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Perceived area plays a dominant role in visual quantity estimation

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Abstract

Many studies have investigated the roles that area and number play in visual quantity estimation. Yet, recent work has shown that *perceived* area is not equal to true, mathematical area. This simple fact calls into question many findings in numerical cognition and suggests a new theoretical perspective: that area estimation plays a dominant role in visual quantity estimation. We examine two 'case studies': (1) a 'general magnitude' account of visual quantity estimation, which posits bi-directional influences between area and number. In contrast with prior work, controlling for perceived area reveals a unidirectional relation between area and number (Experiments 1 and 2), and (2) acuity of area and number estimation (Experiment 3). We show how an understanding of the perception of area forces a reevaluation of several findings concerning the relative acuity of number and area estimation. Combined, and in contrast to many prior studies, our findings suggest a dominant role of area in visual quantity estimation.

Keywords: approximate number, number, area, perception

Introduction

The ability of human adults, infants, and nonhuman animals to rapidly approximate large numbers is a cornerstone of research on numerical cognition. This propensity supposedly relies on an evolutionary ancient system -- the Approximate Number System -- which serves as a foundation for downstream numerical and mathematical ability (Cantlon & Brannon, 2007; Dehaene, 1997; Feigenson et al., 2004; Xu & Spelke, 2000).

Yet this widely accepted notion also raises questions: in our evolutionary environment, how often would number have been the most relevant cue for approximating quantity? Area perception rather than number perception would seem to have been prioritized evolutionarily: if foraging for food, for example, would you prefer to have 100 berries, or 50 berries four times in volume? Nevertheless, approximate area has been vastly understudied relative to approximate number (but see Brannon et al., 2006; Lourenco et al., 2012; Odic et al., 2013). In hundreds of studies, numerosity is assumed to be perceived independently of area (and other continuous dimensions; e.g., average size, density, or convex hull), thereby relegating area manipulations to little more than pesky control conditions in 'bigger' questions about number.

However, visual area approximation has recently emerged as an ability in its own right. Recent work has revealed that the visual approximation of area is guided by a cue other than area (Yousif & Keil, 2019). Instead, visual approximations of area are roughly equivalent to the sum of objects' dimensions rather than their product, resulting in potentially large distortions of perceived space. This continues to be true after accounting for confounds such as numerosity and perimeter. This phenomenon is known as the 'Additive Area Heuristic' (AAH).

An area estimation heuristic raises questions about the relation between area and number. While numerous papers have documented bidirectional 'congruity effects' between area and number (e.g., Hurewitz et al., 2006, Walsh, 2003), perceived area (per the AAH) may not be influenced by numerosity; these past results may arise because of a confound between *perceived* area and numerosity (Yousif & Keil, 2019). Only when unconfounded is it possible to understand the relation between number and area in visual quantity estimation.

The AAH calls into question many other findings in the field of numerical cognition, raising the possibility that many of them can also be explained by a failure to account for perceived area. For example: if numerosity does not influence the perception of area, does the perception of area influence the perception of numerosity? Though this question has been asked before (e.g., Hurewitz et al., 2006), it has operated under a false premise: that true, mathematical area accurately reflects the percept of area. Thus, to the extent that area perception is best captured not by mathematical area but by some other means (e.g., the AAH), this question ought to be revisited.

If perceived area is dissociable from mathematical area, it suggests a reinterpretation – and, in some cases, a reexamination – of many prior findings. The present work explores the relation between number and *perceived* area in the context of two 'case studies': (1) a 'general magnitude' account of number and area, and (2) relative area and

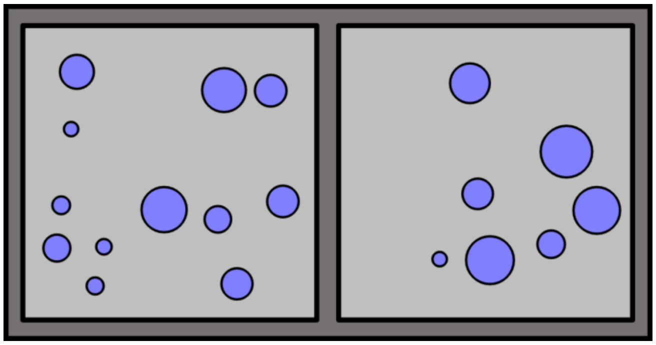

Figure 1. An example display for Experiments 1-3. Most observers report that the left panel is greater in area, despite the fact that the two are equal in true area. However, the left panel is greater in 'Additive Area' (which causes the illusion).

number estimation acuity. In both cases, we demonstrate that accounting for perceived area reveals a qualitatively different pattern from what has been previously observed.

The current study

In a first experiment, we assess the 'general magnitude' account of number and area approximation by examining how increased 'Additive Area' (AA) affects numerosity estimation. To do so, we manipulate AA while number is held constant. Most work has suggested bidirectional interactions between area and number (e.g., Hurewitz et al., 2006), but recent work has shown that manipulating number *does not* influence perceived area (Yousif & Keil, 2019). Here, we show that this relation is in fact unidirectional in that perceived area influences number judgments to a large extent. In a second experiment, we follow up on this by pitting AA and number against each other in a maximally implicit design, by having one group of observers make area judgments and another group of observers make number judgments on the exact same stimuli. Again, we demonstrate influences of area on number perception. In a third experiment, we assess number estimation acuity under different conditions (e.g., controlling AA vs. true, mathematical area). Number acuity appears to differ dramatically depending on how area is controlled.

Experiment 1: Area influences number

Mimicking a design in prior work (Yousif & Keil, 2019), we created stimuli for which additive area, mathematical area (MA), and number could be manipulated independently. AA is used as a proxy for perceived area, given the prior work showing that AA captures perceived area more accurately than MA. Observers viewed two stimuli side-by-side and were simply asked to indicate which was greater in number.

Method

Participants 100 observers were recruited via Amazon Mechanical Turk. Observers were excluded if and only if they began but did not complete the task (5 observers). All observers consented prior to participation, and these studies were approved by the IRB at Yale University.

Stimuli All of the stimuli were generated via custom software written in Python with the PsychoPy libraries (Peirce, 2007). The aim was to create pairs of stimuli that varied in either AA, MA, or number while the other values were equated. For each stimulus pair, we randomly generated an initial set of discs (ranging from 20 pixels to 100 pixels in diameter, with a buffer of at least 10 pixels between any two discs), then pseudo-randomly generated a second set of objects based on a given AA/MA/Number ratio (specific values varied for each experiment; see, e.g., Table 1). The displays always had between 20 and 26 discs (the initial set always having 20). Stimulus pairs were generated randomly until a pair met both the AA, MA, and number criteria, at which point that pair would be rendered another time and saved. The second stimulus always had more area (whether AA or MA) than the initial stimulus. For the details of how AA, MA, and number covaried, see Table 1. All discs were rendered with a thin, black border (4-pixel stroke width). The images depicted in Figure 1 are representative of those used in the experiment.

Procedure The task itself was administered online via Amazon Mechanical Turk, using custom software. On each

Number Ratio	Type	AA Ratio	MA Ratio
1.00	**AA Constant**	1.00	1.00
		1.00	1.05
		1.00	1.10
	MA Constant	1.00	1.10
		1.05	1.10
		1.10	1.10
1.15	**AA Constant**	1.15	1.05
		1.15	1.10
		1.15	1.15
	MA Constant	1.10	1.10
		1.15	1.10
		1.20	1.10
1.30	**AA Constant**	1.20	1.10
		1.20	1.15
		1.20	1.20
	MA Constant	1.15	1.15
		1.20	1.15
		1.25	1.15

Table 1. The number, AA, and MA ratios for Experiment 1.

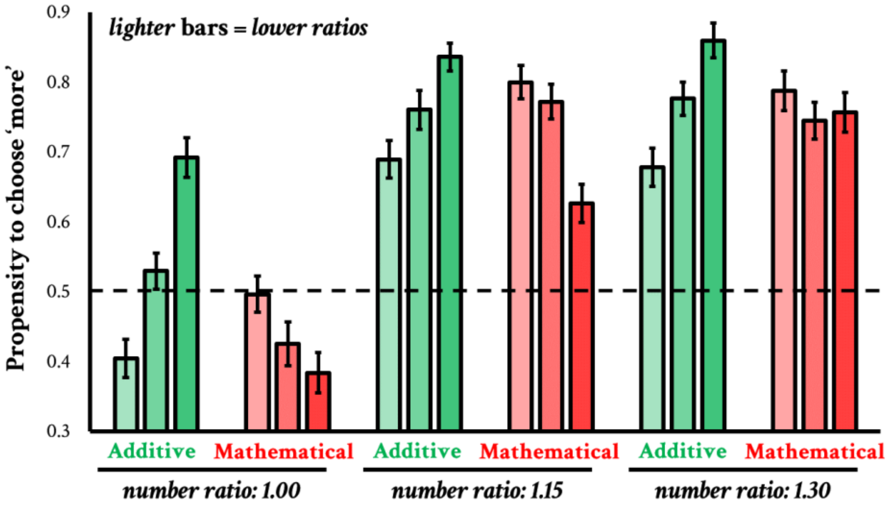

Figure 2. Results from Experiment 1. Three number ratios are represented along the x-axis. Green bars represent MA-controlled sets, where AA varied in three steps. Red bars represent AA-controlled sets, where MA varied in three steps. Lighter bars represent lower ratios. E.g., for the leftmost set of green bars, the lightest bar represents the lowest AA ratio and the darkest bar represents the highest AA ratio. Error bars represent +/- 1 SE. The dashed line represents chance performance.

trial, observers saw two spatially separated sets of lavender-colored dots, presented side-by-side in the center of the screen, with 50 pixels of space in between (see Figure 1). Each stimulus was 400 pixels by 400 pixels. The stimuli were always counterbalanced so that an equal number containing more AA, MA, or number appeared on each side of the screen. Observers were instructed to press 'q' if the image on the left had more cumulative number, and 'p' if the image on the right had more cumulative number. They were also given an additional, explicit warning to respond according to number regardless of area. The stimuli stayed on the screen for 700ms, but there was no time limit on responses. Between each trial, there was a 1000ms ITI. Observers completed 72 trials. All trials were presented in a unique random order for each participant. Observers completed two representative practice trials before beginning the actual task.

Results and Discussion

The results of Experiment 1 are shown in Figure 2. An ANOVA revealed a main effect of numerosity, confirming that observers were able to discriminate on the basis of numerosity, F(2,93)=149.65, p<.001, η_p^2=.61. Further, increased MA generally *decreased* the probability that an observer would select a stimulus as more numerous F(2,93)=12.78, p<.001, η_p^2=.12. Yet, critically, increased AA *did* increase the likelihood that observers would indicate a stimulus was more numerous, F(2,93)=49.08, p<.001, η_p^2=.34 (and this pattern was observed across all ratios, as can be seen in Figure 2). Note that this is in stark contrast to other results showing that changes in numerosity *do not* influence area judgments (Yousif & Keil, 2019).

These results (in combination with prior results) suggest a relation between perceived area and perceived number – but one that is unidirectional (i.e., perceived area influences number, but not vice versa). In contrast to a 'general magnitude' account, which predicts positive relations between various magnitudes, the present results suggest area may play a dominant role in quantity estimation. However, these results do not reveal the extent to which number is perceived independently of AA. The following experiments aim to address that question.

Experiment 2: Number versus area

To understand whether the results of Experiment 1 could be explained by a General Magnitude account (e.g., a 'more-is-more' heuristic), we directly pitted AA and number against each other in a between-subjects experiment. In this way, we can directly assess the effect of increased area on number perception and vice versa. Borrowing from previous work which dissociated AA and MA (Yousif & Keil, 2019we manipulated both AA and number while holding the other constant. In one condition, observers made area judgments; in another condition, a separate group of observers made number judgments.

Method

Participants 200 observers were recruited via Amazon Mechanical Turk (100 for each condition). Observers were excluded if and only if they began but did not complete the task (3 observers, all in the area condition). All observers consented prior to participation, and these studies were approved by the IRB at Yale University.

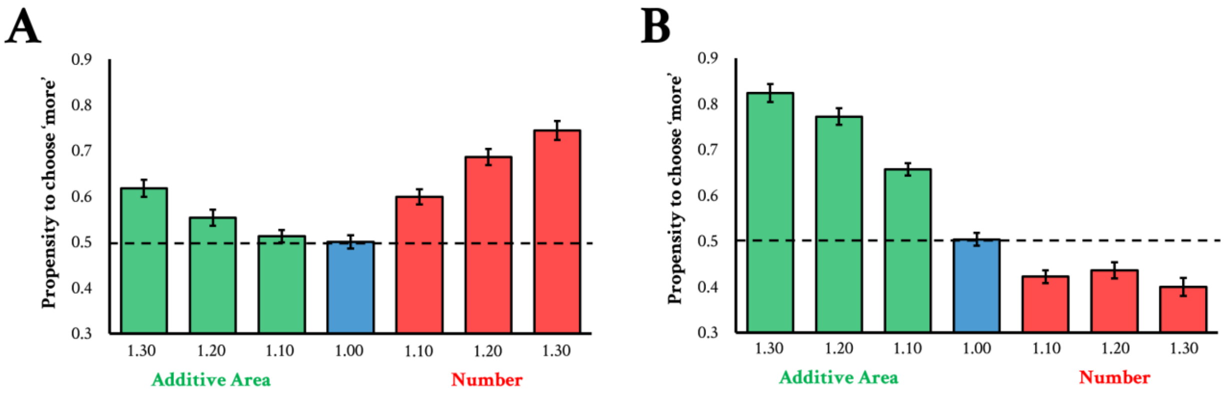

Figure 3. Results from number discriminations (A) and area discriminations (B) in Experiment 2. The green bars represent trials where AA varied (in a 1.1, 1.2, or 1.3 ratio) but number was held constant, while the red bars represent trials where number varied (in a 1.1, 1.2, or 1.3 ratio) while AA was held constant. The y-axis represents the propensity to choose 'more', whether that be more number or more area. Error bars represent +/- 1 SE. The dashed line represents chance performance.

Stimuli The stimuli for this experiment were generated in the same way as those of the prior experiment. The same stimuli were used for each condition. There were seven ratios: three in which number varied (in a 1.1, 1.2, and 1.3 ratio) while AA was held constant, three in which AA varied (in a 1.1, 1.2, and 1.3 ratio) while number was held constant, and one in which both were held constant (to serve as a baseline).

Procedure The procedure is identical to Experiment 1 except that observers completed 84 trials instead of 72. For the number judgment condition, the instructions were the same. For the area judgment condition, observers were told the following: "Your task is simply to indicate which set of circles has **more cumulative area**. In other words: if you printed the images out on a sheet of paper, which would require more total ink?" Later, they were told: "The sets of dots will sometimes vary in number, but the number of dots does not matter. Instead, you should answer only which has more area, regardless of number."

Results and Discussion

The results of the number discrimination condition are shown in Figure 3a. Observers indicated that images containing more discs were more numerous ($t(96)=11.85$, $p<.001$, $d=1.20$). However, observers also indicated that images with greater perceived area (but were equal in number) were more numerous ($t(96)=5.35$, $p<.001$, $d=.54$). In other words, it appears that the perception of area affects the perception of numerosity.

The results of the area discrimination condition are shown in Figure 3b. Observers indicated that images greater in AA were greater in perceived area ($t(96)=17.60$, $p<.001$, $d=1.76$). However, observers were slightly below chance when selecting between displays equal in AA but differing in numerosity ($t(96)=5.81$, $p<.001$, $d=.58$). Thus, all else equal, observers judged displays with more number to have *less* area – replicating the findings of recent work (Yousif & Keil, 2019) but in stark contrast to many existing studies (e.g., Hurewitz et al., 2006).

These results suggest three primary conclusions. First, the results of the number discrimination condition cannot be explained by a response bias to simply pick the image with 'more' on some dimension. Indeed, observers indicated that displays with more number appeared to have less cumulative area. Second, this experiment provides converging evidence with Experiment 1 that perceived area influences perceived numerosity (i.e., people confuse 'more' perceived area for 'more' number). Third, and critically, this experiment shows that number does *not* influence perceived area. This indicates a unidirectional relation between perceived area and number (in contrast to views that posit bidirectional interactions between these domains of magnitude; e.g., Walsh, 2003). There *is* an effect of number on area (such that more number is related to less perceived area) – but our findings challenge a general magnitude account, and are contrary to prior work (e.g., Hurewitz et al., 2006).

Experiment 3: Number and area acuity

A third experiment assessed number discrimination acuity (i.e., the level of precision with which observers can discriminate two non-symbolic numerosities) in a more traditional number acuity task, while controlling for either AA or MA. We predicted that performance will be lower when AA is controlled. The goal of this study is to ascertain whether there is a 'true' number discrimination acuity (or area discrimination acuity, for that matter), as this would bear on studies that have tried to interpret relative acuity in each domain (e.g., Lourenco et al., 2012; Odic et al., 2013).

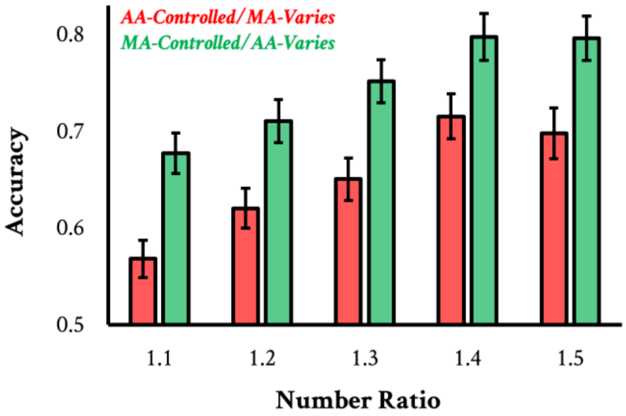

Figure 4. Results from Experiment 3. Five number ratios are represented along the x-axis. Green bars represent MA-controlled sets, where AA varied. Red bars represent AA-controlled sets, where MA varied in three steps. The y-axis represents accuracy for number discriminations, i.e., the proportion of time observers chose the display that was more numerous. Error bars represent +/- 1 SE. The x-axis corresponds to chance performance.

Method

All elements of the experimental design were identical to those of Experiment 1, except as stated below. 80 new observers were tested via Amazon Mechanical Turk. One observer was excluded for failing to complete the task. Observers completed number discriminations at five distinct ratios: 1.10, 1.20, 1.30, 1.40, and 1.50. Half the trials were controlled for AA, and the other half of trials were controlled for MA (while allowing the other dimension to vary). The displays always had between 10 and 30 discs (the initial set having 10 half the time, and 20 the other half of the time). Observers completed 80 trials.

Results and Discussion

The results of Experiment 3 are displayed in Figure 4. Accuracy was indeed lower for the AA-controlled trials, $t(79)=6.97$, $p<.001$, $d=.79$, and this was independently true for each number ratio ($ps<.002$). Of the 80 observers tested, 66 were as good or better at discriminating number in the MA-controlled condition (where AA varied; $p<.001$). Critically, performance across the two different area controls was highly correlated $r(78)=.69$, $p<.001$ – about as highly as performance in each condition was to itself (MA-control: $r=.66$; AA-control: $r=.65$).

Once again, differences in perceived area strongly influenced perceived number. While prior work has made conclusions on the basis of relative acuity (e.g., Lourenco et al., 2012; Odic et al., 2013), these results suggest that comparing acuity across dimensions should be interpreted with caution. In other words: what is 'true' number acuity, if number acuity varies so greatly across different area controls? This is especially relevant for developmental studies which make claims about relative acuity across development (e.g., Odic et al., 2013).

General Discussion

Our first two experiments demonstrate that accounting for perceived area challenges our understanding of the relation between area and number. In particular, we have shown an apparent unidirectional relation between area and number such that area influences number judgments but not the other way around. This contrasts with work documenting a bidirectional relation and forces a reconsideration of the roles of area and number in quantity estimation.

In addition, we have shown how accounting for perceived area challenges our understanding of area and number *acuity*. In particular, number discrimination acuity appears to vary dramatically depending on whether AA or MA is controlled (as revealed explicitly in Experiment 3, but also evident in the results of Experiment 1). This raises questions about prior studies that have interpreted the relative acuity of area and number discriminations (e.g., Lourenco et al., 2012; Odic et al., 2013).

Conclusion: is number special?

Is number special in visual processing? The answer to this question seems obvious: the field of numerical cognition is perhaps one of the largest and most prominent in all of cognitive science, and the ability to discriminate visual number is often thought to be the foundation of our 'core' mathematical competency (Feigenson et al., 2004). Yet, this seemingly obvious conclusion is not evident from first principles. In what evolutionary context would an approximate number system have been more critical for survival than approximate area or volume? Few plausible examples come to mind.

Our studies do not ask whether number is special *somewhere* in the mind. Instead, the question is whether number is special *visually* – or even whether, as more extreme views have suggested, it is a visual feature (like color or orientation; e.g., Anobile et al., 2016; Burr & Ross, 2008). This question has been heavily discussed (e.g., Durgin, 2008; Leibovich et al., 2017). Yet this debate, here and elsewhere, has been plagued by the use of artificial stimuli with a seemingly unbounded number of possible confounds. How can one hope to isolate numbers amidst the continuous dimensions of area, perimeter, convex hull, density, average element size, variance in element size, variability in inter-dot distance, etc. (some of which are often negatively correlated with one another)? This list is only a small subset of all the continuous cues that *may* be related to the perception of number.

The present work is not immune to such confounds. However, our studies do provide clear predictions about a particular cue, AA, (rather than a collection of them) and its relation to numerosity. This prediction is borne out of the theoretical position that visual number estimation is unlikely to have been prioritized in evolution. More consequentially, we find clear influences of area on number, but not the other way around.

What should be said, then, about the perception of number? We have presented evidence for area playing a dominant role in quantity estimation, automatically and irresistibly influencing the estimation of number. Yet, number discrimination ability across very different displays (i.e., displays controlled for either AA or MA), is highly correlated – suggesting that number estimation cannot be explained by perceived area (or by some superficial strategy that operates differently over different sets of stimuli). Thus, while the human visual system is clearly able to extract number, it does not seem to be wired to do so first and foremost. Indeed, area may play the leading role in quantity estimation. This also suggests that number may not be a true visual feature as has been claimed (see Burr & Ross, 2008).

Across several paradigms and stimuli configurations, one salient pattern consistently emerges: area influences number approximation but not the other way around. This is a fundamentally different pattern from what has been observed in tasks that do not control for AA, and these findings offer a new theoretical perspective on the relation between number and area in vision: that number may not be so special after all.

Acknowledgments

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