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EFFECTS OF ANGULAR MOMENTUM
ON KINETIC ENERGY RELEASE IN FISSION*

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Values for the most probable kinetic energy release in fission following compound-nucleus reactions between various targets and ions are reproduced, with a standard deviation of 1.5 MeV, by the expression

$$\overline{E}_K^o \text{ (MeV)} = 0.0042\ Z_o^2/A_o^{1/3} + 0.075\ E_o, \qquad (1)$$

where Z_o, A_o, and E_o are respectively the atomic number, mass number, and the excitation energy of the compound nucleus.

The total kinetic energy, E_K, of the fission fragments at infinite separation depends primarily on their mutual Coulomb potential, V_C, at the scission configuration, and hence on the nucleonic composition and shape at that configuration. This dependence has been studied both theoretically [1] and experimentally [2]. It has been deduced that the scission shape is approximately equal to two spheroids connected with a neck [1,2].

The value of E_K is also expected to be affected by the excitation energy of the fissioning nucleus. This energy appears at the saddle in the form of (a) collective kinetic energy of the motion towards fission, (b) intrinsic excitation (c) vibrational excitation, and (d) rotational excitation. The first three influence the width of the E_K-distribution. The rotational energy may in addition alter the most probable value, $\overline{E}_K$, in the following two ways. First, as the angular momentum increases, the system is forced to break with a decreasingly shorter neck, and hence at an increasingly higher value of V_C. Second, part of the rotational energy is converted into translational energy.

Previous experiments in which fission was induced by the use of ^{16}O ions did not reveal any variation of $\bar{E}_K$ with the excitation energy of the fissioning nucleus. In work described here ^{40}Ar is used, and the excitation energy can be varied over a wider range than was possible with ^{16}O. As will be described below, values for $\bar{E}_K$ can be evaluated from the experimentally determined most probable laboratory-frame (lab) angles between coincident fragment pairs.

The technique used in measuring the angular correlation functions has been described in detail elsewhere [3,4]. The ion beams at an energy of 10.4 MeV/nucleon were furnished by the Berkeley Hilac. Lower energies were obtained by degradation with Al foils. The ion energy spectra were measured with a silicon diode detector. The two silicon diode detectors, in the coincidence experiments, were in the plane with and at opposite sides of the beam axis. In general one was kept at an angle $\psi_2 = 90$ deg to that axis, and the correlation function was then obtained by measuring the fragment-fragment coincidence rate as a function of the lab angular position ψ_1 of the other. For the system $^{20}Ne + {}^{nat}Pd$ this rate was measured as a function of the position ψ, where $\psi = \psi_1 = \psi_2$.

The correlation functions have been shown in general to consist of two peaks [3,4]. The fragments recorded at the narrower one come from fissioning nuclides produced in compound-nucleus reactions, and those arriving at the most probable angles, $\bar{\psi}_1$, for this peak represent the most probable fission event in those reactions. We now assume for this event that (a) the coincident primary fragments have the same nucleonic composition and excitation energy, and (b) nucleons are emitted symmetrically around 90 deg in the center-of-mass system of the emitters, e.g., the compound nucleus and the fragments. Then, throughout the acceleration, the fragments have the same velocity and mass. Let us denote by v and v_o the respective velocities of the fragments at infinite separation when nucleon emission does or does not occur before full acceleration. Let us furthermore denote by $\overline{E}_K$ and $\overline{E}_K^o$ the total kinetic energies of the fragments when their mass is $A_o/2$ and their velocities are v and v_o, respectively. The quantity $\overline{E}_K^o$ is of primary interest, since it represents the kinetic energy release of a nucleus for which the composition, (Z_o,A_o), excitation energy, E_o, and angular momentum distribution can be estimated.

From the law of conservation of linear momentum we obtain for $\overline{E}_K^o$ the expression

$$\overline{E}_K^o = (1-F)\overline{E}_K \ , \tag{2}$$

where

$$\overline{E}_K = (A_I E_I/A_o)(1+4\tan^2\overline{\psi}) \quad \text{for } \overline{\psi}_2 = 90 \text{ deg} \ ; \tag{3a}$$

$$\overline{E}_K = (A_I E_I/A_o) \tan^2\overline{\psi} \qquad \text{for } \overline{\psi}_1 = \overline{\psi}_2 = \overline{\psi} \tag{3b}$$

and

$$F = (4/3)\ N_N/A_o - 2N_p/Z_o + (2R_o/A_o)\left[\sum_{i=1}(1/r_{ni}) + (1-2A_o/Z_o)\sum_{j=1}(1/r_{pj})\right]. \tag{4}$$

Here, Z_o and A_o have been defined before; A_I and E_I are the mass number and lab energy of the ion respectively; N_N and N_p represent respectively the number of nucleons and of protons emitted before scission; and r_{ni} and r_{pj} are the distances between the centra of the fragments at which, respectively, the ith neutron and jth proton are emitted. These distances are given in units of the distance, R_o, between the centra at scission.

For our systems, first-chance fission was estimated to be the most probable event, hence $N_N = N_P = 0$. The distance, r_{xi}, is related to the level width, Γ_{xi}, for the emission of the ith particle x by the expression

$$\hbar/\Gamma_{xi} = (R_o/v)\left[(r_{xi}^2 - r_{xi})^{1/2} + 1/2\ \ln\left\{2(r_{xi}^2 - r_{xi})^{1/2} + 2r_{xi} - 1\right\}\right], \quad (5)$$

where $\hbar$ is Planck's constant divided by 2π.

Approximate values for R_o and v were obtained from the expressions $Z_o^2 e^2/(4R_o) = \overline{E}_K$ and $1/2\ m_o A_o v^2 = \overline{E}_K$, where m_o is the nucleonic mass.

In the estimation of Γ_{xi} we used the equation based on the level density expression $\rho = \rho_o \exp(aE)^{1/2}$ [5]. Here, E is the excitation energy of the nucleus following particle emission, and a is the level density parameter, which was set equal to A/10. In the cascade we assumed the kinetic energy carried off by a neutron to be 4 MeV, and that carried off by a charged particle to be equal to its Coulomb barrier with respect to the residual nucleus.

One can easily show that scattering by target nuclei of the nuclei involved in a fission event does not take place between the time the compound nucleus is formed and the fragments are fully accelerated. Scattering after that time will not change their most probable lab directions. Hence, the values for $\overline{E}_K$, as estimated from Eq. (3), are independent of target thickness--as was also verified experimentally.

Values for $\overline{E}_K$ and $\overline{E}_K^o$ are given in columns 4 and 5 of Table I. The errors in $\overline{E}_K^o$ given in column 6 represent one standard deviation and are estimated from the uncertainties of 1% in E_I and 0.22 deg in $\overline{\Psi}_1$, and the error of 1% introduced by the uncertainty in the values for Γ_{xi}.

The accuracy of $\overline{E}_K^{\,o}$ also depends on the validity of assumptions (a) and (b) given above. Assumption (a) has been shown to be valid for products in (I,xn) reactions [6]. Furthermore, the mass and kinetic-energy distributions of the final fragments in heavy-ion ion-induced fission have been shown to be symmetric [2,7]. These experimental facts strongly suggest that assumptions (a) and (b) are indeed correct.

Included in Table I are many systems for which $\overline{E}_K^{\,o}$ was measured only at full ion energy. These experiments were performed in order to test also the dependence of $\overline{E}_K^{\,o}$ on (Z_o,A_o) and to compare directly values for $\overline{E}_K$ with those obtained previously from the measured most probable lab kinetic energy [2]. The agreement is satisfactory.

It is apparent from Table I that $\overline{E}_K^{\,o}$ increases slightly with increasing E_I. However, only for the system ^{197}Au + ^{40}Ar is this increase outside experimental errors. Using a simple model, we can in the following show that this variation in $\overline{E}_K^{\,o}$ is mainly due to angular momentum effects. This model is based on these six assumptions: (a) the Coulomb potential, V_C, at scission is converted solely into kinetic energy; (b) the scission shape is characterized by two touching collinear spheroids of uniform density; (c) the ratio, C, of the major to the minor axis is the same for both fragments and independent of bombarding energy; (d) the nuclear matter is incompressible and the moments of inertia of the spheroids are those of a rigid body; (e) the fragments are emitted along the symmetry axis, i.e., the major axis; and (f) the most probable value of the projection of the total angular momentum, I, on the symmetry axis is zero.

It follows from these assumptions that experimental values for V_C should be given by

$$V_C = \overline{E}_K^o - BI_p^2 , \qquad (6)$$

and that they should be independent of bombarding energy. Here, I_p is the most probable value of I and [8]

$$B = \frac{25C^{8/3}\hbar^2}{2^{1/3}(1+6C^2)^2 m_o r_o{}^2 A_o{}^{5/3}} , \qquad (7)$$

where for r_o, the nuclear radius parameter, we used the value 1.2×10^{-13} cm, and values for C were taken from the liquid-drop model calculations by Cohen and Swiatecki [1]. Estimated values for B are given in column 7 of Table I.

Since essentially no nucleons are emitted before fission, the I-distribution at scission is equal to that of the compound nucleus. Assuming a sharp cutoff at $I = I_{CN}$, the average value, $\langle I^2 \rangle$, of the square of I can be estimated for such a distribution [9]. A realistic I-distribution is probably rounded near the top in such a way that the value of I_p^2 is somewhere between those of $\langle I^2 \rangle$ and $I_{CN}{}^2$. We shall therefore set

$$I_p^2 = (1/2)\ (\langle I^2 \rangle + I_{CN}{}^2) = 3/2\ \langle I^2 \rangle , \qquad (8)$$

and assign an error 30% to the values for I_p^2.

Values for V_C are given in column 8 of Table I, and we see they are, within errors, independent of bombarding energy. A least-squares analysis of the data gives for V_C the expression

$$V_C = 0.1187\ Z_o^2/A_o^{1/3} . \qquad (9)$$

It is interesting to note that V_C is proportional to the quantity $Z_o^2/A_o^{1/3}$, which is what one should expect for point charges. Values for V_C, as estimated from Eq. (9), can be compared directly with those estimated on the basis of the liquid drop model. Such a comparison, based on data similar to those obtained here, has been performed previously [2], and we shall therefore not discuss this aspect of the results.

Values for $\overline{E}_K^0$ calculated according to Eqs. (6) through (8) fit our experimental data with a standard deviation of about 1.5 MeV. They also fit fairly well experimental values for low-energy fission. This, however, is to be regarded as fortuitous, since at low energy assymetric division is the most probable event, and effects of shell and structure in the mass surface of the fission products play an important role [10].

The value of the quantity I_p^2 increases almost linearly with that of the excitation energy, E_o, of the compound nucleus. Then, if we assume, to a first approximation, the parameter B in Eq. (6) to have a constant value, $\overline{E}_K^0$ will vary linearly with E_o. This is the basis for the empirical expression, Eq. (1), given at the beginning, which is easier to use than Eq. (6), which contains an angular momentum term.

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Table I. Values for various quantities connected with kinetic energy release in fission following compound nucleus reactions between complex nuclei. The quantities in the table have been defined in the text.

System	$Z_0^2/A_0^{1/3}$	E_I (MeV)	$\bar{E}_K$ (MeV)	$\bar{E}_K^0$ (MeV)	St.dev. (MeV)	B (keV)	V_C (MeV)	St.dev. (MeV)
$^{238}U + {}^{40}Ar$	1854	414	231	227	3.8	0.45	219	4.6
$^{238}U + {}^{40}Ar$	1854	348	226	224	3.8	0.45	219	4.2
$^{238}U + {}^{40}Ar$	1854	272	220	220	4.0	0.45	218	4.1
$^{197}Au + {}^{40}Ar$	1520	415	195	191	3.2	0.67	180	4.7
$^{197}Au + {}^{40}Ar$	1520	352	189	187	3.2	0.67	180	4.1
$^{197}Au + {}^{40}Ar$	1520	278	185	185	3.2	0.67	181	3.5
$^{197}Au + {}^{40}Ar$	1520	187	177	177	2.9	0.67	176	3.2
$^{197}Au + {}^{20}Ne$	1318	207	160	160	2.8	0.87	156	3.3
$^{197}Au + {}^{20}Ne$	1318	160	157	157	2.8	0.87	155	2.9
$^{197}Au + {}^{20}Ne$	1318	132	154	154	2.8	0.87	153	2.8
$^{197}Au + {}^{16}O$	1267	165	154	154	2.8	0.93	150	3.0
$^{197}Au + {}^{16}O$	1267	108	148	148	2.7	0.93	147	2.7
$^{165}Ho + {}^{16}O$	994	154	123	123	2.2	1.40	117	2.8
$^{165}Ho + {}^{16}O$	994	122	121	121	2.2	1.40	118	2.4
$^{238}U + {}^{20}Ne$	1634	207	196	196	3.5	0.58	193	3.6
$^{238}U + {}^{16}O$	1579	165	191	191	3.4	0.62	188	3.5
$^{209}Bi + {}^{20}Ne$	1414	207	171	171	3.7	0.76	167	3.9
$^{209}Bi + {}^{16}O$	1362	165	165	166	3.0	0.82	163	3.2
$^{175}Lu + {}^{16}O$	1084	165	134	134	2.4	1.22	129	2.9
$^{159}Tb + {}^{16}O$	953	154	120	120	2.2	1.51	114	3.0
$^{nat}Sb + {}^{20}Ne$	714	207	95.2	95.2	2.4	2.50	82	4.9
$^{nat}Ag + {}^{16}O$	607	165	84.0	84.0	2.2	3.2	72	5.0

References

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