr>
<tr>
<td>Case study</td>
<td>10</td>
</tr>
<tr>
<td>Sensitivity analysis</td>
<td>13</td>
</tr>
<tr>
<td>Optimization of multiple platoons</td>
<td>14</td>
</tr>
<tr>
<td>The constraint between two platoons</td>
<td>14</td>
</tr>
<tr>
<td>Case study</td>
<td>14</td>
</tr>
<tr>
<td>Sensitivity analysis</td>
<td>16</td>
</tr>
<tr>
<td>Conclusions and Discussions</td>
<td>17</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>19</td>
</tr>
<tr>
<td>Reference</td>
<td>20</td>
</tr>
</tbody>
</table>
List of Tables

Table 1: Parameter definitions and values in the fuel consumption model ........................................... 5
Table 2: Average fuel consumption and travel time per vehicle per 100 meters shown in Figure 4 .............................................................................................................................................................................. 12
Table 3: Average fuel consumption and travel time per vehicle per 100 meters shown in Figure 7 ........................................................................................................................................................................... 15
Table 4: Average fuel consumption and travel time per vehicle per 100 meters shown in Figure 11 .............................................................................................................................................................................. 18
List of Figures

Figure 1: Optimal control outcomes of one CAV ................................................................. 6
Figure 2: Illustration of trajectory copying ........................................................................... 8
Figure 3: Illustration of control framework of PTO method .................................................. 8
Figure 4: Trajectories of one platoon entering at different times ........................................... 11
Figure 5: The performance of fuel consumption (ml) and travel time (s) per vehicle per 100 meters with PTO, LTO and AT methods for different entry time ........................................... 12
Figure 6: Sensitivity analysis of one platoon across a signalized intersection with different parameters .................................................................................................................. 13
Figure 7: Trajectories of multiple platoons across a signalized intersection........................ 15
Figure 8: Cumulative probability of fuel consumption (ml) and travel time (s) ..................... 16
Figure 9: Fuel consumption (ml) per vehicle per 100 meters for multiple platoons across a signalized intersection ........................................................................................................ 17
Figure 10: Travel time (s) per vehicle per 100 meters for multiple platoons across a signalized intersection ........................................................................................................... 18
Figure 11: Trajectories of multiple platoons across two signalized intersections................. 19
Energy-aware Trajectory Optimization of CAV Platoons through a Signalized Intersection

Xiao Han, Rui Ma, H. Michael Zhang

Department of Civil and Environmental Engineering, University of California, Davis, 95616, CA, United States

Traffic signals, while serving an important function to coordinate vehicle movements through intersections, also cause frequent stops and delays, particularly when they are not properly timed. Such stops and delays contribute to significant amount of fuel consumption and greenhouse gas emissions. The recent development of connected and automated vehicle (CAV) technology provides new opportunities to enable better control of vehicles and intersections, that in turn reduces fuel consumption and emissions. In this paper, we propose platoon-trajectory-optimization (PTO) to minimize the total fuel consumption of a CAV platoon through a signalized intersection. In this approach, all CAVs in one platoon are considered as a whole, that is, all other CAVs follow the trajectory of the leading one with a time delay and minimum safety gap, which is enabled by vehicle to vehicle communication. Moreover, the leading CAV in the platoon learns of the signal timing plan just after it enters the approach segment through vehicle to infrastructure communication. We compare our PTO control with the other two controls, in which the leading vehicle adopts the optimal trajectory (LTO) or drive with maximum speed (AT), respectively, and the other vehicles follow the leading vehicle with a simplified Gipps’ car-following model. Furthermore, we extend the controls into multiple platoons by considering the interactions between the two platoons. The numerical results demonstrate that PTO has better performance than LTO and AT, particularly when CAVs have enough space and travel time to smooth their trajectories. The reduction of travel time and fuel consumption can be as high as 40% and 30% on average, respectively, in the studied cases, which shows the great potential of CAV technology in reducing congestion and negative environmental impact of automobile transportation.

Keywords: Connected-automated vehicle, platoon, fuel consumption, optimal control

1. Introduction

Transportation is a major consumer of non-renewable energy. In 2018, the U.S. transportation sector alone consumed over 143 billion gallons of motor fuel, and it is predicted...
that the fuel consumption in transportation in the U.S. will remain at a high level in the foreseeable future [1]. Furthermore, the world consumption of transportation fuel is forecast to increase significantly with a steady increase in vehicle ownership as incomes in developing countries rise [2]. There has been a practice of the so-called eco-driving among environmentally conscious drivers, which tries to avoid hard accelerations and decelerations based on real-time driving conditions, particularly on urban streets with numerous traffic lights [3–5]. This practice was shown to reduce personal fuel consumption, but without the advance knowledge of traffic signal status, the practice is based on ad hoc rules and furthermore, its impact on other drivers, and hence at a system level, is not certain. Fortunately, the rapidly evolving connected and autonomous vehicle (CAV) technology can overcome these limitations of eco-driving through better communication and greater vehicle control, and hence provides a powerful tool to reduce both fuel consumption and greenhouse gas emissions more effectively [6–8].

In the transportation system, intersections play a crucial role in assigning and controlling traffic flow. In many cases, traffic streams on arterial roads are controlled by traffic signals at intersections. Vehicles must stop at signals on red, which increases their fuel consumption, emission levels and travel time due to acceleration/deceleration maneuvers and idling required at traffic signals. In this paper, we propose a platoon-trajectory-optimization (PTO) method, to control CAVs moving through a signalized intersection as so to minimize the total fuel consumption of the platoon. In this method, we assume the CAV platoon knows the traffic light’s schedule before entering the approach of the intersection, and consider all CAVs in one platoon as a whole, and classify the scenarios of CAVs passing a signalized intersection into two categories according to whether all CAVs can cross the intersection within one signal cycle or not, i.e., all CAVs passing the intersection within one green light window (Scenario I) and the CAVs passing the intersection in two successive green light windows (Scenario II). In Scenario I, the trajectory of the leading CAVs is copied by the other ones in the platoon with reaction time delay and a safety space gap, enabled by vehicle to vehicle communication. In Scenario II, the platoon must be split into two subplatoons, and the other CAVs in each subplatoon follow the leading one with trajectory copying.

In addition, we compare PTO with other two methods based on a simplified Gipps’ car-following model, i.e., leading-trajectory-optimization (LTO) and aggressive driving (AT). In LTO method, we suppose the leading vehicle is a CAV, and the others are human-driven vehicles. The strategy of the leading CAV is to minimize its fuel consumption with optimal control and pass the signalized intersection without considering the following vehicles. The human-driven vehicles travel across the intersection with a simplified Gipps’ car-following model and stop before the intersection when the red light is on. In AT method, we suppose all vehicles in one platoon are human-driven. The leading vehicle travels with maximum speed and stops before the intersection until the green light is on. As similar as LTO, the other vehicles follow the leading vehicle with a simplified Gipps’ car-following model in AT. Furthermore, we apply the PTO method to control multiple platoons across a signalized intersection in consideration of the intersections between two platoons. A virtual trajectory generated based on the last CAV of the platoon in front is taken as a constraint of the back platoon to ensure safety. The results of case studies and sensitivity analysis demonstrate
PTO outperforms LTO and AT in reducing both fuel consumption and travel time when the CAVs have enough space and traffic throughput to smooth their trajectories.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the results of optimizing one vehicle with optimal control. In Section 4, the frameworks of PTO and the other two methods, LTO and AT, are described. Case studies and sensitivity analysis are conducted to compare the performance of the three methods. In Section 5, we extend the three methods into multiple platoons. As similar as Section 4, we conduct case studies and sensitivity analysis in the multiple-platoon level. Section 6 concludes the paper and discusses some further research directions.

2. Literature review

With the emergence of technologies, such as connected vehicle (CV), autonomous vehicle (AV) and connected autonomous vehicle (CAV), vehicle trajectory control strategies have been proposed to reduce fuel consumption for arterial roads controlled by speed limits or traffic signals in recent years [9–17]. Liu et al. present a fuel-consumption-aware variable-speed limit (FC-VSL) traffic control scheme to minimize the fuel consumption on freeways with the problem formulated as an optimal control problem [10]. He et al. propose a multi-stage optimal control formulation to optimize vehicle trajectory on signalized arterial roads that considers both vehicle queue and traffic light status [11]. Ubiergo and Jin present a hierarchical green driving strategy based on feedback control to smooth stop-and-go traffic in signalized networks with vehicle-to-infrastructure (V2I) communication [12]. With numerical analysis, they demonstrate that their method can save about 15% in travel delays and 8% in fuel consumption and greenhouse gas emissions. Zhou et al. and Ma et al. propose a parsimonious shooting heuristic algorithm to construct vehicle trajectories on a signalized highway segment, in which the trajectories of each vehicle is broken into a few sections that each one is analytically solvable [13, 14]. Li and Zhou propose an intersection automation policy (IAP) to capture complex traffic dynamics and schedule resources (green lights) to serve both CAV and human-driven vehicles [15]. Yao et al. present a trajectory smoothing method based on individual variable speed limits with location optimization (IVSL-LC), and compare the method with the individual advisory speed limits (IASL). They demonstrate IVSL-LC method can greatly increase traffic efficiency and reduce fuel consumption in contrast to IASL [16]. Feng et al. propose a two-stage optimization framework that combines trajectory smoothing and traffic signal control [17]. Simulation results show that the framework can reduce 24% vehicle delay and 13.8% CO2 emissions.

The above studies mainly focus on solving the problem of trajectory smoothing across a signalized intersection at the individual vehicle-level. Moreover, with the development of CAV, it is possible to reduce fuel consumption and travel time at the platoon-level [18–20]. Wei et al. present a set of integer programming and dynamic programming models for scheduling longitudinal trajectories based on a space-time lattice [18]. By adjusting the lead vehicle’s speed and platoon-level reaction time at each time step, their framework can control the complete set of trajectories in a platoon efficiently [18]. Lioris et al. assess the potential mobility benefits of platooning with connected vehicle technology (CVT), and platooning
in CVT environment can double throughput in urban roads [19]. Stebbins et al. propose a trajectory optimization method by optimizing for the delay over the entire trajectory instead of suggesting an individual speed [20]. Moreover, they extend the framework to platoon-level, in which other vehicles follow the leading vehicle with a car-following model.

In this paper, we develop a vehicle trajectory control framework for CAV platoons to reduce fuel consumption and travel delay. To take advantage of vehicle to infrastructure (V2I) and vehicle to vehicle (V2V) communication in a CAV traffic environment, traffic signal timing status is transmitted to the leading CAV vehicle before it enters the intersection, and the platoon leaves the intersection at free-flow speed (or the speed limit of the road), which serves at the final state condition for our formulated optimal trajectory control problem. Our approach first develops the optimal control policy for a single CAV, then extends it to a vehicle platoon, and finally designs a mechanism to control multiple platoons traversing a signalized intersection considering the interactions between platoons.

3. Optimal control of one CAV

First, let us optimize the trajectory of one CAV with optimal control from location $s_0$ to location $s_1$ ($s_1 > s_0$) without traffic signal. Suppose, at time $t_0$, one CAV with maximum speed $v_0$ travel at location $s_0$, and the vehicle must arrive at location $s_1$ at the maximum speed of $v_0$. In this situation, we need to optimize one trajectory to minimize fuel consumption for the vehicle traveling from location $s_0$ to location $s_1$ with speed limit. The framework for solving this problem can be presented as follows.

(1) System Model: For a single vehicle, state vector $x(t)$ is defined as,

$$
\mathbf{x}(t) \triangleq [x_1(t) \ x_2(t)]^T = [s(t) \ v(t)]^T, \quad (1)
$$

where $s(t)$ is the distance from $s_0$, and $v(t)$ is the speed of the vehicle. Those two variables denote the state of the vehicle. The control vector only contains one variable, i.e., the acceleration rate, which is defined as,

$$
\mathbf{u}(t) \triangleq [a(t)]^T. \quad (2)
$$

Therefore, the dynamics of the system can be described with differential equations,

$$
\dot{\mathbf{x}}(t) \triangleq \begin{bmatrix}
\dot{x}_1(t) = v(t) \\
\dot{x}_2(t) = a(t)
\end{bmatrix} \quad (3)
$$

(2) Optimal Control Problem Formulation: The problem of controlling the CAV is formulated to minimize the fuel consumption as follows,

$$
J = \int_{t_0}^{t_f} c(v(t), a(t))dt, \quad (4)
$$

where $t_0$ and $t_f$ are the corresponding time points of $s_0$ and $s_1$, respectively; $c(v(t), a(t))$ is an instantaneous fuel consumption model presented at the Conference of Australian Institutes.
of Transportation Research (CAITR) [10, 21], which is given by,

\[
c(v(t), a(t)) = \begin{cases} 
\alpha, & a(t) \leq -\frac{R_a(t) + R_f(t)}{M_a} \\
\alpha + \beta_1 R_T(t)v(t), & a(t) \in (-\frac{R_a(t) + R_f(t)}{M_a}, 0) \\
\alpha + \beta_1 R_T(t)v(t) + \frac{\beta_2 M_o a(t)^2 v(t)}{1000}, & a(t) \geq 0 
\end{cases}
\]

(5)

where \(R_T(t), R_a(t), \) and \(R_f(t)\) are the tractive, air drag, and rolling resistance, respectively. They can be calculated as follows:

\[
R_T(t) = M_o a(t) + R_a(t) + R_f(t) + R_g(t)
\]

(6)

\[
R_a(t) = \frac{\rho}{2} C_D A_f v(t)^2
\]

(7)

\[
R_f(t) = 0.01 \frac{1 + v(t)}{44.73} M_v g
\]

(8)

The definitions and values of the parameters from Eq. 5 to Eq. 8 are shown in Table 1. Note that the default values of the parameters of the fuel consumption model assume the vehicle travels in on a flat surface (i.e., grade force \(R_g(t) = 0\)) and neglect the wind pressure. However, the fuel consumption model can easily be extended to more general scenarios that can reflect a real environment by adjusting the values of the parameters of \(R_T\). Here, for the sake of simplification, we only consider the parameters shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Idle fuel consumption rate</td>
<td>0.375mL/s</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>Efficiency parameter</td>
<td>0.09mL/kJ</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>Energy-acceleration efficiency parameter</td>
<td>0.03mL/(kJ.m/s²)</td>
</tr>
<tr>
<td>(M_v)</td>
<td>Average vehicle mass</td>
<td>1400kg</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Air density</td>
<td>1.2256kg/m³</td>
</tr>
<tr>
<td>(C_D)</td>
<td>Drag coefficient</td>
<td>0.54</td>
</tr>
<tr>
<td>(A_f)</td>
<td>Average vehicle frontal area</td>
<td>2.1m²</td>
</tr>
<tr>
<td>(g)</td>
<td>Standard gravity</td>
<td>9.8m/s²</td>
</tr>
</tbody>
</table>

The above optimal control problem is challenging to find analytical solutions [10]. Instead, the numerical Gauss pseudospectral method (GPM) is used to discretize a continuous optimal control problem into a nonlinear program (NLP) and obtain the optimal solution. The technique is an orthogonal collocation method where the collocation points are the Legendre-Gauss (LG) points [22]. Here, we employ the General Algebraic Modeling System (GAMS) to obtain the optimal control solution [23].

Figure 1 presents the optimal results with different travel distance, maximum speed, deceleration/acceleration constraint, and LG points. Figure 1(a)-(d) show the relationship between optimal fuel consumption and travel time. The corresponding travel time with
the lowest fuel consumption is a little longer than the shortest travel time with constant maximum speed. At a given travel distance, the optimal fuel consumption decreases firstly and then increases over travel time. Figure 1(e)-(h) show the optimal trajectories with lowest fuel consumption. The CAV traveling with lowest fuel consumption needs to decelerate firstly, and then gradually accelerate to maximum speed. It is a bit counterintuitive at first because it is generally believed that keeping a constant velocity would consume less fuel in contrast to a trajectory with speed variations. However, a closer examination of the fuel consumption model of Eq. 5 reveals the reason for this counterintuitive phenomenon. When the acceleration $a \geq 0$, even though it has a high impact on the fuel consumption in the third term, vehicle speed $v(t)$ dominates in both the second and third terms. The implication is that the effects of the lower speed could offset the impact of high acceleration rate on fuel consumption. Besides, we find the deceleration/acceleration constraint and the number of LG points do not have a significant influence on the performance of optimal control. Therefore, in the following sections, we set the maximum brake deceleration as $a_{br} = -4 m/s^2$, maximum acceleration as $a_{fw} = 2 m/s^2$ and the number of LG points as $N_{LG} = 200$. 

Figure 1: Optimal control outcomes of one CAV. (a-d) The relationship between optimal fuel consumption per 100 meters and travel time; (e-h) The relationship between travel speed and space for optimal trajectories with lowest fuel consumption. (a, e) Optimal results with different control space $s_1$ and the same maximum speed ($v_0 = 20 m/s$) and deceleration/acceleration constraint ($a_{br} = -4 m/s^2, a_{fw} = 2 m/s^2$) and number of LG points ($N_{LG} = 200$). (b, f) Optimal results with different maximum speed and the same control space ($s_1 = 500 m$) and deceleration/acceleration constraint ($a_{br} = -4 m/s^2, a_{fw} = 2 m/s^2$) and number of LG points ($N_{LG} = 200$). (c, g) Optimal results with different deceleration/acceleration constraint and the same control space ($s_1 = 500 m$), maximum speed ($v_0 = 20 m/s$) and number of LG points ($N_{LG} = 200$). (d, h) Optimal results with different number of LG points and the same control space ($s_1 = 500 m$) and maximum speed ($v_0 = 20 m/s$) and deceleration/acceleration constraint ($a_{br} = -4 m/s^2, a_{fw} = 2 m/s^2$).
4. Platoon optimization

4.1. The framework of PTO method

Based on the optimal control framework for one vehicle described in the above section, we propose the PTO method to optimize one platoon across a signalized intersection by considering all CAVs in the platoon as a whole. The components of the PTO method are described as follows.

Road: We only consider one single lane leading to a signalized intersection. The leading CAV in one platoon enters location $s_0$ and arrives location $s_1$ with maximum speed $v_0$. The traffic signal is installed at location $s_1$.

Traffic Signal: The traffic signal we consider here is a fixed signal timing including a sufficient length of $G$ and an effective red time of $R$. Thus, the cycle length of the traffic signal is $C := G + R$.

Platoon: The number of CAVs in one platoon is $N$. The initial state of the platoon (the leading CAV) arriving at location $s_0$ is that all CAVs have the same speed of $v_0$ and the space between two vehicles in the platoon is same. Suppose the reaction time of CAV is $\tau$ and the minimum gap between the two vehicles is $d$. The space between two CAVs at initial state is $l = d + v_0 \tau$, and the total length of the platoon at initial state is $L_p = (d + v_0 \tau)(N - 1)$. Moreover, we suppose all CAVs in one platoon can pass the green light windows if the leading vehicle arrives the intersection at the beginning of the green light window, i.e., $G > \frac{(N-1)l}{v_0}$.

Trajectory copying: The basic idea of PTO method is that all vehicles can copy the trajectory of the leading vehicles with reaction time delay and minimum gap delay. Figure 2 presents an illustration of trajectory copying, in which Trajectory 1 is the trajectory of the leading CAV in one platoon, and the following CAV in the platoon can copy Trajectory 1 with time delay $\tau$ and minimum gap delay $d$, and travel along Trajectory 2.

Two scenarios: For one platoon across a signalized intersection, we divide the process into two scenarios according to whether the platoon can pass the signalized intersection in one green light window. The two scenarios are described as follows.

- Scenario I: The platoon can pass the signalized intersection within one green light window.

- Scenario II: The CAVs in one platoon cannot pass the intersection within a green light window. The platoon must be split into two subplatoons, i.e., Subplatoon A (the former one) and Subplatoon B (the latter one), and pass the signalized intersection in two successive green light windows.

Figure 3 illustrates the operations of controlling one platoon across a signalized intersection with PTO. The platoon is composed of 6 CAVs, which can not pass the intersection within one green light window (Scenario II). The platoon is split into two subplatoons, and 3 CAVs in each subplatoon. As shown in Figure 3, the control space of Subplatoon A (blue trajectories) and Subplatoon B (black trajectories) are $s_1 - s_0$ and $s_1 - s_0 + 3(v_0 \tau + d)$, respectively.
Figure 2: Illustration of trajectory copying.

Figure 3: Illustration of control framework of PTO method.
Taking Scenario I and Scenario II into consideration, the total fuel consumption of one platoon with PTO method across a signalized intersection can be formulated as,

\[
J_p = N_A J_A + N_B J_B,
\]

where \(N_A \geq 0\) and \(N_B \geq 0\); \(N_A (N_B)\) denotes the number of CAVs and \(J_A (J_B)\) the fuel consumption of one vehicle in Subplatoon A (B). Substituting \(J_A\) and \(J_B\) with Eq. 4, we can obtain,

\[
J_p = N_A \int_{t_0}^{t_f^A} c(v(t), a(t))dt + c(v_0, 0)L_p/v_0] + N_B \int_{t_0}^{t_f^B} c(v(t), a(t))dt + c(v_0, 0)(L_p-(v_0\tau + d)N_A/v_0],
\]

where \(t_0\) is the starting time of optimizing the leading CAV in Subplatoon A (B); \(t_f^A\) and \(t_f^B\) are the ending time of optimizing leading CAV in Subplatoon A and Subplatoon B, respectively. Unlike the optimal control of one CAV, the length of the platoon is considered in the control framework of one platoon. The locations of leading CAVs in Subplatoon A and Subplatoon B at \(t_0\) are \(s_0\) and \(s_0 - N_A(v_0\tau + d)\), respectively. The locations of leading CAVs in the two subplatoons at \(t_f^A\) and \(t_f^B\) are both \(s_1\).

The platoon optimization of passing a signalized intersection is to minimize \(J_p\) with all CAVs traveling across the intersection in green light windows. The constraints of guaranteeing all vehicles crossing the intersection within green light windows can be described as,

\[
\begin{align*}
\{ t_f^A\} C &\leq G \\
(t_f^A + (N_A - 1)(\tau + d/v_0)) C &\leq G \\
[t_f^A] &= [t_f^A + (N_A - 1)(\tau + d/v_0)] \\
(t_f^B + (N_B - 1)(\tau + d/v_0)) C &\leq G \\
[t_f^B] &= [t_f^B + (N_B - 1)(\tau + d/v_0)] \\
[t_f^A] + 1 &= [t_f^B]
\end{align*}
\]

The first six equations in Eq. 11 can guarantee all CAVs in Subplatoon A (B) across one green light window, and the last equation can ensure Subplatoon A and Subplatoon B get through the intersection at two successive traffic signal cycles.

In combination of constraint conditions of Eq. 11 and fuel consumption of Eq. 5, we can obtain the optimization trajectories of all vehicles in one platoon with minimizing the total fuel consumption described in Eq. 10.

4.2. Two other methods for comparison

We compare our trajectory optimization framework PTO with two other methods that adopt a simplified Gipps’ car-following model, namely leading-trajectory-optimization (LTO) and aggressive-trajectory (AT). In the LTO method, we assume the leading vehicle in a platoon is a CAV, and optimize its trajectory based on the optimal control framework. The other vehicles in the platoon are human-driven ones and follow the leading vehicle with the simplified Gipps’ car-following model [12, 24, 25]. If the following vehicles arrive at the
signalized intersection in red, they need to stop until the green light is on. For the AT method, we assume there is no CAV in the platoon, and the leading vehicle travel from $s_0$ to $s_1$ with maximum speed $v_0$. If the leading vehicle arrives at the intersection in red, it is forced to wait until the green light is on; otherwise, it travels through the intersection with maximum speed. The other vehicles in the platoon follow the leading vehicle with the simplified Gipps’ car-following model and stop if the red light is on.

The upper limits of acceleration defined in simplified Gipps’ car-following model includes two parts, i.e., free-flow and congested traffic acceleration, which is formulated as,

$$
\begin{align}
    a_i^{\text{free}} &= 2.5a_{fw}(1 - \frac{v_i(t)}{v_0})\sqrt{0.025 + \frac{v_i(t)}{v_0}} \\
    a_i^{\text{cong}} &= \frac{1}{T}\left[\frac{1}{\tau_c}(s_{i-1}(t) - s_i(t)) - d - \frac{(v_i(t))^2}{2a_{br}}\right] - v_i(t)
\end{align}
$$  \hspace{1cm} (12)

where $T$ is the sensitivity coefficient, $\tau_c$ the drivers’ time of reaction, and $d$ the minimum gap between two adjacent vehicles. The acceleration of vehicle $i$ at time $t$ is,

$$
a_i(t) = \max\{a_{br}, \min\{a_i^{\text{free}}(t), a_i^{\text{cong}}(t)\}\}. \hspace{1cm} (13)
$$

The speed and location of one vehicle in the next time step with Gipps’ car-following model are defined as,

$$
\begin{align}
    v_i(t + \Delta t) &= \max\{0, \min\{v_i(t) + a_i(t)\Delta t, v_0\}\} \\
    s_i(t + \Delta t) &= \max\{s_i(t), \min\{s_i(t) + v_0\Delta t, s_i(t) + v_i(t)\Delta t + \frac{a_i(t)\Delta t^2}{2}\}\}
\end{align}
$$  \hspace{1cm} (14)

where $\Delta t$ is the time step between iterations.

4.3. Case study

In this section, we conduct one case study to illustrate the performance of our proposed platoon optimization method PTO and the other two methods, LTO and AT. The parameters in the case study are set as follows: enter location $s_0 = 0$, traffic signal location $s_1 = 500m$, maximum speed $v_0 = 20m/s$, CAV reaction time $\tau = 1.5s$, and driver’s reaction $\tau_c = 2s$. The number of vehicles in the platoon is $N = 6$, and the length of the platoon is $L_p = 200m$ at the initial state. The cycle length of traffic signal is $C = 40s$, and $R = G$.

The parameters in the fuel consumption model are shown in Table 1.

Figure 4 displays trajectories of all vehicles in one platoon with different entry time at location $s_0$. The first, second and third row denote the results of PTO, LTO and AT, respectively. The first two columns are illustrations of Scenario I, in which all vehicles can pass the intersection at one green window. The trajectories in the last column belong to Scenario II, in which one platoon needs to split into two subplatoons and passes the signalized intersection in two successive green light windows. Table 2 shows the average fuel consumption and travel time per vehicle per 100 meters for the platoons shown in Figure 4. We calculate the fuel consumption and travel time of each vehicle from location $s_0$ to location $s_1 + (v_0^2)/2a_{fw}$ with considering the length of a platoon and the acceleration space of human-driven vehicles to compare the performance of PTO, LTO and AT. When entry
time \( t_0 = 5 \), the trajectories with AT are blocked by a red signal and need to wait before the intersection until the light is on green, however, the vehicles with PTO and LTO can adjust their trajectories to avoid stopping before the signalized intersection. In this case, the trajectories with LTO consumes less fuel than PTO and AT because the following vehicles with LTO have more space and time to smooth their trajectories than PTO. When entry time \( t_0 = 15s \), all vehicles can pass the signalized intersection without the influence of red light, and PTO outperforms LTO and AT in fuel consumption. Because all vehicles with AT can travel from \( s_0 \) to \( s_1 \) and cross the intersection with maximum speed, the travel time of AT is lowest among the three methods. When the entry time \( t_0 = 30s \), the trajectories with PTO are optimized in each subplatoon and pass the intersection with maximum speed. In this case, the PTO method can reduce both fuel consumption and travel time in contrast to the other two methods.

![Figure 4: Trajectories of one platoon entering at different times. (a-c) Trajectories with PTO, (d-f) trajectories with LTO and (g-i) trajectories with AT.](image)

Figure 5 (a) presents average fuel consumption per vehicle per 100 meters for one platoon entering location \( s_0 \) at different times \( t_0 \). Overall, the performance of PTO is better than LTO and AT in fuel consumption. The mean values of fuel consumption per vehicle per
Table 2: Average fuel consumption and travel time per vehicle per 100 meters shown in Figure 4.

<table>
<thead>
<tr>
<th></th>
<th>Fuel consumption (ml)</th>
<th>Travel time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t₀ 5 15 30</td>
<td>5 15 30</td>
</tr>
<tr>
<td>PTO</td>
<td>3.39 2.74 3.15</td>
<td>6.25 5.38 6.25</td>
</tr>
<tr>
<td>LTO</td>
<td>3.08 3.12 4.34</td>
<td>6.41 5.53 8.12</td>
</tr>
<tr>
<td>AT</td>
<td>5.17 3.54 4.58</td>
<td>7.04 5.16 6.59</td>
</tr>
</tbody>
</table>

Figure 5: The performance of fuel consumption (ml) and travel time (s) per vehicle per 100 meters with PTO, LTO and AT methods for different entry time. The parameters are set for simulation: \( s_1 = 500m \), \( v_0 = 20m/s \), \( N = 6 \), and \( C = 40s \).

100 meters over different entry times in one cycle of traffic signal with PTO, LTO and AT are 3.14, 3.54 and 4.59 ml, respectively. In contrast to LTO and AT, the fuel consumption with PTO method falls by about 11.30% and 31.59%, respectively. When all vehicles are blocked by red light and need to pass the intersection at next traffic signal cycle, the fuel consumption with LTO may outperform PTO. In this condition, the following vehicles with a simplified Gipps’ car-following model have more space and travel time to smooth their trajectories. Figure 5 (b) depicts the results of average travel time per vehicle per 100 meters. The mean values of travel time over different entry times in one cycle of traffic signal with PTO, LTO and AT are 6.06, 6.60 and 6.52 seconds, respectively. Even though we only take fuel consumption as our optimization objective, the performance of PTO in reducing travel delay is better than LTO and AT because CAVs have less reaction time and pass the signalized intersection with maximum speed in PTO method. Compared with LTO and AT, the travel time reduced about 8.18% and 7.06% in PTO method, respectively. All in all, from the case study, we find the PTO method can not only reduce fuel consumption but also ease traffic congestion and increase traffic efficiency.
4.4. Sensitivity analysis

From the previous case study, we find our PTO method is beneficial for reducing fuel consumption and travel delay. In this section, we analyze how the values of the critical parameters influence the performance of PTO, LTO and AT methods. Figure 6 presents the results of sensitivity analysis about different control space $s_1$, maximum speed $v_0$, the number of vehicles in one platoon $N$, and the length of traffic signal cycle $C$.

Figure 6: Sensitivity analysis of one platoon across a signalized intersection with different parameters. (a-d) Fuel consumption and (e-h) travel time per vehicle per 100 meters. The parameters except for parameters analyzed are set as: $s_1 = 500 m$, $v_0 = 20 m/s$, $N = 6$ and $C = 40 s$. All data points are calculated over different entry times in one traffic signal cycle.

As shown in Figure 6 (a), the average fuel consumption of the three methods all decreases with the increase of control space. The gap in fuel consumption between PTO and the other two methods also increases with the increase of control space. Figure 6 (b) shows the average fuel consumption with different maximum speeds. The average fuel consumption of the three methods all increases with the maximum speed, because travel speed contributes positively to the second and third terms in the fuel consumption model in Eq. 5. Figure 6 (c) shows the sensitivity of the length of the traffic signal cycle on fuel consumption. The fuel consumption of the PTO method increases with the increase of the cycle length. However, for the other two methods, the fuel consumption decreases with the increase of the length of the traffic signal cycle. It is because the fuel consumption with optimal control increases with travel time (see Figure 1). When the traffic signal has a significantly long cycle, the PTO method, by requiring the CAV platoon to arrive at the start of the green interval, does not take full advantage of the the long green time window. Figure 6 (d) shows that the fuel consumption increases slightly with large platoon size, and the number of vehicles in one platoon does not have a significant influence on the fuel consumption of PTO. Figure 6 (e-h) show the results of travel time in different conditions. Even though we only
take fuel consumption as our optimization objective, the PTO method is also beneficial to reduce travel time compared with LTO and AT methods. In summary, our PTO method considerably outperforms the LTO method in reducing fuel consumption and increasing traffic throughput in the situation with longer control distance, lower maximum speed and shorter traffic signal cycle. Moreover, even though only the leading vehicle is CAV in LTO method, it can improve the performance in fuel consumption and travel time in comparison with AT, which is consistent with both theoretical and experimental found in the literature results [12, 16, 26].

5. Optimization of multiple platoons

5.1. The constraint between two platoons

The previous results are for one platoon with different entry times, and no interaction between two platoons is considered. Therefore, in this section, we extend our PTO method to multiple platoons. The probability of the leading vehicle in platoon $k$ entering location $s_0$ at time $t_{k,1}$ according to the time of the last vehicle in platoon $k-1$ entering location $s_0$ is described as,

$$p(t_{k,1}) = \lambda e^{-\lambda[(t_{k,1} - t_{k-1,N}) - \tau_p]},$$

(15)

where $\lambda$ is the average event rate, $\tau_p$ is minimum time headway between two platoons, and $t_{k,1}$ and $t_{k-1,N}$ denotes the time of the leading vehicle in platoon $k$ and the last vehicle in platoon $k-1$ entering location $s_0$, respectively.

For multiple platoons, the behaviors of one platoon will affect the performance of the next platoon. If we optimize the trajectory of platoon by platoon, the trajectory of the last vehicle in platoon $k-1$ may cross with the trajectory of the leading vehicle in platoon $k$. To avoid a crash between two platoons, we suppose one virtual vehicle follow the last vehicle in platoon $k-1$ with time delay $\tau$ and space delay $d$. The trajectory of the virtual vehicle in platoon $k-1$ is the constraint of the leading vehicle in platoon $k$, which can be described as,

$$s_{k-1}^{\text{virtual}}(t) \geq s_{k,1}(t),$$

(16)

where $s_{k-1}^{\text{virtual}}(t)$ and $s_{k,1}(t)$ denote the locations of virtual vehicle in platoon $k-1$ and the leading vehicle in platoon $k$ at time $t$, respectively. Moreover, the constraint between two platoons also is applied to avoid a crash between two subplatoons.

5.2. Case study

In this section, a case study is conducted to compare the performance of PTO, LTO and AT for multiple platoons. The parameters in the case study are set as follows: the number of platoons $N_p = 10$, the average event rate $\lambda = 0.2$, and the minimum time difference between two platoons $\tau_p = 10s$. The other parameters are the same as the case mentioned above for one platoon.

Figure 7 illustrates trajectories of multiple platoons. Overall, the PTO method can reduce congestion and let more vehicles cross the signalized intersection in less traffic signal
cycles in contrast to LTO and AT. According to the trajectories of multiple platoons in Figure 7, we can obtain the average fuel consumption and travel time per vehicle per 100 meters which are shown in Table 3. We can see that more than 30% of fuel consumption and 40% of travel time are reduced with PTO method in contrast to LTO and AT method. However, because LTO only has a local influence on multiple platoons across a signalized intersection, the fuel consumption of the LTO method is not significantly reduced in comparison with AT. In some cases, compared with AT, LTO method may increase traffic congestion.

![Figure 7: Trajectories of multiple platoons across a signalized intersection. (a-c) Trajectories with PTO, (d-f) trajectories with LTO and (g-i) trajectories with AT.](image)

<table>
<thead>
<tr>
<th>$t_0$</th>
<th>Fuel consumption (ml)</th>
<th>Travel time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>PTO</td>
<td>3.21</td>
<td>3.35</td>
</tr>
<tr>
<td>LTO</td>
<td>4.84</td>
<td>4.89</td>
</tr>
<tr>
<td>AT</td>
<td>5.00</td>
<td>4.89</td>
</tr>
</tbody>
</table>
Figure 8 shows the cumulative distribution function (CDF) of fuel consumption and travel time. In Figure 8 (a), the mean values of fuel consumption are 3.32, 4.94 and 4.99 ml per vehicle per 100 meters with PTO, LTO and AT, respectively. In contrast to LTO and AT, the fuel consumption with PTO method falls by about 32.79% and 33.47%, respectively. It is clear that the fuel consumption of most vehicles is less than 4 ml with the PTO method. However, the fuel consumption of most vehicles with LTO and AT is more than 4 ml. In Figure 8 (b), the mean values of travel time are 5.73, 10.97 and 11.10 seconds with PTO, LTO and AT, respectively. In comparison with LTO and AT, the travel time with PTO method decreases about 47.8% and 48.4%, respectively. The travel time of most vehicles is less than 8 seconds with PTO; however, the cumulative probabilities of travel time with LTO and AT are gradually increasing with travel time, indicating PTO method can effectively reduce traffic congestion.

![Cumulative probability](image.png)

Figure 8: Cumulative probability of fuel consumption (ml) and travel time (s). The data is generated by 50 independent simulations.

5.3. sensitivity analysis

From the above case study, we find our PTO method can reduce more than 30% fuel consumption and 40% travel time than the other two methods. In this section, we analyze the influence of key parameters on the performance of PTO, LTO and AT methods. Figure 9 and Figure 10 show the fuel consumption and travel time with different parameters. As shown in Figure 9, some parameters, e.g., $v_0$, $C$, $N$ and $\lambda$, have negative impacts on the fuel consumption of PTO for multiple platoons. As similar as one platoon, in those parameters, it is obvious that the increase of $v_0$ and $C$ will contribute more to the fuel consumption. As shown in Figure 9 (c), when the length of traffic signal $C$ is large enough, The unit fuel consumption of PTO method gradually approaches those of the other two methods, which indicate that our PTO method is better suited for short and moderately long cycles (less than 90 seconds). Moreover, in Figure 9 (d) and (f), more vehicles in one platoon and higher arrival rate of platoons cause the increase of the density of vehicles, leading more fuel consumption. Combing the results in Figure 9 and Figure 10, we find the increase of the length of traffic signal cycle and the density of vehicles go against the performance of
PTO in both fuel consumption and traffic throughput. In general, the PTO method can significantly improve the performance of fuel consumption and traffic throughput in contrast to LTO and AT when vehicles have enough space to smooth their trajectories. However, for multiple platoons, the performance of LTO cannot be significantly improved in comparison with AT in most cases because the impact of CAV would be non-existent or substantially lessened [27].

6. Conclusions and Discussions

In this paper, we propose a platoon-based trajectory optimization method, i.e., PTO, to reduce fuel consumption of vehicles passing through a signalized intersection. In the PTO method, all vehicles are CAVs, and the CAVs in one platoon follow the leading one with a reaction time delay and safety space gap. The method can smooth the trajectories of vehicles, eliminate full stops, economize fuel consumption, and ease traffic congestion. Moreover, we compare the PTO method with the other two methods, LTO and AT. In LTO, only the leading vehicle is a CAV with optimized trajectory, and the other vehicles follow the leading CAV with Gipps’ car-following model. In AT, we simulate the condition that all vehicles are human-driven and no optimization is applied.

Through a series of case studies and sensitivity analysis, we verify that our PTO method has advantages in economizing fuel consumption and reducing travel time over the other two methods. We find there are negative relationships between fuel consumption and the length of the traffic signal cycle, maximum speed, the density of vehicles. Because when those factors have large values, it is equivalent to reducing the space used for trajectory
Figure 10: Travel time (s) per vehicle per 100 meters for multiple platoons across a signalized intersection. The parameters except for parameters analyzed are set as: $s_1 = 500m$, $v_0 = 20m/s$, $N = 6$, $C = 40s$, $N_p = 10$, $\lambda = 0.2$ and $\tau_p = 10s$. All data points are calculated over 50 independent simulations.

Table 4: Average fuel consumption and travel time per vehicle per 100 meters shown in Figure 11.

<table>
<thead>
<tr>
<th>$t_0$</th>
<th>Fuel consumption (ml)</th>
<th>Travel time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>PTO</td>
<td>3.16</td>
<td>3.25</td>
</tr>
<tr>
<td>LTO</td>
<td>4.77</td>
<td>4.77</td>
</tr>
<tr>
<td>AT</td>
<td>4.90</td>
<td>4.76</td>
</tr>
</tbody>
</table>

optimization. From this perspective, the PTO method needs enough space to let all CAVs take optimal trajectories. When the traffic is heavy, and there is not enough space for CAVs to smooth their trajectories, the performance of the PTO method degrades and approaches those of LTO and AT. This indicates that the PTO method is best suited to undersaturated traffic conditions with shorter or moderately long cycles.

In the above analysis, we only consider multiple platoons across an isolated signalized intersection. However, in general, traffic signals are usually coordinated based on a time-distance (T-D) diagram so that platoons can pass the intersections along with a “green wave” without the influence of red light [28]. Figure 11 illustrates the trajectories of multiple platoons across two successive signalized intersections. The offset between the two traffic signals is set as $T_C = (s_2 - s_1)/v_0$, where $s_2$ is the location of the second intersection. As shown in Figure 11 (a-c), all platoons with the PTO method can travel from the first intersection to the second intersection with maximum speed and pass the second intersection.
without stopping. In the case of LTO and AT (Figure 11 (d-i)), however, there are some vehicles that cannot cross the second intersection along with the “green wave”, and need to stop before the second intersection until the light turns green. This highlights the added advantage of the PTO method over LTO and AT methods when traffic lights are coordinated. The results of average fuel consumption and travel time per vehicle per 100 meters for multiple platoons across two intersections in Figure 11 are shown in Table 4.

Several research directions can be pursued to extend this research, which includes, but is not limited to (1) to develop a PTO method for electric vehicles (EV), (2) to extend the PTO method for a network of traffic intersections, and (3) to extend the PTO method with actuated control traffic signals.

7. Acknowledgements

This study was made possible through funding received by the University of California Institute of Transportation Studies from the State of California via the Public Transportation Account and the Road Repair and Accountability Act of 2017 (Senate Bill 1). The authors would like to thank the State of California for its support of university-based research, and especially for the funding received for this project. The authors would also like to
thank CTECH: Center for Transportation, Environment and Community Health, a US DOT funded university center, for its support of this research.

8. Reference


