Title
Effects of Excess Pore Pressure Redistribution in Liquefiable Layers

Journal
Journal of Geotechnical and Geoenvironmental Engineering, 150(4)

ISSN
1090-0241

Authors
Sinha, Sumeet K
Ziotopoulou, Katerina
Kutter, Bruce L

Publication Date
2024-04-01

DOI
10.1061/jggefk.gteng-11857

Peer reviewed

Effects of Excess Pore Pressure Redistribution in Liquefiable Layers

Sumeet K. Sinha, A.M.ASCE[1]; Katerina Ziotopoulou, M.ASCE[2]; and Bruce L. Kutter, M.ASCE[3]

Abstract: Existing simplified procedures for evaluating soil liquefaction potential or for estimating excess pore pressures during earthquakes are typically based on undrained cyclic tests performed on saturated soil samples under controlled loading and boundary conditions. Under such conditions, the effect of excess pore pressure (u_e) dissipation and redistribution to neighboring soil layers cannot be accounted for. Existing simplified procedures treat liquefiable layers as isolated soil layers without any boundary conditions even if dense and loose layers are very thin, permeable, and adjacent to each other. However, redistribution is likely to increase and decrease u_e in the neighboring dense and loose layers respectively. Until now, no procedure short of fully coupled numerical analysis is available to estimate the importance of redistribution. This paper presents an approximate analytical procedure for assessing the effects of u_e redistribution in (1) soil layers that would have liquefied if they were undrained, and (2) soil layers that would have not liquefied even if undrained. It is found that a layer that is initially assumed liquefied under undrained conditions might not even liquefy accounting for the u_e redistribution to neighboring layers. On the other hand, a layer initially assumed to not liquefy can develop significant u_e and can even liquefy due to pore pressure migration from the neighboring layers. Thus, accounting for redistributed u_e is important for liquefaction consequence assessment quantification, particularly in systems that span the depth of these effects like deep foundations. Migration of u toward the tip of a pile can reduce its capacity, even if the tip is embedded in a dense sand layer. On the other hand, if redistribution can result in the reduction of u_e in initially assumed liquefied layers, risks associated with liquefaction might be avoided. A criterion is also developed to evaluate the thicknesses of a layer below which redistribution could prevent liquefaction even if the layer is deemed liquefied according to the existing liquefaction-triggering procedures. Finally, the proposed procedure is illustrated by application to selected shaking events of centrifuge tests involving liquefaction of layered soil profiles. The predictions from the procedure matched the centrifuge test results reasonably. **DOI: 10.1061/JGGEFK.GTENG-11857.**

Author keywords: Excess pore pressure redistribution; Liquefaction; Reconsolidation; Pore pressure dissipation.

Introduction

Over the last decades, there has been significant progress in the refinement of simplified procedures for the assessment of earthquake-induced soil liquefaction (e.g., Youd et al. 2001; Idriss and Boulanger 2008). In addition, methods have been developed to estimate post-liquefaction reconsolidation settlement (e.g., Tokimatsu and Seed 1984) and lateral spreading (Zhang et al. 2004), amongst numerous liquefaction effects. This progress has enabled the study of case histories (Bray and Macedo 2017; Chiaradonna et al. 2015) for assessing the site performance under specified earthquake conditions. The foundation for these procedures is based on undrained cyclic shear tests on uniform soil specimens (e.g., Lee and Albaisa 1974; Nagase et al. 1988), where the effect of excess pore pressure (u_e) dissipation and redistribution due to migration that may be realized to the neighboring layers in the field cannot be accounted for. Undrained tests on saturated specimens (e.g., cyclic triaxial compression or simple shear) have been used to determine the stress conditions under which the soil would generate u_e equal to the applied confining stress (initial liquefaction) and undergo large strains (cyclic mobility). For example, Ishihara and Yoshimine (1992) developed a methodology for estimating liquefaction-induced ground settlement based on many undrained cyclic simple shear tests on uniformly saturated specimens.

Natural soil deposits may often be layered such that u_e migrates from one layer to another during and after shaking. Accounting for redistribution is essential for quantifying liquefaction consequences (e.g., pile foundation capacity) and developing liquefaction mitigation approaches. Sinha et al. (2021a) showed how u_e redistribution from a liquefied layer into a nonliquefied layer will increase u_e and decrease pile tip resistance, potentially causing large settlements. At the same time, the decreasing u_e within the liquefied layers can quickly increase their liquefaction resistance and potentially benefit projects affected by thin and deep liquefiable soil layers by eliminating the need for ground improvement or other liquefaction mitigation efforts.

In this paper, the excess pore pressure ratio-based definition of liquefaction is adopted. We distinguish earthquake-induced undrained excess pore pressure (u_e^u) from excess pore pressure due to redistribution (u_e^d). The superscript u refers to the undrained earthquake loading, and d refers to the drainage or dissipation of u_e from redistribution. Similarly, the excess pore pressure ratio r_u is defined as r_u^u and r_u^d for the undrained condition and accounting for

[1]Assistant Professor, Dept. of Civil Engineering, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110016, India (corresponding author). ORCID: https://orcid.org/0000-0002-2011-4887. Email: sksinha@civil.iitd.ac.in

[2]Associate Professor, Dept. of Civil and Environmental Engineering, Univ. of California Davis, One Shields Ave., Davis, CA 95616. ORCID: https://orcid.org/0000-0001-5494-497X. Email: kziotopoulou@ucdavis.edu

[3]Professor Emeritus, Dept. of Civil and Environmental Engineering, Univ. of California Davis, One Shields Ave., Davis, CA 95616. ORCID: https://orcid.org/0000-0002-0628-1275. Email: blkutter@ucdavis.edu

Note. This manuscript was submitted on April 15, 2023; approved on November 1, 2023; published online on January 30, 2024. Discussion period open until June 30, 2024; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, © ASCE, ISSN 1090-0241.

redistribution, respectively. Accordingly, the liquefiable layers are categorised as NLu, Lu, NLd, and Ld layers as discussed here. The layers that would liquefy (i.e., $r_u^u = 1.0$) assuming the undrained condition and subject to a given earthquake loading are referred to as Lu layers. On the other hand, the soil layers that would not liquefy (i.e., $r_u^u < 1.0$) under undrained earthquake loading are referred to as NLu layers. This aims to make a clear distinction between the description of soil layers as NLu and Lu and is important because potentially all soil layers can be liquefied or not depending on the intensity of shaking and the drainage of water from the adjacent soil layers. For example, the water migration from redistribution can cause liquefaction in NLu layers and, at the same time, also prevent liquefaction in the Lu layers. The portion of an NLu layer with $r_u^d = 1.0$ due to redistribution is referred to as the *Ld* layer. Similarly, the portion of an Lu layer with $r_u^d < 1$ due to redistribution is referred to as the NLd layer. Early on, Seed and Lee (1966) showed that even during a shaking event u_e^u from the Lu layers would dissipate to the adjacent NLu layers, resulting in an increased u_e in NLu layers and decreased u_e in the Lu layers. Yoshimi and Kuwabara (1973) studied u_e^d development in an NLu layer from an overlying reconsolidating Lu layer. They found that as the Lu layer reconsolidated, u_e^d in the NLu layer first increased, attained a peak value, and then decreased. They also found that if the compressibility of the NLu layer was much less than the compressibility of the Lu layer, a very large u_e^d value could be developed in the NLu layer. Seed et al. (1976) developed a numerical model to estimate u_e^d in the soil layers, accounting for the u_e generation from cyclic loading and dissipation from reconsolidation. While these studies have contributed significantly to understanding u_e^d development in the layers adjacent to Lu layers, their usage in simplified procedures has been limited. Mele et al. (2021) developed a simplified model to estimate u_e in soil layers following liquefaction triggering procedures from Idriss and Boulanger (2008); however, they did not consider redistribution effects.

This paper describes a procedure for approximating the effects of redistribution of increased excess pore pressures in the NLu layer and decreased excess pore pressures (hence increased liquefaction resistance) of the Lu layer. It should be noted that redistribution can occur both during and after shaking. First, an analytical framework is developed where redistribution is considered to occur following undrained loading. The framework describes redistribution effects on two primary types of layered systems: the NLu layer below the Lu layer and the NLu above the Lu layer. The developed framework is then used to study the effects of redistribution and estimate peak excess pore pressures in an NLu layer. Later, a procedure for partially drained conditions (where redistribution occurs during shaking) is developed for evaluating the conditions of increased liquefaction resistance of the Lu layer. The procedure is then used to define a criterion on the minimum thickness of the liquefiable layer below which redistribution could prevent liquefaction in that layer. Finally, the proposed procedure is applied to selected shakings of centrifuge tests involving liquefaction of layered soil profiles, and the results are compared.

Analytical Framework–Concepts and Assumptions

Excess pore water pressure redistribution depends on soil properties (i.e., permeability, compressibility, cyclic resistance), soil state (effective stress, initial excess pore pressure), hydraulic boundary conditions, and ground motion characteristics (i.e., intensity, duration, and magnitude). Initially, we consider the system of Fig. 1: an Lu layer that has developed $r_{u-\mathrm{Lu}}^u = 1$ adjacent to an NLu layer with $r_{u-\mathrm{NLu}}^u < 1$ at the end of earthquake shaking surrounded by relatively impermeable soil above and below. In these idealized examples, redistribution is assumed to occur after the undrained loading. As the Lu soil sediments and reconsolidates, water moves from the Lu layer to the NLu layer. The migration of water decreases u_e in the Lu layer and increases u_e in the NLu layer, achieving a peak value in the NLu layer. The peak excess pore pressure in the NLu layer is presumed to occur when u_e at the interface becomes continuous and results in a redistributed excess pore pressure profile as shown in Fig. 1. Interestingly, although the time required for achieving the peak u_e in the two layers depends on the permeability of the Lu and NLu layers, its magnitude can be estimated independently. The u_e^d in the NLu layer can be estimated by tracking the water movement between the NLu and the reconsolidating Lu layers.

On the other hand, accounting for redistribution during shaking (discussed later in the paper) resulting in partially drained conditions, requires knowledge of compressibility and permeability in the soil layers. The maximum u_e in the Lu layer is likely to occur during shaking but may be suppressed if the permeability of the layers is sufficient to drain the u_e through the layer thicknesses on the time scale of the duration of shaking. However, suppose the soil permeabilities are relatively high and the thickness of the Lu layer is smaller than the NLu layer; in that case, dissipation caused by redistribution during shaking could potentially prevent liquefaction (Cubrinovski et al. 2019). In this paper, while the time required to achieve redistributed excess pore pressure is not explicitly modeled, a procedure has been developed using consolidation theory to determine the time required for redistribution. Thus, determination of the thickness of the Lu layer that can be prevented from liquefying requires estimating the u_e generation and dissipation rates within the layer and then integrating them over the entire duration of shaking.

In the following sections, first, an analytical framework is developed to study the effects of redistribution (occurring following the undrained loading) on the idealized two-layered soil profiles as shown in Fig. 1. The results obtained are then used to describe the redistribution effects on multilayered systems. Lastly, a procedure accounting for redistribution during shaking (i.e., partially drained condition) is developed to evaluate its effect on the increased liquefaction resistance of the Lu layer.

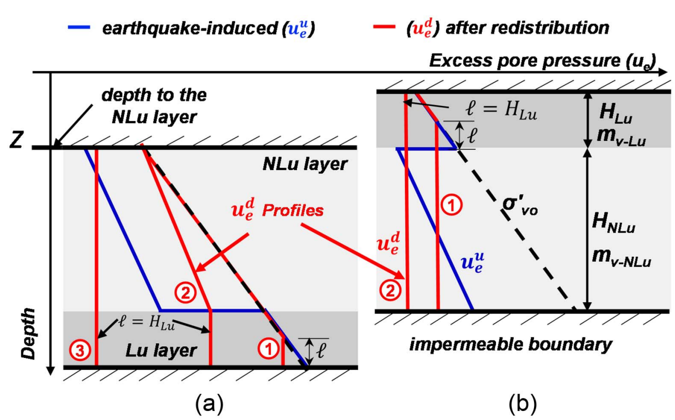

Fig. 1. Illustration of possible redistributed excess pore pressure (u_e^d) profiles (corresponding to the time when peak u_e is developed in the NLu layer) due to migration of excess pore pressures from the reconsolidating Lu layer present (a) below; and (b) above the NLu layer.

Redistributed Excess Pore Pressures (u_e^d) in Two-Layered Systems

The redistribution of u_e is first analyzed for two types of layered systems, depending on whether the Lu layer is below [Fig. 1(a)] or above [Fig. 1(b)] the NLu layer. The two-layered systems are assumed to be surrounded by impermeable layers, neglecting drainage outside these two layers (conservative). Consequently, the effect of soil ejecta is also not modeled. In both cases, the top of the NLu layer is defined at a depth, z. The approximate possible u_e^d profiles (at times when it achieves its peak value in the NLu layer) for the primary two-layered systems are shown by solid red lines in Fig. 1. The approximate u_e^d profiles were estimated based on the numerical procedure defined by Yoshimi and Kuwabara (1973), where Darcy's law is assumed to be valid with a constant permeability within each stratum. The time required to achieve peak u_e^d in the NLu layer is affected by the relative permeability of the soil layers and the hydraulic boundary conditions. Sinha et al. (2022b) conducted several centrifuge tests and observed the redistribution of excess pore pressures within the Lu and NLu layers. The duration of redistribution can be very fast (within seconds) as it requires only a small amount of water to migrate within the Lu and NLu layers to achieve the redistributed excess pore pressure. The prevention of water draining out of the two-layered systems ensures the conservation of the volume of water between the earthquake-induced and redistributed excess pore pressure. When the NLu is above the Lu layer, redistribution can potentially liquefy the NLu layer [Fig. 1(a)]. When the NLu layer is below the Lu layer, the pore pressures will increase but the NLu will never liquefy (i.e., $r_u \neq 1.0$) because the u_e^d in it will not exceed the u_e^u in the overlying Lu layer [Fig. 1(b)]. The results from these two-layered systems act as a basis for u_e^d in multilayered soil profiles (described later in the paper). The average redistributed excess pore pressure ratio (r_u^d) in the NLu and Lu layers can then be obtained by assuming the volume conservation of water between the impermeable boundaries, which can be written as

$$\begin{aligned} &m_{v-\mathrm{Lu}} H_{\mathrm{Lu}} \sigma'_{vo-\mathrm{Lu}} (r^u_{u-\mathrm{Lu}} - r^d_{u-\mathrm{Lu}}) \\ &\quad = m_{v-\mathrm{NLu}} H_{\mathrm{NLu}} \sigma'_{vo-\mathrm{NLu}} (r^d_{u-\mathrm{NLu}} - r^u_{u-\mathrm{NLu}}) \end{aligned} \tag{1}$$

where H_{Lu} and H_{NLu} = thicknesses; $m_{v-\mathrm{Lu}}$ and $m_{v-\mathrm{NLu}}$ = average compressibilities; $\sigma'_{vo-\mathrm{Lu}}$ and $\sigma'_{vo-\mathrm{NLu}}$ = average initial effective stresses; $r^u_{u-\mathrm{Lu}}$ and $r^u_{u-\mathrm{NLu}}$ = average earthquake-induced excess pore pressure ratios [described later in Eq. (20)]; and $r^u_{d-\mathrm{Lu}}$ and $r^u_{d-\mathrm{NLu}}$ = average redistributed excess pore pressure ratios for Lu and NLu layers [described later through Eqs. (3)–(7)] (Fig. 1). Similarly, the terms , $u^u_{e-\mathrm{Lu}}$ and $u^u_{e-\mathrm{NLu}}$ and $u^d_{e-\mathrm{Lu}}$ and $u^d_{e-\mathrm{NLu}}$, represent the average earthquake-induced ($u_e^u = r_u^u \sigma'_{vo}$) and redistributed ($u_e^d = r_u^d \sigma'_{vo}$) excess pore pressures in the Lu and NLu layers, respectively. Solving Eq. (1) for the different soil profiles for the primary two types of layered systems (Fig. 1) can provide estimates of $r^d_{u-\mathrm{Lu}}$ and $r^d_{u-\mathrm{NLu}}$ in the Lu and NLu layers, respectively. While solving Eq. (1), the reconsolidating Lu layer has $r^u_{u-\mathrm{Lu}} = 1.0$.

NLu *Layer above an* Lu *Layer*

When an NLu layer overlies an Lu layer, the high compressibility of the Lu layer and the comparatively lower initial effective stress ($\sigma'_{vo-\mathrm{NLu}}$) of the NLu layer can lead to high $r^d_{u-\mathrm{NLu}}$ (i.e., high volume of water flowing into the NLu layer). For relatively thick Lu layers, giving out a lot of water, $r^d_{u-\mathrm{NLu}}$ values may even reach one [Fig. 1(a)]. Yoshimi and Kuwabara (1973) note that the NLu layer can liquefy due to the redistribution of excess pore pressures from a reconsolidating Lu layer (of relative compressibility 10 times or higher) depending upon the relative H, r_u^u, and Z values of the soil layers. The possible u_e^d profiles, when peak u_e is achieved in the NLu layer, are shown in Fig. 1(a). The redistribution results in the equalization of pore pressures starting from the bottom of the Lu layer. Depending on the magnitude of excess pore pressure redistribution (which in turn depends on the relative compressibility, thickness, and effective stress of the NLu and Lu layers), the u_e^d profile in the NLu layer would change from linearly varying to equalized excess pore pressure with depths as shown in profiles 1, 2, and 3, respectively, in Fig. 1(a).

During reconsolidation, excess pore pressures begin dissipating from the bottom of the Lu layer, as shown in the u_e^d Profile 1 of Fig. 1(a). Redistribution can cause complete liquefaction of an overlying NLu layer if $\ell < H_{\mathrm{Lu}}$, where ℓ is calculated from Eq. (2)

$$\ell = H_{\mathrm{Lu}} \sqrt{2\bar{H}\,\overline{m_v}\,\frac{\sigma'_{vo-\mathrm{NLu}}}{\gamma' H_{\mathrm{Lu}}}\,(1 - r^u_{u-\mathrm{NLu}})} \tag{2}$$

where $\overline{m_v} = m_{v-\mathrm{NLu}}/m_{v-\mathrm{Lu}}$ is the compressibility ratio; $\bar{H} = H_{\mathrm{NLu}}/H_{\mathrm{Lu}}$ is the thickness ratio of the NLu layer with respect to the Lu layer; and γ' is the average effective unit weight of the soil layer.

As the thickness of the NLu layer increases, redistribution could result in u_e^d profile 2 of Fig. 1(a), where $u^d_{e-\mathrm{NLu}}$ equals σ'_{vo} at its top and $u^d_{e-\mathrm{Lu}}$ at its bottom (Yoshimi and Kuwabara 1973). For an even larger thickness of the NLu layer, redistribution would result in the equalization of pore pressures in both the Lu and NLu layers, as shown in u_e^d profile 3 of Fig. 1(a). The resulting r_u^d in the Lu and NLu layers is obtained by solving Eq. (1) for the assumed u_e^d Profiles 2 and 3 and is given as

$$r^d_{u-\mathrm{NLu}} = \begin{cases} 1, & \dfrac{\ell}{H_{\mathrm{Lu}}} < 1 \\[2ex] \dfrac{r^u_{u-\mathrm{NLu}} \bar{H}\,\overline{m_v} + \frac{\sigma'_{vo-\mathrm{Lu}} + \sigma'_{vo-\mathrm{NLu}} - \gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}}}{\bar{H}\,\overline{m_v} + 2}, & 1 - \dfrac{\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}} \le r^d_{u-\mathrm{NLu}} \le 1.0 \\[2ex] \dfrac{r^u_{u-\mathrm{NLu}} \bar{H}\,\overline{m_v} + \frac{\sigma'_{vo-\mathrm{Lu}}}{\sigma'_{vo-\mathrm{NLu}}}}{1 + \bar{H}\,\overline{m_v}}, & r^d_{u-\mathrm{NLu}} \le 1 - \dfrac{\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}} \end{cases} \tag{3}$$

$$r_{u-\mathrm{Lu}}^{d}=\begin{cases}1-\dfrac{\gamma'\ell^{2}}{2H_{\mathrm{Lu}}\sigma'_{vo-\mathrm{Lu}}}, & \dfrac{\ell}{H_{\mathrm{Lu}}}\leq 1\\ \dfrac{\frac{(2r_{u-\mathrm{NLu}}^{u}-1)\sigma'_{vo-\mathrm{NLu}}+\gamma' H_{nliq}/2}{\sigma'_{vo-\mathrm{Lu}}}\bar{H}\,\overline{m_v}+2}{\bar{H}\,\overline{m_v}+2}, & \dfrac{\sigma'_{vo-\mathrm{NLu}}-\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{Lu}}}\leq r_{u-\mathrm{Lu}}^{d}\leq 1-\dfrac{\gamma' H_{\mathrm{Lu}}/2}{\sigma'_{vo-\mathrm{Lu}}}\\ \dfrac{r_{u-\mathrm{NLu}}^{u}\bar{H}\,\overline{m_v}\frac{\sigma'_{vo-\mathrm{NLu}}}{\sigma'_{vo-\mathrm{Lu}}}+1}{1+\bar{H}\,\overline{m_v}}, & r_{u-\mathrm{Lu}}^{d}\leq\dfrac{\sigma'_{vo-\mathrm{NLu}}-\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{Lu}}}\end{cases} \tag{4}$$

It can be observed from Eq. (3) that for a higher compressibility ratio ($\overline{m_v}$), $r_{u-\mathrm{NLu}}^{d}$ decreases. Fig. 2 shows the r_u^d for the NLu and Lu layers as a function of thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) with $\overline{m_v}=1/50$, $r_{u-\mathrm{NLu}}^{u}$ of 0, 0.5, and 0.9, and a unit thickness of the Nlu layer ($H_{\mathrm{NLu}}=1$ m) at a depth of $Z=10$ m. As expected, r_u^d values in both layers (Lu and NLu) decrease as $\bar{H}$ or $\overline{m_v}$ increases. However, for very large values of $\bar{H}\,\overline{m_v}$, $r_{u-\mathrm{NLu}}^{d}$ asymptotically approaches $r_{u-\mathrm{NLu}}^{u}$, whereas $r_{u-\mathrm{Lu}}^{d}$ asymptotically approaches $r_{u-\mathrm{NLu}}^{u}/(1+H_{\mathrm{NLu}}/Z)$ (Fig. 2). The r_u^d in both Lu and NLu layers decreases with depth (Z); however, since NLu is above the Lu Layer, $r_{u-\mathrm{NLu}}^{d}$ is always greater than $r_{u-\mathrm{Lu}}^{d}$ (Fig. 2). As redistribution increased u_e in the NLu layer ($r_{u-\mathrm{NLu}}^{d}>r_{u-\mathrm{NLu}}^{u}$), it decreased u_e in the Lu layer ($r_{u-\mathrm{Lu}}^{d}<r_{u-\mathrm{Lu}}^{u}$). Assuming redistribution occurs during shaking, such a decrease of u_e in the Lu layer will significantly increase its liquefaction resistance, especially for the deep thin Lu layers. A discussion of the role of redistribution in increasing the liquefaction resistance of Lu layers and the factors it depends on is presented later.

NLu *Layer below an* Lu *Layer*

When an NLu layer is under an Lu layer, the movement of water from the Lu layer results in the equalization of excess pore pressures while forming a water film at the impermeable boundary above the Lu layer (Sinha et al. 2022a) [Fig. 1(b)]. With the assumption that reconsolidation of the Lu layer results in water movement only toward the NLu layer with no formation of the water film (above the Lu layer), Eq. (1) can be solved to obtain the r_u^d values in the Lu and NLu layers. Please note, in case of the formation of a water film layer, the redistributed excess pore pressure in NLu would be smaller due to the drainage of some water to the water film. As a result, the assumption of no water film formation results in conservative u_e estimates in the NLu layer. Fig. 1(b) shows the approximate possible u_e^d profiles (1 and 2) when peak u_e is achieved in the NLu layer. The redistribution results in the equalization of pore pressures in the NLu layer. In the Lu layer, equalization of excess pore pressure occurs from the bottom of the layer which slowly progresses through the entire Lu Layer with the increase in magnitude of redistribution as illustrated in profiles 1 and 2 of Fig. 1(b).

During reconsolidation, u_e begins dissipating from the bottom of the Lu layer of a thickness (ℓ) as shown in the u_e^d profile 1 in Fig. 1(b). The maximum possible value of $u_{e-\mathrm{NLu}}^{d}$ is equal to the effective stress at the bottom of the Lu layer (i.e., $u_{e-\mathrm{NLu}}^{d}=\sigma'_{vo-\mathrm{Lu}}+\gamma' H_{\mathrm{Lu}}/2\ for\ \ell\rightarrow 0$). If $u_{e-\mathrm{NLu}}^{u}$ is larger than the effective stress at the bottom of the Lu layer (i.e., $u_{e-\mathrm{NLu}}^{u}>\sigma'_{vo-\mathrm{Lu}}+\gamma' H_{\mathrm{Lu}}/2$), no redistribution can occur toward the NLu layer. The thickness (ℓ) of the Lu layer participating in the redistribution increases with the thickness of the NLu layer [Fig. 1(b)]. Following redistribution, the $u_{e-\mathrm{Lu}}^{d}$ in the thickness ℓ of the Lu layer equalizes and attains a valued equal to the initial $u_{e-\mathrm{Lu}}^{u}$ at a distance ℓ from the bottom of the Lu layer. The equalized redistributed excess pore pressures for the case when ($\ell<H_{\mathrm{Lu}}$) is shown as u_e^d profile 1 in Fig. 1(b). For very thick NLu layers, the entire thickness of the Lu layer ($\ell=H_{\mathrm{Lu}}$) contributes to redistribution and correspondingly results in u_e^d profile 2 as shown in Fig. 1(b). The thickness of the Lu layer contributing to redistribution can be obtained by solving Eq. (1) for u_e^d profile 1 of Fig. 1(b) as

$$\frac{1}{2}\left(\frac{\ell}{H_{\mathrm{Lu}}}\right)^{2}+\overline{m_v}\,\bar{H}\frac{\ell}{H_{\mathrm{Lu}}}-\overline{m_v}\,\bar{H}\left((1-r_{u-\mathrm{NLu}}^{u})\frac{\sigma'_{vo-\mathrm{NLu}}}{\gamma' H_{\mathrm{Lu}}}-\frac{\bar{H}}{2}\right)=0 \tag{5}$$

where a solution of the thickness $\ell\leq H_{\mathrm{Lu}}$ indicates that only a small thickness (ℓ) of the Lu layer participates in redistribution and results in the u_e^d profile 1 of Fig. 1(b). Any other solution would indicate the participation of the full Lu thickness ($\ell=H_{\mathrm{Lu}}$) resulting in u_e^d profile 2 of Fig. 1(b). The resulting r_u^d in the NLu and Lu layers is given as

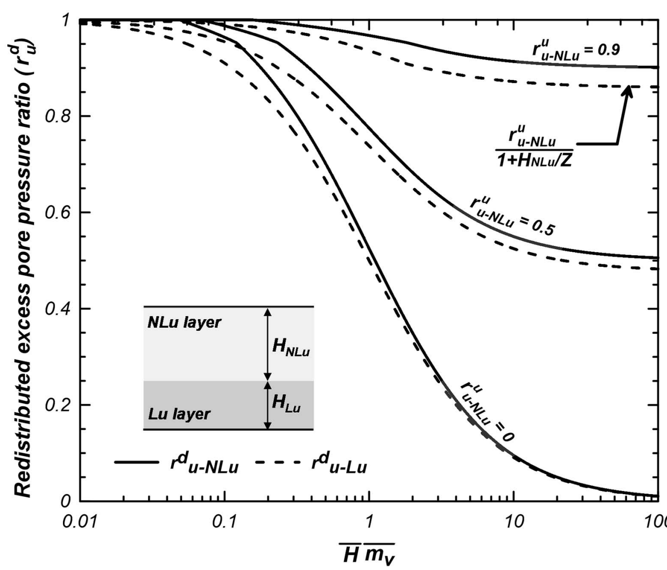

Fig. 2. Redistributed excess pore pressure ratio (r_u^d) in the layered system with an NLu layer above a Lu layer as a function of thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) for earthquake-induced excess pore pressure ratio ($r_{u-\mathrm{NLu}}^{u}$) of 0, 0.5, and 0.9, with a compressibility ratio ($\overline{m_v}$) of 1/50, and a unit thickness of the NLu layer ($H_{\mathrm{NLu}}=1$ m) at a depth of $Z=10$.

$$r^d_{u-\mathrm{NLu}} = \begin{cases} r^u_{u-\mathrm{NLu}}, & r^u_{u-\mathrm{NLu}} \geq 1 - \dfrac{\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}} \\ 1 - \dfrac{\gamma'(\ell + \frac{H_{\mathrm{NLu}}}{2})}{\sigma'_{vo-\mathrm{NLu}}}, & \dfrac{\ell}{H_{\mathrm{Lu}}} < 1, r^u_{u-\mathrm{NLu}} \leq 1 - \dfrac{\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}} \\ \dfrac{\frac{\sigma'_{vo-\mathrm{Lu}}}{\sigma'_{vo-\mathrm{NLu}}} + r^u_{u-\mathrm{NLu}} \bar{H}\,\overline{m_v}}{(1 + \bar{H}\,\overline{m_v})}, & r^d_{u-\mathrm{NLu}} \leq \dfrac{\sigma'_{vo-\mathrm{Lu}} - \gamma' H_{\mathrm{Lu}}/2}{\sigma'_{vo-\mathrm{NLu}}}, r^u_{u-\mathrm{NLu}} \leq 1 - \dfrac{\gamma' H_{\mathrm{NLu}}/2}{\sigma'_{vo-\mathrm{NLu}}} \end{cases} \tag{6}$$

$$r^d_{u-\mathrm{Lu}} = \begin{cases} 1.0, & r^u_{u-\mathrm{NLu}} \geq 1 - \dfrac{\frac{\gamma' H_{\mathrm{NLu}}}{2}}{\sigma'_{vo-\mathrm{NLu}}} \\ 1 - \dfrac{\gamma' \ell^2}{2H_{\mathrm{Lu}} \sigma'_{vo-\mathrm{Lu}}}, & \dfrac{\ell}{H_{\mathrm{Lu}}} < 1, r^u_{u-\mathrm{NLu}} \leq 1 - \dfrac{\frac{\gamma' H_{\mathrm{NLu}}}{2}}{\sigma'_{vo-\mathrm{NLu}}} \\ \dfrac{1 + r^u_{u-\mathrm{NLu}} \bar{H}\,\overline{m_v} \frac{\sigma'_{vo-\mathrm{NLu}}}{\sigma'_{vo-\mathrm{Lu}}}}{(1 + \bar{H}\,\overline{m_v})}, & r^d_{u-\mathrm{Lu}} \leq \dfrac{\sigma'_{vo-\mathrm{Lu}} - \frac{\gamma' H_{\mathrm{Lu}}}{2}}{\sigma'_{vo-\mathrm{Lu}}}, r^u_{u-\mathrm{NLu}} \leq 1 - \dfrac{\frac{\gamma' H_{\mathrm{NLu}}}{2}}{\sigma'_{vo-\mathrm{NLu}}} \end{cases} \tag{7}$$

It can be observed from Eqs. (6) and (7) that $r^d_{u-\mathrm{NLu}}$ increases with $\overline{m_v}$ decreasing and $r^u_{u-\mathrm{NLu}}$ increasing. Fig. 3 shows r^d_u in the Lu and NLu layers as a function of thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) for $\overline{m_v} = 1/50$, r^u_{u-Nlu} of 0, 0.5, and 0.9 and $H_{\mathrm{NLu}} = 1$ m at a depth of $Z = 10$ m. As expected, as $\bar{H}$ or $\overline{m_v}$ increases, r^d_u decreases and asymptotically approaches to $r^d_{u-\mathrm{NLu}} = r^u_{u-\mathrm{NLu}}$ and $r^d_{u-\mathrm{Lu}} = r^u_{u-\mathrm{NLu}}(1 + 0.5H_{\mathrm{NLu}}/Z)$ in the NLu and Lu layers, respectively. Again, r^d_u is higher in the Lu layer since the Lu is above the NLu layer. For this layered profile, redistribution also resulted in decreased u_e in the Lu layer ($r^d_{u-\mathrm{Lu}} < r^u_{u-\mathrm{Lu}}$) and increased u_e in the NLu layer ($r^d_{u-\mathrm{NLu}} > r^u_{u-\mathrm{NLu}}$). The resulting increase in liquefaction resistance of the Lu layer due to redistribution is discussed later.

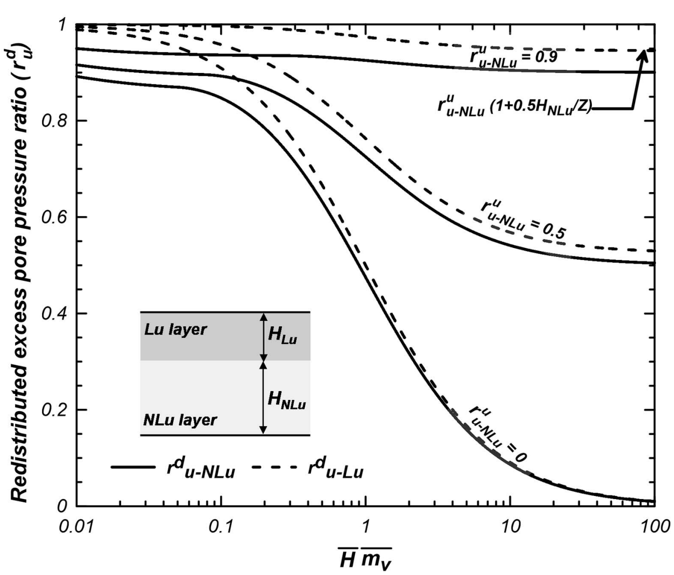

Fig. 3. Redistributed excess pore pressure ratio (r^d_u) in the layered system with an NLu layer below a Lu layer as a function of thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) for earthquake-induced excess pore pressure ratio ($r^u_{u-\mathrm{NLu}}$) of 0, 0.5, and 0.9, with a compressibility ratio ($\overline{m_v}$) of 1/50, and a unit thickness of the NLu layer ($H_{\mathrm{NLu}} = 1$ m) at a depth of $Z = 10$ m.

It should be noted that redistribution always exists between the Lu and NLu layers, resulting in the decrease and increase of u_e in the Lu and NLu layers, respectively. The magnitude of redistribution effects depends on the relative earthquake-induced excess pore pressures ratio ($r^u_{u-\mathrm{Lu}}$), the thickness (H), and the compressibility (m_v) of the soil layers. Increasing the shaking intensity would result in a lower factor of safety against liquefaction (FS_{liq}) and thus larger development of earthquake-induced excess pore pressures (r^u_u) in the Lu and NLu layers. The final redistributed excess pore pressures (r^d_u) would also depend on the relative thickness and compressibility ($\bar{H}\,\overline{m_v}$) of the soil layers. As shown in Figs. 2 and 3, for higher values of thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$), the redistributed excess pore pressures would be similar to the earthquake-induced pore pressures.

Redistributed Excess Pore Pressure in Multi-Layered Systems

Redistributed excess pore pressures in a multilayered system can be far more complex than in simple two-layered systems. While the primary contributing factors are the same as before (soil properties and state, loading conditions), the complexity of drainage conditions gets accentuated proportionally to the number of layers (Bekir Afacan 2020). Furthermore, the timing of liquefaction can vary across multiple Lu layers within such a system thus complicating the redistribution, especially when drainage is considered during shaking.

To apply the developed analytical framework for a multilayered system, it is assumed that it can be decomposed into many primary layered systems (of the NLu layer above or below the Lu layer–Fig. 4) and there exists no migration of excess pore pressure between the primary layered systems. In multilayered layered systems, an increase of u_e in the NLu layer can occur from the Lu layer above and below it. Similarly, the dissipation of u_e from the Lu layer can also occur in either direction: to the NLu layer above and below it. As described earlier, the maximum possible r^d_u in an NLu layer below an Lu layer equals the effective stress at the bottom of the Lu layer. If the NLu layer is above the Lu layer, redistribution can cause liquefaction in the NLu layer ($r^d_{u-\mathrm{NLu}} = 1.0$). Thus, a reasonably simple way to split the multilayered system and prevent double counting the redistribution effect in both

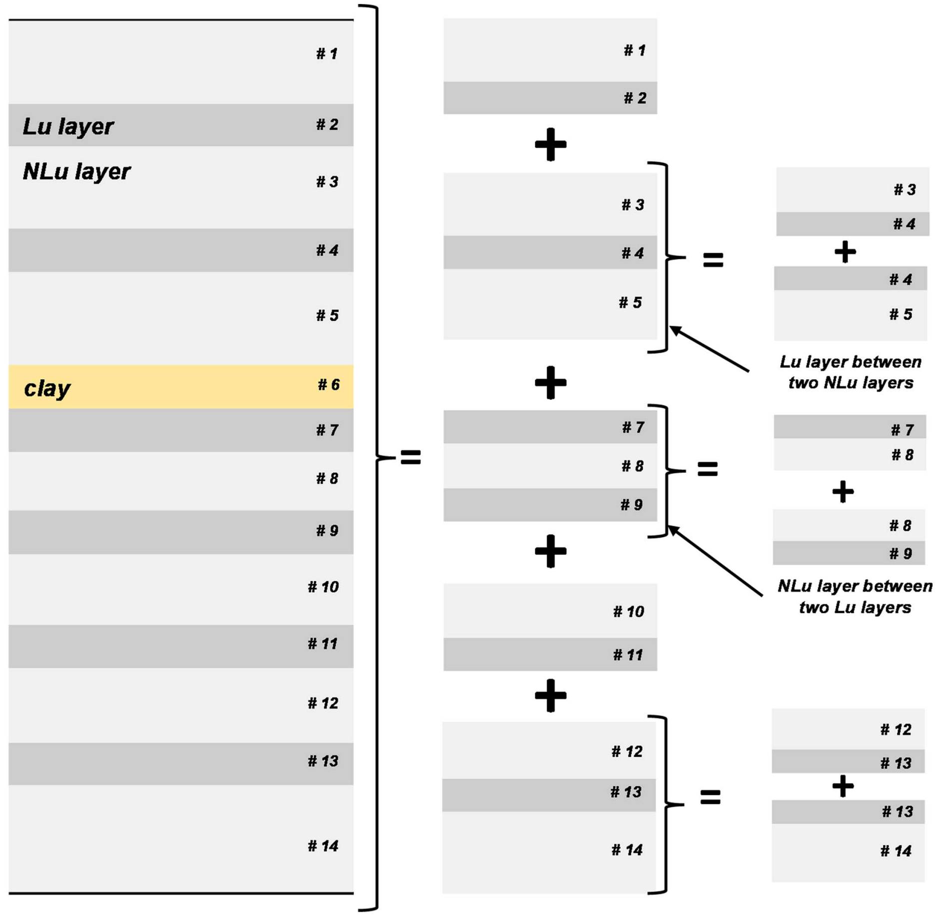

Fig. 4. Illustration of the multi-layered soil systems decomposition into smaller units of the two primary layered soil systems of the NLu layer above/below a Lu layer to estimate redistributed excess pore pressure in the NLu layers.

(upward and downward) directions is to (wherever possible) decompose the multilayered system into multiple units of the primary layered system of an NLu layer above an Lu layer (Fig. 4).

If sublayers cannot be decomposed into a set of NLu layers above Lu layers, then two types of three-layer systems are introduced to account for redistribution: an NLu layer sandwiched between two Lu layers and an Lu layer sandwiched between two NLu layers. For example, in the multilayered system presented in Fig. 4, the presence of the clay layer results in two subsystems: an Lu layer (# 4) sandwiched between the NLu layers (# 3 and 5) and an NLu layer (# 8) sandwiched between the Lu layers (# 7 and 9). Similarly, the no drainage condition beneath layer 14 results in a subsystem with an Lu Layer (# 13) sandwiched between two NLu layers (# 12 and 14). The following subsections describe the estimation of r_u^d for these two additional types of subsystems.

Lu *Layer Sandwiched between* NLu *Layers*

For an Lu layer sandwiched between two NLu layers, $u^d_{e\text{–NLu}}$ can be conservatively estimated by assuming the contribution of the full Lu layer in developing u_e^d in both the NLu layers above and below it. The u_e^d in the NLu layers can be individually calculated accounting for redistribution from the middle Lu layer assuming there is no migration of the excess pore pressures to the other NLu layer. Equivalently, one could further decompose the subsystem into two systems: an NLu layer above the Lu layer and an NLu layer below the Lu layer, and then individually calculate $u^d_{e\text{–NLu}}$ in both NLu layers. The $u^d_{e\text{–Lu}}$ in the sandwiched Lu layer can be conservatively taken equal to the minimum of the $u^d_{e\text{–Lu}}$ calculated from the two primary systems. Since the effect of only one NLu layer (instead of both NLu layers) is considered on the redistribution of excess pore pressures from the Lu layer, it results in a conservative estimate of $u^d_{e\text{–Lu}}$. For example, in Fig. 4, the Lu layer (# 4) sandwiched between two NLu layers (# 3 and # 5) is decomposed into the two systems: an NLu layer above an Lu layer and an NLu layer below an Lu layer. To find $u^d_{e\text{–NLu\#3}}$ and $u^d_{e\text{–NLu\#5}}$ in the NLu layers and $u^d_{e\text{–Lu\#4}}$ in the Lu layer, we assume that $u^{d\text{–Lu\#4}}_{e\text{–NLu\#3}}$ and $u^{d\text{–Lu\#4}}_{e\text{–NLu\#5}}$ represent the u_e^d in NLu layers (# 3 and # 5) due to redistribution from the adjacent Lu layer (# 4). Similarly, $u^{d\text{–NLu\#3}}_{e\text{–Lu\#4}}$ and $u^{d\text{–NLu\#5}}_{e\text{–Lu\#4}}$ represent the u_e^d in the Lu layer (#4) due to redistribution from the adjacent NLu layers (# 3 and # 5). Then, u_e^d in the Lu and NLu layers is taken as

$$u^d_{e\text{–NLu\#3}} = u^{d\text{–Lu\#4}}_{e\text{–NLu\#3}} \tag{8}$$

$$u^d_{e\text{–NLu\#5}} = u^{d\text{–Lu\#4}}_{e\text{–NLu\#5}} \tag{9}$$

$$u^d_{e\text{–Lu\#4}} = \min(u^{d\text{–NLu\#3}}_{e\text{–Lu\#4}}, u^{d\text{–NLu\#5}}_{e\text{–Lu\#4}}) \tag{10}$$

NLu *Layer Sandwiched between* Lu *Layers*

For an NLu layer sandwiched between two Lu layers, $u^d_{e\text{–NLu}}$ can be conservatively estimated by taking the contributions from both Lu layers. First, $u^d_{e\text{–NLu}}$ is estimated from the Lu layer above it.

Then this is assumed as $u^u_{e-\mathrm{NLu}}$ to obtain the final $u^d_{e-\mathrm{NLu}}$ from the Lu layer below it. For example, in Fig. 4, the u^d_e in the NLu layer (# 8) sandwiched between two Lu layers (# 7 and # 9) is obtained by decomposing the profile into two systems: an NLu layer below an Lu layer and an NLu layer above an Lu layer. First, the redistributed excess pore pressure $u^{d-Lu\#7}_{e-NLu\#8}$ in NLu layer # 8 is calculated from the Lu layer # 7 above it. Then, the obtained u^d_e is taken as an earthquake-induced pore pressure (i.e., $u^u_{e-\mathrm{NLu\#8}} = u^{d-\mathrm{Lu\#7}}_{e-\mathrm{NLu\#8}}$) to estimate the final redistributed excess pore pressure in the NLu layer ($u^d_{e-\mathrm{NLu\#8}} = u^{d-\mathrm{Lu\#9}}_{e-\mathrm{NLu\#8}}$) as a result of the Lu layer # 9 below it. The u^d_e in the Lu layers is taken as $u^d_{e-\mathrm{Lu\#7}} = u^{d-\mathrm{NLu\#8}}_{e-\mathrm{Lu\#7}}$ and $u^d_{e-\mathrm{Lu\#9}} = u^{d-\mathrm{NLu\#8}}_{e-\mathrm{Lu\#9}}$. Later in this paper, the developed procedure is applied to centrifuge tests of multilayered soil systems to illustrate its utility and to provide a preliminary partial validation of the approach.

Increased Liquefaction Resistance by Redistribution

This section evaluates the potential of redistribution for preventing liquefaction in an Lu layer. The analytical framework described in the above sections showed that for a large thickness ratio ($\bar{H}$), redistribution can significantly reduce u_e in an Lu layer. During shaking, a partially drained condition can exist, where the undrained earthquake loading generates u_e while redistribution to neighboring layers decreases it. Herein, the rate of excess pore pressure dissipation due to redistribution is the rate at which undrained excess pore pressures are converted to redistributed excess pore pressures. If the dissipation rate from redistribution is fast enough, it can prevent liquefaction in the Lu layer. However, if redistribution occurs too slowly, liquefaction may not be prevented. Thus, determining the rate of excess pore pressure generation (from undrained loading) relative to the rate of dissipation (from redistribution) is required. The net effect of generation and dissipation processes is referred to as the partially drained (pd) excess pore pressure ratio ($r^{pd}_{u-\mathrm{Lu}}$) in the Lu layer.

Rate of Excess Pore Pressure Dissipation from Redistribution ($\dot{r}^d_{u-\mathrm{Lu}}$)

Previously, $r^d_{u-\mathrm{Lu}}$ was calculated at the end of shaking when the Lu layer was reconsolidating from its liquefied state (i.e., $r^u_{u-\mathrm{Lu}} = 1$) [Eqs. (4) and (7)]. Assuming that $r^{pd}_{u-\mathrm{Lu}}(t)$ is known, the simplified and conservative way of estimating the $r^d_{u-\mathrm{Lu}}(t)$ at any time (t) during shaking is the linear scaling of $r^d_{u-\mathrm{Lu}}$ with $r^{pd}_{u-\mathrm{Lu}}(t)$

$$r^d_{u-\mathrm{Lu}}(t) = r^{pd}_{u-\mathrm{Lu}}(t) r^d_{u-\mathrm{Lu}} \tag{11}$$

The assumption of linear scaling of the $r^d_{u-\mathrm{Lu}}$ with $r^{pd}_{u-\mathrm{Lu}}(t)$ would be generally valid for the u^d_e profiles [such as # 3 in Fig. 1(a) and # 2 in Fig. 1(b)] where equalization of u_e occurs between the Lu and NLu layers. The u^d_e profiles #3 and #2 in Figs. 1(a and b), respectively, represent the cases when the u^{u}_e is not enough to liquefy the Lu layer. As a result, during the majority of the shaking period, while u_e is developing in the Lu Layer (i.e., while $r^u_{u-\mathrm{Lu}} < 1$), redistribution would result in equalized excess pore pressures in the Lu and NLu layers as shown in u^d_e profiles # 3 and # 2 of Figs. 1(a and b), respectively. Furthermore, for evaluating the case of increased liquefaction resistance of thin Lu layers adjacent to thick NLu layers, $r^d_{u-\mathrm{Lu}}$ would likely result from fully equalized u^d_e profiles in the Lu and NLu layers. Considering that the $r^d_{u-\mathrm{Lu}}$ is the result from other u^d_e profiles shown in Fig. 1, the assumption of linear scaling would still be conservative as those profiles would result in higher $r^d_{u-\mathrm{Lu}}(\mathrm{t})$. The rate of dissipation from excess pore pressure redistribution ($\dot{r}^d_{u-\mathrm{Lu}}(t)$) can then be estimated as

$$\dot{r}^d_{u-\mathrm{Lu}}(t) = r^{pd}_{u-\mathrm{Lu}}(t)\dot{r}^d_{u-\mathrm{Lu}} \tag{12}$$

$$\dot{r}^d_{u-\mathrm{Lu}} = \frac{(1 - r^d_{u-\mathrm{Lu}})}{t_d} \tag{13}$$

where t_d = time required for redistribution, i.e., the duration required for the earthquake-induced excess pore pressures ($r^u_{u-\mathrm{Lu}}$) to achieve $r^d_{u-\mathrm{Lu}}$ in the reconsolidating Lu layer. As expected, the time required for redistribution (t_d) is larger for smaller $r^d_{u-\mathrm{Lu}}$ values. Later in the paper, a procedure is defined to estimate t_d.

Rate of Excess Pore Pressure Generation from Shaking ($\dot{r}^u_{u-\mathrm{Lu}}$)

The average rate of excess pore pressure ratio generation, $\dot{r}^u_{u-\mathrm{Lu}}(t)$, in the Lu layer can be estimated from the rate of undrained loading (Seed et al. 1976) at an arbitrarily defined stress level (τ) as

$$\dot{r}^u_{u-\mathrm{Lu}}(t) = \dot{r}^u_{u-\mathrm{Lu}} = \left(\frac{N_u}{N_{\mathrm{Lu}}}\right)\frac{1}{t_u} \tag{14}$$

where N_{Lu} = number of shear stress (τ) cycles required to cause liquefaction; N_u = equivalent number of shear stress (τ) cycles; and t_u is the time for $\mathrm{u_e}$ generation. As a simplification, t_u is taken equal to the duration of undrained (or earthquake) loading. The empirical procedure by Idriss and Boulanger (2008) also follows the same philosophy of rate of excess pore pressure generation as defined through Eq. (14) by Seed et al. (1976). The cycle ratio (N_u/N_{Lu}) can be substituted in terms of $FS_{liq-\mathrm{Lu}}$ as defined as the factor of safety against liquefaction and the parameter b defined as the slope of cyclic resistance ratio (CRR) (Idriss and Boulanger 2008) as

$$\dot{r}^u_{u-\mathrm{Lu}} = (FS_{\mathrm{liq-Lu}})^{-1/b}\frac{1}{t_u} \tag{15}$$

Partially Drained Excess Pore Pressure in Lu Layer $\dot{r}^{pd}_{u-\mathrm{Lu}}$

The average rate of partially drained excess pore pressure in the Lu layer, $\dot{r}^{pd}_{u-\mathrm{Lu}}(t)$, is given by the difference in the generation ($\dot{r}^u_{u-\mathrm{Lu}}(t)$) and dissipation ($\dot{r}^d_{u-\mathrm{Lu}}(t)$) rates, resulting in a first-order ordinary differential equation

$$\dot{r}^{pd}_{u-\mathrm{Lu}}(t) = \dot{r}^u_{u-\mathrm{Lu}} - r^{pd}_{u-\mathrm{Lu}}(t)\dot{r}^d_{u-\mathrm{Lu}} \tag{16}$$

This differential equation can be solved with the initial boundary condition of $r^{pd}_{u-\mathrm{Lu}} = 0$ to obtain $r^{pd}_{u-\mathrm{Lu}}(t)$ as

$$r^{pd}_{u-\mathrm{Lu}}(t) = \frac{\dot{r}^u_{u-\mathrm{Lu}}}{\dot{r}^d_{u-\mathrm{Lu}}}(1 - e^{-\dot{r}^d_{u-\mathrm{Lu}}t}), \qquad r^{pd}_{u-\mathrm{Lu}}(t) < 1.0 \tag{17}$$

From Eq. (17), the partially drained excess pore pressure ratio in the Lu layer ($r^{pd}_{u-\mathrm{Lu}}$) at the end of earthquake loading (i.e., $t = t_u$) is given as

$$r^{pd}_{u-\mathrm{Lu}} = (FS_{liq-\mathrm{Lu}})^{-1/b}\frac{1}{(1 - r^d_{u-\mathrm{Lu}})\bar{t}}(1 - e^{-(1-r^d_{u-\mathrm{Lu}})\bar{t}}),$$
$$r^{pd}_{u-\mathrm{Lu}} < 1 \tag{18}$$

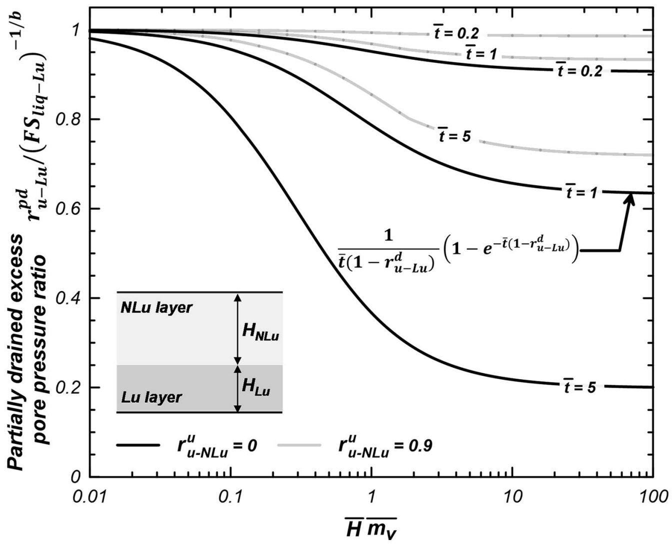

Fig. 5. Partially drained excess pore pressure ratio in the Lu layer $[r^{pd}_{u-\mathrm{Lu}}/(FS_{\mathrm{liq-Lu}})^{-1/b}]$ in the layered system with an NLu layer above a Lu layer as a function of thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ for time ratio $(\bar{t})$ of 0.2, 1, and 5, earthquake-induced excess pore pressure ratio $(r^u_{u-\mathrm{NLu}})$ of 0 and 0.9, and a unit thickness of the NLu layer $(H_{\mathrm{NLu}} = 1\ \mathrm{m})$ at a depth of $Z = 10$ m.

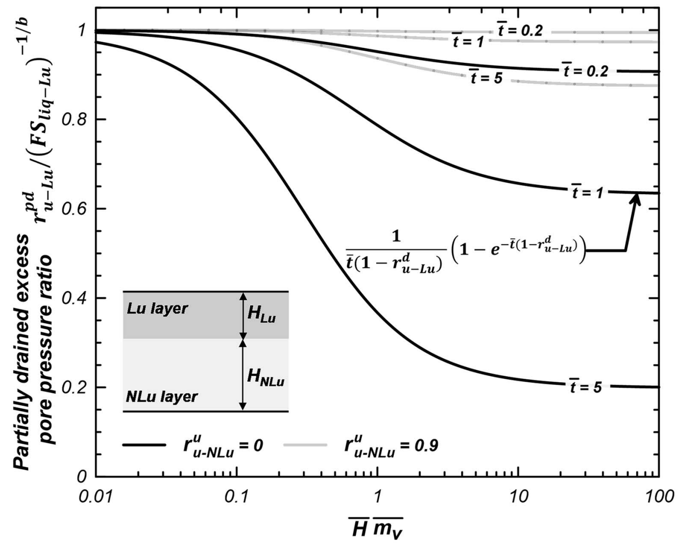

Fig. 6. Partially drained excess pore pressure ratio in the Lu layer $[r^{pd}_{u-\mathrm{Lu}}/(FS_{\mathrm{liq-Lu}})^{-1/b}]$ in the layered system with an NLu layer below a Lu layer as a function of thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ for time ratio $(\bar{t})$ of 0.2, 1, and 5, earthquake-induced excess pore pressure ratio $(r^u_{u-\mathrm{NLu}})$ of 0 and 0.9, and a unit thickness of the NLu layer $(H_{\mathrm{NLu}} = 1\ \mathrm{m})$ at a depth of $Z = 10$ m.

where $\bar{t} = t_u/t_d$ is the time ratio of the u_e generation and dissipation (from redistribution). It should be noted that the dissipation time (i.e., t_d) required for redistribution of excess pore pressures within the Lu and NLu layers can be very fast (even completed within the shaking period) and has been observed in the centrifuge tests by Sinha et al. (2022b). This is because (1) the permeability of the soil can increase up to five times for the liquefied soils (Ueng et al. 2017) and (2) only a small amount of water is required to migrate within the Lu and NLu layers to achieve the redistributed excess pore pressure. In addition to the permeability, t_d is also affected by the thickness, effective stresses, and compressibility of the soil layers, and the hydraulic boundary conditions as shown later in the paper while developing a procedure for its approximate estimation. In Eq. (18), $(FS_{\mathrm{liq-Lu}})^{-1/b}$ accounts for the extent of liquefaction in the Lu layer if it were undrained. It can be observed that for soils with a larger b parameter, redistribution results in smaller $r^{pd}_{u-\mathrm{Lu}}$. The $r^{pd}_{u-\mathrm{Lu}}$ as the function $\overline{m_v}\,\bar{H}$ for $\bar{t}$ of 0.2, 1, and 5, $r^u_{u-\mathrm{NLu}} = 0$ and 0.9, and a $H_{\mathrm{NLu}} = 1$ m (at Z = 10 m) for the primary two-layered systems are shown in Figs. 5 and 6, respectively. As expected, $r^{pd}_{u-\mathrm{Lu}}$ decreases with increases in thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ and time ratio $(\bar{t})$. A smaller $r^u_{u-\mathrm{NLu}}$ leads to smaller $r^{pd}_{u-\mathrm{Lu}}$. For very large values of $\bar{H}\,\overline{m_v}$, $r^{pd}_{u-\mathrm{Lu}}$ asymptotically approaches $r^{pd}_{u-\mathrm{Lu}} = (FS_{liq-\mathrm{Lu}})^{-1/b}$ $(1/\bar{t}(1 - r^d_{u-\mathrm{Lu}}))(1 - e^{-\bar{t}(1-r^d_{u-\mathrm{Lu}})})$.

The Criterion for Liquefaction Prevention in Lu *Layer*

Liquefaction of the Lu layer can be considered prevented if $r^{pd}_{u-\mathrm{Lu}}$ falls below a critical value [Eq. (18)]. For example, the critical value can be assumed to be 0.9. The criterion on the thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ and the time ratio $(\bar{t})$ to prevent liquefaction in the Lu layer for a given undrained loading represented by $(FS_{liq-\mathrm{Lu}})^{-1/b}$ can be obtained by solving Eq. (18) for $r^{pd}_{u-\mathrm{Lu}} = 0.9$. The minimum thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ as a function of the time ratio $(\bar{t})$ required to prevent liquefaction in the

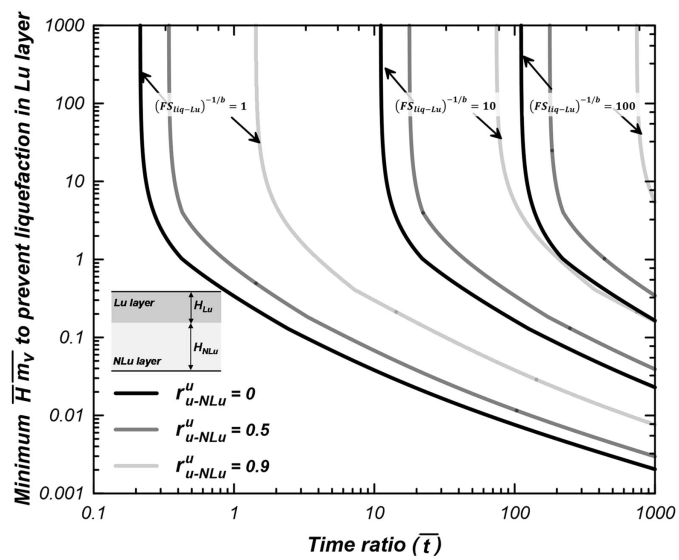

Fig. 7. The minimum thickness and compressibility ratio $(\bar{H}\,\overline{m_v})$ in the layered system with an NLu layer above a Lu layer as a function of the time ratio $(\bar{t})$ for which liquefaction can be prevented in the Lu layer (i.e., $r^{pd}_{u-\mathrm{Lu}} \leq 0.9$) having earthquake-induced excess pore pressure ratio $(r^u_{u-\mathrm{NLu}})$ of 0, 0.5, and 0.9, and a unit thickness of the NLu layer $(H_{\mathrm{NLu}} = 1\ \mathrm{m})$ at a depth of $Z = 10$ m with an undrained loading represented by $(1/FS_{\mathrm{liq-Lu}})^{1/b}$ of 1,10, and 100.

Lu layer for a unit thickness of the NLu layer $(H_{NL} = 1\ \mathrm{m})$ at a depth of $Z = 10$ m for the primary two-layered systems are shown in Figs. 7 and 8, respectively. A larger $\bar{t}$ means faster u_e dissipation from redistribution and thus would prevent liquefaction in a larger thickness of the Lu layer. As a result, the minimum $\bar{H}$ that can prevent liquefaction in Lu is smaller for a larger time ratio $(\bar{t})$. The minimum $\bar{H}$ is also smaller for layers with larger

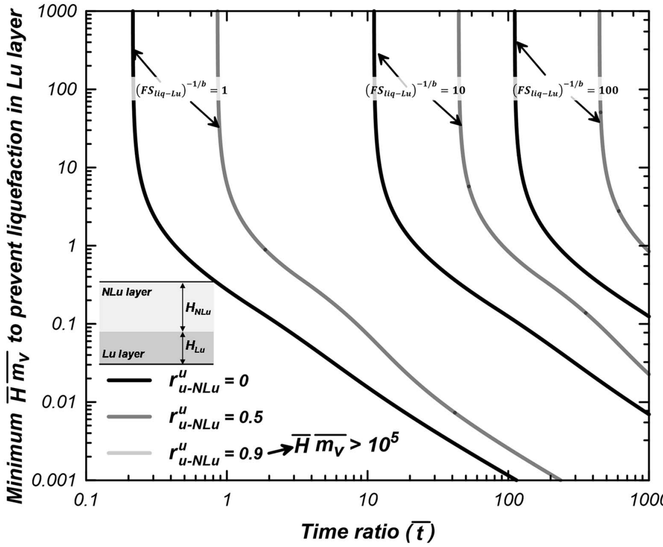

Fig. 8. The minimum thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) in the layered system with an NLu layer below a Lu layer as a function of the time ratio ($\bar{t}$) for which liquefaction can be prevented in the Lu layer (i.e., $r^{pd}_{u-\mathrm{Lu}} \le 0.9$) having earthquake-induced excess pore pressure ratio ($r^u_{u-\mathrm{NLu}}$) of 0, 0.5, and 0.9, and a unit thickness of the NLu layer ($H_{\mathrm{NLu}} = 1$ m) at a depth of $Z = 10$ m with an undrained loading represented by $(1/FS_{\mathrm{liq-Lu}})^{1/b}$ of 1, 10, and 100.

relative compressibility ($\overline{m_v}$), smaller factor of safety against liquefaction ($FS_{liq-\mathrm{Lu}}$), or a larger b parameter (i.e., overall smaller $(FS_{liq-\mathrm{Lu}})^{-1/b}$) and smaller $r^u_{u-\mathrm{NLu}}$. A large value of minimum $\bar{H}$ means redistribution cannot prevent liquefaction in the Lu layer. For example, in Fig. 7, for $r^u_{u-\mathrm{NLu}} = 0$, the minimum $\bar{H}$ approaches infinity for $(FS_{liq-\mathrm{Lu}})^{-1/b} > 10$ and $\bar{t} \le 10$. The figure also shows that reducing the time required for redistribution (t_r) such as by installing earthquake drains, would result in a larger $\bar{t}$, resulting in liquefaction prevention in the Lu layer. For the primary layered system with the NLu layer below the Lu layer, since redistribution does not occur when $u^u_{e-\mathrm{NLu}}$ is greater than the effective stress at the bottom of the Lu layer, for such cases, liquefaction cannot be prevented. For example, in Fig. 8, for $r^u_{u-\mathrm{NLu}} = 0.9$, the minimum $\bar{H}\,\overline{m_v}$ for preventing liquefaction is in the order of 10^5.

From the minimum thickness ratio ($\bar{H}$) estimated from Figs. 7 and 8 [or from Eq. (18)], the maximum thickness of the Lu layer that can be prevented from liquefaction can be computed as $H_{\mathrm{Lu}} = H_{\mathrm{NLu}}/\bar{H}$. For example, in Fig. 8, for $(FS_{liq-\mathrm{Lu}})^{-1/b} = 1$ and $r^u_{u-\mathrm{NLu}} = 0$, and $\bar{t} = 1$, the thickness and compressibility ratio ($\bar{H}\,\overline{m_v}$) is about 0.3. Assuming a compressibility ratio of $\overline{m_v} = 1/20$, the maximum thickness of the Lu layer that can be prevented from liquefaction is about 167 mm (i.e., equal to 16.6% of $H_{\mathrm{NLu}} = 1$ m). Knowing a Lu layer cannot liquefy because of redistribution (as opposed to liquefiable under undrained loading with $FS_{\mathrm{liq\text{-}Lu}} < 1.0$ as predicted by the simplified liquefaction-triggering procedures) can prove to be extremely valuable in reducing the risk of liquefaction-related problems and their remediation costs.

Procedure for Estimating Redistribution Effects

The procedure for estimating redistribution effects in the soil layers involves four steps: (1) determination of Lu and NLu layers, (2) estimation of earthquake-induced excess pore pressures (u^u_e), (3) estimation of redistributed excess pore pressures (u^d_e), and check for liquefaction prevention in the Lu layer, and (4) estimation of partially drained excess pore pressure ($u^{pd}_{e-\mathrm{Lu}}$). The steps are described in what follows.

Determination of Lu *and* NLu *Layers*

A soil liquefaction hazard analysis can identify soil layers as Lu or NLu. Among many methods [e.g., Youd et al. (2001), Idriss and Boulanger (2008), Robertson (2015), and Cetin et al. (2018)], the procedure by Idriss and Boulanger (2008) is widely used for performing a liquefaction hazard analysis for a design earthquake loading, typically quantified via an expected magnitude (M_w) and peak ground acceleration (PGA). These are used to estimate the imposed demand in terms of cyclic stress ratio (CSR) in the various soil layers. The cyclic strength of soil is determined via liquefaction triggering correlations between penetration resistance (standard penetration test N_{160cs} or cone penetration test q_{c1Ncs}) and CRR. The factor of safety against liquefaction ($FS_{liq} = \mathrm{CRR}/\mathrm{CSR}$) is calculated to categorize the layers as "liquefied undrained" (Lu) (with $FS_{\mathrm{liq}} \le 1$) and "non-liquefied undrained" (NLu) (with $FS_{\mathrm{liq}} > 1$). It should be noted that the identification of layers, whether liquefied or non-liquefied, is always in reference to the design earthquake loading. A layer initially identified as NLu for a given shaking may be an Lu layer under a stronger shaking event.

Estimation of Earthquake-Induced Excess Pore Pressures (u^u_e)

Earthquake-induced excess pore pressure (u^u_e) in the soil layers is estimated using the simplified equations by Mele et al. (2021). The method uses FS_{liq} computed in Step (1) to estimate u^u_e as

$$u^u_e = r^u_u \sigma'_{vo} \tag{19}$$

$$r^u_u = \begin{cases} \dfrac{2.0}{\pi} \arcsin\left(FS_{\mathrm{liq}}^{-\frac{1}{2b\beta}}\right) & FS_{\mathrm{liq}} > 1 \\ 1.0 & FS_{\mathrm{liq}} \le 1 \end{cases} \tag{20}$$

where b and β are the parameters defined by Mele et al. (2021) in terms of q_{c1Ncs} and N_{160cs}, respecdtively. The equation for Mele et al. (2021) was slightly modified (following the original equation by Booker et al. 1976) to include the liquefaction triggering condition of $r^u_u = 1.0$ for $FS_{\mathrm{liq}} \le 1$ (instead of $r^u_u = 0.9$ for $FS_{\mathrm{liq}} = 1$).

Estimation of Redistributed Excess Pore Pressures (u^d_e)

The redistributed excess pore pressures (u^d_e) in the Lu and NLu layers depend on the thickness ratio ($\bar{H} = H_{\mathrm{NLu}}/H_{\mathrm{Lu}}$), compressibility ratio ($\overline{m_v} = m_{v-\mathrm{NLu}}/m_{v-\mathrm{Lu}}$), depth to the top of the NLu layer (Z), average effective unit weight (γ'), average initial mean effective stress (σ'_{vo}), and the earthquake-induced pore pressure ratio (r^u_u) computed in Step (2). The compressibility of the soil layers is nonlinearly dependent on the excess pore pressure. When a soil layer liquefies, the compressibility of the layer significantly increases depending upon the extent of liquefaction and initial relative density (Seed et al. 1976). The increase in the compressibility is due to two mechanisms: (1) the nonlinearity of the unloading-reloading compression curve, and (2) volume change that occurs due to sedimentation while the soil is liquefied (i.e., while $r_u \approx 1$) (Scott 1986). As a simplification, the compressibility ratio ($\overline{m_v}$) of

J. Geotech. Geoenviron. Eng., 2024, 150(4): 04024014

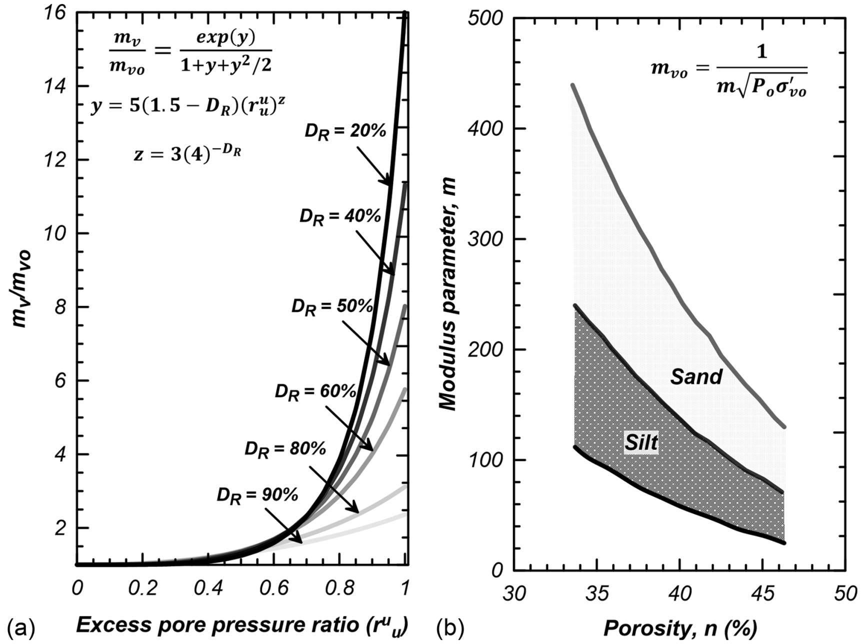

Fig. 9. Estimation of compressibility (m_v) of liquefiable soils using (a) the compressibility ratio (m_v/m_{vo}) relation as a function of earthquake-induced excess pore pressure ratio (r_u^u) from Seed et al. (1976); and (b) compressibility (m_{vo}) of normally consolidated sand and silts at mean effective stress (σ'_{vo}) with $r_u^u = 0$ from Janbu (1985).

the soil layers for the analytical framework is assumed to be constant and is taken for the state of r_u^u in the Lu layer and NLu layers, i.e., the time when redistribution starts. The compressibility (m_v) of the Lu and NLu layers can be estimated using the relation by Seed et al. (1976), which approximates the lab test results from Lee and Albaisa (1974). The relationship models m_v as a function of relative density (D_R) and r_u^u as follows:

$$\frac{m_v}{m_{vo}} = \frac{exp(y)}{1 + y + y^2/2} \tag{21}$$

$$y = 5(1.5 - D_R)(r_u^u)^z \tag{22}$$

$$z = 3(4)^{-D_R} \tag{23}$$

where m_{vo} = compressibility of the normally consolidated sand at mean effective stress (σ'_{vo}); and $r_u^u = 0$ calculated using the empirical correlations from Janbu (1985) as

$$m_{vo} = \frac{1}{m\sqrt{P_{atm}\sigma'_{vo}}} \tag{24}$$

where m = modulus parameter depending upon the porosity (n) of the sand layer (from Fig. 9); and P_{atm} = atmospheric pressure equal to 101.3 kPa. Finally, redistributed excess pore pressures ($u_e^d = r_u^d\sigma'_{vo}$) and their distribution (as shown in Fig. 1) in the Lu and NLu layers are estimated using Eqs. (2)–(7) with the obtained soil layer parameters ($\bar{H}, \overline{m_v}, \sigma'_{vo}, r_u^u, Z, \gamma'$).

Estimation of Partially Drained Excess Pore Pressure in the Lu Layer ($u_{e-\text{Lu}}^{pd}$)

Estimation of partially drained excess pore pressure ($u_{e-\text{Lu}}^{pd}$) in the Lu layer requires estimating the time for excess pore pressure generation (t_u) and redistribution (t_d). The time for u_e generation (t_u) is taken as the duration of earthquake loading. The redistribution time (t_d) can be estimated from the dimensionless time factor ($T_d = (c_v/H^2)t_d$) associated with the degree of consolidation ($U = 1 - r_{u-\text{Lu}}^d$). Taylor (1948) describes the relationship between estimating T_d and U for a single layer as

$$T_d = \begin{cases} \frac{\pi}{4}U^2, & U < 0.6 \\ -0.9332\log_{10}(1-U) - 0.0851, & U \geq 0.6 \end{cases} \tag{25}$$

For a two-layered system, the rate of dissipation depends on the permeability, compressibility, and thickness of both layers. The ratio (c_v/H^2) can be conservatively taken to be equal to the lesser of Lu or NLu ($c_{v-\text{Lu}}/H_{\text{Lu}}^2$ or $c_{v-\text{NLu}}/H_{\text{NLu}}^2$). The time ratio ($\bar{t} = t_u/t_d$) and estimated $r_{u-\text{Lu}}^d$ (in Step (3)) is then used in Eq. (18) to estimate $r_{u-\text{Lu}}^{pd}$. The maximum thickness of the Lu layer that can be prevented from liquefaction (for the assumed liquefaction prevention criteria of $r_{u-\text{Lu}}^{pd} < 0.9$) can be obtained by solving Eq. (18) iteratively with the computed time ratio ($\bar{t}$) and factor of safety against liquefaction (FS_{Lu}).

Application in Centrifuge Model Tests

Sinha et al. (2022b) conducted several centrifuge models tests with loose sand above the dense sand layer and observed redistributed excess pore pressures in the soil layers at the end of shaking. While the simplified liquefaction triggering procedures (Idriss and Boulanger 2008) predicted liquefaction in the loose sand layer and very small excess pore pressures in the dense sand layer; however, because of redistribution during shaking, liquefaction was prevented in the bottom part of the loose sand layer and significant excess pore pressures developed in the dense sand layer.

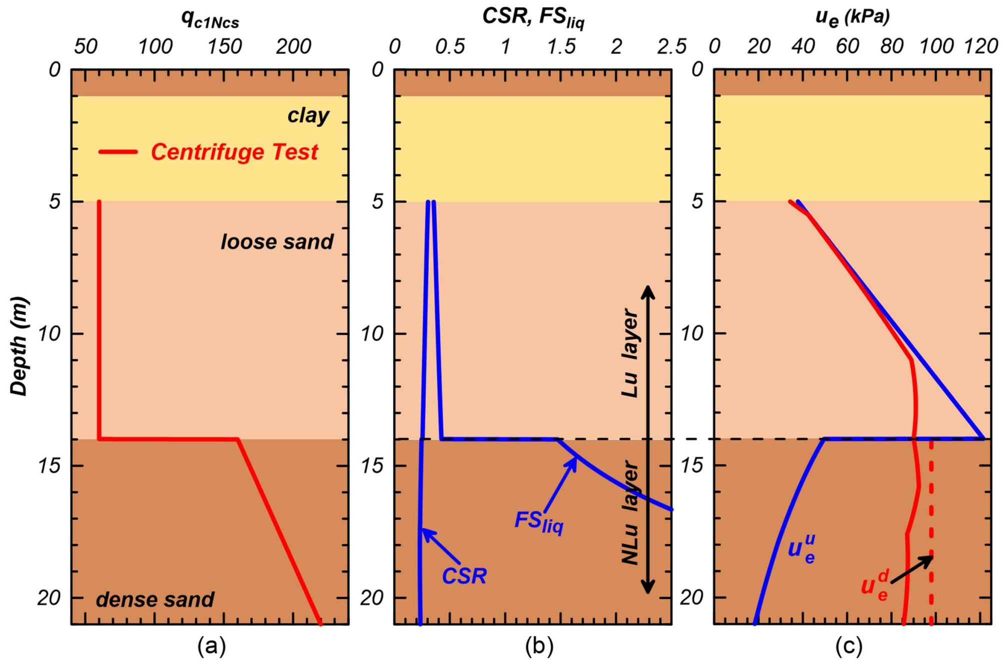

Fig. 10. Redistribution effects for shaking event EQM_3 of centrifuge model test SKS02: (a) measured normalized overburden corrected cone tip resistance (q_{c1Ncs}); (b) cyclic stress ratio (CSR) and factor of safety against liquefaction (FS_{liq}); and (c) comparison of estimated earthquake-induced (u_e^u) and redistributed (u_e^d) excess pore pressures with measured excess pore pressures. (Data from Sinha et al. 2021a.)

Description of Centrifuge Model Tests

Two shaking events, EQM_3 and EQM_4, from two centrifuge model tests, SKS02 (Sinha et al. 2021a) and SKS03 (Sinha et al. 2021b), were chosen to study redistribution effects and as a check on the procedures discussed so far. These centrifuge model tests were conducted on the 9-m radius centrifuge facility at the Center for Geotechnical Modeling at the University of California, Davis. The tests were performed at a centrifugal acceleration of 40 g. All the units reported for the centrifuge test are in the prototype scale following centrifuge scaling laws by Garnier et al. (2007). The models consisted of 21 m of soil with an undrained boundary condition at the bottom of the layers (i.e., at 21 m) because of the impermeable base of the model container. In prototype scale, SKS02 consisted of a 9-m-thick liquefiable loose sand layer ($D_R \approx 43\%$, $n \approx 0.41$) sandwiched between a 4-m-thick layer of over-consolidated clay with an undrained shear strength $s_u \approx 20$ kPa on the top and a dense sand layer ($D_R \approx 85\%$, $n \approx 0.36$) on the bottom (Fig. 10). Above the clay layer, the SKS02 model had a 1 m of Monterey sand layer. The soil profile of SKS03 consisted of 1 m of Monterey sand, 2 m of clay crust ($s_u \approx 28$–35 kPa), 4.7 m of the loose liquefiable sand layer ($D_R \approx 40\%$, $n \approx 0.41$), 1.3 m of a clayey silt layer (20% clay and 80% silt), 4 m of the medium dense sand layer ($D_R \approx 60\%$, $n \approx 0.39$), and a dense sand layer ($D_R \approx 83\%$, $n \approx 0.36$) (Fig. 11). The effective unit weight (γ') of all of the sand layers was very close to 10 $\mathrm{kN/m^3}$. The permeabilities (k) of the loose sand, the medium dense, and the dense sand layer were 0.026 m/s, 0.022 cm/s, and 0.022 cm/s, respectively. The models were shaken with scaled Santa Cruz earthquake motions of $M_w =$ 6.9 from the Loma Prieta 1989 earthquake. The duration of shaking of the earthquake motion was about 30 s. The peak ground accelerations (PGAs) used for the liquefaction potential assessment for the shaking events EQM_3 and EQM_4 were 0.23 g and 0.16 g, respectively (Sinha et al. 2023). The measured normalized overburden corrected cone tip resistance (q_{c1Ncs}) and peak excess pore pressure (u_e) generated in the soil layers during the shaking event are shown in Figs. 10(a and c) and 11(a and c), respectively.

Estimating Redistribution Effects in Lu *and* NLu *Layers*

Results on the cyclic stress ratio (CSR), the factor of safety against liquefaction (FS_{Lu}), and earthquake-induced excess pore pressure (u_e^u) for the selected shaking events from SKS02 and SKS03 are shown in Figs. 10(b and c) and 11(b and c), respectively. The figures also show the categorization of soil layers as Lu and NLu layers depending on whether $FS_{liq} \leq 1$ or $FS_{liq} > 1$, respectively. For EQM_3 in SKS02, the Lu layer consisted of the loose sand layer between the depths of 5 to 14 m (i.e., $H_{\mathrm{Lu}} = 9$ m) and the NLu layer consisted of the dense sand layer below it up to 21 m (i.e., $H_{\mathrm{NLu}} = 7$ m). The average initial mean effective stresses at the mid depth in the Lu and NLu layers were 79.6 kPa and 157.3 kPa, respectively. These layers developed earthquake-induced pore pressures of $u_{e-\mathrm{Lu}}^u = 79.6$ kPa and $u_{e-\mathrm{NLu}}^u =$ 15.03 kPa, resulting in excess pore pressure ratios of 1.0 and 0.096, respectively. The layers resulted in a compressibility ratio of $\overline{m_v} = 1/20$, which was estimated using Eqs. (21) and (24) with compressibility (m_{vo}) computed by taking the mean value of the modulus parameter (m) for the sand layers (Fig. 9). Such a relative compressibility ratio of $\overline{m_v} = 1/20$ for the soil layers is reasonable considering the loose liquefied Lu layer will certainly be much more compressible than the dense NLu layer (see Fig. 9). For EQM_4 in the SKS03 test, the Lu layer consisted of about 1.8 m of the medium dense sand below the relatively impermeable silt layer (i.e., $H_{\mathrm{Lu}} = 1.8$ m) and the NLu layer consisted of 2.2 m of medium dense sand and dense sand layer below it (i.e., $H_{\mathrm{NLu}} =$ 10.2 m with $Z = 10.8$ m). Other values computed for the Lu and NLu layers were 89.54 kPa and 151.8 kPa for their initial mean effective stresses, respectively, 1.0 and 0.24 for their excess

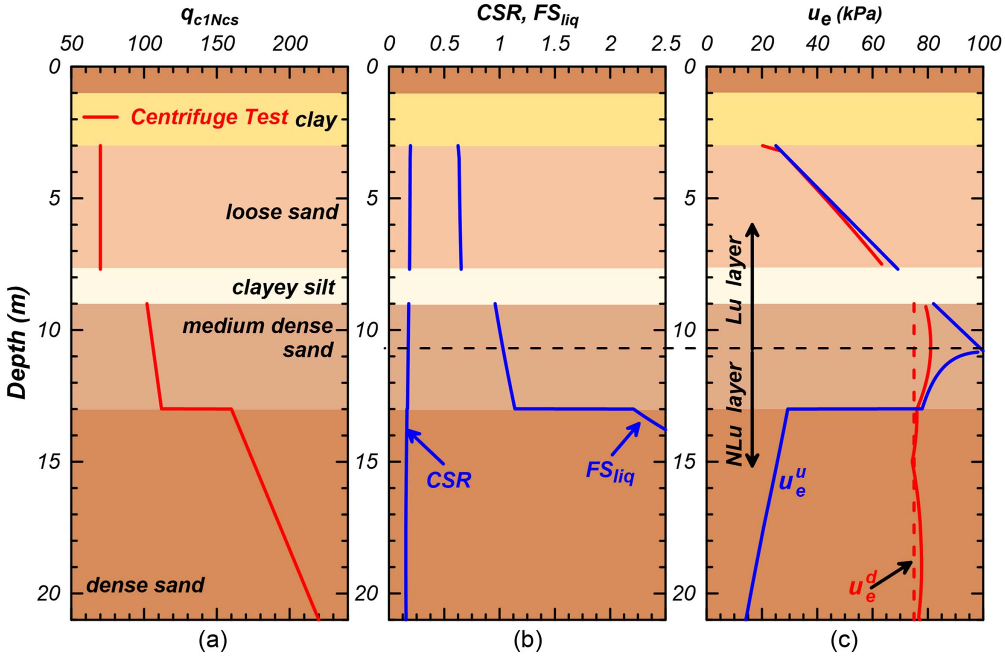

Fig. 11. Redistribution effects for shaking event EQM$_4$ of centrifuge model test SKS03: (a) measured normalized overburden corrected cone tip resistance (q_{c1Ncs}); (b) cyclic stress ratio (CSR) and factor of safety against liquefaction (FS_{liq}); and (c) comparison of estimated earthquake-induced (u_e^u) and redistributed (u_e^d) excess pore pressures with measured excess pore pressures. (Data from Sinha et al. 2021b.)

pore pressure ratios, respectively, $u^u_{e-\mathrm{Lu}} = 89.54$ kPa, $u^u_{e-\mathrm{NLu}} = 36.04$ kPa, and $\overline{m_v} = 1/12.5$.

In both centrifuge models, the NLu layer was below the Lu layer with an impermeable clay or silt layer above the Lu layer and an impermeable boundary condition below the NLu layer. As a result, Eqs. (6) and (7) were used to estimate u_e^d in the NLu and Lu layers. Figs. 10(c) and 11(c) show the estimated u_e^d profiles. Estimated $u^d_{e-\mathrm{NLu}}$ values for EQM$_3$ in SKS02 and EQM$_4$ in SKS03 were 98 kPa and 75 kPa, respectively.

It should be noted that redistribution of excess pore pressures, as well as the complex overall dynamic response due to liquefaction in the adjacent layers, could be the reasons for smaller excess pore pressures in some thicknesses of the loose and medium dense Lu sand layer. However, the large excess pore pressure developed in the underlying dense sand layer (Figs. 10 and 11) clearly demonstrates the significance of redistribution. Similar observations have been made by Cubrinovski et al. (2019) in the system-level response of interbedded liquefiable deposits.

The thickness of the Lu layer for liquefaction prevention ($r_u^{pd} < 0.9$) was found by solving Eq. (18). The u_e generation time (t_u) was taken as 30 s, equal to the duration of earthquake shaking. The sand used in the centrifuge model tests was Ottawa F-65 with parameter "b" equal to 0.15 (Bastidas 2016). For EQM$_3$ of SKS02, the thickness of the Lu layer where liquefaction would be prevented (for calculated $\bar{t} = 42.3$) was predicted to be 1.9 m from the bottom of the loose sand layer. For EQM$_4$ of SKS03, the procedure predicted liquefaction prevention (for calculated $\bar{t} = 10.7$) in the full thickness (1.8 m) of the Lu layer.

Comparison of Results with Centrifuge Test

Excess pore pressures in the NLu layer considering redistribution matched quite well with the centrifuge test. The comparison of estimated u_e^d with the measured u_e for EQM$_3$ in SKS02 and EQM$_4$ in SKS03 are shown in Figs. 10(c) and 11(c), respectively. It can be seen from the figures that the u_e^u (estimated without considering redistribution) significantly underestimates u_e in the NLu layers. In the SKS02 test, $u^u_{e-\mathrm{NLu}} = 15.03$ kPa compared to the measured $u_e \approx 90$ kPa [Fig. 10(c)]. Similarly, in the SKS03 test, $u^u_{e-\mathrm{NLu}} = 36.05$ kPa compared to the measured $u_e \approx 76$ kPa [Fig. 11(c)]. On the other hand, excess pore pressure in the NLu layer considering redistribution ($u^d_{e-\mathrm{NLu}} = 98$ kPa and $u^d_{e-\mathrm{NLu}} = 75$ kPa) matched quite well with both centrifuge test (SKS02 and SKS03) results.

While redistribution increased u_e in the NLu layers, it prevented liquefaction in the Lu layers. The predicted thickness of the Lu layer where redistribution prevented liquefaction is consistent with the observations of the centrifuge test. For EQM$_3$ of SKS02, the developed approximate procedure predicted liquefaction prevention in the 1.9 m thickness of the loose sand layer from its bottom compared to the 3 m thickness as observed in the centrifuge test [Fig. 10(c)]. The difference in the predicted thickness of the Lu layer may be due to the conservativeness in the proposed approximate procedure. For EQM$_4$ of SKS03, the procedure predicted liquefaction prevention in the entire 1.8 m of the Lu layer, like the observations from the centrifuge test [Fig. 11(c)]. It must be noted that more data is required to fully validate the developed procedures.

Summary and Conclusions

This paper described a new procedure to account for the redistribution of excess pore pressures that will either increase u_e in layers determined to be non-liquefied (NLu) (under undrained conditions) or decrease u_e in layers determined to be liquefied (Lu) (under undrained conditions). The development of the procedure involved studying redistributed excess pore pressures (u_e^d) in two types of simple layered systems: an NLu below an Lu layer and an NLu layer above an Lu layer, which formed the basis for estimating redistributed excess pore pressures in multilayered systems. While redistribution increased u_e in the NLu layer, it also decreased u_e in the Lu layers. Estimating accurate u_e values in the NLu layer is critical for estimating the end-bearing resistance of deep foundations. Development of a large u_e at a pile's tip bearing (an NLu) layer can significantly decrease the tip capacity and cause large settlements. At the same time, the decrease of u_e can be beneficial for increasing the liquefaction resistance of the Lu layer. Simplified steps and equations were provided to estimate u_e^d in soil layers. The paper also described a criterion for the maximum thickness of the Lu layer that can be prevented from liquefaction due to redistribution effects. Preventing liquefaction in a deep thin Lu layer because of pore pressure migration to the adjacent NLu layers might prove extremely valuable in reducing the risk of liquefaction-related failures and the cost associated with remediation. Applying the procedure to centrifuge tests showed that the developed analytical procedure reasonably predicted u_e in the soil layers, much better than the existing simplified procedures that ignore redistribution effects. Finally, a Microsoft Excel spreadsheet has been developed (see Supplemental Materials) following the proposed analytical procedure for estimating redistribution effects in liquefiable layers.

Several simplifying assumptions were used to present a complete procedure with many of these assumptions likely be more conservative than necessary. For example, the redistribution of excess pore pressures in the NLu layer assumed no water drainage outside the Lu and NLu layers until redistribution was achieved. This condition would be applicable for the case when the surrounding soil layers are relatively impermeable (such as clay, silt, and sand silt mixtures). For the case of partially or fully drained hydraulic boundary conditions, the presented approximate procedure would result in conservative estimates of redistributed excess pore pressures. Extending the two-layer systems to multilayer systems conservatively assumed that the Lu layer fully contributed to excess pore pressures above and below the layer. In the analytical study of increased liquefaction resistance in the Lu layer from redistribution, the assumption of a constant compressibility ratio might be overly conservative since it increases non-linearly with excess pore pressure development. On the other hand, factors such as the duration of shaking and the extent of liquefaction are not well captured in the present analytical study, which might result in the underestimation of excess pore pressure estimates in the Lu and NLu layers. For long-duration shakings and with prominent liquefaction in the Lu layer, continuous redistribution can occur throughout the shaking resulting in very high excess pore pressures in soil layers. Future refinements of the procedure may be able to estimate redistribution effects better while avoiding some excessive conservatism. For sites where the previously listed factors may play an important role, an advanced 1-D or 2-D site response analysis with a fully coupled analysis of excess pore pressure generation/dissipation models might also be performed to estimate realistic excess pore pressures in soil layers.

In summary, it is recommended that redistribution effects in liquefiable layers are evaluated in practice, especially for the design of deep foundations. The proposed procedures are a first attempt toward the evaluation of redistribution effects. Until the proposed simplified procedures are better validated, they may be regarded as a screening tool to help engineers decide whether more sophisticated analyses are required for this purpose.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available in a repository online per funder data retention policies. All the data used in this study are made available through DesignSafe project PRJ-2828 (Sinha et al. 2021a, b).

Acknowledgments

This work was performed with funding from the California Department of Transportation under Agreement 65A0688. The centrifuge tests were made possible by the staff and facilities of the Center for Geotechnical Modeling at UC Davis. The centrifuge facility at UC Davis is part of the NSF Natural Hazards Research Infrastructure (NHERI) program under Award CMMI 2037883.

Notation

The following symbols are used in this paper:

b = slope of cyclic resistance curve;
c_v = coefficient of consolidation;
D_R = relative density of sand;
d = superscript to denote dissipation from redistribution;
FS_{Lu} = factor of safety against liquefaction;
H = thickness;
$\bar{H}$ = thickness ratio $\bar{H} = H_{\mathrm{NLu}}/H_{\mathrm{Lu}}$;
Ld = subscript to denote a layer that liquefies due to redistribution;
Lu = subscript to denote a layer that would liquefy for a given earthquake loading;
m_v = compressibility;
$\overline{m_v}$ = compressibility ratio $\overline{m_v} = m_{v-\mathrm{NLu}}/m_{v-\mathrm{Lu}}$;
NLd = subscript to denote a layer that will not liquefy due to redistribution;
NLu = subscript to denote a layer that would not liquefy for a given earthquake loading;
n = porosity;
pd = superscript to denote partially drained condition during earthquake loading;
q_{c1Ncs} = overburden corrected cone tip resistance;
r_u = excess pore pressure ratio;
$\dot{r}_u$ = time rate of excess pore pressure ratio;
s_u = undrained shear strength;
T_r = time factor of redistribution;
t = time during a shaking event;
t_d = time required for u_e dissipation from redistribution;
t_u = duration of u_e generation during an undrained loading;
$\bar{t}$ = time ratio $\bar{t} = t_g/t_r$;
U = degree of consolidation;
u = superscript to denote undrained earthquake loading;
u_e = excess pore pressure;
Z = depth to the top of the NLu layer;
σ'_{vo} = initial effective stress;
γ' = effective unit weight of soil; and
γ_w = unit weight of water.

Supplemental Materials

The Microsoft Excel spreadsheet that has been created using the developed analytical procedure for estimating redistribution effects in the liquefiable layers can be found online in the ASCE Library (www.ascelibrary.org).

References

Bastidas, A. M. P. 2016. "Ottawa F-65 sand characterization." Ph.D. dissertation, Dept. of Civil and Environmental Engineering, Univ. of California Davis.

Bekir Afacan, K. 2020. "Estimation of excess pore pressure generation and nonlinear site response of liquefied areas." In *Geotechnical engineering—Advances in soil mechanics and foundation engineering*. London: IntechOpen. https://doi.org/10.5772/intechopen.88682.

Booker, J. R., M. S. Rahman, and H. B. Seed. 1976. *GADFLEA: A computer program for the analysis of pore pressure generation and dissipation during cyclic or earthquake loading*. Berkeley, CA: Earthquake Engineering Research Center.

Bray, J. D., and J. Macedo. 2017. "Simplified procedure for estimating liquefaction-induced building settlement." *Soil Dyn. Earthquake Eng.* 102 (Feb): 215–231. https://doi.org/10.1016/j.soildyn.2017.08.026.

Cetin, K. O., R. B. Seed, R. E. Kayen, R. E. S. Moss, H. T. Bilge, M. Ilgac, and K. Chowdhury. 2018. "SPT-based probabilistic and deterministic assessment of seismic soil liquefaction triggering hazard." *Soil Dyn. Earthquake Eng.* 115 (Jan): 698–709. https://doi.org/10.1016/j.soildyn.2018.09.012.

Chiaradonna, A., G. Tropeano, A. d'Onofrio, F. Silvestri, and D. Park. 2015. "Application of a simplified model for the prediction of pore pressure build-up in sandy soils subjected to seismic loading." In *Proc., 6th Int. Conf. on Earthquake Geotechnical Engineering*. Christchurch, NZ: International Society for Soil Mechanics and Geotechnical Engineering.

Cubrinovski, M., A. Rhodes, N. Ntritsos, and S. Van Ballegooy. 2019. "System response of liquefiable deposits." *Soil Dyn. Earthquake Eng.* 124 (Jun): 212–229. https://doi.org/10.1016/j.soildyn.2018.05.013.

Garnier, J., C. Gaudin, S. M. Springman, P. J. Culligan, D. Goodings, D. Konig, B. Kutter, R. Phillips, M. F. Randolph, and L. Thorel. 2007. "Catalogue of scaling laws and similitude questions in geotechnical centrifuge modeling." *Int. J. Phys. Modell. Geotech.* 7 (3): 1–23. https://doi.org/10.1680/ijpmg.2007.070301.

Idriss, I. M., and R. W. Boulanger. 2008. *Soil liquefaction during earthquakes*. Oakland, CA: Earthquake Engineering Research Institute.

Ishihara, K., and M. Yoshimine. 1992. "Evaluation of settlements in sand deposits following liquefaction during earthquakes." *Soils Found.* 32 (1): 178–188. https://doi.org/10.3208/sandf1972.32.173.

Janbu, N. 1985. "Soil models in offshore engineering." *Géotechnique* 35 (3): 241–281. https://doi.org/10.1680/geot.1985.35.3.241.

Lee, K. L., and A. Albaisa. 1974. "Earthquake induced settlements in saturated sands." *J. Geotech. Eng. Div.* 100 (4): 387–406. https://doi.org/10.1061/AJGEB6.0000034.

Mele, L., A. Chiaradonna, S. Lirer, and A. Flora. 2021. "A robust empirical model to estimate earthquake-induced excess pore water pressure in saturated and non-saturated soils." *Bull. Earthquake Eng.* 19 (Feb): 3865–3893. https://doi.org/10.1007/s10518-020-00970-5.

Nagase, H., K. Ishiharai, and K. Ishihara. 1988. "Liquefaction-induced compaction and settlement of sand during earthquakes." *Soils Found.* 28 (1): 65–76. https://doi.org/10.3208/sandf1972.28.65.

Robertson, P. K. 2015. "Comparing CPT and Vs liquefaction triggering methods." *J. Geotech. Geoenviron. Eng.* 141 (9): 04015037. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001338.

Scott, R. F. 1986. "Solidification and consolidation of a liquefied sand column." *Soils Found.* 26 (4): 23–31. https://doi.org/10.3208/sandf1972.26.4_23.

Seed, H. B., and K. L. Lee. 1966. "Liquefaction of saturated sands during cyclic loading." *J. Soil Mech. Found. Div.* 92 (6): 105–134. https://doi.org/10.1061/JSFEAQ.0000913.

Seed, H. B., P. P. Martin, and J. Lysmer. 1976. "Pore-water pressure changes during soil liquefaction." *J. Geotech. Eng. Div.* 102 (4): 323–346. https://doi.org/10.1061/AJGEB6.0000258.

Sinha, S. K., K. Ziotopoulou, and B. L. Kutter. 2021a. "SKS02: Centrifuge test of liquefaction-induced downdrag in uniform liquefiable deposit." In *Centrifuge testing of liquefaction-induced downdrag on axially loaded piles*. Miami: DesignSafe-CI. https://doi.org/10.17603/ds2-d25m-gg48.

Sinha, S. K., K. Ziotopoulou, and B. L. Kutter. 2021b. "SKS03: Centrifuge test of liquefaction-induced downdrag in interbedded soil deposits." In *Centrifuge testing of liquefaction-induced downdrag on axially loaded piles*. Miami: DesignSafe-CI. https://doi.org/10.17603/ds2-wjgx-tb78.

Sinha, S. K., K. Ziotopoulou, and B. L. Kutter. 2022a. "Centrifuge model tests of liquefaction-induced downdrag on piles in uniform liquefiable deposits." *J. Geotech. Geoenviron. Eng.* 139 (9): 04022048. https://doi.org/10.1061/(ASCE)GT.1943-5606.0002817.

Sinha, S. K., K. Ziotopoulou, and B. L. Kutter. 2022b. "Numerical modeling of liquefaction-induced downdrag: Validation against centrifuge model tests." *J. Geotech. Geoenviron. Eng.* 148 (12): 04022111. https://doi.org/10.1061/(asce)gt.1943-5606.0002930.

Sinha, S. K., K. Ziotopoulou, and B. L. Kutter. 2023. "Displacement-based design of axially loaded piles for seismic loading and liquefaction-induced downdrag." *J. Geotech. Geoenviron. Eng.* 149 (9): 1–15. https://doi.org/10.1061/JGGEFK.GTENG-11178.

Taylor, D. W. 1948. *Fundamentals of soil mechanics*. New York: Wiley.

Tokimatsu, K., and H. B. Seed. 1984. *Simplified procedures for the evaluation of settlements in clean sands*. NSF/CEE-84027. Berkeley, CA: Univ. of California Berkeley.

Ueng, T. S., Z. F. Wang, M. C. Chu, and L. Ge. 2017. "Laboratory tests for permeability of sand during liquefaction." *Soil Dyn. Earthquake Eng.* 100: 249–256. https://doi.org/10.1016/j.soildyn.2017.05.037.

Yoshimi, Y., and F. Kuwabara. 1973. "Effect of subsurface liquefaction on the strength of surface soil." *Soils Found.* 13 (2): 67–81. https://doi.org/10.3208/sandf1972.13.2_67.

Youd, T. L., et al. 2001. "Liquefaction resistance of soils: Summary report from the 1996 NCEER and 1998 NCEER/NSF workshops on evaluation of liquefaction resistance of soils." *J. Geotech. Geoenviron. Eng.* 127 (10): 817–833. https://doi.org/10.1061/(ASCE)1090-0241(2001)127:10(817).

Zhang, G., P. K. Robertson, and R. W. I. Brachman. 2004. "Estimating liquefaction-induced lateral displacements using the standard penetration test or cone penetration test." *J. Geotech. Geoenviron. Eng.* 130 (8): 861–871. https://doi.org/10.1061/(ASCE)1090-0241(2004)130:8(861).

J. Geotech. Geoenviron. Eng., 2024, 150(4): 04024014