Title

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Publication Date

2005-03-01

ROBUST MAINTENANCE POLICIES IN ASSET MANAGEMENT

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ABSTRACT

Asset management systems help public works agencies decide when and how to maintain and rehabilitate infrastructure facilities in a cost-effective manner. Many sources of error, some difficult to quantify, can limit the ability of asset management systems to accurately predict how built systems will deteriorate. This paper introduces the use of robust optimization to deal with epistemic uncertainty. The Hurwicz criterion is employed to ensure management policies are never 'too conservative.' An efficient solution algorithm is developed to solve robust counterparts of the asset management problem. A case study demonstrates how the consideration of uncertainty alters optimal management policies and shows how the proposed approach may reduce maintenance and rehabilitation (M&R) expenditures.

1. INTRODUCTION

The United States has historically made an extraordinary investment in its infrastructure. For instance, the federal government has spent an average of about $59 billion annually since the 1980s on the nation's civilian infrastructure (GAO, 2001). The emphasis of infrastructure investment has shifted in the past 30 years toward maintenance rather than new construction. Of the total expenditure on public works improvements, a larger and larger proportion is being spent on maintenance, with the proportion of public non-capital spending for infrastructure increasing from 39% in 1960 to 57% in 1994 (CBO, 1999). However, the magnitude of maintenance and rehabilitation (M&R) investment has been far from sufficient. Therefore, the critical issue facing public works agencies today is how to allocate limited resources that are available for M&R so as to obtain the best return for their expenditure.

Asset management is the process by which agencies monitor and maintain built systems of facilities, with the objective of providing the best possible service to the users, within the constraints of available resources. More specifically, the asset management process refers to the set of decisions made by a public works agency concerning the allocation of funds among a system of facilities and over time. The primary decisions made by a public works agency are the selection and scheduling of M&R actions to perform on the facilities in the system during a specified planning horizon.

Asset management systems are tools to help public works agencies with these M&R decisions. Experience with asset management systems in the United States shows that the benefits of these systems have been substantial in practice. For example, the Arizona Department of Transportation has reported that the implementation of their Pavement Management System (PMS) has saved them over $200 million in M&R costs over a five-year period (OECD 1987). These savings are achieved because the M&R decisions are made by the PMS with an objective to minimize the life-cycle costs of the pavement sections in the network.

The cost minimization problem solved by the PMS is an asset management problem. In that case, a network of facilities is being managed. Asset management problems may also be formulated at the level of individual facilities. One example of such an optimization problem is presented below:

Formulation 1: Single Facility Long Term Asset Management Markov Decision Problem

Model Parameters:
Let α be the discount rate factor.
Let I be the set of condition states for the asset.
Let A be the set of management actions that may be performed on the asset.
Let i* in I be the initial state of the asset to be managed.
Let c in I x A $\rightarrow$ R relate condition state, action pairs to the sums of corresponding agency and user costs (R = set of real numbers.)
Let π in I x I x A $\rightarrow$ [0,1] relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the action is taken.

Decision Variables:
Let v in I $\rightarrow$ $R_{\geq 0}$ relate condition states to optimal (least) future expected discounted costs.
Let a* in I $\rightarrow$ A relate condition states to optimal actions to take.

Minimize v(i*)
such that $\mathbf{v(i) = c(i,a^*(i)) + \alpha \sum_{j \in I} \pi(i,j,a^*(i))\, v(j)}$ **for all i in I, a in A**

The above optimization may be solved via dynamic programming. Value or policy iteration techniques may be employed to find both an optimal management policy (a*) and optimal future management costs (v).

Note that even for the relatively simple asset management problem presented above, a large amount of error-free data is required as input in order to develop efficient M&R policies.

The most important of these data items relate to the condition of the facility. There are two forms of information on infrastructure condition: information on current condition, provided by facility inspection, and information on future condition, provided by the forecast of a deterioration model. Deterioration models are mathematical relations having as a dependent variable the condition of the facility and as independent variables the facility's age, current condition, level of utilization, environment, historical M&R actions etc.

Both forms of condition information are characterized by a large degree of uncertainty. Inspection output has a number of errors from a variety of sources: technological limitations, data processing errors, errors due to the nature of the infrastructure surface inspected, and errors due to environmental effects. These sources of errors interact and produce measurement biases and random errors. If the magnitudes of the biases are known, then the measurements can be corrected for their presence by suitable subtraction and multiplication. In contrast, the random errors can only be described in terms of the parameters of their statistical distributions, if known, and can not be corrected for (Humplick, 1992). On the other hand, model forecasts are associated with a high degree of uncertainty due to the following factors:

(a) Exogenous factors such as the environment and level of utilization;

(b) Endogenous factors such as facility design and materials;

(c) Statistical factors such as the limited size and scope of data sets used to generate models.

Although we can improve the quality of data by developing more advanced inspection methods and deterioration models, it is impossible to eliminate entirely the uncertainty associated with M&R decision-making. In state-of-the-art asset management systems, the stochastic nature of a facility's deterioration process (intrinsic uncertainty) has been captured through the use of stochastic process models as representations of facility deterioration. On the other hand, the determination of the parameters of these stochastic models is still subject to significant uncertainty. This is what is known as epistemic uncertainty, uncertainty due to lack of knowledge.

2. ROBUST OPTIMIZATION

Robust optimization is employed to address the epistemic uncertainty associated with M&R decision-making. Robust optimization is a modeling methodology to solve optimization problems in which the data are uncertain and only known to belong to some uncertainty set. The approach is to seek optimal (or near optimal) solutions that are not overly sensitive to any realization of uncertainty. Recent reviews on this topic can be found in Mulvey et al (1995), Ben-Tal and Nemirovski (2002) and El Ghaoui (2003) among others.

In robust dynamic programming, no underlying stochastic model of the data is assumed to be known. A robust feasible solution is one that tolerates changes in the problem data, up to a given bound known *a priori*, and a robust optimal solution is a robust feasible solution with the best possible value of the objective function. By carefully constructing and efficiently solving the robust counterpart of the original problem, it is possible to obtain solutions that gracefully trade off performance vs. guaranteed robustness and reliability.

Robust optimization will lead to a new generation of decision-support tools that facilitate the solution of decision problems with uncertainty, based on a set of specifications and a model of uncertainty. Successful applications can be found in many areas, such as finance, telecommunication, structural engineering, transportation etc. No research has been performed to apply robust optimization to asset management. More importantly, previous studies on robust optimization have mainly focused on solving uncertain linear, conic quadratic and semidefinite programming problems (e.g, El Ghaoui and Lebret, 1997; El Ghaoui et al., 1998; Ben-Tal and Nemirovski, 1999). Relatively little research has been done on the subject of robust dynamic programming. El Ghaoui and Nilim (2002) and Iyengar (2002) are two related studies on this topic.

A sample robust optimization problem is presented below. It represents one way that the optimization problem presented in formulation 1 above might be reformulated to consider epistemic uncertainty.

Formulation 2: A MAXIMIN Robust Version of Formulation 1

New Model Parameters:
Let δ be the uncertainty level.
Let Q in I x I x A → [0,1] relate initial condition state, final condition state, action triples to the <u>initially assumed</u> model of probabilities of immediately transitioning between the two states after the management action is taken.

New Decision Variable:
Let P in I x I x A → [0,1] relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, <u>as considered in the robust optimization</u>.

Minimize$_{a^*}$ [Maximize$_P$ [v(i*)]] such that
(1) $v(i) = c(i,a^*(i)) + \alpha \sum_{j \text{ in } I} P(i,j,a^*(i))\, v(j)$ **for all i in I**
(2) $| P(i,j,a) - Q(i,j,a) | \leq \delta$ **for all i, j in I and a in A**

In the example presented in this paper, an "uncertainty level" between 0 and 1 is employed. Setting the uncertainty level to 0 implies no uncertainty, meaning a "likelihood region" is defined that includes only the transition probability matrix given by an initial model. A likelihood region is a set of transition probability matrices, each of which may define the system in question. Increasing the uncertainty level adds new transition probability matrices to the likelihood region. In this example, a transition matrix will be included in the likelihood region if and only if the difference between any element of the transition matrix and the corresponding element of the matrix given by the original medium decay rate is less than or equal to the uncertainty level. Seen in this light, the uncertainty level represents how large an error in transition probabilities is considered possible.

Note that the approach being used in the above model is a MAXIMIN approach. Benefits (costs) are maximized (minimized) considering that nature will act as an opponent. The majority of work in the field of robust optimization uses such an approach. Planning agencies may perceive such an approach to be too conservative. When managing a large network of facilities, it may be too costly, and unrealistic, to manage each one under the assumption that nature is always malevolent. An alternate approach, known as MAXIMAX, involves acting under the assumption that nature will work with decision makers instead of against them. The most realistic point of view would be to recognize that nature will act neither as an adversary nor as an ally, but somewhere in between.

One attractive alternative involves applying the *Hurwicz criterion*. The Hurwicz criterion allows a decision maker to set his or her own 'optimism level.' The optimism level must be a number between 0 and 1. The pessimism level is defined as 1 – the optimism level. Decisions are then made by selecting actions that maximize benefits obtained by summing the optimism level times the greatest possible benefit level with the pessimism level times the least possible benefit level. In the context of asset management, a robust optimization problem that employs the Hurwicz criterion might be defined as follows:

Formulation 3: A Hurwicz Criterion Robust Version of Formulation 1

New Model Parameter:
Let β be the optimism level.

New Decision Variable:
Let P1 in I x I x A → [0,1] relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, as considered in MAXIMAX robust optimization.
Let P2 in I x I x A → [0,1] relate initial condition state, final condition state, action triples to the probabilities of immediately transitioning between the two states after the management action is taken, as considered in MAXIMIN robust optimization.
Let v1 in I → $R_{\geq 0}$ relate condition states to future expected discounted costs, as defined given the transition probabilities considered in MAXIMAX robust optimization.
Let v2 in I → $R_{\geq 0}$ relate condition states to future expected discounted costs, as defined given the transition probabilities considered in MAXIMIN robust optimization.

$\text{Minimize}_{v1,v2,a^*}$ [β Minimize_{P1} [v1(i*)] + (1 – β) Maximize_{P2} [v2(i*)]]
such that
(1) $v1(i) = c(i,a^*(i)) + \alpha \sum_{j \text{ in } I} P1(i,j,a^*(i))\, v1(j)$ for all i in I
(2) $v2(i) = c(i,a^*(i)) + \alpha \sum_{j \text{ in } I} P2(i,j,a^*(i))\, v2(j)$ for all i in I
(3) $| P1(i,j,a) - Q(i,j,a) | \leq \delta$ for all i, j in I and a in A
(4) $| P2(i,j,a) - Q(i,j,a) | \leq \delta$ for all i, j in I and a in A

Using the Hurwicz criterion lets decision makers adjust the optimism level and create management policies as optimistic as they choose. It still might be possible to characterize the above optimization as too conservative on the grounds that certain transitions might be considered in the MAXIMIN part of the above formulation, even though such transitions are considered impossible in real life. It is a relatively simple task to ensure that certain 'impossible' transitions are never considered in robust optimization. For example, a constraint that ensures that any transitions with zero probability in an initial model are given zero probability in any model used in the robust optimization might be incorporated as follows:

Formulation 4: A Constrained Hurwicz Criterion Robust Version of Formulation 1

New Decision Variables:
Let m in I x I x A → {0, 1} relate initial condition state, final condition state, action triples to a variable that ensures whenever the initial model precludes transitions of this form, models considered in the optimization do likewise.

Minimize_{a^*} [β Minimize_{P1} [v1(i*)] + (1 – β) Maximize_{P2} [v2(i*)]] such that

(1) v1(i) = c(i,a*(i)) + α $\sum_{j \text{ in } I}$ P1(i,j,a*(i)) v1(j) for all i in I

(2) v2(i) = c(i,a*(i)) + α $\sum_{j \text{ in } I}$ P2(i,j,a*(i)) v2(j) for all i in I

(3) | P1(i,j,a) – Q(i,j,a) | ≤ δ for all i, j in I and a in A

(4) | P2(i,j,a) – Q(i,j,a) | ≤ δ for all i, j in I and a in A

(5) Q(i,j,a) + m(i,j,a) > 0 for all i, j in I and a in A

(6) P1(i,j,a) m(i,j,a) = 0 for all i, j in I and a in A

(7) P2(i,j,a) m(i,j,a) = 0 for all i, j in I and a in A

Constraints 5 through 7 work as follows. For any i, j in I and a in A, Q(i,j,a) is a given parameter output by some initial infrastructure decay model. If Q(i,j,a) = 0 then m(i,j,a) must be set equal to 1 to satisfy constraint 5. This, in turn, forces both P1(i,j,a) and P2(i,j,a) to 0 to satisfy constraints 6 and 7. If however Q(i,j,a) is not equal to zero, then constraint 5 is not binding on m(i,j,a). Setting m(i,j,a) to 0 will eliminate constraints 6 and 7, while the alternative, setting m(i,j,a) to 1, will not. Thus an optimization solver will set m(i,j,a) to 0, allowing P1(i,j,a) and P2(i,j,a) to be non-zero.

The overall optimization is not as complex as it might appear at first glance. The final formulation, the constrained Hurwicz criterion asset management problem, just combines the costs associated with best case and worst case transition probability matrices. The simplest way to solve this problem is to first solve problems of finding best and worst case transition probabilities and associated cost-to-go functions for all potential policies in all states. In any given state *i*, acting under policy *a*, maximizing costs just implies finding a solution to the objective function **Maximize_P [v(*i*)] = Maximize_P [c(*i*,*a*(*i*)) + α $\sum_{j \text{ in } I}$ P(*i*,j,*a*(*i*)) v(j)]** which can be reduced to **Maximize_P [$\sum_{j \text{ in } I}$ P(*i*,j,*a*(*i*)) v(j)]**. This maximization problem can be solved exactly by altering the initial model transition probability matrix via shifting probability from less costly to more costly states. How much probability can be shifted is determined by conditions on uncertainty. Here this includes constraints 3 – 7 above, combined with the knowledge that transition probabilities can neither be less than 0, nor greater than 1, and that for any state and action pair the sum of its transition probabilities must be 1. Standard optimization software takes very little time solve this problem. Once probabilities have been found, best and worst case costs (v1 and v2 in formulation 4) will be the fixed points of the formulas in constraints 1 and 2 in formulation 4. Finding costs given a transition matrix is identical to the nominal asset management problem and may be solved via dynamic programming. In this way, MAXIMIN and MAXIMAX costs and policies may be computed. The, best and worst case costs are then weighted by the optimism and pessimism levels respectively and summed. Then a policy can be chosen to minimize the total costs, providing the optimal solution to the constrained Hurwicz criterion asset management problem as outlined above.

Hurwicz criterion based robust optimization does require the specification of both an uncertainty and an optimism level. Planning agencies may find it difficult to specify how much uncertainty they have with regards to infrastructure decay rates, or may find it undesirable to have to place a level of optimism on their management strategies. However, asset management clearly does involve managing systems with some degrees of uncertainty. The more uncertainty and the decision of how to manage it are discussed, the more informed asset management policies will be. Since it has been shown that the asset management problem can be made robust without making its computational complexity too great, it would be possible to imagine solving a particular asset management problem numerous times with various uncertainty and optimism levels to see how performance and reliability guarantees can be traded off.

3. COMPUTATIONAL STUDY

In order to illustrate the application of robust dynamic programming algorithms to infrastructure management problems, an example is presented here. A one lane-mile segment of highway pavement is managed according to a policy obtained from infinite horizon robust dynamic programming. Previous research (Golabi et al, 1982; Madanat, 1993; Durango and Madanat, 2002) provides a ready source of data for how pavement deterioration can be modeled via static transition probabilities. However given the uncertainty in these transition probabilities, potential cost savings can be achieved by applying robust dynamic programming to this problem.

3.1 Problem Specification

When managing a section of pavement, the decisions to be made include when and how to maintain, overlay, or reconstruct the pavement. In the example presented here, it is assumed that the choices of actions to take in any given year are those presented by Durango and Madanat (2002). These actions include: (1) do nothing, (2) routine maintenance, (3) 1-in overlay, (4) 2-in overlay, (5) 4-in overlay, (6) 6-in overlay, and (7) reconstruction. The costs of the actions presented here are taken directly from the same reference. These costs vary according to the condition state of the pavement. (A section of pavement is said to be in state 1 if it is unusable and in state 8 if it is brand new, with the intermediate states representing intermediate condition ratings.) Similarly, user costs associated with various pavement condition ratings are included in calculations.

Alongside the costs, Durango and Madanat present three sets of transition probability matrices. The matrix that describes a section of pavement deteriorating at a “medium” rate is meant to reflect the current best estimate of how a given, random section of pavement will deteriorate. The inclusion of alternative “fast” and “slow” rates of deterioration draw attention to the fact that this estimate may under or over estimate decay in meaningful ways. For the purposes of the present example, Durango and Madanat's medium decay rate transition probabilities are used to initialize the robust dynamic programming application.

Various uncertainty and optimism levels between 0 and 1 are considered. The policies obtained by robust optimization, either based on MAXIMIN or MAXIMAX or Hurwicz criteria, are compared to those obtained via non-robust optimization using the medium decay rate. Costs are then calculated in the case that the actual probabilities that guide system dynamics are somewhere between best-case and worst-case transition probabilities.

3.2 Results

In the example case of managing pavement, robust optimal actions were found to differ substantially from actions that would be taken if the "medium" decay rate was assumed to be correct**:**

Table 1: Hurwicz Optimal Actions by Uncertainty and Optimism Levels

Uncertainty level	**Optimism level**	**Optimal action in state 8**	**in state 7**	**in state 6**
0	0 - 1	2	3	4
0.2	0 - 1	2	3	4
0.4	0 - 0.11	3	4	4
	0.12 - 0.41	2	4	4
	0.42 - 0.98	2	3	4
	0.99	2	2	4
	1	2	2	3
0.6	0 - 0.09	4	4	5
	0.1 - 0.14	4	4	2
	0.15 - 0.21	2	4	2
	0.22 - 0.55	2	3	2
	0.56 - 1.00	2	2	2
0.8	0 - 0.08	4	5	6
	0.09 - 0.12	4	5	2
	0.13 - 0.15	4	3	2
	0.16 - 0.38	2	3	2
	0.39 - 0.78	2	2	2
	0.79 - 1.00	1	2	2
1.0	0 - 0.07	4	5	6
	0.08 - 0.12	4	5	2
	0.13 - 0.15	4	3	2
	0.16 - 0.51	2	3	2
	0.52 - 0.81	2	2	2
	0.82 - 1	1	2	2

The actions optimal when uncertainty is set equal to 0 correspond to the actions always chosen by non-robust optimization. The actions optimal at various uncertainty levels when optimism level is set to 0 correspond to the actions chosen by MAXIMIN robust optimization. The management policies optimal in MAXIMIN robust optimization are more conservative than those employed in non-robust optimization, especially as uncertainty becomes more significant. However, the actions prescribed by the MAXIMIN robust dynamic programming algorithm do limit worst case costs, unlike traditional non-robust optimization:

Figure 1: Cost Ranges of Non-Robust Asset Management with Uncertainty

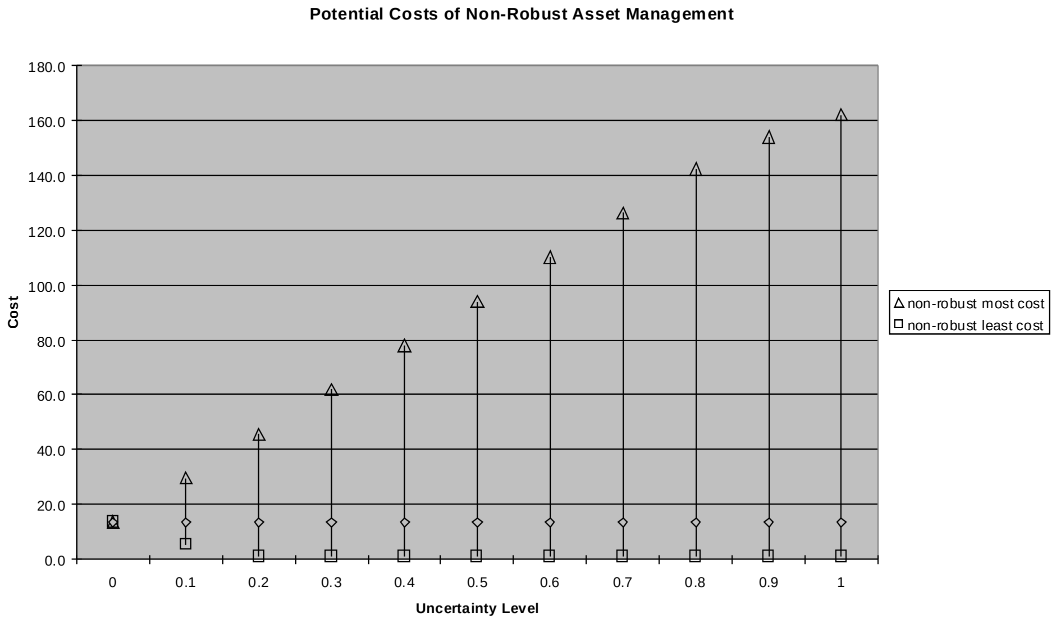
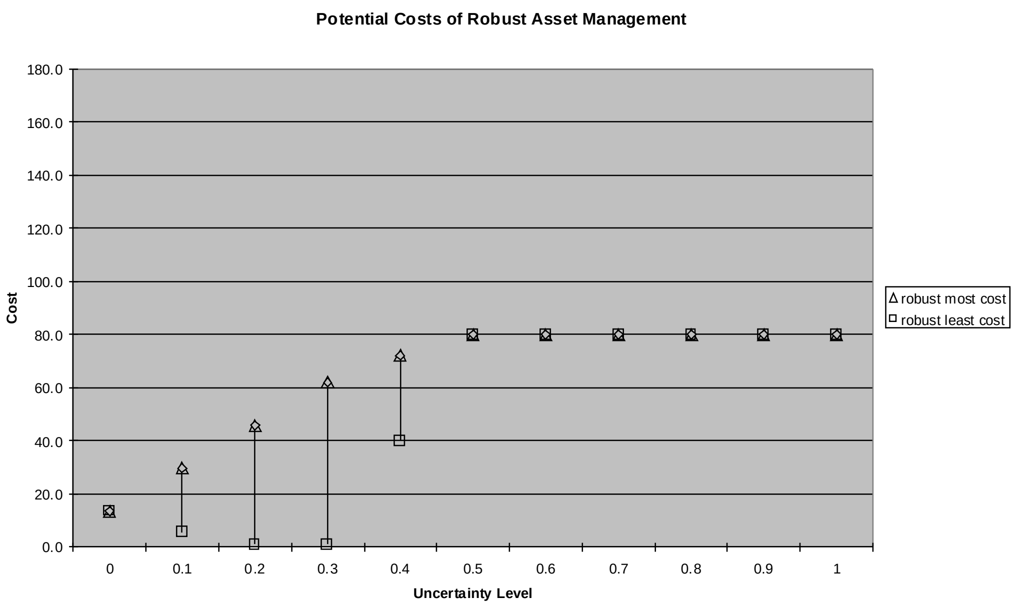
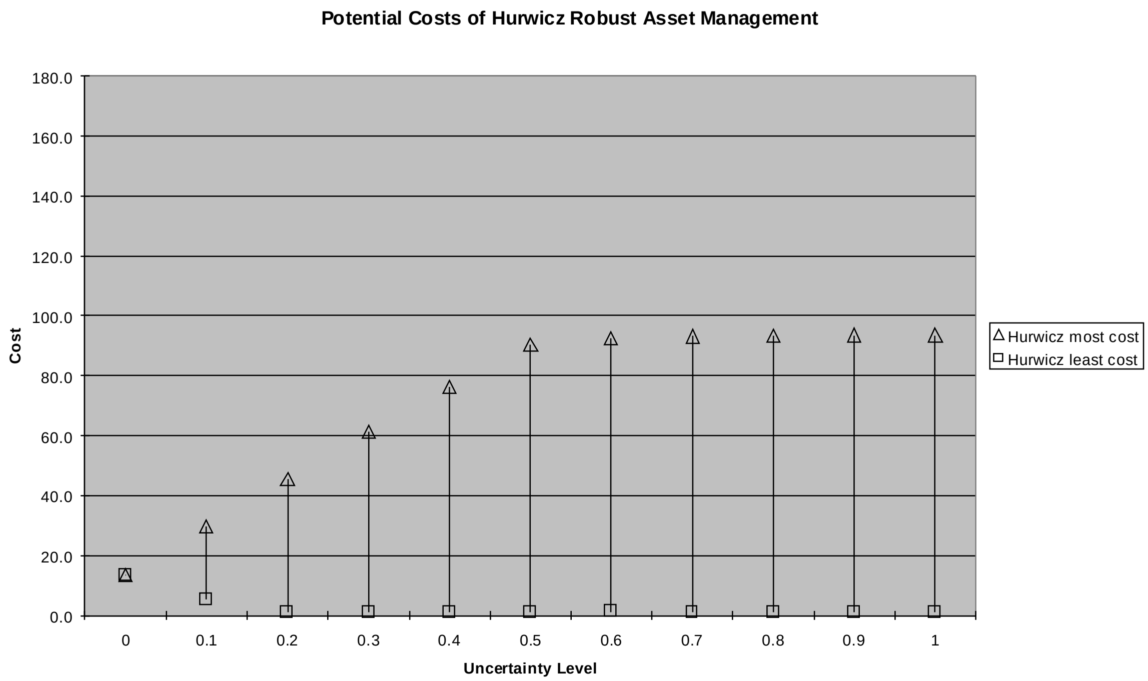


Figure 2: Cost Ranges of MAXIMIN Robust Asset Management with Uncertainty



Clearly MAXIMIN asset management limits the maximum costs, but the above graphs also show that MAXIMIN is unable to lower costs as much as traditional asset management systems can in the best-case situations. This is one of the shortcomings of the MAXIMIN approach, and one area in which the less conservative Hurwicz robust optimization is able to do substantially better.

Figure 3: Cost Ranges of Hurwicz Robust Asset Management (Optimism = 0.5)



Note that the Hurwicz style optimization is able to reap the benefits of best-case transition probabilities, incurring near zero maintenance costs, but also able to limit the worst-case costs. In many ways, the cost ranges observed under this type of asset management offer a suitable compromise between the conservativeness of MAXIMIN style robust optimization and the optimism of MAXIMAX or even traditional DP schemes.

4. CONCLUSION

It has been shown that robust optimization has the potential to significantly reduce lifecycle costs in asset management. It is quite costly to place complete faith in an initial model of infrastructure decay in asset management systems when there is significant uncertainty. If MAXIMIN formulations are deemed too conservative, alternate robust optimization methodologies like the Hurwicz criterion are available. The methodology of robust optimization provides a way to account for knowledge uncertainty within asset management systems. This paper demonstrates a small-scale application of a few robust optimization techniques. Alternative robust methodologies need to be investigated, and robust optimization needs to be extended to multi-facility management problems. It is already quite clear that incorporating a robust optimization methodology will represent an important step forward for asset management systems.

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