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Abstract

The sea-level changes over the last 100-million years due to the decrease in the earth's angular velocity, ω, are calculated on the assumption that the earth is rigid and incompressible. Compressibility is shown to be negligible and the assumption of rigidity is discarded. Comparison of the ellipticity of the earth, as calculated from artificial-satellite observations, and the ellipticity of a hydrostatic model gives upper limits on the changes in sea level. These limits are 200 feet at the poles and -100 feet at the equator.

Introduction. Observations by Eardley [1964] suggest that sea level has risen in the polar regions and fallen near the equator. The change, which has occurred during the last 100-million years, amounts to about 600 feet near the equator and probably more than 600 feet near the poles. The hypothesis has been advanced [Eardley, 1964] that the change in sea level was caused by a change in ω, the earth's rate of rotation.

If a change in ω is to cause a change in sea level, the solid earth must at some time deviate from hydrostatic equilibrium (h. e.) since the oceans are always in h. e. Hence the solid earth and the oceans must respond differently to a change in ω, and we may relate the difference in their responses to the present deviation of the solid earth from hydrostatic equilibrium. We show here that our knowledge of the present deviation of the earth from h. e. (see Caputo, 1965) rules out Eardley's hypothesis.

We first give the magnitude of the sea-level change, assuming that the solid earth is rigid and incompressible and using values of the rate of change of ω given by Munk and MacDonald [1960].

Compressibility is shown to be a negligible effect.

Knowledge of the maximum possible departure of the earth's ellipticity from the h. e. value allows us to calculate the maximum possible effect of a change in ω on sea level.

A rigid, incompressible earth. The theory of the earth's gravitational potential to first order in the ellipticity yields (see, for example, Munk and MacDonald, 1960)

$$f = A \frac{\omega^2 a^3}{GM} ,$$

where a is the equatorial radius of the earth, A depends on the density distribution within the earth, and f is the ellipticity of the ocean's surface. For the present earth, A is observed to be 0.98. (If the density distribution of the solid earth were spherically symmetric, A would be 1/2; if the solid earth and the oceans were homogeneous and of the same density, A would be 5/4.)

Suppose the ocean's surface has ellipticity f_1 at time t_1 and f_2 at time t_2. Let the average densities be ρ_1 and ρ_2 respectively. Conservation of mass implies that

$$\rho_1 a_1^3 (1 - f_1) = \rho_2 a_2^3 (1 - f_2),$$

where a_1 and a_2 are the equatorial radii at times t_1 and t_2. Assuming the earth to be incompressible, we have $\rho_1 = \rho_2$ and

$$\begin{aligned} a_2 - a_1 &= 1/3\, a_2 (f_2 - f_1) \qquad (1) \\ &= 1/3\, a_2 (0.98) \frac{a_2^3}{GM} (\omega_2{}^2 - \omega_1{}^2) \\ a_2 - a_1 &= 2/3\, a_2 f_2 \frac{\Delta\omega}{\omega} \end{aligned}$$

for small $\Delta\omega$, f_1, and f_2. Again to first order, the polar radii b_1 and b_2 satisfy

$$b_2 - b_1 = -2(a_2 - a_1).$$

Munk and MacDonald [1960] have given the fractional rate of change of ω as -2×10^{-10}/year. Taking $t_2 - t_1$ as 100-million years, we have $\Delta\omega/\omega = -0.02$. Hence

$$a_2 - a_1 = -935 \text{ feet}$$

$$b_2 - b_1 = 1870 \text{ feet}.$$

If ϕ is the geocentric latitude, the equations for the shape of the earth at times t_1 and t_2 are, respectively,

$$r_1 = a_1 (1 - f_1 \sin^2\phi), \qquad r_2 = a_2 (1 - f_2 \sin^2\phi),$$

where r is the geocentric radius of the surface of the earth.

We have seen that the change in polar radius and the change in equatorial radius are of opposite sign. Hence there must be a ϕ where $r_1 = r_2$:

$$a_1 (1 - f_1 \sin^2\phi_0) = a_2 (1 - f_2 \sin^2\phi_0),$$

and using equation (1) we find

$$\sin^2\phi_0 = 1/3$$
$$\phi_0 = 35.3° .$$

In this section we have shown that the change in sea level over the last 100-million years would have been -935 feet at the equator, 1870 feet at the poles, and zero at latitudes 35.3°N. and 35.3°S., assuming the solid earth were rigid and incompressible over that period of time.

Compressibility. If k is the bulk modulus of the earth, $k = 10^{12}$ dynes/cm^2 [Jeffreys, 1959]. The absolute change in geocentric radii due to compression is of the order of D near the equator:

$$D = \frac{\rho \omega^2 a^3}{k} \frac{\Delta\omega}{\omega} = 1.6 \text{ km}.$$

One may then wonder whether the equations obtained on the basis of a rigid incompressible earth are at all valid. To see that they are, we need only view the process of change as a shrinking of the earth toward its polar axis, followed by the changes associated with a rigid incompressible Earth. The shrinking corresponds to a small change in scale in the differences between absolute radii and is calculated in the next paragraph. Hence the equations for a rigid incompressible earth can be applied.

We are not interested in the absolute change in the geocentric radius at any point on the earth. We are interested in the difference between the geocentric radii of the solid earth and the ocean's surface, and how it changes with time. The change in the height of the ocean's surface above the solid earth due to compressibility is given by the compression of the ocean's water itself due to the change in pressure at the surface of the earth. This change in height, at the equator, is given by d:

$$d = \frac{\rho \omega^2 a h^2}{k_1} \quad \frac{\Delta\omega}{\omega},$$

where h is the depth of the ocean, and k_1 is the bulk modulus of water. With an average depth of 5 km, we obtain

$$d \approx 5 \text{ cm}.$$

Compression has its maximum effect on the equatorial radius; we have therefore shown that compressibility is negligible.

Rigidity. Many people [e. g., Munk and MacDonald, 1960] have shown that the shape of the earth is very close to the shape of a hydrostatic model. For us to use the results of the equations we have already derived without modification, we must assume that the solid earth is rigid, but just by accident it has a shape and density distribution which fake h.e. at our present rate of rotation. This seems an unreasonable assumption. In fact just the opposite appears more reasonable. That is, we assume the earth approximates a fluid and that it is as close to h.e. as our measurements allow us to accept.

If the earth were in complete h. e. we would see no significant changes in sea level, because the solid earth would have undergone essentially the same change in ellipticity as the oceans. What we want is the possible deviation of the ellipticity of the solid earth from the ellipticity it would have were it in h. e.

Recent satellite observations have provided accurate values for the ellipticity of the earth, independent of any assumption about the internal structure of the earth. These values of the ellipticity can be compared to the ellipticity of the hydrostatic model whose values of $\omega^2 a^3/GM$ and p most closely approximate those observed. (Here $p = J_2^o/H$, J_2^o is the second term in the gravity potential and H is the precessional constant.) Values of f^{-1} are given in Table I.

We see that a discrepancy of about 0.5% exists between the two ellipticities. Thus the ellipticity of the solid earth is at least 0.5% different from its ellipticity were it in h. e., and hence at least 0.5% different from the ellipticity of the ocean's surface. The assumption that the earth is as close to h. e. as measurements allow us to accept leads to the statement that the ellipticity of the solid earth is in fact 0.5% different from the h. e. ellipticity. This difference would allow a change in the sea level to have taken place at the equator in the amount

$$a_2 - a_1 = 1/3\ a_2 f_2 \frac{\Delta f}{f_2} = -100 \text{ feet} \qquad (2)$$

and at the poles,

$$b_2 - b_1 = 200 \text{ feet.} \qquad (3)$$

There is no known reason to suppose that 100-million years ago the ellipticity of the earth was less than the h. e. ellipticity for that time. A decrease in ω would result in the earth's ellipticity being greater than the value for h. e., and would create stresses tending to force the earth to a smaller ellipticity, namely the h. e. value. The mechanism whereby the earth responds to stresses of this sort is little understood. <u>MacDonald</u> [1965] has pointed out the inadequacies of the assumption that the earth is a Newtonian fluid with a uniform viscosity, and no other acceptable models have enough contact with experimental information to make them more than speculative. If, however, we accept the assumption expressed by the first sentence of this paragraph, then equations (2) and (3) present valid upper limits on the rise in sea level at the poles and the fall in sea level at the equator. That is, sea level could not have fallen by more than 100 feet at the equator although it could have risen an unknown amount.

In any case we see that the sea level at the equator cannot have fallen enough to explain Eardley's data.

One proviso must be added to the interpretation of ellipticities as calculated from satellites' data. We illustrate with examples. In the first case suppose the earth were a homogeneous rigid body surrounded by an ocean of equal density. Then a satellite would always measure the value appropriate to h. e. even though the central part of the earth were rigid. If, in the second case, the ocean had a density negligible compared to the density of the earth's central portion, then the satellite would measure the ellipticity of the central portion only, which could then be meaningfully compared to the value for hydrostatic equilibrium. The earth's case lies somewhere in between, and since the density of the ocean is one-sixth that of the solid earth, it seems reasonable to assume that the earth would appear to a satellite much more like the second example than the first. A simple calculation, based on the assumption that the central portion is a sphere of specific gravity 6.0, confirms this idea. [For the central portion $f = 0$. The h. e. value is $(5/4)(\omega^2 a^3/GM)$. A satellite would measure $(1/2)(1/6.0)(\omega^2 a^3/GM)$. Hence the satellite would measure f as about 10% closer to the h. e. value than the central portion actually is.] Hence the solid earth might be slightly farther from h. e. than a satellite measurement might indicate. Such an effect, however, would not be large enough to bring consistency with Eardley's data.

Another comment should be made on the interpretation of ellipticities. The solid earth is, of course, not a perfect ellipsoid, but has an irregular shape. However, the difference between the actual ellipticity term for the solid earth and the ellipticity term it would have were it in h. e. is an order of magnitude larger that the terms representing higher tesseral harmonics [MacDonald, 1963]. Hence a world-wide survey of the changes in sea level, such as the one Eardley has made, should yield as its main feature the properties of the ellipticity term.

Conclusion. If the Earth were rigid and incompressible a rise in sea level of 1870 feet at the poles and a fall of 935 feet at the equator would have occurred over the last 100-million years. Compressibility has a negligible effect on these conclusions. The solid earth is not rigid but is, in fact, very close to being in hydrostatic equilibrium. Calculations based on artificial satellite observations have determined the possible deviation from h. e. which allows us to calculate the maximum possible change in sea level over the last 100-million years. We assume that the earth is as close to h. e.

as measurements allow us to accept. The result is a maximum rise in sea level of 200 feet at the poles and a maximum fall of 100 feet at the equator. Eardley [1964] has observed that the fall at the equator and rise at the poles is about 600 feet. A change in ω, therefore, cannot explain his data.

Table I. Values of f^{-1}.

f^{-1}	Reference
Hydrostatic equilibrium assumed	
300.0	Henriksen [1960]
299.7	Jeffreys [1964]
299.5	Caputo [1965] (Method 4, Model 2)
No assumption about internal structure	
298.4	Henriksen [1960]

FOOTNOTES AND REFERENCES

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