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SHRINC
A COMPUTER PROGRAM FOR NONUNIFORM DRYING SHRINKAGE IN CONCRETE MEMBERS

by

ROBERT H. IDING
BORIS BRESLER

REPORT TO
National Science Foundation
Grant No. ENG. 72 04090
(Formerly GK-35787)

AUGUST 1975

DEPARTMENT OF CIVIL ENGINEERING
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ABSTRACT

This report describes the current version of the computer program SHRINC (SHRINkage in Concrete). The program calculates nonuniform drying shrinkage histories within concrete prismatic members. A nonlinear finite element formulation which considers the moisture dependence of material properties is used in SHRINC. The finite element formulation, the idealization of material properties and boundary conditions, the numerical approach, and practical aspects of using the program are discussed. In the report, a users' manual for SHRINC, a sample problem with listings of input and output, and a complete Fortran listing of the program are included as appendices.

ACKNOWLEDGEMENTS

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1. INTRODUCTION

1.1 Purpose and Scope

This report presents the current version of SHRINC, a computer program designed to calculate the nonuniform drying shrinkage history within concrete prismatic members. The moisture diffusion equation is used to model the shrinkage phenomenon. A finite element method coupled with time step integration was chosen as the solution approach.

The current program determines shrinkage variation in a two-dimensional cross-section. The effects of temperature on shrinkage development are not considered. A version of SHRINC designed to solve three-dimensional, temperature-dependent shrinkage problems will be available at a later date.

In the report, the theoretical model and solution techniques are derived, the organization of the computer program is explained, and a commentary on practical aspects of using the program is made. The appendices contain fully annotated input instructions, a sample problem with input and output, and a Fortran listing of the program.

1.2 Nature of Drying Shrinkage

The physical process of shrinkage is complex and not fully understood. Basically, moisture flows to the specimen's surface and into the ambient drying environment, and concrete contracts due to this moisture loss. Moisture flow through concrete is extremely slow; a large member can be exposed to a long period of drying and show very little shrinkage deep in its interior. For example, in a 40 inch x 40 inch column after three years of drying, shrinkage at the center of the cross-section will be only 1/10 the shrinkage at the surface [2]. Resistance to moisture flow varies both with concrete materials properties, e.g. diffusivity, and with the state of the material, e.g., temperature and moisture content. These same factors control the speed of flow across the specimen surface; other factors controlling the speed of flow are environmental conditions (especially ambient relative humidity) and the characteristics of the surface itself. Ultimate unrestrained shrinkage strain - the contraction experienced by each element of concrete when the drying process is completed - also varies greatly from concrete to concrete and from one set of environmental conditions to another. This ultimate shrinkage can occur only in the laboratory, since under ordinary climatic conditions periods of evaporation alternate with periods of moisture absorption and the drying process is never completed.

Shrinkage is nonuniform and varies throughout the section of a member. Because physical compatibility must be maintained between adjacent material fibers a complex system of stresses is established. The stress pattern is usually tension near the drying surfaces and an equilibrating compression in the interior. Hence, "apparent" shrinkage, the strain actually existing in the specimen, is different from "free" shrinkage, the contraction an element of material would undergo if it were unrestrained by neighboring elements. This is true even in a member not restrained by foundations, steel reinforcement, or adjacent portions of the structure. Shrinkage measured in experiments is apparent shrinkage. Only in a stress-free situation (e.g. in a very small specimen after a long period of drying) is free shrinkage equivalent to apparent shrinkage. The program documented in this report (SHRINC) calculates free shrinkage distributions. Other programs such as SASHTEC [3] calculate apparent shrinkage. Such programs can also calculate stresses in the prism and identify cracked portions of the cross-section.

1.3 Analysis for Drying Shrinkage

Shrinkage significantly affects concrete structures, and shrinkage movements and stresses must thus be considered during the design process. Attempts to analyze these effects began with rough estimates of ultimate shrinkage and progressed to analyses based on the thermodynamics of moisture diffusion [1]. The method underlying program SHRINC logically extends these earlier methods. Because of the nature of concrete as a design material, extreme accuracy will not be attempted. Rather, a practical, roughly accurate analytical description of overall behavior will be sought. This description will hopefully be suitable for shrinkage prediction in practical design situations.

The basic features of the analytical method in program SHRINC can be summarized as follows:

1. The diffusion equation is used to model the moisture flow phenomenon in two-dimensional cross-sections. The free shrinkage strain is assumed to be directly proportional to moisture loss. Although this model has its roots in the theory of moisture diffusion, it cannot be derived directly from the actual physics of the

situation, but is an empirical model that simply describes observed shrinkage behavior. It has been shown [2] that a more refined model is not necessary for predicting approximate shrinkage movements and stresses in actual concrete members and structures.

2. Concrete shrinkage properties can be described by three material parameters: shrinkage diffusivity, surface factor, and ultimate shrinkage strain. Although in certain cases good results have been obtained using constant or time-dependent diffusivity and surface factor, experimental observations show that it is best to use moisture-dependent concrete diffusivity and surface factor [2] in modeling the effect that member size has on shrinkage development. The third material parameter, ultimate shrinkage strain, is assumed to be roughly constant for each concrete mix and to be unaffected by member size. However, ultimate shrinkage may vary with the relative humidity of the ambient drying environment.

3. The diffusion equation is solved by a finite element method. A generalized and versatile method of this type is necessary since the problem is highly nonlinear due to the moisture dependence of diffusivity and surface factor. A step-by-step time integration scheme is used to account for the transient nature (variation of shrinkage with time) of the diffusion equation. One of the principal advantages of the finite element method is that it permits the analysis of cross-sections of any shape and with any combination of boundary conditions, differing in this respect from other earlier methods [5].

4. The solution consists of a discretized description of free shrinkage development at all points within the cross-section. These free shrinkages are output on punched cards so that they may be used as input for a program which analyzes structural behavior due to shrinkage. The program SASHTEC [3] has been developed to accept this free shrinkage input.

Reference [2] contains a discussion of the historical and theoretical background of the above method. This reference also contains a description of experimental results which verify the method by comparison with SHRINC/SASHTEC computer analyses.

2. SHRINKAGE DIFFUSION BOUNDARY VALUE PROBLEM

2.1 Diffusion Equation

In this section, the diffusion equation, which is an initial-boundary value problem, is defined as it is used in shrinkage analysis. Solution techniques are described in Section 3.

The first condition of this boundary value problem is that the partial differential equation

$$K\left\{\frac{\partial^2 S(x,y,t)}{\partial x^2} + \frac{\partial^2 S(x,y,t)}{\partial y^2}\right\} = \frac{\partial S(x,y,t)}{\partial t} - Q_s(x,y,t) \qquad (2.1)$$

be satisfied at all points (x,y) within the two-dimensional concrete cross-section and at all drying times t. The unknown variable S(x,y,t) is shrinkage strain*, the specified quantity $Q_s(x,y,t)$ is external shrinkage flow into the section, and K is a material parameter. Note that a shrinkage field is the principal variable rather than a moisture-content field, even though the theoretical background of the equations stems from the moisture flow phenomenon. The second condition of the problem formulation is that a boundary condition must be specified at each point on the cross-section surface. Boundary conditions are discussed in Section 2.2. The third condition is that shrinkage is zero at the start of the drying process, i.e.,

$$S(x,y,t) = 0, \text{ when } t = 0 \qquad (2.2)$$

---

* "Shrinkage" or "shrinkage strain" will refer to free shrinkage unless otherwise noted.

The solution of the initial-boundary value problem is the shrinkage history $S(x,y,t)$ that satisifies all three conditions: the field equations (2.1), the boundary conditions, and the initial conditions (2.2).

The shrinkage diffusion equation 3.1 is essentially a balance principle. It states that at every point i within a concrete cross-section

$$\left|\begin{array}{l}\text{Rate at which shrink-}\\\text{age flows into point}\\\text{i from the surround-}\\\text{ing material}\end{array}\right| = \left|\begin{array}{l}\text{Rate at which}\\\text{shrinkage in-}\\\text{creases at}\\\text{point i}\end{array}\right| - \left|\begin{array}{l}\text{Rate at which}\\\text{external shrink-}\\\text{age enters point}\\\text{i}\end{array}\right| .$$

2.2 Boundary Conditions

Four types of boundary conditions are possible:

a. S_{BC} = fixed on boundary

b. Q_{BC} = fixed on boundary

c. $\dfrac{\partial S}{\partial n_{BC}} = \dfrac{f}{K} (S_\infty - S)$ on boundary (2.3)

d. $Q_{BC} = f\ (S_\infty - S)$ on boundary

where f and S_∞ are material parameters (see Sec. 2.3).

The boundary conditions are an integral part of the problem formulation and describe the drying behavior of the concrete surface. Only one boundary condition may be specified at each point on the boundary, but different portions of the boundary may have different types of boundary conditions:

a. The amount of shrinkage is fixed at the boundary (Eq. 2.3a).

b. The shrinkage flow across the boundary is fixed (Eq. 2.3b). For a moisture-insulated surface the boundary condition $Q_{BC} = 0$ is used.

c. The boundary shrinkage gradient diminishes as the shrinkage of the surface approaches its final value (Eq. 2.3c). This seems to model best the actual drying behavior of concrete, as discussed in Section 5.4.

d. The shrinkage flow at the boundary diminishes as shrinkage at the surface approaches its final value (Eq. 2.3d). Note that boundary conditions 2.3c and 2.3d are equivalent only in a steady-state shrinkage distribution -- i.e., when the term $\partial S/\partial t$ is zero in Eq. 2.1. For a moisture-insulated surface $\partial S/\partial n = 0$ may be used.

2.3 Material Properties

The parameters K, f, and S_∞ in the diffusion equation and boundary conditions represent the three material properties discussed earlier (Sec. 1.3). They are empirical quantities which are not derivable from principles of physics; they must be determined from experiments on particular materials. The three parameters are:

1. Shrinkage diffusivity K, in units of in^2/day, models the ease with which moisture flows through concrete. Shrinkage diffusivity can be defined as the speed at which shrinkage flows through the material when a unit shrinkage gradient exists. Diffusivity can be constant, time-dependent, or shrinkage-dependent. A shrinkage-dependent model (see Sec. 5.3) seems best to describe an experimentally observed decline in diffusivity as drying proceeds.

2. Surface factor f, in units of inches/day/linear inch, represents the speed of surface evaporation by its control over the shrinkage

gradient or shrinkage flow at drying surfaces (boundary conditions 2.3c-d). Hence, strictly speaking, f can be a surface property as well as a material property. This material parameter can have its own functional form and be independent of the other material parameters. Surface factor can also be time- or shrinkage-dependent.

3. Ultimate shrinkage strain, S_∞, in units of inches/inch, defines the amount of shrinkage an element of concrete can undergo if it is allowed to dry completely and if there is no constraint on its movement by adjacent elements.

3. SOLUTION BY FINITE ELEMENTS AND TIME STEP INTEGRATION

3.1 Discretization by Finite Element Method

The initial-boundary value problem describing shrinkage behavior has no closed-form solution; approximate numerical methods must be used in order to obtain a solution. In program SHRINC, a finite element method discretizes spatial variables and a step-by-step integration technique discretizes the time variable.

The finite element method uses a variational calculus formulation for the boundary value problem. That is, the equations describing the shrinkage process (Eqs. 2.1-2.3) are recast into an equivalent energy functional and the shrinkage field S(x,y,t) that minimizes this functional is sought. The finite element approach represents the continuous shrinkage field S(x,y) within a cross-section by a finite number of nodal shrinkage values. The summation of the shrinkage field due to each nodal shrinkage S_i approximates S(x,y). That is:

$$S(x,y) \doteq \sum_{i=1}^{n} S_i\phi_i(x,y) \qquad (3.1)$$

where

S_i - shrinkage at node i

n - total number of nodes

$\phi_i(x,y)$ - shrinkage field caused by unit shrinkage at node i and zero shrinkage at all other nodes

A typical $\phi_i(x,y)$ is shown in Fig. 3.1. Note that the cross-section in this figure is broken down into 8 nodes and 6 elements. The way in which the cross-section is divided into elements determines the shape of each $\phi_i(x,y)$. The function $\phi_i(x,y)$ shown, along with its 7 companions, makes it possible to define an approximate shrinkage distribution with 8 discrete values of S_i.

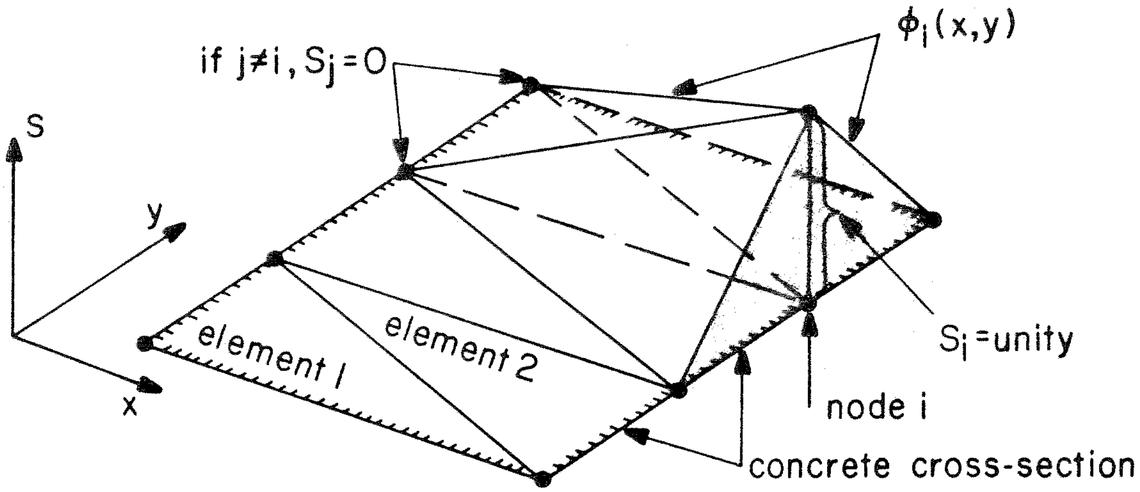


FIGURE 3.1 DISCRETIZATION OF SHRINKAGE FIELD BY FINITE ELEMENTS

By substituting Eq. 3.1 into the energy functional, the problem can be solved by finding the minimizing set of nodal shrinkages, S_i, instead of the minimizing function, $S(x,y)$. This minimum is found by differentiating the functional with respect to each S_i, resulting in the set of equations:

$$[K]\{S\} = [V]\{\dot{S}\} - \{Q\} \qquad (3.2)$$

where

$[K]$ - shrinkage diffusivity matrix

$[V]$ - shrinkage velocity matrix

$\{Q\}$ - external shrinkage flow loading vector

$\{S\}$ - nodal shrinkages

$\{\dot{S}\}$ - nodal shrinkage velocities

In actual applications of the finite element method, the matrices $[K]$, $[V]$, and $\{Q\}$ are assembled directly by calculating each element's contribution to the system matrices. That is, for a specified type of $\phi_i(x,y)$, a specified element size and shape, and specified material properties, an element's contribution to the diffusivity matrix $[K]$ can be explicitly defined. For example, a triangular element with linear $\phi_i(x,y)$ has an element diffusivity, $[K]_{el}$, given by

$$[K]_{el} = \frac{K}{2\lambda} \begin{bmatrix} e^2 + d^2 & y_k e - x_k d & -y_j e + x_j d \\ & y_k^2 + x_k^2 & -y_j y_k - x_j x_k \\ \text{(symmetric)} & & y_j^2 + x_j^2 \end{bmatrix} \qquad (3.3)$$

where K is the shrinkage diffusivity of the material of the element and x_k, x_j, y_k, y_j, e, d, and λ are defined in Fig. 3.2.

Program SHRINC uses quadrilateral elements constructed from four of the above triangles by adding a fifth node. The coordinates of this node are specified as the average of the original four nodes, placing this fifth node at the centroid of the quadrilateral element as in Fig. 3.3. The diffusivities from the four linear triangles (Eq. 3.3) are assembled into an element matrix $[K]_{el}$ (or in expanded notation, $(K_{el})_{i,j}$, i=1, 5, j=1, 5). This 5 x 5 element diffusivity matrix is reduced to a 4 x 4 matrix by static condensation, i.e.,

$$(K_{el})_{i,j} = (K_{el})_{i,j} - \frac{(K_{el})_{i,5} \cdot (K_{el})_{5,j}}{(K_{el})_{5,5}} \qquad (3.4)$$

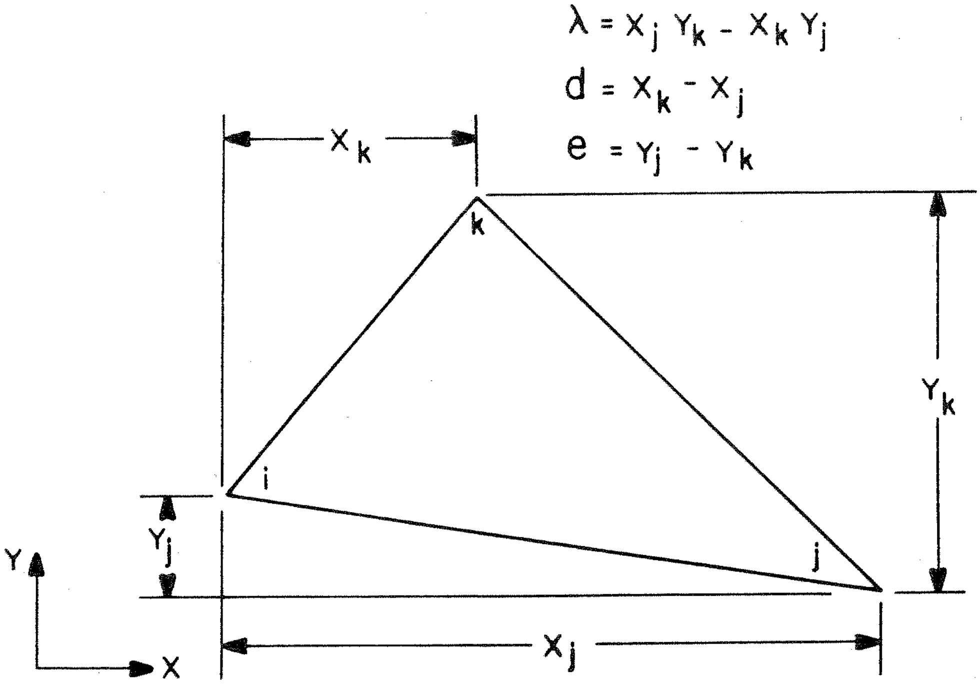
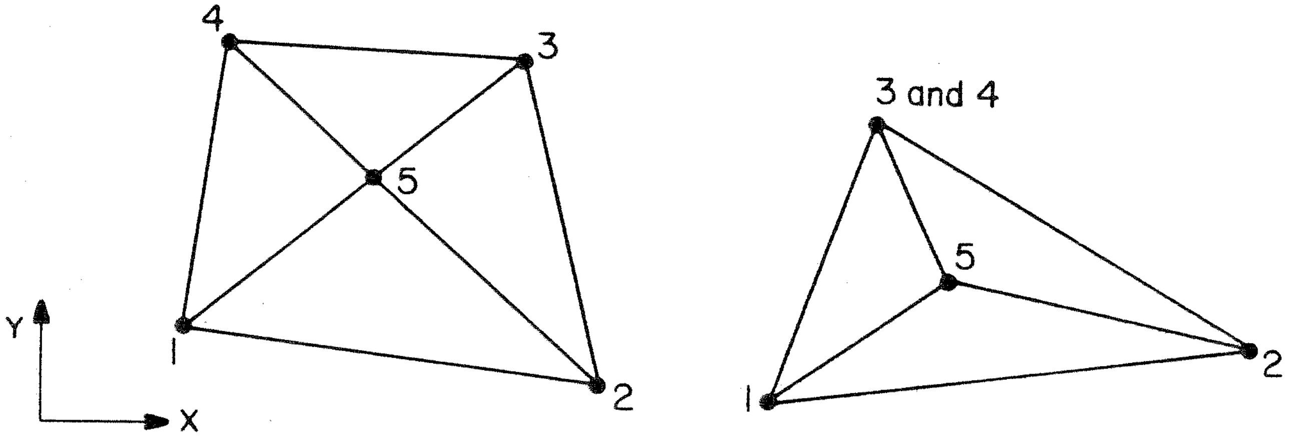


FIGURE 3.2 LINEAR TRIANGULAR ELEMENT

$$X_5 = \frac{X_1 + X_2 + X_3 + X_4}{4} \qquad Y_5 = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$$



a) QUADRILATERAL

b) TRIANGLE
(DEGENERATE QUADRILATERAL)

FIGURE 3.3 QUADRILATERAL ASSEMBLY OF LINEAR TRIANGLES

where i,j=1,4. These element diffusivity matrices are then assembled into the diffusivity matrix for the system:

$$[K] = \sum(K_{el}) \tag{3.5}$$

The velocity matrix [V] is also calculated directly. A matrix consistent with the $\phi_i(x,y)$ used to derive the diffusivity matrix [K] could be used, but it is simpler to idealize the velocity matrix in a way analogous to the lumping of mass in dynamic analyses. This approach has the important advantage of resulting in a diagonal velocity matrix. This lumping is achieved by delineating the area adjacent to a node by a perimeter drawn through the midpoints of element boundaries and the internal nodes previously associated with the diffusivity matrix (see Fig. 3.4). The area tributary to a node is found by adding the contribution of each element bounding the node. The contribution of an element, m, to a particular node, i, (see Fig. 3.4) is given as

$$V_{m,i} = \frac{A_j + A_k}{2}$$

where

A_j, A_k - areas of triangles in element adjacent to node i,

so that the velocity matrix term of node i is

$$V_i = \sum^{m'} V_{m,i}$$

where

m' - all elements adjacent to node i

The diagonal system matrix [V] is assembled by repeating the above calculation for each node.

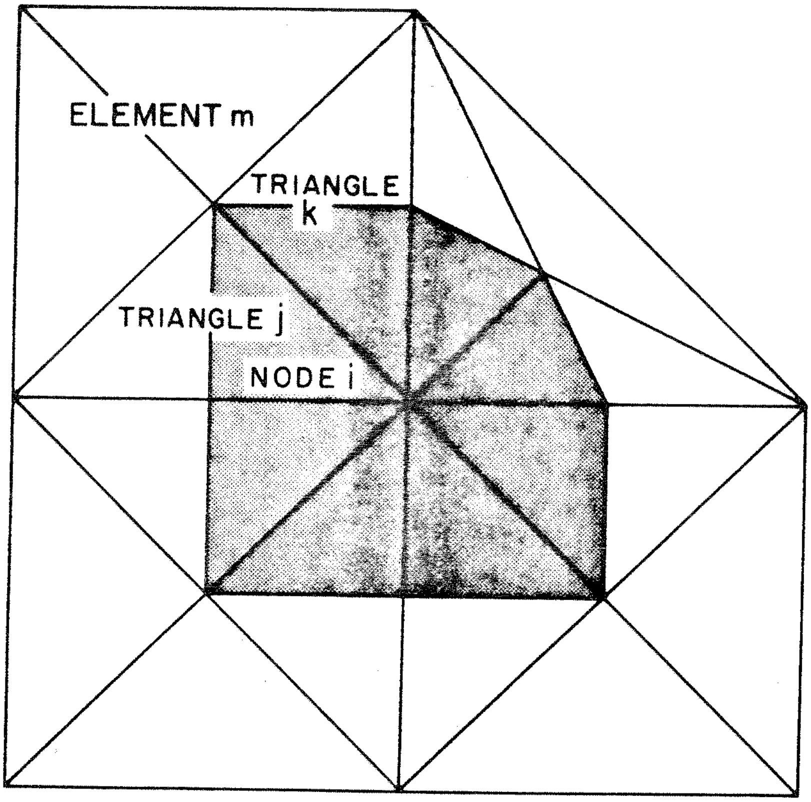


FIGURE 3.4 LUMPED VELOCITY MATRIX IDEALIZATION

The third matrix in the finite element equations 3.2, the load vector {Q}, represents the external shrinkage flow into each node of the system. This term, of course, will be zero at all interior nodes. However, surface nodes can have a nonzero flow term due to boundary conditions.

The unknown nodal shrinkage vector, {S}, in the matrix equations (Eq. 3.2) is a function of time, {S(t)}. Hence, this equation is actually a first order differential equation (in n dimensions) and must be further discretized by step-by-step integration. The nodal shrinkage history {S(t)} is represented by a finite sequence of shrinkage distributions $\{S(t_o)\}$, $\{S(t_1)\}$, $\{S(t_2)\}$, ... By assuming that velocity is constant during a time step, the shrinkage velocity $\{\dot{S}\}$ at any time t_i can be approximated in terms of nodal shrinkages:

$$\{\dot{S}(t_i)\} = \{S(t_i) - S(t_{i-1}))/\Delta t_i\} \tag{3.6}$$

where Δt_i is the time step between time t_{i-1} and t_i. The differential equation (3.2) is now reduced to a set of algebraic equations in the independent variable $\{S(t_i)\}$. There will be a set of equations for each time t_i, i=1,2,..., and the step-by-step solution of each set results in an approximation to $\{S(t)\}$ and, using Equation 3.1, an approximation to the overall shrinkage history $S(x,y,t)$. The accuracy of this finite element step-by-step integration approximation increases with the number of elements and with the smallness of the time step.

3.2 Solution of Matrix Equations

Shrinkage history within a cross-section is found by solving the matrix equations (Eq. 3.2) resulting from finite element discretization of the problem. To facilitate discussion of the solution procedure and the incorporation of boundary conditions into the equations, let Eq. 3.2 be expanded into the following form:

$$\begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{Bmatrix} S \\ S_f \end{Bmatrix} + \begin{bmatrix} V_{aa} & o \\ o & V_{bb} \end{bmatrix} \begin{Bmatrix} \dot{S} \\ o \end{Bmatrix} = \begin{Bmatrix} Q + S_\infty f \\ Q_f \end{Bmatrix} - \begin{Bmatrix} [f]S \\ o \end{Bmatrix} \tag{3.7}$$

where

S_f - nodal shrinkage of nodes which have fixed shrinkage boundary conditions (Eq. 2.3a)

S - nodal shrinkage at all other nodes

Q_f - external flow loading at all nodes with fixed shrinkage boundary conditions

Q - external flow loading at all nodes with shrinkage unfixed

$S_\infty f$ - surface factor x ultimate free shrinkage at all nodes where a shrinkage-dependent flow boundary condition is specified

$[f]S$ - surface factor x nodal shrinkage at all nodes when a shrinkage-dependent flow boundary condition is specified

Note that the additional terms $S_\infty f$ and $[f]S$ in the loading vector are due to the shrinkage-dependent boundary condition (2.3d)

$$Q_{BC} = f(S_\infty - S)$$

Fixed shrinkage boundary conditions (Eq. 2.3a) are specified by the vector S_f, and fixed shrinkage flow boundary conditions (Eq. 2.3b) are specified as part of the vector Q. Note that the vector Q is zero at all nodes except where a fixed shrinkage flow boundary condition is specified. The remaining type of boundary condition, a specified shrinkage gradient (Eq. 2.3c), presents special problems and will be discussed in the next section.

The algebraic manipulation of Eq. 3.7 necessary to find the solution $\{S(t)\}$ is quite straightforward. The quantities $[K]$ and f are evaluated using material properties at $\{S(t_o)\} = \{S\}_o$, where

$\{S\}_o$ is the shrinkage distribution specified as the initial conditions. Then static condensation removes fixed shrinkage nodes from the equations, resulting in

$$[K_{aa} + f]\{S\} + [V_{aa}]\{\dot{S}\} = \{Q\} + \{S_\infty f\} - [K_{ab}]\{S_f\} \qquad (3.8)$$

For the first time step, $\Delta t_1 = t_1 - t_o$, velocity is assumed to be constant; hence

$$\{\dot{S}\}_1 = \frac{\{S\}_1 - \{S\}_o}{\Delta t_1} \qquad (3.9)$$

and the equations valid at the end of the first time step are

$$[K_{aa} + f + \frac{V_{aa}}{\Delta t_1}]\{S\}_1 = \{Q + S_\infty f - [K_{ab}]\{S_f\} + \frac{[V_{aa}]}{\Delta t_1}\{S\}_o\} \qquad (3.10)$$

or in compact form

$$[K]^*\{S\}_1 = \{Q\}^* \qquad (3.11)$$

where

$[K]^*$ - effective shrinkage diffusivity matrix
$\{Q\}^*$ - effective shrinkage loading vector

The nodal shrinkages at t_1, $\{S\}_1$, are found directly by solving Eq. 3.11. If material properties K or f change appreciably due to the shrinkage increase during the time step, the matrices $[K]^*$ and $\{Q\}^*$ can be updated and the equation resolved to find a more accurate $\{S\}_1$. Experience has shown that convergence is very rapid.

When the solution $\{S\}_1$ at the end of the first time step is known, the equations necessary to determine the shrinkage at the end of the second time step, $\{S\}_2$, can be formed. The equations valid at the end of any time step, i, follow the same pattern as those for the first time step, i.e.,

$$[K_{aa} + f + \frac{V_{aa}}{\Delta t_i}]\{S\}_i = \{Q + S_\infty f - [K_{ab}]\{S_f\} + \frac{[V_{aa}]}{\Delta t_i}\{S\}_{i-1}\} \qquad (3.12)$$

or, more compactly,

$$[K]_i^*\{S\}_i = \{Q\}_i^* \qquad (3.13)$$

The quantities K_{aa}, f, and K_{ab} must be reevaluated for each time step if they are time- or shrinkage-dependent. The step-by-step assembly and solution of Eq. 3.13 gradually traces out the shrinkage history.

3.3 Shrinkage Gradient Boundary Conditions

A boundary condition consisting of the prescribed shrinkage gradient

$$\frac{\partial S}{\partial n_{BC}} = \frac{f}{K}(S_\infty - S) \qquad (3.14)$$

is somewhat unnatural to a finite element formulation and presents special numerical difficulties. It could be considered by supplementing the governing equation (Eq. 3.7) with equations that impose constraints on the boundary shrinkage gradient at each node, but this would destroy the symmetry and bandedness of the set of equations and drastically increase computational effort. Using another technique, the specified boundary condition gradient is converted into an "equivalent" shrinkage flow

$$\tilde{Q}_{BC} = K \frac{\partial S}{\partial n_{BC}} = f(S_\infty - S) \qquad (3.15)$$

and this flow is applied at the appropriate boundary nodes in Eq. 3.7. However, as discussed in Sec. 2.3, this flow $\tilde{Q}_{BC}$ is the true shrinkage flow only under steady-state conditions; hence the shrinkage solution obtained using $\tilde{Q}_{BC}$ as loading will not always satisfy the specified boundary condition gradients $\partial S/\partial n_{BC}$. Therefore, the algorithm must examine the error in the gradient, add a proportional correction to $\tilde{Q}_{BC}$ for each node, and resolve for an improved shrinkage distribution. This iterative procedure is repeated until the shrinkage gradient along the boundary satisfies the specified boundary condition. On the last iteration, the corrected equivalent flow $\tilde{Q}_{BC}$ will represent the actual shrinkage flow through the boundary node. The above iterative process requires that the portion of the cross-section surface subject to gradient boundary conditions must be discretized into surface segments. Each pair of boundary nodes is connected with a surface segment as shown in Fig. 3.5. The boundary flow across this segment, specified as inches/day/linear inch of surface, is calculated and lumped at these two adjacent nodes.

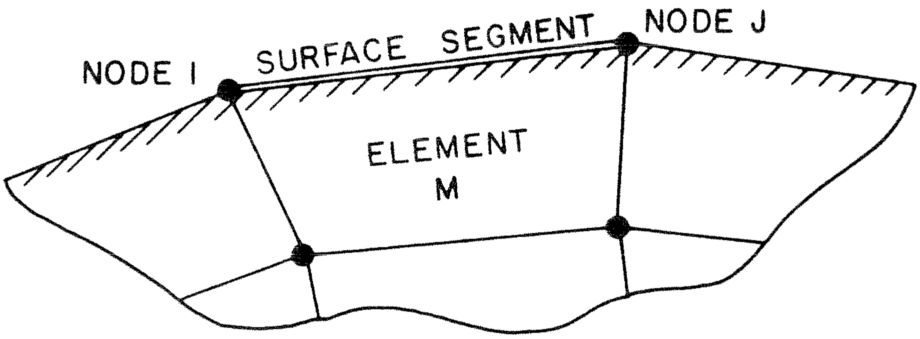


FIGURE 3.5 TYPICAL SURFACE SEGMENT

That is, if

$$Q_{seg} = L_{seg} \cdot f \cdot (S_{\infty} - S_{avg}) \qquad (3.16)$$

where

Q_{seg} - total flow across surface segment

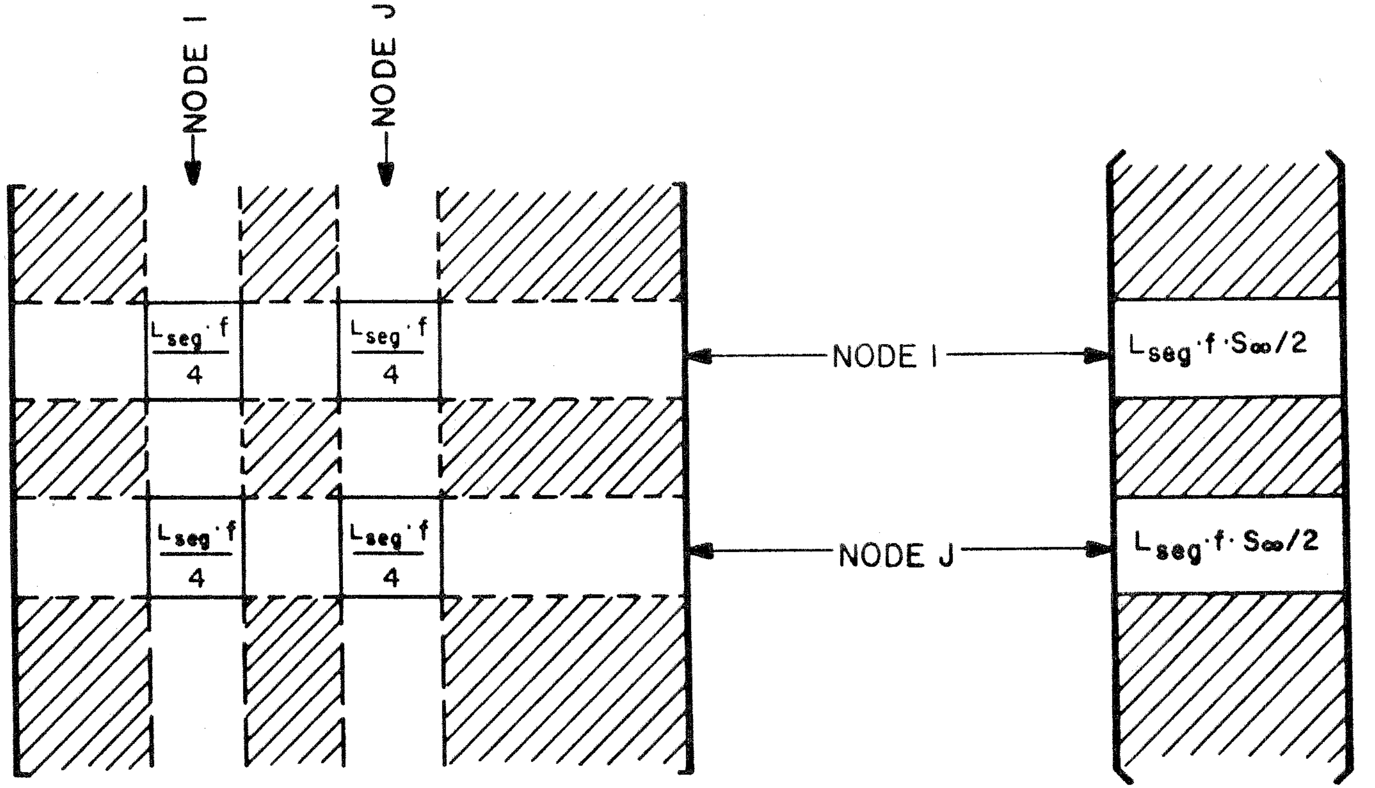
L_{seg} - length of surface segment

f - surface factor of material in element M adjoining surface segment

S_{∞} - ultimate shrinkage of material in element M adjoining surface segment

S_{avg} - average shrinkage strain in surface segment

then the concentrated flow into each boundary node I and J is $Q_{seg}/2$. Since this flow is shrinkage-dependent, it affects both the diffusivity matrix [K] and the loading vector {Q}, as seen in Eq. 3.10. The contribution of a surface segment to these system matrices is shown in Fig. 3.6.

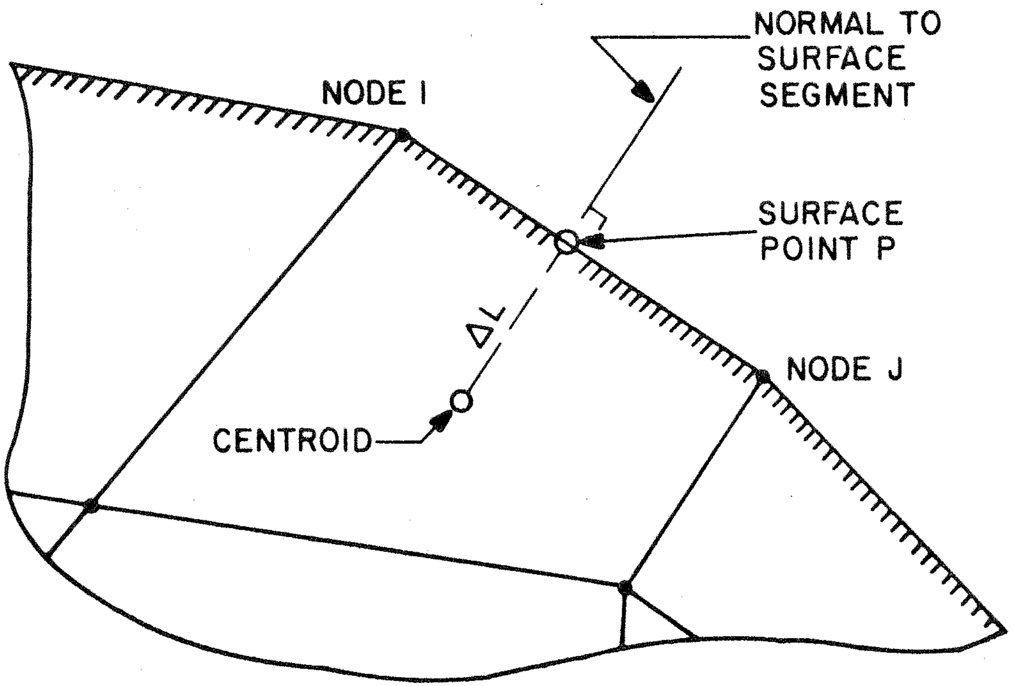


a) DIFFUSIVITY MATRIX $[K]$

b) LOAD VECTOR $\{Q\}$

FIGURE 3.6 EFFECT OF SURFACE SEGMENT BOUNDARY CONDITION FLOW ON SYSTEM MATRICES

The computed shrinkage gradient and specified boundary condition gradient in each surface segment are compared during each iteration. Shrinkage gradient is calculated along the normal line to the centroid of the finite element adjacent to the surface segment as shown in Fig. 3.7. The shrinkage-dependent boundary condition 3.14 is evaluated using the shrinkage value of this same point.



$$\left.\frac{\partial s}{\partial n}\right|_P = \frac{S_{cent} - S_p}{\Delta L}$$

a) SHRINKAGE SLOPE
AT POINT P

$$\left.\frac{\partial s}{\partial n}\right|_{BC} = \frac{f}{K}\left(S_\infty - S_p\right)$$

b) B.C. GRADIENT
AT POINT P

FIGURE 3.7 DISCRETIZATION SCHEME FOR SURFACE GRADIENT

4. STRUCTURE OF COMPUTER PROGRAM SHRINC

4.1 Input Phase

The numerical techniques developed in Chapter 3 are implemented by the computer program SHRINC. The program is written in Fortran IV and was developed on the CDC 6400 computer at the University of California, Berkeley. The program is as general as possible, having options for defining both time-dependent and shrinkage-dependent material properties and for specifying all four types of boundary conditions (Eq. 2.3). Fully annotated Input Instructions are provided in Appendix A.

A flow chart for the program as a whole is shown in Fig. 4.1. The first parts of the program input the data defining the problem. The data is divided into blocks, each block headed by an alphanumeric control card with a key word (to assist the user in assembling the data deck) and necessary control parameters (to guide the program in inputting the remaining data in the block) as shown in Fig. A.1 in Appendix A.

The first item of input is the problem heading to be used in labelling output. The next item is a geometric description of the cross-section to be analyzed, i.e., the locations of nodal points and elements. A list of all surface nodes at which fixed shrinkage boundary conditions are specified is also input during this phase. The program has a mesh generating option (see Appendix A for details) to aid the user in the input of large meshes. The only elements currently included in SHRINC are four-node quadrilaterals and 4-node

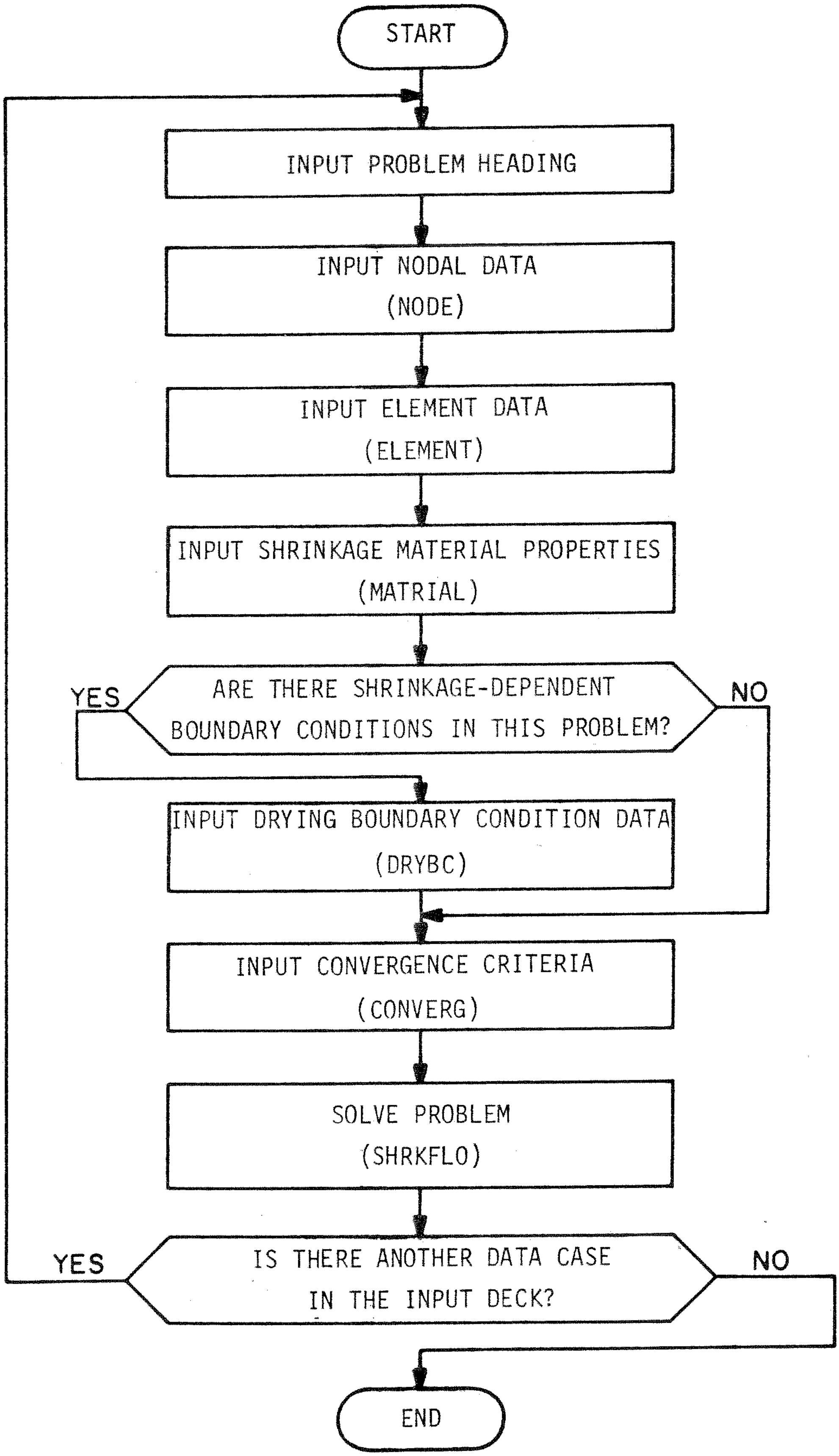


FIGURE 4.1 FLOW CHART FOR PROGRAM SHRINC
(TITLES IN PARENTHESES ARE NAMES OF PROGRAM SUBROUTINES)

triangles (i.e., quadrilaterals in which two nodes have the same location). However, isoparametric or other higher order finite elements could be used in the program if appropriate subroutines were added. During input of the mesh, not only are data read from cards and stored in various arrays for use during the solution phase, but quantities which will recur very often during solution are calculated and stored, e.g., the constant terms of the element diffusivity matrices, SS1-SS9.

The next block of input contains material properties - i.e., the shrinkage diffusivity K, surface factor f, and ultimate shrinkage strain S_∞ of each material in the cross-section. NMAT material types may be input, making it possible to analyze nonhomogeneous sections (e.g. reinforced concrete). These material properties can be time-dependent functions (MATMOD = 1) or shrinkage-dependent functions (MATMOD = 2), and are input as a table of ordered pairs (time-value or shrinkage-value). It is also possible to specify Pickett's formula for shrinkage material properties (Eq. 5.1) by means of an option discussed in Appendix A.

Shrinkage-dependent drying boundary conditions are next input - both flow type

$$Q_{BC} = f(S_\infty - S)$$

and gradient type

$$\frac{\partial S}{\partial n_{BC}} = \frac{f}{K}(S_\infty - S)$$

These boundary conditions are specified by defining NDRYBC surface segments connecting the nodes along those portions of the cross-sectional surface that are exposed to this type of boundary condition. Each surface segment is assigned either a flow (KODEBC = 0) or a gradient type (KODEBC = 1) boundary condition. Note that every surface node must have a boundary condition specified at some point in the input data. Three types are possible:

1. Drying boundary condition, input as described above.
2. Fixed shrinkage boundary condition, input during step-by-step solution phase. The node numbers at which this type of boundary condition exists must be specified along with the nodal point data, as discussed earlier.
3. Fixed flow boundary condition, input during step-by-step solution phase.

Refer to Section 5.4 for a discussion of selecting the types of boundary conditions most useful in practical situations.

The nonlinearities introduced by shrinkage-dependent diffusivity and surface factor or by shrinkage gradient boundary conditions require the input of convergence criteria. Three variables control the iterative solution: the maximum number of permissible iterations (NCONV), the maximum permissible relative error (CONV), and a flag that determines which of the two types of nonlinearities is to be considered within any time step (NREFORM). These control variables are precisely defined in Appendix A.

At this point the program switches to the main step-by-step solution subroutine DHRKFLO, as indicated in Fig. 4.1. However, before proceeding to the actual solution phase, the initial conditions are input. These usually are specified as $\{S\}_0 = \{0\}$ at time zero, but other initial conditions can be specified. For example, when restarting an analysis that must be continued for a longer period of drying, the shrinkage distribution and drying time at which the previous analysis left off can be specified as the initial conditions of the new analysis.

During the input phase all global variables are assigned storage through use of a dynamic dimensioning scheme. That is, the blank common block of storage is partitioned to make room for each array using the key variables NUMNP (number of nodal points), NUMEL (number of elements), NMAT (number of material types), NSTORE (space needed for material data), NDRYBC (number of boundary segments), and NBAND (half-bandwidth of global diffusivity matrix). The space allocation scheme is shown in Fig. 4.2. Dynamic dimensioning permits optimal use of high speed storage; even very large problems can be solved without recourse to out-of-core storage files.

4.2 Step-by-Step Solution Phase

When the input of the data defining the problem is completed, the program begins step-by-step time integration. This phase consists of assembling and solving Eqs. 3.13 at the end of each time step. The overall solution process is controlled by subroutine SHRKFLO, which is illustrated in Fig. 4.3 in the form of a flow chart.

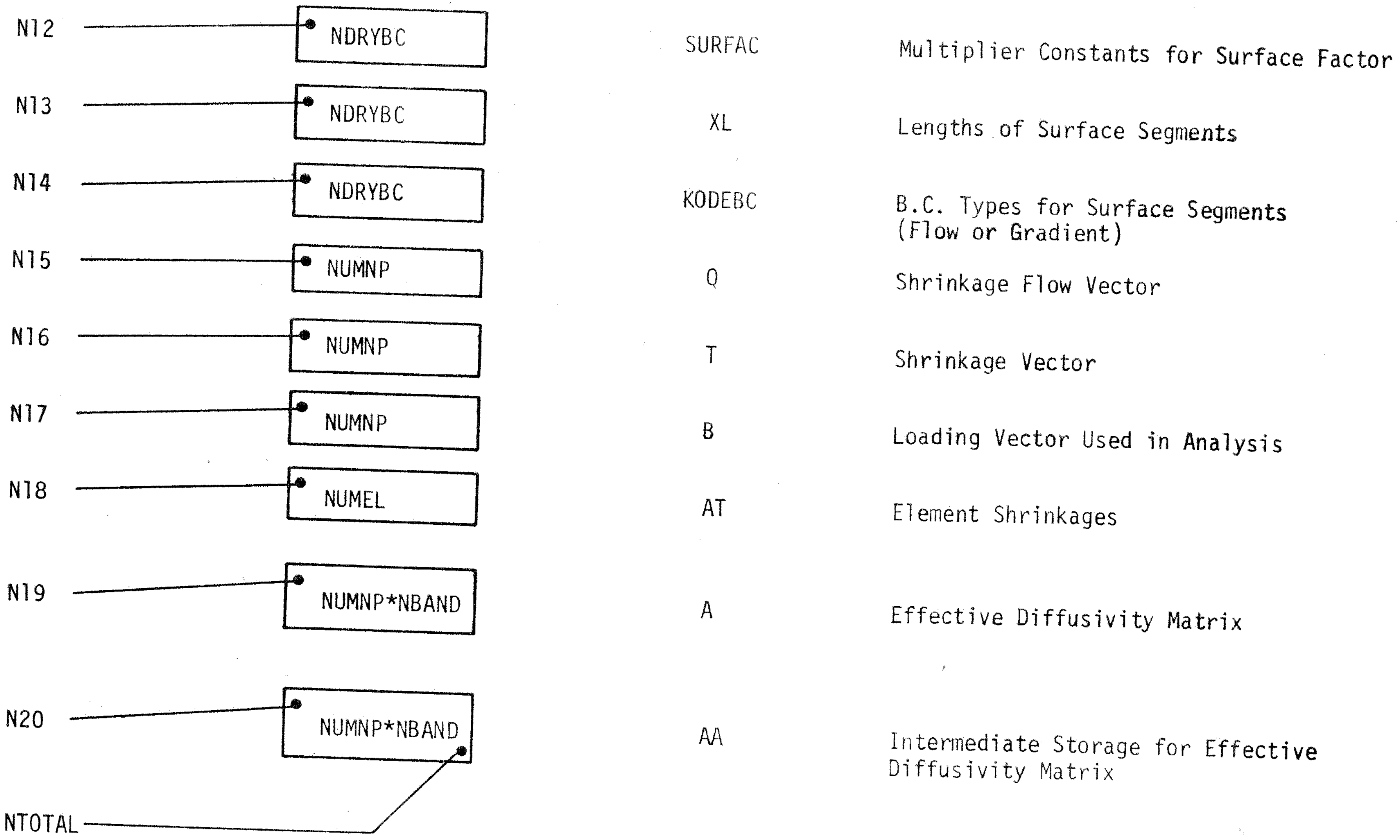
The first step of the procedure for each time step is to input a control card for that particular step. This control card specifies

ADDRESS	DIMENSION	ARRAY	CONTENTS
NS1	4 * NUMEL	SS1	Constant Terms of Element Diffusivity Matrices
NS2	4 * NUMEL	SS2	
NS3	4 * NUMEL	SS3	
NS5	4 * NUMEL	SS5	
NS6	4 * NUMEL	SS6	
NS9	4 * NUMEL	SS9	
N7	6 * NMAT	MATL	Control Data for Material Shrinkage Properties
N8	NSTORE	XYS	Storage for Material Shrinkage Properties
N9	NDRYBC	LI	Nodes Bounding Surface Segments
N10	NDRYBC	LJ	
N11	NDRYBC	LMAT	Elements Bounding Surface Segments

FIGURE 4.2 DYNAMICALLY DIMENSIONED STORAGE ALLOCATION FOR PROGRAM SHRINC

ADDRESS	DIMENSION	ARRAY	CONTENTS
N1=1	NUMNP	X	X-Coordinates of Nodal Points
N2	NUMNP	Y	Y-Coordinates of Nodal Points
N3	NUMNP	KODE	Indicator for Type of B.C. (Flow or (Shrinkage)
ND1	NUMNP	ID or D	Intermediate Storage Needed in Calculations
ND2	NUMNP	MA	
ND3	NUMNP	T1	
ND4	NUMNP	T2	
ND5	NUMNP	T3	
N4	NUMEL	LM	Nodal Point Configurations of Elements
N5	NUMEL	MMTYPE	Material Types of Elements
N6	4 * NUMEL	XLAM	Constant Multiplier for Element Diffusivity Matrices

FIGURE 4.2 (cont.)



N12	NDRYBC	SURFAC	Multiplier Constants for Surface Factor
N13	NDRYBC	XL	Lengths of Surface Segments
N14	NDRYBC	KODEBC	B.C. Types for Surface Segments (Flow or Gradient)
N15	NUMNP	Q	Shrinkage Flow Vector
N16	NUMNP	T	Shrinkage Vector
N17	NUMNP	B	Loading Vector Used in Analysis
N18	NUMEL	AT	Element Shrinkages
N19	NUMNP*NBAND	A	Effective Diffusivity Matrix
N20	NUMNP*NBAND	AA	Intermediate Storage for Effective Diffusivity Matrix
NTOTAL			

FIGURE 4.2 (cont.)

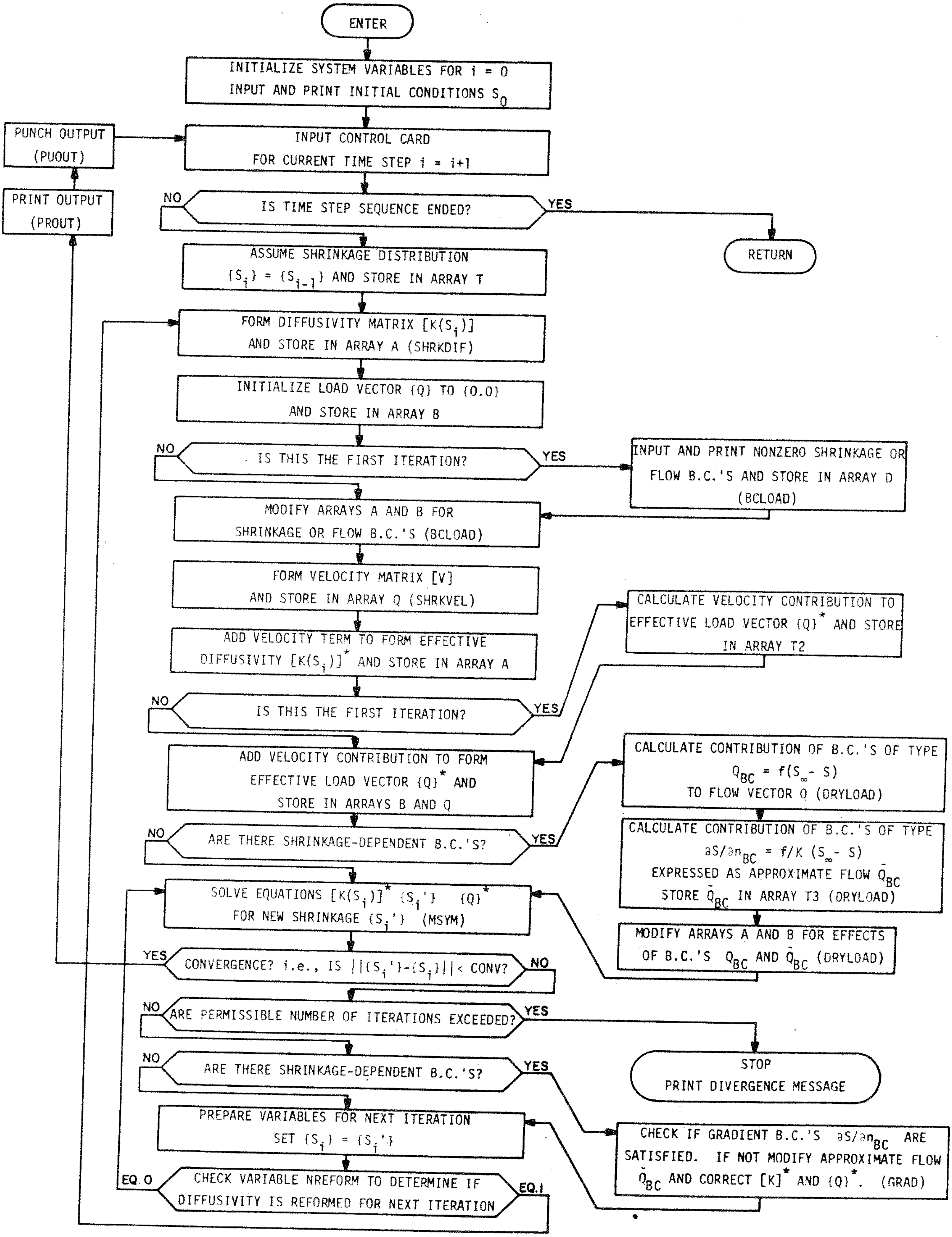


FIGURE 4.3 FLOW CHART FOR SUBROUTINE SHRKFLO
(TITLES IN PARENTHESES ARE NAMES OF PROGRAM SUBROUTINES)

the time step number (as a check in sequencing data cards), time step size, printing and punching options (see Section 4.3), and the number of nodes at which a nonzero shrinkage or flow boundary condition is to be specified. The values of these nonzero boundary conditions immediately follow on data cards. Note that this input formatting permits the user to change boundary conditions as the step-by-step time integration progresses. Also note that only fixed boundary conditions are input at this point; shrinkage-dependent drying boundary conditions are input during the earlier input phase (Section 4.1).

When all data have been input for the time step, the two iteration counters are initialized, the counter MAIN (to record the number of times the diffusivity matrix has been formed or reformed) and the counter NCON (to record the total number of solution iterations). The array T is also initialized to contain $\{S_i\}$, the shrinkage distribution existing in the cross-section at the beginning of the time step.

At this point the program begins to assemble the equations that govern shrinkage dynamic equilibrium at the end of the time step. The diffusivity matrix [K] is formed in subroutine SHRKDIF, as shown in Fig. 4.4. Each 4-node quadrilateral element is divided into four triangles by means of an additional fifth node at its center. A 3 x 3 element diffusivity matrix is calculated for each triangle (Eq. 3.3) and assembled onto the 5 x 5 five-node element diffusivity matrix. This 5 x 5 matrix is reduced to a 4 x 4 quadrilateral

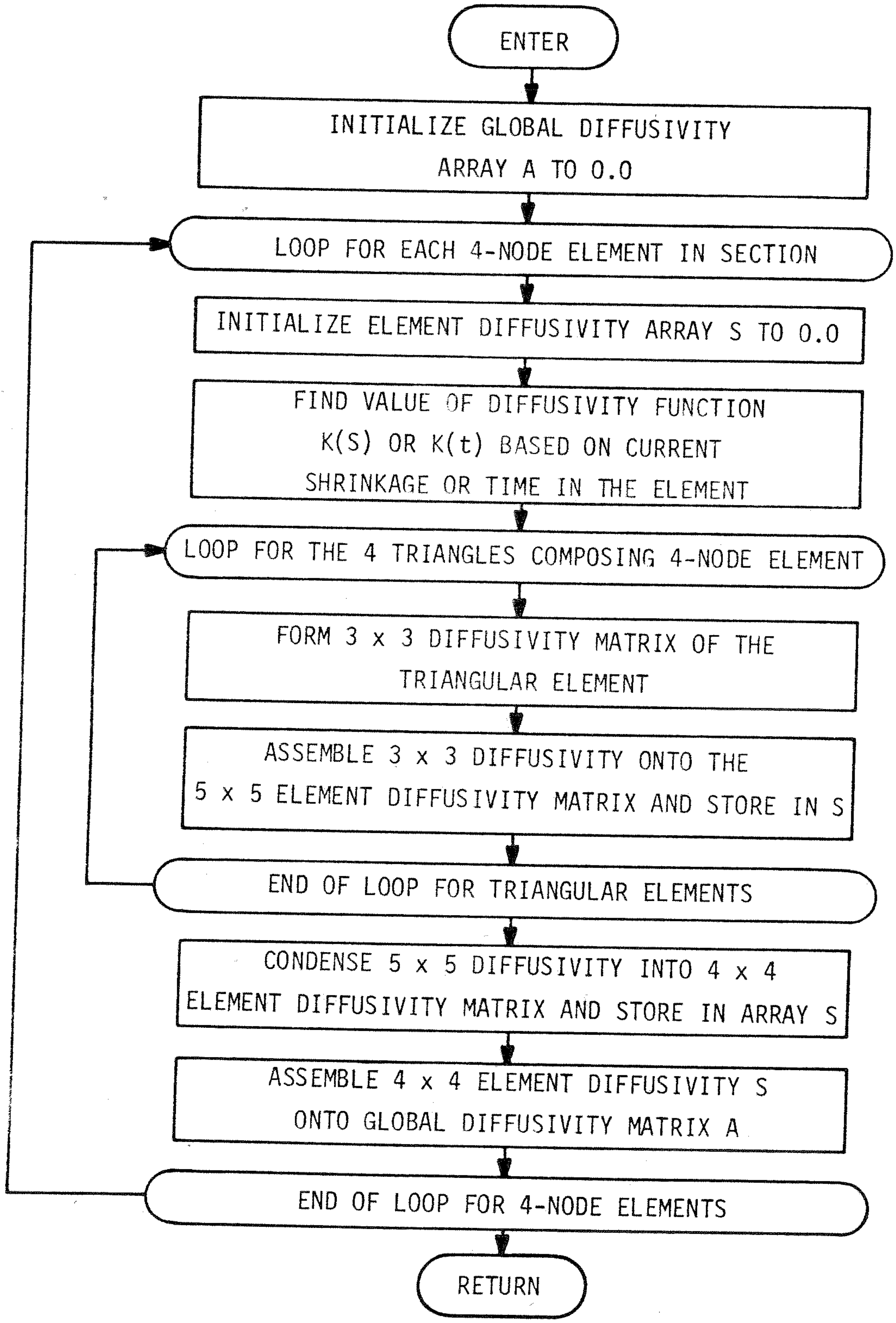


FIGURE 4.4 FLOW CHART FOR SUBROUTINE SHRKDIF

element diffusivity, $[K]_{el}$, by removing the center node by static condensation. All element diffusivities are combined to form an overall section diffusivity matrix, [K], which is stored in banded form in the dynamically dimensioned array A (Fig. 4.2). Note that since the material property K (shrinkage diffusivity) can be time- or shrinkage-dependent, the matrix [K] can also reflect these dependencies. If the matrix is time-dependent, the time at the end of the time step is used when forming it. If the matrix is shrinkage-dependent, the average shrinkage strain existing in each element at the beginning of the time step (array T) is used for the initial formation. Any subsequent reformations of [K] during the iterative solution will use the shrinkage that exists at the start of the current cycle of iteration.

The next quantity to be formed is the load vector Q (in Eq. 3.7), which contains the fixed flow boundary conditions input at the beginning of the time step. Fixed shrinkage boundary conditions are then input to form the vector S_f in Eq. 3.7, and static condensation is performed to remove these fixed shrinkage nodes from the system of equations (Eq. 3.8). The above operations are performed in subroutine BCLOAD, and the condensed load vector, $\{Q\}-[K_{ab}]\ \{S_f\}$, is stored in array B.

The program now assembles the two quantities necessary to convert the static diffusivity and load derived above into the <u>effective</u> diffusivity and load needed in the final governing equations 3.11. Firstly, the velocity matrix V_{aa} is formed in subroutine

SHRKVEL and, since it is a diagonal matrix, temporarily stored in array Q. From this quantity the velocity contribution to the effective diffusivity matrix ($V_{aa}/\Delta t_i$ in Eq. 3.12) is calculated, and the resulting effective diffusivity $[K]^*$ is stored in array A. Also, based on the velocity matrix and the shrinkage solution of the previous time step $\{S_{i-1}\}$ (which is stored in array T at the time of the first iteration), the dynamic contribution to the effective load vector $\{Q\}^*$ is calculated. This term ($[V_{aa}]\{S_{i-1}\}/\Delta t_i$ in Eq. 3.12) is stored in array T2 so that it does not have to be recalculated during subsequent iterations.

Secondly, the effects of shrinkage-dependent drying boundary conditions are added to the effective diffusivity and load, i.e., the terms f (for effective diffusivity) and $S_\infty f$ (for effective load) in Eq. 3.12. The time- or shrinkage-dependent material properties that go into forming these quantities are evaluated as described earlier. That is, they are updated for each iteration if NREFORM is set to "1". Two types of drying boundary conditions exist: flow and gradient. For the flow case, the program simply calculates $Q_{seg} = L_{seg} \cdot f \cdot (S_\infty - S_{avg})$ along each surface segment (as in Eq. 3.16) and contributes $Q_{seg}/2$ to each boundary node. That is, the nodal term $L_{seg} \cdot f \cdot S_\infty/2$ is added to $\{Q\}^*$ and the term $L_{seg} \cdot f/2$ to $[K]^*$. The gradient case is more complex, as discussed in Section 3.3. The term $\tilde{Q}_{BC}$, an approximate boundary flow (Eq. 3.15), must be calculated and added to the effective diffusivity and load as in the flow case. However, the term $\tilde{Q}_{BC}$ is saved in array T3 for use later in the iterative technique needed to satisfy this type of boundary condition. The above operations are carried out in subroutine DRYLOAD, as shown in Fig. 4.3.

At this point the final equations

$$[K]^* \{S_i'\} = \{Q\}^* \tag{4.1}$$

have been formed. The effective load $\{Q\}^*$ is stored in array B and the effective diffusivity $[K]^*$ is stored in array A. Before solution, $[K]^*$ is also transferred to the auxiliary storage array AA, since array A will be modified during triangularization and the array must be preserved intact for use in future iterations. The equations are solved by subroutine MSYM, an efficient solution algorithm for symmetric banded matrices, and the solution $\{S_i'\}$ replaces the loading vector in array B.

A convergence check must be made to determine if the program is to proceed with another iteration. First the difference between the current iterate $\{S_i'\}$ (in array B) and the previous iterate $\{S_i\}$ (in array T) is examined to determine if it is less than the error tolerance CONV. If it is less, iteration ceases and output subroutines (see Section 4.3) are called, after which the program proceeds with the next time step. If the difference between $\{S_i'\}$ and $\{S_i\}$ is greater than the tolerance, a check is made to determine if the permissible number of iterations NCONV has been exceeded. If it has, the analysis is halted and a divergence diagnostic message is printed. If NCONV has not been exceeded, $\{S_i'\}$ is moved into array T and the program goes to the next iteration. Whether shrinkage-dependent quantities are updated during the next iteration is determined by the control variable NREFORM. If NREFORM has been specified as "1", the program returns to reassemble $[K]^*$ and the shrinkage-dependent portions of $\{Q\}^*$, based on the latest iterate $\{S_i'\}$; if NREFORM is "0", the program returns only to modify the gradient boundary conditions. This scheme is shown in the flow chart, Fig. 4.3.

The gradient boundary condition iterative cycle basically consists of modifying the term $\tilde{Q}_{BC}$ (approximate boundary flow - Eq. 3.15) during each iteration until it represents the true flow and hence satisfies the specified gradient boundary condition, as discussed in Section 3.3. The flow $\tilde{Q}_{BC}$ is corrected by comparing (at each surface segment) the gradients due to the shrinkage solution $\{S_i'\}$ from the current iteration with the specified boundary condition gradient $\partial S/\partial n_{BC}$. Then the effective diffusivity $[K]^*$ and effective load $\{Q\}^*$ are corrected accordingly and Eq. 4.1 is resolved. This iterative process is repeated until a solution is obtained that satisfies the gradient boundary conditions. These operations are performed in subroutine GRAD, illustrated in Fig. 4.5.

The step-by-step solution process outlined above continues until a time step control card with a negative step size is encountered. The solution is then considered complete and the program begins consideration of the next data case in the input deck.

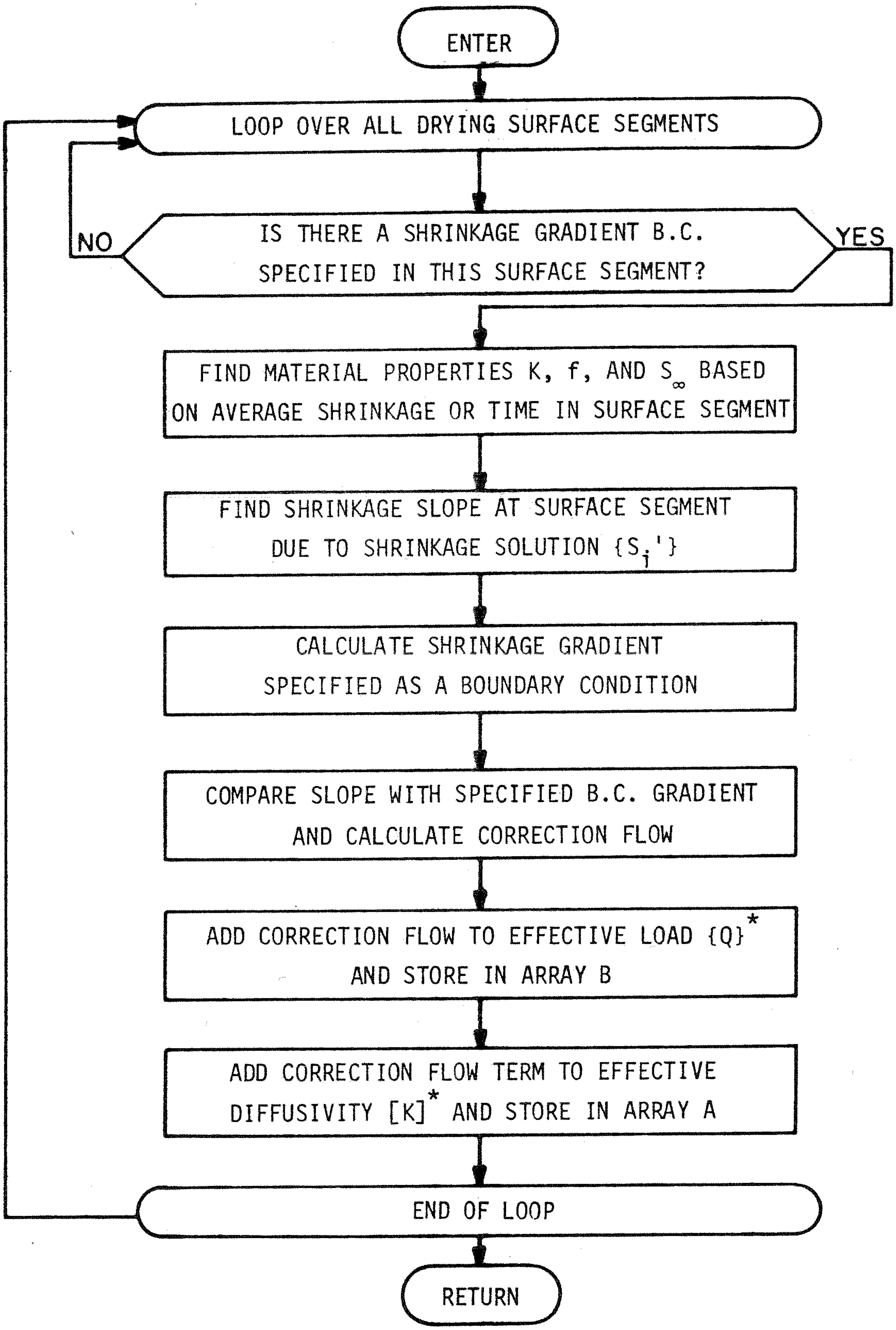


FIGURE 4.5 FLOW CHART FOR SUBROUTINE GRAD

4.3 Output Phase

Two kinds of output can be specified, printed output and punched cards:

1. A printing subroutine (PROUT) is called when convergence is achieved at the end of each time step (see Fig. 4.3). The time step control card may specify whether the nodal shrinkage strains, average element shrinkage strains, or both are printed. Also printed are the fixed shrinkage or flow boundary conditions specified for that time step and the number of iterations needed to effect convergence. See Appendix B for an example of printed output.

   All input data, including generated meshes are also printed. A more detailed output during any particular time step may be specified, i.e., the shrinkage $\{S_i\}$ at various stages of the iterative solution process. This option is useful should the user encounter convergence difficulties in any particular problem set-up.

2. A punching subroutine (PUOUT) is called along with the printing subroutine at the end of each time step (Fig. 4.3), and either nodal or element shrinkages may be punched. This type of output is often needed as input for other computer programs which analyze structural behavior due to shrinkage. For example, the free shrinkages punched by program SHRINC serve as loading for the program SASHTEC [3] which calculates the shrinkage stresses and apparent strains resulting from this drying history. During the input phase the program also punches any generated meshes (both node cards and element cards), so that the mesh is available for use in programs without generating options.

5. COMMENTARY ON USE OF PROGRAM SHRINC

5.1 Design of Finite Element Mesh

One of the first steps in solving a shrinkage problem using program SHRINC is to design a finite element mesh. The efficiency and accuracy of the analysis strongly depends on the skill with which the mesh is laid out. Several considerations should be kept in mind:

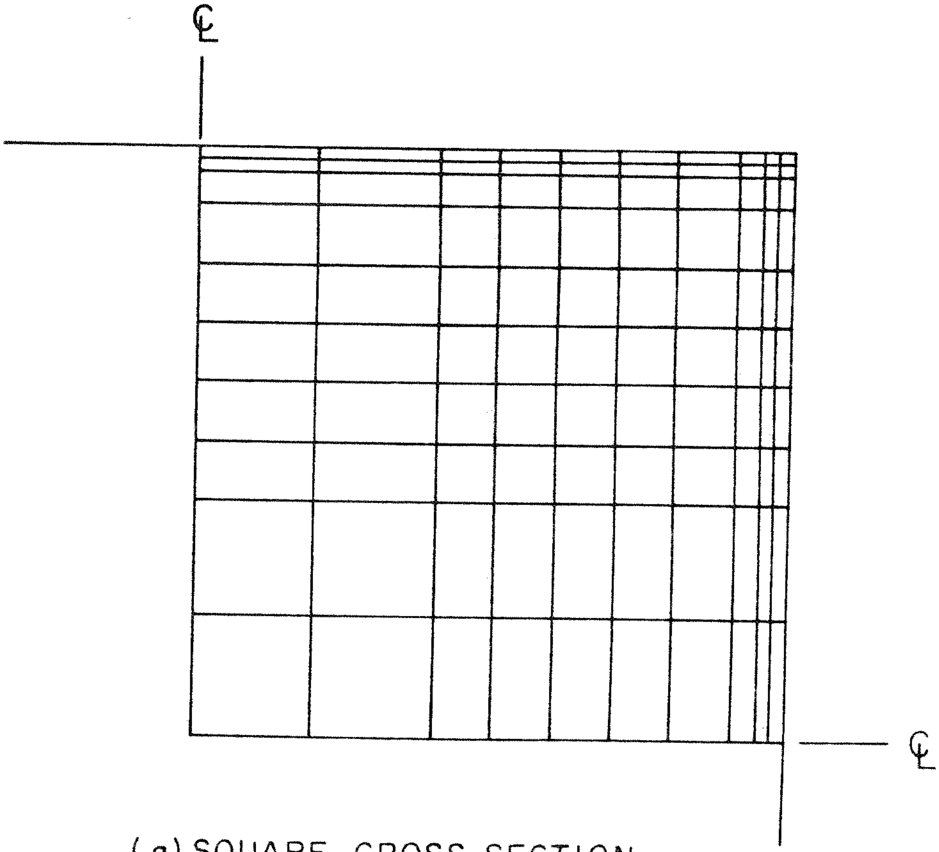
1. A mesh with as few elements as possible should be used since computational effort increases roughly with the cube of the number of nodes. Symmetry should be taken advantage of wherever possible - i.e., a quadrant should be analyzed when the problem lends itself to such simplification. Experience has shown [2] that 100-element meshes are usually sufficient for a regularly-shaped section.

2. Small elements should be used where shrinkage gradients are steep and larger elements where shrinkage is more nearly constant. Gradients tend to be very steep at the surface and meshes should be graduated in a manner similar to that shown in Fig. 5.1.

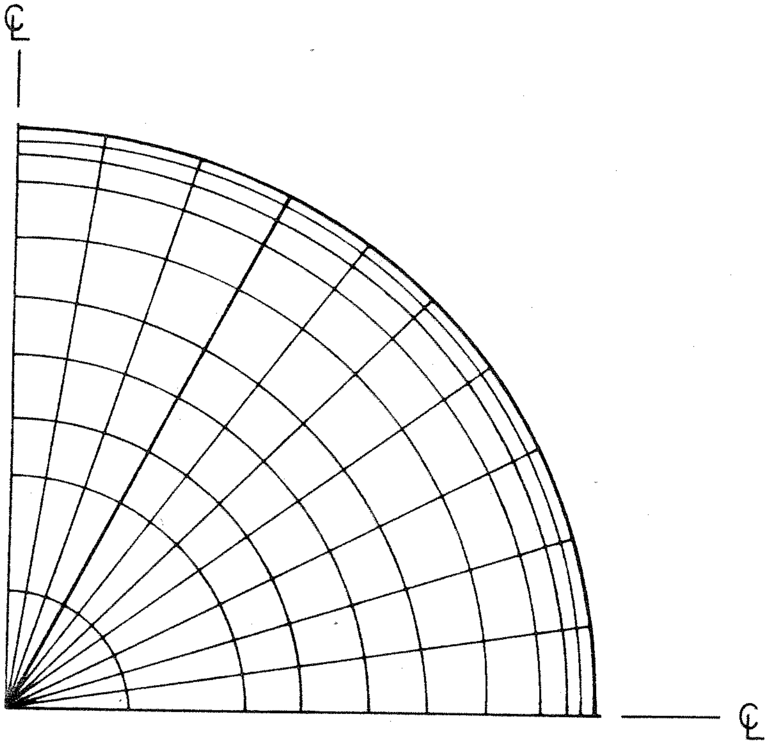
3. Nodes should be numbered in such a way as to minimize bandwidth of the reulting matrix equations, thereby decreasing the computational effort needed to solve the equations.

5.2 Selection of Time Step Sizes

The analyst is free to discretize drying time in the way most suitable for the particular problem being solved. As a general rule, both accuracy and computational effort increase with smaller time steps. The following points should serve as an aid to intuition and experience in choosing a system of time steps:



(a) SQUARE CROSS-SECTION



(b) ROUND CROSS-SECTION

FIGURE 5.1 TYPICAL FINITE ELEMENT MESHES

1. During early drying times, small time steps are needed since the shrinkage distribution within the section changes very fast. Transient behavior during this early period is not adequately represented by using large time steps. At more advanced drying times when steady-state conditions begin to prevail, it is possible to use very large time steps (up to 100 days) with good results.

2. The first time step must be very small (about 1 day), since during this time step shrinkage or shrinkage gradient boundary conditions are imposed upon the system. This boundary condition loading causes $\{S\}_1$, the solution at the end of the first time step, to be very different from $\{S\}_0$, the initial conditions. Convergence difficulties arise if too large an initial time step is used.

3. Small cross-sections tend to need smaller time steps than large cross-sections, since shrinkage develops much more quickly.

5.3 Specification of Material Properties

Material properties are a characteristic of the concrete being analyzed and should be determined experimentally. However, such explicit information is often not available to the analyst, and the following guidelines are offered for such cases:

1. Shrinkage diffusivity, K, tends to decrease as drying proceeds. Parametric studies [2] have shown that if this decrease is modeled as a function of shrinkage strain, better results are obtained than if it is modeled as a function of drying time, especially for large-sized sections. The exact functional form of K(S) seems to vary somewhat from concrete to concrete; however, a useful approximation

to K(S) that has given solutions that correspond well with a wide range of experimental data is shown in Fig. 5.2. Note that the diffusivity curve is very strongly dependent on S_∞, the ultimate shrinkage strain of the concrete. Hence two material functions are shown in Fig. 5.2 - one for a concrete of relatively low shrinkage potential (500 microinches/inch) and one for a concrete of relatively high shrinkage potential (1000 microinches/inch). Approximations of K(S) for other concretes can be found by interpolation.

For small cross-sections (less than 10 in. x 10 in.), quite accurate analyses have been performed using a time-dependent diffusivity suggested by Pickett [4]:

$$K(T) = 0.10 \sqrt{\frac{2}{2+t}} \ \text{in}^2/\text{day}. \qquad (5.1)$$

This K(t) may be specified in program SHRINC by a special input option (see Appendix A).

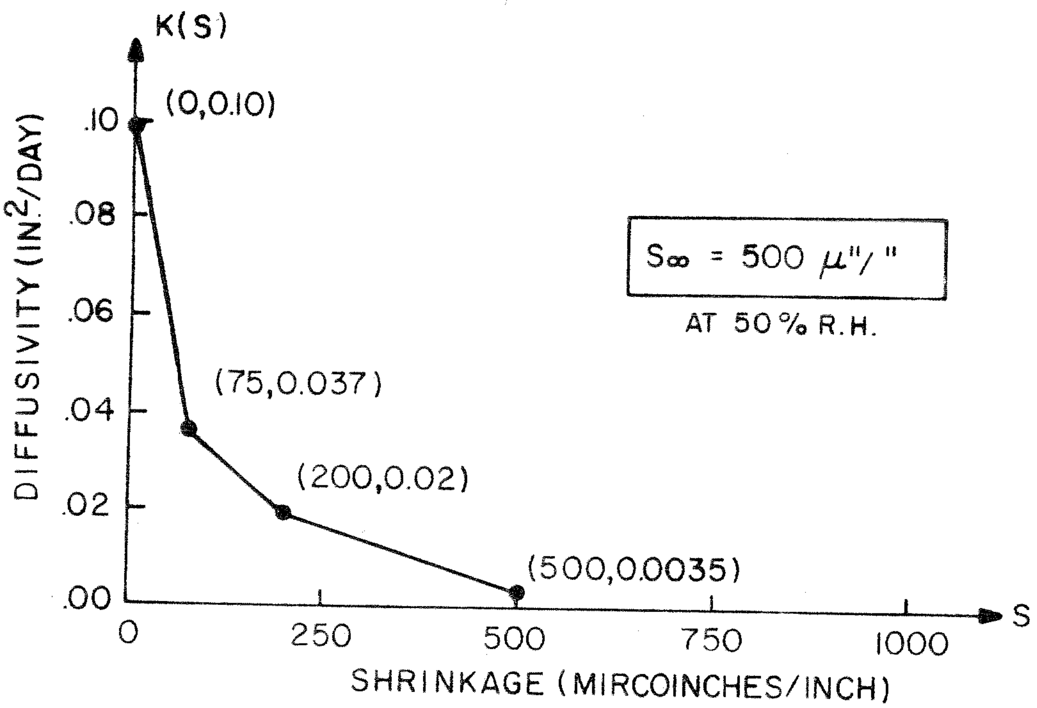
2. <u>Surface factor, f</u>, tends to have a functional form very similar to diffusivity. Hence, in the absence of more specific information on the form f takes, the approximation (in English units)

$$f(S) = 1.67\ K(S)$$

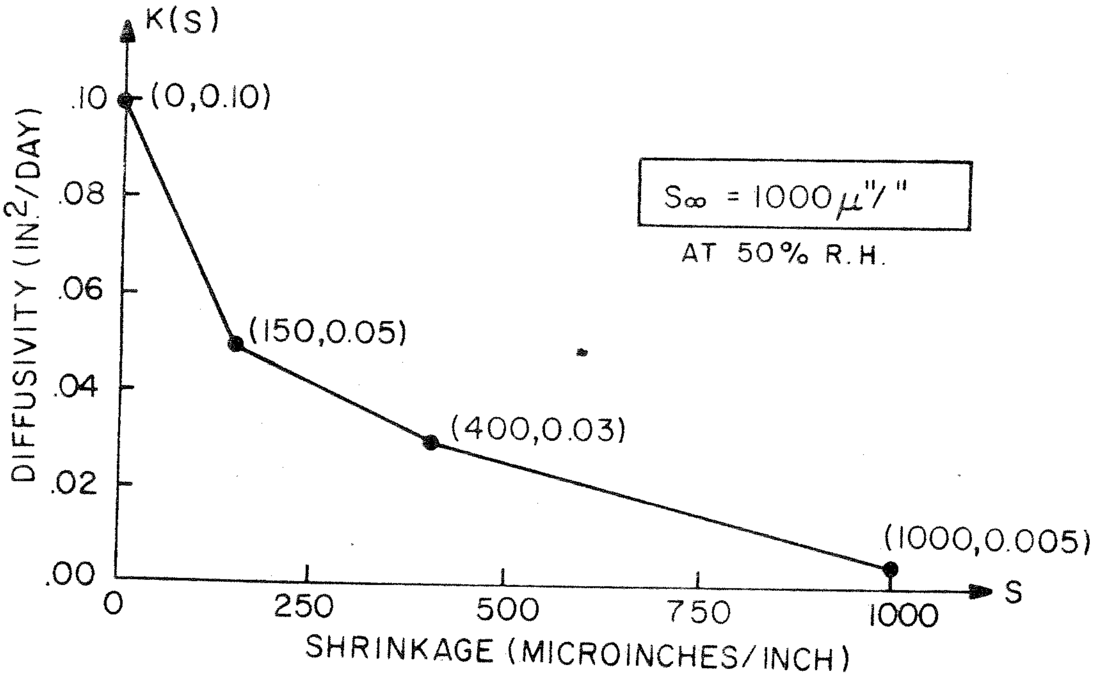
or

$$f(t) = 1.67\ K(t)$$

has given consistently adequate results. For a more extensive discussion of this function see Reference [2].



a) DIFFUSIVITY FOR LOW SHRINKAGE CONCRETE



b) DIFFUSIVITY FOR HIGH SHRINKAGE CONCRETE

FIGURE 5.2 TYPICAL DIFFUSIVITY FUNCTIONS

3. Ultimate shrinkage strain, S_∞, tends to vary greatly from concrete mix to concrete mix. In a 50% relative humidity drying environment, S_∞ can range from about 300 to over 1000 microinches/inch. This is the most sensitive material parameter and its value should be based on some sort of experimental evidence. Thirty- or sixty-day shrinkage tests with a very small diameter specimen of the concrete in question would suffice to obtain an approximate S_∞.

5.4 Specification of Boundary Conditions

Program SHRINC accepts any of the four types of boundary conditions discussed in Section 2 (Eq. 2.3a-d). However, the type that has proved most usable in actual analyses is the shrinkage-dependent gradient boundary condition (Eq. 2.3c), since this seems best to model surface evaporation. It is also possible to specify $S = S_\infty$ at all drying surfaces (Eq. 2.3a), but this only roughly models shrinkage behavior at early drying times. Flow boundary conditions are physically realistic but computationally difficult, since shrinkage flow is not easily measured nor quantitatively specified.

Axes of symmetry can be created by use of the boundary condition $Q_{BC} = 0$ or $\partial S/\partial n_{BC} = 0$.

5.5 Minimization of Cost of Nonlinear Analysis

The analyst is able to control the cost of his solution by choosing which nonlinearities are to be considered. Usually it is not worth the added cost to reform diffusivity and load during iteration within a time step; these quantities change little if a proper time step system has been selected. Iteration is needed to satisfy gradient boundary conditions, but a reasonable convergence criterion (CONV) should be specified to avoid an excessive number of iterations.

The use of shrinkage-dependent material variables does not increase computational effort (if NREFORM = 0), but the use of shrinkage gradient boundary conditions usually triples the cost due to the need for iteration. For a typical analysis the following iteration control variables can be used:

NCONV = 20
CONV = 0.001
NREFORM = 0

REFERENCES

1. Bazant, Z.P. and Najjar, L.J., "Nonlinear Water Diffusion in Nonsaturated Concrete," Materiaux et Constructions (RILEM), v. 5, no. 25, Jan.-Feb. 1972.

2. Iding, R. and Bresler, B., "Analysis of Nonuniform Drying Shrinkage Effects in Reinforced Concrete Frames," UC-SESM Report, Structures and Materials Research, Department of Civil Engineering, University of California, Berkeley (in preparation).

3. Iding, R. and Bresler, B., "SASHTEC - A Computer Program for Structural Analysis of Shrinkage and Temperature Effects in Reinforced Concrete Frames," Structures and Materials Research, Department of Civil Engineering, University of California, Berkeley (in preparation).

4. Pickett, G., "Shrinkage Stresses in Concrete," ACI Journal, Proceedings Vol. 42, Jan.-Feb. 1946.

5. Polivka, R. and Bresler, B., "Time-Dependent Behavior of Reinforced Concrete Columns Including Effects of Shrinkage, Creep and Cracking," Report No. UC-SESM 75-5, Structures and Materials Research, Department of Civil Engineering, University of California, Berkeley, June 1975.

APPENDIX A - INPUT INSTRUCTIONS FOR SHRINC

Contents

I. HEADING CARD (8A10)

note	columns	variable	entry
(1)	1 - 80	ITITLE(8)	Enter the master heading information for use in labelling the output

NOTES/

(1) Begin each new data case with a new heading card. To halt the program, insert two blank cards instead of a heading card.

II. NODAL POINT DATA

A. Control Card (Alphanumeric)

NODES, N1, N2

note	field	variable	entry
(1)	NODES	--	Enter the word "NODES"
(2)	N1	NUMNP	Number of nodal points
(3)	N2	NSBC	Number of nodes with a specified shrinkage boundary condition

NOTES/

(1) This is an alphanumeric control card containing both a key word (NODES) to identify the block of input data to follow and the two main control parameters (N1 and N2) of that block of data. Alphanumeric control cards are left-justified with no blanks in the list. Note the examples in Fig. A.1.

(2) Enter the total number of nodal points in the cross-section.

(3) Enter the total number of boundary nodes at which amount of shrinkage is specified as the boundary condition. Boundary nodes at which shrinkage flow or shrinkage gradient are specified as the boundary condition are not included.

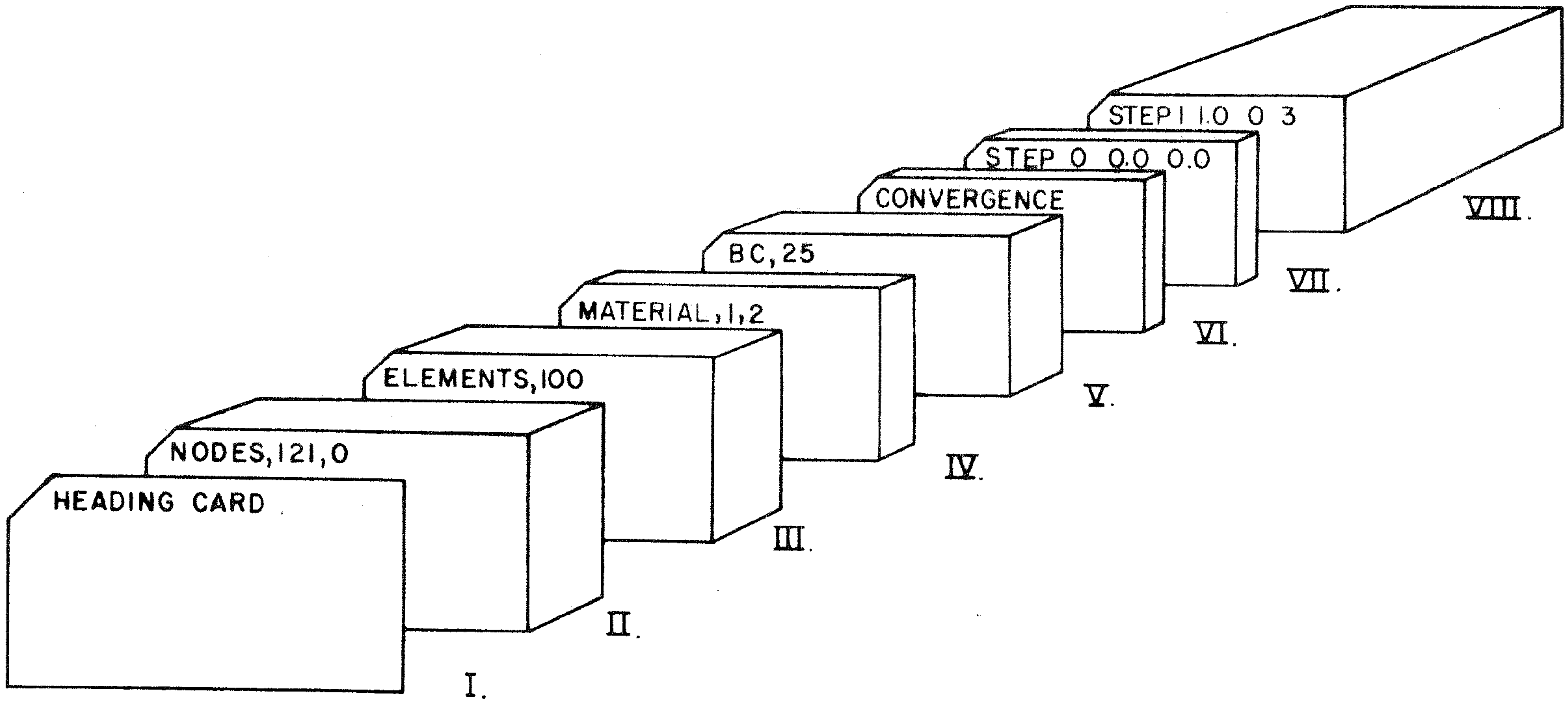


FIGURE A.1 TYPICAL INPUT DECK FOR SHRINC

B. Nodal Data Cards (I5,2F10.0,I5)

note	columns	variable	entry
(1)	1 - 5	N	Nodal point number GE. 1 and LE. NUMNP
(2)	6 - 15	X(N)	X-coordinate
(3)	16 - 25	Y(N)	Y-coordinate
(4)	26 - 30	IRADIAL	Mesh generation flag EQ. 0, generate omitted nodes along straight line EQ. 1, generate omitted nodes along circular curve

NOTES/

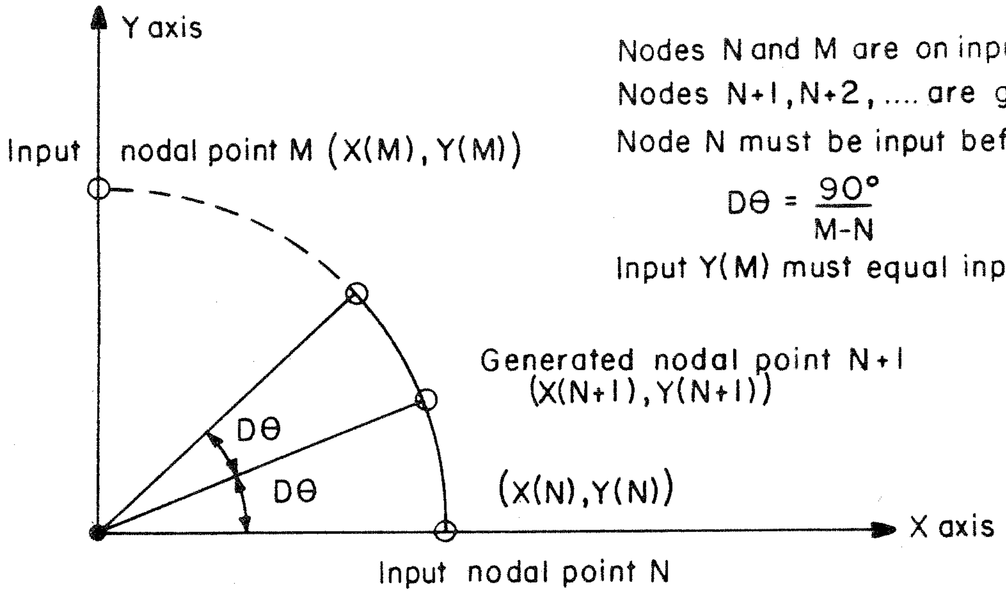
(1) Nodal data must be defined for all (NUMNP) nodes. Nodal data may be input directly (i.e., each node on its own individual card) or the generation option may be used. Nodal point cards must be in ascending numerical sequence. If cards are omitted, the missing nodes are generated at equal intervals along a straight line between the bounding nodal points. Refer to Section 5.1 for more information about mesh design.

(2) Nodal coordinates are input in inches or centimeters. The same system of units must be used when defining other input quantities.

(3) If (IRADIAL) is "0" generation of nodal points missing from the input deck proceeds as discussed in note 1. Setting (IRADIAL) equal to "1" activates a scheme for generating circular quadrants. Coordinates originate at the center of the cross-section and input nodes lie along the positive X and Y axes, as shown in Fig. A.2. Omitted nodes are generated at equal distances along the arc connecting the bounding nodes.

C. Fixed Shrinkage Boundary Condition Data (16I5)

note	columns	variable	entry
(1)	1 - 5	ID(1)	Nodal number of first fixed shrinkage boundary condition node
	6 - 10	ID(2)	Nodal number of second fixed shrinkage boundary condition node
	.	.	.
	.	.	.
	.	.	.
		ID(N2)	Continue until N2 nodes are input



Nodes N and M are on input cards.
Nodes N+1, N+2, ... are generated.
Node N must be input before node M.

$$D\Theta = \frac{90°}{M-N}$$

Input Y(M) must equal input X(N)

FIGURE A.2 GENERATION OF CIRCULAR NODAL MESH

NOTES/

(1) Enter the node number of each boundary node at which shrinkage is fixed as the boundary condition. Input 16 shrinkage nodes per card and use as many cards as necessary to contain

all (N2) nodes. If only flow or gradient boundary conditions are to be specified in a data case, set (N2) equal to "0" and omit this card.

III. ELEMENT DATA

A. Control Card (Alphanumeric)

ELEMENTS, N1

note	field	variable	entry
(1)	ELEMENTS	--	Enter the word "ELEMENTS"
(2)	N1	NUMEL	Number of elements

NOTES/

(1) This is an alphanumeric control card with key word (ELEMENTS) and main control parameter (N1), as in Fig. A.1.

(2) Enter the total number of quadrilateral elements in the cross-section. Include both concrete and reinforcing steel elements.

B. Element Data Cards (615)

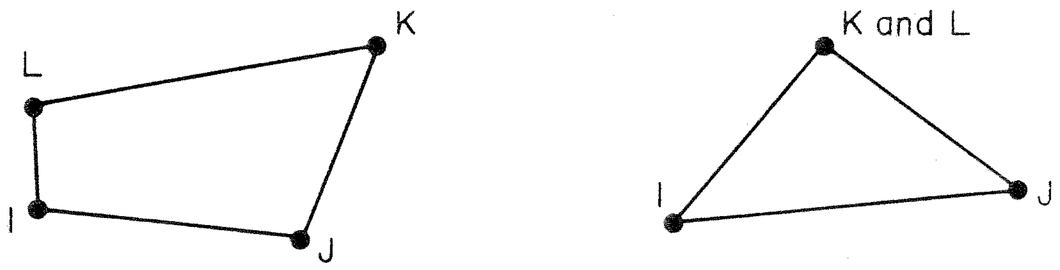
note	columns	variable	entry
(1)	1 - 5	N	Element number
(2)	6 - 10	LM(1,N)	Nodal point I
(2)	11 - 15	LM(2,N)	Nodal point J
(2)	16 - 20	LM(3,N)	Nodal point K
(2)	21 - 25	LM(4,N)	Nodal point L
(3)	26 - 30	MMTYPE(N)	Material type for this element EQ.0, default value of "1" used

NOTES/

(1) Elements must be input in ascending element number order. If element cards are missing, the program generates the missing elements by incrementing N and nodal points I, J, K, and L. A generated element assumes the material type of the element immediately preceding it. The last element in the mesh cannot be generated.

(2) The program uses quadrilateral elements defined by their corner nodes (I,J,K,L). Enter the global node number corresponding to each of these four corner nodes in counterclockwise order, as shown in Fig. A.3. Triangular elements can be formed by specifying a degenerate quadrilateral, i.e., letting K and L be defined by the same global nodal point (see Fig. A.3).

(3) One or more sets of material properties are input in the next data block, each of them labelled by an identification number. Enter the identification number of the material this element is composed of.



a) QUADRILATERAL b) TRIANGLE

FIGURE A.3 FOUR-NODE FINITE ELEMENTS

IV. MATERIAL PROPERTY DATA

A. Control Card (Alphanumeric)

MATERIAL, N1, N2

note	field	variable	entry
(1)	MATERIAL	--	Enter the word "MATERIAL"
(2)	N1	NMAT	Number of material types
(3)	N2	MATMOD	Material model used EQ. 1, Time-dependent material properties EQ. 2, Shrinkage-dependent material properties.

NOTES/

(1) This is an alphanumeric control card with key word (MATERIAL) and main control parameters (N1) and (N2), as in Fig. A.1.

(2) Enter the number of different materials that make up the cross-section. Include each type of steel and each type of concrete.

(3) This control parameter specifies whether the materials are to have time-dependent properties or shrinkage-dependent properties (see Section 5.3 for discussion of material modeling). The choice of model applies to all (N) material types.

B. Material Data

Input the following set of cards for each material type:

1. Control Card (2I5)

note	columns	variable	entry
(1)	1 - 5	MM	Number of points used to define shrinkage diffusivity function. EQ. 0, constant diffusivity
(1)	6 - 10	MMM	Number of points used to define surface factor function EQ. 0, constant surface factor

NOTES/

(1) Each material type is defined by three (3) material parameters. One of them, ultimate shrinkage strain, is always input as a constant; however, the other two, shrinkage diffusivity and surface factor, may be input either as constants or as tabular functions of drying time or shrinkage strain. At least two points are needed to define each function, and linear interpolation is used between points.

If (MATMOD) has been set for time-dependent material properties and it is wished to specify Pickett's equations

$$K(t) = 0.10\sqrt{\frac{2}{2+t}} \quad in^2/day$$

$$f(t) = 1.67\ K(t) \quad in/day$$

Enter "-1" for both MM and MMM.

2. Shrinkage diffusivity (8E10.0)

note	columns	variable	entry
(1)	1 - 10	X(1)	Time/shrinkage strain of point 1
	11 - 20	Y(1)	Value of diffusivity function at point 1
	21 - 30	X(2)	Time/shrinkage strain of point 2
	⋮	⋮	⋮
		Y(MM)	Continue until all MM points are input

NOTES/

(1) Enter the table that defines the shrinkage diffusivity function. Each point is described by an ordered pair (X,Y), where X is drying time in days (if MATMOD. EQ. 1) or shrinkage strain in inches/inch (if MATMOD. EQ. 2) and Y is the value of diffusivity in in^2/day or cm^2/day, as in Fig. A.4. Input 4 such pairs per card and use as many cards as necessary. The table for the function must be defined over the entire range of time or shrinkage to be considered in the

solution process - i.e., extrapolation below the lowest point or beyond the highest point is not permitted. For information about choosing a suitable diffusivity function, refer to Section 5.3. If shrinkage diffusivity was specified constant (MM. EQ. 0) enter the constant value in columns 1-10 in units of in^2/day or cm^2/day.

If Pickett's relation is to be used (MM. EQ. -1) enter in columns 1-10 the value 0.10 (English units) or 0.645 (metric units).

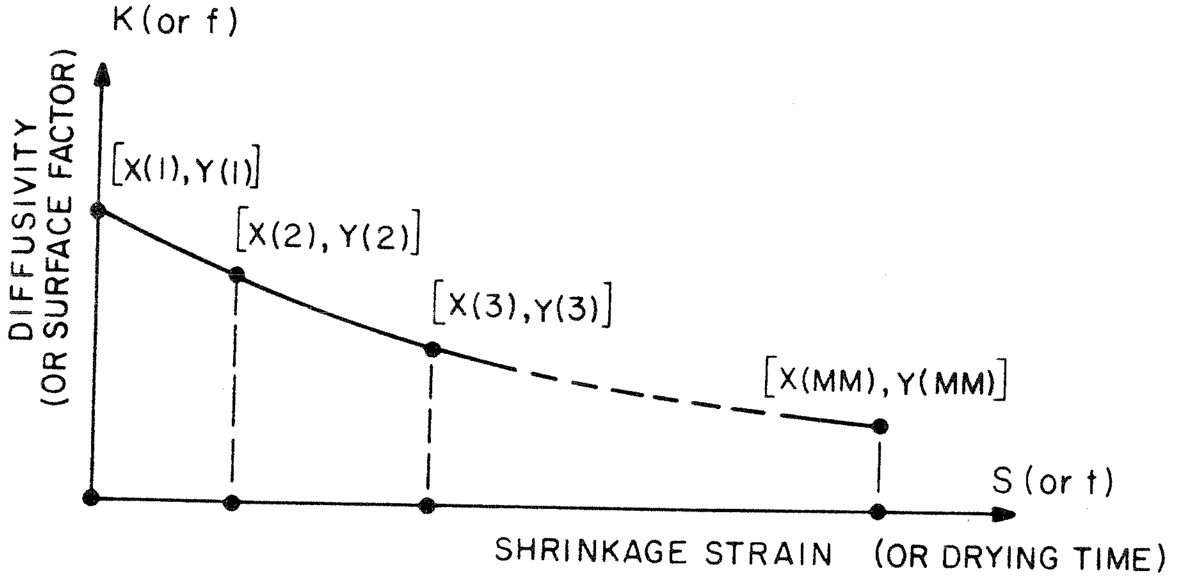


FIGURE A.4 - MATERIAL PROPERTY TABLE FORMAT

3. Surface factor (8E10.0)

note	columns	variable	entry
(1)	1 - 10	X(1)	Time/shrinkage strain of point 1
	11 - 20	Y(1)	Value of surface factor function at point 1
	.	.	.
	.	.	.
	.	.	.
	.	.	.
	.	Y(MMM)	Continue until all MMM points are input

NOTES/

(1) Enter the table that defines the surface factor function in the same way that shrinkage diffusivity was input. Surface factor is input in units of inches/day or centimeters/day. For information on selecting an appropriate surface factor, refer to Section 5.3. If surface factor is constant (MMM. EQ. 0) enter the constant value in columns 1-10 in units of in/day or cm/day.

If Pickett's relation is to be used (MMM. EQ. -1) enter in columns 1-10 the value 1.67 (English units) or 0.66 (metric units).

4. Ultimate shrinkage strain (E10.0)

note	columns	variable	entry
(1)	1 - 10	SINF	Ultimate shrinkage strain

NOTES/

(1) This parameter is always considered a constant. Units of inches/inch (or, equivalently, cm/cm) are used.

V. DRYING BOUNDARY CONDITIONS DATA

A. Control Card (Alphanumeric)

BC, N1

note	field	variable	entry
(1)	BC	--	Enter the letters "BC"
(2)	N1	NDRYBC	Number of surface segments exposed to a shrinkage-dependent drying boundary condition

NOTES/

(1) This is an alphanumeric control card with key symbol (BC) and main control parameter (N1), as in Fig. A.1.

(2) A drying boundary condition is one in which the prescribed surface flow or gradient is of the form

$$Q_{BC} = f(S_\infty - S) \tag{A.1}$$

$$\frac{dS}{dn_{BC}} = \frac{f}{K}(S_\infty - S) \tag{A.2}$$

The total surface exposed to this type of boundary condition is divided into segments - each segment bounded by two nodal points along the surface, as in Fig. A.5. If drying boundary conditions are not used, input this control card but set (N1) equal to zero.

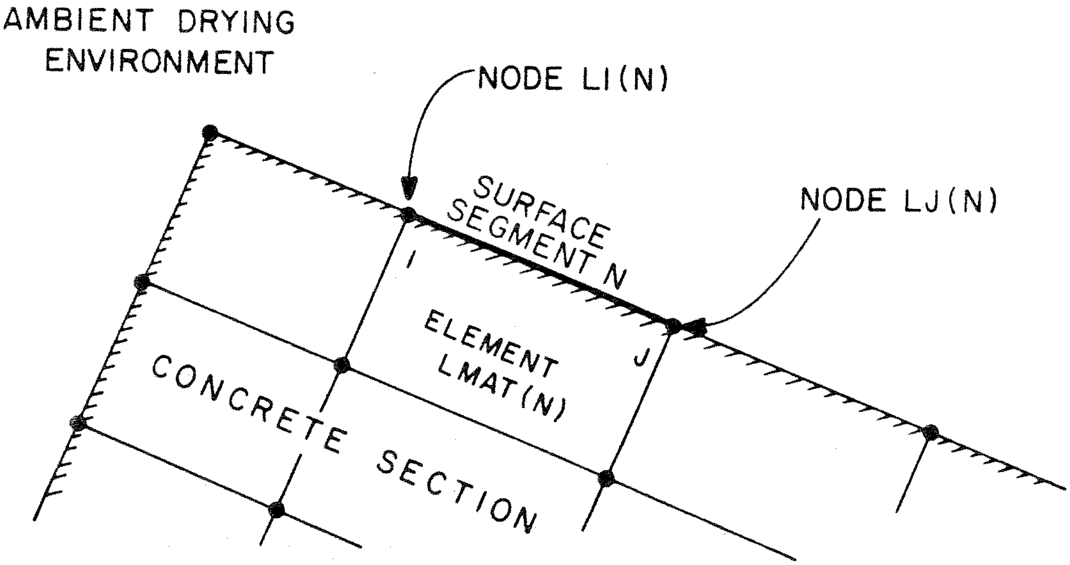


FIGURE A.5 SURFACE SEGMENT FOR DRYING BOUNDARY CONDITION

B. Surface Segment Data (4(3I4, F4.0, I4))

note	columns	variable	entry
- - - - - - - - - - - - - - for surface segment No. 1 - - - - - - - - - - -			
(1)	1 - 4	LI(1)	Node number of segment end I
(1)	5 - 8	LJ(1)	Node number of segment end J
(2)	9 - 12	LMAT(1)	Element number of element between I and J
(3)	13 - 16	SURFAC(1)	Multiplier for material surface factor
(4)	17 - 20	KODEBC(1)	Type of drying boundary condition EQ. 0, flow Q_{BC} Eq. 1, gradient dS/dn_{BC}
- - - - - - - - - - - - - - for surface segment No. 2 - - - - - - - - - - -			
(5)	21 - 24	LI(2)	Node number of segment end I
	⋮	⋮	⋮
		KODEBC(N1)	Continue until data for all N1 surface segments is input

NOTES/

(1) Enter the global node numbers of the two surface nodes bounding the segment, as shown in Fig. A.5.

(2) The edge of one of the quadrilateral elements composing the cross-section mesh lies between nodes I and J. Enter the number of that element so that the surface factor assigned to its material may be used in calculating the boundary condition between I and J.

(3) The boundary condition due to the material properties (diffusivity and surface factor) can be modified by a constant multiplier. For example, setting SURFAC equal to "0.0" models an insulated surface. A positive factor causes drying (shrinkage flow into the surface). To specify the

usual drying boundary condition (Eq. A.1 or A.2) set (SURFAC) equal to "1.0".

(4) Each surface segment may represent either a flow boundary condition (Eq. A.1) or a shrinkage gradient boundary condition (Eq. A.2). See Section 5.4 for a discussion of drying boundary conditions.

(5) Input on each card data for four (4) surface segments. Use as many cards as needed in order to input all (N1) segments. If (N1) was specified as zero omit this card.

VI. CONVERGENCE CRITERIA

A. Control Card (Alphanumeric)

CONVERGENCE

note	field	variable	entry
(1)	CONVERGENCE	--	Enter the word "CONVERGENCE"

NOTES/

(1) This is an alphanumeric control card with key word (CONVERGENCE) and no control parameters, as in Fig. A.1.

B. Convergence Criteria Card (I5, F10.0, I5)

note	columns	variable	entry
(1)	1 - 5	NCONV	Maximum number of iterations permitted in each time step EQ. 0, no iteration
(1)	6 - 15	CONV	Permissible error EQ.0, default value of 0.001 used
(2)	16 - 20	NREFORM	Matrix reforming flag EQ.0, update nonlinear diffusivity and flow matrices at start of each time step only EQ.1, update matrices during each iteration

NOTES/

(1) During the step-by-step analysis, iteration is necessary to find a solution whenever the problem is nonlinear - i.e., whenever diffusivity and surface factor are shrinkage-dependent or whenever gradient boundary conditions are specified (See Section 4.2). Iterative cycling will be continued until

$$\left\| \frac{(S_i - S_{i-1})}{S_{\infty\,avg}} \right\| < \quad (CONV) \qquad (A.3)$$

at each node in the cross section, where

S_i = shrinkage solution obtained during this iteration

S_{i-1} = shrinkage solution obtained during previous iteration

$S_{\infty\,avg}$ = average (over all material types) ultimate shrinkage strain

If convergence according to Eq. A.3 is not obtained after (NCONV) iterations, the program is halted and an error diagnostic written.

If the problem being input is linear or if no iteration is desired, set (NCONV) equal to zero and disregard the remainder of this card.

(2) If diffusivity and surface factor are shrinkage-dependent variables, their values will change after each iteration. Hence the effective diffusivity matrix [K]* and effective load matrix {Q}* must be recalculated after each iteration, as discussed in Section 4.2. This is quite expensive computationally and quite often it is desired to recalculate the matrices only at the beginning of each time step and not during each iteration.

If material properties are constant or time-dependent ignore this variable.

VII. INITIAL CONDITIONS

A. Time Step Control Card (A4, I6, 2F10.0, A3)

note	columns	variable	entry
	1 - 4	IA	Enter the word "STEP"
(1)	5 - 10	MDT	Initial number in sequencing of time steps
(1)	11 - 20	TIME	Initial time (i.e., base time) in days
(2)	21 - 30	SIC	Uniform initial shrinkage. (If initial shrinkage is nonuniform, insert any negative number here; nodal values are input on next data card)
(3)	31 - 33	JP	3-symbol alphanumeric code that will appear in columns 74-76 of punched output

NOTES/

(1) The usual starting point for time step sequencing is "0" and the usual initial time is "0.0". However, nonzero values may be specified if desired.

(2) The usual initial condition for a drying process is a uniform zero-shrinkage distribution (SIC = 0.0). However, nonzero or nonuniform initial conditions may be specified if desired.

(3) If these columns are left blank the code "NODE" will appear on punched nodal shrinkages and the code "ELEM" on punched element shrinkages.

B. Initial Shrinkage Distribution Data (6(4X, F8.6))

Omit this card if (SIC) in columns 21-30 of the preceding card is not negative.

note	columns	variable	entry
(1)	1 - 4	--	blank
	5 - 12	T(1)	Initial shrinkage strain at node 1.
	13 - 16	--	blank
	17 - 24	T(2)	Initial shrinkage strain at node 2
	.	.	.
	.	.	.
	.	.	.
	.	.	.
	.	T(NUMNP)	Continue until all nodal points are input

NOTES/

(1) Enter the initial nodal shrinkages, six (6) nodes per data card, using as many cards as necessary. This format is compatible with punched output from the program; hence, this option can be used to restart a previous analysis that must be continued for a longer period of drying. For a restarted analysis change the initial time (TIME) on the control card to the time at which the previous analysis ended.

VIII. TIME STEP DATA

For each time step input the following block of data:

A. Time Step Control Card (A4, I6, F10.0, 4I5)

note	columns	variable	entry
	1 - 4	IA	Enter the word "STEP"
(1)	5 - 10	NDT	Time step number
(2)	11 - 20	DT	Time step interval
(3)	21 - 25	ISOF	Number of nodes with a nonzero flow shrinkage boundary condition

	26 - 30	I1	Printed output desired for this particular timestep. EQ. 0, no output EQ. 1, nodal point shrinkages EQ. 2, element shrinkages EQ. 3, both nodal and element shrinkages
	31 - 35	I2	Punches output desired for this particular time step EQ. 0, no punched output EQ. 1, nodal point shrinkages EQ. 2, element shrinkages EQ. 3, both nodal and element shrinkages
(4)	36 - 40	I6	Intermediate printout for debugging purposes EQ. 0, no printout EQ. 1, debugging printout

NOTES/

(1) Time step cards must be input in ascending sequence with no time steps omitted.

(2) Input the length of the time step in days. To end a data case input a negative time step size in these columns and then proceed to the next data case (heading card).

(3) Note that each surface node of the cross-section must have one of the following boundary conditions:

1. Shrinkage strain specified. Such nodes are identified on cards II.C, and the shrinkage values are specified in the following data block (VIII.B). These values may change from time step to time step if so desired.

2. Shrinkage flow specified (inches/day). Zero flow ($Q_n = 0$) is the default boundary condition; if no other input is given all surface nodes are assumed to have this boundary condition. If a nonzero value is specified, it is input in data block VIII.B. These flow values may change from time step to time step if so desired.

3. Shrinkage flow or shrinkage gradient specified as a drying boundary condition, as in Eq. A.1 or A.2. Data for these nodes are input in data block V.

The total number of nodes that have a nonzero boundary condition of type 1 or 2 is entered in columns 21-25; the actual values are entered on the following card.

(4) If convergence difficulties are encountered in any particular analysis, this option may be called to print various data during each iteration.

B. <u>Nonzero Boundary Condition Data</u> (5(I5, F10.0))

Omit this card if (ISOF) in columns 21-25 of the preceeding card is zero.

note	columns	variable	entry
(1)	1 - 5	J(1)	Node number
	6 - 15	FT(1)	Specified shrinkage or flow at that node
	16 - 20	J(2)	Node number
	21 - 30	FT(2)	Specified shrinkage or flow at that node
	.	.	.
	.	.	.
	.	.	.
	.	FT(ISOF)	Continue until all (ISOF) boundary nodes are input

NOTES/

(1) Input the global node number and the specified shrinkage boundary condition (in/in) or specified flow boundary condition (in/day) for each of the (ISOF) boundary nodes. Enter five (5) nodes per data card and use as many cards as necessary. This data must be input for each time step - even if the boundary conditions do not change from time step to time step.

APPENDIX B - SAMPLE PROBLEM

A 16 inch by 16 inch square concrete column reinforced with 10 #9 bars (placed as shown in Fig. B.1) is analyzed for free shrinkage development. Because of symmetry only a quadrant of the column need be considered. The finite element grid shown in Fig. B.2 and the time step system shown in Fig. B.3 are used to discretize the problem. Note that the round reinforcing bars are idealized as rectangles of equivalent area. The two sides of the quadrant exposed to drying are assumed to have shrinkage-dependent gradient boundary conditions (Eq. 2.3c) and the two sides along the axes of symmetry are assumed insulated ($Q_{BC} = 0$).

Shrinkage-dependent material parameters are utilized. For concrete, the diffusivity function shown in Fig. 5.2a is assumed, and surface factor is defined by the relation

$$f(S) = 1.67\ K(S).$$

Ultimate shrinkage, S_∞, is specified as 390 microinches/inch. Steel, of course, is assumed to undergo no shrinkage and to have zero shrinkage diffusivity.

Input data for this problem is shown in Table B.1. Note that the time step data have been abbreviated.

Typical output is shown in Table B.2. The free shrinkage solution at selected times is plotted in Figs. B.4 and B.5. Figure B.4 shows shrinkage contours within the cross-section and Fig. B.5 shows shrinkage gradients along one of the axes of symmetry.

An average of four iterations was needed in each time step to satisfy the gradient boundary conditions. The central processing time (CDC 6400) required for the entire analysis was 167 seconds.

```
*** 16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***
NODES,136,0
    1      0.        0.
    3      3.2       0.
    4      4.0       0.
    5      4.8       0.
    6      5.0       0.
    8      6.2       0.
    9      6.4       0.
   10      7.2       0.
   11      7.6       0.
   13      8.0       0.
   14      0.0       1.6
   16      3.2       1.6
   18      4.8       1.6
   19      5.0     0.417
   20      5.6     0.417
   21      6.2     0.417
   22      6.4       1.6
   23      7.2       1.6
   24      7.6       1.6
   26      8.0       1.6
   27      5.6       1.6
   28      0.0       3.2
   30      3.2       3.2
   35      7.2       3.2
   36      7.6       3.2
   38      8.0       3.2
   39      0.0       4.0
   41      3.2       4.0
   46      7.2       4.0
   47      7.6       4.0
   49      8.0       4.0
   50      0.0       4.8
   52      3.2       4.8
   57      7.2       4.8
   58      7.6       4.8
   60      8.0       4.8
   61      0.0       5.6
   63      3.2       5.6
   64      4.0       5.6
   65      4.2       5.2
   66      4.8       5.2
   67      5.6       5.2
   68      6.2       5.2
   69      4.2       5.6
   70      4.8       5.6
   71      5.6       5.6
   72      6.2       5.6
   73      6.4       5.6
   74      7.2       5.6
   75      7.6       5.6
   77      8.0       5.6
   78      0.0       6.4
   80      3.2       6.4
   81      4.0       6.4
   82      4.2       6.2
   83      4.8       6.2
   84      5.6       6.2
   85      6.2       6.2
   86      4.8       6.4
   89      7.2       6.4
   90      7.6       6.4
   92      8.0       6.4
   93      0.0       7.2
   95      3.2       7.2
  100      7.2       7.2
  101      7.6       7.2
  103      8.0       7.2
  104      0.0       7.6
```

```
  106      3.2       7.6
  111      7.2       7.6
  112      7.6       7.6
  114      8.0       7.6
  115      0.0       7.8
  117      3.2       7.8
  122      7.2       7.8
  123      7.6       7.8
  125      8.0       7.8
  126      0.0       8.0
  128      3.2       8.0
  133      7.2       8.0
  134      7.6       8.0
  136      8.0       8.0
ELEMENTS,114
    1    1    2   15   14    1
    6   19   20   27   18    1
    7   20   21   22   27    1
    8    8    9   22   21    1
   13   14   15   29   28    1
   17   18   27   33   32    1
   18   27   22   34   33    1
   19   22   23   35   34    1
   23   28   29   40   39    1
   33   39   40   51   50    1
   43   50   51   62   61    1
   46   53   65   69   64    1
   47   53   54   66   65    1
   50   68   56   73   72    1
   51   56   57   74   73    1
   55   61   62   79   78    1
   58   64   69   82   81    1
   59   72   73   88   85    1
   60   73   74   89   88    1
   64   82   83   86   81    1
   65   83   84   87   86    1
   67   78   79   94   93    1
   70   81   86   97   96    1
   71   86   87   98   97    1
   77   93   94  105  104    1
   87  104  105  116  115    1
   97  115  116  127  126    1
  107    6    7   20   19    2
  108    7    8   21   20    2
  109   65   66   70   69    2
  112   69   70   83   82    2
  114   71   72   85   84    2
MATERIAL,2,2
    4    4
        0.      0.10  0.000075     0.037  0.000200      0.02  0.000500    0.0035
        0.     0.167  0.000075     0.062  0.000200     0.033  0.000500    0.0055
  .390E-03
    0    0
        0.
        0.
        0.
BC,20
   13   26   12  1.    1   26   38   22  1.    1   38   49   32  1.    1   49   60   42  1.    1
   60   77   54  1.    1   77   92   63  1.    1   92  103   76  1.    1  103  114   86  1.    1
  114  125   96  1.    1  125  136  106  1.    1  126  127   97  1.    1  127  128   98  1.    1
  128  129   99  1.    1  129  130  100  1.    1  130  131  101  1.    1  131  132  102  1.    1
  132  133  103  1.    1  133  134  104  1.    1  134  135  105  1.    1  135  136  106  1.    1
CONVERGENCE
   30     0.002    0
STEP     0        0.         0.
STEP     1       1.0    0    3
STEP     2       2.0    0    3
STEP     3       2.0    0    3
STEP     4       2.0    0    3
STEP     5       2.0    0    3
```

TABLE B.1 INPUT FOR SAMPLE PROBLEM (TIME STEP DATA TRUNCATED)

```
************************************************************************

                          * S H R I N C *


          NON-UNIFORM DRYING SHRINKAGE OF TWO-DIMENSIONAL CROSS-SECTIONS
          WITH SHRINKAGE-DEPENDENT MATERIAL PROPERTIES


************************************************************************


                    - - - TITLE OF RUN - - -

***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

************************************************************************
```

```
************************************************************************

    *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

     ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

    GEOMETRIC DESCRIPTION OF SYSTEM TO BE ANALYZED

************************************************************************


 . . . . THERE ARE   136 NODAL POINTS . . . .

NODAL           COORDINATES                 BOUNDARY
POINT          X               Y            CONDITION

    1        0.              0.               FLOW
    2        1.6000          0.               FLOW
    3        3.2000          0.               FLOW
    4        4.0000          0.               FLOW
    5        4.8000          0.               FLOW
    6        5.0000          0.               FLOW
    7        5.6000          0.               FLOW
    8        6.2000          0.               FLOW
    9        6.4000          0.               FLOW
   10        7.2000          0.               FLOW
   11        7.6000          0.               FLOW
   12        7.8000          0.               FLOW
   13        8.0000          0.               FLOW
   14        0.              1.6000           FLOW
   15        1.6000          1.6000           FLOW
   16        3.2000          1.6000           FLOW
   17        4.0000          1.6000           FLOW
   18        4.8000          1.6000           FLOW
   19        5.0000           .4170           FLOW
   20        5.6000           .4170           FLOW
   21        6.2000           .4170           FLOW
   22        6.4000          1.6000           FLOW
   23        7.2000          1.6000           FLOW
   24        7.6000          1.6000           FLOW
   25        7.8000          1.6000           FLOW
   26        8.0000          1.6000           FLOW
   27        5.6000          1.6000           FLOW
   28        0.              3.2000           FLOW
   29        1.6000          3.2000           FLOW
   30        3.2000          3.2000           FLOW
   31        4.0000          3.2000           FLOW
   32        4.8000          3.2000           FLOW
   33        5.6000          3.2000           FLOW
   34        6.4000          3.2000           FLOW
   35        7.2000          3.2000           FLOW
   36        7.6000          3.2000           FLOW
   37        7.8000          3.2000           FLOW
   38        8.0000          3.2000           FLOW
```

TABLE B.2 OUTPUT FOR SAMPLE PROBLEM

```
  39        0.          4.0000      FLOW
  40        1.6000      4.0000      FLOW
  41        3.2000      4.0000      FLOW
  42        4.0000      4.0000      FLOW
  43        4.8000      4.0000      FLOW
  44        5.6000      4.0000      FLOW
  45        6.4000      4.0000      FLOW
  46        7.2000      4.0000      FLOW
  47        7.6000      4.0000      FLOW
  48        7.8000      4.0000      FLOW
  49        8.0000      4.0000      FLOW
  50        0.          4.8000      FLOW
  51        1.6000      4.8000      FLOW
  52        3.2000      4.8000      FLOW
  53        4.0000      4.8000      FLOW
  54        4.8000      4.8000      FLOW
  55        5.6000      4.8000      FLOW
  56        6.4000      4.8000      FLOW
  57        7.2000      4.8000      FLOW
  58        7.6000      4.8000      FLOW
  59        7.8000      4.8000      FLOW
  60        8.0000      4.8000      FLOW
  61        0.          5.6000      FLOW
  62        1.6000      5.6000      FLOW
  63        3.2000      5.6000      FLOW
  64        4.0000      5.6000      FLOW
  65        4.2000      5.2000      FLOW
  66        4.8000      5.2000      FLOW
  67        5.6000      5.2000      FLOW
  68        6.2000      5.2000      FLOW
  69        4.2000      5.6000      FLOW
  70        4.8000      5.6000      FLOW
  71        5.6000      5.6000      FLOW
  72        6.2000      5.6000      FLOW
  73        6.4000      5.6000      FLOW
  74        7.2000      5.6000      FLOW
  75        7.6000      5.6000      FLOW
  76        7.8000      5.6000      FLOW
  77        8.0000      5.6000      FLOW
  78        0.          6.4000      FLOW
  79        1.6000      6.4000      FLOW
  80        3.2000      6.4000      FLOW
  81        4.0000      6.4000      FLOW
  82        4.2000      6.2000      FLOW
  83        4.8000      6.2000      FLOW
  84        5.6000      6.2000      FLOW
  85        6.2000      6.2000      FLOW
  86        4.8000      6.4000      FLOW
  87        5.6000      6.4000      FLOW
  88        6.4000      6.4000      FLOW
  89        7.2000      6.4000      FLOW
  90        7.6000      6.4000      FLOW
  91        7.8000      6.4000      FLOW
  92        8.0000      6.4000      FLOW
  93        0.          7.2000      FLOW
  94        1.6000      7.2000      FLOW
```

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  95        3.2000      7.2000      FLOW
  96        4.0000      7.2000      FLOW
  97        4.8000      7.2000      FLOW
  98        5.6000      7.2000      FLOW
  99        6.4000      7.2000      FLOW
 100        7.2000      7.2000      FLOW
 101        7.6000      7.2000      FLOW
 102        7.8000      7.2000      FLOW
 103        8.0000      7.2000      FLOW
 104        0.          7.6000      FLOW
 105        1.6000      7.6000      FLOW
 106        3.2000      7.6000      FLOW
 107        4.0000      7.6000      FLOW
 108        4.8000      7.6000      FLOW
 109        5.6000      7.6000      FLOW
 110        6.4000      7.6000      FLOW
 111        7.2000      7.6000      FLOW
 112        7.6000      7.6000      FLOW
 113        7.8000      7.6000      FLOW
 114        8.0000      7.6000      FLOW
 115        0.          7.8000      FLOW
 116        1.6000      7.8000      FLOW
 117        3.2000      7.8000      FLOW
 118        4.0000      7.8000      FLOW
 119        4.8000      7.8000      FLOW
 120        5.6000      7.8000      FLOW
 121        6.4000      7.8000      FLOW
 122        7.2000      7.8000      FLOW
 123        7.6000      7.8000      FLOW
 124        7.8000      7.8000      FLOW
 125        8.0000      7.8000      FLOW
 126        0.          8.0000      FLOW
 127        1.6000      8.0000      FLOW
 128        3.2000      8.0000      FLOW
 129        4.0000      8.0000      FLOW
 130        4.8000      8.0000      FLOW
 131        5.6000      8.0000      FLOW
 132        6.4000      8.0000      FLOW
 133        7.2000      8.0000      FLOW
 134        7.6000      8.0000      FLOW
 135        7.8000      8.0000      FLOW
 136        8.0000      8.0000      FLOW


. . . . THERE ARE   114 ELEMENTS . . . .

ELEMENT          NODAL LOCATION             MATERIAL
NUMBER          I       J       K       L      TYPE

     1          1       2      15      14       1
     2          2       3      16      15       1
     3          3       4      17      16       1
     4          4       5      18      17       1
     5          5       6      19      18       1
```

```
  6      19    20    27    18      1
  7      20    21    22    27      1
  8       8     9    22    21      1
  9       9    10    23    22      1
 10      10    11    24    23      1
 11      11    12    25    24      1
 12      12    13    26    25      1
 13      14    15    29    28      1
 14      15    16    30    29      1
 15      16    17    31    30      1
 16      17    18    32    31      1
 17      18    27    33    32      1
 18      27    22    34    33      1
 19      22    23    35    34      1
 20      23    24    36    35      1
 21      24    25    37    36      1
 22      25    26    38    37      1
 23      28    29    40    39      1
 24      29    30    41    40      1
 25      30    31    42    41      1
 26      31    32    43    42      1
 27      32    33    44    43      1
 28      33    34    45    44      1
 29      34    35    46    45      1
 30      35    36    47    46      1
 31      36    37    48    47      1
 32      37    38    49    48      1
 33      39    40    51    50      1
 34      40    41    52    51      1
 35      41    42    53    52      1
 36      42    43    54    53      1
 37      43    44    55    54      1
 38      44    45    56    55      1
 39      45    46    57    56      1
 40      46    47    58    57      1
 41      47    48    59    58      1
 42      48    49    60    59      1
 43      50    51    62    61      1
 44      51    52    63    62      1
 45      52    53    64    63      1
 46      53    65    69    64      1
 47      53    54    66    65      1
 48      54    55    67    66      1
 49      55    56    68    67      1
 50      68    56    73    72      1
 51      56    57    74    73      1
 52      57    58    75    74      1
 53      58    59    76    75      1
 54      59    60    77    76      1
 55      61    62    79    78      1
 56      62    63    80    79      1
 57      63    64    81    80      1
 58      64    69    82    81      1
 59      72    73    88    85      1
 60      73    74    89    88      1
 61      74    75    90    89      1
```

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 62      75    76    91    90      1
 63      76    77    92    91      1
 64      82    83    86    81      1
 65      83    84    87    86      1
 66      84    85    88    87      1
 67      78    79    94    93      1
 68      79    80    95    94      1
 69      80    81    96    95      1
 70      81    86    97    96      1
 71      86    87    98    97      1
 72      87    88    99    98      1
 73      88    89   100    99      1
 74      89    90   101   100      1
 75      90    91   102   101      1
 76      91    92   103   102      1
 77      93    94   105   104      1
 78      94    95   106   105      1
 79      95    96   107   106      1
 80      96    97   108   107      1
 81      97    98   109   108      1
 82      98    99   110   109      1
 83      99   100   111   110      1
 84     100   101   112   111      1
 85     101   102   113   112      1
 86     102   103   114   113      1
 87     104   105   116   115      1
 88     105   106   117   116      1
 89     106   107   118   117      1
 90     107   108   119   118      1
 91     108   109   120   119      1
 92     109   110   121   120      1
 93     110   111   122   121      1
 94     111   112   123   122      1
 95     112   113   124   123      1
 96     113   114   125   124      1
 97     115   116   127   126      1
 98     116   117   128   127      1
 99     117   118   129   128      1
100     118   119   130   129      1
101     119   120   131   130      1
102     120   121   132   131      1
103     121   122   133   132      1
104     122   123   134   133      1
105     123   124   135   134      1
106     124   125   136   135      1
107       6     7    20    19      2
108       7     8    21    20      2
109      65    66    70    69      2
110      66    67    71    70      2
111      67    68    72    71      2
112      69    70    83    82      2
113      70    71    84    83      2
114      71    72    85    84      2

 . . . MAXIMUM BANDWIDTH IS   19 . . .
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*********************************************************************************

    *SHRINC* - NON-UNIFORM SHRINKAGE OF STRUCTURES

     ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

    SHRINKAGE PROPERTIES OF MATERIALS USED IN ANALYSIS

    THERE ARE    2 DIFFERENT MATERIALS

*********************************************************************************


 . . . . MATERIAL MODEL USED  . . . . . . .              2
         EQ.1  -  TIME-DEPENDENT
         EQ.2  -  SHRINKAGE-DEPENDENT


 . . . . MATERIAL NUMBER    1  . . . .

       . . . SHRINKAGE DIFFUSIVITY . . . . .

    NODE        TIME/SHR        VALUE        SLOPE

      1        0.              .100
                                         -.840E+03
      2          .750E-04      .370E-01
                                         -.136E+03
      3          .200E-03      .200E-01
                                        -55.000
      4          .500E-03      .350E-02

       . . . SURFACE FACTOR  . . . . . . . .

    NODE        TIME/SHR        VALUE        SLOPE

      1        0.              .167
                                         -.140E+04
      2          .750E-04      .620E-01
                                         -.232E+03
      3          .200E-03      .330E-01
                                        -91.667
      4          .500E-03      .550E-02

       . . ULTIMATE SHRINKAGE STRAIN . . . .

 MATERIAL PARAMETER OF CONSTANT VALUE     .390E-03
```

```
 . . . . MATERIAL NUMBER    2  . . . .


      . . . SHRINKAGE DIFFUSIVITY . . . . .

 MATERIAL PARAMETER OF CONSTANT VALUE    0.


      . . . SURFACE FACTOR  . . . . . . . .

 MATERIAL PARAMETER OF CONSTANT VALUE    0.


      . . ULTIMATE SHRINKAGE STRAIN . . . .

 MATERIAL PARAMETER OF CONSTANT VALUE    0.



 . . . . THERE ARE    20 SURFACE ELEMENTS EXPOSED TO DRYING  . . . . . . . .

 . . . . THERE ARE     0 SURFACE ELEMENTS WITH FLOW BOUNDARY CONDITIONS
 . . . . THERE ARE    20 SURFACE ELEMENTS WITH GRADIENT BOUNDARY CONDITIONS


       DESCRIPTION OF SURFACE EXPOSED TO DRYING

   DRYINGBC     NODE TO NODE    ELEMENT      SURFACE             LENGTH      KODEBC
   SURFACE        I       J     NUMBER       MULTIPLIER

       1         13      26       12         1.00000             1.600          1
       2         26      38       22         1.00000             1.600          1
       3         38      49       32         1.00000              .800          1
       4         49      60       42         1.00000              .800          1
       5         60      77       54         1.00000              .800          1
       6         77      92       63         1.00000              .800          1
       7         92     103       76         1.00000              .800          1
       8        103     114       86         1.00000              .400          1
       9        114     125       96         1.00000              .200          1
      10        125     136      106         1.00000              .200          1
      11        126     127       97         1.00000             1.600          1
      12        127     128       98         1.00000             1.600          1
      13        128     129       99         1.00000              .800          1
      14        129     130      100         1.00000              .800          1
      15        130     131      101         1.00000              .800          1
      16        131     132      102         1.00000              .800          1
      17        132     133      103         1.00000              .800          1
      18        133     134      104         1.00000              .400          1
      19        134     135      105         1.00000              .200          1
      20        135     136      106         1.00000              .200          1
```

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*******************************************************************************

   *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

    ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

   INFORMATION RELEVANT TO THE ANALYSIS PROCEDURE

*******************************************************************************


 . . . . CONVERGENCE CRITERIA . . . .

 CONVERGENCE CRITERIA FOR EACH TIME STEP

   PERMISSIBLE ERROR                                 =    .00200

   MAXIMUM NUMBER OF ITERATIONS                      =      30

   SHRINKAGE-DEPENDENT MATRICES REFORMATION FLAG =          0
      EQ.0,   UPDATE AT START OF EACH
              TIME STEP ONLY
      EQ.1,   UPDATE FOR EACH ITERATION



 . . . . STORAGE REQUIREMENT FOR BLANK COMMON . . . .

    SIZE BLANK COMMON =       10816 (DECIMAL)
                      =     0025100 (OCTAL)
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*******************************************************************************

   *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

    ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

   INITIAL SEQUENCE NUMBER IS     0 AND THE INITIAL TIME IS      0.

*******************************************************************************

------------ NODAL POINT SHRINKAGES ----------------


    N     SHRK.       N     SHRK.       N     SHRK.       N     SHRK.
    1  0.             2  0.             3  0.             4  0.
    5  0.             6  0.             7  0.             8  0.
    9  0.            10  0.            11  0.            12  0.
   13  0.            14  0.            15  0.            16  0.
   17  0.            18  0.            19  0.            20  0.
   21  0.            22  0.            23  0.            24  0.
   25  0.            26  0.            27  0.            28  0.
   29  0.            30  0.            31  0.            32  0.
   33  0.            34  0.            35  0.            36  0.
   37  0.            38  0.            39  0.            40  0.
   41  0.            42  0.            43  0.            44  0.
   45  0.            46  0.            47  0.            48  0.
   49  0.            50  0.            51  0.            52  0.
   53  0.            54  0.            55  0.            56  0.
   57  0.            58  0.            59  0.            60  0.
   61  0.            62  0.            63  0.            64  0.
   65  0.            66  0.            67  0.            68  0.
   69  0.            70  0.            71  0.            72  0.
   73  0.            74  0.            75  0.            76  0.
   77  0.            78  0.            79  0.            80  0.
   81  0.            82  0.            83  0.            84  0.
   85  0.            86  0.            87  0.            88  0.
   89  0.            90  0.            91  0.            92  0.
   93  0.            94  0.            95  0.            96  0.
   97  0.            98  0.            99  0.           100  0.
  101  0.           102  0.           103  0.           104  0.
  105  0.           106  0.           107  0.           108  0.
  109  0.           110  0.           111  0.           112  0.
  113  0.           114  0.           115  0.           116  0.
  117  0.           118  0.           119  0.           120  0.
  121  0.           122  0.           123  0.           124  0.
  125  0.           126  0.           127  0.           128  0.
  129  0.           130  0.           131  0.           132  0.
  133  0.           134  0.           135  0.           136  0.
```

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*********************************************************************************

   *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

    ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

   TIME STEP NUMBER     1 - TIME    1.000 - TIME STEP    1.000

*********************************************************************************

    NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY CONDITIONS     0

    NUMBER OF ITERATIONS TO EFFECT CONVERGENCE     4

------------ NODAL POINT SHRINKAGES -----------------


   N      SHRK.        N      SHRK.        N      SHRK.        N      SHRK.
   1    .000000        2    .000000        3    .000000        4    .000000
   5    .000000        6    .000000        7  0.                 8    .000001
   9    .000002       10    .000011       11    .000043       12    .000085
  13    .000162       14    .000000       15    .000000       16    .000000
  17    .000000       18    .000000       19    .000000       20    .000000
  21    .000001       22    .000001       23    .000011       24    .000043
  25    .000085       26    .000162       27    .000000       28    .000000
  29    .000000       30    .000000       31    .000000       32    .000000
  33    .000000       34    .000001       35    .000011       36    .000043
  37    .000085       38    .000162       39    .000000       40    .000000
  41    .000000       42    .000000       43    .000000       44    .000000
  45    .000001       46    .000011       47    .000043       48    .000085
  49    .000162       50    .000000       51    .000000       52    .000000
  53    .000000       54    .000000       55    .000000       56    .000001
  57    .000011       58    .000043       59    .000085       60    .000162
  61    .000000       62    .000000       63    .000000       64    .000000
  65    .000000       66    .000000       67    .000000       68    .000001
  69    .000000       70  0.                71  0.                72    .000001
  73    .000002       74    .000011       75    .000043       76    .000086
  77    .000162       78    .000001       79    .000001       80    .000001
  81    .000001       82    .000001       83    .000001       84    .000001
  85    .000002       86    .000002       87    .000002       88    .000003
  89    .000012       90    .000044       91    .000086       92    .000162
  93    .000011       94    .000011       95    .000011       96    .000011
  97    .000011       98    .000011       99    .000012      100    .000021
 101    .000052      102    .000092      103    .000167      104    .000043
 105    .000043      106    .000043      107    .000043      108    .000043
 109    .000043      110    .000044      111    .000052      112    .000078
 113    .000114      114    .000184      115    .000085      116    .000085
 117    .000085      118    .000085      119    .000085      120    .000086
 121    .000086      122    .000092      123    .000114      124    .000145
 125    .000205      126    .000162      127    .000162      128    .000162
 129    .000162      130    .000162      131    .000162      132    .000162
 133    .000167      134    .000184      135    .000205      136    .000253
```

```
-------------- SHRINKAGE OF ELEMENTS ---------------


   N      SHRK.        N      SHRK.        N      SHRK.        N      SHRK.
   1    .000000        2    .000000        3    .000000        4    .000000
   5    .000000        6    .000000        7    .000001        8    .000001
   9    .000006       10    .000027       11    .000064       12    .000124
  13    .000000       14    .000000       15    .000000       16    .000000
  17    .000000       18    .000001       19    .000006       20    .000027
  21    .000064       22    .000124       23    .000000       24    .000000
  25    .000000       26    .000000       27    .000000       28    .000001
  29    .000006       30    .000027       31    .000064       32    .000124
  33    .000000       34    .000000       35    .000000       36    .000000
  37    .000000       38    .000001       39    .000006       40    .000027
  41    .000064       42    .000124       43    .000000       44    .000000
  45    .000000       46    .000000       47    .000000       48    .000000
  49    .000001       50    .000001       51    .000006       52    .000027
  53    .000064       54    .000124       55    .000001       56    .000001
  57    .000001       58    .000001       59    .000002       60    .000007
  61    .000028       62    .000065       63    .000124       64    .000001
  65    .000001       66    .000002       67    .000006       68    .000006
  69    .000006       70    .000006       71    .000007       72    .000007
  73    .000012       74    .000032       75    .000069       76    .000127
  77    .000027       78    .000027       79    .000027       80    .000027
  81    .000027       82    .000028       83    .000032       84    .000051
  85    .000084       86    .000139       87    .000064       88    .000064
  89    .000064       90    .000064       91    .000064       92    .000065
  93    .000069       94    .000084       95    .000113       96    .000162
  97    .000124       98    .000124       99    .000124      100    .000124
 101    .000124      102    .000124      103    .000127      104    .000139
 105    .000162      106    .000202      107    .000000      108    .000000
 109    .000000      110    .000000      111    .000001      112    .000000
 113    .000000      114    .000001
```

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*******************************************************************************

   *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

    ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

   TIME STEP NUMBER    2 - TIME    3.000 - TIME STEP   2.000

*******************************************************************************

    NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY CONDITIONS     0

    NUMBER OF ITERATIONS TO EFFECT CONVERGENCE    5

----------- NODAL POINT SHRINKAGES ---------------

    N      SHRK.      N      SHRK.      N      SHRK.      N      SHRK.
    1    .000000      2    .000000      3    .000000      4    .000000
    5    .000000      6    .000000      7  0.                8    .000005
    9    .000007     10    .000026     11    .000061     12    .000101
   13    .000173     14    .000000     15    .000000     16    .000000
   17    .000000     18    .000000     19    .000000     20    .000001
   21    .000004     22    .000006     23    .000026     24    .000061
   25    .000101     26    .000173     27    .000001     28    .000000
   29    .000000     30    .000000     31    .000000     32    .000000
   33    .000001     34    .000006     35    .000026     36    .000061
   37    .000101     38    .000173     39    .000000     40    .000000
   41    .000000     42    .000000     43    .000000     44    .000001
   45    .000006     46    .000026     47    .000061     48    .000101
   49    .000173     50    .000000     51    .000000     52    .000000
   53    .000000     54    .000000     55    .000001     56    .000006
   57    .000026     58    .000061     59    .000102     60    .000173
   61    .000001     62    .000001     63    .000001     64    .000001
   65    .000000     66    .000000     67    .000001     68    .000005
   69    .000001     70  0.               71  0.               72    .000006
   73    .000008     74    .000027     75    .000061     76    .000102
   77    .000174     78    .000006     79    .000006     80    .000006
   81    .000006     82    .000005     83    .000005     84    .000006
   85    .000009     86    .000007     87    .000008     88    .000012
   89    .000030     90    .000064     91    .000105     92    .000176
   93    .000026     94    .000026     95    .000026     96    .000026
   97    .000026     98    .000027     99    .000030    100    .000046
  101    .000078    102    .000118    103    .000186    104    .000061
  105    .000061    106    .000061    107    .000061    108    .000061
  109    .000061    110    .000064    111    .000078    112    .000110
  113    .000147    114    .000208    115    .000101    116    .000101
  117    .000101    118    .000101    119    .000102    120    .000102
  121    .000105    122    .000118    123    .000147    124    .000177
  125    .000229    126    .000173    127    .000173    128    .000173
  129    .000173    130    .000173    131    .000174    132    .000176
  133    .000186    134    .000208    135    .000229    136    .000270

-------------- SHRINKAGE OF ELEMENTS ---------------

    N      SHRK.      N      SHRK.      N      SHRK.      N      SHRK.
    1    .000000      2    .000000      3    .000000      4    .000000
    5    .000000      6    .000001      7    .000003      8    .000005
    9    .000016     10    .000043     11    .000081     12    .000137
   13    .000000     14    .000000     15    .000000     16    .000000
   17    .000001     18    .000004     19    .000016     20    .000043
   21    .000081     22    .000137     23    .000000     24    .000000
   25    .000000     26    .000000     27    .000001     28    .000004
   29    .000016     30    .000043     31    .000081     32    .000137
   33    .000000     34    .000000     35    .000000     36    .000000
   37    .000001     38    .000004     39    .000016     40    .000043
   41    .000081     42    .000137     43    .000001     44    .000001
   45    .000001     46    .000001     47    .000000     48    .000001
   49    .000003     50    .000006     51    .000017     52    .000044
   53    .000081     54    .000138     55    .000004     56    .000004
   57    .000004     58    .000003     59    .000009     60    .000019
   61    .000046     62    .000083     63    .000139     64    .000006
   65    .000007     66    .000009     67    .000016     68    .000016
   69    .000016     70    .000016     71    .000017     72    .000019
   73    .000029     74    .000055     75    .000091     76    .000146
   77    .000043     78    .000043     79    .000043     80    .000043
   81    .000044     82    .000046     83    .000055     84    .000078
   85    .000113     86    .000165     87    .000081     88    .000081
   89    .000081     90    .000081     91    .000082     92    .000083
   93    .000091     94    .000113     95    .000145     96    .000190
   97    .000137     98    .000137     99    .000137    100    .000138
  101    .000138    102    .000139    103    .000146    104    .000165
  105    .000190    106    .000226    107    .000000    108    .000003
  109    .000000    110    .000000    111    .000003    112    .000003
  113    .000003    114    .000005
```

```
*******************************************************************************

     *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

      ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

     TIME STEP NUMBER   10 - TIME   30.000 - TIME STEP   5.000

*******************************************************************************

      NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY CONDITIONS     0

      NUMBER OF ITERATIONS TO EFFECT CONVERGENCE    4

------------ NODAL POINT SHRINKAGES ---------------

    N     SHRK.      N     SHRK.      N     SHRK.      N     SHRK.
    1   .000001      2   .000001      3   .000004      4   .000007
    5   .000012      6   .000012      7  0.              8   .000053
    9   .000057     10   .000105     11   .000155     12   .000191
   13   .000240     14   .000002     15   .000002     16   .000005
   17   .000008     18   .000015     19   .000015     20   .000027
   21   .000048     22   .000049     23   .000103     24   .000154
   25   .000190     26   .000240     27   .000028     28   .000004
   29   .000005     30   .000007     31   .000010     32   .000017
   33   .000029     34   .000050     35   .000102     36   .000154
   37   .000190     38   .000240     39   .000008     40   .000009
   41   .000010     42   .000013     43   .000019     44   .000030
   45   .000052     46   .000105     47   .000155     48   .000191
   49   .000241     50   .000015     51   .000016     52   .000017
   53   .000017     54   .000020     55   .000032     56   .000056
   57   .000110     58   .000160     59   .000195     60   .000243
   61   .000028     62   .000028     63   .000029     64   .000029
   65   .000021     66   .000020     67   .000031     68   .000056
   69   .000028     70  0.             71  0.             72   .000071
   73   .000074     74   .000121     75   .000168     76   .000202
   77   .000249     78   .000049     79   .000050     80   .000051
   81   .000056     82   .000053     83   .000069     84   .000078
   85   .000089     86   .000071     87   .000082     88   .000098
   89   .000140     90   .000183     91   .000216     92   .000259
   93   .000101     94   .000102     95   .000104     96   .000109
   97   .000116     98   .000125     99   .000141    100   .000179
  101   .000218    102   .000247    103   .000283    104   .000153
  105   .000153    106   .000155    107   .000158    108   .000163
  109   .000171    110   .000184    111   .000218    112   .000253
  113   .000276    114   .000304    115   .000189    116   .000189
  117   .000191    118   .000194    119   .000198    120   .000205
  121   .000217    122   .000247    123   .000276    124   .000295
  125   .000318    126   .000239    127   .000239    128   .000240
  129   .000242    130   .000245    131   .000251    132   .000260
  133   .000283    134   .000304    135   .000318    136   .000336
```

```
-------------- SHRINKAGE OF ELEMENTS ---------------

    N     SHRK.      N     SHRK.      N     SHRK.      N     SHRK.
    1   .000001      2   .000003      3   .000006      4   .000011
    5   .000014      6   .000021      7   .000038      8   .000052
    9   .000079     10   .000129     11   .000173     12   .000215
   13   .000003     14   .000005     15   .000008     16   .000013
   17   .000022     18   .000039     19   .000076     20   .000128
   21   .000172     22   .000215     23   .000007     24   .000008
   25   .000010     26   .000015     27   .000024     28   .000040
   29   .000077     30   .000129     31   .000173     32   .000215
   33   .000012     34   .000013     35   .000014     36   .000017
   37   .000025     38   .000042     39   .000081     40   .000132
   41   .000175     42   .000218     43   .000022     44   .000022
   45   .000023     46   .000024     47   .000020     48   .000026
   49   .000044     50   .000064     51   .000090     52   .000140
   53   .000181     54   .000222     55   .000039     56   .000039
   57   .000041     58   .000041     59   .000083     60   .000108
   61   .000153     62   .000192     63   .000232     64   .000062
   65   .000075     66   .000087     67   .000076     68   .000077
   69   .000080     70   .000088     71   .000099     72   .000111
   73   .000139     74   .000180     75   .000216     76   .000251
   77   .000127     78   .000128     79   .000131     80   .000137
   81   .000144     82   .000155     83   .000181     84   .000217
   85   .000248     86   .000277     87   .000171     88   .000172
   89   .000174     90   .000178     91   .000184     92   .000194
   93   .000217     94   .000248     95   .000275     96   .000298
   97   .000214     98   .000215     99   .000217    100   .000220
  101   .000225    102   .000233    103   .000252    104   .000277
  105   .000298    106   .000317    107   .000014    108   .000032
  109   .000017    110   .000013    111   .000039    112   .000037
  113   .000037    114   .000060
```

```
*************************************************************************************

   *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

    ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

   TIME STEP NUMBER   23 - TIME  180.000 - TIME STEP  20.000

*************************************************************************************

    NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY CONDITIONS   -0

    NUMBER OF ITERATIONS TO EFFECT CONVERGENCE    3

------------ NODAL POINT SHRINKAGES ---------------

     N     SHRK.       N     SHRK.       N     SHRK.       N     SHRK.
     1   .000045       2   .000048       3   .000057       4   .000066
     5   .000077       6   .000079       7  0.               8   .000160
     9   .000165      10   .000218      11   .000258      12   .000281
    13   .000309      14   .000048      15   .000051      16   .000061
    17   .000070      18   .000087      19   .000085      20   .000112
    21   .000151      22   .000153      23   .000213      24   .000255
    25   .000279      26   .000306      27   .000114      28   .000059
    29   .000061      30   .000071      31   .000080      32   .000097
    33   .000122      34   .000159      35   .000218      36   .000258
    37   .000282      38   .000309      39   .000069      40   .000071
    41   .000080      42   .000089      43   .000104      44   .000128
    45   .000166      46   .000225      47   .000264      48   .000287
    49   .000312      50   .000086      51   .000089      52   .000097
    53   .000101      54   .000110      55   .000135      56   .000180
    57   .000239      58   .000275      59   .000296      60   .000319
    61   .000112      62   .000115      63   .000123      64   .000126
    65   .000111      66   .000110      67   .000135      68   .000184
    69   .000125      70  0.              71  0.              72   .000217
    73   .000220      74   .000261      75   .000291      76   .000309
    77   .000329      78   .000150      79   .000152      80   .000162
    81   .000177      82   .000173      83   .000214      84   .000240
    85   .000249      86   .000217      87   .000243      88   .000259
    89   .000287      90   .000311      91   .000325      92   .000341
    93   .000209      94   .000211      95   .000222      96   .000236
    97   .000255      98   .000274      99   .000292     100   .000317
   101   .000335     102   .000345     103   .000356     104   .000252
   105   .000253     106   .000262     107   .000272     108   .000286
   109   .000300     110   .000315     111   .000335     112   .000349
   113   .000356     114   .000364     115   .000277     116   .000278
   117   .000285     118   .000294     119   .000305     120   .000316
   121   .000329     122   .000346     123   .000356     124   .000362
   125   .000369     126   .000305     127   .000305     128   .000311
   129   .000317     130   .000326     131   .000335     132   .000344
   133   .000357     134   .000365     135   .000369     136   .000374
```

```
-------------- SHRINKAGE OF ELEMENTS ---------------

     N     SHRK.       N     SHRK.       N     SHRK.       N     SHRK.
     1   .000048       2   .000054       3   .000063       4   .000075
     5   .000082       6   .000100       7   .000133       8   .000157
     9   .000187      10   .000236      11   .000268      12   .000294
    13   .000055      14   .000061      15   .000070      16   .000084
    17   .000105      18   .000137      19   .000186      20   .000236
    21   .000269      22   .000294      23   .000065      24   .000071
    25   .000080      26   .000092      27   .000113      28   .000144
    29   .000192      30   .000241      31   .000273      32   .000298
    33   .000079      34   .000084      35   .000092      36   .000101
    37   .000119      38   .000152      39   .000203      40   .000251
    41   .000281      42   .000304      43   .000100      44   .000106
    45   .000112      46   .000116      47   .000108      48   .000123
    49   .000158      50   .000200      51   .000225      52   .000267
    53   .000293      54   .000313      55   .000132      56   .000138
    57   .000147      58   .000151      59   .000236      60   .000257
    61   .000288      62   .000309      63   .000326      64   .000195
    65   .000228      66   .000248      67   .000181      68   .000187
    69   .000199      70   .000221      71   .000247      72   .000267
    73   .000289      74   .000313      75   .000329      76   .000342
    77   .000232      78   .000237      79   .000248      80   .000262
    81   .000279      82   .000295      83   .000315      84   .000334
    85   .000346      86   .000355      87   .000265      88   .000270
    89   .000278      90   .000289      91   .000302      92   .000315
    93   .000331      94   .000346      95   .000356      96   .000363
    97   .000291      98   .000295      99   .000302     100   .000310
   101   .000320     102   .000331     103   .000344     104   .000356
   105   .000363     106   .000369     107   .000069     108   .000106
   109   .000087     110   .000061     111   .000134     112   .000128
   113   .000114     114   .000176
```

```
*****************************************************************************

    *SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTURES

     ***  16 INCH BY 16 INCH COLUMN  -  SHRINKAGE ANALYSIS  ***

    TIME STEP NUMBER   40 - TIME 1000.000 - TIME STEP  50.000

*****************************************************************************

     NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY CONDITIONS   -0

     NUMBER OF ITERATIONS TO EFFECT CONVERGENCE    3

------------ NODAL POINT SHRINKAGES -----------------


    N     SHRK.       N     SHRK.       N     SHRK.       N     SHRK.
    1   .000211       2   .000215       3   .000228       4   .000239
    5   .000250       6   .000251       7  0.               8   .000305
    9   .000307      10   .000331      11   .000346      12   .000355
   13   .000363      14   .000215      15   .000220      16   .000233
   17   .000244      18   .000259      19   .000256      20   .000276
   21   .000300      22   .000301      23   .000330      24   .000345
   25   .000354      26   .000363      27   .000278      28   .000230
   29   .000234      30   .000246      31   .000256      32   .000269
   33   .000286      34   .000308      35   .000334      36   .000348
   37   .000356      38   .000365      39   .000242      40   .000245
   41   .000256      42   .000264      43   .000275      44   .000292
   45   .000313      46   .000338      47   .000352      48   .000359
   49   .000367      50   .000257      51   .000260      52   .000269
   53   .000273      54   .000280      55   .000297      56   .000322
   57   .000346      58   .000358      59   .000364      60   .000370
   61   .000276      62   .000279      63   .000287      64   .000290
   65   .000280      66   .000281      67   .000297      68   .000324
   69   .000290      70  0.              71  0.              72   .000341
   73   .000342      74   .000356      75   .000365      76   .000370
   77   .000375      78   .000299      79   .000301      80   .000310
   81   .000320      82   .000318      83   .000341      84   .000353
   85   .000355      86   .000341      87   .000354      88   .000359
   89   .000366      90   .000372      91   .000376      92   .000379
   93   .000327      94   .000329      95   .000336      96   .000344
   97   .000354      98   .000363      99   .000369     100   .000375
  101   .000379     102   .000381     103   .000383     104   .000343
  105   .000345     106   .000351     107   .000356     108   .000363
  109   .000370     110   .000374     111   .000379     112   .000382
  113   .000384     114   .000385     115   .000352     116   .000353
  117   .000358     118   .000363     119   .000368     120   .000373
  121   .000377     122   .000382     123   .000384     124   .000385
  125   .000386     126   .000362     127   .000362     128   .000366
  129   .000369     130   .000374     131   .000377     132   .000381
  133   .000384     134   .000385     135   .000386     136   .000387
```

```
--------------- SHRINKAGE OF ELEMENTS ----------------

    N     SHRK.       N     SHRK.       N     SHRK.       N     SHRK.
    1   .000215       2   .000224       3   .000236       4   .000248
    5   .000254       6   .000267       7   .000289       8   .000303
    9   .000317      10   .000338      11   .000350      12   .000359
   13   .000225      14   .000233      15   .000245      16   .000257
   17   .000273      18   .000293      19   .000318      20   .000339
   21   .000351      22   .000359      23   .000238      24   .000245
   25   .000255      26   .000266      27   .000280      28   .000300
   29   .000323      30   .000343      31   .000354      32   .000362
   33   .000251      34   .000258      35   .000265      36   .000273
   37   .000286      38   .000306      39   .000330      40   .000348
   41   .000358      42   .000365      43   .000268      44   .000274
   45   .000280      46   .000283      47   .000279      48   .000289
   49   .000310      50   .000332      51   .000341      52   .000356
   53   .000364      54   .000370      55   .000289      56   .000294
   57   .000302      58   .000305      59   .000349      60   .000356
   61   .000365      62   .000371      63   .000375      64   .000330
   65   .000347      66   .000355      67   .000314      68   .000319
   69   .000327      70   .000340      71   .000353      72   .000361
   73   .000367      74   .000373      75   .000377      76   .000380
   77   .000336      78   .000340      79   .000347      80   .000354
   81   .000362      82   .000369      83   .000375      84   .000379
   85   .000382      86   .000383      87   .000348      88   .000352
   89   .000357      90   .000363      91   .000369      92   .000374
   93   .000378      94   .000382      95   .000384      96   .000385
   97   .000357      98   .000360      99   .000364     100   .000368
  101   .000373     102   .000377     103   .000381     104   .000384
  105   .000385     106   .000386     107   .000196     108   .000220
  109   .000213     110   .000145     111   .000241     112   .000237
  113   .000174     114   .000262


SOLUTION TERMINATED - PROBLEM COMPLETE




INPUT DATA COMPLETE FOR THIS RUN
```

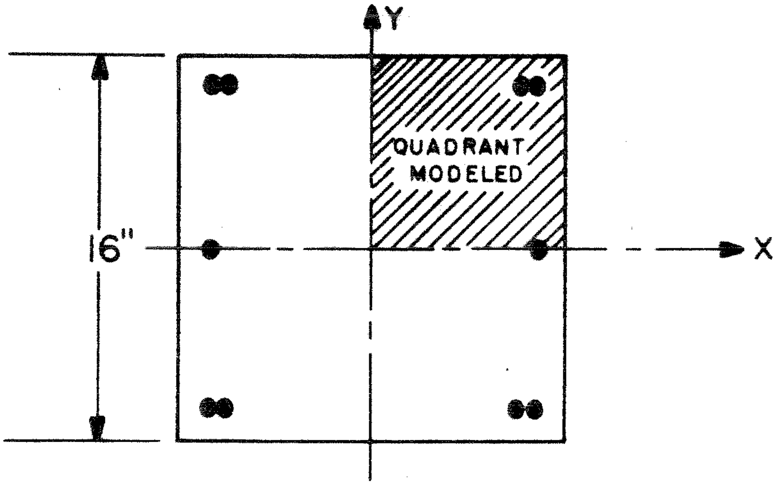


FIGURE B.1 SQUARE COLUMN CROSS-SECTION

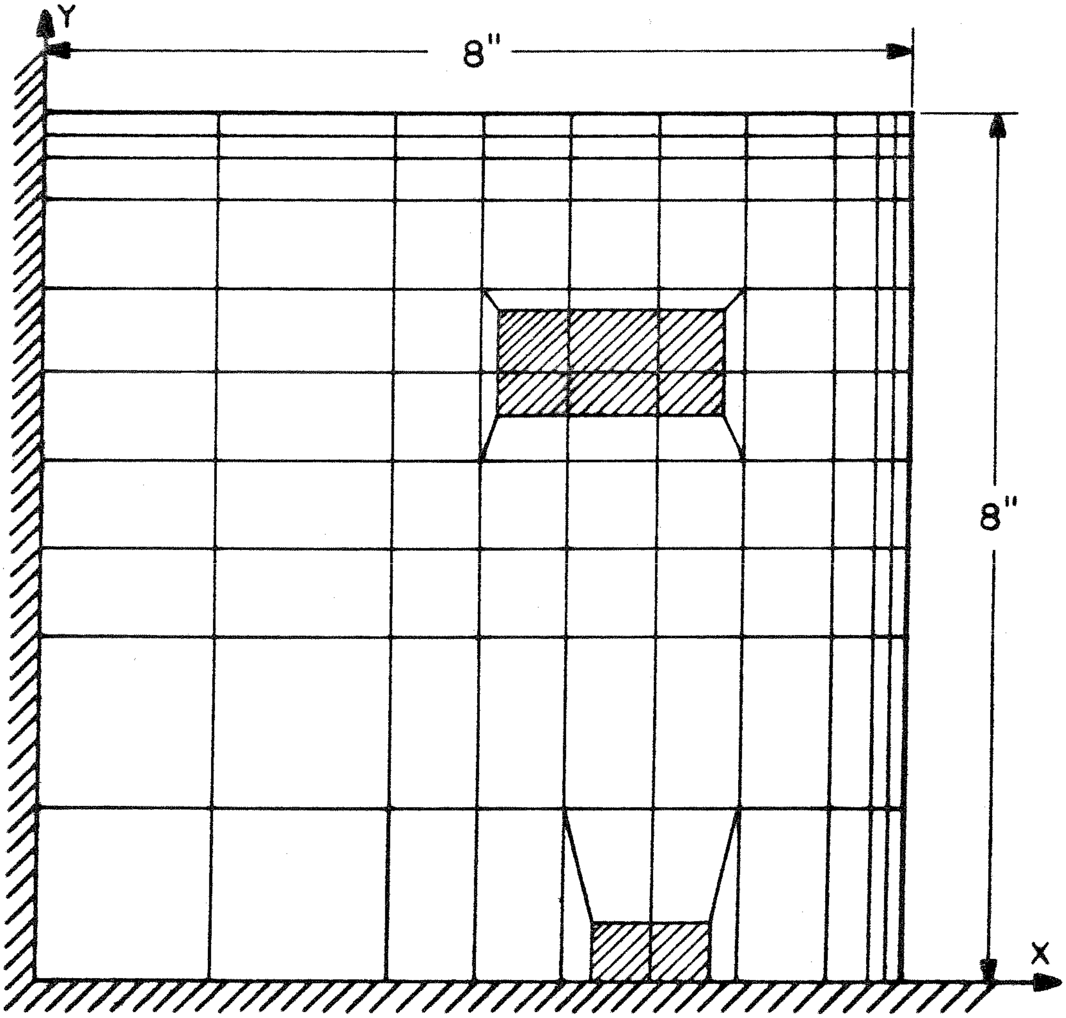


FIGURE B.2 FINITE ELEMENT MESH FOR COLUMN QUADRANT

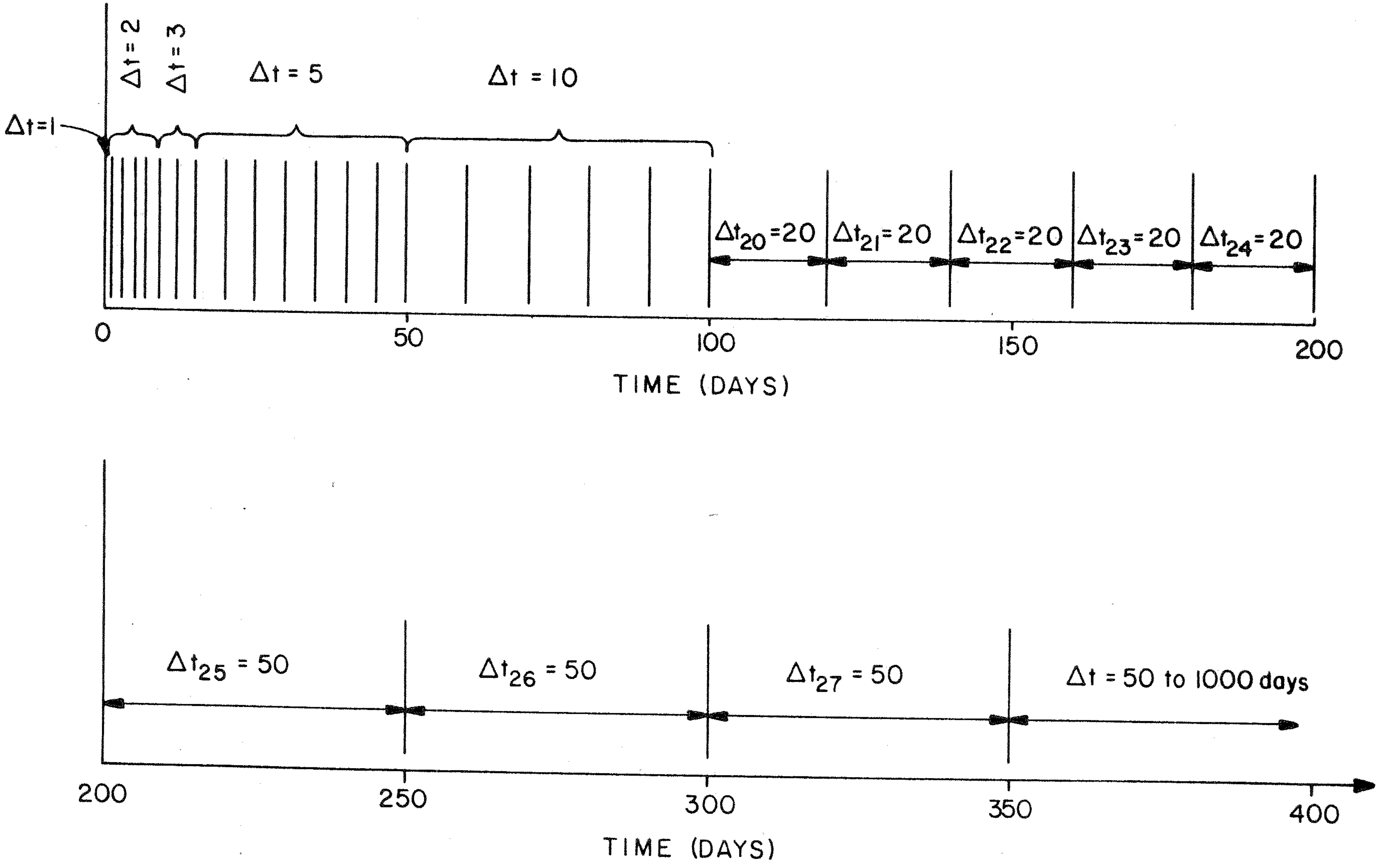
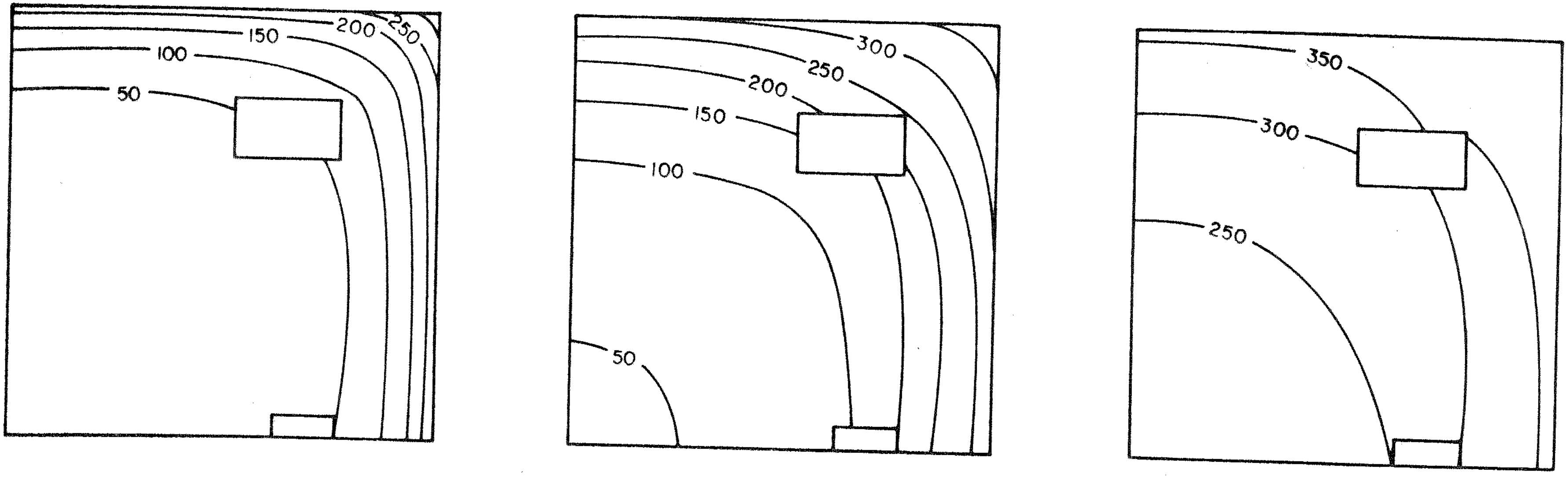


FIGURE B.3 TIME STEP SYSTEM FOR SAMPLE PROBLEM



a) AT T = 30 DAYS

b) AT T = 180 DAYS

c) AT T = 1000 DAYS

(CONTOUR INTERVAL = 50 MICROINCHES/INCH)

FIGURE B.4 SAMPLE PROBLEM AT 30, 180, and 1000 DAYS

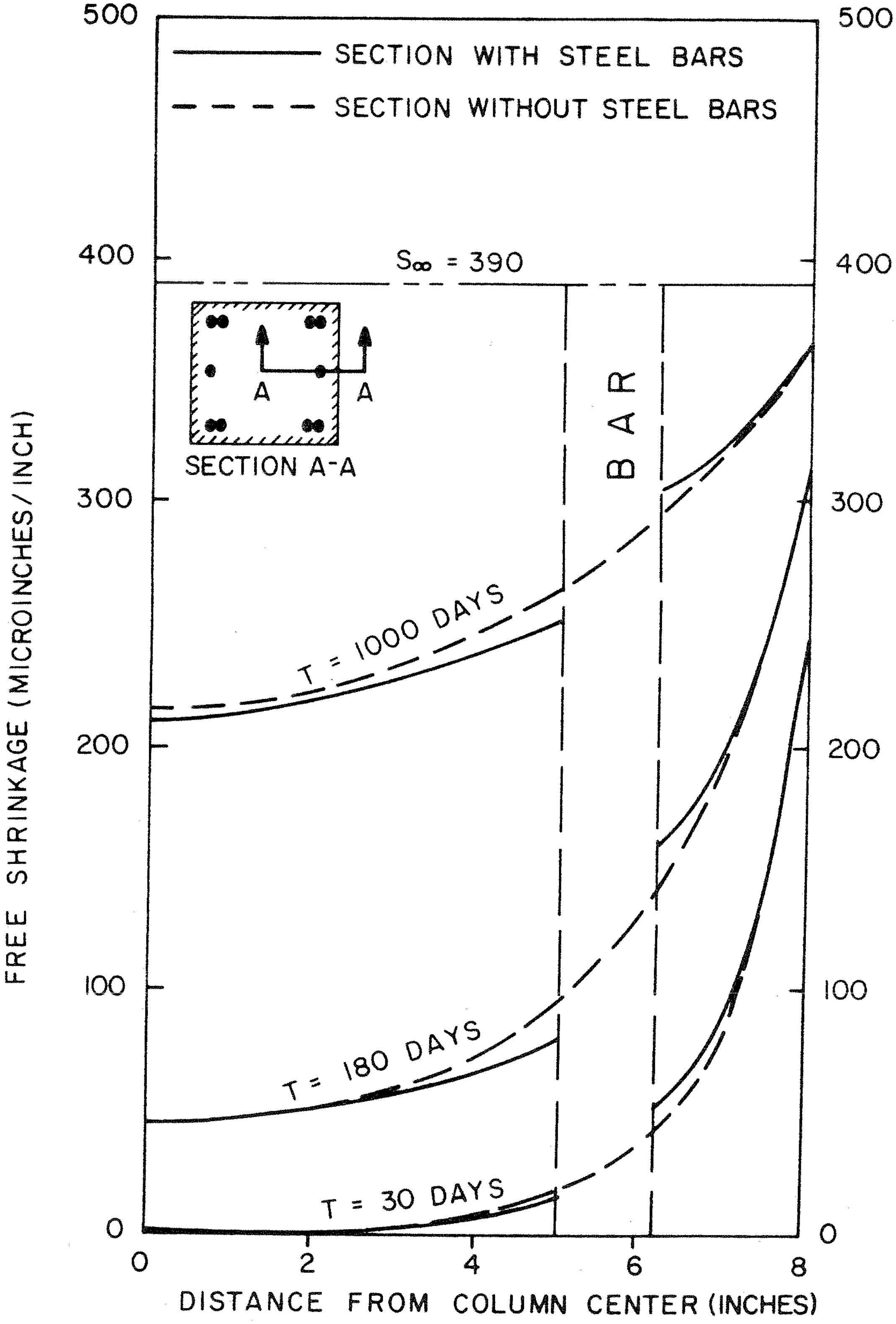


FIGURE B.5 SHRINKAGE GRADIENTS AT SELECTED TIMES THROUGH SECTION A-A

APPENDIX C - LISTING OF PROGRAM SHRINC

The following listing is the operational version of the computer program SHRINC, as of August 1975. Although the program has been tested, no warranty is made regarding its accuracy or reliability.

```
      PROGRAM SHRINC (INPUT,OUTPUT,PUNCH,TAPE1=INPUT,TAPE2=OUTPUT,TAPE3=
     1PUNCH)
C
C
C
C
C     ******************************************************************
C     *                                                                *
C     *                                                                *
C     *    S H R I N C   IS A PROGRAM TO SOLVE THE NONLINEAR, TWO-     *
C     *    DIMENSIONAL SHRINKAGE DIFFUSION BOUNDARY VALUE PROBLEM.     *
C     *    THE PROBLEM IS SOLVED ANALYTICALLY BY A FINITE ELEMENT      *
C     *    METHOD COUPLED WITH A TIME STEP INTEGRATION.                *
C     *                                                                *
C     *    MATERIAL PROPERTIES CAN BE FUNCTIONS OF DRYING SHRINKAGE    *
C     *    OR OF DRYING TIME.                                          *
C     *                                                                *
C     *                                                                *
C     *       U N I V E R S I T Y   O F   C A L I F O R N I A          *
C     *                                                                *
C     *                      M A Y   1 9 7 5                           *
C     *                                                                *
C     *                                                                *
C     ******************************************************************
C
C
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      COMMON C(20000)
C
C
      NIN=1
      NOUT=2
      NPUNCH=3
C
  100 NDRYBC=0
      NGRADBC=0
C
C     INPUT TITLE OF PROBLEM
C
      READ (NIN,160) ITITLE
C
C     INPUT OF PROBLEM DESCRIPTION DONE ON BASIS OF ALPHA-NUMERIC
C     CONTROL CARDS IN WHICH THE FIRST LETTER OF THE CONTROL
C     WORD IS CHECKED FOR PROPER INPUT SEQUENCE
C
      READ (NIN,310) IREAD
      IF (IREAD(1).NE.16B) GO TO 155
      N=1
C
C     OUTPUT TITLE PAGE
C
      WRITE (NOUT,170)
      WRITE (NOUT,180)
      WRITE (NOUT,190)
      WRITE (NOUT,290)
      WRITE (NOUT,180)
      WRITE (NOUT,300) ITITLE
      WRITE (NOUT,180)
C
C     INPUT OF NODAL DATA
C
C        NUMNP - NUMBER OF NODAL POINTS
C        NSBC - NUMBER OF FIXED SHRINKAGE BOUNDARY CONDITIONS
C               DEFAULT IS FIXED FLOW BOUNDARY CONDITIONS OR
C               FIXED SHRINKAGE GRADIENT BOUNDARY CONDITIONS
C        X(I)=C(N1) - X COORDINATE OF NODAL POINT
C        Y(I)=C(N2) - Y COORDINATE OF NODAL POINT
C        KODE(I)=C(N3) - INDICATES TYPE OF BOUNDARY CONDITION
C                        FLOW - SHRINKAGE FLOW IS KNOWN
C                               OR SHRINKAGE GRADIENT IS KNOWN
C                        SHRK - SHRINKAGE IS KNOWN
C        DUMI(I)=C(NDI) - DUMMY VARIABLES REQUIRED IN CALCULATION
C                         I VARIES FROM 1 TO 5
C
      NUMNP=NUMBER(N)
      IF (NUMNP.LE.0) GO TO 150
      NSBC=NUMBER(N)
      IF (NSBC.GT.NUMNP) GO TO 150
      N1=1
      N2=N1+NUMNP
      N3=N2+NUMNP
      ND1=N3+NUMNP
      ND2=ND1+NUMNP
      ND3=ND2+NUMNP
      ND4=ND3+NUMNP
      ND5=ND4+NUMNP
      N4=ND5+NUMNP
C
      CALL NODE (C(N1),C(N2),C(N3),C(ND1),NSBC)
C
      READ (NIN,310) IREAD
      IF (IREAD(1).NE.5B) GO TO 150
      N=1
C
C     INPUT OF ELEMENT DATA
C
C        NUMEL - NUMBER OF ELEMENTS
C        MBAND - MAXIMUM BANDWIDTH OF DIFFUSIVITY MATRIX
C        LM(1,I)=C(N4) - ELEMENT NODAL POINT CONFIGURATION
C        MMTYPE(I)=C(N5) - ELEMENT MATERIAL TYPE
C        XLAM(I)=C(N6) - MULTIPLIER CONSTANT FOR DIFFUSIVITY MATRIX
C        SSI(1,I)=C(NSI) - CONSTANT TERMS OF DIFFUSIVITY MATRIX
C                          I HAS THE FOLLOWING VALUES 1,2,3,5,6,9
C
      NUMEL=NUMBER(N)
      IF (NUMEL.LE.0) GO TO 150
      N5=N4+5*NUMEL
      N6=N5+NUMEL
      N=4*NUMEL
      NS1=N6+N
      NS2=NS1+N
      NS3=NS2+N
      NS5=NS3+N
      NS6=NS5+N
      NS9=NS6+N
      N7=NS9+N
C
      CALL ELEMENT (C(N1),C(N2),C(N4),C(N5),C(N6),C(NS1),C(NS2),C(NS3),C
     1(NS5),C(NS6),C(NS9))
C
      READ (NIN,310) IREAD
```

```
      IF (IREAD(1).NE.15B) GO TO 150
      N=1
C
C     INPUT OF SHRINKAGE PROPERTIES FOR DIFFERENT MATERIALS
C
C        NMAT - NUMBER OF DIFFERENT MATERIALS
C        MATMOD - MATERIAL MODEL
C                 EQ.1,    TIME-DEPENDENT
C                 EQ.2,    SHRINKAGE-DEPENDENT
C        MATL(1)=C(N7) - CONTROL DATA REQUIRED FOR CALCULATION OF
C                        SHRINKAGE PROPERTIES
C        XYS(1)=C(N8) - FUNCTION VALUES FOR SHRINKAGE PROPERTIES
C                       CONTAINS  X COORDINATE - TIME/SHRINKAGE
C                                 Y COORDINATE - FUNCTION VALUE
C                                 S - SLOPE OF LINES CONNECTING X,Y PAIR
C
      NMAT=NUMBER(N)
      IF (NMAT.EQ.0) GO TO 150
      MATMOD=NUMBER(N)
      IF (MATMOD.NE.1 .AND. MATMOD.NE.2) GO TO 150
      N8=N7+6*NMAT
C
      CALL MATRIAL (C(N5),C(N7),C(N8),N)
      N9=N8+N
C
      READ (NIN,310) IREAD
      N=1
      IF (IREAD(1).NE.2B) GO TO 150
C
C     INPUT DRYING BOUNDARY CONDITIONS DATA
C     ( I.E., SHRINKAGE-DEPENDENT BOUNDARY CONDITIONS )
C
C        NDRYBC - TOTAL NUMBER OF DRYING BOUNDARY COND. SURFACE SEGMENTS
C        NGRADBC - NUMBER OF SEGMENTS IN WHICH SHRINKAGE GRADIENTS
C                  ARE SPECIFIED (REQUIRING ITERATIVE SOLUTION PROCESS)
C        LI(1)=C(N9)  - NODE THAT BOUNDS ONE END OF SURFACE SEGMENT
C        LJ(1)=C(N10) - NODE BOUNDING OTHER END OF SURFACE SEGMENT
C        LMAT(1)=C(N11) - ELEMENT IDENTIFICATION NUMBER
C        SURFAC(1)=C(N12) - MULTIPLIER FOR MATERIAL SURFACE FACTOR
C        XL(1)=C(N13) - LENGTH OF SURFACE I-J
C        KODEBC(1)=C(N14) - TYPE OF DRYING BOUNDARY CONDITION
C                           EQ.0, FLOW BOUNDARY CONDITION
C                                 Q = F*(S(ULTIMATE) - S)
C                           EQ.1, SHRINKAGE GRADIENT BOUNDARY CONDITION
C                                 DS/DN = (F/K)*(S(ULTIMATE) - S)
C
      NDRYBC=NUMBER(N)
      IF (NDRYBC.EQ.0) GO TO 130
      N10=N9+NDRYBC
      N11=N10+NDRYBC
      N12=N11+NDRYBC
      N13=N12+NDRYBC
      N14=N13+NDRYBC
      N15=N14+NDRYBC
C
      CALL DRYBC (C(N1),C(N2),C(N3),C(N9),C(N10),C(N11),C(N12),C(N13),
     1C(N14))
      GO TO 140
C
  130 CONTINUE
C
C     WHEN THERE ARE NO DRYING BOUNDARY CONDITIONS IT IS
C     STILL NECESSARY TO ESTABLISH A MINIMAL STORAGE FOR THE DRYING
C     BOUNDARY CONDITION RELATED VARIABLES
C
      N10=N9+1
      N11=N10+1
      N12=N11+1
      N13=N12+1
      N14=N13+1
      N15=N14+1
C
  140 CONTINUE
C
C     ESTABLISH ADDITIONAL DYNAMICALLY DIMENSIONED VARIABLES REQUIRED
C     IN THE TIME-STEP SHRINKAGE ANALYSIS
C
C         Q(1)=C(N15) - FLOW VECTOR
C         T(1)=C(N16) - SHRINKAGE VECTOR
C         B(1)=C(N17) - LOADING (FLOW) VECTOR USED IN ANALYSIS
C         AT(1)=C(N18) - ELEMENT SHRINKAGES
C         A(1,1)=C(N19) - EFFECTIVE DIFFUSIVITY MATRIX
C         AA(1,1)=C(N20) - EFFECTIVE DIFFUSIVITY MATRIX WORKING SPACE
C         NTOTAL - TOTAL AMOUNT OF STORAGE REQUIRED FOR BLANK COMMON
C
      N16=N15+NUMNP
      N17=N16+NUMNP
      N18=N17+NUMNP
      N19=N18+NUMEL
      N20=N19+NUMNP*MBAND
      NTOTAL=N20+NUMNP*MBAND
C
      READ (NIN,310) IREAD
      IF (IREAD(1).NE.3B) GO TO 150
C
C     INPUT CONVERGENCE CONTROL INFORMATION
C
      CALL CONVERG (NTOTAL)
C
C     TRANSFER CONTROL OF PROGRAM TO TIME-STEP ANALYSIS ROUTINE
C
      CALL SHRKFLO (C(N1),C(N2),C(N3),C(N4),C(N5),C(N6),C(N7),C(N8),C(N9
     1),C(N10),C(N11),C(N12),C(N13),C(N14),C(N15),C(N16),C(N17),C(N18),
     2C(N19),NUMNP,C(NS1),C(NS2),C(NS3),C(NS5),C(NS6),C(NS9),C(ND1),
     3C(ND2),C(ND3),C(ND4),C(ND5),C(N20))
C
C     PROBLEM COMPLETE  -  RETURN TO START NEXT DATA CASE
C
      GO TO 100
C
  150 CONTINUE
C
C     ERROR WAS DETECTED IN CONTROL WORD OR CONTROL VARIABLE
C     HAS IMPROPER VALUE AND PROGRAM IS TERMINATED
C
      WRITE (NOUT,320) IREAD
      STOP
C
  155 CONTINUE
C
C     NO DATA WAS SUPPLIED FOR NEXT PROBLEM AND PROGRAM IS HALTED
C
      WRITE (NOUT,330)
      STOP
```

```
C
C
C
  160 FORMAT (8A10)
  170 FORMAT (1H6,13(/))
  180 FORMAT (80H ********************************************************
     1************************)
  190 FORMAT (///32X,15H* S H R I N C *,///)
  290 FORMAT (    16X,48HNON-UNIFORM DRYING SHRINKAGE OF TWO-DIMENSIONAL
     1,15HCROSS-SECTIONS  /16X,34HWITH SHRINKAGE-DEPENDENT MATERIAL
     2,12HPROPERTIES      ///)
  300 FORMAT (///20X,24H- - - TITLE OF RUN - - -,//1X,8A10///)
  310 FORMAT (80R1)
  320 FORMAT (6(/),45H - - - PROGRAM TERMINATED - INPUT ERROR - - -,//1X
     1,80R1)
  330 FORMAT (6(/),34H INPUT DATA COMPLETE FOR THIS RUN     )
      END
```

```
      SUBROUTINE NODE (X,Y,KODE,ID,NSBC)
C
C
C     SUBROUTINE NODE INPUTS THE NODAL COORDINATES AND SETS
C     THE FLOW AND SHRINKAGE BOUNDARY CONDITIONS
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION X(1), Y(1), KODE(1), ID(1)
C
C     OUTPUT PAGE HEADING
C
      WRITE (NOUT,200)
      WRITE (NOUT,210)
      WRITE (NOUT,220) ITITLE
      WRITE (NOUT,230)
      WRITE (NOUT,210)
      WRITE (NOUT,240) NUMNP
      WRITE (NOUT,250)
C
C     INITIALIZE BOUNDARY CONDITIONS TO FLOW
C
      DO 110 I=1,NUMNP
  110 KODE(I)=4HFLOW
C
C     INPUT NODAL COORDINATES
C
      PI=3.1415926535898
      IRADTL=0
      L=1
  120 READ (NIN,260) N,X(N),Y(N),IRADIAL
      IRADTL=IRADTL + IRADIAL
      IF (N.GE.L) GO TO 140
C
  130 WRITE (NOUT,270) N,X(N),Y(N)
      STOP
C
  140 IF (N.EQ.L) GO TO 160
      IF (L.EQ.1) GO TO 130
C
C     WHEN N.GE.L THE PROGRAM GENFRATES INTERMEDIATE NODES AT INTERVALS
C     OF DX AND DY
C
C     IF CIRCULAR CROSS-SECTION (IRADIAL.EQ.1) IS SPECIFIED THEN FIRST
C     NODE IN SERIES MUST BE ALONG +X AXIS OF POSITIVE QUADRANT AND
C     SECOND NODE IN SERIES MUST LIE ALONG +Y AXIS OF QUADRANT
C     INTERMEDIATE NODES ARE GENFRATED ALONG CONNECTING ARC
C
      DIFF=N+1-L
      IF (IRADIAL.EQ.0) GO TO 149
      IL=N - L
      DR=X(L-1)
      DIFR=Y(N) - DR
      IF (DIFR.LT.1.E-6) GO TO 141
      WRITE (NOUT,270) N,X(N),Y(N)
      STOP
  141 ANG=(PI/2.)/DIFF
      ANCUM=0.
      DO 142 I=1,IL
      ANCUM=ANCUM + ANG
      X(L)=DR*COS(ANCUM)
      Y(L)=DR*SIN(ANCUM)
  142 L=L + 1
      GO TO 160
  149 DX=(X(N)-X(L-1))/DIFF
      DY=(Y(N)-Y(L-1))/DIFF
  150 X(L)=X(L-1)+DX
      Y(L)=Y(L-1)+DY
      L=L+1
      IF (N.GT.L) GO TO 150
  160 L=L + 1
  161 IF (N.LT.NUMNP) GO TO 120
      IF (N.NE.NUMNP) GO TO 130
      IF (NSBC.EQ.0) GO TO 180
C
C     INPUT NODES AT WHICH SHRINKAGE BOUNDARY CONDITIONS ARE SPECIFIED
C
      READ (NIN,280) (ID(I),I=1,NSBC)
      DO 170 I=1,NSBC
      J=ID(I)
      IF (J.LE.0.OR.J.GT.NUMNP) GO TO 190
  170 KODE(J)=4HSHRK
C
C     OUTPUT NODAL COORDINATES AND BOUNDARY CONDITION CODE
C
  180 WRITE (NOUT,290) (I,X(I),Y(I),KODE(I),I=1,NUMNP)
C
C     PUNCH NODAL MESH IF CIRCULAR MESH WAS GENERATED
C
      IF (IRADTL.EQ.0) RETURN
      DO 181 I=1,NUMNP
  181 WRITE (NPUNCH,310) I, X(I), Y(I)
      RETURN
C
  190 WRITE (NOUT,300)
      STOP
C
C
C
  200 FORMAT (1H6,5(/))
  210 FORMAT (80H ********************************************************
```

```
     1***************************)
  220 FORMAT (/5X,43H*SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF ,
     110HSTRUCTURES,//5X,8A10)
  230 FORMAT (/5X,46HGEOMETRIC DESCRIPTION OF SYSTEM TO BE ANALYZED,/)
  240 FORMAT (//,20H  . . . . THERE ARE ,I5,22H NODAL POINTS . . . . )
  250 FORMAT (//,7H  NODAL,11X,11HCOORDINATES,13X,8HBOUNDARY,/,7H  POINT
     1,10X,1HX,14X,1HY,9X,9HCONDITION,/)
  260 FORMAT (I5,2F10.0,I5)
  270 FORMAT (5(/),51H  - - - PROGRAM TERMINATED - NODE INPUT ERROR - -
     1-,//1X,I5,2F10.4)
  280 FORMAT (16I5)
  290 FORMAT (I7,2F15.4,8X,A4)
  300 FORMAT (5(/),33H  - - - PROGRAM TERMINATED - - - ,/,47H  ERROR IN
     1SHRINKAGE BOUNDARY CONDITION INPUT     )
  310 FORMAT (I5,2F10.6)
      END
```

```
      SUBROUTINE ELEMENT (X,Y,LM,MMTYPE,XLAM,SS1,SS2,SS3,SS5,SS6,SS9)
C
C
C     SUBROUTINE ELEMENT INPUTS DATA DESCRIBING THE ELEMENTS IN THE
C     SYSTEM TO BE ANALYZED AND ALSO CALCULATES THE CONSTANT TERMS
C     ASSOCIATED WITH THE DIFFUSIVITY MATRIX.  QUADRILATERAL ELEMENTS
C     ARE FORMED FROM 4 LINEAR TRIANGULAR ELEMENTS, THUS THE CONSTANT
C     TERMS CALCULATED AND STORED ARE THOSE OF THE RELEVANT TRIANGULAR
C     ELEMENT.  IN ADDITION TRIANGULAR ELEMENTS ARE CREATED FROM
C     THREE LINEAR TRIANGULAR ELEMENTS.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION X(1), Y(1), LM(5,1), MMTYPE(1), XLAM(4,1), SS1(4,1), SS2
     1(4,1), SS3(4,1), SS5(4,1), SS6(4,1), SS9(4,1)
C
C     OUTPUT HEADING FOR ELEMENT DATA
C
      WRITE (NOUT,250) NUMEL
      WRITE (NOUT,260)
      MBAND=0
      NUM=0
C
      DO 240 N=1,NUMEL
      IF (NUM-N) 110,120,120
C
C     READ IN ELEMENT CARD
C
  110 READ (NIN,270) NUM,K1,K2,K3,K4,MTYPE
      IF (MTYPE.EQ.0) MTYPE=1
      IF (NUM.GT.NUMEL) GO TO 150
C
      IF (N.EQ.1) GO TO 140
C
C     SHIFT ELEMENT DESCRIPTION ONE LOCATION
C
  120 DO 130 I=1,4
  130 LM(I,N)=LM(I,N-1)+1
      MMTYPE(N)=MMTYPE(N-1)
C
  140 IF (NUM-N) 150,160,170
C
  150 CONTINUE
      WRITE (NOUT,280) NUM,K1,K2,K3,K4,MTYPE
      STOP
C
  160 CONTINUE
      LM(1,N)=K1
      LM(2,N)=K2
      LM(3,N)=K3
      LM(4,N)=K4
      MMTYPE(N)=MTYPE
  170 CONTINUE
C
C     CALCULATE THE LOCATION OF THE ELEMENTS CENTER OF GRAVITY
C     C. G. COORDINATES ARE XX AND YY
C
      I=LM(1,N)
      J=LM(2,N)
      K=LM(3,N)
      L=LM(4,N)
      LM(5,N)=I
      IF (K.EQ.L) GO TO 180
C
      XX=(X(I)+X(J)+X(K)+X(L))/4.
      YY=(Y(I)+Y(J)+Y(K)+Y(L))/4.
      GO TO 190
  180 XX=(X(I)+X(J)+X(K))/3.0
      YY=(Y(I)+Y(J)+Y(K))/3.0
  190 CONTINUE
C
C     CALCULATE DATA REQUIRED IN DETERMINING THE CONSTANT TERMS
C     FOR THE DIFFUSIVITY MATRIX
C
      DO 210 K=1,4
C
      I=LM(K,N)
      J=LM(K+1,N)
      IF (I-J) 200,210,200
  200 AJ=X(J)-X(I)
      AK=XX-X(I)
      BJ=Y(J)-Y(I)
      BK=YY-Y(I)
      C=BJ-BK
      DX=AK-AJ
C
C
C     CALCULATE THE CONSTANT TERMS FOR EACH ELEMENTS DIFFUSIVITY MATRIX
C     WHERE
C                                                I                 I
C        K(TRIANGLE) =  K(TIME/SHRK)   I SS1   SS2   SS3 I
C                                ---------------  I                 I
C                                     2*XLAM     I       SS5   SS6 I
C                                                I                 I
C                                                I             SS9 I
C                                                I                 I
C
      XLAM(K,N)=AJ*BK-AK*BJ
      SS1(K,N)=C**2+DX**2
      SS2(K,N)=BK*C-AK*DX
      SS3(K,N)=-BJ*C+AJ*DX
      SS5(K,N)=BK**2+AK**2
```

```
      SS6(K,N)=-BJ*BK-AJ*AK
      SS9(K,N)=BJ**2+AJ**2
  210 CONTINUE
C
C     DETERMINE BANDWIDTH AND STORE IF MAXIMUM
C
      DO 230 L=1,4
      I=LM(L,N)
      DO 230 M=1,4
      J=LM(M,N)-I+1
      IF (MBAND-J) 220,230,230
  220 MBAND=J
  230 CONTINUE
  240 CONTINUE
C
C     PRINT OUT ELEMENT DATA AND MAXIMUM BANDWIDTH
C
      WRITE (NOUT,290) (I,(LM(J,I),J=1,4),MMTYPE(I),I=1,NUMEL)
      WRITE (NOUT,300) MBAND
      RETURN
C
C
C
  250 FORMAT (3(/),20H  . . . . THERE ARE ,I5,18H ELEMENTS . . . . ,//)
  260 FORMAT (2X,7HELEMENT,10X,14HNODAL LOCATION,11X,8HMATERIAL/2X,6HNUM
     1BER,10X,1HI,6X,1HJ,6X,1HK,6X,1HL,5X,4HTYPE/)
  270 FORMAT (6I5)
  280 FORMAT (5(/),50H  - - - - PROGRAM TERMINATED - ELEMENT INPUT ERROR
     1,/1X,6I5)
  290 FORMAT (I8,4X,4I7,I8)
  300 FORMAT (///,31H     . . . MAXIMUM BANDWIDTH IS ,I4,6H . . .)
      END


      FUNCTION NUMBER (I)
C
C
C     NUMBER IS A FUNCTION THAT OBTAINS AN INTEGER CONSTANT CONTAINED
C     ON AN ALPHA-NUMERIC CONTROL CARD.  THE INTEGER CONSTANT MUST
C     BE CONTAINED BETWEEN COMMAS OR BETWEEN A COMMA AND A BLANK.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
C
      K=0
  110 J=IREAD(I)
      IF (J.EQ.56B) GO TO 120
      IF (J.GT.32B) GO TO 140
      I=I+1
      IF (I.EQ.81) GO TO 130
      GO TO 110
  120 I=I+1
      J=IREAD(I)
      IF (J.EQ.55B.OR.J.EQ.56B) GO TO 130
      IF (J.LT.33B.OR.J.GT.44B) GO TO 140
      J=J-33B
      K=K*10+J
      GO TO 120
```

```
  130 NUMBER=K
      RETURN
C
  140 CONTINUE
      PRINT 150, (IREAD(J),J=1,80)
      STOP
C
C
C
  150 FORMAT (///,33H - - - PROGRAM TERMINATED - - - -,//,13H  INPUT ERR
     1OR,/1X,80R1)
      END


      SUBROUTINE MATRIAL (MMTYPE,MATL,XYS,NSTORE)
C
C
C     SUBROUTINE MATRIAL INPUTS THE NECESSARY MATERIAL PROPERTIES.
C     THE VALUES OF THESE PROPERTIES ARE EITHER IN THE FORM OF A
C     CONSTANT OR A LINEARIZED FUNCTION OF TIME OR SHRINKAGE STRAIN.
C     NSTORE CONTAINS THE STARTING LOCATION FOR STORING THE LINEARIZED
C     MATERIAL PROPERTY FUNCTIONS AND ALSO RETURNS THE REQUIRED STORAGE
C     FOR XYS TO THE MAIN PROGRAM.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION MMTYPE(1), MATL(1), XYS(1)
C
C
      NSTORE=1
      SINFAV=0.0
C
C     OUTPUT PAGE HEADING
C
      WRITE (NOUT,150)
      WRITE (NOUT,160)
      WRITE (NOUT,170) ITITLE
      WRITE (NOUT,180) NMAT
      WRITE (NOUT,160)
      WRITE (NOUT,206) MATMOD
C
C     CHECK DECLARED MATERIAL TYPE REQUIREMENTS AGAINST INTENDED INPUT
C
      DO 110 I=1,NUMEL
      IF (MMTYPE(I).GT.NMAT) GO TO 120
  110 CONTINUE
      GO TO 130
C
  120 WRITE (NOUT,190) I,MMTYPE(I),NMAT
      STOP
C
  130 CONTINUE
C
C     LOOP OVER ALL MATERIAL TYPES
C
      DO 140 M=1,NMAT
      WRITE (NOUT,200) M
C
```

```
      WRITE (NOUT,170)
C
C     OUTPUT THE FUNCTION IF IT IS VARIABLE
C
      DO 130 I=1,M
      WRITE (NOUT,180) I,X(I),Y(I)
      WRITE (NOUT,190) S(I)
  130 CONTINUE
      WRITE (NOUT,180) K,X(K),Y(K)
      RETURN
C
C
  140 WRITE (NOUT,200) K
      STOP
C
C
C
  150 FORMAT (8E10.0)
  160 FORMAT (/,39H  MATERIAL PARAMETER OF CONSTANT VALUE ,G11.3)
  161 FORMAT (/,44H  MATERIAL PARAMETER FOR PICKETTS RELATION     ,G11.3)
  170 FORMAT (/5X,19HNODE          TIME/SHR,6X,5HVALUE,7X,5HSLOPE,/)
  180 FORMAT (I9,G17.3,2X,G11.3)
  190 FORMAT (39X,G11.3)
  200 FORMAT (///,44H  ------------PROGRAM TERMINATED------------,/,14H
     1  INPUT ERROR,/,23H   CONTROL CONSTANT IS ,I5,22H AND IT MUST BE E
     2ITHER,/,34H   0 OR GREATER THAN OR EQUAL TO 2)
      END



      SUBROUTINE DRYBC (X,Y,KODE,LI,LJ,LMAT,SURFAC,XL,KODEBC)
C
C
C     SUBROUTINE DRYBC INPUTS THE GEOMETRIC DESCRIPTION OF THE
C     SURFACE OF THE SYSTEM THAT WILL BE EXPOSED TO A
C     SHRINKAGE-DEPENDENT DRYING PROCESS
C
C     I.E.,
C          FLOW CASE        Q = F*(S(ULTIMATE) - S(CURRENT STEP))
C          GRADIENT CASE    DS/DN = (F/K)*(S(ULTIMATE) - S(CURRENT))
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION X(1),Y(1),KODE(1),LI(1),LJ(1),LMAT(1),SURFAC(1),XL(1)
     1,KODEBC(1)
C
C
C     INPUT DRYING BC DATA - FOUR SURFACE SEGMENTS TO AN INPUT CARD
C
      READ (NIN,170) (LI(I),LJ(I),LMAT(I),SURFAC(I),KODEBC(I),
     1I=1,NDRYBC)
C
C     CHECK VALIDITY OF MATERIAL REQUIREMENTS AND PREVIOUSLY DECLARED
C     BOUNDARY CONDITIONS.  FOR A SURFACE TO BE CONSIDERED A DRYING B.C.
C     IT MUST BE BOUNDED BY NODES THAT HAVE A DECLARED FLOW B.C.
C     OR A DECLARED SHRINKAGE GRADIENT B.C.
C
      DO 110 I=1,NDRYBC
      IF (KODEBC(I).EQ.1) NGRADBC=NGRADBC+1
      IF (LMAT(I).GT.NUMEL) GO TO 120
      II=LI(I)
      JJ=LJ(I)
      IF (KODE(II).EQ.4HSHRK.OR.KODE(JJ).EQ.4HSHRK) GO TO 120
  110 CONTINUE
      GO TO 130
C
  120 CONTINUE
      WRITE (NOUT,180) I,LI(I),LJ(I),LMAT(I),SURFAC(I)
      STOP
C
  130 CONTINUE
C
C     CALCULATE THE LENGTH OF THE DRYING BC SURFACE SEGMENTS
C
      DO 140 I=1,NDRYBC
      II=LI(I)
      JJ=LJ(I)
      DX=X(II)-X(JJ)
      DY=Y(II)-Y(JJ)
      D=DX*DX+DY*DY
      XL(I)=SQRT(D)
  140 CONTINUE
C
C     WRITE HEADING FOR DRYING BC OUTPUT
C
      WRITE (NOUT,160) NDRYBC
      NDUM=NDRYBC - NGRADBC
      WRITE (NOUT,166) NDUM
      WRITE (NOUT,165) NGRADBC
      WRITE (NOUT,190)
C
C     OUTPUT DRYING BC DATA
C
      WRITE (NOUT,200) (I,LI(I),LJ(I),LMAT(I),SURFAC(I),XL(I),KODEBC(I),
     1I=1,NDRYBC)
C
C     REDUCE THE SURFACE LENGTH BY 1/2 TO AID IN FUTURE COMPUTATION
C
      DO 150 I=1,NDRYBC
  150 XL(I)=0.5*XL(I)
C
      RETURN
C
C
C
  160 FORMAT (////,21H    . . . . THERE ARE ,I4,52H SURFACE ELEMENTS EXPO
     1SED TO DRYING  . . . . . . . .)
  165 FORMAT (    ,21H    . . . . THERE ARE ,I4,52H SURFACE ELEMENTS WITH
     1 GRADIENT BOUNDARY CONDITIONS          )
  166 FORMAT (   /,21H    . . . . THERE ARE ,I4,52H SURFACE ELEMENTS WITH
     1 FLOW BOUNDARY CONDITIONS              )
  170 FORMAT (4(3I4,F4.0,I4))
  180 FORMAT (5(/),54H  - - PROGRAM TERMINATED - DRYING BC INPUT ERROR -
     1 - -,1X,4I5,F10.0)
  190 FORMAT (//9X,46HDESCRIPTION OF SURFACE EXPOSED TO DRYING       ,//4
     1X,8HDRYINGBC,4X,12HNODE TO NODE,2X,8H ELEMENT,5X,7HSURFACE,11X,
     26HLENGTH,5X,6HKODEBC/4X,7HSURFACE,6X,1HI,7X,1HJ,5X,6HNUMBER,6X,
     310HMULTIPLIER/)
  200 FORMAT (I8,2X,2I8,I9,F17.5,F14.3,I10)
      END
```

```
C     READ IN CONTROL VARIABLES. IF INTEGER VARIABLE IS ZERO THEN
C     MATERIAL VALUE IS CONSTANT. IF INTEGER CONSTANT IS GREATER THAN
C     2, THEN IT IS THE NUMBER OF POINTS REQUIRED TO DESCRIBE THE
C     LINEARIZED TIME-DEPENDENT OR SHRINKAGE-DEPENDENT FUNCTION.
C
C     IF INTEGER CONSTANT IS -1, THEN USE PICKETTS RELATION
C
C        K = DIFFUSIVITY CONSTANT * SQRT((2./(2. + TIME))
C        F = SURFACE FACTOR CONSTANT * K
C
      READ (NIN,210) MM,MMM
      IF (MM.EQ.-1 .AND. MATMOD.EQ.2) STOP
      IF (MMM.EQ.-1 .AND. MATMOD.EQ.2) STOP
      MS=(M-1)*6
C
C     INPUT SHRINKAGE DIFFUSIVITY DATA
C
      WRITE (NOUT,220)
      MATL(MS+1)=NSTORE
      MATL(MS+2)=MM
      MD=MM
      IF (MM.EQ.-1) MD=0
      CALL MATIN (MM,XYS(NSTORE),XYS(NSTORE+MD),XYS(NSTORE+MD+MD))
      NSTORE=NSTORE+3*MD
      IF (MD.EQ.0) NSTORE=NSTORE+1
C
C     INPUT SURFACE FACTOR
C
      WRITE (NOUT,230)
      MATL(MS+3)=NSTORE
      MATL(MS+4)=MMM
      MSF=MMM
      IF (MMM.EQ.-1) MSF=0
      CALL MATIN (MMM,XYS(NSTORE),XYS(NSTORE+MSF),XYS(NSTORE+MSF+MSF))
      NSTORE=NSTORE+3*MSF
      IF (MSF.EQ.0) NSTORE=NSTORE+1
C
C     INPUT ULTIMATE SHRINKAGE STRAIN
C
      WRITE (NOUT,240)
      MATL(MS+5)=NSTORE
      MATL(MS+6)=0
      CALL MATIN (0,XYS(NSTORE),XYS(NSTORE),XYS(NSTORE))
      SINFAV=SINFAV + XYS(NSTORE)
      NSTORE=NSTORF+1
C
  140 CONTINUE
C
C     FIND AVERAGE ULTIMATE UNRESTRAINED SHRINKAGE STRAIN
C
      SINFAV=SINFAV/NMAT
C
      RETURN
C
C
C
  150 FORMAT (1H6,5(/))
  160 FORMAT (80H *****************************************************
     1**************************)
  170 FORMAT (/5X,46H*SHRINC* - NON-UNIFORM SHRINKAGE OF STRUCTURES ,
     1//5X,8A10)
  180 FORMAT (/5X,50HSHRINKAGE PROPERTIES OF MATERIALS USED IN ANALYSIS
     2//5X,10HTHERE ARE ,I3,20H DIFFERENT MATERIALS,/)
  190 FORMAT (5(/),32H  - - - PROGRAM TERMINATED - - -,/,17H  ELEMENT NU
     1MBER ,I5,24H HAS A MATERIAL TYPE OF ,I5,22H WHICH IS GREATER THAN,
     2/,24H  THE INTENDED INPUT OF ,I5,10H MATERIALS)
  200 FORMAT (////,26H  . . . . MATERIAL NUMBER ,I4,9H  . . . .)
  205 FORMAT (I5,F10.0)
  206 FORMAT (///,45H  . . . . MATERIAL MODEL USED  . . . . . . . .  ,8X,
     2        I5/  45H            EQ.1  -  TIME-DEPENDENT
     3          /  45H            EQ.2  -  SHRINKAGE-DEPENDENT          )
  210 FORMAT (2I5)
  220 FORMAT (//7X,40H . . . SHRINKAGE DIFFUSIVITY . . . . .   )
  230 FORMAT (//7X,40H . . . SURFACE FACTOR  . . . . . . . .  )
  240 FORMAT (//7X,42H . . ULTIMATE SHRINKAGE STRAIN . . . .      )
C
      END



      SUBROUTINE MATIN (K,X,Y,S)
C
C
C     SUBROUTINE MATIN INPUTS A MATERIALS TIME- OR SHRINKAGE-DEPENDENT
C     PROPERTIES, CALCULATES THEIR SLOPES, AND DISPLAYS RESULTS.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION X(1), Y(1), S(1)
C
      IF (K.NE.-1) GO TO 100
C
C     INPUT CONSTANT PARAMETERS FOR PICKETTS RELATION
C
      READ (NIN,150) X(1)
      WRITE (NOUT,161) X(1)
      RETURN
C
  100 IF (K.NE.0) GO TO 110
C
C     INPUT CONSTANT VALUE FUNCTION
C
      READ (NIN,150) X(1)
      WRITE (NOUT,160) X(1)
      RETURN
C
C     INPUT VARIABLE VALUE FUNCTION
C
  110 CONTINUE
C
C     CHECK K FOR AN ALLOWABLE NUMBER OF POINTS IN MATERIAL FUNCTION
C
      IF (K.EQ.1.OR.K.LT.0) GO TO 140
      READ (NIN,150) (X(I),Y(I),I=1,K)
      M=K-1
C
C     DETERMINE THE SLOPES OF THE LINEAR SEGMENTS OF THE MATERIAL
C     FUNCTIONS
C
      DO 120 I=1,M
  120 S(I)=(Y(I+1)-Y(I))/(X(I+1)-X(I))
```

```
      SUBROUTINE CONVERG (NTOTAL)
C
C
C     SUBROUTINE CONVERG INPUTS THE CONVERGENCE CRITERIA THAT CONTROL
C     THE ITERATIVE PROCESS NECESSARY TO FIND THE SHRINKAGE SOLUTION
C     THAT SATISFIES GRADIENT BOUNDARY CONDITIONS AND
C     SHRINKAGE-DEPENDENT MATERIAL PROPERTIES.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      COMMON /CONRG/ NCONV,CONV,NREFORM
C
C     OUTPUT PAGE HEADING
C
      WRITE (NOUT,120)
      WRITE (NOUT,130)
      WRITE (NOUT,140) ITITLE
      WRITE (NOUT,150)
      WRITE (NOUT,130)
      WRITE (NOUT,160)
C
C     CONVERGENCE ASSUMED EFFECTED WHEN
C
C          ABS (T(I)-T(I-1) / S(ULTIMATE) ) .LESS THAN. CONV
C
C        WHERE
C
C              NCONV    - NUMBER OF PERMISSIBLE
C                         ITERATIONS (INPUT 0 IF
C                         ITERATION IS NOT DESIRED)
C              CONV     - PERMISSIBLE ERROR RELATIVE TO MAGNITUDE OF
C                         FINAL UNRESTRAINED SHRINKAGE STRAIN
C
C     INPUT CONVERGENCE CRITERIA
C
      READ (NIN,170) NCONV,CONV,NREFORM
      IF (CONV.EQ.0.0) CONV=0.001
      IF (MATMOD.EQ.1) NREFORM=0
C
C     OUTPUT CONVERGENCE CRITERIA
C
      WRITE (NOUT,180) CONV,NCONV,NREFORM
C
C     STORAGE REQUIREMENT FOR BLANK COMMON IS NOW PRINTED
C
      WRITE (NOUT,200) NTOTAL,NTOTAL
      RETURN
C
C
C
  120 FORMAT (1H6,5(/))
  130 FORMAT (80H *******************************************************
     1*************************)
  140 FORMAT (/5X,51H*SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTUR
     1,2HES,//5X,8A10)
  150 FORMAT (/5X,46HINFORMATION RELEVANT TO THE ANALYSIS PROCEDURE,/)
  160 FORMAT (////38H  . . . . CONVERGENCE CRITERIA . . . .)
  170 FORMAT (I5,F10.0,I5)
  180 FORMAT (//41H  CONVERGENCE CRITERIA FOR EACH TIME STEP///22H    PE
     1RMISSIBLE ERROR ,23X,1H=,F10.5//33H     MAXIMUM NUMBER OF ITERATION
     2S ,17X,1H=,I5//
     3,51H    SHRINKAGE-DEPENDENT MATRICES REFORMATION FLAG =,I5/
     4,51H         EQ.0,   UPDATE AT START OF EACH                /
     5,51H                 TIME STEP ONLY                         /
     6,51H         EQ.1,   UPDATE FOR EACH ITERATION              )
  200 FORMAT (////,54H  . . . . STORAGE REQUIREMENT FOR BLANK COMMON . .
     1 . .,//5X,20HSIZE BLANK COMMON = ,I10,10H (DECIMAL),/23X,5H=     ,O
     27,8H (OCTAL))
       END



       SUBROUTINE SHRKFLO (X,Y,KODE,LM,MMTYPE,XLAM,MATL,XYS,LI,LJ,LMAT,
     1SURFAC,XL,KODEBC,Q,T,B,AT,A,NP,SS1,SS2,SS3,SS5,SS6,SS9,D,MA,T1,T2,
     2T3,AA)
C
C
C      SUBROUTINE SHRKFLO IS THE SHRINKAGE TIME-STEP ANALYSIS CONTROLLER
C
C
       COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
       COMMON /CONRG/ NCONV,CONV,NREFORM
       DIMENSION X(1), Y(1), KODE(1), LM(5,1), MMTYPE(1), XLAM(4,1), MATL
     1(1), XYS(1), LI(1), LJ(1), LMAT(1), SURFAC(1), KODEBC(1), XL(
     21), Q(1), T(1), B(1), AT(1), A(NP,1), SS1(4,1), SS2(4,1), SS3(4,1)
     3, SS5(4,1), SS6(4,1), SS9(4,1), D(1), MA(1), T1(1), T2(1), T3(1)
     4, AA(NP,1)
C
C      INITIALIZE THE SYSTEM
C
       IP1=0
       IP2=0
       SDT=0.0
       DS=0.0
C
C      INPUT INITIAL TIME STEP CARD
C
C      WHERE    IA - CONTROL WORD (STEP)
C              MOT - SEQUENCE COUNTER - INITIAL VALUE
C             TIME - SYSTEM CLOCK - INITIAL VALUE
C              SIC - INDICATOR FOR INITIAL SHRINKAGE - IF NON-NEGATIVE
C                    THEN TAKEN AS UNIFORM INITIAL SHRINKAGE
C                    THROUGHOUT CROSS-SECTION
C               JP - IDENTIFIER FOR PUNCHED OUTPUT
C
       READ (NIN,450) IA,MOT,TIME,SIC,JP
C
C      OUTPUT HEADING FOR INITIAL TIME STEP
C
       WRITE (NOUT,460)
       WRITE (NOUT,470)
       WRITE (NOUT,480) ITITLE
C
       IF (IA.NE.4HSTEP) GO TO 140
C
       WRITE (NOUT,490) MOT,TIME
       WRITE (NOUT,470)
C
C      INITIALIZE SHRINKAGES
C
```

```
      IF (SIC.GE.0.0) GO TO 110
      READ (NIN,500) (T(I),I=1,NUMNP)
      GO TO 130
  110 DO 120 I=1,NUMNP
  120 T(I)=SIC
C
C     OUTPUT INITIAL SHRINKAGES
C
  130 CALL PROUT (4,T,AT,T1,B,MAIN,NCON,1)
      GO TO 150
C
  140 CONTINUE
C
C     TERMINATE PROGRAM - INITIAL TIME STEP CARD ERROR
C
      WRITE (NOUT,510) IA,MDT,TIME,SIC,JP
      STOP
C
  150 CONTINUE
C
C     START STEP-BY-STEP TIME INTEGRATION
C
C     READ TIME STEP CARD
C
C     WHERE  .IA - CONTROL WORD (STEP)
C             NDT - SEQUENCE NUMBER
C             DT - TIME STEP INTERVAL
C             ISOF - NUMBER OF NON-ZERO SHRINKAGE OR FLOW B.C.S
C             I1 - PRINTED OUTPUT OPTION
C                  0 - NO PRINTED OUTPUT
C                  1 - OUTPUT NODAL POINT SHRINKAGES
C                  2 - OUTPUT ELEMENT CENTROID SHRINKAGES
C                  3 - OUTPUT BOTH NODAL AND ELEMENT SHRINKAGES
C             I2 - PUNCHED OUTPUT OPTION
C                  CODE SAME AS FOR PRINTED DATA
C             I6 - DEBUGGING OUTPUT OPTION
C
  160 READ (NIN,520) IA,NDT,DT,ISOF,I1,I2,I6
      MAIN=0
      NCON=0
      MDT=MDT+1
C
C     CHECK SEQUENCING OF TIME STEP CARD
C
      IF (NDT.EQ.MDT.AND.IA.EQ.4HSTEP) GO TO 190
C
C     PROGRAM TERMINATED IF SEQUENCING ERROR
C
      WRITE (NOUT,530)
  170 WRITE (NOUT,540) IA,NDT,DT,ISOF,I1,I2,I6
      STOP
  180 WRITE (NOUT,550)
      GO TO 170
C
C     ESTABLISH TIME INTERVAL DT
C
  190 IF (DT) 200,210,220
  200 WRITE (NOUT,560)
      RETURN
C
  210 IF (DS.EQ.0.0) GO TO 180
      DT=DS
      GO TO 230
  220 DS=DT
  230 CONTINUE
C
      TIME=TIME+DT
C
C     OUTPUT HEADING FOR TIME STEP
C
      WRITE (NOUT,460)
      WRITE (NOUT,470)
      WRITE (NOUT,480) ITITLE
      WRITE (NOUT,570) NDT,TIME,DT
      WRITE (NOUT,470)
      WRITE (NOUT,580) ISOF
      DT2=1./DT
C
C     BEGINNING OF ITERATIVE CYCLE IF DIFFUSIVITY TEMPERATURE- OR
C     SHRINKAGE-DEPENDENT.  IF THESE QUANTITIES ARE ONLY TIME-DEPENDENT
C     OR IF (NRFFORM) WAS SPECIFIED AS ZERO ONLY ONE CYCLE IS EXECUTED.
C
  250 MAIN=MAIN+1
C
C     SAVE THE INITIALLY ASSUMED SHRINKAGES IN VECTOR T1
C
      DO 260 N=1,NUMNP
  260 T1(N)=T(N)
C
C     CALCULATE THE ELEMENTS SHRINKAGES
C        ( FOR USE IF MATERIAL PARAMETERS ARE SHRINKAGE-DEPENDENT )
C
      DO 270 N=1,NUMEL
      SCALE=0.333333333333333
      LL1=LM(1,N)
      LL2=LM(2,N)
      LL3=LM(3,N)
      ATS=T(LL1)+T(LL2)+T(LL3)
      LL4=LM(4,N)
      IF (LL3.EQ.LL4) GO TO 270
      ATS=ATS+T(LL4)
      SCALE=0.2500
  270 AT(N)=SCALE*ATS
C
C     FORM DIFFUSIVITY MATRIX AND STORE IN ARRAY A
C
      CALL SHRKDIF (MMTYPE,LM,SS1,SS2,SS3,SS5,SS6,SS9,AT,A,NUMNP,XLAM,
     1MATL,XYS,TIME)
C
C     INITIALIZE LOADING VECTOR ( B ) TO 0.0
C
      DO 280 I=1,NUMNP
  280 B(I)=0.0
C
C     INPUT ANY NON-ZERO SHRINKAGE AND FLOW BOUNDARY CONDITIONS
C     AND ADD THE RELATED TERMS TO MATRIX A AND VECTOR B
C
      CALL BCLOAD (ISOF,D,KODE,B,A,NUMNP,MAIN,T3,Q)
C
C     FORM VELOCITY MATRIX Q
C
      CALL SHRKVEL (MMTYPE,Q,AT,XLAM,LM,MATL,XYS)
C
C     ADD VELOCITY TERMS TO DIFFUSIVITY MATRIX A
```

```
C     TO OBTAIN EFFECTIVE DIFFUSIVITY
C
      DO 300 N=1,NUMNP
      IF (KODE(N).EQ.4HSHRK) GO TO 300
      IF (Q(N)) 290,300,290
  290 A(N,1)=A(N,1)+Q(N)*DT2
  300 CONTINUE
C
      IF (MAIN.NE.1) GO TO 320
C
C     CALCULATE EFFECTIVE LOAD VECTOR B
C
C     CALCULATE VELOCITY MATRIX CONTRIBUTION TO THE EFFECTIVE LOAD
C     AND SAVE IN VECTOR T2
C
      DO 310 II=1,NUMNP
      IF (KODE(II).EQ.4HSHRK) GO TO 310
      T2(II)=Q(II)*T(II)*DT2
  310 CONTINUE
  320 DO 330 II=1,NUMNP
      IF (KODE(II).EQ.4HSHRK) GO TO 330
      B(II)=B(II)+T2(II)
  330 Q(II)=B(II)
C
C     ADD EFFECTS OF SHRINKAGE GRADIENT OR FLOW DRYING BOUNDARY
C     CONDITIONS TO LOAD VECTOR B AND DIFFUSIVITY MATRIX A
C
      IF (NDRYBC.EQ.0) GO TO 340
      CALL DRYLOAD (LI,LJ,LMAT,SURFAC,XL,KODEBC,MMTYPE,MATL,XYS,A,B,Q,
     1T3,TIME,NUMNP,T1,MAIN)
C
C     BEGINNING OF ITERATION DUE TO SHRINKAGE GRADIENT BOUNDARY
C     CONDITIONS.
C
  340 CONTINUE
C
      NCON=NCON+1
C
C     CALCULATE THE SHRINKAGE SOLUTION FOR THIS TIME STEP (OR ITERATION)
C
      DO 345 II=1,NUMNP
      DO 345 JJ=1,MBAND
  345 AA(II,JJ)=A(II,JJ)
C
  350 CALL MSYM (1,B,MA,AA,NUMNP)
C
      IF (NCONV.EQ.0) GO TO 405
      IF (I6.NE.0) CALL PROUT (2,T,AT,T1,B,MAIN,NCON,I1,LM)
C
C     CHECK FOR CONVERGENCE OF ITERATION AGAINST PERMISSIBLE ERROR
C
      DO 360 N=1,NUMNP
      DX=ABS(B(N)-T(N))
      DY=CONV*SINFAV
      IF (DX.GT.DY) GO TO 380
  360 CONTINUE
  405 DO 370 N=1,NUMNP
  370 T(N)=B(N)
      GO TO 400
C
C     IF CONVERGENCE NOT OBTAINED CHECK FOR POSSIBILITY OF EXCEEDING
C     PERMISSIBLE NUMBER OF CYCLES
C
  380 IF (NCON.GT.NCONV) CALL PROUT (3,T,AT,T1,B,MAIN,NCON,I1,LM)
C
C     CHECK ACCURACY OF GRADIENT BOUNDARY CONDITIONS AND
C     ESTIMATE NEW APPROXIMATION OF B.C. FLOW FOR NEXT CYCLE
C
      DO 390 JJ=1,NUMNP
  390 T(JJ)=B(JJ)
C
      CALL GRAD (LI,LJ,LMAT,SURFAC,KODEBC,MMTYPE,MATL,XYS,A,B,Q,T3,T,
     1LM,X,Y,TIME,NUMNP,T1)
C
      IF (NREFORM.EQ.1) GO TO 250
      GO TO 340
C
  400 CONTINUE
C
C     CHECK FOR DESIRED OUTPUT
C
      WRITE (NOUT,590) NCON
      IF (I1.NE.0) CALL PROUT (4,T,AT,T1,B,MAIN,NCON,I1,LM)
      IF (I2.NE.0) CALL PUNOUT (I1,I2,T,AT,LM,X,Y,MMTYPE,TIME,IP1,IP2,D,T
     13,JP)
C
      GO TO 160
C
C
C
  450 FORMAT (A4,I6,2F10.0,A3)
  460 FORMAT (1H6,5(/))
  470 FORMAT (80H *****************************************************
     1**************************)
  480 FORMAT (/5X,53H*SHRINC* - NON-UNIFORM DRYING SHRINKAGE OF STRUCTUR
     1ES,//5X,8A10)
  490 FORMAT (/5X,27HINITIAL SEQUENCE NUMBER IS ,I4,25H AND THE INITIAL
     1TIME IS ,F8.2/)
  500 FORMAT (6(4X,F8.6))
  510 FORMAT (5(/),69H - - - - PROGRAM TERMINATED - ERROR IN INITIAL TIM
     1E STEP CARD - - - -,//1X,A4,I6,2F10.2,A3)
  520 FORMAT (A4,I6,F10.0,4I5)
  530 FORMAT (///,43H  TIME STEP CARD OUT OF SEQUENCE - CARD NO.,I5/,12H
     1  INPUT CARD)
  540 FORMAT (//,57H  - - - - PROGRAM TERMINATED - TIME STEP CARD WAS -
     1- - -,//1X,A4,I6,F10.2,4I5)
  550 FORMAT (///,30H  NO TIME INTERVAL ESTABLISHED)
  560 FORMAT (///,39H SOLUTION TERMINATED - PROBLEM COMPLETE )
  570 FORMAT (/5X,17HTIME STEP NUMBER ,I4,8H - TIME ,F8.3,13H - TIME STE
     1P ,F7.3/)
  580 FORMAT (/5X,57H NUMBER OF NON-ZERO FLOW OR SHRINKAGE BOUNDARY COND
     1ITIONS,I5)
  590 FORMAT (/5X,43H NUMBER OF ITERATIONS TO EFFECT CONVERGENCE,I5)
      END
```

```
      SUBROUTINE MSYM (KKK,B,MA,A,NP)
C
C
C     MODIFIED SYMSOL
C     VARIABLE BANDWIDTH,ZEROS WITHIN BAND
```

```
C
C              KKK.EQ.1       SOLVE
C              KKK.EQ.2       BACKSUBSTITUTE ONLY
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION B(1), MA(1), A(NP,1)
C
      NEQ=NUMNP
C
      GO TO (110,170), KKK
C
C     ****************************************************************
C
C     REDUCE MATRIX..... A
C
C     ****************************************************************
C
  110 NEQQ=NEQ-1
      DO 160 N=1,NEQQ
C
      M=MBAND
      DO 120 I=2,MBAND
      IF (A(N,M).NE.0.) GO TO 130
  120 M=M-1
  130 MA(N)=M
C
      I=N
      DO 150 L=2,M
      I=I+1
      CC=A(N,L)/A(N,1)
      IF (CC.EQ.0.) GO TO 150
      J=0
      DO 140 K=L,M
      J=J+1
  140 A(I,J)=A(I,J)-CC*A(N,K)
      A(N,L)=CC
  150 CONTINUE
  160 CONTINUE
C
C     ****************************************************************
C
C     REDUCE VECTOR..... B AND BACKSUBSTITUTE
C
C     ****************************************************************
C
  170 DO 190 N=1,NEQQ
      CC=B(N)
      IF (CC.EQ.0.) GO TO 190
      M=MA(N)
      I=N
      DO 180 L=2,M
      I=I+1
  180 B(I)=B(I)-CC*A(N,L)
      B(N)=CC/A(N,1)
  190 CONTINUE
      B(NEQ)=B(NEQ)/A(NEQ,1)
C
      NN=NEQ
      DO 210 N=1,NEQQ
      NN=NN-1
      M=MA(NN)
      I=NN
      DO 200 K=2,M
      I=I+1
  200 B(NN)=B(NN)-A(NN,K)*B(I)
  210 CONTINUE
  220 RETURN
C
      END
```

```
      SUBROUTINE PROUT (K,T,AT,T1,B,MAIN,NCON,I1,LM)
C
C
C     SUBROUTINE PROUT DOES ALL PRINTING OF SHRINKAGE
C     DISTRIBUTIONS, BOTH NODAL AND ELEMENTAL
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION T(1), AT(1), T1(1), B(1), LM(5,1)
C
C
      GO TO (120,120,130,140), K
C
C
  120 CONTINUE
C
C     DEBUGGING OUTPUT SHRINKAGES FOR EACH ITERATION
C
      WRITE (NOUT,210) NCON
      WRITE (NOUT,290) (N,B(N),N=1,NUMNP)
      RETURN
C
  130 CONTINUE
C
C     OUTPUT DATA FOR DUMP WHEN PROBLEM HAS NOT CONVERGED AFTER
C     PERMISSIBLE NUMBER OF CYCLES
C
      WRITE (NOUT,220)
      WRITE (NOUT,230) MAIN,NCON
      WRITE (NOUT,240)
      WRITE (NOUT,290) (N,T1(N),N=1,NUMNP)
      WRITE (NOUT,250)
      WRITE (NOUT,290) (N,T(N),N=1,NUMNP)
      WRITE (NOUT,260)
      WRITE (NOUT,290) (N,B(N),N=1,NUMNP)
      STOP
C
  140 CONTINUE
C
C     OUTPUT OF RESULTS FOR A TIME STEP
C
      IF (I1.EQ.1.OR.I1.EQ.3) 150,160
  150 CONTINUE
C
C     OUTPUT NODAL POINT SHRINKAGES
C
      WRITE (NOUT,270)
```

```
      WRITE (NOUT,280)
      WRITE (NOUT,290) (N,T(N),N=1,NUMNP)
  160 CONTINUE
      IF (I1.EQ.2.OR.I1.EQ.3) 170,190
C
C     OUTPUT ELEMENT SHRINKAGES
C
  170 WRITE (NOUT,300)
      WRITE (NOUT,280)
C
C     CALCULATE THE AVERAGE ELEMENT SHRINKAGES
      DO 180 JI=1,NUMEL
C
      SCALE=0.33333333333
      IL1=LM(1,JI)
      IL2=LM(2,JI)
      IL3=LM(3,JI)
      ATS=T(IL1)+T(IL2)+T(IL3)
      IL4=LM(4,JI)
      IF (IL3.EQ.IL4) GO TO 180
      SCALE=0.25000
      ATS=ATS+T(IL4)
  180 AT(JI)=ATS*SCALE
C
      WRITE (NOUT,290) (N,AT(N),N=1,NUMEL)
  190 CONTINUE
C
      RETURN
C
C
C
  210 FORMAT (/,40H  NODAL POINT SHRINKAGES FOR ITERATION  ,I5/)
  220 FORMAT (//,20H  PROGRAM TERMINATED,/,59H  CONVERGENCE NOT OBTAINED
     1 IN REQUIRED NUMBER OF ITERATIONS)
  230 FORMAT (/,15H  SYSTEM CYCLE ,I5,16H AND B.C. CYCLE ,I5)
  240 FORMAT (//,31H  SYSTEM NODAL POINT SHRINKAGES)
  250 FORMAT (//,50H  NODAL POINT SHRINKAGES AT BEGINNING OF ITERATION)
  260 FORMAT (//,50H  NODAL POINT SHRINKAGES AT END OF ITERATION       )
  270 FORMAT (/,54H ------------ NODAL POINT SHRINKAGES ----------------
     1-/)
  280 FORMAT (/1X,4(16H     N      SHRK. ))
  290 FORMAT (4(I6,F10.6))
  300 FORMAT (////,53H -------------- SHRINKAGE OF ELEMENTS ------------
     1---/)
      END


      SUBROUTINE SHRKDIF (MMTYPE,LM,SS1,SS2,SS3,SS5,SS6,SS9,AT,A,NP,XLAM
     1,MATL,XYS,TIME)
C
C
C     THIS SUBROUTINE FORMS THE SHRINKAGE DIFFUSIVITY MATRIX
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION MMTYPE(1), LM(5,1), SS1(4,1), SS2(4,1), SS3(4,1), SS5(4,
     11), SS6(4,1), SS9(4,1), AT(1), A(NP,1), XLAM(4,1), MATL(1), XYS(1)
      DIMENSION E(3,3), S(5,5), IX(3)
C
      MB=MBAND*NUMNP
C
C     INITIALIZE THE SYSTEMS DIFFUSIVITY MATRIX
C
      DO 110 I=1,MB
  110 A(I)=0.0
C
C     FORM ELEMENT DIFFUSIVITY MATRIX
C
      DO 210 N=1,NUMEL
      DO 120 I=1,25
  120 S(I)=0.0
C
C     DETERMINE TIME- OR SHRINKAGE-DEPENDENT SHRINKAGE DIFFUSIVITY
C
      IF (MATMOD.EQ.1) ARG=TIME
      IF (MATMOD.EQ.2) ARG=AT(N)
      MS=MMTYPE(N)
      MS=(MS-1)*6
      J=MATL(MS+1)
      L=MATL(MS+2)
      K=L
      IF (L.EQ.-1) K=0
      DIFS=VMAT(L,XYS(J),XYS(J+K),XYS(J+K+K),ARG,10H      K(T) )
      IF (L.EQ.-1) DIFS=DIFS*SQRT(2./(2.+ARG))
C
C     DETERMINE THE DIFFUSIVITY CONTRIBUTION OF EACH SUBELEMENT AND
C     ADD TO ELEMENT STIFFNESS
C
      DO 170 K=1,4
      IK=LM(K,N)
      JK=LM(K+1,N)
C
C     CHECK FOR TRIANGULAR ELEMENT
C
      IF (IK-JK) 130,170,130
C
C     RECALL CONSTANT TERMS OF DIFFUSIVITY MATRIX FOR TRIANGLE
C
  130 E(1,1)=SS1(K,N)
      E(1,2)=SS2(K,N)
      E(1,3)=SS3(K,N)
      E(2,1)=SS2(K,N)
      E(2,2)=SS5(K,N)
      E(2,3)=SS6(K,N)
      F(3,1)=SS3(K,N)
      E(3,2)=SS6(K,N)
      E(3,3)=SS9(K,N)
C
C     CALCULATE MATRIX CONSTANT
C
      COMM=0.5*DIFS/XLAM(K,N)
C
C     ADD TERMS FROM TRIANGULAR DIFFUSIVITY MATRIX TO THAT OF THE
C     ELEMENT (IE, THE FLEMENTS 5X5 MATRIX S)
C
      IX(1)=K
      IX(2)=K+1
      IF (K-4) 150,140,150
  140 IX(2)=1
  150 IX(3)=5
```

```
      DO 160 I=1,3
      II=IX(I)
      DO 160 J=1,3
      JJ=IX(J)
  160 S(II,JJ)=S(II,JJ)+E(I,J)*COMM
C
  170 CONTINUE
C
C     CONDENSE S FROM 5X5 TO 4X4 ELIMINATING THE CENTER NODE
C
      IF (S(5,5).EQ.0.0) GO TO 210
      DO 180 I=1,4
      DO 180 J=1,4
  180 S(I,J)=S(I,J)-S(I,5)*S(J,5)/S(5,5)
C
C     ADD ELEMENT DIFFUSIVITY TO THE SYSTEMS DIFFUSIVITY MATRIX ( A )
C
      DO 200 L=1,4
      I=LM(L,N)
      DO 200 M=1,4
      J=LM(M,N)-I+1
      IF (J) 200,200,190
  190 A(I,J)=A(I,J)+S(L,M)
  200 CONTINUE
  210 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE BCLOAD (ISOF,D,KODE,B,A,NP,MAIN,FT,J)
C
C
C     SUBROUTINE WHICH APPLIES THE SHRINKAGE OR FLOW BOUNDARY
C     CONDITIONS.
C     FIXED BOUNDARY CONDITIONS ONLY  -  SHRINKAGE-DEPENDENT BOUNDARY
C     CONDITIONS ADDED IN BY SUBROUTINE *DRYLOAD*
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION D(1), KODE(1), B(1), A(NP,1), FT(1), J(1)
C
      IF (MAIN.NE.1) GO TO 130
C
C     INITIALIZE SHRINKAGE AND FLOW B.C.S TO 0.0
C
      DO 110 I=1,NUMNP
  110 D(I)=0.0
C
      IF (ISOF.EQ.0) GO TO 130
C
      WRITE (NOUT,200)
C
C     INPUT NON-ZERO SHRINKAGE AND FLOW B.C.S
C
      READ (NIN,210) (J(I),FT(I),I=1,ISOF)
      WRITE (NOUT,220)
C
C     OUTPUT THE NON-ZERO BOUNDARY CONDITIONS AND STORE IN MATRIX D
C
      DO 120 I=1,ISOF
      II=J(I)
      D(II)=FT(I)
      JJ=KODE(II)
      WRITE (NOUT,230) II,JJ,D(II)
  120 CONTINUE
C
  130 DO 190 N=1,NUMNP
C
C     MODIFY MATRIX B FOR FLOW B.C.
C
      B(N)=B(N)+D(N)
      IF (KODE(N).EQ.4HFLOW) GO TO 190
C
C     MODIFY A AND B MATRIX FOR SHRINKAGE B.C.S
C
      DO 180 M=2,MBAND
      K=N-M+1
      IF (K) 150,150,140
  140 B(K)=B(K)-A(K,M)*D(N)
      A(K,M)=0.0
  150 L=N+M-1
      IF (NUMNP-L) 170,160,160
  160 B(L)=B(L)-A(N,M)*D(N)
  170 A(N,M)=0.0
  180 CONTINUE
      A(N,1)=1.0
      B(N)=D(N)
  190 CONTINUE
C
      RETURN
C
C
C
  200 FORMAT (//5X,62HVALUES OF SHRINKAGES OR FLOWS FOR NON-ZERO BOUNDAR
     1Y CONDITIONS)
  210 FORMAT (5(I5,F10.2))
  220 FORMAT (/,8H    NODE,7X,4HTYPE,10X,5HVALUE)
  230 FORMAT (I8,7X,A4,5X,F10.7)
      END
```

```
      SUBROUTINE SHRKVEL (MMTYPE,Q,AT,XLAM,LM,MATL,XYS)
C
C
C     THIS SUBROUTINE FORMS THE SHRINKAGE VELOCITY MATRIX
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION MMTYPE(1), Q(1), AT(1), XLAM(4,1), LM(5,1), MATL(1), XYS
     1(1)
C
C     INITIALIZE THE VELOCITY MATRIX TO 0.0
C
      DO 110 I=1,NUMNP
  110 Q(I)=0.0
```

```
C
C     DETERMINE THE CONTRIBUTION OF EACH ELEMENT TO THE VELOCITY MATRIX
C
      DO 140 N=1,NUMEL
C
      DO 130 K=1,4
      IK=LM(K,N)
      JK=LM(K+1,N)
      IF (IK-JK) 120,130,120
C
C     CALCULATE THE ELEMENTS AREA CONTRIBUTION
C
  120 CONTINUE
      QSTORE=0.25*XLAM(K,N)
C
C     ADD VELOCITY TERM TO SYSTEM VELOCITY MATRIX
C
      Q(IK)=Q(IK)+QSTORE
      Q(JK)=Q(JK)+QSTORE
  130 CONTINUE
  140 CONTINUE
C
      RETURN
      END
```

```
      FUNCTION VMAT (K,X,Y,S,T,NAME)
C
C
C     FUNCTION VMAT CALCULATES THE VALUE OF THE TIME-DEPENDENT
C     OR SHRINKAGE-DEPENDENT VARIABLES REQUIRED IN THE ANALYSIS
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION X(1), Y(1), S(1)
C
      IF (K.GT.0) GO TO 110
      VMAT=X(1)
      RETURN
C
  110 I=0
  120 I=I+1
      IF (I.GT.K) GO TO 150
      IF (T-X(I)) 140,130,120
  130 VMAT=Y(I)
      RETURN
C
  140 IF (I.EQ.1) GO TO 150
      VMAT=Y(I-1)+S(I-1)*(T-X(I-1))
      RETURN
C
  150 WRITE (NOUT,160) NAME,T,X(1),X(K)
      STOP
C
C
C
  160 FORMAT (///,48H  -------------PROGRAM TERMINATED --------------,/,
     150H    BOUNDS OF CURVE DESCRIBING MATERIAL PARAMETER ,A10,19H HAVE
     2 BEEN EXCEEDED,/,23H    THE TIME/SHRK    WAS ,F10.3,20H THE LOWER BO
     3UND IS ,F10.3,24H AND THE UPPER BOUND IS ,F10.3)
      END
```

```
      SUBROUTINE DRYLOAD (LI,LJ,LMAT,SURFAC,XL,KODEBC,MMTYPE,MATL,XYS,
     1A,B,Q,T3,TIME,NP,T1,MAIN)
C
C
C     SUBROUTINE DRYLOAD ADDS DRYING BOUNDARY CONDITIONS
C     ( ALWAYS EXPRESSED AS AN EXACT OR APPROXIMATE FLOW )
C     TO THE EFFECTIVE DIFFUSIVITY AND EFFECTIVE LOAD
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      DIMENSION LI(1),LJ(1),LMAT(1),SURFAC(1),XL(1),KODEBC(1),MMTYPE(1),
     1MATL(1),XYS(1),A(NP,1),B(1),Q(1),T3(1),T1(1)
C
C     INITIALIZE T3, WHICH WILL HOLD THE GRADIENT BOUNDARY CONDITION
C     EXPRESSED AS AN APPROXIMATE FLOW LOAD
C
      IF (MAIN.NE.1) GO TO 20
      DO 10 I=1,NDRYBC
   10 T3(I)=0.0
C
C     LOOP OVER ALL BOUNDARY ELEMENTS
C
C     FIND SHRINKAGE FLOW
C
   20 DO 100 I=1,NDRYBC
      II=LI(I)
      JJ=LJ(I)
      SAVG=(T1(II)+T1(JJ))/2.
      IF (MATMOD.EQ.1) ARG=TIME
      IF (MATMOD.EQ.2) ARG=SAVG
      M=LMAT(I)
      MS=MMTYPE(M)
      MS=(MS - 1)*6
      J=MATL(MS+3)
      L=MATL(MS+4)
      K=L
      IF (L.EQ.-1) K=0
      FAC=VMAT(K,XYS(J),XYS(J+K),XYS(J+K+K),ARG,10H      F(T) )
      IF (L.NE.-1) GO TO 101
      J=MATL(MS+1)
      K=0
      DIFS=VMAT(K,XYS(J),XYS(J+K),XYS(J+K+K),ARG,10H      K(T) )
      FAC=FAC*DIFS*SQRT(2./(2.+ARG))
  101 CONTINUE
      QB=SURFAC(I)*FAC*XL(I)
C
C     ADD TO EFFECTIVE LOAD
C
      J=MATL(MS+5)
      SINF=VMAT(0,XYS(J),XYS(J),XYS(J),ARG,10H      SINF )
      II=LI(I)
      JJ=LJ(I)
      IF (KODEBC(I) .EQ. 1 .AND. MAIN .NE. 1) GO TO 400
```

```
      B(II)=B(II) + QB*SINF
      B(JJ)=B(JJ) + QB*SINF
      Q(II)=B(II)
      Q(JJ)=B(JJ)
C
C     ADD TO EFFECTIVE DIFFUSIVITY
C
      A(II,1)=A(II,1) + QB/2.
      A(JJ,1)=A(JJ,1) + QB/2.
      IF (JJ.LT.II) GO TO 200
      A(II,JJ-II+1)=A(II,JJ-II+1) + QB/2.
  200 IF (II.LT.JJ) GO TO 300
      A(JJ,II-JJ+1)=A(JJ,II-JJ+1) + QB/2.
  300 CONTINUE
C
C     STORE PORTION OF FLOW LOAD DUE TO GRADIENT BOUNDARY CONDITIONS
C     ON VECTOR T3
C
      IF (KODEBC(I).EQ.0) GO TO 100
      T3(I)=QB
      GO TO 100
C
C     IN THE CASE OF  MAIN .NE. 1
C     MODIFY A AND B USING T3 DIRECTLY
C
  400 B(II)=B(II) + T3(I)*SINF
      B(JJ)=B(JJ) + T3(I)*SINF
      Q(II)=B(II)
      Q(JJ)=B(JJ)
      A(II,1)=A(II,1) + T3(I)/2.
      A(JJ,1)=A(JJ,1) + T3(I)/2.
      IF (JJ.LT.II) GO TO 500
      A(II,JJ-II+1)=A(II,JJ-II+1) + T3(I)/2.
  500 IF (II.LT.JJ) GO TO 600
      A(JJ,II-JJ+1)=A(JJ,II-JJ+1) + T3(I)/2.
  600 CONTINUE
C
  100 CONTINUE
C
      RETURN
      END


      SUBROUTINE GRAD (LI,LJ,LMAT,SURFAC,KODEBC,MMTYPE,MATL,XYS,A,B,Q,T3
     1,T,LM,X,Y,TIME,NP,T1)
C
C
C     I T E R A T I V E    S U B R O U T I N E
C
C     SUBROUTINE GRAD COMPARES SLOPES OF ALL BOUNDARY ELEMENTS
C     ( CALCULATED FROM THE SHRINKAGE DISTRIBUTION OF THIS ITERATION )
C     WITH GRADIENT B.C.S AND ADDS CORRECTION FLOWS TO EFFECTIVE LOAD
C     AND EFFECTIVE DIFFUSIVITY TO ADJUST FOR ANY DISCREPANCY.
C
C
      COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
     1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
      COMMON /CONRG/ NCONV,CONV,NREFORM
      DIMENSION LI(1),LJ(1),LMAT(1),SURFAC(1),KODEBC(1),MMTYPE(1),
     1MATL(1),XYS(1),A(NP,1),B(1),Q(1),T3(1),T(1),LM(5,1),X(1),Y(1),
     2T1(1)
      INTEGER FLG
C
C     LOOP OVER ALL DRYING SURFACE SEGMENTS WITH GRADIENT B.C.S
C
      DO 100 I=1,NDRYBC
      IF (KODEBC(I).EQ.0) GO TO 100
C
C     FIND MATERIAL PROPERTIES FOR THIS BOUNDARY SEGMENT
C
      KI=LI(I)
      KJ=LJ(I)
      SAVG=(T1(KI)+T1(KJ))/2.
      IF (NREFORM.EQ.1) SAVG=(T(KI) + T(KJ))/2.
      IF (MATMOD.EQ.1) ARG=TIME
      IF (MATMOD.EQ.2) ARG=SAVG
      MEL=LMAT(I)
      MS=MMTYPE(MEL)
      MS=(MS - 1)*6
      J1=MATL(MS+1)
      J2=MATL(MS+2)
      JJ=J2
      IF (J2.EQ.-1) JJ=0
      J3=MATL(MS+3)
      J4=MATL(MS+4)
      JI=J4
      IF (J4.EQ.-1) JI=0
      J5=MATL(MS+5)
      DIFS=VMAT(J2,XYS(J1),XYS(J1+JJ),XYS(J1+JJ+JJ),ARG,10H     K(T) )
      FAC=VMAT(J4,XYS(J3),XYS(J3+JI),XYS(J3+JI+JI),ARG,10H     F(T) )
      SINF=VMAT(0,XYS(J5),XYS(J5),XYS(J5),ARG,10H   S(INF) )
C
C     PICKETTS MATERIAL PROPERTIES
C
      IF (J2.NE.-1) GO TO 5
      DIFS=DIFS*SQRT(2./(2.+ARG))
      FAC=FAC*DIFS
C
C     FIND SHRINKAGE SLOPE OF THIS BOUNDARY ELEMENT THAT RESULTS
C     FROM THE SHRINKAGE SOLUTION OF THE CURRENT ITERATION
C
    5 K1=LM(1,MEL)
      K2=LM(2,MEL)
      K3=LM(3,MEL)
      K4=LM(4,MEL)
      TCEN=(T(K1)+T(K2)+T(K3)+T(K4))/4.
      XCEN=(X(K1)+X(K2)+X(K3)+X(K4))/4.
      YCEN=(Y(K1)+Y(K2)+Y(K3)+Y(K4))/4.
      KI=LI(I)
      KJ=LJ(I)
      SL1=(XCEN-X(KI))**2 + (YCEN-Y(KI))**2
      SL1=SQRT(SL1)
      SL2=(X(KI)-X(KJ))**2 + (Y(KI)-Y(KJ))**2
      SL2=SQRT(SL2)
      RATIO=SL1/SL2
      XS=X(KI) + RATIO*(X(KJ)-X(KI))
      YS=Y(KI) + RATIO*(Y(KJ)-Y(KI))
      SL3=(XCEN-XS)**2 + (YCEN-YS)**2
      SL3=0.5*SQRT(SL3)
      ALF=2.*ASIN(SL3/SL1)
      DL=SL1*SIN(ALF)
```

```
SHRINC 1785          IF (ALF.LE.1.58) GO TO 10
SHRINC 1786          WRITE (NOUT,1000)
SHRINC 1767          STOP
SHRINC 1788       10 DB=SL1*COS(ALF)
SHRINC 1789          SMID=T(KI) + (DB/SL2)*(T(KJ)-T(KI))
SHRINC 1790          SGRAD=(SMID-TCEN)/DL
SHRINC 1791   C
SHRINC 1792   C      FIND THE GRADIENT WHICH WAS SPECIFIED AS A BOUNDARY CONDITION
SHRINC 1793   C
SHRINC 1794          SGRADBC= (FAC*SURFAC(I)/DIFS)*(SINF - SMID)
SHRINC 1795   C
SHRINC 1796   C      COMPARE COMPUTED SLOPE AND B.C. SLOPE
SHRINC 1797   C      CALCULATF CORRECTION FLOW TO EQUALIZE THE TWO FOR NEXT ITERATION
SHRINC 1798   C
SHRINC 1799          QB=T3(I)
SHRINC 1800          RATIO=SGRADBC/SGRAD
SHRINC 1801          QBCOR=QB*(RATIO - 1.0)
SHRINC 1802   C
SHRINC 1803   C      ADD CORRECTION FLOW TO EFFECTIVE LOAD
SHRINC 1804   C
SHRINC 1805          Q(KI)=Q(KI) + QBCOR*SINF
SHRINC 1806          Q(KJ)=Q(KJ) + QBCOR*SINF
SHRINC 1807   C
SHRINC 1808   C      ADD CORRECTION FLOW TO EFFECTIVE DIFFUSIVITY
SHRINC 1809   C
SHRINC 1810          A(KI,1)=A(KI,1) + QBCOR/2.
SHRINC 1811          A(KJ,1)=A(KJ,1) + QBCOR/2.
SHRINC 1812          IF (KJ.LT.KI) GO TO 200
SHRINC 1813          A(KI,KJ-KI+1)=A(KI,KJ-KI+1) + QBCOR/2.
SHRINC 1814      200 IF (KI.LT.KJ) GO TO 300
SHRINC 1815          A(KJ,KI-KJ+1)=A(KJ,KI-KJ+1) + QBCOR/2.
SHRINC 1816      300 CONTINUE
SHRINC 1817   C
SHRINC 1818   C      RESET T3 FOR NEXT ITERATION
SHRINC 1819   C
SHRINC 1820          T3(I)=QBCOR + QB
SHRINC 1821      100 CONTINUE
SHRINC 1822   C
SHRINC 1823   C      STORE NEW EFFECTIVE LOAD ON B
SHRINC 1824   C
SHRINC 1825          DO 400 I=1,NUMNP
SHRINC 1826      400 B(I)=Q(I)
SHRINC 1827   C
SHRINC 1828          RETURN
SHRINC 1829   C
SHRINC 1830   C
SHRINC 1831   C
SHRINC 1832     1000 FORMAT (//30H BOUNDARY ELEMENTS TOC IRREGULAR - STOP
SHRINC 1833   C
SHRINC 1834          END



SHRINC 1835          SUBROUTINE PUOUT (I1,I2,T,AT,LM,X,Y,MMTYPE,TIME,IP1,IP2,XX,YY,JP)
SHRINC 1836   C
SHRINC 1837   C
SHRINC 1838   C      SUBROUTINE PUOUT PUNCHES THE SHRINKAGE DISTRIBUTIONS THAT
SHRINC 1839   C      RESULT FROM THE ANALYSIS DONE IN THE PROGRAM.
SHRINC 1840   C         JP - IDENTIFIER TO BE USED IN LAST 8 COLUMNS
SHRINC 1841   C         IP1 - COUNTER FOR NODAL DATA CARDS PUNCHED
SHRINC 1842   C         IP2 - COUNTER FOR ELEMENT DATA CARDS PUNCHED
SHRINC 1843   C
SHRINC 1844   C
SHRINC 1845          COMMON /CONTROL/ ITITLE(8),IREAD(80),NIN,NOUT,NPUNCH,NUMNP,NUMEL,M
SHRINC 1846         1BAND,NMAT,NDRYBC,NGRADBC,SINFAV,DT,MATMOD
SHRINC 1847          DIMENSION T(1), AT(1), LM(5,1), X(1), Y(1), MMTYPE(1), XX(1), YY(1
SHRINC 1848         1),NL(3)
SHRINC 1849          INTEGER ELEM
SHRINC 1850   C
SHRINC 1851          NP=NPUNCH
SHRINC 1852   C
SHRINC 1853          IF (I2.EQ.1.OR.I2.EQ.3) 110,210
SHRINC 1854   C
SHRINC 1855   C      PUNCHING NODAL DATA
SHRINC 1856   C
SHRINC 1857      110 WRITE (NOUT,370)
SHRINC 1858          NODE=4HNODE
SHRINC 1859          IF (JP.EQ.3H   ) GO TO 120
SHRINC 1860          ENCODE (4,380,NODE) JP
SHRINC 1861      120 CONTINUE
SHRINC 1862          IF (IP1.NE.0) GO TO 170
SHRINC 1863   C
SHRINC 1864   C      NODAL COORDINATES PUNCHED THE FIRST TIME SHRINKAGE DATA IS
SHRINC 1865   C      REQUESTED
SHRINC 1866   C
SHRINC 1867          DO 130 I=1,NUMNP
SHRINC 1868          XX(I)=X(I)+0.000001
SHRINC 1869      130 YY(I)=Y(I)+0.000001
SHRINC 1870   C
SHRINC 1871          IP1=IP1+1
SHRINC 1872          WRITE (NP,390) ITITLE,NODE,IP1
SHRINC 1873          IP1=IP1+1
SHRINC 1874          WRITE (NP,400) NODE,IP1
SHRINC 1875          IP1=IP1+1
SHRINC 1876          WRITE (NP,410) NODE,IP1
SHRINC 1877          N1=1
SHRINC 1878      140 N2=N1+3
SHRINC 1879          IP1=IP1+1
SHRINC 1880          IF (NUMNP-N2) 160,150,150
SHRINC 1881      150 WRITE (NP,420) (I,XX(I),YY(I),I=N1,N2),NODE,IP1
SHRINC 1882          IF (N2.EQ.NUMNP) GO TO 170
SHRINC 1883          N1=N2+1
SHRINC 1884          GO TO 140
SHRINC 1885      160 N2=NUMNP
SHRINC 1886          N=NUMNP+1-N1
SHRINC 1887          M=72-N*16
SHRINC 1888          ENCODE (30,430,L) N,M
SHRINC 1889          WRITE (NP,L) (I,XX(I),YY(I),I=N1,N2),NODE,IP1
SHRINC 1890   C
SHRINC 1891   C      PUNCHING NODAL POINT SHRINKAGES
SHRINC 1892   C
SHRINC 1893      170 IP1=IP1+1
SHRINC 1894          WRITE (NP,440) TIME,NUMNP,NODE,IP1
SHRINC 1895          N1=1
SHRINC 1896      180 N2=N1+5
SHRINC 1897          IP1=IP1+1
SHRINC 1898          IF (NUMNP-N2) 200,190,190
SHRINC 1899      190 WRITE (NP,450) (I,T(I),I=N1,N2),NODE,IP1
SHRINC 1900          IF (N2.EQ.NUMNP) GO TO 210
SHRINC 1901          N1=N2+1
SHRINC 1902          GO TO 180
SHRINC 1903      200 N2=NUMNP
```

```
      N=NUMNP+1-N1
      M=72-N*12
      ENCODE (30,460,L) N,M
      WRITE (NP,L) (I,T(I),I=N1,N2),NODE,IP1
  210 IF (I2.EQ.2.OR.I2.EQ.3) 220,360
C
C     PUNCHING ELEMENT DATA
C
  220 WRITE (NOUT,470)
      IF (IP2.NE.0) GO TO 290
      ELEM=4HELEM
      IF (JP.EQ.3H   ) GO TO 230
      ENCODE (4,480,ELEM) JP
  230 CONTINUE
C
C     PUNCH OUT THE ELEMENTS CENTROID COORDINATES DURING THE FIRST
C     REQUEST FOR ELEMENTAL DATA
C
C     CALCULATE THE CENTROIDS OF EACH ELEMENT
C
      DO 250 I=1,NUMEL
      J=LM(1,I)
      K=LM(2,I)
      M=LM(3,I)
      N=LM(4,I)
      IF (M.EQ.N) GO TO 240
      XX(I)=(X(J)+X(K)+X(M)+X(N))/4.0
      YY(I)=(Y(J)+Y(K)+Y(M)+Y(N))/4.0
      GO TO 250
  240 XX(I)=(X(J)+X(K)+X(M))/3.0
      YY(I)=(Y(J)+Y(K)+Y(M))/3.0
  250 CONTINUE
      IP2=IP2+1
      WRITE (NP,390) ITITLE,ELEM,IP2
      IP2=IP2+1
      WRITE (NP,490) ELEM,IP2
      IP2=IP2+1
      WRITE (NP,500) ELEM,IP2
      N1=1
  260 N2=N1+3
      IP2=IP2+1
      IF (NUMEL-N2) 280,270,270
  270 WRITE (NP,420) (I,XX(I),YY(I),I=N1,N2),ELEM,IP2
      IF (N2.EQ.NUMEL) GO TO 290
      N1=N2+1
      GO TO 260
  280 N2=NUMEL
      N=NUMEL+1-N1
      M=72-N*16
      ENCODE (30,430,L) N,M
      WRITE (NP,L) (I,XX(I),YY(I),I=N1,N2),ELEM,IP2
C
C     PUNCHING ELEMENT SHRINKAGES
C
  290 IP2=IP2+1
      WRITE (NP,510) TIME,NUMEL,ELEM,IP2
      IF (I1.EQ.2.OR.I1.EQ.3) GO TO 320
C
C     CALCULATE THE AVERAGE ELEMENT SHRINKAGE
C
      DO 310 I=1,NUMEL
      J=LM(1,I)
      K=LM(2,I)
      M=LM(3,I)
      N=LM(4,I)
      IF (M.EQ.N) GO TO 300
      AT(I)=(T(J)+T(K)+(M)+T(N))/4.0
      GO TO 310
  300 CONTINUE
      AT(I)=(T(J)+T(K)+T(M))/3.0
  310 CONTINUE
  320 CONTINUE
      N1=1
  330 N2=N1+5
      IP2=IP2+1
      IF (NUMEL-N2) 350,340,340
  340 WRITE (NP,450) (I,AT(I),I=N1,N2),ELEM,IP2
      IF (N2.EQ.NUMEL) GO TO 360
      N1=N2+1
      GO TO 330
  350 N2=NUMEL
      M=72-N*12
      N=NUMEL+1-N1
      ENCODE (30,460,L) N,M
      WRITE (NP,L) (I,AT(I),I=N1,N2),ELEM,IP2
  360 CONTINUE
C
      RETURN
C
C
C
  370 FORMAT (////,37H  . . . . PUNCHING NODAL DATA . . . .)
  380 FORMAT (1HN,A3)
  390 FORMAT (7A10,A2,A4,I4)
  400 FORMAT (52H  NODAL POINT SHRINKAGES FOR SELECTED TIME INTERVALS,20
     1X,A4,I4)
  410 FORMAT (38H  NODAL POINT COORDINATES - NUMBER,X,Y,34X,A4,I4)
  420 FORMAT (4(I4,2F6.3),8X,A4,I4)
  430 FORMAT (1H(,I1,11H(I4,2F6.3),,I2,8HX,A4,I4))
  440 FORMAT (41H  ----NODAL POINT SHRINKAGES AT TIME   = ,F7.3,3H - ,I3
     1,6H NODES,12X,A4,I4)
  450 FORMAT (6(I4,F8.6),A4,I4)
  460 FORMAT (1H(,I1,10H(I4,F8.6),,I2,8HX,A4,I4))
  470 FORMAT (////,39H  . . . . PUNCHING ELEMENT DATA . . . .)
  480 FORMAT (1HE,A3)
  490 FORMAT (48H  ELEMENT SHRINKAGES FOR SELECTED TIME INTERVALS,24X,A4
     1,I4)
  500 FORMAT (43H  ELEMENT CENTROID COORDINATES - NUMBER,X,Y,29X,A4,I4)
  510 FORMAT (37H  ----ELEMENT SHRINKAGES AT TIME   = ,F7.3,3H - ,I3,9H
     1ELEMENTS,13X,A4,I4)
C
      END
```