Title
Asymmetric dark matter with a possible Bose-Einstein condensate

Journal
Physical Review D, 99(2)

ISSN
2470-0010

Authors
HajiSadeghi, Shakiba
Smolenski, S
Wudka, J

Publication Date
2019-01-15

DOI
10.1103/physrevd.99.023514

Peer reviewed

Prepared for submission to JHEP

Asymmetric dark matter with a possible Bose-Einstein condensate.

S. HajiSadeghi, S. Smolenski, and J. Wudka

Department of Physics and Astronomy, UC Riverside, Riverside, CA 92521-0413, U.S.A

E-mail: shakiba.hajisadeghi@email.ucr.edu, sarah.smolenski@email.ucr.edu, jose.wudka@ucr.eduv

Abstract: We investigate the properties of a Bose gas with a conserved charge as a dark matter candidate, taking into account the restrictions imposed by relic abundance, direct and indirect detection limits, big-bang nucleosynthesis and large scale structure formation constraints. We consider both the WIMP-like scenario of dark matter masses $\gtrsim 1$ GeV, and the small mass scenario, with masses $\lesssim 10^{-11}$ eV. We determine that a Bose-Einstein condensate will be present at sufficiently early times, but only for the small-mass scenario it will remain at the present epoch.

Contents

1 Introduction

Understanding the nature of dark matter (DM) remains one of the most pressing contemporary issues in astroparticle physics and cosmology. To date all DM properties have been inferred form its gravitational effects [1]; other probes, such as direct [2–5] and indirect [6–8] detection experiments and LHC measurements [9] have produced only limits. These constraints have led to a significant shrinkage of the allowed parameter space in many theoretically favored models [10–12], and this has spurred interest in alternative models involving dark sectors of varied complexity [13–18].

A large number of model for DM assume a dark sector that contains one or more dark scalars, which in some cases are the main contributors to the relic abundance required by the CMB experiments [19]. Having such scalar relics opens the possibility of such particles undergoing a transition to a Bose-condensed phase; in fact, a variety of models of this kind have been studied in the literature. In some cases the condensate can appear only in the non-relativistic regime, as it happens in axion [20–22] and axion-like [23–47] models, where the scalars are assumed to be extremely light. The effects of Bose condensates in such cases have been studied extensively in cosmology [23–37, 40] and in astrophysics [38–47], especially in the context of galactic dynamics, where quantum effects of these very light scalars address the cusp vs. core [48] and "too big to fail" [49] problems if the scalar mass is $\sim O(10^{-22})$ eV (though simulations including both baryonic and Bose-gas components is still lacking). Recently, the authors of Ref. [50] investigated the effects of these light bosons on the Lyman α forest and gave a lower bound on the scalar mass $\gtrsim O(10^{-20})$ eV that excludes the favored mass range, though this result is still being debated [51].

In this paper we will consider a DM model that involves a new continuous symmetry under which all SM particles are singlets. Given the appropriate conditions and particle content, this symmetry allows for a Bose-Einstein condensate (BEc) to form. This situation differs form most models involving BEc in that the symmetry involved is assumed to be exact, and so the presence of a condensate is not restricted to the case where DM is non-relativistic. We will assume a flat, homogeneous and isotropic universe; effects of fluctuations will be discussed in a future publication.

The simplest model that generates a Bose condensate involves a single complex scalar field χ: the associated $U(1)$ symmetry,

$$\chi \to e^{i\alpha}\chi\,, \quad (\alpha = \text{const.}) \tag{1.1}$$

leads to the required conservation law. Models without an exact conservation law can of course also condense, but only in the non-relativistic regime, where particle number plays the role of a conserved charge; without a conserved charge the condensate will necessarily disappear as the temperature approaches the particle mass. In contrast, the presence or absence of a condensate in models with a conserved charge is determined by the temperature and density of the gas, in particular, relativistic gases of this sort can condense if the density is sufficiently high.

In this paper we will study several aspects of dark matter models that contain an exactly conserved charge. The thermodynamic parameters then will include the corresponding chemical potential μ that is bound by the particle mass $|\mu| \le m_{\tt be}$; a condensate will be present whenever the equality holds [1]. The condition $\mu \ne 0$ presupposes the presence of a primordial charge whose possible origin we will not discuss in this paper. We will consider two mass regions for the DM: *(i)* $m_{\tt be} \ge 1$ GeV where the behavior in many situations is WIMP like; and *(ii)* $m_{\tt be} \lesssim 2\times 10^{-11}$ eV where the gas can exhibit a condensate at the present epoch.

The model we consider has then the Lagrangian

$$\mathcal{L} = |\partial\chi|^2 - {m_{\tt be}}^2|\chi|^2 - \frac{1}{2}\lambda_{\tt be}|\chi|^4 + \epsilon|\chi|^2|\phi|^2 + \mathcal{L}_{\tt sm} \tag{1.2}$$

[1]The explicit definition of μ is given in eq. (2.1) below; in the non-relativistic regime it si customary to define a shifted quantity $\mu' = \mu - m_{\tt be}$ so that condensation corresponds to the condition $\mu' = 0$.

where ϕ denotes the SM scalar isodoublet. This is a simple extension of the usual Higgs-portal models that involve a real scalar field. Various cosmological aspects of this type of models have been studied [23–37, 40], with emphasis on the cosmological aspects of the theory and the low mass regime. Here we will be interested in a much wider range of masses. in various aspects of the detection of the model, and in studying the conditions under which a Bose-Einstein condensate can occur.

In the usual Higgs-portal models [52, 53], for a given choice of DM mass, the relic abundance and direct detection constraints impose, respectively, lower and upper limits on the DM self coupling constant, and these limits are consistent only for a relatively small range of masses ($55\ \text{GeV} < m_{\tt be} < 62\ \text{GeV}$ or $m_{\tt be} > 400\ \text{GeV}$) [54]; in particular, light masses are excluded. The model eq. (1.2) sidesteps some of these difficulties because of the presence of a chemical potential μ: the relic abundance depends on the mass $m_{\tt be}$, the coupling ϵ and μ; the possibility of adjusting the latter relaxes the constraints on the first two parameters (the more severe restrictions found in the simplest Higgs-portal models reappear if one imposes the constraint $\mu = 0$).

We will assume that the self-coupling $\lambda_{\tt be}$ in eq. (1.2) is sufficiently large to ensure that the gas remains in equilibrium and yet small enough to ensure that the λ-dependent contributions to thermodynamic quantities represent small corrections to the free-gas expressions. Under these circumstances these thermodynamic quantities take the naive expressions found in textbooks [55]. It is also worth noting that as a statistical system the BE gas may or may not be in equilibrium with the SM, this is determined by the strength of the coupling ϵ in eq. (1.2) as well as by the environment, specifically, by the rate of expansion of the universe. As long the gas and the SM are in equilibrium they will have the same temperature; when gas and SM are not in equilibrium they can have different temperatures, but even then the gas will be in equilibrium with itself and behave as a regular statistical system.

In most publications the relic abundance is calculated using the Boltzmann equation to determine the DM abundance through the decoupling era and into the late universe. We will follow a different approach based on the Kubo formalism [56, 57] that can be used to describe the decoupling of two statistical systems; since the Bose gas remains a statistical system after decoupling such an approach is desirable. For the relic abundance calculation we will use the naive criterion, where decoupling occurs when the interaction rate falls below the Hubble parameter. We do this for simplicity, but also because the presence of a chemical potential allows us to adjust the relic abundance to the experimentally required value, so the full calculation using the kinetics of a Bose gas is not warranted.

We assume throughout that the model is in the perturbative regime: if the Higgs self-coupling takes the form $\mathcal{L}_{\tt sm} \supset -\lambda_{\tt sm}|\phi|^4/2$, then (tree-level) vacuum stability requires $\epsilon > -\sqrt{\lambda_{\tt be}\,\lambda_{\tt sm}}$, while the model remains perturbative provided $4\pi \gtrsim \lambda_{\tt sm},\, \lambda_{\tt be} > 0$. As noted above, we will study the Bose gas in two mass regimes: the WIMP case where $m_{\tt be} \gtrsim 1$ GeV and the low mass scenario $m_{\tt be} \lesssim 2 \times 10^{-11}$ eV. The remaining model parameter, μ, is restricted by $|\mu| \le m_{\tt be}$ (to lowest order in $\lambda_{\tt be}$).

The rest of the paper is organized as follows: in the next section we discuss the cosmology of a Bose gas to lowest (zeroth) order in λ_{be} and discuss some aspects of the conditions under which a condensate is present. We next consider relic abundance and the decoupling transition (section 4) and direct detection (section 5) in the WIMP regime. We discuss the low-mass scenario in section 6, including constraints form large scale structure formation and big-bang nucleosynthesis. Section 7 contains parting comments and conclusions while the appendices involve some formulae used in the text.

2 Cosmology with a Bose gas

As mentioned in the introduction we will consider the behavior of a Bose gas in an expanding universe, including the possibility that a Bose-Einstein condensate (BEc) may be present at some time. We will assume that the rate of expansion of the universe is sufficiently slow that the gas will be in local thermodynamic equilibrium [2], so that the well-know expressions for an ideal Bose gas [55] can be used as reasonable approximations (to order λ_{be}^0, see eq. (1.2))

We denote the occupation numbers for particles and antiparticles are given by

$$\begin{aligned} n_{\mathsf{be}}^+ &= \left(e^{(E-\mu)/T}-1\right)^{-1} = \left(e^{\mathrm{x}(\sqrt{u^2+1}-\varpi)}-1\right)^{-1}; \quad \mathrm{x} = \frac{m_{\mathsf{be}}}{T}\,,\ \varpi = \frac{\mu}{m_{\mathsf{be}}} \\ n_{\mathsf{be}}^- &= \left(e^{(E+\mu)/T}-1\right)^{-1} = \left(e^{\mathrm{x}(\sqrt{u^2+1}+\varpi)}-1\right)^{-1}, \end{aligned} \tag{2.1}$$

where $E=\sqrt{\mathbf{p}^2+m^2}$ and $u=|\mathbf{p}|/m_{\mathsf{be}}$. Also

$$|\mu| \le m_{\mathsf{be}} \quad \Rightarrow \quad |\varpi| \le 1. \tag{2.2}$$

In terms of these the conserved charge associated with the symmetry eq. (1.1) is given by

$$\begin{aligned} q_{\mathsf{be}} &= q_{\mathsf{be}}^{(c)} + q_{\mathsf{be}}^{(e)} \\ &= q_{\mathsf{be}}^{(c)} + {m_{\mathsf{be}}}^3 \nu_{\mathsf{be}}\,; \quad \nu_{\mathsf{be}} = \frac{1}{2\pi^2}\int_0^\infty du\, u^2 (n_{\mathsf{be}}^+ - n_{\mathsf{be}}^-)\,, \end{aligned} \tag{2.3}$$

where $q_{\mathsf{be}}^{(e,c)}$ are the charge densities in the excited states and in the condensate (if present). Without loss of generality we will assume $q_{\mathsf{be}}^{(c)} \ge 0$; if there is a condensate then $\varpi=1$.

The entropy and energy densities for the Bose gas are given by

$$\begin{aligned} s_{\mathsf{be}} &= {m_{\mathsf{be}}}^3\sigma_{\mathsf{be}}\,; &\sigma_{\mathsf{be}} &= \frac{1}{2\pi^2}\int_0^\infty du\, u^2 \sum_{n=n_{\mathsf{be}}^+,n_{\mathsf{be}}^-} \left[(1+n)\ln(1+n) - n\ln n\right]\,, \\ \rho_{\mathsf{be}} &= m_{\mathsf{be}} q_{\mathsf{be}}^0 + {m_{\mathsf{be}}}^4 r_{\mathsf{be}}\,; &r_{\mathsf{be}} &= \frac{1}{2\pi^2}\int_0^\infty du\, u^2\sqrt{u^2+1}(n_{\mathsf{be}}^+ + n_{\mathsf{be}}^-)\,. \end{aligned} \tag{2.4}$$

For future use we note the following approximations

$$\varpi=1,\ \mathrm{x}\ll 1: \qquad r_{\mathsf{be}}(\mathrm{x}) \simeq \frac{\pi^2}{15\mathrm{x}^4}\,, \qquad \nu_{\mathsf{be}}(\mathrm{x}) \simeq \frac{1}{3\mathrm{x}^2}\,, \qquad \sigma_{\mathsf{be}}(\mathrm{x}) \simeq \frac{4\pi^2}{45\mathrm{x}^3}\,. \tag{2.5}$$

$$\varpi=1,\ \mathrm{x}\gg 1: \qquad r_{\mathsf{be}}(\mathrm{x}) \simeq \frac{\zeta_{3/2}}{(2\pi\mathrm{x})^{3/2}}\,, \quad \nu_{\mathsf{be}}(\mathrm{x}) \simeq \frac{\zeta_{3/2}}{(2\pi\mathrm{x})^{3/2}}\,, \quad \sigma_{\mathsf{be}}(\mathrm{x}) \simeq \frac{(5/2)\zeta_{5/2}}{(2\pi\mathrm{x})^{3/2}}\,. \tag{2.6}$$

[2] This is discussed in detail in [36].

The Standard Model energy and entropy densities are approximately given by [59]

$$\rho_{\tt sm} = \frac{\pi^2}{30} T^4 g_\star(T)\,, \qquad s_{\rm sm} = \frac{2\pi^2}{45} T^3 g_{\star\tt s}(T)\,, \tag{2.7}$$

where

$$\begin{aligned} g_\star(T) &\simeq \sum_{\rm bosons} g_i \left(\frac{T_i}{T}\right)^4 \theta(T-m_i) + \frac{7}{8}\sum_{\rm fermions} g_i \left(\frac{T_i}{T}\right)^4 \theta(T-m_i)\,, \\ g_{\star\tt s}(T) &\simeq \sum_{\rm bosons} g_i \left(\frac{T_i}{T}\right)^3 \theta(T-m_i) + \frac{7}{8}\sum_{\rm fermions} g_i \left(\frac{T_i}{T}\right)^3 \theta(T-m_i)\,, \end{aligned} \tag{2.8}$$

where g_i denotes the number of internal degrees of freedom, and T_i the temperature for each particle; we assumed a zero chemical potential for the SM particles.

In the discussion below we repeatedly use the fact that when the SM and Bose gas are in equilibrium the ratio $q_{\tt be}/s_{\tt tot}$ is conserved, where $s_{\tt tot} = s_{\rm be} + s_{\rm sm}$ is the total entropy. When the SM and Bose gas are not in equilibrium the ratios $q_{\tt be}/s_{\rm sm}$ and $s_{\rm be}/s_{\rm sm}$ are separately conserved (in this case $q_{\tt be}/s_{\tt tot}$ is also conserved, but it is not independent of these quantities).

3 The Bose-Einstein condensate

As noted above, whether the SM and gas are in equilibrium or not, the ratio

$$Y = \frac{q_{\tt be}}{s_{\tt tot}} = Y^{(e)} + Y^{(c)}\,; \quad Y^{(e,c)} = \frac{q_{\tt be}^{(e,c)}}{s_{\tt tot}} \tag{3.1}$$

is conserved (though the (e) and (c) contributions in general are not). A condensate will be present whenever the total charge cannot be accommodated in the excited states, that is,

$$q_{\tt be}^{(c)} \neq 0 \quad \text{if} \quad Y > Y^{(e)}\Big|_{\varpi=1} = \left.\frac{\nu_{\tt be}}{\sigma_{\tt be} + s_{\rm sm}/{m_{\tt be}}^3}\right|_{\varpi=1}\,. \tag{3.2}$$

Now, since $s_{\rm sm} > 0$ we have the following inequality:

$$Y^{(e)}\Big|_{\varpi=1} < \left.\frac{\nu_{\tt be}}{\sigma_{\tt be}}\right|_{\varpi=1} < \left.\frac{\nu_{\tt be}}{\sigma_{\tt be}}\right|_{\varpi=1\,,T\to 0} = \frac{\zeta_{3/2}}{(5/2)\zeta_{5/2}} \simeq 0.78\,. \tag{3.3}$$

Therefore, a condensate will be always present if $Y > 0.78$. Note also (cf. eq. (2.5)) that for large temperatures [3] $\nu \sim T^2$ (since the leading particle and antiparticle contributions to $\nu_{\tt be}$ in eq. (2.3) cancel), while $\sigma \sim T^3$. It follows that $Y^{(e)} \to 0$ as $T \to \infty$, in particular, a condensate will always be present at sufficiently high temperatures [4] [61]. These properties are illustrated in figure 1.

[3]The Bose gas entropy and charge are not exponentially suppressed as $T \to 0$ because of the condition $\varpi = 1$.

[4]This holds whether the SM and Bose gas are in equilibrium or not.

To clarify this behavior note that $T \to \infty$ corresponds to $a \to 0$, where a denotes the distance scale in the Robertson-Walker metric: a contracting co-moving volume accompanies an increasing temperature. There are then two competing effects on the Bose gas, the reduction of volume favors the formation of the condensate, while the increase in temperature tends to destroy it; the above results indicate that the volume effect dominates.

Figure 1 also shows that $Y^{(e)} \to 0$ as $x \to 0$; it follows that even if $Y < 0.78$, a condensate will be present at a sufficiently early time (*i.e.* for T large enough); but it will cease to exist when $Y = Y^{(e)}(\varpi = 1)$, thereafter we will have $|\mu| < m_{\mathtt{be}}$. Note also that if a condesate is present at a particular time, it will then be present at all earlier epochs.

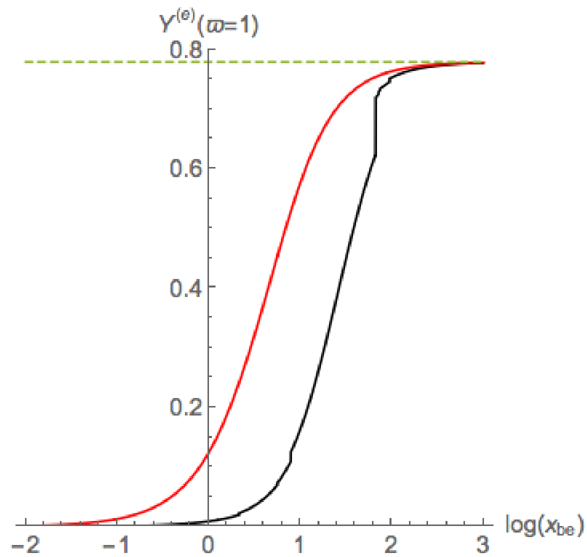

Figure 1. Plot of the Bose charge in the excited states per entropy when $m_{\mathtt{be}} = 10^{-12}$ eV (top curve) and $m_{\mathtt{be}} = 10$ GeV (bottom curve); the dashed line corresponds to the bound in eq. (3.3). For illustration purposes we assumed the Bose gas and the SM have the same temperature. The drop as $T \to \infty$ ocurs because $\nu_{\mathtt{be}} \sim T^2$ while $s_{\mathtt{tot}} \sim T^3$, see eq. (2.5).

Because of the exact $U(1)$ symmetry of the dark sector, the presence of this condensate does not require the gas to be non-relativistic (in which case particle number is conserved). We will see later (see section 6) that experimental constraints allow for the condensate to persist to the present day only if $m_{\mathtt{be}}$ is in the pico- eV range; for WIMP scenarios ($m_{\mathtt{be}} \gtrsim 1$ GeV) the condensate disappears already in the very early universe.

3.1 Conditions for a BEc at decoupling

We will show below that for WIMP-like masses ($m_{\mathtt{be}} \gtrsim 1$ GeV) the gas will be non-relativistic at T_d; it then follows that it will also be non-relativistic at present. Then

$$\frac{q_{\mathtt{be}}}{s_{\mathrm{sm}}} \simeq \frac{1}{m_{\mathtt{be}}} \frac{\rho_{\mathrm{DM}}}{s_{\mathrm{sm}}} = \frac{0.4 \ \mathrm{eV}}{m_{\mathtt{be}}} \qquad (T < T_d)\,, \tag{3.4}$$

where we used the known value of the SM entropy now, and the fact that for a non-relativistic gas $\rho_{\rm DM} = m_{\tt be} q_{\tt be}$; as noted in section 2, the left hand side of eq. (3.4) is conserved below T_d. This can be used to determine whether a Bose condensate would have been present at the decoupling temperature. The condition for the presence of a condensate is

$$\frac{q_{\tt be}(T_d)}{(m_{\tt be} T_d)^{3/2}} > \frac{\zeta_{3/2}}{(2\pi)^{3/2}} \simeq 0.166\,. \tag{3.5}$$

Next, using eq. (3.4) to eliminate $q_{\tt be}(T_d)$ and eq. (2.7) for the SM entropy, we find

$$\frac{T_d^{3/2}}{m_{\tt be}{}^{5/2}} g_{\star{\tt s}}(T_d) > \frac{1}{1.06 \ {\rm eV}}\,. \tag{3.6}$$

Finally, since for a non-relativistic gas $m_{\tt be} > T_d$ and $g_{\star{\tt s}} < 106.75$ we find (using 3σ errors)

$$m_{\tt be} < 1.3 \ {\rm keV} \tag{3.7}$$

A condensate can occur at decoupling only for light Bose particles.

3.2 Conditions for a BEc to exist at present

Before proceeding with the calculation of the cross section relevant for direct detection, we study the possibility that the Bose gas supports a condensate at present. To this end we note first that a non-relativistic Bose gas will have a condensate provided $q_{\tt be}(m_{\tt be}T)^{-3/2} > \zeta_{3/2}(2\pi)^{-3/2}$, see eq. (3.5); denoting the current gas temperature by $T_{\tt now}$ it follows that a condensate will be currently present if

$$\left(\frac{0.0215 \ {\rm eV}}{m_{\tt be}}\right)^{5/3} {}^o{\rm K} > T_{\tt now} \tag{3.8}$$

Now we use the fact that the conservation of $s_{\rm be}/s_{\rm sm}$ allows us to obtain relation between $T_{\tt now}$ and the decoupling temperature T_d, noting that the gas is non relativistic at T_d, and that a condensate at $T_{\tt now}$ implies a condensate was also present at T_d (see section 2), we find

$$\frac{4.3\,{}^o{\rm K}}{g_{\star{\tt s}}(T_d)^{1/3}} = \sqrt{T_d T_{\tt now}} \tag{3.9}$$

where we used eq. (2.4) and eq. (2.7). Combining eq. (3.8) and using eq. (2.6) and eq. (3.9),

$$[g_{\star{\tt s}}(T_d)]^{2/3}\, T_d \gtrsim \left(\frac{m_{\tt be}}{0.009 \ {\rm eV}}\right)^{5/3} {}^o{\rm K} \tag{3.10}$$

and since $m_{\tt be} > T_d$, this gives

$$9.5\, g_{\star{\tt s}}(T_d) \ {\rm eV} \gtrsim m_{\tt be} \quad \Rightarrow \quad 88 \ {\rm eV} > m_{\tt be} \tag{3.11}$$

It follows that a WIMP-like Bose gas will not exhibit a condensate at the present era (nonetheless, for completeness we include in Appendix B the expressions for the cross section when a condensate does occur). The case of a light Bose gas with a condensate will be considered in section 6.

3.3 The BEc transition temperature

For WIMP-like masses we will show (section 4) that the SM and Bose gas will be in equilibrium down to a decoupling temperature T_d, below T_d the ratios $q_{\tt be}/s_{\rm sm}$ and $s_{\rm be}/s_{\rm sm}$ will be separately conserved; above T_d only $q_{\tt be}/s_{\tt tot}$ is conserved; we will also show that in this case the gas was non-relativistic at $T = T_d$ and that the relic abundance constraint reduces to the simple relation $q_{\tt be} = 0.4,\ {\rm eV}(s_{\rm sm}/m_{\tt be})$ (cf. eq. (3.4)). Combining these results we find that the temperature $T_{\tt BEC}$ at which the condensate forms (the same for the gas and SM since $T_{\tt BEC} > T_d$) is given by

$$[2 + g_{*s}(T_{\tt BEC})]\, T_{\tt BEC} = \frac{15}{2\pi^2}\left[\frac{5}{2} - \ln z_d + \frac{m_{\tt be}}{0.4\ \text{eV}}\right] m_{\tt be} \quad\Rightarrow\quad T_{\tt BEC} \simeq {m_{\tt be}}^2 \frac{1.9\ \text{eV}^{-1}}{g_{\star s}(T_{\tt BEC}) + 2}\,, \tag{3.12}$$

where [5] $z = \exp[(\varpi - 1)x]$, and we used the fact that $|\ln z| \ll m_{\tt be}/(0.4\ \text{eV})$ for all cases being considered.

For example, $T_{\tt BEC} \sim 10^7$ GeV if $g_\star(T_{\tt BEC}) \sim 100$ and $m_{\tt be} \sim 1$ GeV (though, of course, the number of relativistic degrees of freedom at these high temperatures may be much higher); while $T_{\tt BEC} \sim 1.75$ TeV if $g_\star(T_{\tt BEC}) = 106.75$ and $m_{\tt be} \sim 10$ MeV. It is worth noting that for the WIMP-like scenario, the condensate, when it forms, will hold a small fraction of the total energy density of the gas: using eq. (2.5) and eq. (2.6) and the above conservation laws we find,

$$\left.\frac{m_{\tt be} q_{\tt be}}{\rho_{\tt be}}\right|_{T > T_{\tt BEC}} = \frac{2 + g_{*s}(T) - (5/\pi^2) A\, x}{2 + g_{*s}(T) + A\, x^{-1}}\,, \quad A = \frac{3}{2}\left[\frac{5}{2} - \ln z + \frac{m_{\tt be}}{0.4\ \text{eV}}\right] \simeq \frac{m_{\tt be}}{0.27\ \text{eV}} \tag{3.13}$$

$$\simeq (0.27\ \text{eV}) \frac{2 + g_{*s}(T)}{T} \qquad \text{for } x \ll 0.4\ \text{eV}/m_{\tt be} \tag{3.14}$$

So in the early universe $Y^{(e)} \to 0$ but $\rho^{(e)}_{\tt be}/\rho_{\tt be} \to 1$: the charge is mainly in the condensate, but the energy is mainly in the excited states.

For an ultra-light DM ($m_{\tt be} \sim 10^{-12}$ eV) the situation is completely different, we discuss this in section 6.

4 Relic abundance

In obtaining the relic abundance we will follow an approximate method that will not involve solving the Boltzmann equation. Instead we imagine the Bose gas and the SM to be in equilibrium at some early time and describe their decoupling using the Kubo formalism [56].

The total Hamiltonian for the system is of the form

$$H = H_{\tt sm} + H_{\tt be} - H'\,, \quad H' = -\epsilon \int d^3x\, \mathcal{O}_{\tt sm} \mathcal{O}_{\tt be}\,, \tag{4.1}$$

[5] It follows from eq. (2.6) and the conservation laws that z is constant below T_d for a non-relativistic gas without a condensate.

where $\mathcal{O}_{\tt sm} = |\phi|^2$ $\mathcal{O}_{\tt be} = |\chi|^2$ and ϵ is defined in eq. (1.2). Using the same arguments as in [57] (for a different situation), we find that a possible temperature difference (and hence a lack of equilibrium) between the SM and Bose gas obeys

$$\dot{\vartheta} + 4\mathbb{H}\vartheta = -\Gamma\vartheta\,; \quad \vartheta = T_{\tt be} - T_{\tt sm}\,, \tag{4.2}$$

where $\mathbb{H}$ is the Hubble parameter. This expression is valid when the temperature difference is small, so the width Γ can be evaluated at the (almost) common temperature T. We use this expression to define the temperature T_d at which the SM and Bose gas decouple by the condition

$$T = T_d \quad \Rightarrow \quad \Gamma = \mathbb{H}\,. \tag{4.3}$$

Explicitly we have [57] (for a different situation),

$$\Gamma = \left(\frac{1}{c_{\tt be}} + \frac{1}{c_{\tt sm}}\right)\frac{\epsilon^2 G}{T}\,, \tag{4.4}$$

where $c_{\tt sm}$, $c_{\tt be}$ denote the heat capacities per unit volume, T the common temperature, and

$$G = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle \mathcal{O}_{\rm BE}(-is,\mathbf{x})\dot{\mathcal{O}}_{\rm BE}(t,\mathbf{0})\right\rangle \left\langle \mathcal{O}_{\rm SM}(-is,\mathbf{x})\dot{\mathcal{O}}_{\rm SM}(t,\mathbf{0})\right\rangle\,. \tag{4.5}$$

The heat capacities are given by

$$\begin{aligned} c_{\tt sm} &= \frac{4\pi^2}{30}T^3 g_{\star s}\,; \\ c_{\tt be} &= \left(\frac{m_{\tt be}T}{2\pi}\right)^{3/2} \times \begin{cases} (15/4){\rm Li}_{5/2}(1) & \text{(BEc)}\,, \\ (15/4){\rm Li}_{5/2}(z) - (9/4)[{\rm Li}_{3/2}(z)]^2/{\rm Li}_{1/2}(z) & \text{(no BEc)}\,, \end{cases} \end{aligned} \tag{4.6}$$

where Li denotes the Poly-logarithmic function and $z = \exp[(\mu - m_{\tt be})/T]$.

4.1 Evaluation of G

In the presence of a condensate we follow [60] and write $\chi = [(A_1 + C) + iA_2]/\sqrt{2}$, $A_{1,2}$ denote the fields and C the condensate amplitude. We also assume that decoupling occurs below the electroweak phase transition so that $|\phi|^2 = (v+H)^2/2$, where v is the SM vacuum expectation value, and H the Higgs field. Substituting in eq. (4.5) we find, after an appropriate renormalization,

$$G_{\tt BEc} = \left[v^2C^2G_{2-2} + \frac{1}{4}C^2G_{2-4} + \frac{1}{4}v^2G_{4-2} + \frac{1}{16}G_{4-4}\right]_{\mu=m_{\tt be}}\,, \tag{4.7}$$

where

$$G_{2-2} = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle A_1(-is,\mathbf{x}) \frac{dA_1(t,\mathbf{0})}{dt} \right\rangle \left\langle H(-is,\mathbf{x}) \frac{dH(t,\mathbf{0})}{dt} \right\rangle ,$$

$$G_{2-4} = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle A_1(-is,\mathbf{x}) \frac{dA_1(t,\mathbf{0})}{dt} \right\rangle \left\langle H^2(-is,\mathbf{x}) \frac{dH^2(t,\mathbf{0})}{dt} \right\rangle ,$$

$$G_{4-2} = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle {}^2(-is,\mathbf{x}) \frac{d^2(t,\mathbf{0})}{dt} \right\rangle \left\langle H(-is,\mathbf{x}) \frac{dH(t,\mathbf{0})}{dt} \right\rangle ,$$

$$G_{4-4} = \int_0^\beta ds \int_0^\infty dt \int d^3\mathbf{x} \left\langle {}^2(-is,\mathbf{x}) \frac{d^2(t,\mathbf{0})}{dt} \right\rangle \left\langle H^2(-is,\mathbf{x}) \frac{dH^2(t,\mathbf{0})}{dt} \right\rangle . \tag{4.8}$$

In the absence of a condensate we have

$$G_{\mathtt{BEc}} = \frac{1}{4} v^2 G_{4-2} + \frac{1}{16} G_{4-4} , \tag{4.9}$$

evaluated at a chemical potential $|\mu| < m_{\mathsf{be}}$

We evaluate the G_{n-m} using the standard Feynman rules for the real-time formalism of finite-temperature field theory (see for example [58]) and the propagators derived in appendix A. The calculation is straightforward but tedious; to simplify the expressions we use the following shortcuts:

$$\begin{array}{llll} E = E_{\mathbf{k}} , & E' = E_{\mathbf{k}'} , & \bar{E} = \bar{E}_{\mathbf{q}} , & \bar{E}' = \bar{E}_{\mathbf{q}'} , \\ n_{\mathtt{H}} = n_{\mathtt{H}}(E_{\mathbf{k}}) , & n'_{\mathtt{H}} = n_{\mathtt{H}}(E_{\mathbf{k}'}) , & n^{\pm}_{\mathsf{be}} = n^{\pm}_{\mathsf{be}}(\bar{E}_{\mathbf{q}}) , & n^{\pm}_{\mathsf{be}}{}' = n^{\pm}_{\mathsf{be}}(\bar{E}_{\mathbf{q}'}) , \end{array} \tag{4.10}$$

and

$$d\tilde{\mathbf{k}} = \frac{d^3\mathbf{k}}{2E_{\mathbf{k}}(2\pi)^3} , \qquad d\tilde{\mathbf{q}} = \frac{d^3\mathbf{q}}{2\bar{E}_{\mathbf{q}}(2\pi)^3} \tag{4.11}$$

where

$$E_{\mathbf{k}} = \sqrt{m_{\mathtt{H}}{}^2 + \mathbf{k}^2} , \qquad \bar{E}_{\mathbf{q}} = \sqrt{m_{\mathsf{be}}{}^2 + \mathbf{q}^2} ; \qquad n^{(\pm)}_{\mathsf{be}}(\bar{E}) = \left[e^{\beta(\bar{E} \mp \mu)} - 1 \right]^{-1} , \tag{4.12}$$

and $m_{\mathtt{H}}$ denotes the Higgs mass.

Then the G_{n-m} (for arbitrary μ) are given by

- G_{4-4}

$$G_{4-4} = 16\pi\beta \int d\tilde{\mathbf{k}}\, d\tilde{\mathbf{k}'}\, d\tilde{\mathbf{q}}\, d\tilde{\mathbf{q}'}\, (2\pi)^3\, \delta^{(3)}(\mathbf{k} + \mathbf{k}' + \mathbf{q} + \mathbf{q}') \mathcal{G}_{4-4} ;$$

$$\begin{aligned} \mathcal{G}_{4-4} = {} & \frac{1}{2}(1 + n_{\mathtt{H}})(1 + n'_{\mathtt{H}}) n^{+}_{\mathsf{be}} n^{-}_{\mathsf{be}}{}'\, \delta(E + E' - \bar{E} - \bar{E}')\, (E + E')^2 \\ & + \frac{1}{2}(1 + n^{+}_{\mathsf{be}})(1 + n^{-}_{\mathsf{be}}{}') n_{\mathtt{H}} n'_{\mathtt{H}}\, \delta(E + E' - \bar{E} - \bar{E}')\, (E + E')^2 \\ & + (1 + n_{\mathtt{H}})(1 + n^{+}_{\mathsf{be}}) n'_{\mathtt{H}} n^{+}_{\mathsf{be}}{}'\, \delta(E + \bar{E} - E' - \bar{E}')\, (E - E')^2 \\ & + (1 + n_{\mathtt{H}})(1 + n^{-}_{\mathsf{be}}) n'_{\mathtt{H}} n^{-}_{\mathsf{be}}{}'\, \delta(E + \bar{E} - E' - \bar{E}')\, (E - E')^2 , \end{aligned} \tag{4.13}$$

where the 4 terms represent the processes $HH \leftrightarrow \chi\chi^\dagger$, $H\chi \to H\chi$ and $H\chi^\dagger \to H\chi^\dagger$ respectively; the factors of 1/2 are due to Bose statistics.

- G_{2-4}

$$G_{2-4} = 2\pi\beta \int d\tilde{\mathbf{k}}\, d\tilde{\mathbf{k}}\,' d\tilde{\mathbf{q}}\, (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{k}'+\mathbf{q}) \mathcal{G}_{2-4}\,;$$

$$\begin{aligned} \mathcal{G}_{2-4} = {} & \frac{1}{2}(1+n_{\mathtt{H}})\ (1+n'_{\mathtt{H}})n^-_{\mathsf{be}}\, \delta(E+E'-\bar{E}-m_{\mathsf{be}})\,(E+E')^2 \\ & + \frac{1}{2}(1+n^-_{\mathsf{be}})n_{\mathtt{H}}n'_{\mathtt{H}}\, \delta(E+E'-\bar{E}-m_{\mathsf{be}})\,(E+E')^2 \\ & + (1+n_{\mathtt{H}})n'_{\mathtt{H}}n^+_{\mathsf{be}}\, \delta(E+m_{\mathsf{be}}-E'-\bar{E})\,(E-E')^2 \\ & + (1+n_{\mathtt{H}})(1+n^+_{\mathsf{be}})n'_{\mathtt{H}}\, \delta(E+\bar{E}-E'-m_{\mathsf{be}})\,(E-E')^2\,, \end{aligned} \tag{4.14}$$

these 4 terms represent the processes $HH \leftrightarrow C\chi^\dagger$ and $HC \leftrightarrow H\chi$, where C corresponds to a particle in the condensate (mass m_{be} and zero momentum); the factors of 1/2 are due to Bose statistics.

- G_{4-2}

$$G_{2-4} = 4\pi\beta \int d\tilde{\mathbf{k}}\, d\tilde{\mathbf{q}}\, d\tilde{\mathbf{q}}\,'(2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{q}+\mathbf{q}') \mathcal{G}_{4-2}\,;$$

$$\mathcal{G}_{4-2} = \left[(1+n^+_{\mathsf{be}})(1+n^-_{\mathsf{be}}{}')n_{\mathtt{H}} + (1+n_{\mathtt{H}})\ n^+_{\mathsf{be}}\ n^-_{\mathsf{be}}{}'\right] E^2 \delta(\bar{E}+\bar{E}'-E)\,, \tag{4.15}$$

these 2 terms represent the processes $H \leftrightarrow \chi\chi^\dagger$.

- G_{2-2}

$$G_{2-2} = \frac{1}{2}\pi\beta \int d\tilde{\mathbf{k}}\, d\tilde{\mathbf{q}}\, (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{q}) \mathcal{G}_{2-2}\,;$$

$$\mathcal{G}_{2-2} = \left[(1+n_{\mathtt{H}})n^-_{\mathsf{be}} + (1+n^-_{\mathsf{be}})n_{\mathtt{H}}(E)\right] E^2 \delta(E-m_{\mathsf{be}}-\bar{E})\,, \tag{4.16}$$

these 2 terms represent the processes $H \leftrightarrow C\chi^\dagger$.

In the non-relativistic limit, where $m_{\mathtt{be}}$, $m_{\mathtt{H}} \gg T$ we find [6]

$$
\begin{aligned}
G_{2-2}^{(\mathrm{NR})}\Big|_{\mu=m_{\mathtt{be}}} &\simeq \frac{m_{\mathtt{H}}}{r\sqrt{(2\pi)^3 x}} \frac{2\epsilon_\Gamma\, e^{-2x}}{\epsilon_\Gamma^2 + (r^2-4)^2}\,; \\
G_{2-4}^{(\mathrm{NR})}\Big|_{\mu=m_{\mathtt{be}}} &\simeq \left(\frac{m_{\mathtt{H}}}{2\pi r x}\right)^3 \left[2r^2x^2\rho K_1(\rho) + \zeta_3\left(\frac{(r+1)^2}{4r}\right)\right] e^{-rx}\,; \\
G_{4-2}^{(\mathrm{NR})} &\simeq \left(\frac{m_{\mathtt{H}}}{2\pi}\right)^3 \frac{4}{x^2 r^3}\left[e^{-rx}\sqrt{\pi\left(\frac{rx}{2}\right)^3\left(\frac{r^2}{4}-1\right)}\,\theta(r-2) + \frac{\mathrm{Li}_{3/2}(z)}{z}\frac{2\epsilon_\Gamma\, e^{-2x}}{\epsilon_\Gamma^2+(r^2-4)^2}\right]; \\
G_{4-4}^{(\mathrm{NR})} &\simeq \frac{1}{16}\frac{{m_{\mathtt{H}}}^5}{r^3(1+r)^{7/2}}\left(\frac{2}{\pi x}\right)^{9/2} e^{-rx}\left(z + \frac{1}{z}e^{-2x}\right), \qquad (4.17)
\end{aligned}
$$

where $K_1(u)$, $\zeta_3(u)$ and Li denote the usual Bessel, zeta and Poly-logarithmic functions, and where we defined

$$
r = \frac{m_{\mathtt{H}}}{m_{\mathtt{be}}}, \quad \rho = \frac{4r|r-1|x}{\sqrt{2(r^2+1)}}, \quad \epsilon_\Gamma = r^2\frac{\Gamma_{\mathtt{sm}}}{m_{\mathtt{H}}}, \quad z = e^{\beta(\mu - m_{\mathtt{be}})}\,. \qquad (4.18)
$$

Before continuing it is worth pointing out a slight difference between the expression for Γ derived from eq. (4.5) and eq. (4.4) , and the corresponding expression usually found in the literature (see *e.g.* [59]). The expression eq. (4.4) describes the energy transfer between the SM and the Bose gas, as indicated by the factors of $(E \pm E')^2$ in eqs. (4.13) to (4.16). As a result Γ in eq. (4.4) has a factor $\sim$ (mass/T)^2 compared to the usual expressions, which calculate the change in the DM particle number. As a consequence the decoupling temperature obtained from eq. (4.3) will be somewhat higher than usual; this difference, however, is not significant given that the criterion eq. (4.3) itself is not sharply defined.

4.2 The decoupling temperature

For a non-relativistic at $T = T_d$, we have from eq. (3.4)

$$
\frac{0.4\ \mathrm{eV}}{m_{\mathtt{be}}} s_{\mathrm{sm}}(T_d) \simeq 2(m_{\mathtt{be}}T_d/2\pi)^{3/2}\cosh(\mu/T_d)\, e^{-m_{\mathtt{be}}/T_d}\,. \qquad (4.19)
$$

We will use this expression to eliminate μ in eq. (4.3); in doing this we implement the requirement that the Bose gas generates the correct DM relic abundance [7]

Using then eq. (4.19) to eliminate μ, the condition $\Gamma = \mathbb{H}$ in eq. (4.3) provides a relation between T_d, $m_{\mathtt{be}}$ and ϵ, which we plot in Fig. 2. We see that, as we assumed, the Bose gas is non-relativistic at T_d for a wide range of couplings ϵ. The resonance effects are broadened below $m_{\mathtt{H}}/2$ due to the effects of the non-resonant term in G_{4-2} that are proportional to

[6] $G_{2-2,\,2-4}$ contribute only when there is condensate, so we evaluate then them only for $\mu = m_{\mathtt{be}}$; the expressions for $G_{4-2,\,4-4}$ are valid for all μ.

[7] This calculation can yield $|\mu| > m_{\mathtt{be}}$ for some choice of $m_{\mathtt{be}}$ and T_d, this only means that such masses and temperatures are excluded by the relic abundance constraint.

$\theta(r-2)$. The rapid change in curvature observed for $m_{\mathsf{be}} \sim 100$ GeV is produced by G_{4-4} that dominates Γ for large masses. We also see that for the range of couplings being considered $T_d \lesssim m_{\mathsf{be}}/10$ so that the gas is non-relativistic at decoupling, as was assumed above.

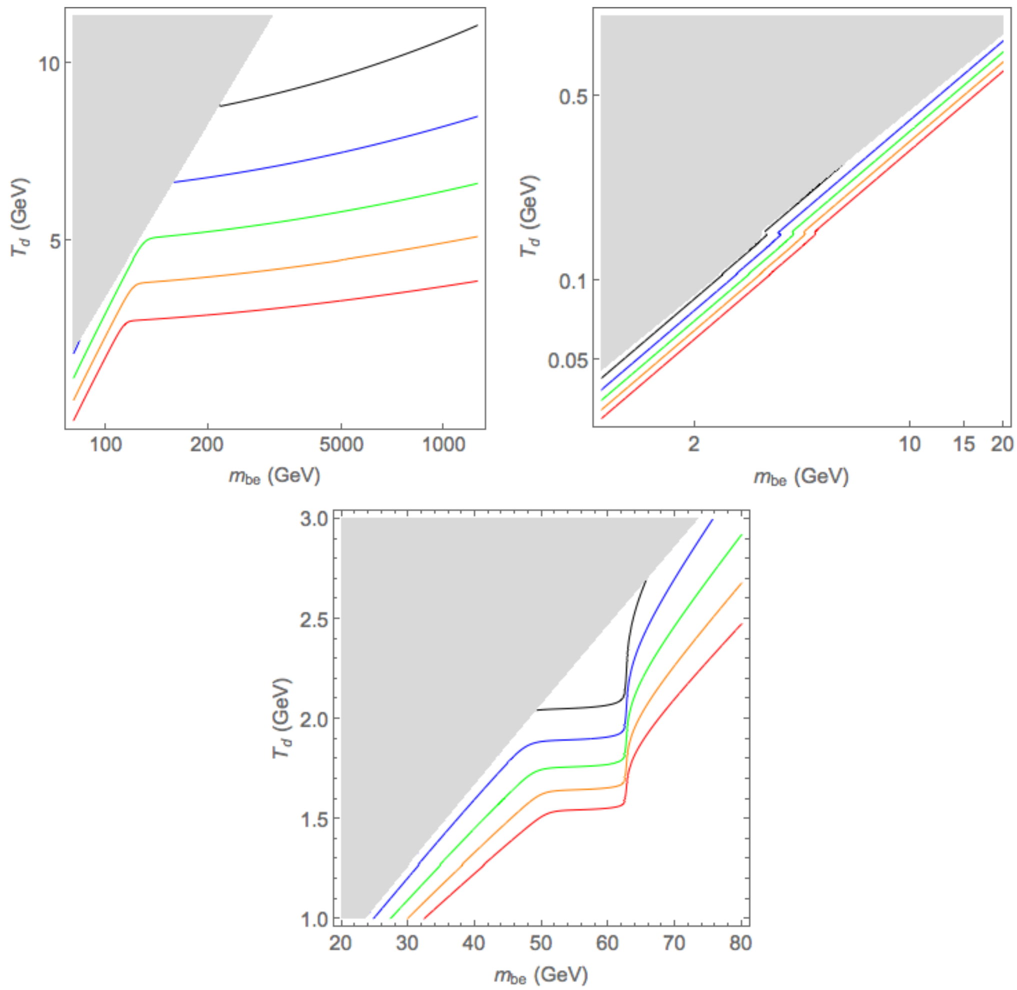

Figure 2. Values of T_d satisfying the decoupling condition eq. (4.3) as a function of m_{be} for $\epsilon = 0.001, 0.01, 0.1, 1, 10$ (bottom to top curves) and for low and high values of m_{be} (top left and right graphs, respectively), and in the resonance region (bottom graph). The peak at $m_{\mathsf{be}} \simeq 62.5$ GeV corresponds to the effects of the Higgs resonance. The shaded region is excluded by the relic abundance constraint.

5 Direct detection

We first calculate the cross section for the process $\eta\chi \to \eta\chi$, where η denotes a neutral scalar coupled to the Bose gas via an interaction

$$\mathcal{L}_{\eta-\chi} = \frac{1}{2} g \eta^2 |\chi|^2 \,. \tag{5.1}$$

The interesting case of nucleon scattering will reduce to the expressions obtained for η in the non-relativistic limit, for an appropriate choice of g, except for a spin multiplicity factor.

The transition probability is given by

$$W_{i\to f} = \left|{}_{\tt out}\langle f\,|i\rangle_{\tt in}\right|^2 , \tag{5.2}$$

where the initial state consists of an η particle with momentum $\mathbf{p}$ and the Bose gas in state X: $|i\rangle_{\tt in} = a_{\mathbf{p}}^{{\tt in}\,\dagger}\,|0;X\rangle$ (where 0 denotes the perturbative vacuum for the η); the final state has an η of momentum $\mathbf{q}$ and the Bose gas in a state Y: $|f\rangle_{\tt out} = a_{\mathbf{q}}^{{\tt out}\,\dagger}\,|0;Y\rangle$. We require $\mathbf{p}\neq\mathbf{q}$, since we are looking for non-trivial interactions.

Using the standard LSZ reduction formula we find

$$\begin{aligned}
{}_{\tt out}\langle f\,|i\rangle_{\tt in} &= \langle 0;Y\,|\Theta_{\mathbf{p},\mathbf{q}}|\,0;X\rangle\ , \\
\Theta_{\mathbf{p},\mathbf{q}} &= -\int d^4x\,d^4x' e^{-ip\cdot x+iq\cdot x'}(\Box_x+m^2)(\Box_{x'}+m^2)\,\mathbb{T}\left[\eta(x)\,\eta(x')\right]\ ,
\end{aligned} \tag{5.3}$$

where $\mathbb{T}$ is the time-ordering operator and we ignored a wave-function renormalization factor (we will be working to lowest non-trivial order where this factor is one). In order to obtain the cross section, we sum over the final gas states (Y) and thermally average over initial gas states (X); this gives

$$\begin{aligned}
\langle W_{i\to f}\rangle_\beta = \int & d^4x\,d^4x'\,d^4y\,d^4y' e^{i(p\cdot y-q\cdot y'-p\cdot x+q\cdot x')}(\Box_x+m^2)(\Box_{x'}+m^2) \\
&\times(\Box_y+m^2)(\Box_{y'}+m^2)\left\langle\mathbb{T}\left[\eta(x^0-i\beta,\mathbf{x})\eta(x'^0-i\beta,\mathbf{x}')\eta(y^0,\mathbf{y})\eta(y'^0,\mathbf{y}')\right]\right\rangle_\beta
\end{aligned} \tag{5.4}$$

where $\langle\ldots\rangle_\beta$ indicates a thermal average at temperature $1/\beta$. $\langle W_{i\to f}\rangle_\beta$ can be evaluated using standard techniques of the real-time formulation of finite-temperature field theory [8] [58], while the optical theorem relates this quantity to the desired cross section:

$$\sigma = \frac{1}{2q_{\tt be}|\mathbf{p}|}\left(\frac{1}{\mathcal{V}}\int' \frac{d^3\mathbf{q}}{2E_{\mathbf{q}}\,(2\pi)^3}, \langle W_{i\to f}\rangle_\beta\right)\ , \tag{5.5}$$

where $E_{\mathbf{q}}$ is the energy of the outgoing η, $q_{\tt be}$ the number density of Bose gas particles, and $\mathcal{V}$ denotes the volume of space-time; the prime indicates that the region $p\simeq q$ is to be excluded.

To lowest order in λ (see eq. (5.1)) we have

$$\langle W_{i\to f}\rangle_\beta = g^2\int\frac{d^4k}{(2\pi)^4}\left[D^<(k+P)\right]_{ij}\left[D^>(k)\right]_{ij}\Big|_{C=0}\ ;\quad P=p-q\,, \tag{5.6}$$

where the propagators are given in eq. (A.12) and eq. (A.14), and $C=0$ implements the absence of a condensate. The evaluation of this expression is straightforward, we find

$$\begin{aligned}
\langle W_{i\to f}\rangle_\beta &= \frac{g^2Tf(-P_0)}{2\pi|\mathbf{P}|}\ln\left|\frac{1+n_{\tt be}^+(E_-)}{1+n_{\tt be}^+(E_+)}\frac{1+n_{\tt be}^-(E_-)}{1+n_{\tt be}^-(E_+)}\right|\ , \\
&\simeq \frac{g^2}{4\pi|\mathbf{P}|\beta}e^{-\beta E_-}\,\cosh(\beta\mu)\,;\quad E_\pm = \frac{1}{2}\left[|\mathbf{P}|\sqrt{1-\frac{4m_{\tt be}{}^2}{P^2}}\mp P_0\right]\ ,
\end{aligned} \tag{5.7}$$

[8] In particular,under $\mathbb{T}$, the complex times in eq. (5.4) are later than the real ones.

where $n_{\tt be}^{(\pm)}$ are defined in eq. (4.10), and f in eq. (A.12); the second expression is valid in the non-relativistic limit. Substituting this into eq. (5.5) gives

$$\begin{aligned}\sigma &= \left[\frac{1}{\sqrt{\pi}\,u}e^{-u^2} + \left(1+\frac{1}{2u^2}\right)\mathrm{Erf}(u)\right]\sigma_0\,; \quad u = \frac{|\mathbf{p}|}{m_{\tt H}}\sqrt{\frac{m_{\tt be}}{2T}}\,, \\ &= \left[1+\frac{1}{2u^2} + O\left(u^{-5}e^{-u^2}\right)\right]\sigma_0\,, \quad (u\to\infty)\end{aligned} \tag{5.8}$$

where σ_0 is the $T=0$ non-relativistic cross section, and we used

$$n = 2\left(\frac{m_{\tt be}T}{2\pi}\right)^{3/2} e^{-\beta m_{\tt be}}\cosh(\beta\mu) \tag{5.9}$$

in eq. (5.5).

The above expression for $\langle W_{i\to f}\rangle_\beta$ holds also for non-relativistic nucleons, except for a factor of $2m_N^2$, where m_N is the nucleon mass. Also, since for the direct-detection reactions the momentum transfer for this process is very small, the coupling g will be given by

$$g \to \frac{\epsilon\, v}{m_H^2} g_{\tt N-H} \quad \Rightarrow \quad \sigma_0 = \frac{1}{8\pi {m_{\tt be}}^2}\left[\frac{m_{\tt be}m_N}{m_{\tt be}+m_N}\,\frac{\epsilon\, g_{\tt N-H} v}{{m_{\tt H}}^2}\right]^2, \tag{5.10}$$

where v denotes the SM vacuum expectation value m_N the nucleon mass, and $g_{\tt N-H} \simeq 0.0034$ the Higgs-nucleon coupling [12, 62, 63].

For the range of parameters we consider the present Bose gas temperature is very small so that

$$\begin{aligned}\sigma &= \frac{\epsilon^2}{8\pi {m_{\tt be}}^2}\left(\frac{m_{\tt be}/m_N}{1+m_{\tt be}/m_N}\,\frac{g_{\tt N-H} v\, m_N}{{m_{\tt H}}^2}\right)^2 \left(1 + r^2\frac{T}{m_{\tt be}\mathsf{v}^2}\right), \\ &\simeq 6.93\times 10^{-34}\left(\frac{\epsilon}{1+m_{\tt be}/m_N}\right)^2\left(1+\frac{m_N^3}{{m_{\tt be}}^3}\frac{T}{600\,^oK}\right)\ \mathrm{cm}^2\,,\end{aligned} \tag{5.11}$$

where r is defined in eq. (4.18), and $\mathsf{v}\simeq 10^{-3}$ is the nucleon-dark matter relative velocity and, as above, $r = m_{\tt H}/m_{\tt be}$.

These results can be compared to the most recent XENON [4] and CDMSLite [64] constraints, we present the results in Fig.3. We find that the leading temperature correction in eq. (5.11) is negligible except for very small $m_{\tt be}$, in this case, however the cross section itself is very small.

The graphs in Fig. 3 represent the strongest constraints on the model parameters. If the parameters are allowed by the direct-detection constraint the model will will satisfy the relic abundance requirement for an appropriate choice of μ.

6 Bose condensate in the small mass region

As noted above, a condensate can occur when the gas has sub-eV masses. In this case, however, there are additional constraints stemming form the possible effects of such light particles on

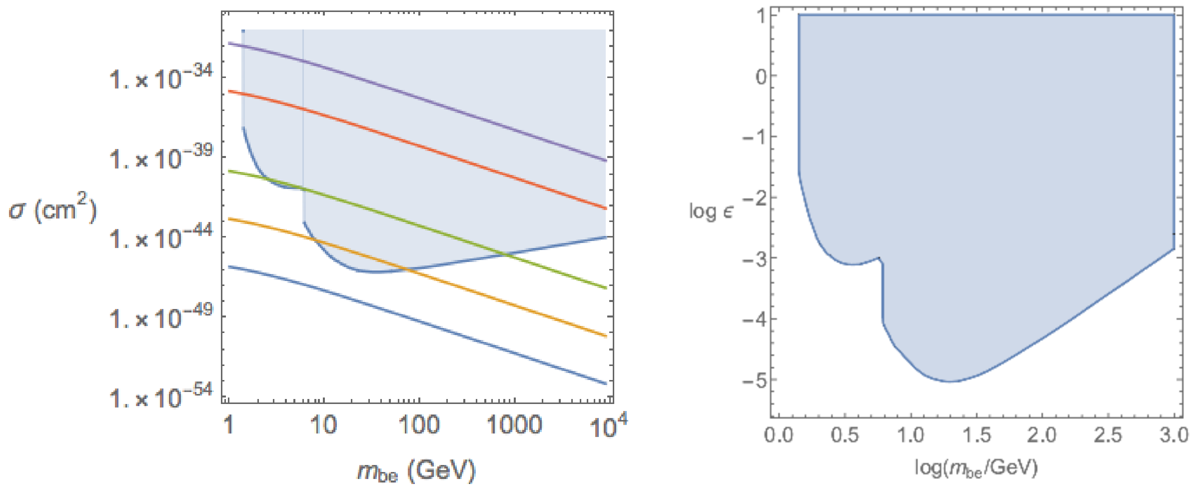

Figure 3. Left: the curves give the direct-detection cross section eq. (5.11) for (lower to upper curves, respectively) $\log\epsilon = -6,\ -4.5,\ -3,\ -0.5,\ 1$ with the shaded area denoting the region excluded by the XENON and CDMSLite experiments. Right: the shaded area denotes the region of the $m_{\tt be} - \epsilon$ plane excluded by direct-detection.

large scale structure (LSS) formation and on big-bang nucleosynthesis (BBN). In this section we will investigate the regions in parameter space allowed by these constraints assuming that the gas is currently condensed, (as noted in section 2 this ensures the presence of a condensate in earlier times).

For the small masses needed to ensure the presence of a BEc now (see below) the expressions derived in section 4 would require a coupling ϵ orders of magnitude above the perturbativity limit (see sect. 1), hence in this case the gas is decoupled form the SM during the BBN and LSS epochs.

LSS formation occurred at redshift $z_{\tt LSS} \sim 3400$ as the matter-dominated era began [59]. To ensure that the Bose gas does not interfere with the formation of structure we require it to be non-relativistic at that time, in addition, since we also assume the presence of a BEc at present, a BEc was present at the LSS epoch (sect. 3). Then the conservation of $a^3 s_{\tt be}$ gives, using eq. (2.6), $a^3 x^{-3/2} =$ constant (a denotes the scale factor in the Robertson-Walker metric); equivalently,

$$\left.\frac{a^2}{x}\right|_{\tt now} = \left.\frac{a^2}{x}\right|_{\tt LSS} \quad \Rightarrow \quad x_{\tt now} = (1 + z_{\tt LSS})^2\, x_{\tt LSS}\,. \tag{6.1}$$

Since the gas must be non-relativistic during the LSS epoch, $x_{\tt LSS} > 3$, so we have

$$x_{\tt now} > 3.5 \times 10^7\,. \tag{6.2}$$

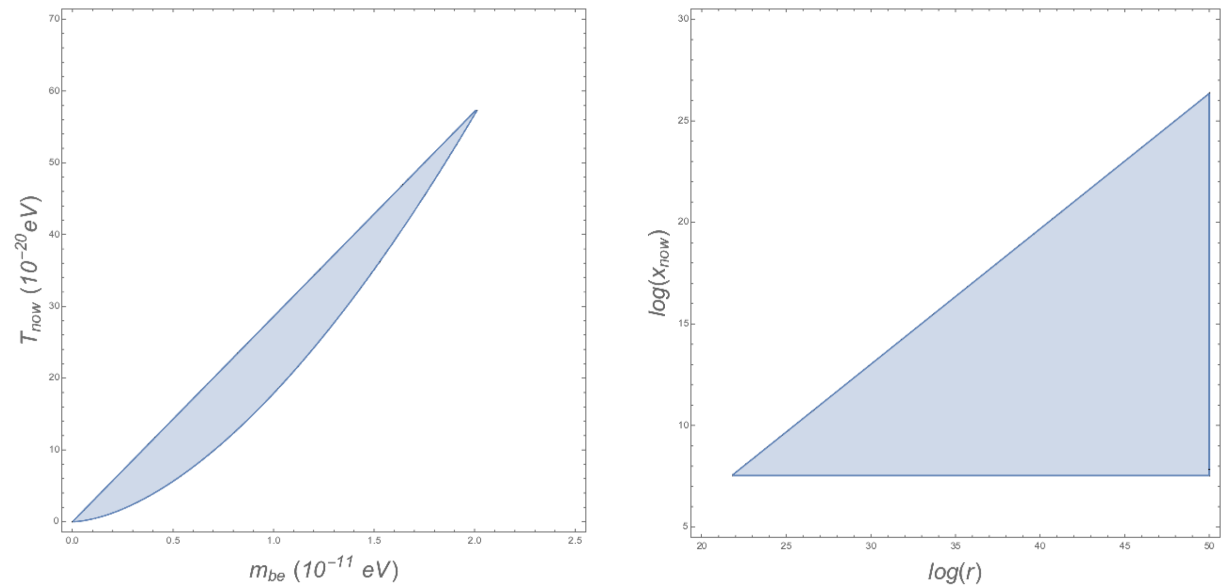

Figure 4. Regions of the $m_{\tt be} - T$ and $r - x$ planes where a non-relativistic Bose condensate occurs consistent with the LSS constraint of eq. (6.2). On the left-hand graph the low-T limit results form eq. (6.3), while the upper limit is due to eq. (6.2).

In addition, the requirement that a BEc be present now implies

$$\frac{0.4 \text{ eV}}{m_{\tt be}} \, s_{\rm sm}\big|_{\tt now} > \left(\frac{{m_{\tt be}}^2}{2\pi \mathrm{x}_{\tt now}} \right)^{3/2} \zeta_{3/2} \,, \tag{6.3}$$

where we used the fact that the gas is currently non-relativistic.

The regions in the $m_{\tt be} - T$ and $m_{\tt be} - \mathrm{x}$ planes allowed by eq. (6.2) and eq. (6.3) are given in Figure 4 (here T refers to the gas temperature). It is worth noting that if these conditions occur at present, most of the gas will be in the condensate: using eq. (3.4) and eq. (6.2) the gas fraction in the excited states is given by

$$\left. \frac{q_{\tt be}^{(e)}}{q_{\tt be}} \right|_{\tt now} < \left(\frac{m_{\tt be}}{1.82 \text{ eV}} \right)^4 \,, \tag{6.4}$$

which is negligible in view of the range of masses being here considered (see figure 4).

We now turn to the BBN constraints. We write the contributions from the gas to the energy density in the form of an effective number of neutrino species ΔN_ν:

$$\rho_{\tt be}|_{\tt BBN} = \frac{3}{\pi^2} \frac{7}{4} \left(\frac{4}{11} \right)^{4/3} \Delta N_\nu \, T_\gamma^4 \simeq 0.138 \Delta N_\nu \, T_\gamma^4 \,; \quad s_{\rm sm} = \frac{2\pi^2}{45} g_{*s}(T_\gamma) T_\gamma^3 \,, \tag{6.5}$$

where $T_\gamma \simeq 0.06$ MeV denotes the photon temperature during BBN [65]. Imposing the relic-abundance constraint eq. (3.4) we find, using eq. (2.1) and eq. (2.4),

$$\Delta N_\nu = 7.2 \times 10^{-5} + 7.24 \frac{m_{\tt be}{}^4}{T_\gamma^4} \left[r_{\tt be}(x_{\tt BBN}) - \nu_{\tt be}(x_{\tt BBN}) \right]_{\varpi=1} , \tag{6.6}$$

where we used the above value of $T_\gamma \simeq 60$ keV, and $g_{*s}(T_\gamma) = 3.38$, and where $r_{\tt be}$ and $\nu_{\tt be}$ are defined in eq. (2.4) and eq. (2.3) respectively. From the LSS constraint $m_{\tt be} < 2 \times 10^{-11}$ eV (see Fig. 4), so that $(m_{\tt be}/T_\gamma) \lesssim 10^{-62}$. It follows that ΔN_ν could be significant only when $r_{\tt be} - \nu_{\tt be}$ is large, which occurs if $x \to 0$, that is, when the gas is relativistic. Using eq. (2.5) we find

$$\frac{s_{\rm be}}{s_{\rm sm}} = {\rm const} = \left. \frac{\frac{5}{2}\zeta_{5/2}(2\pi x)^{-3/2} m_{\tt be}{}^3}{s_{\rm sm}} \right|_{\tt now} \quad \Rightarrow \quad x_{\tt BBN} \simeq \sqrt{\frac{x_{\tt now}}{2.3 \times 10^{16}}} , \tag{6.7}$$

whence (we dropped the contribution from $\nu_{\tt be}$ which is subdominant)

$$\Delta N_\nu \le 7.2 \times 10^{-5} + \left(\frac{4.4 \times 10^{-18}}{x_{\tt now}} \right)^2 \left(\frac{m_{\tt be}}{10^{-11}\ {\rm eV}} \right)^4 , \tag{6.8}$$

which is negligible, since $x_{\tt now} > 3.5 \times 10^7$ and $m_{\tt be} < 2 \times 10^{-11}$ eV. It is worth noting that the gas can be relativistic during BBN.

It remains to see whether a gas satisfying eq. (6.2) can be in equilibrium with the SM (at an epoch earlier than that of BBN), however, given the small range for $m_{\tt be}$ and the large values of $x_{\tt now}$, such equilibrium could have occurred only when the gas was ultra-relativistic, in which environment the presence or absence of a condensate will have no effect. The situation then reduces to that of a standard Higgs-portal model with DM masses in the pico-eV range. Concerning direct detection experiments it is clear that for the very small masses being considered in this section the cross sections will be negligible. We will not consider these points further.

7 Comments and conclusions

In this paper we considered a complex scalar model of dark matter and studied the possible presence of a Bose condensate which can occur even in the relativistic regime due to the presence of a conserved quantum number, associated with the "dark" $U(1)$ symmetry

We showed that a Bose condensate will indeed be present at sufficiently early times, but will persist until the present only if the dark matter mass is in the pico-eV range if the constraints from large scale structure formation are imposed.

The model can meet the relic-density constraint for all masses in the cold dark-matter regime ($m_{\tt be} \gtrsim 1$ GeV) provided the portal coupling $\epsilon \ge 0.1$ (for smaller values the mass range is somewhat restricted, see Figure 2). The limits derived from direct-detection experiments are much more restrictive allowing only small couplings and/or small masses (figure 3), still the allowed region in parameter space is considerably extended compared to the usual Higgs-portal model [54] because of the presence of a chemical potential that can be adjusted to ensure the correct relic density.

For WIMP-like masses we have shown above that there is no condensate for $T < T_d$ but that a condensate forms in the early universe; at very high temperatures the condensate then carries the net charge of the gas, but most of the energy density is carried by the excited states (section 2). In contrast, for very small masses, $m_{\mathtt{be}} \sim 10^{-12}$ eV the gas can form a condensate even at present temperatures while also satisfying the relic abundance requirement. In this case, however, there Bose gas and the SM are never in equilibrium (assuming natural values of ϵ).

We have not discussed indirect detection constraints because, for WIMP-like masses they will be identical to those derived for the standard Higgs portal models [66].

A Appendix: Thermodynamics of a Bose gas

In this appendix we provide for completeness a summary of the Bose gas thermodynamics. We begin with the Lagrangian

$$\mathcal{L} = |\partial\chi|^2 - m^2|\chi|^2 - \frac{1}{2}\lambda|\chi|^4 \tag{A.1}$$

and write $\chi = (A_1 + iA_2)/\sqrt{2}$. Then the Hamiltonian and total conserved charge $Q_{\mathtt{be}}$ are given by

$$H = \int d^3\mathbf{x}\left[\frac{1}{2}\boldsymbol{\pi}^2 + \frac{1}{2}|\nabla|^2 + V\right], \qquad Q_{\mathtt{be}} = -\int d^3\mathbf{x}\,(A_1\pi_2 - A_2\pi_1)\,, \tag{A.2}$$

where π_i is the canonical momentum conjugate to A_i.

To include the possibility of a Bose condensate we replace $A_1 \to A_1 + C$; using then standard techniques of finite-temperature field theory (we use the Matsubara formalism) [67] we find that the pressure P is given by [60, 61]

$$P = \frac{1}{2}(\mu^2 - m^2)C^2 - \frac{1}{8}\lambda C^4 - T\int d_3\mathbf{p}\left[\ln\left(1 - e^{-\beta\Omega_+}\right) + \ln\left(1 - e^{-\beta\Omega_-}\right) + \frac{1}{2}\beta E_0\right], \tag{A.3}$$

where

$$\begin{aligned} \Omega_\pm^2 &= \mu^2 + \mathbf{p}^2 + m^2 + \lambda C^2 \pm \sqrt{4\mu^2(\mathbf{p}^2 + m^2 + \lambda C^2) + (\lambda C^2/2)^2}\,, \\ E_0 &= 2(\mu^2 + \mathbf{p}^2 + m^2 + \lambda C^2) - \sqrt{\mathbf{p}^2 + m^2 + 3\lambda C^2/2} - \sqrt{\mathbf{p}^2 + m^2 + \lambda C^2/2}\,. \end{aligned} \tag{A.4}$$

We asume that $m^2 + \lambda C^2/2 > \mu^2$, which ensures $\Omega_\pm^2 > 0$.

The zero-momentum component C is determined by the condition that it minimizes the thermodynamic potential $-P(C, \mu, T)$:

$$\left(\frac{\partial P}{\partial C}\right) = 0\,. \tag{A.5}$$

A condensate occurs if the above equation has a solution with $C \neq 0$. A sufficient condition for this to happen is $-\partial P/\partial C^2|_{C=0} < 0$; while if $-\partial P/\partial C^2|_{C=0} > 0$ there will be no condensate, at least for small enough λ. The transition occurs at the critical temperature:

$$T = T_c \quad \Rightarrow \quad 0 = \left(\frac{\partial P}{\partial C^2}\right)_{C=0} . \tag{A.6}$$

Using the above expressions this gives

$$\begin{aligned} x_c^2 - \varpi_c^2 &= \frac{\lambda}{2\pi^2}\int_0^\infty du \left(\frac{u^2}{B}\right)\frac{\cosh\varpi_c - e^{-B}}{\cosh\varpi_c - \cosh B}\,; \\ B = \sqrt{x_c^2+u^2}, &\quad x_c = \frac{m_{\mathsf{be}}}{T_c}, \quad \varpi_c = \frac{\mu}{T_c}\,. \end{aligned} \tag{A.7}$$

Approximating the integral on the right hand side by setting $m = \mu = 0$ we find

$$T_c^2 \simeq \frac{6}{\lambda}(\mu^2 - {m_{\mathsf{be}}}^2)\,, \tag{A.8}$$

which is a known result [60, 61].

A.1 χ propagator.

The above Hamiltonian and charge operators can be used to derive the propagator and Feynman rules in the real-time formalism, which we use in our calculations. Defining, as usual [9]

$$D^>_{ij}(x-x') = \left\langle A_i(x)A_j(x')\right\rangle_\beta\,, \quad D^<_{ij}(x-x') = \left\langle A_j(x')A_i(x)\right\rangle_\beta\,, \tag{A.9}$$

(so that $D^<_{ij}(x-x') = D^>_{ji}(x'-x)$) where

$$\langle\cdots\rangle_\beta = \frac{\mathrm{tr}\left\{e^{-\beta H}\cdots\right\}}{\mathrm{tr}\left\{e^{-\beta H}\right\}}\,. \tag{A.10}$$

Then if,

$$\rho_{ij}(k) = D^>_{ij}(k) - D^<_{ij}(k)\,; \quad D^{\gtrless}_{ij}(k) = \int d^4x\, e^{+ik.x} D^{\gtrless}_{ij}(x)\,, \tag{A.11}$$

we have

$$D^<_{ij}(k) = f(k_0)\rho_{ij}(k)\,, \quad D^>_{ij}(k) = -f(-k_0)\rho_{ij}(k)\,; \qquad f(k_0) = \left(e^{k_0\beta}-1\right)^{-1}\,. \tag{A.12}$$

A straightforward (though tedious) calculation yields

$$\begin{aligned} \rho(k) =& 2\pi\varepsilon(k_0)\left[\frac{\delta(\omega^2-\Omega_+^2)-\delta(\omega^2-\Omega_-^2)}{\Omega_+^2-\Omega_-^2}\right]\mathbb{R}(k)\,, \\ \mathbb{R}(k) =& \begin{pmatrix} k^2+\mu^2-m^2-\lambda C^2/2 & -2i\mu k_0 \\ 2i\mu k_0 & k^2+\mu^2-m^2-3\lambda C^2/2 \end{pmatrix}. \end{aligned} \tag{A.13}$$

[9] We follow the conventions of LeBellac [58]

This has the expected form when $\mu = 0$. For the calculations in this paper we only need the expression when $\lambda = 0$:

$$\rho(k)|_{\lambda=0} = \pi \sum_{s=\pm 1} (\mathbb{1} \pm \tau_2)\varepsilon(k_0 \mp \mu)\delta((k_0 \mp \mu)^2 - \bar{E}_{\mathbf{k}}^2)\,, \tag{A.14}$$

where $\bar{E}_{\mathbf{k}} = \sqrt{m_{\mathtt{be}}{}^2 + \mathbf{k}^2}$. This expression is also valid in the presence of a condensate, when $\mu = m_{\mathtt{be}}$.

A.2 Higgs propagator and resonant contributions

When the SM and the Bose gas are in thermal equilibrium a similar expression can be derived for the Higgs propagator, however, this approach misses an important resonant contribution which can occur when $m_{\mathtt{H}} = 2m_{\mathtt{be}}$; to include it we replace

$$2\pi\delta(p^2 - m_{\mathtt{H}}{}^2) \to \frac{2\Gamma_{\mathtt{H}}\, m_{\mathtt{H}}}{(p^2 - m_{\mathtt{H}}{}^2)^2 + (\Gamma_{\mathtt{H}}\, m_{\mathtt{H}})^2} \tag{A.15}$$

in $D_{\mathtt{H}}^{\gtrless}$, where $\Gamma_{\mathtt{H}}$ denotes the Higgs width.

B Appendix: Cross section in the presence of a condensate

In this case, writing again $\chi \to [(A_1 + C) + iA_2]/\sqrt{2}$ we find, to lowest order,

$$\begin{aligned} \langle W_{i\to f}\rangle_\beta = C^2 \int d^4x\, d^4y e^{-i(p-q).(x-y)}\, \langle T_c\, [A_1(t - i\beta, \mathbf{x}) A_1(y)] \rangle_\beta \\ + \frac{1}{4} \int d^4x\, d^4y e^{-i(p-q).(x-y)} \left[\left\langle T_c \left[{}^2(t - i\beta, \mathbf{x}) {}^2(y) \right] \right\rangle_\beta - \left\langle {}^2 \right\rangle_\beta^2 \right] , \end{aligned} \tag{B.1}$$

where $\langle W_{i\to f}\rangle$ is defined in eq. (5.4), $\mathcal{V}$ denotes the volume of space time, and we assumed that the incoming momentum p of the SM particle is different form its outoging momentum q. Now, using eq. (A.12) and eq. (A.14) we find

$$\frac{1}{\mathcal{V}} \langle W_{i\to f}\rangle = C^2 D_{11}^{>}(P)|_{\mu=m_{\mathtt{be}}} + \frac{g^2 T f(-P_0)}{2\pi|\mathbf{P}|} \ln \left| \frac{1 + n_{\mathtt{be}}^{+}(E_-)}{1 + n_{\mathtt{be}}^{+}(E_+)} \frac{1 + n_{\mathtt{be}}^{-}(E_-)}{1 + n_{\mathtt{be}}^{-}(E_+)} \right|_{\mu=m_{\mathtt{be}}} , \tag{B.2}$$

where $n_{\mathtt{be}}^{(\pm)}$ are defined in eq. (4.10), $E_\pm$ in eq. (5.7), and $P = p - q$. Then

$$\begin{aligned} \sigma &= \sigma^{(1)} + \sigma^{(2)}\,, \\ \sigma^{(1)} &= \frac{q_{\mathtt{be}}^{(c)}}{2m_{\mathtt{be}}|\mathbf{p}|q_{\mathtt{be}}} \int' \frac{d^3\mathbf{q}}{2E_{\mathbf{q}}(2\pi)^3} D_{11}^{>}(P)|_{\mu=m_{\mathtt{be}}}\,; \quad E_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + m_\eta{}^2}\,, \\ \sigma^{(2)} &= \frac{1}{2q_{\mathtt{be}}|\mathbf{p}|} \int' \frac{d^3\mathbf{q}}{2E_{\mathbf{q}}(2\pi)^3} \frac{g^2 T f(-P_0)}{2\pi|\mathbf{P}|} \ln \left| \frac{1 + n_{\mathtt{be}}^{+}(E_-)}{1 + n_{\mathtt{be}}^{+}(E_+)} \frac{1 + n_{\mathtt{be}}^{-}(E_-)}{1 + n_{\mathtt{be}}^{-}(E_+)} \right|_{\mu=m_{\mathtt{be}}} , \end{aligned} \tag{B.3}$$

where $E_{\mathbf{q}}$ is the energy of the outgoing η, q_{be} the number density of Bose gas particles, and we used $q_{\mathsf{be}}^{(c)} = m_{\mathsf{be}} C^2$ for the number density in the condensate; the prime indicates that the region $p = q$ should be excluded.

In the non-relativistic limit, and for $m_{\mathsf{be}} \neq m_\eta$, we find

$$\sigma^{(1)} = -\frac{T n_0/n}{32\pi m_{\mathsf{be}} \mathbf{p}^2} \ln |f(-\mathcal{E}_-) f(\mathcal{E}_+)| \; ; \quad \mathcal{E}_\pm = \frac{2 m_{\mathsf{be}} \mathbf{p}^2}{{m_{\mathsf{be}}}^2 + {m_\eta}^2 \pm 2 m_{\mathsf{be}} \bar{E}_{\mathbf{p}}} \, , \tag{B.4}$$

where $\bar{E}$ is defined in eq. (4.12), and f in eq. (A.12). For $T \to 0$ (so that $q_{\mathsf{be}}^{(c)} \to q_{\mathsf{be}}$) this reduces to the standard result, $\sigma^{(1)} \to [16\pi(m_{\mathsf{be}} + m_\eta)^2]^{-1}$; also, $\sigma^{(1)} > 0$ for all parameters of interest.

The evaluation of $\sigma^{(2)}$ is more involved. We begin with the non-relativistic expression for $E_\pm$:

$$E_\pm = m_{\mathsf{be}} + \frac{1}{8 m_{\mathsf{be}} |\mathbf{P}|^2} \left[|\mathbf{P}|^2 \mp \frac{m_{\mathsf{be}}}{m_\eta} (\mathbf{p}^2 - \mathbf{q}^2) \right]^2 . \tag{B.5}$$

Then, defining new integration variables

$$w = \frac{|\mathbf{P}|}{|\mathbf{p}|} \, , \quad z = \frac{1}{w} \left(\frac{|\mathbf{q}|^2}{|\mathbf{p}|^2} - 1 \right) \frac{m_\eta}{m_{\mathsf{be}}} \, , \tag{B.6}$$

we find

$$\sigma^{(2)} = \frac{T |\mathbf{p}|}{256 \pi^3 q_{\mathsf{be}} \, m_{\mathsf{be}}} \int_0^\infty dw \, w \int_{(w-2) m_{\mathsf{be}}/m_\eta}^{(w+2) m_{\mathsf{be}}/m_\eta} \frac{dz}{\exp\{4\ell w z\} - 1} \ln \left| \frac{1 - \exp\{-\ell (w+z)^2\}}{1 - \exp\{-\ell (w-z)^2\}} \right| , \tag{B.7}$$

where $\ell = \beta |\mathbf{p}|^2/(8 m_{\mathsf{be}})$. This must be evaluated numerically for moderate values of ℓ, while for $\ell \to \infty$ gives eq. (5.11).

References

[1] G. Bertone, D. Hooper and J. Silk, Phys. Rept. **405** (2005) 279 [hep-ph/0404175].

[2] B. Sadoulet, NATO Sci. Ser. C **511** (1998) 517.

[3] P. Beltrame [LUX Collaboration],

[4] E. Aprile *et al.* [XENON Collaboration], arXiv:1705.06655 [astro-ph.CO]

[5] B. Serfass [CDMS and SuperCDMS Collaborations], J. Low. Temp. Phys. **167** (2012) no.5-6, 1119.

[6] P. Salati, arXiv:1605.01218 [astro-ph.HE].

[7] V. Vitale *et al.* [Fermi-LAT Collaboration], arXiv:0912.3828 [astro-ph.HE].

[8] L. Bergstrom, J. Edsjo and P. Gondolo, Phys. Rev. D **58** (1998) 103519 [hep-ph/9806293].

[9] Pazzini J. [ATLAS and CMS Collaborations], Nuovo Cim. C **40** (2017) no.1, 3.

[10] P. Huang, R. A. Roglans, D. D. Spiegel, Y. Sun and C. E. M. Wagner, Phys. Rev. D **95** (2017) no.9, 095021 [arXiv:1701.02737 [hep-ph]].

[11] K. A. Olive, PoS DSU **2015** (2016) 035 [arXiv:1604.07336 [hep-ph]].

[12] G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. **267** (1996) 195 [hep-ph/9506380].

[13] J. K. Hwang, Mod. Phys. Lett. A **32** (2017) no.26, 1730023.

[14] B. Sahoo, M. K. Parida and M. Chakraborty, arXiv:1707.01286 [hep-ph].

[15] Y. Kajiyama, H. Okada and T. Toma, Phys. Rev. D **88** (2013) no.1, 015029 [arXiv:1303.7356 [hep-ph]].

[16] Q. H. Cao, C. R. Chen, C. S. Li and H. Zhang, JHEP **1108** (2011) 018 [arXiv:0912.4511 [hep-ph]].

[17] V. González-Macías, J. I. Illana and J. Wudka, JHEP **1605** (2016) 171 [arXiv:1601.05051 [hep-ph]].

[18] A. Drozd, B. Grzadkowski and J. Wudka, JHEP **1204** (2012) 006 Erratum: [JHEP **1411** (2014) 130] [arXiv:1112.2582 [hep-ph]].

[19] P. A. R. Ade *et al.* [Planck Collaboration], Astron. Astrophys. **594** (2016) A13 [arXiv:1502.01589 [astro-ph.CO]].

[20] Y. Nomura and J. Thaler, Phys. Rev. D **79** (2009) 075008 [arXiv:0810.5397 [hep-ph]].

[21] P. Sikivie and Q. Yang, Phys. Rev. Lett. **103** (2009) 111301 [arXiv:0901.1106 [hep-ph]].

[22] A. Ringwald, PoS NOW **2016** (2016) 081 [arXiv:1612.08933 [hep-ph]].

[23] T. Matos and L. A. Urena-Lopez, Phys. Rev. D **63** (2001) 063506 [astro-ph/0006024].

[24] B. Kain and H. Y. Ling, Phys. Rev. D **85** (2012) 023527 [arXiv:1112.4169 [hep-ph]].

[25] L. A. Urena-Lopez, JCAP **0901** (2009) 014 [arXiv:0806.3093 [gr-qc]].

[26] A. P. Lundgren, M. Bondarescu, R. Bondarescu and J. Balakrishna, Astrophys. J. **715** (2010) L35 [arXiv:1001.0051 [astro-ph.CO]].

[27] D. J. E. Marsh and P. G. Ferreira, Phys. Rev. D **82** (2010) 103528 [arXiv:1009.3501 [hep-ph]].

[28] I. Rodriguez-Montoya, J. Magana, T. Matos and A. Perez-Lorenzana, Astrophys. J. **721** (2010) 1509 [arXiv:0908.0054 [astro-ph.CO]].

[29] I. Rodríguez-Montoya, A. Pérez-Lorenzana, E. De La Cruz-Burelo, Y. Giraud-Héraud and T. Matos, Phys. Rev. D **87** (2013) no.2, 025009 [arXiv:1110.2751 [astro-ph.CO]].

[30] A. Suarez and T. Matos, Mon. Not. Roy. Astron. Soc. **416** (2011) 87 [arXiv:1101.4039 [gr-qc]].

[31] T. Harko, Phys. Rev. D **83** (2011) 123515 [arXiv:1105.5189 [gr-qc]].

[32] T. Harko, Mon. Not. Roy. Astron. Soc. **413** (2011) 3095 [arXiv:1101.3655 [gr-qc]].

[33] T. Matos, A. Vazquez-Gonzalez and J. Magana, Mon. Not. Roy. Astron. Soc. **393** (2009) 1359 [arXiv:0806.0683 [astro-ph]].

[34] P. H. Chavanis, Astron. Astrophys. **537** (2012) A127 [arXiv:1103.2698 [astro-ph.CO]].

[35] H. Velten and E. Wamba, Phys. Lett. B **709** (2012) 1 [arXiv:1111.2032 [astro-ph.CO]].

[36] A. Aguirre and A. Diez-Tejedor, JCAP **1604** (2016) no.04, 019 [arXiv:1502.07354 [astro-ph.CO]].

[37] J. Magana, T. Matos, A. Suarez and F. J. Sanchez-Salcedo, JCAP **1210** (2012) 003 [arXiv:1204.5255 [astro-ph.CO]].

[38] A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. D **64** (2001) 123528 [astro-ph/0105564].

[39] W. Hu, R. Barkana and A. Gruzinov, Phys. Rev. Lett. **85** (2000) 1158 [astro-ph/0003365].

[40] A. Suárez, V. H. Robles and T. Matos, Astrophys. Space Sci. Proc. **38** (2014) 107 [arXiv:1302.0903 [astro-ph.CO]].

[41] S. J. Sin, Phys. Rev. D **50** (1994) 3650 [hep-ph/9205208].

[42] J. w. Lee and I. g. Koh, Phys. Rev. D **53** (1996) 2236 [hep-ph/9507385].

[43] J. Goodman, New Astron. **5** (2000) 103 [astro-ph/0003018].

[44] T. Rindler-Daller and P. R. Shapiro, Mon. Not. Roy. Astron. Soc. **422** (2012) 135 [arXiv:1106.1256 [astro-ph.CO]].

[45] T. Rindler-Daller and P. R. Shapiro, Mod. Phys. Lett. A **29** (2014) no.2, 1430002 [arXiv:1312.1734 [astro-ph.CO]].

[46] C. G. Boehmer and T. Harko, JCAP **0706** (2007) 025 [arXiv:0705.4158 [astro-ph]].

[47] T. Harko and E. J. M. Madarassy, JCAP **1201** (2012) 020 [arXiv:1110.2829 [astro-ph.GA]].

[48] W. J. G. de Blok, Adv. Astron. **2010** (2010) 789293 [arXiv:0910.3538 [astro-ph.CO]].

[49] M. Boylan-Kolchin, J. S. Bullock and M. Kaplinghat, Mon. Not. Roy. Astron. Soc. **415** (2011) L40 [arXiv:1103.0007 [astro-ph.CO]].

[50] V. Irsic, M. Viel, M. G. Haehnelt, J. S. Bolton and G. D. Becker, Phys. Rev. Lett. **119** (2017) no.3, 031302 [arXiv:1703.04683 [astro-ph.CO]].

[51] J. Zhang, J. L. Kuo, H. Liu, Y. L. S. Tsai, K. Cheung and M. C. Chu, arXiv:1708.04389 [astro-ph.CO].

[52] B. Patt and F. Wilczek, hep-ph/0605188.

[53] J. McDonald, Phys. Rev. D **50** (1994) 3637 [hep-ph/0702143 [HEP-PH]].

[54] P. Athron *et al.* [GAMBIT Collaboration], arXiv:1705.07931 [hep-ph].

[55] R. K. Pathria and P. D. Beale, *Statistical Mechanics* (Academic Press; 3 edition; 2011) ISBN-10: 0123821886; ISBN-13: 978-0123821881

[56] R. Kubo, J. Phys. Soc. Jap. **12** (1957) 570.

[57] B. Grzadkowski and J. Wudka, Phys. Rev. D **80** (2009) 103518 [arXiv:0809.0977 [hep-ph]].

[58] M. L. Bellac, *Thermal Field Theory*, (Cambridge University Press, 2000), ISBN-10: 0521654777, ISBN-13: 978-0521654777

[59] E. W. Kolb and M. S. Turner, *The Early Universe*, (Westview Press, 1994); ASIN: B019NE1EQC.

[60] J. I. Kapusta, Phys. Rev. D **24** (1981) 426.

[61] H. E. Haber and H. A. Weldon, Phys. Rev. D **25** (1982) 502.

[62] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. **78B** (1978) 443.

[63] S. Dawson and H. E. Haber, SCIPP-89/14.

[64] R. Agnese *et al.* [SuperCDMS Collaboration], [arXiv:1707.01632 [astro-ph.CO]].

[65] D. Baumann, *Lectures on Cosmology*,
http://www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf

[66] M. Ackermann *et al.* [Fermi-LAT Collaboration], Astrophys. J. **840** (2017) no.1, 43 doi:10.3847/1538-4357/aa6cab [arXiv:1704.03910 [astro-ph.HE]].

[67] J. I. Kapusta and C. Gale, *Finite-Temperature Field Theory: Principles and Applications*, (Cambridge University Press; 2 edition , 2011); ISBN-10: 0521173221, ISBN-13: 978-0521173223.