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Parameter Estimation of Extended Nonlinear Muskingum Models with the Weed Optimization Algorithm

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Abstract: The nonlinear Muskingum model is a hydrologic flood-routing method useful when the storage flow relation departs from the classic linear assumption. This paper extends versions of the nonlinear Muskingum model by introducing a parameterized initial storage condition. The extended nonlinear Muskingum values have an increased number of degrees of freedom that allows an enhanced capacity to accurately predict outflow hydrographs provided that parameter estimation is optimized as proposed in this work. The parameters of the nonlinear Muskingum models are estimated with the weed optimization algorithm (WOA), and the excellent performance of the extended nonlinear Muskingum models is demonstrated with several types of hydrographs using several criteria of statistical efficiency. The implementation results show that the nonlinear Muskingum model's predictions outperform those of the best results reported with other routing models for the examples presented in this paper. **DOI:** 10.1061/(ASCE)IR.1943-4774.0001095. *© 2016 American Society of Civil Engineers.*

Author keywords: Flood hydrograph routing; Nonlinear Muskingum model; Weed optimization algorithm.

Introduction

Despite the vast range of recent investigations related to optimization algorithms in various domains of water resources systems such as reservoir operation (Ahmadi et al. 2014; Bolouri-Yazdeli et al. 2014; Ashofteh et al. 2013a, 2015a), groundwater resources (Bozorg-Haddad et al. 2013; Fallah-Mehdipour et al. 2013b), conjunctive use operation (Fallah-Mehdipour et al. 2013a), design-operation of pumped-storage and hydropower systems (Bozorg-Haddad et al. 2014), flood management (Bozorg-Haddad et al. 2015b), water project management (Orouji et al. 2014), hydrology (Ashofteh et al. 2013b), qualitative management of water resources systems, (Orouji et al. 2013b; Bozorg-Haddad et al. 2015a; Shokri et al. 2014), water distribution systems (Seifollahi-Aghmiuni et al. 2013; Soltanjalili et al. 2013; Beygi et al. 2014; Solgi et al. 2015, 2016; Bozorg-Haddad et al. 2016a), agricultural crops (Ashofteh et al. 2015c), sedimentation (Shokri et al. 2013), and algorithmic developments (Ashofteh et al. 2015b), estimation of parameters of the extended nonlinear Muskingum models with the weed optimization algorithm (WOA) has not been pursued yet.

The Muskingum model hydrologic flood-routing method was introduced by McCarthy (1938) in studies of floods of the Ohio River in the United States. Application of the Muskingum model involves a calibration phase and a prediction phase (Das 2004). The calibration parameters of the Muskingum model are normally obtained from observed input and output hydrographs in a river reach or reservoir. The output hydrograph is calculated with the Muskingum routing formulas in the prediction phase. The Muskingum model's basic equations are the continuity and storage equations, which are given by Eqs. (1) and (2), respectively

$$\frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} = I - O \tag{1}$$

$$S = K[XI + (1 - X)O] \tag{2}$$

where S = storage; I = inflow; O = outflow; t = time; $\Delta S/\Delta t$ = rate of change of storage during a time interval Δt; K = storage: time constant for the river reach; and X = dimensionless weighting factor that represents the inflow-outflow effects on storage. X ranges between 0 and 0.5 for reservoir storage and 0 and 0.3 for stream channels (Mohan 1997). The Muskingum method assumes that the water surface in the reach is a uniform unbroken surface profile between upstream and downstream ends of the section. It also assumes that K and X are constant through the range of flows. The Muskingum parameters (K and X) are best derived from streamflow measurements and are not easily related to channel characteristics (Veissmann and Lewis 2003).

Eq. (2) implies a linear relation between storage and inflow/outflow. In many cases that relation is nonlinear, in which case a nonlinear version of the Muskingum model is more suitable for flood routing. The nonlinear Muskingum model, however, involves

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more unknowns and more complex parameter-estimation algorithms (Yoon and Padmanahban 1993). Several models have been proposed for the nonlinear relation between store size and flow magnitude. The first nonlinear Muskingum model (NL1) was reported by Chow (1959) and is expressed by Eq. (3) for the storage/flow function. Gill (1978) applied the exponent β to the linear storage equation [Eq. (4)], which constitutes the second nonlinear Muskingum model (NL2). The third nonlinear Muskingum model (NL3) was introduced by Hamedi et al. (2014a, b) using Eq. (5). More recently, Bozorg-Haddad et al. (2015c) introduced a fourth nonlinear Muskingum model (NL4) expressed by [Eq. (6)]. The latter authors showed that the NL4 produced better (smaller) sum of squared deviations (SSQ), sum of the absolute deviations (SAD), and deviation of the peak of routed from the observed outflows (DPO) than other nonlinear Muskingum models, and attributed the NL4's superior performance to its larger number of degrees of freedom compared to other nonlinear Muskingum models

$$S = K[XI^{\alpha} + (1-X)O^{\alpha}] \qquad \text{(NL1)} \tag{3}$$

$$S = K[XI + (1-X)O]^{\beta} \qquad \text{(NL2)} \tag{4}$$

$$S = K[XI^{\alpha} + (1-X)O^{\alpha}]^{\beta} \qquad \text{(NL3)} \tag{5}$$

$$S = K[X(C_1 I^{\alpha_1}) + (1-X)(C_2 O^{\alpha_2})]^{\beta} \qquad \text{(NL4)} \tag{6}$$

Tung (1985) simulated hydrographs with the nonlinear Muskingum model using the Euler numerical method. The initial boundary condition (I_0, O_0) has a strong influence on the calculation of the storage using the Muskingum model. The flow at the start of a flood may be in steady state ($O_0 = I_0$) or be unsteady ($O_0 \neq I_0$).Tung (1985) considered the initial computational inflow equal to the initial observations outflow to estimate the initial storage ($\hat{O}_0 = I_0$).

The complexity of parameter estimation increases with the increasing number of parameters in the nonlinear Muskingum model. Therefore, using optimization methods for estimating such parameters becomes unavoidable. Optimization methods for parameter estimation can be broadly divided into two categories (Geem 2006). The methods in the first category are based on mathematical techniques, such as the segmented least squares (S-LSQ) (Gill 1978), nonlinear least squares (N-LSQ) (Yoon and Padmanabhan 1993), MARquardt method (MAR) (Papamichail and Georgiou 1994), Lagrange multiplier (LM) (Das 2004, 2007), Broyden–Fletcher–Goldfarb–Shanno (BFGS) (Geem 2006), the NMS approach (Barati 2011), and the generalized reduced gradient (GRG) (Barati 2013a). The cited mathematical methods have several drawbacks, such as the need for calculating derivatives and the specification of an initial estimate of the solution.

The second category involves methods based on phenomena-mimicking algorithms (PMAs), such as pattern search (PS) (Tung 1985), genetic algorithms (GA) (Mohan 1997), standard search (SS) (Gavilan and Houck 1985), harmony search (HS) (Kim et al. 2001), particle swarm optimization algorithm (PSO) (Chu and Chang 2009), parameter-setting-free harmony search algorithm (PSF-HS) (Geem 2011), immune clonal selection algorithm (ICSA) (Luo and Xie 2010), differential evolution (DE) (Xu et al. 2012), simulated annealing algorithm (SA) (Orouji et al. 2013a), and the shuffled frog-leaping algorithm (SFLA) (Orouji et al. 2013a). There are advantages of using these algorithms compared with mathematical techniques, such as (1) there is no need to calculate derivatives, (2) there is no need for initial estimation of the solution, and (3) there is a better chance for finding a global optimum compared with nonlinear programming methods. However, one of critical disadvantage of the PMAs is that users need to determine the value of the algorithms' parameters carefully (Geem 2011).

The WOA is a metaheuristics method. It was introduced by Mehrabian and Lucas (2006) and is inspired by the spreading and growth characteristics of weeds in nature. The latter authors compared the WOA with other metaheuristics algorithms by solving two sample problems. Their results showed better efficiency of the WOA than the other metaheuristics algorithms. Krishnanand et al. (2009) compared the efficiency of the WOA with that of the GA, PSO algorithm, artificial bee colony algorithm (RBC), and artificial immune algorithm (AI) using five standard mathematical problems with multivariable functions. They found that the WOA achieved the best efficiency. Several other studies have implemented the WOA in various fields of science that have demonstrated its high capacity to achieve global optima (Mehrabian and Yousefi-Koma 2007; Mallahzadeh et al. 2008; Sahraei Ardakani et al. 2008; Roshanaei et al. 2009; Zhang et al. 2010; Sharma et al. 2011; Kostrzewa and Josiński 2012; Abu-Al-Nadi et al. 2013; Sang and Pan 2013; Saravanan et al. 2014). Recently, Asgari et al. (2015) used WOA for optimal operation of various reservoir systems. They considered the benchmark problem of optimal operation of a four-reservoir system in continuous and discrete domains plus the Bazoft single-reservoir system and compared their results to those obtained by GA, LP, and NLP.

Various authors have modified the structure of the nonlinear Muskingum model and increased the number of degrees of freedom in the model to estimate output hydrographs more accurately. This paper extends previous versions of the nonlinear Muskingum model by introducing a parameterized initial boundary condition and an increased number of model parameters for improved hydrograph prediction. The weed optimization algorithm is implemented for parameter estimation of the proposed nonlinear Muskingum model.

Extended Nonlinear Muskingum Models with Parameterized Initial Storage

The nonlinear Muskingum model was implemented with the Euler method for hydrograph simulation. Hydrograph simulation requires initial conditions to start the routing simulations. The authors specify the initial storage as a parameter ($\hat{S}_0 = \theta$) that is estimated with optimization techniques. The treatment of the initial storage as a parameter introduces flexibility in handling the initial routing condition, and raises the number of degrees of freedom of the nonlinear Muskingum approach with which to improve the calculation of the storage compared with other methods that do not parameterize the initial storage. The nonlinear Muskingum models with parameterized initial storage are herein named extended nonlinear Muskingum models. Their counterparts without parameterized initial conditions are called standard nonlinear Muskingum models.

The NL2, NL3, and NL4 models were used in this work for comparison with the proposed routing model because of their common application and good performance. The simulation method of the proposed nonlinear Muskingum models is as follows: N denotes the number of time steps of flood routing and S_i, I_i, $\hat{O}_i$ denote respectively storage, the observed inflow, and calculated outflow in time steps i, where $i = 0, 1, 2, \ldots, N-1$

1. Specify the initial values of the parameters present in the nonlinear Muskingum models, which are estimated by the optimization algorithm in each simulation (K, X, β in the NL2 model; β, α, X, K in NL3 model; and $C_2, C_1, \beta, \alpha_2, \alpha_1, X, K$ in the NL4 model);

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2. Assume the initial storage value as a parameter, which is estimated by the optimization algorithm in each simulation: ($\hat{S}_0 = \theta$). Moreover, $0 < \theta < V\max$($V\max$ = maximum volume of the inflow hydrograph) is added as a constrained to the optimization model;
3. Calculate the rate of change of storage ($\Delta S_i/\Delta t$) in the NL2, NL3, and NL4 models using Eqs. (7)–(9), respectively;

$$\frac{\Delta S_i}{\Delta t} = -\left(\frac{1}{1-X}\right)\left(\frac{S_i}{K}\right)^{1/\beta} + \left(\frac{1}{1-X}\right)I_i \quad \text{NL2} \quad i = 1, \ldots, N-1 \tag{7}$$

$$\frac{\Delta S_i}{\Delta t} = I_i - \left\{\left[\frac{1}{(1-X)}\right]\left(\frac{S_i}{K}\right)^{1/\beta} - \left[\frac{X}{(1-X)}\right]I_i^{\alpha}\right\}^{1/\alpha} \quad \text{NL3} \quad i = 1, \ldots, N-1 \tag{8}$$

$$\frac{\Delta S_i}{\Delta t} = I_i - \left\{\left[\frac{1}{C_2(1-X)}\right]\left(\frac{S_i}{K}\right)^{1/\beta} - \left[\frac{1}{C_2(1-X)}\right][X(C_1 I_i^{\alpha_1})]\right\}^{1/\alpha_2} \quad \text{NL4} \quad i = 1, \ldots, N-1 \tag{9}$$

4. Calculate the storage with Eq. (10)

$$S_i = S_{i-1} + \Delta t\left(\frac{\Delta S_{i-1}}{\Delta t}\right) \quad i = 0, 1, \ldots, N-1 \tag{10}$$

5. Calculate the outflow in the NL2, NL3, and NL4 models respectively with Eqs. (11)–(13)

$$\hat{O}_i = \left(\frac{1}{1-X}\right)\left(\frac{S_i}{K}\right)^{1/\beta} - \left(\frac{X}{1-X}\right)I_i \quad \text{NL2} \quad i = 0, 1, \ldots, N-1 \tag{11}$$

$$\hat{O}_i = \left\{\left[\frac{1}{(1-X)}\right]\left(\frac{S_i}{K}\right)^{1/\beta} - \left[\frac{X}{(1-X)}\right]I_{i-1}^{\alpha}\right\}^{1/\alpha} \quad \text{NL3} \quad i = 0, 1, \ldots, N-1 \tag{12}$$

$$\hat{O}_i = \left\{\left[\frac{1}{C_2(1-X)}\right]\left(\frac{S_i}{K}\right)^{1/\beta} - \left[\frac{1}{C_2(1-X)}\right][X(C_1 I_{i-1}^{\alpha_1})]\right\}^{1/\alpha_2} \quad \text{NL4} \quad i = 1, \ldots, N-1 \tag{13}$$

6. Repeat Steps (3)–(5) until reaching the end of the outflow simulation at $i = N-1$.

Testing of the quality of the optimized parameters is carried out by minimizing the SSQ, the SAD, and the difference between the peak routed and actual flows (DPO). In other words, DPO is the absolute value of the difference between the maximum value of the input hydrograph and that of the routed hydrographs. The performance criteria SSQ, SAD, and DPO are compared with observed outflow (O_i) and calculated outflow ($\hat{O}_i$) using Eqs. (14)–(16), respectively

$$\text{SSQ} = \sum_{i=1}^{N-1}(O_i - \hat{O}_i)^2 \tag{14}$$

$$SAD = \sum_{i=1}^{N-1}|O_i - \hat{O}_i| \tag{15}$$

$$\text{DPO} = |\text{peak}_{\text{routed}} - \text{peak}_{\text{observed}}| \tag{16}$$

Weed Optimization Algorithm

The WOA was developed based on weeds' growth characteristics (Mehrabian and Lucas 2006). The WOA's simulation of weed behavior relies on the following four stages:

1. Loading (initialization): a limited number of seeds are distributed in the search area;
2. Reproduction: every seed turns into weed, which, in turn, produces seeds according to its qualities;
3. Distribution: produced seeds spread randomly in the environment and create new weeds; and
4. Competitive exclusion: following seed production the competition for life starts between the weeds. In each stage the weeds with lower quality are removed to assure the survival of the fittest seeds. This process of selection is repeated until the maximum number of iteration is reached. At this stage, there is a strong possibility of production of weeds of the highest quality. This means solutions of at or very near the global optimum.

Each of previously discussed stages is described in detail as follows:

- Start with the generation of an initial population: the initial population (P_{initial}) is randomly generated and distributed in a d-dimensional search area. Each weed is a solution of the optimization problem. A colony is formed by a number of weeds. In this problem each weed is one set of K, X, β in the NL2 model; β, α, X, K in NL3 model; and $C_2, C_1, \beta, \alpha_2, \alpha_1, X, K$ in the NL4 model.
- Reproduction: at this stage the weeds with the best and worst qualities in the colony, and the smallest and largest numbers of seed production ($\text{NoS}_{\min}$, and $\text{NoS}_{\max}$ respectively, which are chosen by the user) are generated. In the WOA new generated solution is known as seeds. Seed production uses a linear function according to Eq. (17). Each seed intrinsically is the same weed (such as parents and children in the GA)

$$\text{NoSweed}_i = \left[\frac{OF_i - OF_{\min}}{OF_{\max} - OF_{\min}} \times (\text{NoS}_{\max} - \text{NoS}_{\min})\right] + \text{NoS}_{\min} \tag{17}$$

where NoSweed_i = number of calculated seeds produced for weed i; OF_i = value of objective function (quality) for weed i; $OF_{\min}$ = smallest value of the objective function in a colony; and $OF_{\max}$ = largest objective function in a colony. This step allows the more frequent reproduction of the better-quality weeds, selecting them for survival in a colony over less adept weeds.
- Distribution of seeds: adaptation and randomness are included in this stage of the algorithm. The produced seeds are distributed randomly and normally with zero average in a d-dimensional area occupied by weeds. The standard deviation is reduced in each repetition from the initial predetermined value (maximum) to a final predetermined value (minimum). This reduction in the simulation of weeds is applied in nonlinear form according to Eq. (18). Eq. (18) causes the first weeds produced to be spread widely about their parent weeds. As the iterations of the WOA proceed the calculated standard deviation by Eq. (18) is reduced and the produced weeds approach their parent weeds

$$\sigma_{\text{iter}} = \frac{(\text{iter}_{\max} - \text{iter})^n}{(\text{iter}_{\max})^n}(\sigma_{\text{initial}} - \sigma_{\text{final}}) + \sigma_{\text{final}} \tag{18}$$

in which σ_{iter} = standard deviation of the current iteration; $\text{iter}_{\max}$ = maximum iteration number (reproduction stages); iter = iteration number; σ_{initial}, σ_{final} = initial and the final

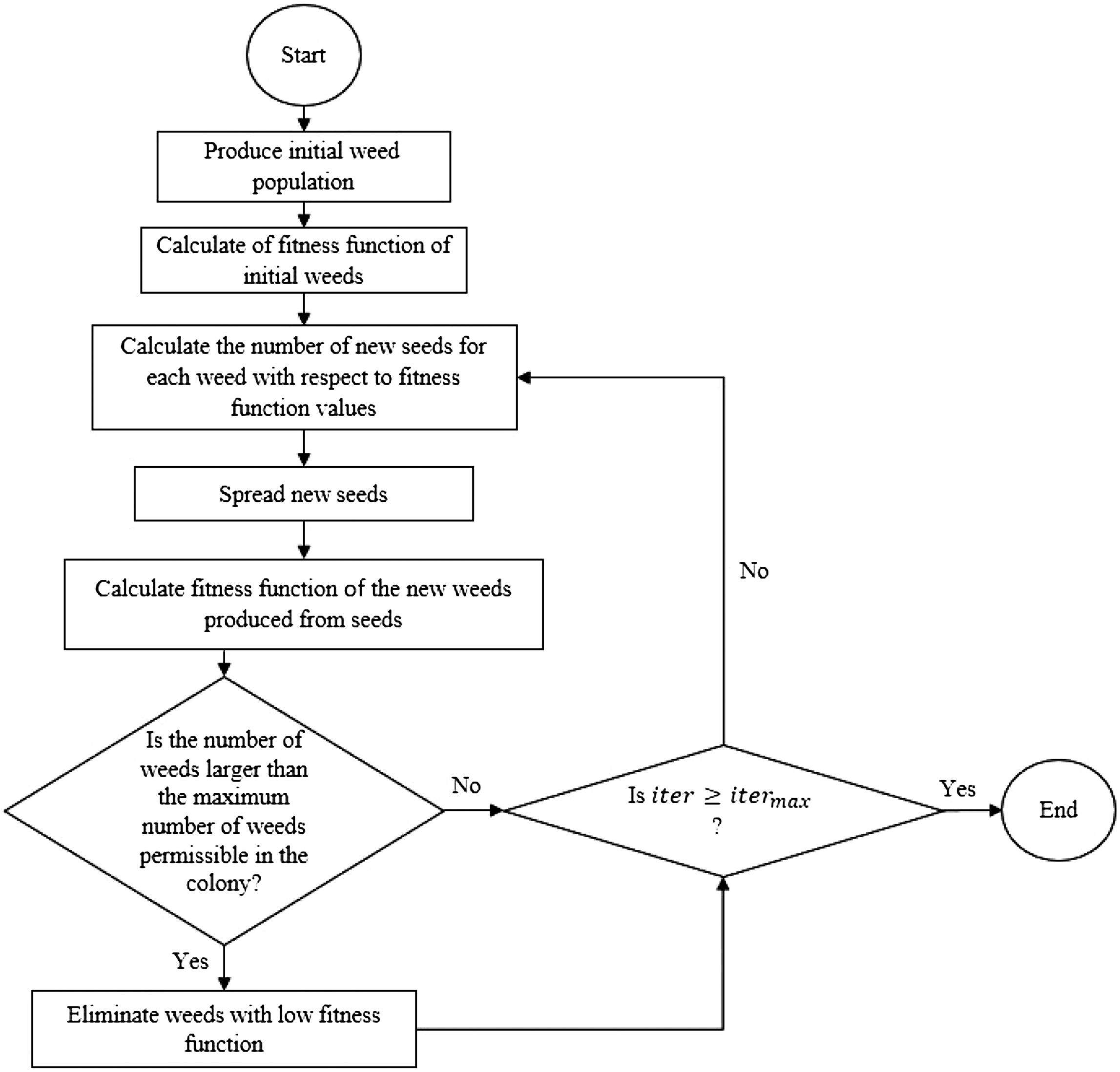

Fig. 1. Flowchart of the WOA

standard deviation, respectively, which are chosen by the user, = the final standard deviation, and n = a nonlinear module (nonlinear modulation index) which is chosen by the user.

- Competitive exclusion: The number of weeds present in the colony must be limited, and this is achieved by removing some of them. After the number of weeds in the colony reaches its maximum value ($P_{\max}$) and each of them produces and spreads seeds in the search area, then the weeds with less fit function are removed and the weeds with more fit function are carried through the next selection iteration. This process is repeated until reaching the end of the algorithm. Fig. 1 shows a flowchart of the WOA.

Results and Discussion

Three hydrograph types (experimental, real, and multipeaks) were chosen in this study to assess the efficiency of the nonlinear Muskingum models NL2, NL3, and NL4 with parameterized initial boundary condition. The reasons for choosing these case studies are (1) relations between S value and $[XI + (1 - X)O]$ differs among the example hydrographs, and (2) nonlinear Muskingum models have been applied to these case studies by different researchers and their results serve as a baseline for comparison. The WOA was initialized in each case with $P_{\text{initial}} = 5$, $P_{\max} = 15$, $\sigma_{\text{initial}} = 2$, $\sigma_{\text{final}} = 0.001$, $n = 3$, $\text{NoS}_{\max} = 3$, $\text{NoS}_{\min} = 0$, and $\text{iter}_{\max} = 100$. Also, the reported answers are the best results of five implementations of the WOA.

Results for the First Case Study

Wilson (1974) introduced this example as a benchmark problem. The data reported by Wilson suggests a nonlinear relation between S and $[XI + (1 - X)O]$ (Yoon and Padmanabhan 1993). The input and output observed hydrographs are identified in Fig. 2 as observed inflow and outflow, respectively. The calculation time steps equals 6 h ($\Delta t = 6$) and the simulation involves $N = 21$ time steps. This case has a steady flow at the beginning of the flood. Different nonlinear Muskingum models have been tested with this case study, such as those by Gill (1978), Galvin and Hook (1985), Tung (1985), Yoon and Padmanabhan (1993), Mohan (1997), Kim et al. (2001), Das (2004, 2007), Geem, (2006, 2011), Chow and Chang (2009), Lo and Xi (2010), Barati (2011, 2012, 2013a, 2013b), Xu et al. (2012), Orouji et al. (2013a), Karahan et al. (2013),

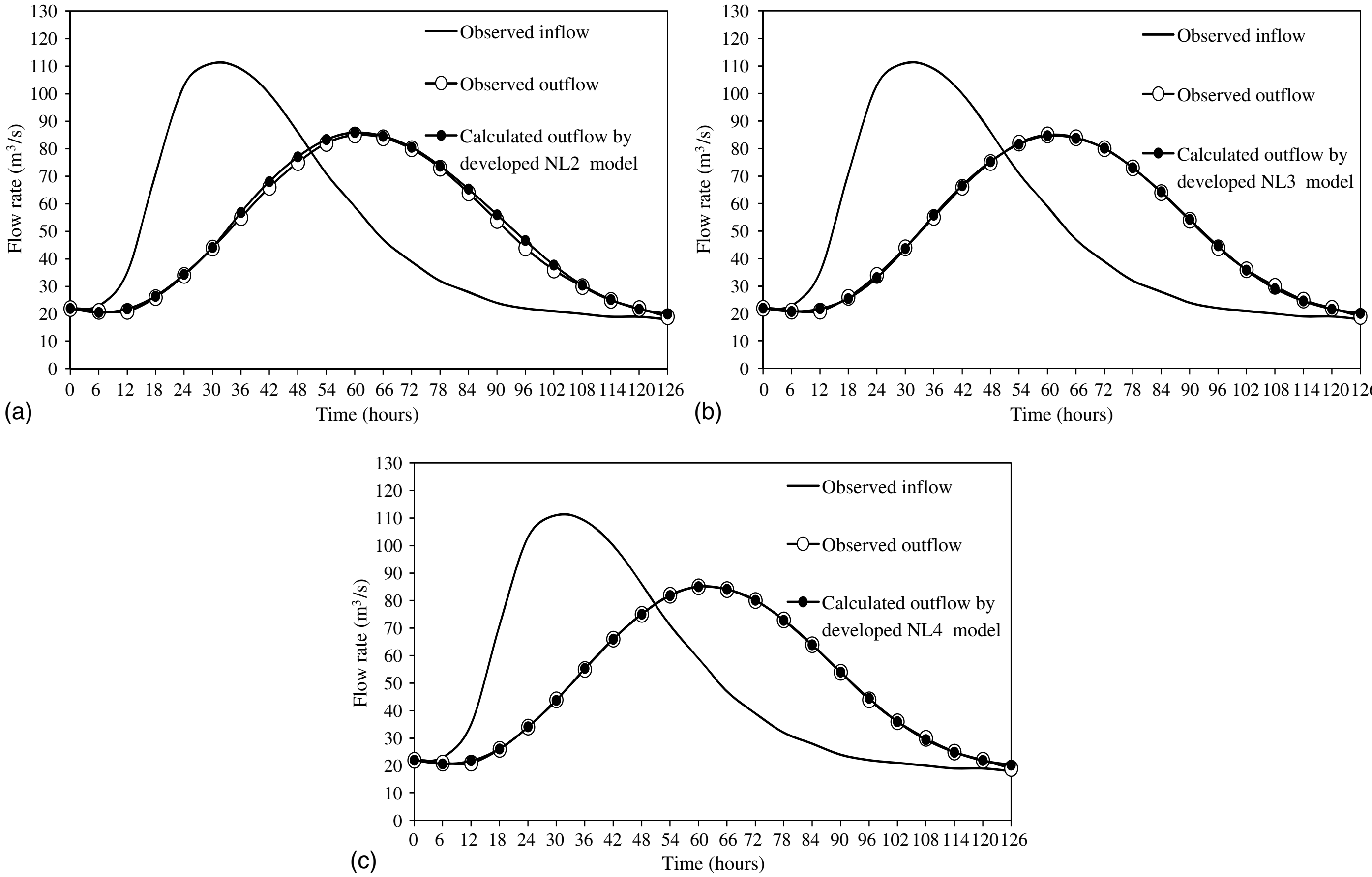

Fig. 2. Comparison between the observed hydrograph and the calculated hydrograph obtained with the extended nonlinear Muskingum models for the first case study: (a) with NL2 model; (b) with NL3 model; (c) with NL4 model

Hamedi et al. (2014a, b), and Bozorg-Haddad et al. (2015c, 2016b). The calculated SSQ, SAD, and DPO values obtained with the NL2, NL3, and NL4 models are respectively 34.12, 21.99, 0.88, and 5.78, 9.37, 0.31, and 3.19, 5.35, 0.07.

Fig. 2 presents a comparison between the observed hydrograph and the routed outflow hydrograph computed with the extended nonlinear Muskingum models for the first case study. It is seen in Fig. 2 that the nonlinear Muskingum models approximate the observed outflow hydrograph very well, and that the accuracy of the approximation increases with increasing order of the nonlinear Muskingum models (that is, NL4 better than NL3 better than NL2).

Table 1 presents the best values of SSQ, SAD, and DPO calculated by the standard and extended NL2, NL3, and NL4 models. The parameters of the NL2 Muskingum model were presented by Barati (2013b), which were estimated using with EV-GRG. Furthermore, the parameters of the NL3 and NL4 Muskingum models were calculated by Bozorg-Haddad et al. (2015c) with SFLA-NMS. The parameters of the extended nonlinear Muskingum models were estimated with the WOA as proposed in this study.

The values listed in Table 1 demonstrate improved SSQ, SAD, and DPO values with the extended nonlinear Muskingum model introduced in this work. The values of the SSQ, SAD, and DPO obtained with the extended NL2 model were decreased (improved) respectively 7, 6, and 2% compared to the standard NL2 model. The values of SSQ and SAD obtained with the extended NL3 model were decreased (improved) respectively 23 and 9% in comparison with the standard NL3 model. The values of DPO obtained with the extended and standard NL3 models are nearly identical. The SSQ and SAD values obtained with the extended NL4 model were decreased (improved) respectively 42 and 20% in comparison with the standard NL4 model. The DPO estimation with the extended NL4 model is less accurate than that of the standard NL4 model. The results of Table 1 display that the developed NL4 is the

Table 1. Results of the Standard and Extended Nonlinear Muskingum Models for the First Case Study

Model type	K	X	θ	α	α_1	α_2	β	C_1	C_2	SSQ	SAD	DPO
NL2	0.52	0.287	161	—	—	—	1.86	—	—	36.77	23.47	0.90
Extended NL2	0.52	0.287	12	—	—	—	1.87	—	—	34.12	21.99	0.88
NL3	0.83	0.296	190	0.43	—	—	4.079	—	—	7.67	10.31	0.31
Extended NL3	0.80	0.295	164	0.44	—	—	3.99	—	—	5.87	9.37	0.31
NL4	0.48	0.083	28	—	0.70	0.425	3.82	0.619	0.735	5.44	6.69	0.05
Extended NL4	0.49	0.811	24	—	0.73	0.436	3.73	0.58	0.72	3.19	5.35	0.07

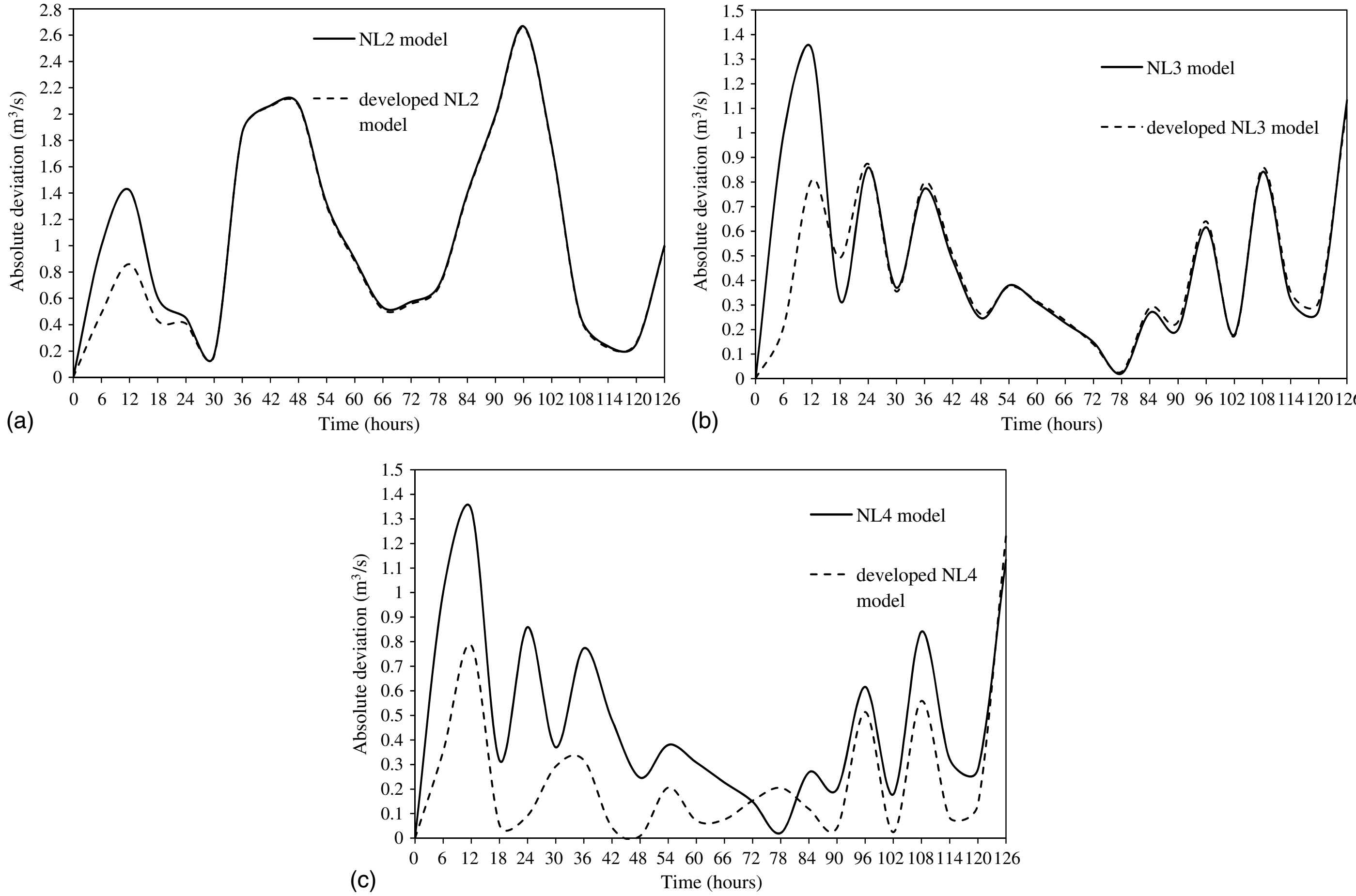

Fig. 3. Comparison of the ADs between observed and routed outflows obtained with the standard and extended nonlinear Muskingum model for the first case study: (a) NL2 model; (b) NL3 model; (c) NL4 model

best among the extended nonlinear Muskingum model in estimating the output hydrograph for the first case study. The values of SSQ, SAD, and DPO with the extended NL4 model were decreased (improved) respectively 45, 43, and 77% compared to the best corresponding values of the standard NL3 model reported by other authors.

Fig. 3 depicts a comparison between the absolute deviation (AD) of observed outflow and routed outflow calculated with the standard and extended nonlinear Muskingum models for the first case study. It is shown in Fig. 3 that the extended nonlinear Muskingum models improved the AD values in the initial hours of routing. This improvement reflects the influence of the parameterized initial boundary condition.

Results for the Second Case Study

The second case study is a flood that occurred in 1960 on the River Wye, England, over a length of 69.75 km between Erwood and Belmont. This case study was reported by O'Donnell (1985) with the use of linear Muskingum models, and exhibits a nonlinear relation between S and $[XI + (I - X)O]$ (Barati 2013b). The time step is 6 h long ($\Delta t = 6$), and the number of time steps in the simulation equals 33 ($N - 1 = 33$). The beginning flow condition is unsteady. The parameters of the NL2 Muskingum model were reported by Karahan et al. (2013), Barati (2013a), Hamedi et al. (2014a). The NL3 and NL4 parameters were estimated respectively with GA-GRG and SFLA-NMS by Easa (2013) and Bozorg-Haddad et al. (2015c). The outflow hydrographs were routed with extended nonlinear Muskingum models introduced in this paper.

Fig. 4 depicts a comparison between the observed hydrograph and the extended routed (computed) outflow hydrograph obtained with the extended nonlinear Muskingum models. It is observed in Fig. 4 that the extended nonlinear Muskingum models estimated the outflow hydrograph of the second case study of with acceptable accuracy.

Table 2 lists a the best values of SSQ, SAD, and DPO calculated with the extended and the standard NL2, NL3, and NL4 models. The parameters of the standard NL2, NL3, and NL4 Muskingum models were calculated by Bozorg-Haddad et al. (2015c) using SFLA-NMS. The SSQ, SAD, and DPO values were respectively 32,718, 788, and 90 from the extended NL2 nonlinear Muskingum model, they equaled 31,260, 722, and 67 from the extended NL3 nonlinear Muskingum model, and 30,804, 723, and 73 from the extended NL4 nonlinear Muskingum model.

The values of SSQ and SAD listed in Table 2 that were calculated with the extended NL2 model were decreased (improved) respectively 0.2 and 0.6% compared with those obtained with the standard NL2 model. The values of DPO obtained with the standard and extended NL2 models are nearly identical. The SSQ, SAD, DPO values obtained with the extended NL3 model were decreased (improved) respectively by 3, 3, and 11% in comparison with those calculated with the standard NL3 model. The values of SSQ and SAD computed with the extended NL4 model were

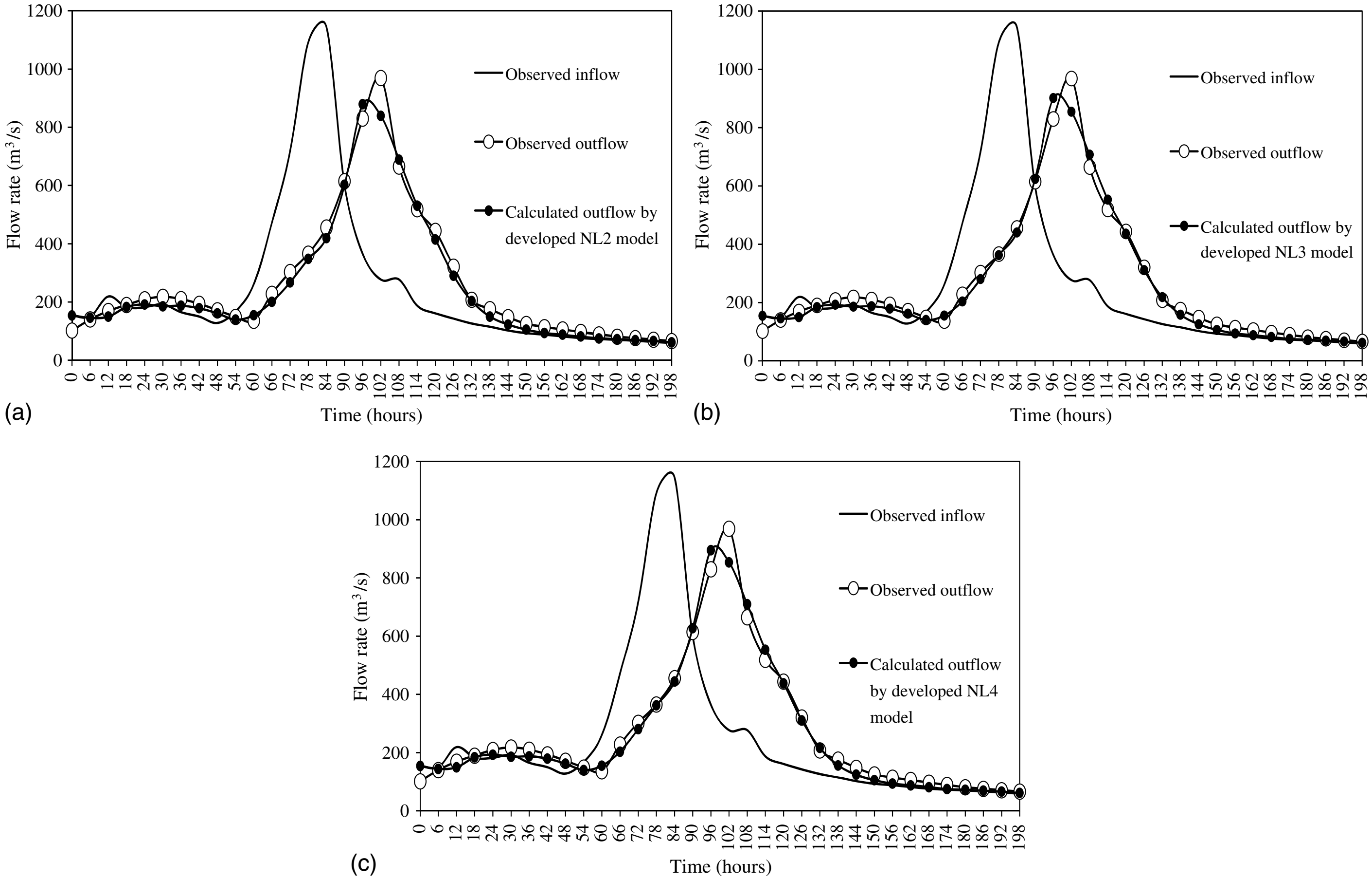

Fig. 4. Comparison of the ADs between observed hydrograph and routed hydrograph obtained with the standard and extended nonlinear Muskingum models for the first case study: (a) NL2 model; (b) NL3 model; (c) NL4 model

Table 2. Results of the Standard and Extended Nonlinear Muskingum Models for the Second Case Study

Model type	K	X	θ	α	α_1	α_2	β	C_1	C_2	SSQ	SAD	DPO
NL2	0.08	0.415	939	—	—	—	1.59	—	—	34,789	793	90
Extended NL2	0.44	0.415	1,029	—	—	—	1.59	—	—	32,718	788	90
NL3	0.44	0.404	1,023	1.20	—	—	1.33	—	—	32,299	743	76
Extended NL3	0.45	0.414	1,027	1.17	—	—	1.36	—	—	31,260	722	67
NL4	0.60	0.609	1,052	—	1.056	1.16	1.40	1.00	1.00	30,894	732	73
Extended NL4	0.61	0.615	1,024	—	1.051	1.16	1.40	1.02	1.008	30,804	723	73

(improved) respectively by 0.3 and 1%. The value of DPO calculated with the extended and standard NL2 models are nearly identical. The results listed in Table 2 demonstrate that the extended NL4 is the best model among the extended nonlinear Muskingum models for the purpose of estimating the output hydrographs in the second case study. The extended NL3 model has lower (better) SAD and DPO values than those from other extended Muskingum models. However, the extended NL4 model is better than the extended NL3 model in estimating output hydrographs judged by the SSQ objective.

Fig. 5 shows a comparison between the AD of observed outflow and the routed outflow calculated with the standard and extended nonlinear Muskingum models for the second case study. The results of Fig. 5 establish that the extended NL2, NL3, and NL4 Muskingum models improved the AD values in comparison with the standard NL2, NL3, and NL4 Muskingum models.

Results for the Third Case Study

The third case study is a multipeaked flood hydrograph reported, which is a synthetic problem, by Viessman and Lewis (2003). The time step equals 1 h ($\Delta t = 1$) and the number of time steps is 23 ($N - 1 = 23$). The hydrograph by Viessman and Lewis (2003) has two peaks at 10:00 and 17:00. This hydrograph has unsteady flow at the beginning of the flood. Fig. 6 graphs a comparison between the observed hydrograph and the routed (computed) outflow hydrograph obtained with the extended Muskingum models for the third case study. It is seen in Fig. 6 that the extended Muskingum models routed the outflow hydrograph very well. The accuracy of estimation of the outflow hydrograph and the DPO value increases with increasing order of the nonlinear Muskingum models in the third case study.

Table 3 lists the best values of SSQ, SAD, and DPO calculated with the standard and extended nonlinear Muskingum models in

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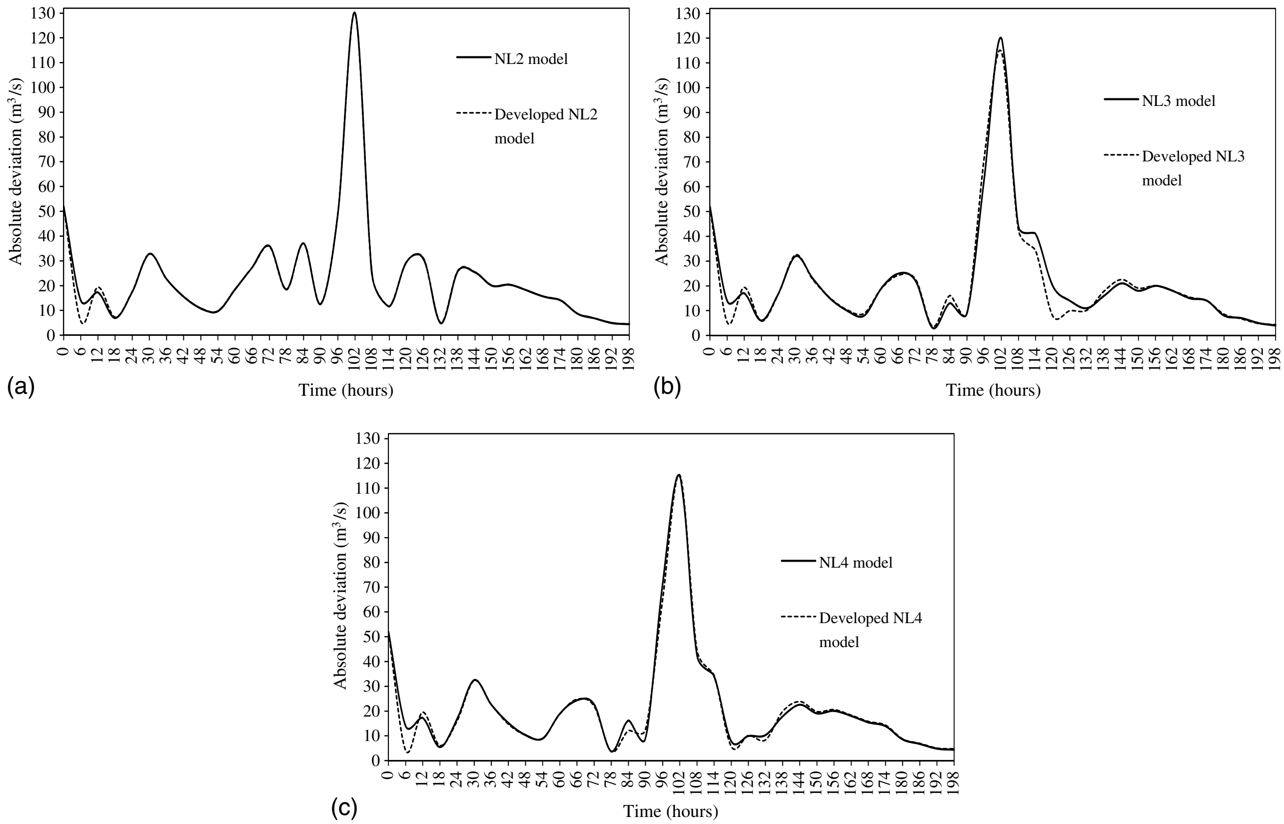

Fig. 5. The ADs between observed outflow and routed outflow obtained with the standard and extended nonlinear Muskingum models for the second case study: (a) NL2 model; (b) NL3 model; (c) NL4 model

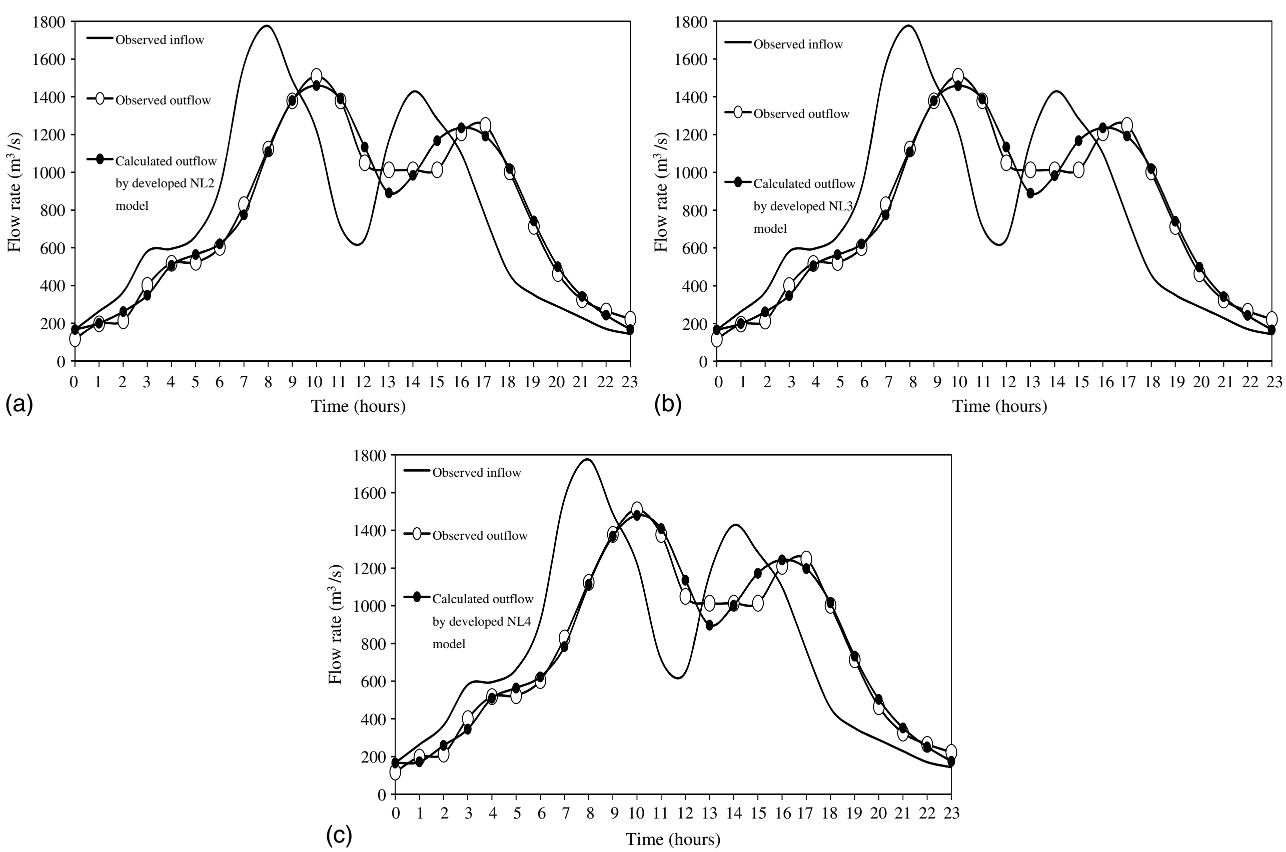

Fig. 6. Comparison between the observed hydrograph and calculated hydrograph obtained with the extended nonlinear Muskingum models for third case study: (a) for NL2 model; (b) for NL3 model; (c) for NL4 model

Table 3. Results of the Standard and Extended Nonlinear Muskingum Models for the Third Case Study

Model type	K	X	θ	α	α_1	α_2	β	C_1	C_2	SSQ	SAD	DPO
NL2	0.077	0.167	123	—	—	—	1.45	—	—	73,399	1,037	56
Extended NL2	0.069	0.166	32	—	—	—	1.46	—	—	72,210	1,003	50
NL3	0.077	0.167	124	0.921	—	—	1.57	—	—	73,379	1,033	56
Extended NL3	0.070	0.166	31	0.945	—	—	1.54	—	—	72,215	1,001	51
NL4	0.007	5×10^{-6}	111	—	3.12	1.42	1.00	1.00	1.00	69,861	993	30
Extended NL4	0.078	5×10^{-7}	69	—	3.12	1.42	1.00	1.00	1.00	69,538	988	30

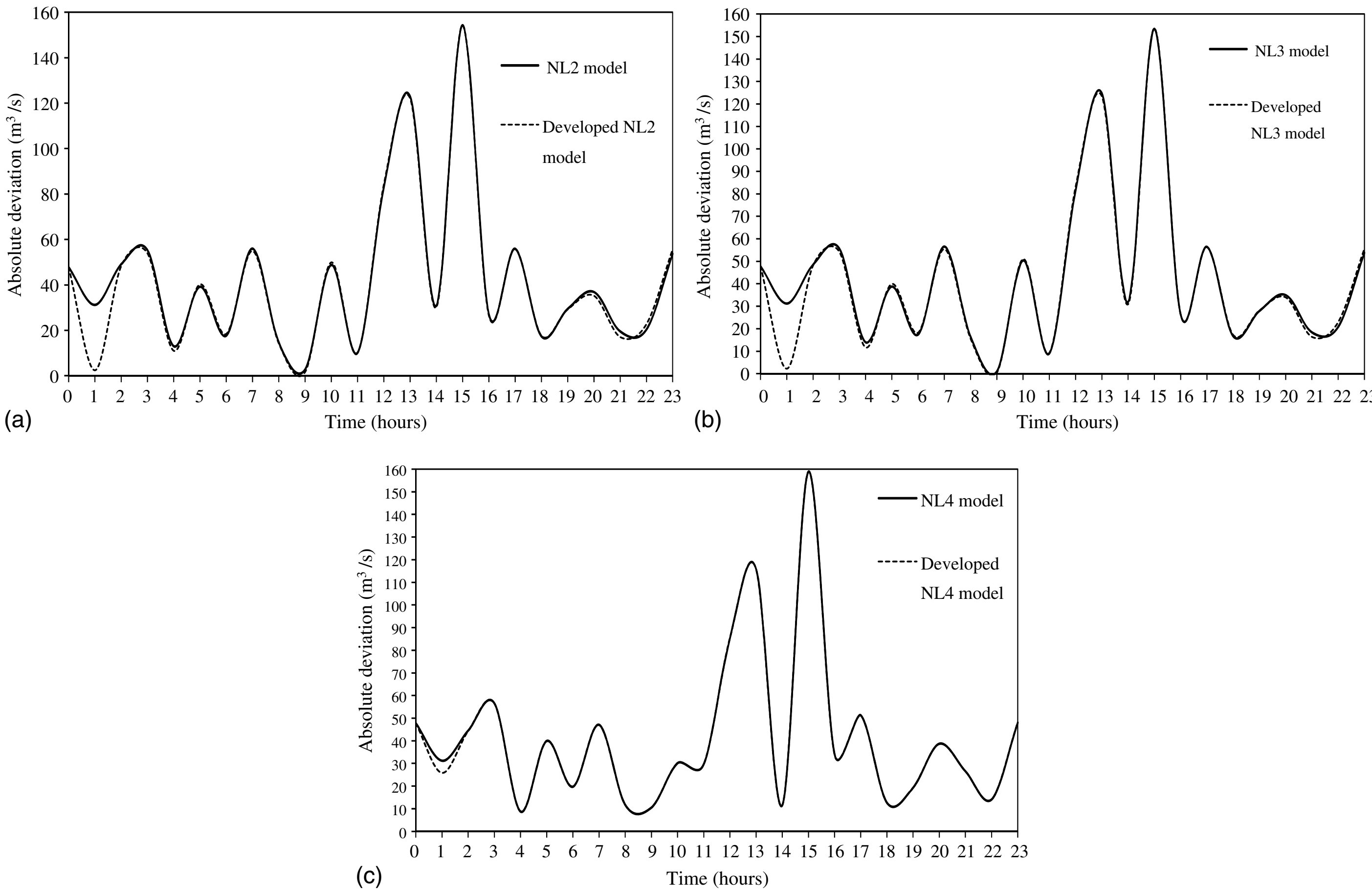

Fig. 7. Comparison between the observed hydrograph and calculated hydrograph obtained with the standard and extended nonlinear Muskingum models for the third case study: (a) NL2 model; (b) NL3 model; (c) NL4 model

the third case study. The parameters of the NL2, NL3, and NL4 Muskingum model were calculated by Bozorg-Haddad et al. (2015c) with SFLA-NMS. The parameters of the extended nonlinear Muskingum models were estimated with WOA algorithm introduced in this study. The SSQ, SAD, and DPO values equaled respectively 72, 210, 1,003, and 50 for the extended NL2 nonlinear Muskingum model, 72,215, 1,001, and 51 for the extended NL3 nonlinear Muskingum model, and 69,598, 988, and 30 for the extended NL4 nonlinear Muskingum model.

The results in Table 3 establish that the values of SSQ, SAD, and DPO from the extended NL2 model were decreased (improved) respectively 2, 3, 11% compared to those from the standard NL2 model, the values of SSQ, SAD, DPO from the extended NL3 model were decreased (improved) respectively 2, 3, and 9% in comparison with those from the standard NL3 model, and the values of SSQ and SAD from the extended NL4 model were decreased (improved) respectively 0.5 and 0.5%. The values of DPO from the extended and standard NL2 models were nearly identical. The results from Table 3 indicated that the extended NL4 is the best among the extended nonlinear Muskingum model for the purpose of estimating the output hydrograph in the third case study. The values of SSQ, SAD, and DPO from the extended NL4 model were decreased (improved) respectively 4, 1, and 41% compared to those from the extended NL3 model.

Fig. 7 shows a comparison of the ADs between the observed outflow and the routed outflow from the standard and extended nonlinear Muskingum models for the third case study. The graphs of Fig. 7 demonstrate that the extended Muskingum models improved the AD values near the initial times of the routed hydrograph compared with the standard NL1, NL2, and NL3 Muskingum models.

Concluding Remarks

This study extended existing versions of the nonlinear Muskingum model by treating the initial storage condition as a parameterized variable subjected to optimized estimation. The extended nonlinear Muskingum models have more degrees of freedom in comparison with their standard versions, affording them more flexibility in estimating output hydrographs, as shown by three case studies in this work. This work applied the WOA in the estimation of the parameters of the extended nonlinear Muskingum models. The efficiency of the WOA in parameters estimation and the resulting accuracy of outflow hydrograph prediction were demonstrated with the types of nonlinear outflow hydrographs. Specifically, the results from the extended nonlinear Muskingum models in routing hydrographs showed that the extended NL2 model in the first, second, and third case studies reduced (improved) 7, 0.2, and 2% the SSQ compared with the best results previously reported for the standard NL2 in these three case studies. The extended NL3 model in the first, second, and third case studies reduced (improved) 23, 3, and 2% the SSQ compared with the best results previously reported for the standard NL3 in these three case studies. The extended NL4 model in the first, second, and third case studies reduced (improved) 42, 0.3, and 0.5% the SSQ compared with the best results previously reported for the standard NL4. The results for all the case studies show that the WOA convergences to global optima rapidly. The best solutions in all the case studies were achieved with computational times under one minute.

The extended Muskingum models improved the SAD and DPO values in with respect to the majority of the chosen estimation criteria compared with the best corresponding results from standard nonlinear Muskingum models. The authors' results indicate that the extended nonlinear Muskingum models featuring a parameterized initial storage condition are preferable to their standard counterparts for the purpose of routing outflow hydrographs, especially in the case of steady initial condition.

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