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Thermal Conductivity of Fluid-Saturated Rocks

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Summary

A theoretical model is developed to predict the thermal conductivity of fluid-saturated rocks. Rock is idealized as being composed of a connected mineral phase permeated with an assemblage of non-intersecting oblate spheroidal pores. The pores are assumed to be completely saturated with a single-phase fluid. An effective medium theory is used to predict the overall conductivity in terms of the conductivities of the solid and fluid phases, the porosity, and an average aspect ratio of the spheroidal pores. In the limiting case where the pores are thin penny-shaped cracks, the predicted conductivity coincides with one of the Hashin-Shtrikman bounds, while the limiting case of spherical pores reproduces the other Hashin-Shtrikman bound. The theory is shown to allow the prediction of liquid-saturated conductivities based on gas-saturated measurements, without the need for any arbitrary parameters. Agreement between theory and experimental data is found to be typically within 10%, for both sedimentary and crystalline rocks.

Keywords: Thermal conductivity, microcracks, porous rocks.

1 Introduction

The thermal conductivity of rock is a physical parameter that is relevant to many areas of geophysics. Knowledge of the numerical value of the thermal conductivity is necessary for the estimation of the crustal heat flux (Lachenbruch 1978), for modelling heat and mass transfer in hydrocarbon reservoirs (Preuss & Narasimhan 1985), and for predicting the performance of underground nuclear waste repositories (Pollock 1986). Crustal rocks typically are permeated with cracks and pores, to one degree or another, and are saturated with one or more fluid components. The thermal conductivity of such a rock depends on the conductivity of its mineral components, the conductivity of the pore fluids, and the extent and geometry of the pore space. Various empirical equations have been proposed to relate the thermal conductivity to these parameters (see Somerton 1958; Sugawara & Yoshizawa 1962), and some attempts have been made to predict the conductivity theoretically (i.e., Walsh & Decker 1966). The purpose of this paper is to provide the basis for a systematic approach to the prediction of the thermal conductivity of fluid-saturated porous rocks.

2 Bounds on the Conductivity

In the most general sense, a fluid-saturated porous rock can be thought of as a heterogeneous mixture of numerous solid and fluid components. For example, a particular sandstone might be composed of quartz, feldspar, and calcite, and be saturated with an oil/water or oil/gas mixture. Each of these components has its own intrinsic thermal conductivity, and each can be characterized by a volume fraction c_i , where $0 < c_i < 1$, and $\sum c_i = 1$. This paper will concentrate on the special case where there is only one solid (mineral) component, and one fluid component, so that the volume fraction of the fluid always equals the porosity ϕ . Note that while the extension of the results derived below to the case of more than one mineral component poses no new problems, the case of multiple fluid components or phases entails additional considerations (involving

wettability), and will not be treated here.

The effective conductivity of such a two-phase solid/fluid mixture will (intuitively) be expected to lie somewhere between the conductivities of the solid and of the fluid. In almost all cases of practical interest, the solid conductivity will exceed that of the saturating fluid. If the saturant is a liquid, the ratio k_f/k_s will usually be in the range 0.01-0.50, while for gas-saturated rocks this ratio may be as low as 0.001. Since the effective conductivity k (no subscript) will depend on k_s and k_f , along with the volume fractions, weighted averages immediately suggest themselves. The simplest models are weighted averages of the conductivities or the resistivities, which correspond to so-called "parallel" and "series" models, respectively, (Fig. 1). For these two models, the effective conductivities k are given by

$$k = \phi k_f + (1-\phi)k_s \quad (\text{parallel}) \quad (1)$$

$$\frac{1}{k} = \frac{\phi}{k_f} + \frac{1-\phi}{k_s} \quad (\text{series}) . \quad (2)$$

These two expressions were shown by Wiener to represent upper and lower bounds on the effective conductivity (Böttcher 1952). Among the problems with these models is that they are physically unrealistic, since a body consisting of alternating slabs of solid and fluid material would possess no structural rigidity. Furthermore, the series and parallel models are not isotropic, whereas isotropy (or at least very mild anisotropy) is fairly common in rocks. These two bounds are also usually too far apart to provide useful predictions. For example, Fig. 2 shows the series and parallel bounds for $k_f/k_s = 0.2$, which is a reasonable value for liquid saturation. Note that in the practically important range of $\phi = 0.00-0.40$, the bounds are quite far apart. Using the geometric mean as a best estimate would still entail a possible error of as much as

$\pm 40\%$.

The series and parallel bounds are not, however, the sharpest known bounds that are still independent of pore structure. Hashin & Shtrikman (1962) derived bounds which are always tighter than the series and parallel bounds. Normalized with respect to k_s , they are

$$k_f + \frac{3k_f(k_s - k_f)(1 - \phi)}{3k_f + (k_s - k_f)\phi} \leq k \leq k_s + \frac{3k_s(k_f - k_s)\phi}{3k_s + (k_f - k_s)(1 - \phi)} \quad (3)$$

These bounds are plotted in Fig. 2 for $k_f/k_s = 0.2$, where it is seen that they offer an improvement over the series/parallel bounds. It is worth noting that these bounds can be physically realized by a space-filling assemblage of composite spherical shells, each of which having the same volume fraction of solid and fluid (Hashin & Shtrikman 1962).

For cases such as that shown in Fig. 2, the H-S bounds are close enough that the geometric mean always furnishes an estimate of effective conductivity that is accurate to within $\pm 11\%$. For many purposes this would suffice. Unfortunately, for a value of k_f/k_s representative of gas-saturation, even the H-S bounds are quite far apart. If $k_f/k_s = 0.01$, the geometric mean of the H-S bounds might still be in error by as much as a factor of two (Fig. 3). This example illustrates the need for having a more accurate means of estimating the effective conductivity. Since the H-S bounds can be achieved by the afore-mentioned spherical-shell assemblage, it is clear that these are the best geometry-independent bounds possible. Hence the need for a theory that accounts for pore structure.

3 Spheroidal Pore Model

The idealization of a rock as being composed of a solid permeated with non-intersecting pores of spheroidal shape has been used with great success in modeling mechanical and acoustical properties (e.g., Cheng & Toksöz 1979; Zimmerman 1984). A spheroid is a degenerate ellipsoid which has (at least) two axes of equal length. The parameter which characterizes the shape of a spheroid is the aspect ratio, which is defined here to be the ratio of the length of the unequal axis to the length of one of the equal axes. In its limiting cases, the spheroid can represent a needle-like pore ($\alpha \rightarrow \infty$), a spherical pore ($\alpha \rightarrow 1$), or a thin, "penny-shaped" crack ($\alpha \rightarrow 0$). This model would at first seem to suffer from the defect that the pores are not connected, as they would often be in a real rock; however, this is known not to pose a problem as far as mechanical properties are concerned, and will presently be shown not to hinder the application of this model to the prediction of thermal conductivity.

The general approach to determining the effective properties of a composite material consisting of inclusions of any shape is first to calculate the effect of an infinitesimally small concentration of inclusions, and then use some approximate method to account for finite concentration. In the present problem of thermal conductivity, this requires calculation of the perturbative effect that a single spheroidal "inclusion" has on an otherwise uniform temperature gradient in a solid (non-porous) rock. This effect is then averaged over all possible spatial orientations of the pore with respect to the temperature gradient. (In general, the distribution of pore orientations may be anisotropic; the discussion here is restricted to the case where there are no preferred directions of pore alignment). It should be noted that, because of the small dimensions of most pores, thermal convection within an individual pore can be ignored. In this event the thermal conductivity is the only relevant property of the pore fluid; mathematically, therefore, the inclusion may just as well be solid as fluid.

For the general case of a spheroidal pore of arbitrary aspect ratio, the relevant boundary-value problem to be solved is not trivial. Fortunately, this mathematical problem has already been solved, in the context of electrical conductivity (Fricke 1924). For small concentrations of porosity, the effective conductivity is given by:

$$\frac{k}{k_s} = 1 - \beta\phi, \quad \text{where} \quad \beta = \frac{1-r}{3} \left[\frac{4}{2 + (r-1)M} + \frac{1}{1 + (r-1)(1-M)} \right], \quad (4)$$

$r = k_f/k_s$, and M is a factor that depends on the aspect ratio α of the pore. For the two regimes of oblate ($\alpha < 1$), and prolate ($\alpha > 1$) spheroids, this factor is given by

$$\text{oblate:} \quad M = \frac{(2\theta - \sin 2\theta)}{2 \tan \theta \sin^2 \theta}, \quad \text{where } \theta = \arccos(\alpha), \quad (5)$$

$$\text{prolate:} \quad M = \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{2 \sin^3 \theta} \ln \left[\frac{1 + \sin \theta}{1 - \sin \theta} \right], \quad \text{where } \theta = \arccos(1/\alpha). \quad (6)$$

The factor β is plotted in Fig. 4 for a range of conductivity ratios and aspect ratios. For a fixed aspect ratio, β increases as the conductivity ratio decreases, as would be expected. For a fixed conductivity ratio, β is a minimum for spherical pores, and approaches an asymptotic value in the limiting case of thin crack-like pores. As has been found previously in the analysis of the effect of pore shape on mechanical compressibility (Zimmerman 1984), the aspect ratio of a prolate spheroid has little effect, while the results are very sensitive to aspect ratio if the pore is oblate.

Analytical expressions for various limiting cases can be found by the usual methods of Taylor expansions and l'Hôpital's rule. Some of the more interesting and useful limiting cases are:

$$\text{thin cracks: } \beta \rightarrow \frac{(1-r)(1+2r)}{3r} \quad \text{as } \alpha \rightarrow 0, \quad (7)$$

$$\text{spherical pores: } \beta = \frac{3(1-r)}{2+r}, \quad (8)$$

$$\text{needle-like pores: } \beta \rightarrow \frac{(1-r)(5+r)}{3(1+r)} \quad \text{as } \alpha \rightarrow \infty. \quad (9)$$

As a rough rule of thumb, α must be less than $r/10$ for a pore to effectively function as a "thin crack". For water saturation, this amounts to saying that all pores for which $\alpha < 10^{-2}$ are crack-like, whereas for gas saturation the thin-crack approximation holds only when $\alpha < 10^{-3}$. Since many rocks do have pores with aspect ratios on the order of 10^{-2} to 10^{-3} , the thin-crack approximation cannot be used indiscriminately.

The effective thermal conductivity given by (4) is known to be precisely correct only in the limit of very low porosity, since its derivation was based on the consideration of the effect that a single fluid-filled pore has on the heat flux in an otherwise solid, homogeneous material. In order to arrive at an expression that is applicable to realistic cases of rocks with finite porosity, some sort of "effective medium" theory must be invoked to account for the interactions of nearby pores. There is as of yet no generally agreed upon methodology for predicting the effective properties of heterogeneous materials with finite concentrations of the inclusion phase (cf. Hashin 1983; Watt, Davies & O'Connell 1976). Since the purpose of this paper is not to determine definitively which effective medium theory is most accurate, but merely to show that such an approach can provide useful predictions of thermal conductivity for fluid-filled rocks, one particular effective medium theory that can be shown to have some plausibility will be utilized.

One of the earliest effective medium theories was proposed by Maxwell (1892), and applied to the estimation of the effective electrical conductivity of a dispersion of spherical inclusions in a matrix. This theory is based on determining the perturbed temperature field due to a large number of small inclusions, and then equating this to the temperature perturbation due to a single inclusion which has the proper "effective conductivity". When applied to dispersions of spheroids, this method predicts an effective thermal conductivity of

$$\frac{k}{k_s} = \frac{(1-\phi)(1-r) + r\beta\phi}{(1-\phi)(1-r) + \beta\phi}, \quad (10)$$

where r and β are defined as before.

Expression (10) for the effective thermal conductivity has the interesting property, apparently unnoticed heretofore, that the entire range of values permitted by the H-S bounds is covered as the aspect ratio varies from 1 to 0 (Fig. 5). This can be verified by merely inserting the limiting cases (7) and (8) into (10), from which, after some algebraic manipulation, the bounds (3) emerge. Spherical pores cause the minimum change in thermal conductivity, while crack-like pores have the maximum possible effect. Contrary to the impression that might be gained from some published discussions of this topic (i.e., Schärli & Rybach 1984, Fig. 2), and in contrast to the closely related electrical conductivity problem (in which $r \rightarrow \infty$ when a rock is saturated with an electrolytic fluid), the question of interconnectedness of the pores is of little consequence. So whereas most of the porosity in rocks is typically interconnected, the lack of interconnectivity implicit in the spheroidal pore model does not limit its applicability to real rocks. Coupled with the fact that this method is by necessity correct to first order in the limit of small ϕ , it seems plausible to assume that (10) will be reasonably accurate for finite values of ϕ . In this regard, it is worth noting that many other

effective medium theories would predict conductivities that violate the H-S bounds at certain porosities.

4 Application of Model to Experimental Data

A major potential use of a thermal conductivity model such as the one described above might be to predict the conductivity of a liquid-saturated rock based on laboratory measurements made under air-saturated conditions. Appropriate data for which this procedure can be attempted is found in the work of Woodside & Messmer (1961), who presented conductivity measurements on six different consolidated sandstones, using air, water, and n-heptane as the pore saturants. The porosities of these six sandstones ranged from 0.03 to 0.59, and each consisted of at least 85% quartz. To simplify the calculations, therefore, k_s will be taken to be the conductivity of quartz, which is given by Woodside & Messmer as 8.4 W/mK. (This is an average value, at 30C, of the conductivities in the different crystallographic directions). More precise analysis could account for the small fractions of silica and feldspar, which have lower conductivities than does quartz, but the accuracy of the measurements probably does not justify this step. The thermal conductivities of the three saturants, (air, water, and heptane) are 0.026, 0.63, and 0.13 W/mK, respectively, corresponding to r values of 0.0031, 0.075, and 0.016. The most extensive measurements were made under a net confining pressure of zero, so these data will be used to test the model.

For each of the sandstones, the air-saturated k/k_s value is used, along with r and ϕ , to solve for β , after which (4) and (5) are solved for α (assuming that the pores are all similarly-shaped oblate spheroids). Note that most of this inversion can be done in closed-form, leaving only (5) to be solved numerically. Once the average aspect ratio α is "known", (4,5,10) are used to predict the effective conductivity for other values of r (i.e., different pore fluids). The values thus predicted are in each case somewhat lower than those measured, by an average of 7.5% for water saturation, and 16.0% for

heptane saturation (Fig. 6). As a reference point to judge this method, note that when the rocks are saturated with either water or heptane, the H-S bounds are quite far apart. As an example, using the geometric mean of the two H-S bounds for the water-saturated tripolite specimen (which has the lowest of the conductivities in Fig. 6) would entail a possible error of $\pm 42\%$, while the prediction described above is in error by only 11.5%.

This scheme for modeling the thermal conductivity can also be tested against data on low porosity, crystalline rocks. Such data can be found in the work of Schärli & Rybach (1984), who measured the conductivities of five air- and water-saturated granitic rocks. Each sample had substantial amounts of quartz, feldspar, plagioclase, and biotite, and a porosity of slightly less than 0.01. The conductivities of the mineral phases were calculated from the single mineral conductivities and mineral volume fractions as lying between 3.03 and 3.44 W/mK, while the conductivities of the air and water are 0.026 and 0.63 W/mK. It will be assumed that the "thin crack" limit can be used for these rocks, which requires the typical aspect ratio to be less than about 10^{-3} , which is plausible. If the procedure described above is applied to these data, the ratio of air-saturated conductivity to liquid-saturated conductivity is predicted to be

$$\frac{k_{as}}{k_{ws}} = \frac{(1-0.661\phi)(1+1.37\phi)}{(1+41.3\phi)(1-0.538\phi)} \quad (11)$$

The predictions are extremely accurate, with the average error being less than 3% (Fig. 7). Since the differences between the air-saturated and liquid-saturated conductivities are in each case about 30%, these predictions are non-trivial. Note that this procedure is similar to that carried out by Schärli & Rybach, but is done here in a more general framework; their eq. (3) can be derived from (4,5,10) of the present work by considering the limiting case where α , r , and ϕ are each very small.

As a final example of the use of this model, consider the thermal conductivity measurements on Casco granite made by Walsh & Decker (1966). They presented both compressibility and conductivity data, over the range of pressure from 0 to 100 MPa. Crack porosity will be derived from the stress-strain curve as a function of pressure, and then used as input to the thermal conductivity model to predict the variation of k with pressure. In order to focus on the effect of the closable cracks, the mineral phase plus any non-closable pores will be treated as the "matrix", and only the cracks will be considered as the inclusion phase. For this analysis, the data for sample B will be treated, because it exhibits a greater variation of conductivity with pressure than does sample A. The air-saturated conductivities will be considered, since a pore saturant with lower conductivity accentuates the effect of the cracks.

At pressures high enough so that the stress-strain curve is linear, it is clear that all of the thin cracks have been closed. This occurs at about 100 MPa, at which pressure sample B has a "matrix" conductivity of 3.23 W/mK. The conductivity of air is 0.026, so $r=0.0080$. The pressure required to close a crack is roughly equal to αE_s , where E_s is the Young's modulus of the rock matrix (Walsh & Decker 1966; Zimmerman 1984). Since the stress-strain curve levels off at about 100 MPa, and a reasonable value for E_s is about 100 GPa, the crack aspect ratios are all less than about 10^{-3} . Since $r=0.0080$, this value of $\alpha_{\max}$ is just about on the edge of the range for which the thin-crack approximation holds. For the purposes of demonstrating the use of this method in a relatively simple manner, this approximation (7) will be used in the subsequent calculations.

As explained by Walsh & Decker (1966), the amount of porosity contained in thin, closable cracks can be determined directly from the nonlinear portion of the stress-strain curve (their Fig. 2). With the crack porosity ϕ_c thus known as a function of pressure, $r=0.0080$, and $k_s=3.23$ W/mK, (7) and (10) can be used to yield

$$k = 3.23 \text{ W/mK} \frac{1-0.661\phi_c(P)}{1+41.3\phi_c(P)} . \quad (12)$$

The crack porosity can be fit, accurately enough for the present purposes, with an exponential function of pressure:

$$\phi_c(P) = 0.0038 e^{-(P/2.0 \text{ MPa})} . \quad (13)$$

The result of inserting (13) into (12) yields the predicted thermal conductivity as a function of pressure (Fig. 8). The predicted values are extremely accurate, over the entire range of pressures. Note that this procedure avoids the extensive numerical integration required for the method used by Walsh & Decker (1966).

5 Conclusions

The thermal conductivity of a fluid-saturated porous rock depends on the mineral and fluid conductivities, the porosity, and the pore structure. If no assumptions are made concerning the pore structure, the Hashin-Shtrikman bounds furnish the only available "prediction" of the effective conductivity. In many cases of geophysical interest, these bounds are too far apart to be of much use. If the pores are assumed to be oblate spheroids, an effective medium theory can be used to predict the conductivity. This prediction involves an additional parameter which represents the average aspect ratio of the pores. The predicted values are mathematically exact to at least the first order in porosity. If the effective medium theory proposed by Maxwell (1892) is used, the entire range of conductivities allowed by the H-S bounds are spanned as the average aspect ratio varies from 0 (thin cracks) to 1 (spherical pores). The model has been successfully tested against laboratory data on both sedimentary and crystalline rocks by predicting liquid-saturated conductivities from air-saturated values. The

model was also tested on Casco granite by determining crack porosity (as a function of pressure) from compressibility tests, and then using this information to predict the pressure variation of thermal conductivity. In general, this approach seems to successfully correlate thermal conductivity values under different saturation conditions, and can potentially provide a link with other physical properties (such as permeability and compressibility) that also depend on pore structure.

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References

- Böttcher, C.J.F., 1952, *Theory of Electric Polarization*, Elsevier, Houston.
- Cheng, C.H. & Toksöz, M.N., 1979. Inversion of seismic velocities for the pore aspect ratio distribution of a rock, *J. Geophys. Res.*, **84**, 7534-7543.
- Fricke, H., 1924. A mathematical treatment of the electric conductivity and capacity of disperse systems, *Phys. Rev.*, **24**, 575-587.
- Hashin, Z., 1983. Analysis of composite materials - a survey, *J. Appl. Mech.*, **50**, 481-505.
- Hashin, Z. & Shtrikman, H., 1962. A variational approach to the theory of the effective magnetic permeability of multiphase materials, *J. Appl. Phys.*, **33**, 3125-3131.
- Lachenbruch, A.H., 1978. Heat flow in the Basin and Range and thermal effects of tectonic extension, *Pure and Appl. Geoph.*, **117**, 34-50.
- Maxwell, J.C., 1892, *A Treatise on Electricity and Magnetism, Vol. I, 3rd ed.*, Clarendon Press, Oxford.
- Pollock, D.W., 1986. Simulation of fluid flow and energy transport processes associated with high-level radioactive waste disposal in unsaturated alluvium, *Water Resour. Res.*, **22**, 765-775.
- Preuss, K., & Narasimhan, T.N., 1985. A practical method for modeling fluid and heat flow in fractured porous media, *Soc. Pet. Eng. J.*, **25**, 14-26.
- Schärli, U. & Rybach, L., 1984. On the thermal conductivity of low-porosity crystalline rocks, *Tectonophysics*, **103**, 307-313.
- Somerton, W.H., 1958. Some thermal characteristics of porous rocks, *Trans. AIME*, **213**, 61-64.
- Sugawara, A. & Yoshizawa, Y., 1962. An experimental investigation on the thermal conductivity of consolidated porous materials, *J. Appl. Phys.*, **33**, 3135-3138.

- Walsh, J.B., & Decker, E.R., 1966. Effect of pressure and saturating fluid on the thermal conductivity of compact rock, *J. Geophys. Res.*, **71**, 3053-3061.
- Watt, J.P., Davies, G.F., & O'Connell, R.J., 1976. The elastic properties of composite materials, *Rev. Geophys. Space Phys.*, **14**, 541-563.
- Woodside, W. & Messmer, J.H., 1961. Thermal conductivity of porous media. II. Consolidated rocks, *J. Appl. Phys.*, **32**, 1699-1706.
- Zimmerman, R.W., 1984. *The Effect of Pore Structure on the Pore and Bulk Compressibilities of Consolidated Sandstones*, Ph.D. Thesis, University of California at Berkeley.

Figure Captions

Figure 1. Series and parallel models for the effective conductivity of a fluid-saturated rock. Models correspond to the upper and lower Wiener bounds.

Figure 2. Upper and lower Wiener and Hashin-Shtrikman bounds on the effective conductivity. Ratio of fluid conductivity to solid conductivity is 0.20.

Figure 3. Same as Figure 2, except that the ratio of fluid conductivity to solid conductivity is 0.01.

Figure 4. Beta factors for a dilute concentration of randomly oriented spheroidal pores. Different curves are for different values of the conductivity ratio.

Figure 5. Normalized effective conductivity, according to a Maxwell-type effective medium theory. As the pore aspect ratio varies from 1 to 0, the full range of effective conductivities allowed by the Hashin-Shtrikman bounds are obtained.

Figure 6. Prediction of the conductivities of liquid-saturated sedimentary rocks, based on the air-saturated conductivities.

Figure 7. Prediction of the conductivities of liquid-saturated granitic rocks, based on the air-saturated conductivities. The "infinitely-thin crack" approximation is used here.

Figure 8. Thermal conductivity of a Casco granite, as a function of confining pressure. Predicted values are based on the crack porosity, as inferred from compressibility measurements, and are constrained to agree with measured values at high pressures (when all cracks are closed).

SERIES MODEL

PARALLEL MODEL

XBL 883-10150

Figure 1. Series and parallel models for the effective conductivity of a fluid-saturated rock. Models correspond to the upper and lower Wiener bounds.

XBL 883-10143

Figure 2. Upper and lower Wiener and Hashin-Shtrikman bounds on the effective conductivity. Ratio of fluid conductivity to solid conductivity is 0.20.

XBL 883-10144

Figure 3. Same as Figure 2, except that the ratio of fluid conductivity to solid conductivity is 0.01.

XBL 883-10146

Figure 4. Beta factors for a dilute concentration of randomly oriented spheroidal pores. Different curves are for different values of the conductivity ratio.

XBL 883-10145

Figure 5. Normalized effective conductivity, according to a Maxwell-type effective medium theory. As the pore aspect ratio varies from 1 to 0, the full range of effective conductivities allowed by the Hashin-Shtrikman bounds are obtained.

XBL 883-10148

Figure 6. Prediction of the conductivities of liquid-saturated sedimentary rocks, based on the air-saturated conductivities.

XBL 883-10147

Figure 7. Prediction of the conductivities of liquid-saturated granitic rocks, based on the air-saturated conductivities. The “infinitely thin crack” approximation is used here.

XBL 883-10149

Figure 8. Thermal conductivity of a Casco granite, as a function of confining pressure. Predicted values are based on the crack porosity, as inferred from compressibility measurements, and are constrained to agree with measured values at high pressures (when all cracks are closed).

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