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Conformal Defects in Gauged Supergravity

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ABSTRACT OF THE DISSERTATION

Conformal Defects in Gauged Supergravity

by

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In this dissertation, we explore 1/2-BPS conformal defects that are holographically realized as a warped product of anti-de Sitter spacetime and a circle in gauged supergravity. These solutions can be obtained as the double analytic continuation of BPS black holes with hyperbolic horizons. Observables including the expectation value of the defect and one-point functions of fields in the presence of the defect are calculated.

In Chapter 1, we present a brief review of the AdS/CFT correspondence and its supergravity approximation together with an introduction to conformal defects. In Chapter 2, we construct a singular spacetime in pure $D = 4, N = 2$ gauged supergravity dual to a 1/2-BPS conformal line defect. In Chapter 3, we show that the coupling of vector multiplets to the previous solution is capable of removing the singularity and present several examples. In Chapter 4, we construct solutions in $D = 5, N = 4$ gauged supergravity dual to 1/2-BPS surface operators in $\mathcal{N} = 2$ superconformal field theories.

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CONTRIBUTION OF AUTHORS

Chapter 2 is based on [1] in collaboration with Michael Gutperle. Chapter 3 is based on [2] in collaboration with Kevin Chen and Michael Gutperle. Chapter 4 is based on [3] in collaboration with Michael Gutperle.

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CHAPTER 1

Introduction

Understanding how to combine the principles of quantum mechanics and general relativity into a theory of quantum gravity has been one of the major problems of theoretical physics for quite some time. A theory of quantum gravity is necessary for understanding properties of the very early universe and the behavior of black holes. The singularities appearing in solutions of general relativity, such as the Big Bang or black hole singularities, are expected to be cured by a theory of quantum gravity. The most straightforward approach of treating general relativity as a classical field theory and directly quantizing fails due to issues of non-renormalizability. By replacing zero-dimensional point particles with one-dimensional strings, string theory gives rise to the only known perturbatively finite theory of quantum gravity. Extra dimensions, supersymmetry, and higher dimensional extended objects known as branes also play a significant role in string theory.

By studying the near-horizon limit of a stack of branes, Maldacena conjectured that string theory on negatively curved anti-de Sitter (AdS) spacetime is equivalent to a conformal field theory (CFT) in one fewer dimension [4]. Maldacena's conjecture is a realization of the holographic principle of quantum gravity in which spacetime can be described by a lower dimensional boundary theory [5, 6]. This correspondence provides a non-perturbative definition of string theory and allows one to study strongly coupled quantum field theories using only semiclassical string theory. The AdS/CFT correspondence has led to new results about strongly coupled quantum field theories appearing not only in high energy physics, but in condensed matter physics as well [7, 8]. Taking a step back from its string theory origin, the AdS/CFT correspondence is the best understood example of a gauge/gravity duality relating a gravitational theory to a non-gravitational gauge theory.

1.1 The AdS/CFT Correspondence

The AdS/CFT correspondence [4] is a duality between two different types of physical theories. One side of the duality involves string theory on asymptotically anti-de Sitter spacetimes and is therefore a theory of gravity. The second side of the duality contains a special class of quantum field theories known as conformal field theories. The AdS/CFT correspondence is holographic in that it relates a $(d + 1)$-dimensional theory of gravity to a d-dimensional quantum field theory. The CFT can be thought of as living on the conformal boundary of the dual asymptotically AdS spacetime. This gauge/gravity correspondence is a strong-weak duality that allows us to probe strongly coupled phenomena of CFTs using semiclassical string theory. Studying the gravitational side of the correspondence has suggested new strong coupling results about the dual gauge theory that were later confirmed using other methods such as localization. Review articles and lecture notes devoted to the AdS/CFT correspondence include [9–12].

1.1.1 Anti-de Sitter Space

Anti-de Sitter space [13, 14] is the maximally symmetric spacetime with constant negative curvature and is a solution of the Einstein field equations with a negative cosmological constant. AdS arises as a solution of gauged supergravity theories [15,16] and in dimensional reductions of higher dimensional gravity theories [17]. The isometries of AdS_{d+1} can be made manifest by embedding the spacetime as a hyperboloid in $(d+2)$-dimensional flat space with metric $\eta_{AB} = \mathrm{diag}(-1, 1, 1, \ldots, 1, -1)$. The hyperboloid defining AdS_{d+1} is expressed in terms of Cartesian coordinates X^A with $A = 0, 1, \ldots, d+1$ as

$$\eta_{AB} X^A X^B = -L^2, \tag{1.1}$$

where L is the radius of curvature. In this formulation, the SO(d, 2) isometry group of AdS_{d+1} becomes manifest.

Having an intrinsic $(d+1)$-dimensional description of AdS_{d+1} is desirable and several convenient coordinate systems exist. Global coordinates are defined by the transformation

$$\begin{aligned} X^0 &= \sqrt{r^2+L^2}\sin(t/L), & X^{d+1} &= \sqrt{r^2+L^2}\cos(t/L), \\ X^i &= r n^i, & \sum_{i=1}^{d} \left(n^i\right)^2 &= 1, \end{aligned} \tag{1.2}$$

where n^i is a vector parameterizing the $(d-1)$-dimensional unit sphere S^{d-1}. The induced metric obtained from the ambient line element $ds^2 = \eta_{AB} dX^A dX^B$ is

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right) dt^2 + \left(1+\frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-1}^2, \tag{1.3}$$

where $d\Omega_{d-1}^2$ is the line element of the unit sphere S^{d-1}. This choice of coordinate system covers the entirety of AdS_{d+1} and makes the time translation and rotational symmetries manifest. Due to the coordinate transformation (1.2), the time coordinate t is periodic with range $0 \le t < 2\pi L$. The physical spacetime is obtained by unwrapping this circle and allowing the coordinate range to be $-\infty < t < \infty$. The conformal boundary in global coordinates occurs as $r \to \infty$ and has the topology of a cylinder $\mathbb{R} \times S^{d-1}$. Gravitational solutions in global coordinates will therefore be dual to CFTs on spheres.

Another useful set of intrinsic coordinates describing AdS_{d+1} is the Poincaré patch defined by

$$\begin{aligned} X^0 &= \frac{L}{z} x^0, & X^j &= \frac{L}{z} x^j, \\ X^d &= \frac{z}{2}\left(\frac{L^2 - x^2}{z^2} - 1\right), & X^{d+1} &= \frac{z}{2}\left(\frac{L^2 + x^2}{z^2} + 1\right), \end{aligned} \tag{1.4}$$

where $x^2 = -(x^0)^2 + \sum_{j=1}^{d-1}(x^j)^2$ and the coordinate range of z is restricted to $0 < z < \infty$. For Lorentzian signature, the Poincaré patch only partially covers the spacetime. However, the patch does cover all of Euclidean AdS_{d+1}. The corresponding metric is conformally flat and takes the form

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 - (dx^0)^2 + (dx^1)^2 + \ldots (dx^{d-1})^2\right], \tag{1.5}$$

making the d-dimensional Poincaré symmetry of the $\{x^0, x^1, \ldots, x^{d-1}\}$ coordinates manifest. The metric furthermore makes the scaling symmetry $z \to \lambda z$, $x^0 \to \lambda x^0$, $x^j \to \lambda x^j$ apparent. The conformal boundary of the Poincaré patch occurs as $z \to 0$ and is given by d-dimensional Minkowski space where the dual CFT occurs.

Pure AdS spacetimes are dual to CFTs in their vacuum state. In order to study CFTs in other states, we must consider asymptotically AdS spacetimes whose metrics approach that of AdS near their boundaries. A simple example is provided by the AdS-Schwarzschild black hole

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-1}^2, \\
f &= 1 + \frac{r^2}{L^2} - \left(\frac{r_0}{r}\right)^{d-2},
\end{aligned}
\tag{1.6}
$$

where r_0 is related to the black hole mass. In the limit $r \to \infty$, we recover AdS_{d+1} in the global coordinates (1.3). As an aside, the $(d+1)$-dimensional asymptotically flat Schwarzschild black hole can be obtained from this solution in the limit of infinite radius of curvature $L \to \infty$. Similar to the asymptotically flat case, charged and rotating asymptotically AdS black hole solutions exist. In the case of black holes, the dual CFT is in a thermal state whose temperature is identified with the Hawking temperature $T = \kappa/2\pi$, where κ is the surface gravity of the black hole [18].

1.1.2 Conformal Field Theory

Conformal field theories are quantum field theories invariant under conformal transformations. They appear as fixed points of the renormalization group flow, some of which describe condensed matter systems at criticality. Ordinary quantum field theories can be viewed as points along the renormalization group flow arising from perturbing some CFT. An introduction to CFTs is provided by [19].

A conformal transformation preserves the form of the metric up to a possibly position dependent scale factor

$$
\begin{aligned}
x^\mu &\to \tilde{x}^\mu, \\
g_{\mu\nu} &\to \tilde{g}_{\mu\nu} = \Omega(x)^2 g_{\mu\nu}.
\end{aligned}
\tag{1.7}
$$

These transformations preserve angles and the causal structure of spacetime, but not necessarily distances. For d-dimensional Euclidean space with $d > 2$, the most general infinitesimal conformal transformation is given by

$$x^\mu \to x^\mu + a^\mu + \omega^{\mu\nu} x_\nu + \lambda x^\mu + x^2 b^\mu - 2(x \cdot b) x^\mu, \tag{1.8}$$

where a^μ is the parameter corresponding to translations, $\omega_{\mu\nu} = \omega_{[\mu\nu]}$ to rotations, λ to scaling transformations, and b^μ to special conformal transformations. A special conformal transformation can be obtained from an inversion $x^\mu \to x^\mu / x^2$, followed by a translation $x^\mu \to x^\mu - b^\mu$, and then finally an additional inversion. In $d = 2$, conformal transformations are described by the infinite dimensional set of holomorphic functions.

Using the action of conformal transformations on the spacetime coordinates x^μ, one can determine the corresponding generators of the conformal algebra and their commutation relations. Doing so gives the commutation relations

$$\begin{gathered}
[M_{\mu\nu}, M_{\rho\sigma}] = i\left(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\sigma} M_{\mu\rho}\right), \\
[M_{\mu\nu}, P_\rho] = i\left(\eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu\right), \qquad [M_{\mu\nu}, K_\rho] = i\left(\eta_{\nu\rho} K_\mu - \eta_{\mu\rho} K_\nu\right), \\
[K_\mu, P_\nu] = 2i\left(\eta_{\mu\nu} D - M_{\mu\nu}\right), \qquad [D, P_\mu] = i P_\mu, \qquad [D, K_\mu] = -i K_\mu,
\end{gathered} \tag{1.9}$$

where $M_{\mu\nu}$ and P_μ are the generators of the special Euclidean group, D is the dilatation operator, and K_μ are the special conformal generators. These commutation relations are isomorphic to the commutation relations of SO$(d+1,\, 1)$

$$[J_{ab}, J_{cd}] = i\left(\eta_{bc} J_{ad} - \eta_{ac} J_{bd} + \eta_{ad} J_{bc} - \eta_{bd} J_{ac}\right), \tag{1.10}$$

under the identifications

$$
\begin{aligned}
J_{\mu\nu} &= M_{\mu\nu}, \qquad & J_{-1,\mu} &= \frac{1}{2}\left(P_\mu - K_\mu\right), \\
J_{-1,0} &= D, \qquad & J_{0,\mu} &= \frac{1}{2}\left(P_\mu + K_\mu\right),
\end{aligned}
\tag{1.11}
$$

with $a, b \in \{-1, 0, 1, \ldots, d\}$ and $\eta_{ab} = \mathrm{diag}\,(-1, 1, 1, \ldots, 1)$. After a Wick rotation to Minkowski space, the conformal group becomes $\mathrm{SO}(d, 2)$.

Irreducible representations of the conformal group are labeled by their spin s and scaling dimension Δ, the eigenvalue of the dilatation operator D. Operators annihilated by K_μ at the origin are called primary and transform covariantly under conformal transformations. Primary operators behave as highest weight states with respect to $-\Delta$ and all states can be generated by repeatedly applying the momentum operator P_μ to obtain descendant states. As can be seen from the conformal algebra (1.9), the derivative operator P_μ acts as a raising operator for the scaling dimension and K_μ acts as a lowering operator. Unitarity bounds restrict the allowed scaling dimensions of operators such as $\Delta \geq (d-2)/2$ for scalar operators other than the identity for which $\Delta = 0$.

Conformal symmetry strongly constrains the form of correlation functions. For a collection of scalar primary operators $\mathcal{O}_{\Delta_i}(x)$, the one-point, two-point, and three-point correlation functions are fixed up to field re-definitions to be

$$
\begin{aligned}
\langle \mathcal{O}_{\Delta_i}(x) \rangle &= 0, \\
\langle \mathcal{O}_{\Delta_i}(x) \mathcal{O}_{\Delta_j}(y) \rangle &= \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}, \\
\langle \mathcal{O}_{\Delta_i}(x) \mathcal{O}_{\Delta_j}(y) \mathcal{O}_{\Delta_k}(z) \rangle &= \frac{c_{ijk}}{|x-y|^{\Delta - 2\Delta_k} |x-z|^{\Delta - 2\Delta_j} |y-z|^{\Delta - 2\Delta_i}},
\end{aligned}
\tag{1.12}
$$

where $\Delta = \Delta_i + \Delta_j + \Delta_k$ for operators with non-zero scaling dimension. The dynamics of the

CFT are contained in the collection of scaling dimensions Δ_i and the three-point coefficients c_{ijk}. Operators in a CFT satisfy the operator product expansion (OPE)

$$\mathcal{O}_{\Delta_i}(x)\mathcal{O}_{\Delta_j}(0) = \sum_k \frac{c_{ijk}}{|x|^{\Delta_i+\Delta_j-\Delta_k}} \left[\mathcal{O}_{\Delta_k}(0) + \alpha_1 x^\mu \partial_\mu \mathcal{O}_{\Delta_k}(0) + \ldots\right], \tag{1.13}$$

where the sum is over primary operators and the coefficients of the descendant operators are fixed by conformal symmetry [20]. The OPE is understood to hold inside correlation functions and operators with spin will also appear in the OPE of two scalar operators in general. The OPE has a radius of convergence equal to the distance to the nearest operator insertion other that $\mathcal{O}_{\Delta_i}(x)$. Using the OPE, higher-point correlations functions can be reduced to expressions involving the correlation functions appearing in (1.12) and their derivatives. Applying the OPE to a four-point correlation function and decomposing it into different channels leads to strong constraints on the allowed spectrum and OPE coefficients of the CFT and forms the basis of the conformal bootstrap approach [20–23].

An important operator present in all CFTs is the symmetric and conserved stress tensor $T_{\mu\nu}$ with scaling dimension $\Delta = d$. The stress tensor appears in the Ward identities following from conformal symmetry and encodes how the theory couples to a background metric. Conformal symmetry implies the stress tensor is traceless $T^\mu_\mu = 0$ and constrains the two-point function to be

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle = \left[2\pi_{\mu\nu}\pi_{\rho\sigma} - (d-1)(\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\nu\rho})\right] \frac{c}{|x-y|^{2d-4}}, \tag{1.14}$$

where $\pi_{\mu\nu} = \partial_\mu\partial_\nu - \eta_{\mu\nu}\Box$ and c is a central charge of the CFT. When coupled to a curved background metric, the stress tensor suffers a trace anomaly in even dimensions given by

$$\begin{aligned} d=2: &\quad \langle T^\mu_\mu \rangle = -\frac{c}{12}R, \\ d=4: &\quad \langle T^\mu_\mu \rangle = \frac{c}{16\pi^2}W^2 - \frac{a}{16\pi^2}E^2, \end{aligned} \tag{1.15}$$

where a is an additional central charge, R is the Ricci scalar, $W_{\mu\nu\rho\sigma}$ the Weyl tensor, and $E_{\mu\nu\rho\sigma}$ the Euler density of the background metric. The central charge a also appears in the three-point function of the stress tensor [24].

Conformal symmetry is compatible with supersymmetry and superconformal field theories (SCFTs) exist in dimensions $d \leq 6$ [25]. These theories are invariant under a superconformal algebra that contains additional fermionic generators S_α that are related to the conformal generators D and K_μ. We will be primarily interested in superconformal field theories arising in three and four dimensions. In $d = (2+1)$ dimensions, the superconformal group is $\mathrm{OSp}(\mathcal{N}|4)$ and in $d = (3+1)$ dimensions, it is $\mathrm{SU}(2,2|\mathcal{N})$. The classification of superconformal groups in other dimensions can be found in [25, 26].

1.1.3 AdS/CFT from the Top-Down

The canonical example of the AdS/CFT correspondence is the duality between type IIB string theory on an $\mathrm{AdS}_5 \times S^5$ background and $\mathcal{N} = 4$ Super Yang-Mills theory. The bosonic symmetries of these two theories can be matched in a straightforward manner. The four-dimensional conformal group SO(4, 2) and the $\mathrm{SU}(4) \cong \mathrm{SO}(6)$ R-symmetry of $\mathcal{N} = 4$ Super Yang-Mills are realized by the isometries of the AdS_5 and S^5 factors, respectively. This matching of bosonic symmetries further extends to the fermionic symmetries with both theories invariant under the supergroup SU(2,2|4). After performing a Kaluza-Klein reduction on S^5, the SU(2,2|4) representations of the type IIB fields were matched to the spectrum of operators in $\mathcal{N}=4$ Super Yang-Mills in [27, 28]. By studying the near-horizon limit of a stack of N coincident D3-branes, Maldacena showed that the parameters of the two theories are related by

$$L^4 = \alpha'^2 g_{YM}^2 N, \qquad g_s = \frac{g_{YM}^2}{4\pi}, \tag{1.16}$$

where L is the radius of curvature for both the AdS_5 and S^5 factors, $(2\pi\alpha')^{-1}$ is the string tension, g_{YM} the Yang-Mills coupling constant, N the number of colors, and g_s the string coupling [4].

Type IIB supergravity is a good approximation to type IIB string theory when higher derivative corrections proportional to powers of α' can be ignored. This requires $\alpha' \ll L^2$ or equivalently large 't Hooft coupling $g_{YM}^2 N \gg 1$. Quantum effects in type IIB supergravity will be suppressed for $g_s \ll 1$ or $g_{YM}^2 \ll 1$. Therefore, classical type IIB supergravity will be a good approximation to $\mathcal{N}=4$ Super Yang-Mills in the limit of large 't Hooft coupling and many colors. By studying type IIB supergravity in this limit, it was conjectured that the radiative corrections to the three-point coefficients of a class of chiral primary operators vanished. This conjecture was later confirmed and represented one of the first new results about $\mathcal{N}=4$ Super Yang-Mills obtained from the AdS/CFT correspondence [29–34].

1.1.4 AdS/CFT from the Bottom-Up

The AdS/CFT correspondence provides a mapping between classical fields in the bulk and operators in the dual CFT. The boundary values of the bulk fields determine the one-point correlation functions and sources of the dual operators [35]. In Poincaré coordinates, the metric of Euclidean AdS_{d+1} with unit radius of curvature takes the form

$$ds^2 = \frac{1}{z^2}\left(dz^2 + dx_1^2 + \cdots + dx_d^2\right), \tag{1.17}$$

with $z > 0$. The conformal boundary of Euclidean AdS_{d+1} is d-dimensional Euclidean space and occurs as $z \to 0$. As a simple example, consider a free massive scalar with action

$$S_{\mathrm{bulk}}[\phi] = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}m^2\phi^2\right), \tag{1.18}$$

where we ignore the backreaction on the metric. By expanding about the boundary $z = 0$ and solving the classical equations of motion for ϕ, we obtain two linearly independent solutions

$$\phi = \phi_0(x)z^{d-\Delta} + \phi_1(x)z^{\Delta} + \ldots\ , \tag{1.19}$$

where

$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2 + 4m^2}\right), \tag{1.20}$$

is the scaling dimension of the scalar operator $\mathcal{O}_\Delta(x)$ dual to ϕ. Similar formulas relating the scaling dimension of the dual operator to the mass of the bulk field exist for higher spin fields. Unitary requires Δ to be real which implies the Breitenlohner-Freedman bound $m^2 \geq -d^2/4$ [36, 37]. The coefficient $\phi_0(x)$ of the non-normalizable term $z^{d-\Delta}$ is identified with the source of $\mathcal{O}_\Delta(x)$ and appears in the CFT action as $\int d^dx\ \phi_0(x)\mathcal{O}_\Delta(x)$. The coefficient $\phi_1(x)$ of the normalizable term z^Δ is identified with the vacuum expectation value $\langle\mathcal{O}_\Delta(x)\rangle$. By adding interactions such as $\lambda_3\phi^3$ or $\lambda_4\phi^4$ to the action (1.18), the operator $\mathcal{O}_\Delta(x)$ develops an anomalous dimension.

Higher-point correlation functions can be computed using the identification of the on-shell bulk action with the CFT partition function [35]

$$e^{-S_{\text{on-shell}}[\phi]} = Z_{\text{CFT}} = \left\langle e^{-\int d^d x \ \phi_0(x) \mathcal{O}_\Delta(x)} \right\rangle. \tag{1.21}$$

Correlation functions are then given by varying the on-shell bulk action with respect to the boundary values of the field

$$\langle \mathcal{O}_\Delta(x_1) \dots \mathcal{O}_\Delta(x_n) \rangle = (-1)^{n+1} \frac{\delta}{\delta \phi_0(x_1)} \cdots \frac{\delta}{\delta \phi_0(x_n)} S_{\text{on-shell}}[\phi] \Big|_{\phi_0 = 0} . \tag{1.22}$$

The two previous expressions are formally divergent and require renormalization. The on-shell action $S_{\text{on-shell}}[\phi]$ diverges due to an integration over infinite space, but can be holographically renormalized. The action is first regularized by imposing a cutoff $z \geq \epsilon$ and then renormalized by adding covariant, local counterterms on the boundary $z = \epsilon$ to absorb divergences [38, 39]. Generically, the divergences are of the form $1/\epsilon^k$, but for even dimensions d there are logarithmic divergences $(\log \epsilon)^n$ related to the trace anomaly [40]. In addition to the counterterms, the Gibbons-Hawking-York term [41, 42] must be included in the regularized action in order to maintain the variational principle. Taking the limit $\epsilon \to 0$ then gives finite correlation functions and on-shell action.

In addition to correlation functions of local observables, the AdS/CFT correspondence allows us to compute non-local quantities such as the entanglement entropy which is of interest in condensed matter systems and quantum information theory. The entanglement entropy is a measure of the amount of entanglement occurring between two subsystems. Given a bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ and a state specified by the density matrix $\hat{\rho}$, the reduced density matrix $\hat{\rho}_A$ describing solely the subregion A is obtained by tracing out the degrees of freedom contained in B

$$\hat{\rho}_A = \mathrm{tr}_B\, \hat{\rho}. \tag{1.23}$$

Even if the state $\hat{\rho}$ is pure, the reduced state $\hat{\rho}_A$ is generically mixed. The entanglement entropy S_A of region A is then given by the von Neumann entropy

$$S_A = -\mathrm{tr}\, \hat{\rho}_A \log \hat{\rho}_A. \tag{1.24}$$

For quantum field theories, the entanglement entropy diverges due to the short range correlations between degrees of freedom near the boundary of A and B. In two-dimensional CFTs, the entanglement entropies of vacuum and thermal states have been computed by making use of the replica trick and exhibit a universal form proportional to the central charge [43,44].

Entanglement entropies can be computed holographically through the AdS/CFT correspondence using the Ryu-Takayanagi prescription [45, 46]. The leading contribution to the entanglement entropy of a subregion A in the CFT is given by the expression

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}, \tag{1.25}$$

where γ_A is an extremal codimension-two bulk surface intersecting the boundary on ∂A such that γ_A is homologous to A. If multiple surfaces satisfying these criteria exist, the surface with minimal area is chosen. The Ryu-Takayanagi prescription is reminiscent of the Bekenstein-Hawking formula for the entropy of a black hole [47]. Intuitively, the entangling surface γ_A acts as a horizon shielding the region A from the rest of spacetime. The Ryu-Takayanagi prescription gives a divergent entanglement entropy due to the surface γ_A stretching to the infinitely distant boundary. This prescription satisfies the property of strong subadditivity [48] from which many properties of entanglement entropy can be derived. Using the AdS/CFT correspondence, this holographic prescription was proven in [49], where additional subleading corrections were found.

1.2 Supergravity

Supergravity is the supersymmetric extension of Einstein's theory of general relativity. In addition to the massless spin-2 graviton, all supergravity theories contain at least one massless spin-3/2 superpartner called the gravitino. These gravitinos can be viewed as the gauge field associated with local supersymmetry transformations. The number of gravitinos present is determined by the amount of supersymmetry. Supergravity theories may also contain additional fields such as vectors, fermions, and scalars that necessarily form supermultiplets of the relevant supersymmetry algebra. In minimal supergravity, only the graviton supermultiplet is present. Matter-coupled supergravities contain additional supermultiplets such as chiral, vector, hyper or tensor multiplets in addition to the graviton multiplet. An introduction to supergravity is provided by [50].

Although supersymmetry improves the ultraviolet behavior of scattering amplitudes in the quantized theory, supergravity theories are still non-renormalizable. In the following, we treat supergravity as an unquantized classical field theory. For a purely bosonic background, the condition of unbroken supersymmetry is determined by the vanishing of fermionic supersymmetry variations. These are first-order differential equations that can be simpler to solve than the second-order equations of motion. The supersymmetry equations imply a a subset of the equations of motion and spacetimes satisfying the supersymmetry conditions often satisfy all the equations of motion.

1.2.1 Minimal $D = 4, N = 1$ Supergravity

Supergravity theories are specified by the spacetime dimension, type of supersymmetry, and the presence or absence of coupled matter multiplets. One of the simplest and most familiar supergravity theories is minimal $D = 4, N = 1$ supergravity [51]. Both chiral and vector multiplets can be coupled to this minimal supergravity. The only fields present are a graviton e_μ^a and a Majorana gravitino ψ_μ. The frame field e_μ^a is related to the metric $g_{\mu\nu}$ by

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}, \tag{1.26}$$

where η_{ab} is the Minkowski metric. By convention, curved/spacetime indices are denoted by Greek letters and flat/tangent space indices by Latin letters with both ranging over four possible values. Curved and flat indices are raised/lowered with the metrics $g_{\mu\nu}$ and η_{ab}, respectively. In order to write the action or discuss spinors in curved space, it is necessary to introduce the Clifford algebra

$$\gamma_a\gamma_b + \gamma_b\gamma_a = 2\eta_{ab}, \tag{1.27}$$

where γ_a are complex four by four matrices. By convention, gamma matrices with multiple indices are antisymmetrized with weight one such as $\gamma_{ab} = \gamma_{[ab]} = \frac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a)$. The curved gamma matrices γ_μ are related to the Clifford algebra basis γ_a through the vielbein by $\gamma_\mu = e_\mu^a\gamma_a$.

The supergravity action is given by

$$\begin{aligned} S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big(R - \bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{1}{16}\left(\bar{\psi}^\rho\gamma^\mu\psi^\nu\right)\left(\bar{\psi}_\rho\gamma_\mu\psi_\nu + 2\bar{\psi}_\rho\gamma_\nu\psi_\mu\right) \\ + \frac{1}{4}\left(\bar{\psi}_\mu\gamma_\nu\psi^\nu\right)\left(\bar{\psi}^\mu\gamma_\rho\psi^\rho\right)\Big), \end{aligned} \tag{1.28}$$

where $D_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\psi_\nu$ and ω_μ^{ab} is the torsion-free spin connection defined by the structure equation $de^a + \omega^a{}_b \wedge e^b = 0$. The first two terms appearing in this action are the Einstein-Hilbert and Rarita-Schwinger Lagrangians describing spin-2 and spin-3/2 fields, respectively. The presence of the additional four-gravitino interaction terms are required by supersymmetry. The action is invariant under the $N = 1$ supersymmetry transformations

$$\begin{aligned} \delta e_\mu^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \\ \delta\psi_\mu &= \partial_\mu\epsilon + \frac{1}{4}\left(\omega_\mu^{ab} + K_\mu^{ab}\right)\gamma_{ab}\epsilon, \end{aligned} \tag{1.29}$$

with $\epsilon(x)$ a Majorana spinor. In the above, the variation of the gravitino includes torsion with

$$K_{\mu\nu\rho} = -\frac{1}{4}\left(\bar{\psi}_\mu\gamma_\rho\psi_\nu - \bar{\psi}_\nu\gamma_\mu\psi_\rho + \bar{\psi}_\rho\gamma_\nu\psi_\mu\right). \tag{1.30}$$

We will be interested in classical solutions for which all fermionic fields vanish and therefore the torsion vanishes as well. The gravitino variation then takes the form of a gauge transformation and shifts by a covariant derivative. For supergravity theories in general, the variation of bosonic (fermionic) fields only involve fermionic (bosonic) fields for classical solutions.

A solution is called supersymmetric if there exists a Majorana spinor $\epsilon(x)$ such that all supersymmetry variations vanish. For a classical solution, the variations of all bosonic fields automatically vanish. The amount of unbroken supersymmetry is then determined by the number of linearly independent Majorana spinors $\epsilon(x)$ such that $\delta\psi_\mu = 0$ and can be expressed as a fraction of the total number of real supercharges. Minkowski space which has vanishing spin connection is a maximally supersymmetric vacuum of this theory with $\epsilon(x) = \epsilon_0$ a constant Majorana spinor.

Motivated by the AdS/CFT correspondence, we will be interested in supergravity theories with a supersymmetric AdS vacuum. A simple modification of the theory (1.28) given by

$$D_\mu \to \hat{D}_\mu = D_\mu - \frac{1}{2L}\gamma_\mu, \tag{1.31}$$

leads to a theory with an AdS vacuum with radius of curvature L [52]. Ignoring four-gravitino interaction terms, the action is given by

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R - \bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\mu - \frac{1}{L}\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \frac{6}{L^2}\right), \tag{1.32}$$

where a negative cosmological constant has been added to maintain supersymmetry. The action is invariant under the $N = 1$ supersymmetry transformations

$$\begin{aligned} \delta e^a_\mu &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \\ \delta\psi_\mu &= \partial_\mu\epsilon + \frac{1}{4}\omega^{ab}_\mu\gamma_{ab}\epsilon - \frac{1}{2L}\gamma_\mu\epsilon, \end{aligned} \tag{1.33}$$

and contains a mass-like term for the gravitino with $m = L^{-1}$. In the AdS_4 vacuum, however, the equations of motion for the field ψ_μ are those of a massless particle [53]. AdS_4 is a maximally supersymmetric vacuum of this theory and the supersymmetry parameter $\epsilon(x)$ was found in [36].

1.2.2 Higher Dimensional Supergravity Theories

The largest spacetime dimension permitting the existence of a supergravity theory with one timelike direction is $D = 11$ [54]. Theories with $N = 1$ supersymmetry in larger spacetime dimensions have more than thirty-two real supercharges. After dimensional reduction to $D = 4$, these theories would lead to interacting theories of massless particles with spin greater than two for which no consistent interactions are known [25]. The $D = 11$ supergravity theory can be viewed as the low energy effective field theory of M-theory. The theory contains a three-form field $A_{\mu\nu\rho}$ under which the M2-brane and M5-brane are charged. The maximal $D = 11$ supergravity is fixed and does not contain any matter multiplets or possible modifications such as a cosmological constant. Similar to the $D = 11$ theory, type IIA and IIB supergravity in $D = 10$ are the low energy effective theories of type IIA and IIB string theory, respectively. Both theories contain higher-form fields under which the relevant branes are charged.

By performing a Kaluza-Klein reduction of these higher dimensional theories, one can generate many lower dimensional supergravity theories. This procedure will lead to an infinite tower of Kaluza-Klein modes for each higher dimensional field. As a simple example, consider the dimensional reduction of a free complex scalar field ϕ on $\mathbb{R}^{1,D-2} \times S^1$ where the circle has radius R. If we parameterize the circle by y with $0 \leq y < 2\pi R$ and expand the scalar in modes

$$\phi(x^\mu, y) = \sum_{k=-\infty}^{\infty} e^{iky/R} \phi_k(x^\mu), \tag{1.34}$$

the D-dimensional Klein-Gordon equation $(\Box_D - m^2)\,\phi(x^\mu, y) = 0$ reduces to

$$\left(\Box_{D-1} - \frac{k^2}{R^2} - m^2\right) \phi_k(x^\mu) = 0, \tag{1.35}$$

for each mode k. The higher dimensional scalar field $\phi(x^\mu, y)$ has given rise to the infinite collection of lower dimensional scalar fields $\phi_k(x^\mu)$ with masses $m_k^2 = m^2 + k^2/R^2$. Similar results apply to reductions on spheres S^n where the scalar field is expanded in the relevant spherical harmonics. The reduction of non-scalar fields proceeds similarly and leads to a collection of lower dimensional fields transforming in multiple representations of the Lorentz group. For example, the S^1 reduction of a D-dimensional metric includes fields transforming as a $(D-1)$-dimensional metric, vector, and scalar field.

In special cases, one can consistently truncate this infinite tower of Kaluza-Klein fields to some finite subset. For example, the maximally supersymmetric $\mathrm{AdS}_4 \times S^7$ Freund-Rubin vacuum of $D = 11$ supergravity and the $\mathrm{AdS}_5 \times S^5$ vacuum of type IIB supergravity lead to the maximally supersymmetric SO(8) and SO(6) gauged supergravity theories in dimensions $D = 4$ and $D = 5$, respectively [55–59]. In order for this to be possible, the surviving fields may not appear as sources for the removed fields in the equations of motion. Determining the reduction formulas for how the higher dimensional fields descend into the lower dimensional fields is highly non-trivial. When a consistent truncation exists, solutions of the lower dimensional theory can be uplifted to the higher dimensional theory. Finding solutions in lower dimensional supergravity theories is often convenient due to the ability of choosing simpler ansätze. Even if the lower dimensional theory is not a consistent truncation, it is still worthwhile to study its solutions which can teach us about strongly coupled CFTs and potentially provide a guide for constructing higher dimensional solutions.

1.2.3 Gauged Supergravity

Motivated by the AdS/CFT correspondence, we will be interested in gauged supergravity theories with an AdS vacuum. In a gauged supergravity theory, a cosmological constant or scalar potential must be added to the Lagrangian in order to maintain supersymmetry. A supergravity theory is considered gauged if a subgroup G_0 of the global symmetry group G is gauged. If the lower dimensional theory is a truncation of some higher dimensional theory, the gauge group can arise from the isometries of the compactified manifold. For example, type IIB supergravity on the $\mathrm{AdS}_5 \times S^5$ vacuum can be truncated to maximal SO(6) gauged supergravity in $D = 5$.

A simple situation in which a supergravity theory can be gauged is provided by minimal

$D = 4, N = 2$ supergravity [60]. The field content of this theory is a graviton e_μ^a, two Majorana gravitinos (ψ_μ^1, ψ_μ^2), and a U(1) gauge field A_μ. It is convenient to combine the two Majorana gravitinos into a single Dirac gravitino $\psi_\mu = \psi_\mu^1 + i\psi_\mu^2$. The R-symmetry of the $D = 4, N = 2$ supersymmetry algebra is SU(2). If the vector A_μ gauges a U(1) subgroup of this SU(2), the Dirac gravitino ψ_μ becomes charged and in order to preserve supersymmetry of the action, a negative cosmological constant must be added to the Lagrangian. In this gauged theory, AdS_4 is a maximally supersymmetric vacuum.

In supergravity theories with sufficient supersymmetry, the scalars parameterize a symmetric space G/H where G is a non-compact global symmetry group and H is its maximal compact subgroup. For theories with large global symmetry groups G, it is convenient to specify the gauging through the use of embedding tensors. The embedding tensors project the full global symmetry group G down to a subgroup G_0 and explicitly parameterize the group generators that are gauged. The embedding tensors satisfy consistency conditions of which several are reminiscent of the Jacobi identity.

1.3 Conformal Defects

Quantum field theories often contain interesting non-local operators in addition to their usual local operators. Examples include Wilson lines [61, 62] or surface defects [63, 64] in $\mathcal{N} = 4$ Super Yang-Mills, surface defects in six-dimensional (0,2) theories [65], and conformal defects in two-dimensional CFTs [66, 67]. These extended objects can serve as order [68] or disorder parameters [63] classifying the phases of the theory and are capable of probing non-perturbative phenomena. A special class of non-local operators in CFTs known as (super)conformal defects preserve a subgroup of the (super)conformal symmetry. The correlation functions of local operators in the presence of conformal defects can be calculated holographically through the AdS/CFT correspondence.

1.3.1 Non-Local Operators in Gauge Theory

SU(N) gauge theory in $d = 4$ contains both electric and magnetic gauge-invariant line operators. For the electric case, the Wilson line operator [69] is given by

$$W(C) = \mathrm{Tr}\,\mathcal{P}\exp\left(i\oint_C A_\mu dx^\mu\right), \tag{1.36}$$

where $\mathcal{P}$ denotes path-ordering, C is a closed loop, and A_μ is the gauge field in an irreducible representation R of SU(N) that is not necessarily the adjoint representation. The Wilson line is an an electric operator since it is constructed from fundamental fields appearing in the path integral. It can be thought of as creating a loop of electric flux and serves as an order parameter for confinement. In a confining phase, the expectation value of the Wilson line operator behaves as

$$\log\langle W(C)\rangle = -\sigma\ \mathrm{Area}(C), \tag{1.37}$$

where Area(C) is the area of the minimal surface with boundary C and σ is some constant with units of inverse area. In the Higgs phase, the expectation value takes the form

$$\log\langle W(C)\rangle = -\mu\ \mathrm{Length}(C), \tag{1.38}$$

where Length(C) is the length of the loop C and μ has units of inverse length.

't Hooft operators [70] are the magnetic dual of Wilson lines and correspond to the insertion of a magnetic monopole. They can not be constructed from the fundamental fields appearing in the path integral as was the case for Wilson lines. 't Hooft operators can be inserted into correlation functions by specifying the singularities of the integrated fields that appear in the path integral such as

$$A_\phi^N \to +\frac{g}{4\pi r}\frac{1-\cos\theta}{\sin\theta},$$
$$A_\phi^S \to -\frac{g}{4\pi r}\frac{1+\cos\theta}{\sin\theta}, \tag{1.39}$$

as $r \to 0$ for a U(1) gauge field. In the above, g is the charge of the magnetic monopole located at the origin $r = 0$ in spherical coordinates $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$. The superscripts N and S refer to the northern and southern hemispheres, respectively. The gauge field A_μ differs by a gauge transformation between the northern and southern hemispheres. Since the insertion of a 't Hooft operator creates singularities in the fundamental field A_μ, it is known as a disorder operator.

Surface or codimension-two operators in $d = 4$ gauge theories are special as they can be both electrically and magnetically charged due to electromagnetic duality [71]. Surface operators can be thought of as probing a theory with charged strings. Since the field strength F is a two-form, it can be integrated over a surface Σ and inserted into the path integral in the gauge-invariant form

$$\exp\left(i\eta\int_\Sigma F\right), \tag{1.40}$$

for a U(1) gauge theory with $F = dA$ analogous to Wilson lines (1.36). For Σ an infinite plane, it is convenient to choose coordinates

$$ds^2 = dx_1^2 + dx_2^2 + dr^2 + r^2 d\theta^2, \tag{1.41}$$

with x_1, x_2 parallel to Σ and r, θ polar coordinates around Σ. The surface operator will be magnetically charged if we integrate over fields in the path integral with the singularity

$$A = \alpha\, d\theta, \tag{1.42}$$

where $d(d\theta) = 2\pi\delta(D)$ is the two-form Dirac delta function supported on Σ. The classification of 1/2-BPS surface operators in $\mathcal{N} = 4$ Super Yang-Mills was performed in [64] and depends on additional parameters β and γ related to singularities appearing in the scalar fields.

1.3.2 Conformally Invariant Extended Operators

Due to their non-local nature, it is possible for extended operators to preserve a conformal subgroup. A p-dimensional conformal defect is an extended operator in a d-dimensional CFT that produces the symmetry breaking pattern

$$\mathrm{SO}(d,2) \times G \to \mathrm{SO}(p,2) \times \mathrm{SO}(d-p) \times H, \tag{1.43}$$

where $\mathrm{SO}(p,2)$ and $\mathrm{SO}(d-p)$ are the preserved conformal and transverse rotational symmetries of the defect, respectively. The CFT may also be invariant under some flavor or R-symmetry group G that is broken to a subgroup $H \subset G$. If the CFT is supersymmetric and the defect furthermore preserves a superconformal subgroup, it is called a superconformal defect. As an example, 1/2-BPS superconformal surface operators in $d = 4, \mathcal{N} = 2$ SCFTs induce the symmetry breaking pattern

$$\mathrm{SU}(2,2|2) \to \mathrm{SU}(1,1|1) \times \mathrm{SU}(1,1|1) \times \mathrm{U}(1), \tag{1.44}$$

corresponding to (2,2) supersymmetry in $d = 2$ [72].

The conformal defect can be extended along a plane or sphere, but we will focus on the planar case. Following the notation of [73], let $x_{||}^a$ and $x_\perp^i$ be the coordinates parallel and perpendicular to the defect, respectively. The defect contains local operators $\hat{\mathcal{O}}_{\hat{\Delta}_i}(x_{||})$ satisfying the properties of a p-dimensional CFT and forming representations of the SO$(d-p)$ global symmetry. In particular, the defect scaling dimensions $\hat{\Delta}_i$ are independent of the bulk scaling dimensions Δ_i and represent additional data. When a bulk operator is brought near the defect, it satisfies a bulk-to-defect OPE of the form

$$\mathcal{O}_\Delta\left(x_{||}, x_\perp\right) = \frac{b_{\mathcal{O}\hat{\mathcal{O}}}}{|x_\perp|^{\Delta-\hat{\Delta}}} \hat{\mathcal{O}}_{\hat{\Delta}}\left(x_{||}\right) + \dots \ , \tag{1.45}$$

where the bulk-to-defect coefficients $b_{\mathcal{O}\hat{\mathcal{O}}}$ further characterize the defect. Using this OPE, the bulk operators can acquire one-point expectation values in the presence of the defect given by

$$\langle \mathcal{O}_\Delta\left(x_{||}, x_\perp\right)\rangle = \frac{b_{\mathcal{O}\hat{\mathbb{1}}}}{|x_\perp|^{\Delta}}, \tag{1.46}$$

where $\hat{\mathbb{1}}$ is the identity operator on the defect with $\hat{\Delta} = 0$.

Correlation functions involving bulk and defect primaries are constrained due to conformal symmetry. Using the embedding formalism, one can show the two-point correlation functions of a bulk and defect primary are fixed by conformal symmetry up to a collection of coefficients [73]. In the embedding formalism, one maps d-dimensional Euclidean space to the light-cone

$$P \cdot P = 0, \tag{1.47}$$

of $(d+2)$-dimensional Minkowski space where the conformal group SO$(d+1, 1)$ acts linearly.

To remove the remaining redundancy, points differing by a rescaling are identified under

$$\lambda P \sim P, \qquad \lambda > 0, \tag{1.48}$$

which is compatible with the restriction to the light-cone. Since the conformal symmetry acts linearly on the embedding space P, it is then possible to write down all the tensor structures allowed by conformal symmetry. After these tensor structures are known, they can then be projected back down to the original physical spacetime.

1.3.3 Holographic Realization

Conformal defects can be constructed and studied holographically using the AdS/CFT correspondence. Motivated by the symmetry breaking pattern (1.43), a conformal defect will be dual to a warped product of the form

$$ds^2 = f_1(r) ds^2_{\mathrm{AdS}_{p+1}} + f_2(r) d\Omega^2_{d-p-1} + f_3(r) dr^2, \tag{1.49}$$

such as the Janus ansatz [74]. The isometry group of this spacetime matches the preserved conformal and rotational symmetries of the defect. If the defect preserves additional flavor or R-symmetries, these can be realized geometrically as the isometry group of some compact space or global symmetry of bulk fields. Superconformal defects will correspond to spacetimes of this form with the relevant amount of unbroken supersymmetry.

As an example, codimension-one operators in $\mathcal{N} = 4$ Super Yang-Mills can be described by the insertion of an $\mathrm{AdS}_4 \times S^2$ probe D5-brane in the $\mathrm{AdS}_5 \times S^5$ vacuum of type IIB supergravity [75]. Outside of the probe approximation, one must take into account the backreaction of the branes on the metric and solve the full set of supersymmetry equations.

These solutions were found in [76] as a product of $\mathrm{AdS}_4 \times S^2 \times S^2$ warped over a Riemann surface Σ that preserves sixteen of the thirty-two supersymmetries. Additional supergravity solutions corresponding to superconformal defects in various theories include [77–88].

CHAPTER 2

Minimal $D = 4, N = 2$ Gauged Supergravity

Four-dimensional AdS gravitational theories with gauge fields and scalars have been used to model many strongly coupled three-dimensional condensed matter systems including superfluids and superconductors, see e.g. [89–91]. A simple model for strongly coupled three-dimensional CFTs is four-dimensional AdS Einstein-Maxwell theory. The presence of a gauge field allows the construction of charged defect solutions which on the CFT side corresponds to turning on a position dependent chemical potential for the charge dual to the gauge field. Such solutions were constructed in [92] and for general forms of the chemical potential, it was found that the solutions break conformal invariance. However, for a special choice of the gauge field there is the possibility to preserve an $\mathrm{SO}(2,1)\times$ $\mathrm{SO}(2)$ subgroup of the three-dimensional conformal group $\mathrm{SO}(3,2)$ and such a solution is of the Janus type.

The goal of the present chapter is to construct four-dimensional conformal defect solutions in gauged $N = 2$ supergravity. In Section 2.1, we embed the solutions [92] into minimal $N = 2$ gauged supergravity [60] and generalize the solution to have both non-trivial electric and magnetic fields. In Section 2.2, we analyze the BPS conditions for the conformal defect solution and show that there is a clash between supersymmetry and regularity of the geometry. We show that the solution breaks supersymmetry if we demand that there is no conical singularity present in both the bulk metric and the boundary metric. In Section 2.3, we prove that a more general ansatz for a conformal defect starting with an AdS_2 factor warped over a Riemann surface Σ with boundary reduces to the ansatz used above, i.e. supersymmetry implies the presence of an additional U(1) isometry and hence the spacetime reduces to $AdS_2 \times S^1$ warped over one spatial coordinate. This result is in line with classification theorems found in [93, 94]. In Section 2.4, we summarize the results of this chapter. We present our Clifford algebra conventions and basis for AdS_2 Killing spinors in Appendix A.

2.1 Dyonic Conformal Defect Solution

The action for Einstein-Maxwell theory with a negative cosmological constant is given by

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{3}{2L^2}\right), \tag{2.1}$$

with the equations of motion taking the following form

$$\begin{aligned} R_{\mu\nu} - 2(F_\mu{}^\rho F_{\nu\rho} - \tfrac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}) + \frac{3}{L^2}g_{\mu\nu} = 0, \\ \partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0. \end{aligned} \tag{2.2}$$

The conformal defect solution in the boundary CFT exhibits an SO(2, 1)× SO(2) isometry which can be realized by taking an $AdS_2 \times S^1$ space warped over a radial coordinate. We construct the most general solution which is asymptotically AdS_4 and has non-zero electric and magnetic components in the field strength $F_{\mu\nu}$

$$ds^2 = \frac{L^2}{\lambda^2(1-y^2)^2}\left[\rho^2\frac{-dt^2+d\eta^2}{\eta^2} + y^2 f(y)d\phi^2 + \frac{4\lambda^2 dy^2}{f(y)}\right], \tag{2.3}$$

$$A = \frac{L\alpha}{\eta}dt + L\lambda\beta y^2 d\phi, \tag{2.4}$$

$$f(y) = 3 - 3y^2 + y^4 - \frac{\lambda^2}{\rho^2}(1-y^2)^2 + \frac{\lambda^4}{\rho^4}(\alpha^2+\beta^2\rho^4)(1-y^2)^3. \tag{2.5}$$

This solution is that of the analytically continued Reissner-Nordström black hole [93]

$$\begin{aligned} ds^2 &= \psi^2\left(A_0 r^2 dv^2 + 2dvdr\right) + \frac{d\psi^2}{P(\psi)} + P(\psi)d\phi^2, \\ F &= a\ dr \wedge dv + b\psi^{-2}d\psi \wedge d\phi, \end{aligned} \tag{2.6}$$

with $P(\psi) = A_0 - c(2\psi)^{-1} - (a^2+b^2)\psi^{-2} - \Lambda\psi^2/3$ under the identifications

$$\begin{aligned} A_0 &= -\rho^{-2}, \\ c &= -\frac{2\left(\lambda^2 - \rho^2 + \lambda^4(\alpha^2\rho^{-2} + \beta^2\rho^2)\right)}{\lambda^3\rho^2}, \\ a &= \alpha\rho^{-2}, \\ b &= \beta, \end{aligned} \tag{2.7}$$

for $\psi^{-1} = \lambda(1-y^2)$ and $L = 1$. The condition that at $y = 0$ the space closes off smoothly without a conical deficit, imposes the condition

$$\frac{\lambda^4}{\rho^4}\left(\alpha^2 + \beta^2\rho^4\right) = 2\lambda + \frac{\lambda^2}{\rho^2} - 3, \tag{2.8}$$

on the four parameters α, β, ρ and λ. With this condition imposed, the function f only depends on ρ and λ

$$f(y) = 3 - 3y^2 + y^4 - \frac{\lambda^2}{\rho^2}(1-y^2)^2 + \left(2\lambda + \frac{\lambda^2}{\rho^2} - 3\right)(1-y^2)^3. \tag{2.9}$$

In general, there is an allowed parameter range for ρ and λ outside of which the function $f(y)$ develops a zero in the range $[0,1]$ and the solution becomes singular. For example, the allowed parameter range for λ is given by $1 \leq \lambda \lesssim 4.43$ for $\rho^2 = 1$. The choice $\lambda = \rho = 1$ corresponds to the AdS_4 vacuum with vanishing electromagnetic fields in an $\mathrm{AdS}_2 \times S^1$ slicing. As we shall see in the next section, solutions with $\rho^2 \neq 1$ correspond to boundary spaces with a conical defect.

2.1.1 Holography

The conformal boundary of the metric (2.3) is located at $y = 1$, but the metric is not in Fefferman-Graham (FG) form. However, it is straightforward to construct a FG coordinate z near the boundary as a power series solution to

$$\frac{dz}{z} = \frac{-2dy}{(1-y^2)\sqrt{f(y)}}. \tag{2.10}$$

This equation can be solved perturbatively to yield

$$y = 1 - z - \frac{z^2}{2} + \frac{2\lambda^2 - \rho^2}{2\rho^2} z^3 - \frac{32\lambda^4(\alpha^2 + \beta^2\rho^4) + 8\lambda^2\rho^2 - 17\rho^4}{24\rho^4} z^4 + O(z^5), \tag{2.11}$$

and the metric (2.3) becomes

$$ds^2 = L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2}\Big(g^{(0)}_{ij} + z^2 g^{(2)}_{ij} + z^3 g^{(3)}_{ij} + O(z^4)\Big) dx^i dx^j, \tag{2.12}$$

with

$$\begin{aligned} g^{(0)}_{ij} dx^i dx^j &= \frac{1}{4\lambda^2}\left(\rho^2 \frac{-dt^2 + d\eta^2}{\eta^2} + d\phi^2\right), \\ g^{(2)}_{ij} dx^i dx^j &= \frac{1}{2}\left(\frac{-dt^2 + d\eta^2}{\eta^2} - \frac{d\phi^2}{\rho^2}\right), \\ g^{(3)}_{ij} dx^i dx^j &= \frac{2(\lambda^4(\alpha^2 + \beta^2\rho^4) + \lambda^2\rho^2 - \rho^4)}{3\lambda^2\rho^2}\left(\frac{dt^2 - d\eta^2}{\eta^2} + \frac{2}{\rho^2} d\phi^2\right). \end{aligned} \tag{2.13}$$

Following the standard holographic dictionary, $g^{(0)}$ is the metric of the $\mathrm{AdS}_2 \times S^1$ boundary and $g^{(2)}$ is determined by $g^{(0)}$. Note that for $\rho^2 \neq 1$, the boundary is conformal to $\mathbb{R}^{1,2}$ with a conical defect at $\eta = 0$. The next term in the FG expansion $g^{(3)}$ determines the expectation value of the stress tensor. Since we have an odd-dimensional boundary, there is no conformal anomaly and

$$\langle T_{ij} \rangle = \frac{3}{16\pi G_N} g^{(3)}_{ij}. \tag{2.14}$$

Note that the stress tensor is indeed traceless in agreement with conformal symmetry. The near boundary behavior of the gauge field in FG coordinates is given by

$$A = \frac{L\alpha}{\eta} dt + L\lambda\beta(1 - 2z) d\phi + O(z^2). \tag{2.15}$$

The standard holographic dictionary for a gauge fields identifies the z^0 term as a source and the z^1 term as an expectation value of the dual current j_μ in the CFT. As discussed in [92], the first term in (2.15) can be interpreted as a chemical potential for the current. After a conformal transformation from $\text{AdS}_2 \times S^1$ to $\mathbb{R}^{1,2}$, it takes the form $\mu(r) = La_\lambda/r$. This corresponds to a point charge defect localized at the origin $r = 0$. The second term in (2.15) can be interpreted as a source and expectation value of j^ϕ [95–98]. As in [92], the entanglement entropy of the defect can be analyzed using the Ryu-Takayanagi prescription [45]. Extremal surfaces centered on the defect at $\eta = 0$ are given by $\eta = \eta_0$. The entanglement entropy of a region defined by $\eta \leq \eta_0$ is given by

$$S = \frac{1}{4G_N} \int d\phi dy \sqrt{g_{\phi\phi} g_{yy}} = \frac{\pi L^2}{\lambda G_N} \int_0^{y_\Lambda} dy \frac{y}{(1-y^2)^2}, \tag{2.16}$$

with $y_\Lambda \sim 1$ a UV cutoff. Note, the entanglement entropy S does not depend on η. We can study the entanglement entropy of the defect alone by considering the quantity

$$\Delta S = S(\lambda) - S(1). \tag{2.17}$$

Matching the circumference of circles near the asymptotic boundary requires $\lambda(1 - \tilde{y}_\Lambda^2) = 1 - y_\Lambda^2$ and leads to a defect entanglement entropy of

$$\Delta S = \frac{\pi L^2}{2G_N} \left(1 - \frac{1}{\lambda}\right). \tag{2.18}$$

2.2 Minimal Gauged Supergravity

The field content of minimal $D = 4, N = 2$ gauged supergravity consists of a graviton $g_{\mu\nu}$, a pair of Majorana gravitini (ψ_μ^1, ψ_μ^2), and a photon A_μ. The Einstein-Maxwell action (2.1) is the action of the purely bosonic sector. The full action includes the additional fermionic terms

$$\frac{1}{\sqrt{-g}}(\mathcal{L}_\psi + \mathcal{L}_{\text{int}}) = -\frac{1}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\mathcal{D}_\nu \psi_\rho - \frac{i}{8}\left(F + \hat{F}\right)^{\mu\nu} \bar{\psi}_\rho \gamma_{[\mu}\gamma^{\rho\sigma}\gamma_{\nu]}\psi_\sigma + \frac{1}{2L}\bar{\psi}_\mu \gamma^{\mu\nu}\psi_\nu, \quad (2.19)$$

with gauge covariant derivative $\mathcal{D}_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} - \frac{i}{L}A_\mu$. In the above, the gravitini have been combined into a Dirac spinor $\psi_\mu = \psi_\mu^1 + i\psi_\mu^2$ with charge $e = 1/L$ and $\hat{F}_{\mu\nu}$ is defined by $\hat{F}_{\mu\nu} = F_{\mu\nu} - \text{Im}(\bar{\psi}_\mu \psi_\nu)$. For classical solutions, $\psi_\mu = 0$ and the condition for unbroken supersymmetry of the background is the vanishing of the gravitino supersymmetry variation

$$\delta\psi_\mu = \hat{\nabla}_\mu \epsilon = \left(\partial_\mu + \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - \tfrac{i}{L}A_\mu + \tfrac{1}{2L}\gamma_\mu + \tfrac{i}{4}F_{ab}\gamma^{ab}\gamma_\mu\right)\epsilon = 0. \quad (2.20)$$

Given the AdS_2 isometry, we decompose the Killing spinors as a tensor product

$$\epsilon = \sum_{k,\kappa} \psi_k^{(\kappa)}(t,\eta) \otimes \chi_k^{(\kappa)}(y,\phi), \quad (2.21)$$

where $\psi_k^{(\kappa)}$ satisfies the AdS_2 Killing spinor equation

$$\left(\partial_{\hat{\mu}} + \frac{1}{4}\hat{\omega}_{\hat{\mu}}^{\hat{a}\hat{b}}\tilde{\gamma}_{\hat{a}\hat{b}} + \frac{\kappa}{2}\tilde{\gamma}_{\hat{\mu}}\right)\psi_k^{(\kappa)} = 0, \quad (2.22)$$

with hatted indices denoting AdS_2 directions. Integrability requires $\kappa^2 = 1$ and for each κ there are two linearly independent solutions labeled by k. A Clifford algebra basis adapted to this decomposition and the explicit form of $\psi_k^\pm$ can be found in Appendix A. Multiplying the AdS_2 Killing spinor equation by the chirality matrix $\tilde{\gamma}_*$ shows we can choose $\psi_k^\pm$ to satisfy $\tilde{\gamma}_*\psi_k^\pm = \psi_k^\mp$ which will be needed in the reduction. The reduction will give identical and decoupled equations for $k = 1, 2$ so any solution is automatically 1/2-BPS. In what follows, we drop the subscript k.

Applying the reduction to the following combination of BPS variations

$$\delta\psi_t - \gamma_{01}\delta\psi_\eta = -\frac{i\alpha}{\eta}\epsilon, \tag{2.23}$$

shows that $\alpha = 0$ and only a purely magnetic solution can be supersymmetric. The AdS$_2$ components of the BPS equation (2.20) gives

$$\left[\left(\frac{1}{2} \mp \frac{\lambda(1-y^2)}{2\rho}\right) - \frac{\lambda(1-y^2)^2}{4Ly}A'_\phi\sigma_2\right]\chi^\pm = -\frac{\sqrt{f}}{4\rho}(1-y^2)^2\frac{d}{dy}\left(\frac{\rho}{1-y^2}\right)\sigma_3\chi^\mp, \tag{2.24}$$

which requires $\chi^\pm \propto |\pm\rangle_y$ modulo the discrete symmetry $\lambda \to -\lambda$ and $\chi^\pm \to \sigma_3\chi^\pm$. We therefore choose $\chi^\pm = h_\pm(y)e^{in\phi/2}|\pm\rangle_y$ with $n \in \mathbb{Z}$. The ϕ and y components give the algebraic and differential equations

$$\begin{aligned}\left(\frac{n}{2} - \frac{1}{L}A_\phi \mp \frac{\sqrt{f}(1-y^2)}{4\lambda}\frac{d}{dy}\left(\frac{y\sqrt{f}}{1-y^2}\right)\right)h_\pm &= \left(-\frac{\sqrt{f}(1-y^2)}{4L}A'_\phi \pm \frac{y\sqrt{f}}{2\lambda(1-y^2)}\right)h_\mp,\\ \frac{dh_\pm}{dy} &= \left(-\frac{1}{\sqrt{f}(1-y^2)} \pm \frac{\lambda(1-y^2)}{2Ly\sqrt{f}}A'_\phi\right)h_\mp,\end{aligned} \tag{2.25}$$

respectively. These equations are solved by

$$\begin{aligned}h_+^2 + h_-^2 &= \frac{\rho}{1-y^2},\\ h_+^2 - h_-^2 &= \lambda + \frac{1}{2}\rho\lambda\left(n - \rho^{-1}\right)(1-y^2),\end{aligned} \tag{2.26}$$

subject to the conditions

$$\begin{aligned}\lambda &= 2(n+\rho^{-1})^{-1},\\ 4\beta &= n^2 - \rho^{-2}.\end{aligned} \tag{2.27}$$

Thus we have a family of supersymmetric solutions labeled by the parameters n and ρ. The conditions of no conical defect in the bulk (2.8)

$$\lambda^4\beta^2 = 2\lambda + \frac{\lambda^2}{\rho^2} - 3,$$

and no conical defect in the boundary (2.13)

$$\rho^2 = 1,$$

are mutually incompatible with the constraints from supersymmetry (2.27) with the exception of vacuum AdS_4 with $\lambda = \rho = n = 1$ and $\beta = 0$. Thus a non-trivial supersymmetric solution must either have a conical defect in the bulk at $y = 0$ or in the boundary at $\eta = 0$.

2.3 Emergence of U(1) Isometry

A more general ansatz incorporating a Riemann surface Σ is given by

$$\begin{aligned} ds^2 &= L^2\rho(z,\bar{z})^2\left(\frac{-dt^2 + d\eta^2}{\eta^2}\right) + L^2 f(z,\bar{z})^2\, dzd\bar{z}, \\ A &= A_z(z,\bar{z})\, dz + A_{\bar{z}}(z,\bar{z})\, d\bar{z}. \end{aligned} \tag{2.28}$$

Applying the Killing spinor decomposition (2.21) to the AdS_2 components of the BPS equations (2.20) gives the projection conditions $\chi^{\pm} = h_{\pm}(z,\bar{z})\,|\pm\rangle_y$ modulo a discrete symmetry. In what follows, we drop the subscript k since identical equations hold for $k = 1, 2$. The solution will therefore automatically be 1/2-BPS. As before, a component of the gauge field of the form $A_t(\eta)$ is inconsistent with the BPS equations. The Riemann surface components of the BPS equations $\delta\psi_z = \overline{\delta\psi_{\bar z}} = 0$ give

$$\frac{\partial h_+}{\partial z} + \frac{1}{2f}\frac{\partial f}{\partial z}h_+ - \frac{i}{L}A_z h_+ - if\left(\frac{1}{2} + \frac{i}{Lf^2}\left(\frac{\partial A_z}{\partial \bar z} - \frac{\partial A_{\bar z}}{\partial z}\right)\right)h_- = 0, \tag{2.29}$$

$$\frac{\partial h_-}{\partial z} - \frac{1}{2f}\frac{\partial f}{\partial z}h_- - \frac{i}{L}A_z h_- = 0, \tag{2.30}$$

$$\frac{\partial \bar h_+}{\partial z} - \frac{1}{2f}\frac{\partial f}{\partial z}\bar h_+ + \frac{i}{L}A_z \bar h_+ = 0, \tag{2.31}$$

$$\frac{\partial \bar h_-}{\partial z} + \frac{1}{2f}\frac{\partial f}{\partial z}\bar h_- + \frac{i}{L}A_z \bar h_- - if\left(\frac{1}{2} - \frac{i}{Lf^2}\left(\frac{\partial A_z}{\partial \bar z} - \frac{\partial A_{\bar z}}{\partial z}\right)\right)\bar h_+ = 0. \tag{2.32}$$

Equations (2.30) and (2.31) can be solved to yield

$$\frac{i}{L}A_z = \frac{\partial}{\partial z}\ln\left(\frac{h_-}{\sqrt{f}}\right) = \frac{\partial}{\partial z}\ln\left(\frac{\sqrt{f}}{\bar h_+}\right). \tag{2.33}$$

This implies we must have

$$\bar h_+ h_- = \bar g_1(\bar z) f, \tag{2.34}$$

for some anti-holomorphic function $\bar g_1(\bar z)$. Taking the linear combination $\bar h_+$(2.29)+ $\bar h_-$(2.30) + h_+(2.31) + h_-(2.32) gives

$$g_1(z)\frac{\partial}{\partial z}\left(|h_+|^2 + |h_-|^2\right) = i|h_+|^2|h_-|^2, \tag{2.35}$$

where we have used (2.34) and $\bar f = f$. Taking the linear combination $\bar h_+$(2.29) $-$ $\bar h_-$(2.30) + h_+(2.31) $-$ h_-(2.32) gives

$$\frac{\partial}{\partial z}\left(|h_+|^2 - |h_-|^2\right) = i\bar g_1(\bar z)\frac{\partial}{\partial z}\frac{\partial}{\partial \bar z}\ln\left(\frac{|h_-|^2}{|h_+|^2}\right), \tag{2.36}$$

which can be integrated in z. To obtain this equation, one needs to make use of both (2.33) and (2.34). Setting $u = |h_+|^2$, $v = |h_-|^2$, and then integrating and complex conjugating the previous equation gives

$$
\begin{aligned}
g_1(z)\frac{\partial}{\partial z}(u+v) &= iuv, \\
u - v + ig_1(z)\frac{\partial}{\partial z}\ln\left(\frac{v}{u}\right) &= \frac{d\tilde{g}_2(z)}{dz},
\end{aligned}
\tag{2.37}
$$

for some holomorphic function $\tilde{g}_2(z)$. After performing the change of variables

$$
u = e^{X+Y}, \qquad\qquad v = e^{X-Y}, \tag{2.38}
$$

and change of coordinates defined by $g_1(z) = dz/dw$, equation (2.37) becomes

$$
\begin{aligned}
\frac{\partial}{\partial w}\left(2e^X \cosh Y\right) &= ie^{2X}, \\
2e^X \sinh Y - 2i\frac{\partial Y}{\partial w} &= \frac{dg_2(w)}{dw}.
\end{aligned}
\tag{2.39}
$$

Reality of X and Y requires

$$
2i\frac{\partial Y}{\partial w} + \frac{dg_2}{dw} = -2i\frac{\partial Y}{\partial \bar{w}} + \frac{d\bar{g}_2}{d\bar{w}}, \tag{2.40}
$$

which is solved by

$$
Y = \frac{i}{2}\left(g_2(w) - \bar{g}_2(\bar{w})\right) + \tilde{y}(w - \bar{w}). \tag{2.41}
$$

Solving for e^X using the second line of (2.39) and substituting into the first gives

$$
0 = \left(\frac{\partial \tilde{y}}{\partial w}\right)^2 + i\frac{dg_2}{dw}\frac{\partial \tilde{y}}{\partial w} - \sinh\left(i(g_2 - \bar{g}_2) + 2\tilde{y}\right)\frac{\partial^2 \tilde{y}}{\partial w^2}. \tag{2.42}
$$

Forming the combination (2.42)$-\overline{(2.42)}$ gives

$$i\left(\frac{dg_2}{dw} - \frac{d\bar{g}_2}{d\bar{w}}\right)\frac{\partial \tilde{y}}{\partial w} = 0. \tag{2.43}$$

This equation is solved by $g_2(w) = aw + b$ with $a \in \mathbb{R}$. The case $\partial\tilde{y}/\partial w = 0$ corresponds to the field strength vanishing identically. Thus, all the fields are only a function of $w - \bar{w} \propto y$. The Riemann ansatz simplifies to

$$\begin{aligned} ds^2 &= L^2\rho(y)^2\left(\frac{-dt^2 + d\eta^2}{\eta^2}\right) + L^2 f(y)^2\,(d\phi^2 + dy^2), \\ A &= A(y)\,d\phi. \end{aligned} \tag{2.44}$$

This ansatz is simple enough to use the Einstein-Maxwell equations directly. The ϕ component is the only non-trivial component of Maxwell's equation and is solved by

$$f(y) = c_1\rho(y)^2\frac{dA}{dy}. \tag{2.45}$$

Next, the difference of the $\phi\phi$ and yy components of Einstein's field equation is solved by

$$\frac{dA}{dy} = c_2\frac{2\rho'(y)}{\rho(y)^2}. \tag{2.46}$$

Thus, we only have 1 unknown function $\rho(y)$ and solving the BPS equations will recover the previous magnetic defect solution. In summary, we showed that a more general ansatz of an AdS_2 factor warped over a Riemann surface reduces to $\mathrm{AdS}_2 \times S^1$ warped over a one-dimensional interval.

2.4 Discussion

In the present chapter, we investigated the question of whether supersymmetric conformal defect solutions exist in four-dimensional AdS gauged supergravity. We considered a simple ansatz where the four-dimensional geometry is given by an $AdS_2 \times S^1$ factor warped over a single coordinate with non-trivial electric and magnetic field components. For minimal gauged supergravity without additional matter multiplets, the most general solution of the equations of motion are double analytic continuations of black hole solutions. We showed that no supersymmetric solutions other than AdS_4 exist if we demand the absence of a conical defect in both the bulk and boundary metrics. An interesting question is whether a more generalized setup allows for supersymmetric solutions with a non-singular geometry. Two possible generalizations include adding vector multiplets or hypermultiplets and considering a non-abelian gauging of their scalar manifold isometries. In the subsequent chapter, we consider coupling vector multiplets in an attempt to remove the conical singularities.

CHAPTER 3

Matter-Coupled $D = 4, N = 2$ Gauged Supergravity

In this chapter, we consider matter-coupled $D = 4, N = 2$ gauged supergravity. We construct 1/2-BPS supergravity solutions which are dual to line defects in three-dimensional $\mathcal{N} = 2$ superconformal field theories. The metric ansatz is given by $\mathrm{AdS}_2 \times S^1$ warped over an interval. The examples appearing in this chapter are generalizations of the solution in the previous chapter and are free of conical singularities.

The structure of the chapter is as follows. In Section 3.1, we review our conventions for four-dimensional $N = 2$ gauged supergravity coupled to vector multiplets. In Section 3.2, we give a general solution describing a 1/2-BPS line defect, obtained by a double analytic continuation of the black hole solutions first found by Sabra [99]. Since the behavior of the vector multiplet scalars can only be determined implicitly, we consider three examples, namely a single scalar model, the gauged STU model, and the SU(1,n) coset model to obtain explicit solutions. In Section 3.3, we use the machinery of holographic renormalization to calculate holographic observables for the solutions, namely the on-shell action and the expectation values of operators dual to the supergravity fields. In Section 3.4, we explore the conditions for a regular geometry and determine their consequences. In Section 3.5, we summarize the results of this chapter. Our conventions and some details of the calculations presented in the main body of the chapter are relegated to Appendices B and C.

3.1 Coupling of Vector Multiplets

In this section, we review four-dimensional $N = 2$ gauged supergravity coupled to n vector multiplets. We use the conventions and notations of [94, 100, 101].

The field content of the gauged supergravity theory is as follows. The supergravity contains one graviton e^a_μ, two gravitinos ψ^i_μ, and one graviphoton A^0_μ. The gravity multiplet can be coupled to $N = 2$ matter and in particular, we consider n vector multiplets labeled by an index $\alpha = 1, 2, \ldots, n$. Each vector multiplet contains one vector field A^α_μ, two gauginos λ^α_i, and one complex scalar τ^α. In this chapter we do not consider adding $N = 2$ hypermultiplets.

It is convenient to introduce a new index $I = 0, 1, \ldots, n$ and include the graviphoton with the other vector fields as A^I_μ. The complex scalars τ^α parameterize a special Kähler manifold equipped with a holomorphic symplectic vector

$$v(\tau) = \begin{pmatrix} Z^I(\tau) \\ \mathcal{F}_I(\tau) \end{pmatrix} , \tag{3.1}$$

where the Kähler potential $\mathcal{K}(\tau, \bar{\tau})$ is determined by

$$e^{-\mathcal{K}(\tau,\bar{\tau})} = -i \langle v, \bar{v} \rangle \equiv -i(Z^I \bar{\mathcal{F}}_I - \mathcal{F}_I \bar{Z}^I) . \tag{3.2}$$

In the models we will consider, there exists a holomorphic function $\mathcal{F}(Z)$, called the prepotential, that is homogeneous of second-order in Z such that

$$\mathcal{F}_I(\tau) = \frac{\partial}{\partial Z^I} \mathcal{F}\big(Z(\tau)\big) . \tag{3.3}$$

The supergravity theory is fully specified by the prepotential $\mathcal{F}(Z)$ and the choice of gauging of the SU(2) R-symmetry. We will choose the U(1) Fayet-Iliopoulos (FI) gauging. The only charged fields of the theory are the gravitinos, which couple to the gauge fields through the linear combination $\xi_I A^I$, for some real constants ξ_I. The two gravitinos have opposite charges $\pm g\xi_I$ for each U(1) gauge factor, where g is the gauge coupling.

The bosonic action is

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos}} &= \frac{1}{2}R - g_{\alpha\bar{\beta}}\partial^\mu\tau^\alpha\partial_\mu\bar{\tau}^{\bar{\beta}} - V(\tau,\bar{\tau}) \\ &\quad + \frac{1}{4}(\operatorname{Im}\mathcal{N})_{IJ}F^{I\mu\nu}F^J_{\mu\nu} - \frac{1}{8}(\operatorname{Re}\mathcal{N})_{IJ}e^{-1}\epsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma} \ , \end{aligned} \tag{3.4}$$

where $8\pi G_{\text{N}} = 1$, $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ are the field strengths, and $g_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}\mathcal{K}$ is the Kähler metric of the scalar manifold. We use $G_{\mu\nu}$ to denote the four-dimensional metric, so $e = \sqrt{-\det G}$. The scalar potential is

$$V(\tau,\bar{\tau}) = -2g^2\xi_I\xi_J\Big((\operatorname{Im}\mathcal{N})^{-1|IJ} + 8e^{\mathcal{K}}\bar{Z}^I Z^J\Big) \ , \tag{3.5}$$

where the kinetic matrix $\mathcal{N}_{IJ}$ is given by

$$\mathcal{N}_{IJ}(\tau,\bar{\tau}) = \bar{\mathcal{F}}_{IJ} + 2i\frac{(\operatorname{Im}\mathcal{F}_{IL})(\operatorname{Im}\mathcal{F}_{JK})Z^L Z^K}{(\operatorname{Im}\mathcal{F}_{MN})Z^M Z^N} \ , \qquad \mathcal{F}_{IJ} \equiv \frac{\partial}{\partial Z^I}\frac{\partial}{\partial Z^J}\mathcal{F}(Z) \ . \tag{3.6}$$

This is equivalently defined as the matrix which solves the equations

$$\mathcal{F}_I = \mathcal{N}_{IJ}Z^J \ , \qquad \mathcal{D}_{\bar{\alpha}}\bar{\mathcal{F}}_I = \mathcal{N}_{IJ}\mathcal{D}_{\bar{\alpha}}\bar{Z}^J \ , \tag{3.7}$$

where $\mathcal{D}$ is the Kähler covariant derivative

$$\begin{aligned}
\mathcal{D}_\alpha v &= (\partial_\alpha + \partial_\alpha \mathcal{K}) v \ , \\
\mathcal{D}_{\bar\alpha} \bar v &= (\partial_{\bar\alpha} + \partial_{\bar\alpha} \mathcal{K}) \bar v \ , \\
\mathcal{D}_\alpha \bar v &= \partial_\alpha \bar v = 0 \ , \\
\mathcal{D}_{\bar\alpha} v &= \partial_{\bar\alpha} v = 0 \ .
\end{aligned} \tag{3.8}$$

The equations of motion are obtained by varying the Lagrangian (3.4)

$$\begin{aligned}
R_{\mu\nu} &= 2 g_{\alpha\bar\beta} \partial_\mu \tau^\alpha \partial_\nu \bar\tau^{\bar\beta} + V G_{\mu\nu} + (\mathrm{Im}\,\mathcal{N})_{IJ} \left(-F^I{}_\mu{}^\rho F^J{}_{\nu\rho} + \frac{1}{4} F^{I\rho\sigma} F^J{}_{\rho\sigma} G_{\mu\nu} \right) \ , \\
\partial_\mu \left(e g_{\alpha\bar\beta} \partial^\mu \bar\tau^{\bar\beta} \right) &= e \left((\partial_\alpha g_{\beta\bar\gamma}) \partial^\mu \tau^\beta \partial_\mu \bar\tau^{\bar\gamma} - \frac{1}{4} \partial_\alpha (\mathrm{Im}\,\mathcal{N})_{IJ} F^{I\mu\nu} F^J_{\mu\nu} + \partial_\alpha V \right) \\
&\quad + \frac{1}{8} \partial_\alpha (\mathrm{Re}\,\mathcal{N})_{IJ} \epsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \ , \\
0 &= \partial_\mu \left(e (\mathrm{Im}\,\mathcal{N})_{IJ} F^{J\mu\nu} - \frac{1}{2} (\mathrm{Re}\,\mathcal{N})_{IJ} \epsilon^{\mu\nu\rho\sigma} F^J_{\rho\sigma} \right) \ .
\end{aligned} \tag{3.9}$$

The supersymmetry transformations are given in Appendix B.

3.2 Line Defect Solution

In this section, we give a general solution describing a 1/2-BPS line defect in four-dimensional $N = 2$ gauged supergravity and construct the solution for three specific choices of the prepotential.

3.2.1 Holographic Line Defects

A conformal line defect in three dimensions is a codimension-two defect which breaks the three-dimensional conformal group $\mathrm{SO}(3,2)$ down to an $\mathrm{SO}(2,1) \times \mathrm{SO}(2)$ subgroup. The subgroup factors represent the unbroken conformal symmetry along the defect and transverse rotations about the defect, respectively. Minkowski space $\mathbb{R}^{1,2}$ is related by a Weyl transformation to $\mathrm{AdS}_2 \times S^1$, namely

$$- \mathrm{d}t^2 + \mathrm{d}r^2 + r^2\, \mathrm{d}\phi^2 = \Omega(r)^2 \left(\frac{- \mathrm{d}t^2 + \mathrm{d}r^2}{r^2} + \mathrm{d}\phi^2 \right) . \tag{3.10}$$

Hence in the holographic dual, the $\mathrm{SO}(2,1) \times \mathrm{SO}(2)$ symmetry can be realized as the isometries of $\mathrm{AdS}_2 \times S^1$, which we choose as the boundary of the four-dimensional asymptotically anti-de Sitter space. Therefore we consider a metric ansatz with $\mathrm{AdS}_2 \times S^1$ warped over a radial coordinate. We note that the location of the defect at $r = 0$ in Minkowski space gets mapped to the boundary of AdS_2 in the $\mathrm{AdS}_2 \times S^1$ geometry. Secondly, the absence of a conical singularity on the boundary fixes the periodicity of the angle ϕ to be 2π.

The superconformal algebras in three dimensions are $\mathrm{OSp}(\mathcal{N}|4)$ where $\mathcal{N} = 1, 2, \ldots, 6, 8$. For the CFT dual of four-dimensional $N = 2$ gauged supergravity, the relevant superalgebra is $\mathrm{OSp}(2|4)$ which has four Poincaré and four conformal supercharges. A conformal line defect is called superconformal if it preserves some supersymmetry. In the present chapter, we will consider 1/2-BPS defects which preserve an $\mathrm{OSp}(2|2)$ superalgebra and hence four of the eight supersymmetries.

3.2.2 General Solution

Four-dimensional $N = 2$, U(1) FI gauged supergravity admits 1/2-BPS black hole solutions first found in [99]. The line defect solutions with $\mathrm{AdS}_2 \times S^1$ geometry are constructed by a double analytic continuation of these black hole solution. The metric and gauge fields are given by

$$\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H(r)}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f(r)}{\sqrt{H(r)}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H(r)}}{f(r)}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + 8g^2r^2H(r) \ , \\
H(r)^{1/4} &= \frac{1}{\sqrt{2}}e^{\mathcal{K}/2}Z^I H_I(r) \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0, 1, \ldots, n \ , \\
A^I &= \left(-2H(r)^{-1/4}e^{\mathcal{K}/2}Z^I + \mu^I\right)\mathrm{d}\theta \ , \qquad I = 0, 1, \ldots, n \ ,
\end{aligned} \tag{3.11}$$

for some real constants q_I and μ_I, where $Z^I = \bar{Z}^I$. Given a prepotential $\mathcal{F}(Z)$ and choice of parameterization of the symplectic sections $Z^I(\tau)$, the scalars τ^α are given implicitly by the equation

$$iH^{1/4}e^{\mathcal{K}/2}(\mathcal{F}_I - \bar{\mathcal{F}}_I) = \frac{1}{\sqrt{2}}H_I \ . \tag{3.12}$$

At the conformal boundary where $r \to \infty$, in order to have asymptotic AdS_4 we need $2\sqrt{2}g\theta$ to be 2π-periodic, i.e. $\theta \sim \theta + \pi/\sqrt{2}g$. The AdS_4 length scale is then given by

$$L^{-2} = 8g^2H(r = \infty)^{1/2} \ . \tag{3.13}$$

We will set $8g^2 = 1$ to obtain the usual S^1 periodicity $\theta \sim \theta + 2\pi$.

The center of the space $r = r_+$ corresponds to the largest value of r where $f(r) = 0$. In the black hole geometry, this previously corresponded to the location of the horizon. We consider radii taking values in the range $r \in [r_+, \infty)$. Demanding a regular geometry requires $r_+ > 0$ and the absence of a conical singularity, both of which can be accomplished by tuning the q_I and ξ_I parameters. This is explored in further detail in Section 3.4.

3.2.3 Examples

For a general prepotential, equation (3.12) is complicated and can only be solved numerically. Consequently, we work out the line defect solution for three specific prepotentials for which we can find explicit expressions for the scalars. An important requirement is the existence of an AdS_4 vacuum, which not all prepotentials admit, see e.g. [100, 102].

As our first model, consider a single ($n = 1$) vector multiplet with the prepotential $\mathcal{F}(Z) = -iZ^0 Z^1$. This theory has a single complex scalar τ and the scalar manifold is SU(1,1)/U(1). Using the parameterization $(Z^0, Z^1) = (1, \tau)$, we can calculate the Kähler potential, kinetic matrix, and scalar potential

$$
\begin{aligned}
e^{\mathcal{K}(\tau,\bar{\tau})} &= \frac{1}{2(\tau+\bar{\tau})} \ , \\
\mathcal{N}(\tau,\bar{\tau}) &= -i \begin{pmatrix} \tau & 0 \\ 0 & 1/\tau \end{pmatrix} \ , \\
V(\tau,\bar{\tau}) &= -\frac{1}{2(\tau+\bar{\tau})}\left(\xi_0^2 + 2\xi_0\xi_1(\tau+\bar{\tau}) + \xi_1^2 \tau\bar{\tau}\right) \ . \qquad (3.14)
\end{aligned}
$$

The potential has extrema at $\tau = \pm\xi_0/\xi_1$, but only $\tau = \xi_0/\xi_1$ maintains $e^{\mathcal{K}} > 0$ for $\xi_I > 0$.

The cosmological constant at this extremum gives the AdS_4 length scale

$$L^{-2} = \frac{1}{2}\xi_0\xi_1. \tag{3.15}$$

We choose $\xi_1 = 2/\xi_0$ to set the AdS_4 length scale to unity. The line defect solution (3.11) has the explicit form

$$\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + r^2 H(r) \ , \\
\sqrt{H(r)} &= \frac{1}{2}H_0H_1 \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0,1 \ , \\
A^I &= \left(-\frac{\sqrt{2}}{H_I} + \mu^I\right)\mathrm{d}\theta \ , \qquad I = 0,1 \ .
\end{aligned} \tag{3.16}$$

The scalar is given by

$$\tau = \frac{H_0}{H_1} \ . \tag{3.17}$$

We have verified that the above fields obey the equations of motion (3.9).

For the second model, we choose the STU model given by considering $n = 3$ vector multiplets with the prepotential

$$\mathcal{F}(Z) = -2i\sqrt{Z^0Z^1Z^2Z^3}. \tag{3.18}$$

This theory has three complex scalars τ^1, τ^2, τ^3 and the scalar manifold is three copies of SU(1,1)/U(1). When all $\xi_I = \xi > 0$ are equal, this theory is a consistent truncation of $N = 8$ gauged supergravity [103, 104]. For reference on this model, see [105]. Using the parameterization $(Z^0, Z^1, Z^2, Z^3) = (1, \tau^2\tau^3, \tau^1\tau^3, \tau^1\tau^2)$, the Kähler potential is

$$e^{\mathcal{K}(\tau,\bar{\tau})} = \frac{1}{(\tau^1 + \bar{\tau}^1)(\tau^2 + \bar{\tau}^2)(\tau^3 + \bar{\tau}^3)} \ . \tag{3.19}$$

The expressions for the kinetic matrix and scalar potential are complicated, but simplify for real scalars $\tau^\alpha = \bar{\tau}^{\bar{\alpha}}$, which will be the case for the line defect solution

$$\begin{aligned} \mathcal{N}(\tau, \bar{\tau} = \tau) &= -i \,\mathrm{diag}\left(\tau^1\tau^2\tau^3, \frac{\tau^1}{\tau^2\tau^3}, \frac{\tau^2}{\tau^1\tau^3}, \frac{\tau^3}{\tau^1\tau^2}\right) \ , \\ V(\tau, \bar{\tau} = \tau) &= -\frac{1}{2}\left(\xi_0\left(\frac{\xi_1}{\tau^1} + \frac{\xi_2}{\tau^2} + \frac{\xi_3}{\tau^3}\right) + \left(\tau^1\xi_2\xi_3 + \xi_1\tau^2\xi_3 + \xi_1\xi_2\tau^3\right)\right) \ . \end{aligned} \tag{3.20}$$

The potential has extrema at

$$\tau^1 = \pm\sqrt{\frac{\xi_0\xi_1}{\xi_2\xi_3}} \ , \qquad \tau^2 = \pm\sqrt{\frac{\xi_0\xi_2}{\xi_1\xi_3}} \ , \qquad \tau^3 = \pm\sqrt{\frac{\xi_0\xi_3}{\xi_1\xi_2}} \ . \tag{3.21}$$

Positivity of $e^{\mathcal{K}}$ requires us to choose the positive root. The cosmological constant at this extremum gives the AdS_4 length scale

$$L^{-2} = \sqrt{\xi_0\xi_1\xi_2\xi_3} \ . \tag{3.22}$$

We choose the non-zero constants ξ_I in a way that sets the AdS_4 length scale to unity. The line defect solution (3.11) has the explicit form

$$
\begin{aligned}
\mathrm{d}s^2 &= r^2\sqrt{H}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ , \\
f(r) &= -1 + r^2 H(r) \ , \\
H(r) &= H_0 H_1 H_2 H_3 \ , \\
H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0,1,2,3 \ , \\
A^I &= \left(-\frac{1}{\sqrt{2}H_I} + \mu^I\right)\mathrm{d}\theta \ , \qquad I = 0,1,2,3 \ .
\end{aligned} \tag{3.23}
$$

The scalars are

$$
\tau^1 = \sqrt{\frac{H_0 H_1}{H_2 H_3}} \ , \qquad \tau^2 = \sqrt{\frac{H_0 H_2}{H_1 H_3}} \ , \qquad \tau^3 = \sqrt{\frac{H_0 H_3}{H_1 H_2}} \ . \tag{3.24}
$$

This solution is also the double analytic continuation of the hyperbolic black hole solution in [106]. As consistency checks, we have verified that the above solution obeys the equations of motion (3.9) and is 1/2-BPS. The latter was done by a direct calculation, independent of [99], which can be found in Appendix B.

Another model which admits an AdS_4 vacuum has the prepotential $\mathcal{F}(Z) = \frac{i}{4} Z^I \eta_{IJ} Z^J$ and can be formulated with any number of vector multiplets. η_{IJ} is the Minkowski metric which we choose to be $\eta_{IJ} = \mathrm{diag}(-1, 1, \ldots, 1)$. The scalar manifold of this theory is SU(1,n)/U(1)×SU(n). Using the parameterization $(Z^0, Z^\alpha) = (1, \tau^\alpha)$, the Kähler potential is

$$
e^{\mathcal{K}(\tau,\bar{\tau})} = \frac{1}{1 - \sum_\alpha \tau^\alpha \bar{\tau}^\alpha} \ . \tag{3.25}
$$

Similarly, the kinetic matrix and scalar potential have simpler forms for real scalars $\tau^\alpha = \bar{\tau}^{\bar{\alpha}}$.

The matrix η_{IJ} is used to lower indices, e.g. $Z_I = \eta_{IJ} Z^J$, and

$$\begin{aligned} \mathcal{N}_{IJ}(\tau, \bar{\tau} = \tau) &= -\frac{i}{2}\eta_{IJ} - i e^{\mathcal{K}(\tau,\tau)} Z_I Z_J \ , \\ V(\tau, \bar{\tau} = \tau) &= \frac{1}{2}\xi_I \eta^{IJ} \xi_J - \frac{(\xi_0 + \sum_\alpha \xi_\alpha \tau^\alpha)^2}{1 - \sum_\alpha (\tau^\alpha)^2} \ . \end{aligned} \tag{3.26}$$

This potential has an extremum at $\tau^\alpha = -\xi_\alpha/\xi_0$. The other extrema at $\xi_0 + \sum_\alpha \xi_\alpha \tau^\alpha = 0$ do not admit AdS_4 vacua while maintaining $e^{\mathcal{K}}$ positive. The cosmological constant at this extremum gives us the AdS_4 length scale

$$L^{-2} = -\xi^2/2 \ , \tag{3.27}$$

where $\xi^2 = \xi_I \eta^{IJ} \xi_J$. We choose a time-like ξ_I with $\xi^2 = -2$ that will set the AdS_4 length scale to unity. The line defect solution (3.11) has the explicit form

$$\begin{aligned} \mathrm{d}s^2 &= r^2 \sqrt{H}\, \mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f}{\sqrt{H}}\, \mathrm{d}s^2_{S^1} + \frac{\sqrt{H}}{f}\, \mathrm{d}r^2 \ , \\ f(r) &= -1 + r^2 H(r) \ , \\ \sqrt{H(r)} &= -\frac{1}{2} H_I \eta^{IJ} H_J \ , \\ H_I(r) &= \xi_I + \frac{q_I}{r} \ , \qquad I = 0, 1, \ldots, n \ , \\ A^I &= \left(\frac{\sqrt{2}\eta^{IJ} H_J}{\sqrt{H}} + \mu^I \right) \mathrm{d}\theta \ , \qquad I = 0, 1, \ldots, n \ . \end{aligned} \tag{3.28}$$

The scalars are

$$\tau^\alpha = -\frac{H_\alpha}{H_0} \ . \tag{3.29}$$

We have verified that the above fields obey the equations of motion (3.9).

3.3 Holographic Calculations

In this section, we use the machinery of holographic renormalization [38, 39] to calculate the on-shell action and one-point functions of CFT operators in the presence of the defect, namely the stress tensor, scalars, and currents. This is done explicitly for the three examples in Section 3.2.

3.3.1 General Procedure

First, we put the metric into the Fefferman-Graham (FG) form

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(\mathrm{d}z^2 + g_{ij}(x,z)\,\mathrm{d}x^i\,\mathrm{d}x^j\right) , \tag{3.30}$$

where $i, j = 1, 2, 3$ run over the AdS_2 and S^1 indices and $z \to 0$ is the conformal boundary. This is done by taking $z = z(r)$ so that the appropriate coordinate change is obtained by the solution to the ordinary differential equation

$$-\frac{H(r)^{1/4}}{f(r)^{1/2}}\,\mathrm{d}r = \frac{\mathrm{d}z}{z} , \tag{3.31}$$

which can be integrated perturbatively in $1/r$. This coordinate change gives the FG expansions of the fields, which we assume take the form

$$\begin{aligned}
g_{ij} &= g_{0ij} + z^2 g_{2ij} + z^3 g_{3ij} + O(z^4) , \\
A^I &= A^I_0 + zA^I_1 + O(z^2) , \\
\tau^\alpha &= \tau^\alpha_0 + z\tau^\alpha_1 + z^2\tau^\alpha_2 + O(z^3) , \\
\bar{\tau}^{\bar{\alpha}} &= \tau^\alpha_0 + z\tau^\alpha_1 + z^2\tau^\alpha_2 + O(z^3) ,
\end{aligned} \tag{3.32}$$

where A_0^I and A_1^I are one-forms on the x^1, x^2, x^3 coordinates. The constants τ_0^α are the AdS_4 vacuum values of the scalars, which depend on the model. There is no gravitational conformal anomaly (i.e. a term proportional to $z^3 \log z$ in the expansion of g_{ij}) since $d = 3$ is odd.

In the three-dimensional boundary CFT, the conformal dimensions of the dual operators corresponding to the scalars τ^α and vector fields A^I are determined by the linearized bulk equations of motion near the AdS boundary. For instance, using the expansion $\tau^\alpha \sim \tau_0^\alpha + z^{\Delta_\tau}$ in the linearized equation of motion for the scalar, we find that the scaling dimension of the dual operator is related to the squared mass of the field by the equation

$$\Delta_\tau(\Delta_\tau - 3) = -2 \ . \tag{3.33}$$

The squared mass is -2 for all scalars of the three examples considered in this chapter. This mass is within the window where both standard and alternative quantization are possible [107], which implies that the scaling dimension of the dual operator can be either $\Delta_\tau = 1$ or $\Delta_\tau = 2$. Similarly, using the expansion $A^I \sim z^{\Delta_A - 1}\, d\theta$ in the linearized equation of motion for the vector field gives us

$$(\Delta_A - 1)(\Delta_A - 2) = 0 \ . \tag{3.34}$$

We must have $\Delta_A = 2$ as the vector field sources a conserved current of the boundary CFT.

These scaling dimensions naturally fit into the flavor current multiplet $A_2\overline{A}_2[0]_1^{(0)}$ of the $d = 3$, $\mathcal{N} = 2$ boundary CFT, using the notation of [108]. This short multiplet contains, in addition to the spin-1 operator $[2]_2^{(0)}$ with scaling dimension $\Delta = 2$, two scalar operators $[0]_1^{(0)}$ and $[0]_2^{(0)}$ as bottom and top components with scaling dimensions $\Delta = 1$ and $\Delta = 2$, respectively. The stress tensor multiplet $A_1\overline{A}_1[2]_2^{(0)}$ is also present as usual.

For a well-defined variational principle of the metric in the four-dimensional gauged supergravity, we must add the Gibbons-Hawking-York boundary term

$$\begin{aligned} I_{\rm bulk} &= \int_M \mathrm{d}^4x\, \mathcal{L}_{\rm bos} \ , \\ I_{\rm GH} &= \int_{\partial M} \mathrm{d}^3x \sqrt{-h}\, \mathrm{Tr}(h^{-1}K) \ , \end{aligned} \tag{3.35}$$

to the bulk action where h_{ij} is the induced metric on the boundary and K_{ij} is the extrinsic curvature. In FG coordinates, these take the form

$$h_{ij} = \frac{1}{z^2} g_{ij} \ , \qquad\qquad K_{ij} = -\frac{z}{2}\partial_z h_{ij} \ . \tag{3.36}$$

The action $I_{\rm bulk} + I_{\rm GH}$ diverges due to the infinite volume of integration. To regulate the theory, we restrict the bulk integral to the region $z \geq \epsilon$ and evaluate the boundary term at $z = \epsilon$. Divergences in the action then appear as poles $1/\epsilon^k$. Counterterms $I_{\rm ct}$ are added on the boundary which subtract these divergent terms. The counterterms have been constructed in [105] and are compatible with supersymmetry. They are

$$I_{\rm ct} = \int_{\partial M} \mathrm{d}^3x \sqrt{-h} \left(\mathcal{W} - \frac{1}{2} R[h] \right) \ , \qquad\qquad \mathcal{W} \equiv -\sqrt{2} e^{\mathcal{K}/2} \big| \xi_I Z^I \big| \ , \tag{3.37}$$

where $R[h]$ is the Ricci scalar of the boundary metric and $\mathcal{W}$ is the superpotential. In all, the renormalized action

$$I_{\rm ren} = I_{\rm bulk} + I_{\rm GH} + I_{\rm ct} \ , \tag{3.38}$$

is finite. We can now perform functional derivatives to obtain finite expectation values of the dual CFT operators. Let T_{ij} be the boundary stress tensor, $\mathcal{O}_\alpha$ be the operators dual to τ^α, and J_{Ii} be the current operators dual to A^I_μ.

The expectation value of the boundary stress tensor is defined to be [109]

$$\langle T_{ij} \rangle \equiv \frac{-2}{\sqrt{-g_0}} \frac{\delta I_{\rm ren}}{\delta g_0^{ij}} \ . \tag{3.39}$$

The variation decomposes into two contributions: one coming from the regularized action and one coming from the counterterms. As usual [110], the former is given by

$$T^{\rm reg}_{ij}[h] \equiv \frac{-2}{\sqrt{-h}} \frac{\delta(I_{\rm bulk} + I_{\rm GH})}{\delta h^{ij}} = -K_{ij} + h_{ij} {\rm Tr}(h^{-1}K) \ . \tag{3.40}$$

The latter is straightforward to compute and is given by

$$T^{\rm ct}_{ij}[h] \equiv \frac{-2}{\sqrt{-h}} \frac{\delta I_{\rm ct}}{\delta h^{ij}} = h_{ij} \left(\mathcal{W} - \frac{1}{2} R[h] \right) + R_{ij}[h] \ . \tag{3.41}$$

Therefore,

$$\langle T_{ij} \rangle = \lim_{\epsilon \to 0} \left[\epsilon^{-1} \left(T^{\rm reg}_{ij}[h] + T^{\rm ct}_{ij}[h] \right) \Big|_{z=\epsilon} \right] \ . \tag{3.42}$$

By construction of the counterterms, this limit is finite.

The expectation value of the operator $\mathcal{O}_\alpha$ is similarly defined by

$$\langle \mathcal{O}_\alpha \rangle \equiv \frac{1}{\sqrt{-g_0}} \frac{\delta I_{\rm ren}}{\delta \tau^\alpha_1} = \lim_{\epsilon \to 0} \left[\epsilon^{-2} \ \frac{1}{\sqrt{-h}} \frac{\delta I_{\rm ren}}{\delta \tau^\alpha} \Big|_{z=\epsilon} \right] \ . \tag{3.43}$$

The variation has contributions from the bulk action and the counterterms, and is

$$\frac{1}{\sqrt{-h}}\frac{\delta I_{\text{ren}}}{\delta\tau^{\alpha}} = g_{\alpha\bar{\beta}} z\partial_z \bar{\tau}^{\bar{\beta}} + \partial_\alpha \mathcal{W} \ . \tag{3.44}$$

For real scalars, supersymmetry implies $\langle \mathcal{O}_\alpha \rangle = 0$. A proof of this statement can be found in Appendix C.

The expectation value of the current operator J_I is defined by

$$\left\langle J_I^i \right\rangle \equiv \frac{1}{\sqrt{-g_0}}\frac{\delta I_{\text{ren}}}{\delta A_{0i}^I} = \lim_{\epsilon\to 0}\left[\epsilon^{-3}\,\frac{1}{\sqrt{-h}}\frac{\delta I_{\text{ren}}}{\delta A_i^I}\bigg|_{z=\epsilon}\right] . \tag{3.45}$$

The only contribution to the variation comes from the bulk action and is

$$\frac{1}{\sqrt{-h}}\frac{\delta I_{\text{ren}}}{\delta A_i^I} = -(\text{Im}\,\mathcal{N})_{IJ} h^{ij} z\partial_z A_j^J \ . \tag{3.46}$$

We can evaluate the on-shell action for the line defect solution by further simplifying the bulk action to a total derivative [111]

$$I_{\text{bulk}}\bigg|_{\text{on-shell}} = \text{Vol}(\text{AdS}_2)\text{Vol}(S^1)\left[-\frac{H'(r)}{4H(r)}r^2 f(r) - r\big(f(r)+1\big)\right]\bigg|_{r_+}^{\infty} , \tag{3.47}$$

where $\text{Vol}(S^1) = 2\pi$ and $\text{Vol}(\text{AdS}_2) = -2\pi$ is the regularized volume of AdS_2.

3.3.2 Examples

In this section, we use the general expressions derived in Section 3.3.1 to compute observables for the three examples considered in this chapter. Let us consider the defect solution (3.16, 3.17) of the single scalar model. The FG expansion of the radial coordinate r found from solving the ordinary differential equation (3.31) is

$$\begin{aligned}\frac{1}{r} &= z + \frac{1}{2}\left(\sum_{I=0}^{1}\frac{q_I}{\xi_I}\right)z^2 + \frac{-16+(3q_1\xi_0+q_0\xi_1)(3q_0\xi_1+q_1\xi_0)}{64}z^3 \\ &\quad + \frac{(q_1\xi_0+q_0\xi_1)(-16+12q_0q_1\xi_0\xi_1+3(q_0\xi_1+q_1\xi_0)^2)}{384}z^4 + O(z^5)\ .\end{aligned} \tag{3.48}$$

Using this coordinate change, the metric, gauge fields, and scalar can be expanded in FG coordinates. The one-point functions in the presence of the line defect can then be evaluated by computing the limits (3.42, 3.43, 3.45) directly. For the renormalized on-shell action (3.38), the finite terms at the conformal boundary cancel, leaving just the term obtained by evaluating (3.47) at $r = r_+$. In the end, we obtain the following expectation values

$$\begin{aligned} I_{\text{ren}} &= \text{Vol}(\text{AdS}_2)\text{Vol}(S^1)r_+\ , \\ \langle T_{ij}\rangle &= \frac{1}{2}\left(\sum_{I=0}^{1}\frac{q_I}{\xi_I}\right)\begin{pmatrix}-g_{\text{AdS}_2} & 0\\ 0 & 2g_{S^1}\end{pmatrix}_{ij}\ , \\ \langle T_i^i\rangle &= 0\ , \\ \langle \mathcal{O}\rangle &= 0\ , \\ \langle J_{Ii}\rangle &= \frac{q_I}{\sqrt{2}}\delta_{i\theta}\ . \end{aligned} \tag{3.49}$$

Next we consider the STU model defect (3.23, 3.24) for which some of the calculations are identical to those found in [106]. The FG expansion of the radial coordinate r obtained from solving the ordinary differential equation (3.31) is

$$\frac{1}{r} = z + \frac{A}{4}z^2 + \frac{-16+B_1+10B_2}{64}z^3 + \frac{-16A+C_1+11C_2+62C_3}{384}z^4 + O(z^5)\ , \tag{3.50}$$

where we have defined the constants

$$A = \sum_{I=0}^{3} \frac{q_I}{\xi_I} \ , \qquad B_1 = \sum_{I=0}^{3} \left(\frac{q_I}{\xi_I}\right)^2 \ , \qquad B_2 = \sum_{I<J} \frac{q_I q_J}{\xi_I \xi_J} \ ,$$
$$C_1 = \sum_{I=0}^{3} \left(\frac{q_I}{\xi_I}\right)^3 \ , \qquad C_2 = \sum_{I\neq J} \left(\frac{q_I}{\xi_I}\right)^2 \frac{q_J}{\xi_J} \ , \qquad C_3 = \sum_{I<J<K} \frac{q_I q_J q_K}{\xi_I \xi_J \xi_K} \ . \tag{3.51}$$

Using this coordinate change, the fields of the defect solution can be expanded in FG coordinates. We obtain the following on-shell action and one-point functions

$$\begin{aligned} I_{\text{ren}} &= \text{Vol}(\text{AdS}_2)\text{Vol}(S^1) r_+ \ , \\ \langle T_{ij} \rangle &= \frac{1}{4} \left(\sum_{I=0}^{3} \frac{q_I}{\xi_I} \right) \begin{pmatrix} -g_{\text{AdS}_2} & 0 \\ 0 & 2 g_{S^1} \end{pmatrix}_{ij} \ , \\ \langle T^i_i \rangle &= 0 \ , \\ \langle \mathcal{O}_1 \rangle = \langle \mathcal{O}_2 \rangle = \langle \mathcal{O}_3 \rangle &= 0 \ , \\ \langle J_{Ii} \rangle &= \frac{q_I}{\sqrt{2}} \delta_{i\theta} \ . \end{aligned} \tag{3.52}$$

Note that the expression for I_{ren} is identical to that of the single scalar model, but the radius $r_+ = r_+(\xi_I, q_I)$ will be different.

For the SU(1,n) coset defect solution (3.28, 3.29), the FG expansion of the radial coordinate r is

$$\begin{aligned} \frac{1}{r} = z &- \frac{1}{2} q_I \xi^I z^2 - \frac{1}{4} \left[1 + \frac{1}{2} q_I q^I - \frac{3}{4} (q_I \xi^I)^2 \right] z^3 \\ &+ \frac{1}{12} q_I \xi^I \left[1 + \frac{3}{2} q_I q^I - \frac{3}{4} (q_I \xi^I)^2 \right] z^4 + O(z^5) \ , \end{aligned} \tag{3.53}$$

where η^{IJ} is used to raise the indices of ξ_I and q_I. Using this coordinate change and expanding the fields in FG coordinates, the on-shell action and one-point functions are

$$
\begin{aligned}
I_{\text{ren}} &= \text{Vol}(\text{AdS}_2)\text{Vol}(S^1)r_+ \ , \\
\langle T_{ij} \rangle &= -\frac{q_I \xi^I}{2} \begin{pmatrix} -g_{\text{AdS}_2} & 0 \\ 0 & 2g_{S^1} \end{pmatrix}_{ij} \ , \\
\left\langle T^i_i \right\rangle &= 0 \ , \\
\langle \mathcal{O}_\alpha \rangle &= 0 \ , \\
\langle J_{Ii} \rangle &= \frac{q_I}{\sqrt{2}} \delta_{i\theta} \ .
\end{aligned}
\tag{3.54}
$$

The expression for I_{ren} is again unchanged, but the functional form of $r_+ = r_+ (\xi_I, q_I)$ changes.

3.4 Regularity

In this section, we impose two regularity conditions on the solutions. First, we demand that the geometry smoothly closes off at the largest positive zero of $f(r)$ without a conical singularity in the bulk spacetime. This condition is analogous to the regularity condition imposed on Euclidean black hole solutions. Second, we fix the periodicity of the S^1 at the conformal boundary such that when the $\text{AdS}_2 \times S^1$ boundary is conformally mapped to $\mathbb{R}^{1,2}$ there is no conical deficit on the boundary. This condition is different from the one imposed in the holographic calculation of supersymmetric Rényi entropies [112–115], which use solutions that are related by double analytic continuation. For these solutions, the periodicity is related to the Rényi index n.

The regularity conditions will impose constraints on the parameters of the solutions. Since the general solution is only implicit, a detailed analysis is performed for the examples presented in this chapter. We will show that for the single scalar and coset models, these conditions imply a bound on the expectation value of the boundary stress tensor.

3.4.1 General Statements

Given the metric

$$\mathrm{d}s^2 = r^2\sqrt{H(r)}\,\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{f(r)}{\sqrt{H(r)}}\,\mathrm{d}s^2_{S^1} + \frac{\sqrt{H(r)}}{f(r)}\,\mathrm{d}r^2 \ , \tag{3.55}$$

the center of the space $r = r_+$ is defined to be the largest zero of $f(r) = r^2 H(r) - 1$. We can identify four criteria a regular geometry should satisfy:

(a) Positivity of the root: $r_+ > 0$,

(b) $0 < H(r) < \infty$ for $r \in [r_+, \infty)$,

(c) $0 < f(r) < \infty$ for $r \in (r_+, \infty)$,

(d) Absence of a conical singularity at $r = r_+$.

Criteria (b) and (c) are satisfied if $H(r)$ is continuous: the AdS length scale (3.13) is well-defined if and only if the limit $H(r = \infty)$ is positive and finite. Since a zero of $H(r)$ occurs at $f(r) < 0$, positivity of $H(r)$ at large r and continuity imply that the spacetime closes off before a zero of $H(r)$ is ever encountered.

By expanding the metric around the center of the space, criterion (d) is satisfied when

$$f'(r_+)^2 = 4H(r_+) \ . \tag{3.56}$$

This can be simplified to

$$H'(r_+)(r_+^2 f'(r_+) + 2r_+) = 0 \ . \tag{3.57}$$

As the second factor is the sum of two positive quantities, a conical singularity can be avoided if we satisfy the condition $H'(r_+) = 0$. As r_+ is determined implicitly in terms of q_I and ξ_I through the equation $f(r_+) = 0$, this condition can be viewed as a constraint on the possible values q_I and ξ_I can take. Additionally, we will see that criterion (a) manifests as an inequality on q_I and ξ_I that must be satisfied.

3.4.2 Single Scalar Model

The single scalar model is simple enough that the conditions for a regular geometry can be solved exactly. Let us define $x_I \equiv q_I/\xi_I$, but still set the AdS length scale to be unity, i.e. keep $\xi_0\xi_1 = 2$. The metric functions become

$$\begin{aligned} H(r) &= \Big(1 + \frac{x_0}{r}\Big)^2 \Big(1 + \frac{x_1}{r}\Big)^2 \ , \\ f(r) &= -1 + \frac{1}{r^2}(r + x_0)^2(r + x_1)^2 \ . \end{aligned} \tag{3.58}$$

Let us first satisfy the criterion $r_+ > 0$. Solving $f(r) = 0$, gives

$$0 = \big(r^2 + r(x_0 + x_1 - 1) + x_0x_1\big)\big(r^2 + r(x_0 + x_1 + 1) + x_0x_1\big) \ . \tag{3.59}$$

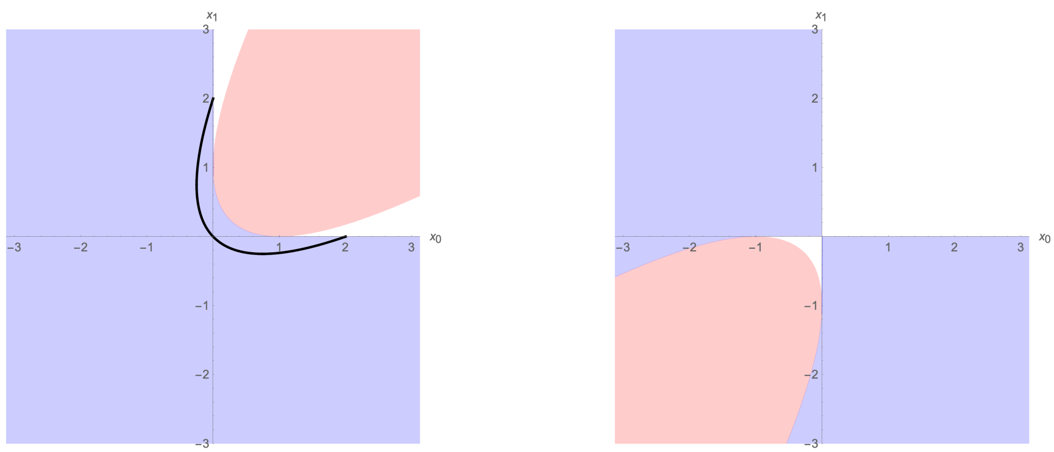
When the first factor is zero, we have a solution

$$r_1 = \frac{1}{2}\Big(-(x_0 + x_1 - 1) + \sqrt{(x_0 + x_1 - 1)^2 - 4x_0x_1}\Big) \ , \tag{3.60}$$

where we choose the positive branch of the square root. This solution exists whenever $(x_0 + x_1 - 1)^2 - 4x_0x_1 \geq 0$, which is a region in the x_0x_1-plane bounded by a parabola as shown in Figure 3.1a. The red shaded region indicates where r_1 does not exist and the blue shaded region indicates where $r_1 > 0$. When the second factor of (3.59) is zero, we have another solution

$$r_2 = \frac{1}{2}\Big(-(x_0 + x_1 + 1) + \sqrt{(x_0 + x_1 + 1)^2 - 4x_0x_1}\Big) , \qquad (3.61)$$

where we again choose the positive branch of the square root. We have also displayed the regions where this solution exists and is positive in Figure 3.1b. In regions where r_1 and r_2 both exist and $r_1 > 0$, we have $r_1 > r_2$. Therefore, we have $r_+ = r_1$ and restrict the (x_0, x_1) parameter space to the blue shaded region of Figure 3.1a.



(a) r_1 (b) r_2

Figure 3.1: Potential r_+ values for the single scalar model.

Let us now avoid the conical singularity by satisfying $H'(r_+) = 0$. Calculating the derivative of $H(r)$ in (3.58) and plugging in $r_+ = r_1$ from (3.60), we obtain the condition

$$0 = (x_0 - x_1)^2 - 2(x_0 + x_1) \ . \tag{3.62}$$

This is a parabola indicated by the black curve in Figure 3.1a in the region where $r_+ > 0$. For the single scalar model to admit a regular geometry, the parameters $x_I = q_I/\xi_I$ must satisfy this condition. As a corollary, we can note that

$$0 \leq x_0 + x_1 < 2 \ . \tag{3.63}$$

This implies that the components of the boundary stress tensor (3.49) have bounded expectation value. Additionally, the pure AdS$_4$ vacuum ($x_0 = x_1 = 0$) is the only solution with regular geometry and $\langle T_{ij} \rangle = 0$.

3.4.3 SU(1, n) Coset Model

The coset model is also simple enough that the conditions for a regular geometry can be solved exactly. We note that

$$H(r) = \left(1 - \frac{q_I \xi^I}{r} - \frac{q_I q^I}{2r^2}\right)^2 , \tag{3.64}$$

has the same form as (3.58) where

$$x_0 = \frac{-q_I\xi^I - \sqrt{(q_I\xi^I)^2 + 2q_I q^I}}{2} \ , \qquad x_1 = \frac{-q_I\xi^I + \sqrt{(q_I\xi^I)^2 + 2q_I q^I}}{2} \ . \tag{3.65}$$

This map is always well-defined as $(q_I\xi^I)^2 + 2q_Iq^I \geq 0$, which can be checked by rotating to the frame where $\xi_I = (\sqrt{2}, 0, 0, \ldots, 0)$. Thus all our results for the single scalar model can be carried over. The bound (3.63) for the single scalar model translates to the same bound on $\langle T_{ij}\rangle$ for the coset model:

$$0 \leq -q_I \xi^I < 2 \ . \tag{3.66}$$

The condition (3.62) for a regular geometry translates to

$$0 = (q_I \xi^I)^2 + 2q_I q^I + 2q_I \xi^I \ . \tag{3.67}$$

We can show that the only regular geometry with vanishing $\langle T_{ij}\rangle$ is the AdS_4 vacuum. If we rotate to the frame where $\xi_I = (\sqrt{2}, 0, 0, \ldots, 0)$, the only q which satisfies $q_I \xi^I = 0$ and $q_I q^I = 0$ is $q_I = 0$. A general ξ then has a q in the orbit of $q_I = 0$, which is still the zero vector.

3.4.4 Gauged STU Model

For the gauged STU model, it is not practical to solve $f(r) = 0$ to find r_+ as f is a quartic polynomial. However, we still expect the criterion $r_+ > 0$ to impose an inequality on the four-dimensional parameter space (x_0, x_1, x_2, x_3) and the condition of avoiding a conical singularity to reduce this to a three-dimensional hypersurface. However, note that unlike the single scalar and coset models, the expectation value $\langle T_{ij}\rangle$ is unbounded for the STU model.

A construction of STU models with regular geometry and arbitrarily large $x_0+x_1+x_2+x_3$ can be found by an approach that is different from that of Section 3.4.2. Instead of solving the condition $f = 0$ and then $H' = 0$, we first solve $H' = 0$ and then $f = 0$. The advantage of this approach is that H' is a lower-degree polynomial that is simpler to solve. However, the downside is that this generates spurious solutions: it is possible that the value r we obtain is not the largest root r_+ and that r_+ does not satisfy the equation $H' = 0$. These spurious solutions then need to be removed by hand. To summarize the approach, consider the following construction:

1. Let x_0 be any positive number.

2. Numerically solve the equation

$$27x_1(x_0 - x_1)^4 = -16x_0(x_0 + 3x_1)^2 \ . \tag{3.68}$$

   Let x_1 be the unique solution satisfying $-x_0/3 < x_1 < 0$.

3. Consider an STU model with unit AdS_4 length scale where

$$x_0 = \frac{q_0}{\xi_0} \ , \qquad x_1 = \frac{q_1}{\xi_1} = \frac{q_2}{\xi_2} = \frac{q_3}{\xi_3} \ . \tag{3.69}$$

   Numerically solve the equation $f(r) = 0$ for r,

$$(r + x_0)(r + x_1)^3 = r^2 \ . \tag{3.70}$$

   There exist exactly two solutions: a positive solution greater than $-x_1$, and a negative solution less than $-x_0$. Let r_+ be the positive solution.

4. Check that $H'(r_+) = 0$. This is guaranteed by the following argument. Consider $r^* = -4x_0x_1/(x_0 + 3x_1) > 0$ which satisfies $H'(r^*) = 0$. This also satisfies $f(r^*) = 0$, as plugging $r = r^*$ into (3.70) simplifies to (3.68), which is satisfied by construction of x_1. But as the positive solution to $f = 0$ is unique, we must have $r_+ = r^*$.

The steps above give a STU model with regular geometry. To prove that x_0+3x_1 is arbitrarily large, we need a better bound than $-x_0/3 < x_1 < 0$. To satisfy (3.68) for large x_0, we have

$$x_1 \sim -\frac{16}{27x_0}. \tag{3.71}$$

Therefore $x_0+3x_1 \approx x_0$ and the expectation value of the stress tensor can be made arbitrarily large.

3.5 Discussion

In this chapter, we constructed solutions of four-dimensional $N = 2$ gauged supergravity by a double analytic continuation of the 1/2-BPS black hole solutions first found by Sabra [99]. While the black hole solutions exist for arbitrary prepotentials, explicit expressions for the scalars fields involve algebraic equations which can only be solved numerically in general. We considered three explicit examples of matter-coupled gauged supergravities, namely the single scalar model, the gauged STU model, and the $\mathrm{SU}(1,n)/\mathrm{U}(1) \times \mathrm{SU}(n)$ coset model to find solutions and calculate holographic observables.

The solutions we found are holographic duals to line defects in three-dimensional SCFTs. The defect is characterized by a non-trivial expectation value of the R-symmetry and flavor currents along the S^1 factor in the $\mathrm{AdS}_2 \times S^1$ description of the defect. After conformally mapping to Minkowski space, this corresponds to a holonomy when encircling the line defect. The expectation values of the real scalar operators vanish for general models as a consequence of supersymmetry.

For a conformal defect on $\mathrm{AdS}_2 \times S^1$, the expectation value of the stress tensor can be parameterized by a single coefficient h,

$$\langle T_{ab} \rangle = h \, g_{ab}^{\mathrm{AdS}_2} \ , \quad \langle T_{\theta\theta} \rangle = -2h \, g_{\theta\theta} \ , \tag{3.72}$$

in analogy to the scaling dimension of local operators [116, 117]. However, there are in general no unitarity bounds on h which follow from the superconformal algebra. For line operators in $\mathcal{N} = 4$ Super Yang-Mills and ABJM theories, h can be related to the so-called Bremsstrahlung function B [118–122] which has been used in the application of conformal bootstrap techniques to the study of defects [73, 123–125]. For the single scalar and coset models studied in this chapter, we found that $-2 < h \leq 0$, where the upper bound is saturated only by the AdS_4 vacuum. However, such a bound does not seem to generally hold, since for the gauged STU model, h can become arbitrarily negative. Based on numerical searches, we conjecture that only the AdS_4 vacuum has vanishing h. Recently the relation of h and B, as well as the negativity of h, has been established on the CFT side for various defect theories [126–129] and the arguments should carry over to defects dual to the solutions studied in this chapter.

The solutions we found are related to supergravity solutions [106, 113–115] which are holographic duals for a supersymmetric version of Rényi entropy first formulated in [112]. We note two differences. First, the solutions we found in Minkowski time signature have real gauge fields unlike the duals cited above. After analytic continuation to Euclidean signature, the gauge fields in both cases are real. Second, we imposed the condition that the periodicity of the circle in $\mathrm{AdS}_2 \times S^1$ boundary is such that after a conformal map we obtain flat space without a conical singularity. On the other hand, in the holographic duals to the Super-Rényi entropy, the conical singularity is related to the Rényi index n. We note that in [106, 113–115] the holographic calculation of the Rényi entropy was compared to a localization calculation and agreement was found. It would be interesting to see whether such a calculation can be performed for the holonomy defects described in this chapter.

Another interesting question is whether more general solutions going beyond the examples discussed in this chapter can be found. First, it would be interesting to study (numerical) solutions for more complicated prepotentials. Second, it would be interesting to see whether one can go beyond the gauged supergravity approximation and find solutions dual to holonomy defects in ten- or eleven-dimensional duals of $\mathcal{N} = 2$ SCFTs. Uplifting the solutions found in this chapter might prove to be a useful guide in this direction [104].

CHAPTER 4

Matter-Coupled $D = 5, N = 4$ Gauged Supergravity

$N = 4$ gauged supergravities in five dimensions have sixteen supersymmetries and their AdS_5 vacua can be used to describe four-dimensional $\mathcal{N} = 2$ SCFTs. The pure gauged supergravity was constructed in [130, 131], whereas the addition of matter multiplets and general gaugings were constructed in [132,133]. The AdS_5 vacua and moduli spaces for these theories were analyzed in [134]. Some recent papers studying solutions in these theories can be found in [135–139].

In the present chapter, we study $D = 5, N = 4$ gauged supergravity solutions which are dual to surface defects in the $\mathcal{N} = 2$ SCFTs. The structure of the chapter is as follows: In Section 4.1, we briefly review the pure $D = 5, N = 4$ gauged supergravity of Romans. We consider an ansatz for the defect solution of the form $AdS_3 \times S^1$ warped over an interval. Such an ansatz can be related to a charged black hole by double analytic continuation and it is shown that there is no global regular solution for the defect as a conical deficit or excess in either the bulk or boundary cannot be removed. In Section 4.2, we review the matter-coupled theory and its gaugings, and show that completely regular solutions can be constructed for this theory. In Section 4.3, we utilize these solutions to calculate holographic observables, namely the one-point functions of operators in the presence of the defect as well as the on-shell supergravity action which is related to the free energy in the presence of the defect. In Section 4.4, we summarize the results of this chapter. In Appendix D, we present details of the spin connection and the form of supersymmetry transformations used in the main part of the chapter. We also show that the solution in Section 4.2 preserves eight of the sixteen supersymmetries. In Appendix E, we present a solution corresponding to a line defect in the Euclidean $N = 4$ gauged supergravity.

4.1 Romans' Gauged $N = 4$ Supergravity

The field content of Romans' gauged supergravity [130, 131] is given by the $N = 4$ gauged supergravity multiplet

$$\left(e_\mu{}^r, \psi_{\mu a}, a_\mu, A^I_\mu, B^\alpha_{\mu\nu}, \chi_a, \phi\right), \tag{4.1}$$

which contains the graviton $e_\mu{}^r$, four gravitini $\psi_{\mu a}$, a U(1) gauge field a_μ, an SU(2) Yang-Mills gauge field A^I_μ, two antisymmetric tensor fields $B^\alpha_{\mu\nu}$, four spin-1/2 fermions χ_a, and a single scalar ϕ. In the above, indices $a, b = 1, 2, 3, 4$ are Spin(5) $\cong$ USp(4) indices; $I, J, K = 1, 2, 3$ are SU(2) adjoint indices; and $\alpha, \beta = 4, 5$ are SO(2) $\cong$ U(1) indices. All fermionic fields satisfy the symplectic Majorana condition. We review our conventions in Appendix D.

The bosonic Lagrangian is given by

$$\begin{aligned} e^{-1}\mathcal{L} = &-\frac{1}{4}R - \frac{1}{4}\xi^{-4} f^{\mu\nu} f_{\mu\nu} - \frac{1}{4}\xi^2 \left(F^{\mu\nu I} F^I_{\mu\nu} + B^{\mu\nu\alpha} B^\alpha_{\mu\nu}\right) + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi \\ &+ \frac{1}{4}e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}\left(\frac{1}{g_1}\epsilon_{\alpha\beta}B^\alpha_{\mu\nu}D_\rho B^\beta_{\sigma\tau} - F^I_{\mu\nu}F^I_{\rho\sigma}a_\tau\right) + V(\phi), \end{aligned} \tag{4.2}$$

where the field strengths and scalar potential take the form

$$
\begin{aligned}
f_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu, \\
F^I_{\mu\nu} &= \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + g_2 \epsilon^{IJK} A^J_\mu A^K_\nu, \\
V &= \frac{g_2}{8}\left(g_2 \xi^{-2} + 2\sqrt{2} g_1 \xi\right), \\
\xi &= \exp\left(\sqrt{\frac{2}{3}}\phi\right).
\end{aligned}
\tag{4.3}
$$

The Lagrangian (4.2) leads to the equations of motion

$$
\begin{aligned}
R_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi - \frac{4}{3}V(\phi)g_{\mu\nu} + \xi^{-4}\left(2 f_{\mu\rho} f_\nu{}^\rho - \frac{1}{3} g_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma}\right)& \\
+\xi^2\left(2 F^I_{\mu\rho} F^{I\ \rho}_\nu + 2 B^\alpha_{\mu\rho} B^{\alpha\ \rho}_\nu - \frac{1}{3} g_{\mu\nu}\left(F^I_{\rho\sigma} F^{I\rho\sigma} + B^\alpha_{\rho\sigma} B^{\rho\sigma\alpha}\right)\right) &= 0, \\
-\Box\phi + \frac{\partial V}{\partial\phi} + \sqrt{\frac{2}{3}}\xi^{-4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{\sqrt{6}}\left(F^I_{\mu\nu} F^{I\mu\nu} + B^\alpha_{\mu\nu} B^{\mu\nu\alpha}\right) &= 0, \\
D_\nu\left(\xi^{-4} f^{\nu\mu}\right) - \frac{1}{4} e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}\left(F^I_{\nu\rho} F^I_{\sigma\tau} + B^\alpha_{\nu\rho} B^\alpha_{\sigma\tau}\right) &= 0, \\
D_\nu\left(\xi^2 F^{\nu\mu I}\right) - \frac{1}{2} e^{-1}\epsilon^{\mu\nu\rho\sigma\tau} F^I_{\nu\rho} f_{\sigma\tau} &= 0, \\
e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}\epsilon^{\alpha\beta}\mathcal{D}_\rho B^\beta_{\sigma\tau} - g_1 \xi^2 B^{\alpha\mu\nu} &= 0,
\end{aligned}
\tag{4.4}
$$

where the covariant derivative acting on a vector representation is

$$
D_\mu V^{I\alpha} = \nabla_\mu V^{I\alpha} + g_1 a_\mu \epsilon^{\alpha\beta} V^{I\beta} + g_2 \epsilon^{IJK} A^J_\mu V^{K\alpha}. \tag{4.5}
$$

The supersymmetry transformation of the fermions are

$$
\begin{aligned}
\delta\psi_{\mu a} &= D_\mu \epsilon_a + \gamma_\mu T_{ab}\epsilon^b - \frac{1}{6\sqrt{2}}(\gamma_\mu{}^{\nu\rho} - 4\delta^\nu_\mu \gamma^\rho)\left(H_{\nu\rho ab} + \frac{1}{\sqrt{2}} h_{\nu\rho ab}\right)\epsilon^b, \\
\delta\chi_a &= \frac{1}{\sqrt{2}}\gamma^\mu \partial_\mu \phi \epsilon_a + A_{ab}\epsilon^b - \frac{1}{2\sqrt{6}}\gamma^{\mu\nu}(H_{\mu\nu ab} - \sqrt{2} h_{\mu\nu ab})\epsilon^b,
\end{aligned}
\tag{4.6}
$$

where the action of the covariant derivative on a spinor is

$$D_\mu \epsilon_a = \nabla_\mu \epsilon_a + \frac{1}{2} g_1 a_\mu (\Gamma_{45})_a{}^b \epsilon_b + \frac{1}{2} g_2 A_\mu^I (\Gamma_{I45})_a{}^b \epsilon_b, \tag{4.7}$$

and

$$\begin{aligned} H_{\mu\nu}^{ab} &= \xi (F_{\mu\nu}^I (\Gamma_I)^{ab} + B_{\mu\nu}^\alpha (\Gamma_\alpha)^{ab}), \\ h_{\mu\nu}^{ab} &= \xi^{-2} \Omega^{ab} f_{\mu\nu}, \\ T^{ab} &= \frac{1}{6} \left(\frac{1}{\sqrt{2}} g_2 \xi^{-1} + \frac{1}{2} g_1 \xi^2 \right) (\Gamma_{45})^{ab}, \\ A^{ab} &= \frac{1}{2\sqrt{3}} \left(\frac{1}{\sqrt{2}} g_2 \xi^{-1} - g_1 \xi^2 \right) (\Gamma_{45})^{ab}. \end{aligned} \tag{4.8}$$

The matrices Γ_i satisfy the $D = 5$ Euclidean Clifford algebra

$$(\Gamma_i)_a{}^b (\Gamma_j)_b{}^c + (\Gamma_j)_a{}^b (\Gamma_i)_b{}^c = 2\delta_{ij} \delta_a^c, \tag{4.9}$$

and the charge conjugation matrix $\Omega^{ab} = -\Omega^{ba}$ can be used to raise or lower spinor indices

$$\epsilon^a = \Omega^{ab} \epsilon_b, \qquad \epsilon_a = \Omega_{ab} \epsilon^b, \tag{4.10}$$

so that $\Omega_{ab}\Omega^{bc} = \delta_a^c$ for consistency. Γ_5 is chosen such that $(\Gamma_{12345})_a{}^b = \delta_a^b$. As discussed in [130], different choices of the parameters g_1 and g_2 correspond to different gauged supergravities. For the choice $g_2 = \sqrt{2} g_1 = 2\sqrt{2}$, the theory has an anti-de Sitter vacuum with radius of curvature $L_{AdS} = 1$ and preserves sixteen supersymmetries. These values of the couplings are used in what follows. The bosonic and fermionic supersymmetries combine into the supergroup SU$(2, 2|2)$ which is also the superconformal group of $d = 4, \mathcal{N} = 2$ SCFTs.

4.1.1 1/2-BPS Surface Defect in Romans' Theory

The supergroup SU(2, 2|2) contains a subgroup SU(1, 1|1)×SU(1, 1|1)×U(1), which has eight odd generators and an even SO(2, 1)× SO(2, 1)×U(1)^3 ≅ SO(2, 2)×U(1)^3 subgroup. Unbroken superalgebras of this form correspond to 1/2-BPS superconformal surface operators in $d = 4, \mathcal{N} = 2$ SCFTs [72]. The even part of the subgroup can be realized holographically by the ansatz

$$\begin{aligned} ds^2 &= f_1(r)^2 ds^2_{AdS_3} - f_2(r)^2 ds^2_{S^1} - f_3(r)^2 dr^2, \\ A^I &= \delta^{I3} A(r) d\theta. \end{aligned} \tag{4.11}$$

A solution of this form can be generated by performing a double Wick rotation of the BPS black hole solution [140, 141] used in [115] to calculate Super-Rényi entropies. The solution to the equations of motion is then given by

$$\begin{aligned} ds^2 &= r^2 H(r)^{2/3} \left(\cosh^2\rho\, dt^2 - d\rho^2 - \sinh^2\rho\, d\varphi^2\right) - \frac{f(r)}{H(r)^{4/3}} d\theta^2 - \frac{H(r)^{2/3}}{f(r)} dr^2, \\ H &= 1 + \frac{q}{r^2}, \qquad f = r^2 H^2 - 1, \\ \xi &= H^{1/3}, \qquad A^I = \delta^{I3}\left(\mu - \frac{q}{\sqrt{2}(r^2+q)}\right) d\theta. \end{aligned} \tag{4.12}$$

This solution preserves eight of the original sixteen supersymmetries of the AdS$_5$ vacuum of Romans' theory and is a special case of the matter-coupled solution that is presented in the following section. The number of supersymmetries and the verification of the equations of motion follow from the more general case considered there.

The minimal value of the radial coordinate r_0 is determined by the largest root of $f(r)$ which previously corresponded to the outer horizon of the BPS black hole. Expanding about the origin r_0 leads to

$$\begin{aligned} ds^2 &\sim d\tilde{r}^2 + (1-4q)\,\tilde{r}^2 d\theta^2, \\ \tilde{r} &= r - r_0 = r - \frac{1}{2}\left(1+\sqrt{1-4q}\right). \end{aligned} \tag{4.13}$$

The boundary metric is conformal to flat space

$$ds^2_\partial = \cosh^2\rho\, dt^2 - d\rho^2 - \sinh^2\rho\, d\varphi^2 - d\theta^2 = ds^2_{AdS_3} - d\theta^2, \tag{4.14}$$

which implies that there will be an angular deficit or excess in either the bulk metric or the boundary metric unless $q = 0$. Regularity at the origin can be restored by coupling vector multiplets.

4.2 Matter-Coupled Theory

It is possible to add matter multiplets to the pure Romans' theory. The $N = 4$ vector multiplet

$$(A_\mu, \lambda_i, \phi^m)\,, \tag{4.15}$$

contains a vector field A_μ, four fermions λ_i, and five scalars ϕ^m. The indices $i = 1, \ldots, 4$ and $m = 1, \ldots, 5$ are USp(4) and SO(5) indices, respectively. The matter couplings and gaugings are completely determined in terms of embedding tensors ξ_{MN} and f_{MNP} [132, 133]. The supersymmetric vacua of such theories were investigated in [134].

These embedding tensors satisfy the quadratic constraints

$$f_{R[MN}f_{PQ]}{}^R = 0, \qquad\qquad \xi_M{}^Q f_{QNP} = 0, \tag{4.16}$$

and determine the gauging of the R-symmetry. It is convenient to introduce a composite index $\mathcal{M} = \{0, M\}$ such that the covariant derivative acting on a vector representation is given by

$$\begin{aligned} D_\mu V^{\mathcal{M}} &= \nabla_\mu V^{\mathcal{M}} + g A_\mu^{\mathcal{N}} X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{M}} V^{\mathcal{P}}, \\ X_{MN}{}^P &= -f_{MN}{}^P, \qquad\qquad X_{0M}{}^N = -\xi_M{}^N. \end{aligned} \tag{4.17}$$

The coupling of n vector multiplets is described by a coset representative $\mathcal{V}$ of the group $\mathrm{SO}(5,n)/\mathrm{SO}(5)\times\mathrm{SO}(n)$. The coset representative $\mathcal{V}$ decomposes as

$$\mathcal{V} = \left(\mathcal{V}_M{}^m, \mathcal{V}_M{}^a\right), \tag{4.18}$$

where $m = 1, \ldots, 5$ and $a = 1, \ldots, n$ are SO(5) and SO(n) indices respectively. As an element of SO($5, n$), $\mathcal{V}$ must satisfy

$$\eta_{MN} = \mathcal{V}_M{}^P \eta_{PQ} \mathcal{V}_N{}^Q = -\mathcal{V}_M{}^m \mathcal{V}_N{}^m + \mathcal{V}_M{}^a \mathcal{V}_M{}^a, \tag{4.19}$$

where $\eta_{MN} = \mathrm{diag}(-1,-1,-1,-1,-1,1,\ldots,1)$. The scalar kinetic terms are expressed in terms of the matrix

$$M_{MN} = \mathcal{V}_M{}^m \mathcal{V}_N{}^m + \mathcal{V}_M{}^a \mathcal{V}_M{}^a, \tag{4.20}$$

and the bosonic Lagrangian is given by

$$\begin{aligned} e^{-1}\mathcal{L} =& \frac{1}{2}R - \frac{1}{4}\Sigma^2 M_{MN} \mathcal{H}^M_{\mu\nu} \mathcal{H}^{N\mu\nu} - \frac{1}{4}\Sigma^{-4} \mathcal{H}^0_{\mu\nu} \mathcal{H}^{0\mu\nu} \\ & - \frac{3}{2}\Sigma^2 \left(\partial_\mu \Sigma\right)^2 + \frac{1}{16}\left(D_\mu M_{MN}\right)\left(D^\mu M^{MN}\right) - g^2 V + e^{-1}\mathcal{L}_{\mathrm{top}}, \end{aligned} \tag{4.21}$$

where $\mathcal{L}_{\text{top}}$ is a topological term. The covariant field strengths are

$$\begin{aligned}\mathcal{H}^{\mathcal{M}}_{\mu\nu} &= \partial_\mu A^{\mathcal{M}}_\nu - \partial_\nu A^{\mathcal{M}}_\mu + g X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{M}} A^{\mathcal{N}}_\mu A^{\mathcal{P}}_\nu + g Z^{\mathcal{M}\mathcal{N}} B_{\mu\nu\mathcal{N}},\\ Z^{MN} &= \frac{1}{2}\xi^{MN},\end{aligned} \tag{4.22}$$

where $B_{\mu\nu\mathcal{M}}$ are two-form fields that are introduced in the process of gauging the theory. The scalar potential is

$$\begin{aligned}V &= V_1 + V_2 + V_3,\\ V_1 &= \frac{1}{4} f_{MNP} f_{QRS} \Sigma^{-2}\left(\frac{1}{12} M^{MQ} M^{NR} M^{PS} - \frac{1}{4} M^{MQ} \eta^{NR} \eta^{PS} + \frac{1}{6} \eta^{MQ} \eta^{NR} \eta^{PS}\right),\\ V_2 &= \frac{1}{16} \xi_{MN} \xi_{PQ} \Sigma^4 \left(M^{MP} M^{NQ} - \eta^{MP} \eta^{NQ}\right),\\ V_3 &= \frac{1}{6\sqrt{2}} f_{MNP} \xi_{QR} \Sigma M^{MNPQR},\end{aligned} \tag{4.23}$$

with the completely antisymmetric matrix M_{MNPQR} taking the form

$$M_{MNPQR} = \epsilon_{mnopq} \mathcal{V}_M{}^m \mathcal{V}_N{}^n \mathcal{V}_P{}^o \mathcal{V}_Q{}^p \mathcal{V}_R{}^q\,. \tag{4.24}$$

The SO(5) index M of $\mathcal{V}_M$ can be converted to a pair of antisymmetric USp(4) indices ij through the formulas

$$\mathcal{V}_M{}^{ij} = \frac{1}{2} \mathcal{V}_M{}^m \Gamma^{ij}_m, \qquad \mathcal{V}_{ij}{}^M = \frac{1}{2} \mathcal{V}_m{}^M \Gamma^{kl}_m \Omega_{ki} \Omega_{lj}, \tag{4.25}$$

with a sum over m. The matrices

$$
\begin{aligned}
\zeta^{ij} &= \sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M{}^{ik}\mathcal{V}_N{}^{jl}\xi^{MN}, \\
\zeta^{aij} &= \Sigma^2\mathcal{V}_M{}^a\mathcal{V}_N{}^{ij}\xi^{MN}, \\
\rho^{ij} &= -\frac{2}{3}\Sigma^{-1}\mathcal{V}_M{}^{ik}\mathcal{V}_N{}^{jl}\mathcal{V}^P{}_{kl}f^{MN}{}_P, \\
\rho^{aij} &= \sqrt{2}\Sigma^{-1}\Omega_{kl}\mathcal{V}_M{}^a\mathcal{V}_N{}^{ik}\mathcal{V}_P{}^{jl}f^{MNP},
\end{aligned}
\tag{4.26}
$$

appear in the fermion shift matrices

$$
\begin{aligned}
A_1^{ij} &= \frac{1}{\sqrt{6}}\left(-\zeta^{ij} + 2\rho^{ij}\right), \\
A_2^{ij} &= -\frac{1}{\sqrt{6}}\left(\zeta^{ij} + \rho^{ij}\right), \\
A_2^{aij} &= \frac{1}{2}\left(-\zeta^{aij} + \rho^{aij}\right).
\end{aligned}
\tag{4.27}
$$

A minus sign has been inserted into A_2^{ij} relative to [133] to match the BPS equations of Romans' supergravity in a mostly plus signature as in [132]. The BPS equations are

$$
\begin{aligned}
\delta\psi_{\mu i} &= D_\mu\epsilon_i - \frac{i}{6}\left(\Omega_{ij}\Sigma\mathcal{V}_M{}^{ik}\mathcal{H}^M_{\nu\rho} - \frac{1}{2\sqrt{2}}\delta_i^k\Sigma^{-2}\mathcal{H}^0_{\nu\rho}\right)\left(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu\gamma^\rho\right)\epsilon_k \\
&\quad + \frac{ig}{\sqrt{6}}\Omega_{ij}A_1^{jk}\gamma_\mu\epsilon_k, \\
\delta\chi_i &= -i\frac{\sqrt{3}}{2}\left(\Sigma^{-1}\partial_\mu\Sigma\right)\gamma^\mu\epsilon_i - \frac{1}{2\sqrt{3}}\left(\Sigma\Omega_{ij}\mathcal{V}_M{}^{jk}\mathcal{H}^M_{\mu\nu} + \frac{1}{\sqrt{2}}\Sigma^{-2}\delta_i^k\mathcal{H}^0_{\mu\nu}\right)\gamma^{\mu\nu}\epsilon_k \\
&\quad + \sqrt{2}g\Omega_{ij}A_2^{kj}\epsilon_k, \\
\delta\lambda_i^a &= i\Omega^{jk}\left(\mathcal{V}_M{}^a D_\mu\mathcal{V}_{ij}{}^M\right)\gamma^\mu\epsilon_k - \frac{1}{4}\Sigma\mathcal{V}_M{}^a\mathcal{H}^M_{\mu\nu}\gamma^{\mu\nu}\epsilon_i + \sqrt{2}g\Omega_{ij}A_2^{akj}\epsilon_k,
\end{aligned}
\tag{4.28}
$$

with the action of the covariant derivative on a spinor given by

$$
D_\mu\epsilon_i = \nabla_\mu\epsilon_i - \mathcal{V}^M{}_{ik}\partial_\mu\mathcal{V}_M{}^{kj}\epsilon_j - gA^0_\mu\xi^{MN}\mathcal{V}_{Mik}\mathcal{V}_N{}^{kj}\epsilon_j + gA^M_\mu f_{MNP}\mathcal{V}^N_{ik}\mathcal{V}^{Pkj}\epsilon_j. \tag{4.29}
$$

4.2.1 1/2-BPS Surface Defect in the Matter-Coupled Theory

The gauging corresponding to Romans' supergravity with $L_{AdS} = 1$ is given by

$$\begin{aligned} f_{MNP} &= -\frac{1}{\sqrt{2}}\epsilon_{MNP}, \qquad M, N, P \in \{1, 2, 3\}, \\ \xi_{MN} &= -\frac{1}{2}\left(\delta^4_M \delta^5_N - \delta^4_N \delta^5_M\right). \end{aligned} \tag{4.30}$$

We will couple one vector multiplet and choose the coset element

$$\mathcal{V} = \exp(\phi_3 Y_3), \tag{4.31}$$

with the non-compact generator $(Y_3)_{mn} = \delta_{3m}\delta_{6n} + \delta_{3n}\delta_{6m}$. The scalar ϕ_3 is a singlet under gauge transformations generated by $\sigma_3 \in \mathrm{su}(2)$. The theory can be truncated to $\Sigma, \phi_3, A^3_\mu, A^6_\mu, g_{\mu\nu}$ and the Lagrangian is

$$\begin{aligned} e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}\Sigma^2 &\left[\frac{1}{2}e^{2\phi_3}\left(F^3_{\mu\nu} + F^6_{\mu\nu}\right)^2 + \frac{1}{2}e^{-2\phi_3}\left(F^3_{\mu\nu} - F^6_{\mu\nu}\right)^2\right] \\ &- \frac{3}{2}\Sigma^{-2}\left(\partial_\mu \Sigma\right)^2 - \frac{1}{2}\left(\partial_\mu \phi_3\right)^2 + 2\left(\Sigma^{-2} + \Sigma\left(e^{\phi_3} + e^{-\phi_3}\right)\right), \end{aligned} \tag{4.32}$$

where $A^6_\mu = A_\mu$ is the vector from the vector multiplet. For $\phi_3 = A^6_\mu = 0$, we recover Romans' theory with the gauge field A^3_μ rescaled. The STU model [140] can be embedded into the matter-coupled theory with the identifications

$$\begin{aligned} T &= \frac{1}{\Sigma}e^{-\phi_3}, \\ U &= \frac{1}{\Sigma}e^{\phi_3}, \\ F_{\mu\nu} &= F^3_{\mu\nu} + F^6_{\mu\nu}, \\ G_{\mu\nu} &= F^3_{\mu\nu} - F^6_{\mu\nu}. \end{aligned} \tag{4.33}$$

The equations of motion are

$$
\begin{aligned}
&R_{\mu\nu} + \frac{1}{2}\Sigma^2\left(e^{2\phi_3}F_\mu^{\ \alpha}F_{\alpha\nu} + e^{-2\phi_3}G_\mu^{\ \alpha}G_{\alpha\nu}\right) - 3\Sigma^{-2}\partial_\mu\Sigma\partial_\nu\Sigma - \partial_\mu\phi_3\partial_\nu\phi_3 \\
&+g_{\mu\nu}\left(\frac{1}{12}\Sigma^2\left(e^{2\phi_3}F^{\alpha\beta}F_{\alpha\beta} + e^{-2\phi_3}G^{\alpha\beta}G_{\alpha\beta}\right) + \frac{4}{3}\left(\Sigma^{-2} + \Sigma\left(e^{\phi_3} + e^{-\phi_3}\right)\right)\right) = 0, \\
&\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\Sigma^{-2}\partial^\mu\Sigma\right) + \Sigma^{-3}\left(\partial_\mu\Sigma\right)^2 - \frac{1}{12}\Sigma\left(e^{2\phi_3}F^{\mu\nu}F_{\mu\nu} + e^{-2\phi_3}G^{\mu\nu}G_{\mu\nu}\right) \\
&\qquad + \frac{2}{3}\left(e^{\phi_3} + e^{-\phi_3} - 2\Sigma^{-3}\right) = 0, \qquad (4.34) \\
&\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi_3\right) - \frac{1}{4}\Sigma^2\left(e^{2\phi_3}F^{\mu\nu}F_{\mu\nu} - e^{-2\phi_3}G^{\mu\nu}G_{\mu\nu}\right) + 2\Sigma\left(e^{\phi_3} - e^{-\phi_3}\right) = 0, \\
&\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\Sigma^2 e^{2\phi_3}F^{\mu\nu}\right) = 0, \\
&\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\Sigma^2 e^{-2\phi_3}G^{\mu\nu}\right) = 0.
\end{aligned}
$$

It is straightforward to verify that the equations are solved by the double Wick rotated two charge solution of [140]

$$
\begin{aligned}
&ds^2 = r^2(H_1H_2)^{1/3}\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\varphi^2\right) + \frac{f}{(H_1H_2)^{2/3}}d\theta^2 + \frac{(H_1H_2)^{1/3}}{f}dr^2, \\
&H_1 = 1 + \frac{Q}{r^2}, \qquad H_2 = 1 + \frac{q}{r^2}, \qquad f = r^2H_1H_2 - 1, \\
&\Sigma = (H_1H_2)^{1/6}, \qquad e^{2\phi_3} = \frac{H_1}{H_2}, \\
&A^3 + A^6 = \left(\mu_3 + \mu_6 - \frac{Q}{r^2+Q}\right)d\theta, \qquad A^3 - A^6 = \left(\mu_3 - \mu_6 - \frac{q}{r^2+q}\right)d\theta,
\end{aligned}
\tag{4.35}
$$

where μ_3 and μ_6 are the chemical potentials for A^3 and A^6 respectively. For $Q = q$ and $\mu_6 = 0$, this solution reduces to that of the previous section (4.12) upon identifying $A_{\text{new}} = \sqrt{2}A_{\text{old}}$. As before, the spacetime closes at the largest root r_0 of $f(r)$ which is now given by

$$
r_0^2 = \frac{1-q-Q}{2} + \frac{1}{2}\sqrt{1+(Q-q)^2 - 2(Q+q)}. \tag{4.36}
$$

After expanding the bulk metric about r_0, the absence of an angular deficit or excess in both the bulk metric and the boundary metric requires

$$(Q - q)^2 = 2(Q + q). \tag{4.37}$$

It is convenient to redefine the integration constants q and Q as

$$\begin{aligned} Q &= q_1 + q_2, \\ q &= q_1 - q_2, \end{aligned} \tag{4.38}$$

so that regularity at the origin requires $q_1 = q_2^2$ and the spacetime closes at $r_0^2 = 1 - q_2^2$. The spacetime develops a singularity at $r = 0$, but this value will be excluded from the physical range of the radial coordinate for $q_2^2 \leq 1$.

In the solution (4.35), both scalars have a non-trivial profile. The dilaton Σ is regular at the origin, but the additional scalar ϕ_3 contains a kink

$$\Sigma'(r_0) = 0, \qquad\qquad \phi_3'(r_0) \neq 0. \tag{4.39}$$

For generic chemical potentials, the gauge fields have a non-zero holonomy around $r = r_0$. We show in Appendix D that the bosonic background (4.35) preserves eight of the sixteen supersymmertries of the gauged supergravity.

Since our solution has only two non-zero gauge fields and scalars, it can be related to solutions in $D = 5, N = 2$ gauged supergravity [140, 141]. It has been shown in [104] that these solutions can be uplifted to ten and eleven dimensions, which means that our solution can be uplifted as well. It was argued in [142] that the truncation used in this chapter falls into a class of truncations of gauged $N = 8$ supergravity which can be uplifted to ten dimensions [143].

4.3 Holographic Observables

In this section, we use holographic renormalization [38, 39] to calculate holographic observables, namely the free energy and vacuum expectation values of operators in the presence of a surface defect.

4.3.1 Free Energy

Using the equations of motion, the on-shell action takes the form

$$S_{\text{bulk}} = -\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{12}\Sigma^2\left(e^{2\phi_3}F^{\mu\nu}F_{\mu\nu} + e^{-2\phi_3}G^{\mu\nu}G_{\mu\nu}\right) + \frac{4}{3}\left(\Sigma^{-2} + \Sigma\left(e^{\phi_3} + e^{-\phi_3}\right)\right)\right).$$

The bulk action is divergent and can be renormalized by imposing a cutoff on the spacetime. In Fefferman-Graham coordinates

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}g_{ij}dx^i dx^j, \tag{4.40}$$

one imposes the cutoff $z = \epsilon$ and adds boundary counterterms. Since the regularized spacetime contains a boundary, the Gibbons-Hawking-York term

$$S_{\text{GH}} = \int_{\partial\mathcal{M}} d^4x\sqrt{-h}K = -\int_{\partial\mathcal{M}} d^4x\, z\,\partial_z\sqrt{-h}, \tag{4.41}$$

must be included to maintain the variational principle of the metric. In the above formula, $h_{\mu\nu}$ is the induced metric on the boundary and K is the trace of the extrinsic curvature. In the notation of [137], the bulk fields are expanded as

$$
\begin{aligned}
g_{ij} &= g_{ij}^{(0)} + z^2 g_{ij}^{(2)} + z^4 \left(g_{ij}^{(4)} + (\log z)^2 \, h_{ij}^{(0)} + \log z \, h_{ij}^{(1)} \right) + \dots \ , \\
\Sigma &= 1 + z^2 \left(b_1 \log z + b_2 \right) + \dots \ , \\
\phi_3 &= z^2 \left(c_1 \log z + c_2 \right) + \dots \ , \\
F &= d \left(A_1 + A_2 z^2 + A_3 z^2 \log z + \dots \right) , \\
G &= d \left(a_1 + a_2 z^2 + a_3 z^2 \log z + \dots \right) ,
\end{aligned}
\tag{4.42}
$$

and the equations of motion are solved order by order in z. The expansion of the Ricci tensor is

$$
\begin{aligned}
R_{zz} &= -\frac{4}{z^2} - \frac{1}{2} \mathrm{Tr} \left[g^{-1} g'' \right] + \frac{1}{2z} \mathrm{Tr} \left[g^{-1} g' \right] + \frac{1}{4} \mathrm{Tr} \left[g^{-1} g' g^{-1} g' \right] , \\
R_{ij} &= -\frac{4}{z^2} g_{ij} - \frac{1}{2} g''_{ij} + \frac{3}{2z} g'_{ij} + \frac{1}{2} \left(g' g^{-1} g' \right)_{ij} - \frac{1}{4} \mathrm{Tr} \left[g^{-1} g' \right] g'_{ij} \\
&\quad + R[g]_{ij} + \frac{1}{2z} \mathrm{Tr} \left[g^{-1} g' \right] g_{ij} ,
\end{aligned}
\tag{4.43}
$$

where $R[g]_{ij}$ is the boundary Ricci tensor and primes denote derivatives with respect to z. The expansion of the volume element

$$
\begin{aligned}
\frac{\sqrt{-g}}{\sqrt{-g^{(0)}}} &= 1 + \frac{z^2}{2} t^{(2)} + \frac{z^4}{2} \left(t^{(4)} - \frac{1}{2} t^{(2,2)} + \frac{1}{4} (t^{(2)})^2 + (\log z)^2 \, u^{(0)} + \log z \, u^{(1)} \right) + \dots \ , \\
t^{(n)} &= \mathrm{Tr} \left[\left(g^{(0)} \right)^{-1} g^{(n)} \right] , \qquad t^{(2,2)} = \mathrm{Tr} \left[\left(g^{(0)} \right)^{-1} g^{(2)} \left(g^{(0)} \right)^{-1} g^{(2)} \right] , \\
u^{(n)} &= \mathrm{Tr} \left[\left(g^{(0)} \right)^{-1} h^{(n)} \right] ,
\end{aligned}
\tag{4.44}
$$

will be needed when expanding the action. The ij component of the Einstein field equation to order $O(z^0)$ is solved by

$$g_{ij}^{(2)} = -\frac{1}{2}\left(R[g^{(0)}]_{ij} - \frac{1}{6}R[g^{(0)}]g_{ij}^{(0)}\right), \tag{4.45}$$

which implies

$$\begin{aligned} t^{(2)} &= -\frac{1}{6}R[g^{(0)}], \\ t^{(2,2)} &= \frac{1}{4}\left(R[g^{(0)}]_{ij}R[g^{(0)}]^{ij} - \frac{2}{9}R[g^{(0)}]^2\right). \end{aligned} \tag{4.46}$$

The zz component of the Einstein field equation to order $O(z^2)$ is solved by

$$\begin{aligned} u^{(0)} &= -\frac{2}{3}\left(3b_1^2 + c_1^2\right), \\ u^{(1)} &= -\frac{4}{3}\left(3b_1b_2 + c_1c_2\right), \\ 4t^{(4)} &= t^{(2,2)} - u^{(0)} - 3u^{(1)} - \left(3b_1^2 + c_1^2\right) - \frac{8}{3}\left(3b_2^2 + c_2^2\right) - 4\left(3b_1b_2 + c_1c_2\right) \\ &\quad + \frac{1}{12}\left(|F|^2_{g^{(0)}} + |G|^2_{g^{(0)}}\right), \end{aligned} \tag{4.47}$$

where $|F|^2_{g^{(0)}} = F_{ij}F_{kl}g^{(0)ik}g^{(0)jl}$ is the norm of the boundary field strength and similarly for $|G|^2_{g^{(0)}}$. The leading divergence takes the form

$$\frac{1}{\epsilon^4}\int_{\partial\mathcal{M}} d^4x\sqrt{-g^{(0)}}\left(-1+4\right), \tag{4.48}$$

where the coefficients come from S_{bulk} and S_{GH}, respectively. This is canceled by the counterterm $\delta S_1 = -3\int_{\partial\mathcal{M}} d^4x\sqrt{-h}$. The subleading divergences are

$$\frac{1}{\epsilon^2}\int_{\partial\mathcal{M}} d^4x\sqrt{-g^{(0)}}\left(-1+1-\frac{3}{2}\right)t^{(2)}, \tag{4.49}$$

where the coefficients come from $S_{\mathrm{bulk}}, S_{\mathrm{GH}}$, and δS_1 respectively. This can be canceled by the counterterm $\delta S_2 = -\frac{1}{4}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}R[h]$. The logarithmic divergences are given by

$$\begin{aligned} S_{\mathrm{bulk}} &\sim \left[\frac{1}{2}\left(\left(t^{(2)}\right)^2 - t^{(2,2)}\right) - \frac{1}{6}\left(3b_1^2 + c_1^2\right) + \frac{1}{8}\left(|F|^2_{g^{(0)}} + |f|^2_{g^{(0)}}\right)\right]\log\epsilon, \\ S_{\mathrm{GH}} &\sim \frac{2}{3}\left(3b_1^2 + c_1^2\right)\log\epsilon, \\ \delta S_1 &\sim \left(3b_1^2 + c_1^2\right)(\log\epsilon)^2 + 2\left(3b_1b_2 + c_1c_2\right)\log\epsilon, \\ \delta S_2 &\sim 0\cdot\log\epsilon. \end{aligned} \tag{4.50}$$

The logarithmic divergences are canceled by the counterterms

$$\begin{aligned} \delta S_3 =& \frac{1}{8}\int d^4x\sqrt{-h}\log\epsilon\left[\left(R[h]^{ij}R[h]_{ij} - \frac{1}{3}R[h]^2\right) - F^{ij}F_{ij} - G^{ij}G_{ij}\right] \\ &+ \int d^4x\sqrt{-h}\left[-3(\Sigma-1)^2 - \frac{3}{2\log\epsilon}(\Sigma-1)^2 - \phi_3^2 - \frac{1}{2\log\epsilon}\phi_3^2\right]. \end{aligned} \tag{4.51}$$

Putting together the different contributions, the renormalized action

$$S_{\mathrm{ren}} = \lim_{\epsilon\to 0}\left(S_{\mathrm{bulk}} + S_{\mathrm{GH}} + \delta S_1 + \delta S_2 + \delta S_3\right), \tag{4.52}$$

evaluates to

$$S_{\mathrm{ren}} = \left(\frac{5}{8} - q_2^2\right)\mathrm{Vol}(\mathrm{AdS}_3)\mathrm{Vol}(S^1), \tag{4.53}$$

for the surface defect where $\mathrm{Vol}(\mathrm{AdS}_3)$ is the regularized volume of the AdS_3 factor.

4.3.2 Vacuum Expectation Values

Using the renormalized action (4.52), the vacuum expectation values can be computed through differentiation

$$\begin{aligned}
\langle \mathcal{O}_\Sigma \rangle &= \frac{1}{\sqrt{-g^{(0)}}} \frac{\delta S_{\text{ren}}}{\delta b_1}\bigg|_{b_1=0} = -3b_2, \\
\langle \mathcal{O}_{\phi_3} \rangle &= \frac{1}{\sqrt{-g^{(0)}}} \frac{\delta S_{\text{ren}}}{\delta c_1}\bigg|_{c_1=0} = -c_2, \\
\langle \mathcal{J}^i \rangle &= \frac{1}{\sqrt{-g^{(0)}}} \frac{\delta S_{\text{ren}}}{\delta A_{1i}}\bigg|_{A_1=0} = \frac{1}{2}\left(A_3 + 2A_2\right)^i, \\
\langle j^i \rangle &= \frac{1}{\sqrt{-g^{(0)}}} \frac{\delta S_{\text{ren}}}{\delta a_{1i}}\bigg|_{a_1=0} = \frac{1}{2}\left(a_3 + 2a_2\right)^i, \\
\langle T_{ij} \rangle &= -\frac{2}{\sqrt{-g^{(0)}}} \frac{\delta S_{\text{ren}}}{\delta g^{(0)ij}} = \lim_{\epsilon\to 0}\left(\frac{1}{\epsilon^2} T[h]_{ij}\bigg|_{z=\epsilon}\right),
\end{aligned} \tag{4.54}$$

where $T[h]_{ij}$ is the boundary stress tensor. For the surface defect solution, the asymptotic expansion is

$$r = \frac{1}{z} + \left(\frac{1}{4} - \frac{q_2^2}{3}\right) z - \frac{q_2^4}{36} z^3 + \frac{108q_2^2 + 63q_2^4 - 20q_2^6}{3888} z^5 + O(z^6), \tag{4.55}$$

and the expectation values are

$$\begin{aligned}
\langle \mathcal{O}_\Sigma \rangle &= -q_2^2, \\
\langle \mathcal{O}_{\phi_3} \rangle &= -q_2, \\
\langle \mathcal{J}_\theta \rangle &= q_2(1+q_2), \\
\langle j_\theta \rangle &= q_2(1-q_2), \\
\langle T_{ij} \rangle &= \left(\frac{3}{8} - 2q_2^2\right) \begin{pmatrix} -\frac{1}{3} g_{AdS_3} & 0 \\ 0 & g_{S^1} \end{pmatrix}_{ij},
\end{aligned} \tag{4.56}$$

so that there are no conformal anomalies: $\langle T^i_i \rangle = 0$. Note that the solution does not contain any logarithmic divergences and the boundary stress tensor is therefore given by

$$T[h]_{ij} = K_{ij} - K h_{ij} + 3h_{ij} - \frac{1}{2}\left(R[h]_{ij} - \frac{1}{2}R[h]h_{ij}\right) + \left(3(\Sigma - 1)^2 + \phi_3^2\right) h_{ij}. \qquad (4.57)$$

4.4 Discussion

In this chapter, we investigated solutions of $D = 5$, $N = 4$ gauged supergravity that are holographic duals of 1/2-BPS conformal surface defects in $\mathcal{N} = 2$ SCFTs. The ansatz for the solution is informed by the unbroken symmetries of such defects and is given by $\mathrm{AdS}_3 \times S^1$ warped over an interval with non-trivial gauge potentials along S^1. We showed for pure Romans' theory that the only solution in this class which is non-singular is the AdS_5 vacuum; all non-trivial solutions suffer from a conical defect. This situation can be improved by coupling vector multiplets to $N = 4$ gauged supergravity. The simplest case of one additional vector multiplet already allows for the construction of a one parameter family of regular solutions dual to conformal surface defects preserving eight of the sixteen supersymmetries of the vacuum.

An important question is whether solutions of lower dimensional gauged supergravities can be uplifted to ten- or eleven-dimensional solutions for which the dual SCFTs are in general known from decoupling limits of brane configurations. It has been shown that pure Romans' theory is a consistent truncation of type IIB [144, 145], type IIA [104] and M-theory [146] and hence solutions of this theory can be uplifted. Much less is known about uplifts of matter-coupled $D = 5, N = 4$ gauged supergravity. In [142], it was argued that Romans' theory coupled to two tensor multiplets is a consistent truncation of an orbifold of $\mathrm{AdS}_5 \times S^5$. The rigidity of supersymmetric $N = 4$ vacua [134] makes the existence of other consistent truncations likely.

Since our solution involves only two gauge fields and scalars, it can be related to solutions in $D = 5, N = 2$ gauged supergravity [140, 141]. It has been shown in [104] that these solutions can be uplifted to ten and eleven dimensions, which means that our solution can be uplifted as well. It was argued in [142] that the truncation used in this chapter falls into a class of truncations of gauged $N = 8$ supergravity which can be uplifted to ten dimensions [143]. One could also consider applying the construction in this chapter to the general class of gauged supergravities of [142] which describe $\mathbb{Z}_N$ orbifolds and investigate whether in the field theory, the surface operators of the orbifold theory can be obtained from surface operators of $\mathcal{N} = 4$ Super Yang-Mills [63, 147, 148].

APPENDIX A

Minimal $D = 4, N = 2$ Supergravity Conventions

We use the metric conventions $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$. A convenient basis of the four-dimensional Clifford algebra is given by

$$\gamma_0 = i\sigma_2 \otimes 1, \qquad \gamma_1 = \sigma_1 \otimes 1, \qquad \gamma_2 = \sigma_3 \otimes \sigma_1, \qquad \gamma_3 = \sigma_3 \otimes \sigma_3, \tag{A.1}$$

where $\tilde{\gamma}_0 = i\sigma_2$ and $\tilde{\gamma}_1 = \sigma_1$ form a basis of the two-dimensional Clifford algebra with chirality matrix $\tilde{\gamma}_* = \sigma_3$. In this basis, the Killing spinors of

$$ds^2_{AdS_2} = L^2 \left(\frac{-dt^2 + d\eta^2}{\eta^2} \right), \tag{A.2}$$

are given by

$$\psi_1^{\pm} = \frac{1}{\sqrt{\eta}} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}, \qquad \psi_2^{\pm} = \frac{1}{\sqrt{\eta}} \begin{pmatrix} t + \eta \\ \pm(t - \eta) \end{pmatrix}. \tag{A.3}$$

APPENDIX B

Matter-Coupled $D = 4, N = 2$ Supergravity Conventions and Calculations

We use the metric conventions $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$ and $\epsilon_{0123} = -\epsilon^{0123} = 1$. The chirality matrix γ_5 is defined as

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3. \tag{B.1}$$

The two chiral gravitinos can be written in terms of a single complex Dirac spinor ψ_μ and likewise for the gauginos λ^α. The supersymmetry transformations of the four-dimensional gauged supergravity are [94]

$$\begin{aligned}
\delta\psi_\mu = &\Big(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{i}{2}Q_\mu\gamma_5 + ig\xi_I A_\mu^I + ge^{\mathcal{K}/2}\gamma_\mu\xi_I\big(\mathrm{Im}\, Z^I + i\gamma_5\,\mathrm{Re}\, Z^I\big) \\
&+ \frac{i}{4}e^{\mathcal{K}/2}\gamma^{ab}(\mathrm{Im}\,\mathcal{N})_{IJ}\big(\mathrm{Im}(F_{ab}^{-I}Z^J) - i\gamma_5\,\mathrm{Re}(F_{ab}^{-I}Z^J)\big)\gamma_\mu\Big)\epsilon\ , \\
\delta\lambda^\alpha = &\Big(\gamma^\mu\partial_\mu(\mathrm{Re}\, z^\alpha - i\gamma_5\,\mathrm{Im}\, z^\alpha) + 2ge^{\mathcal{K}/2}\xi_I\Big(\mathrm{Im}(\mathcal{D}_{\bar\beta}\bar Z^I g^{\alpha\bar\beta}) - i\gamma_5\,\mathrm{Re}(\mathcal{D}_{\bar\beta}\bar Z^I g^{\alpha\bar\beta})\Big) \\
&+ \frac{i}{2}e^{\mathcal{K}/2}\gamma^{ab}(\mathrm{Im}\,\mathcal{N})_{IJ}\Big(\mathrm{Im}(F_{ab}^{-I}\mathcal{D}_{\bar\beta}\bar Z^J g^{\alpha\bar\beta}) - i\gamma_5\,\mathrm{Re}(F_{ab}^{-I}\mathcal{D}_{\bar\beta}\bar Z^J g^{\alpha\bar\beta})\Big)\Big)\epsilon\ ,
\end{aligned} \tag{B.2}$$

where ϵ is a complex spinor and we have defined

$$F^{\pm I}_{ab} \equiv \frac{1}{2}(F^I_{ab} \pm \tilde{F}^I_{ab}) \ , \qquad \tilde{F}^I_{ab} \equiv -\frac{i}{2}\epsilon_{abcd}F^{Icd} \ . \tag{B.3}$$

The Kähler connection Q_μ is

$$Q_\mu = -\frac{i}{2}(\partial_\mu \tau^\alpha \partial_\alpha \mathcal{K} - \partial_\mu \bar{\tau}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) \ . \tag{B.4}$$

For the gauged STU model defect solution (3.23), we can work with the explicit coordinates $(x^0, x^1, x^2, x^3) = (t, \eta, \theta, r)$ and the metric

$$\mathrm{d}s^2 = r^2\sqrt{H}\left(\frac{-\,\mathrm{d}t^2 + \mathrm{d}\eta^2}{\eta^2}\right) + \frac{f}{\sqrt{H}}\,\mathrm{d}\theta^2 + \frac{\sqrt{H}}{f}\,\mathrm{d}r^2 \ . \tag{B.5}$$

The non-vanishing spin connection one-forms of the metric are

$$\begin{aligned}
\omega^{01} &= -\frac{\mathrm{d}t}{\eta} \ , & \omega^{03} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}\left(rH^{1/4}\right)\frac{\mathrm{d}t}{\eta} \ , \\
\omega^{13} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}\left(rH^{1/4}\right)\frac{\mathrm{d}\eta}{\eta} \ , & \omega^{23} &= \frac{f^{1/2}}{H^{1/4}}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{f^{1/2}}{H^{1/4}}\right)\mathrm{d}\theta \ .
\end{aligned} \tag{B.6}$$

For the following calculations, we use the parameterization $(Z^0, Z^1, Z^2, Z^3) = (i, iz^2z^3, iz^1z^3, iz^1z^2)$. The BPS equations (B.2) simplify to

$$\begin{aligned}
0 = \delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + ig\xi_I A^I_\mu + \sqrt{2}g\gamma_\mu \frac{\mathrm{d}}{\mathrm{d}r}(rH^{1/4}) - \frac{i}{2}\gamma_{23}\gamma_\mu\frac{\mathrm{d}}{\mathrm{d}r}(H^{-1/4})\right) \ , \\
0 = \delta\lambda^\alpha &= \frac{\mathrm{d}z^\alpha}{\mathrm{d}r}\left(\frac{f^{1/2}}{H^{1/4}}\gamma_3 + 2\sqrt{2}grH^{1/4} + \frac{i}{H^{1/4}}\gamma_{23}\right)\epsilon \ .
\end{aligned} \tag{B.7}$$

The gaugino equation implies the projector

$$0 = \left(1 + \frac{2\sqrt{2}gr\sqrt{H}}{\sqrt{f}}\gamma_3 - \frac{i}{\sqrt{f}}\gamma_2\right)\epsilon \ . \tag{B.8}$$

The $\mu = t, \eta, \theta$ components of the gravitino equation then simplify to

$$\begin{aligned}
0 &= \left(\partial_t - \frac{1}{2\eta}\gamma_{01} - \frac{i}{2\eta}\gamma_{023}\right)\epsilon \ , \\
0 &= \left(\partial_\eta - \frac{i}{2\eta}\gamma_{123}\right)\epsilon \ , \\
0 &= \left(\partial_\theta + i\sqrt{2}g\left(-1 + \frac{1}{\sqrt{2}}\xi_I\mu^I\right)\right)\epsilon \ .
\end{aligned} \tag{B.9}$$

These can be integrated to

$$\epsilon = \exp\left(-i\sqrt{2}g\theta\left(-1 + \frac{1}{\sqrt{2}}\xi_I\mu^I\right)\right)\exp\left(\frac{i}{2}\gamma_{123}\ln\eta\right)\exp\left(\frac{t}{2}(\gamma_{01} + i\gamma_{023})\right)\tilde{\epsilon}(r) \ . \tag{B.10}$$

We can see that we need $\xi_I\mu^I \in 2\sqrt{2}\mathbb{Z}$ in order for ϵ to be anti-periodic under the identification $\theta \sim \theta + \pi/\sqrt{2}g$. The $\mu = r$ component of the gravitino equation simplifies to

$$\left(\partial_r + \frac{1}{8}\frac{H'}{H} + \frac{f'}{8\sqrt{2}gr\sqrt{H}\sqrt{f}}\gamma_3\right)\epsilon = 0. \tag{B.11}$$

The gaugino projector (B.8) and the radial equation (B.11) take the form of the equation solved in the appendix of [149] by identifying

$$\begin{aligned}
x &\equiv \frac{2\sqrt{2}gr\sqrt{H}}{\sqrt{f}} \ , \qquad & y &\equiv \frac{-i}{\sqrt{f}} \ , \\
\Gamma_1 &\equiv \gamma_3 \ , \qquad & \Gamma_2 &\equiv \gamma_2 \ .
\end{aligned} \tag{B.12}$$

The solution is

$$\tilde{\epsilon}(r) = \frac{1}{H^{1/8}}\left(\sqrt{\sqrt{f} + 2\sqrt{2}gr\sqrt{H}} - \gamma_2\sqrt{\sqrt{f} - 2\sqrt{2}gr\sqrt{H}}\right)(1-\gamma_3)\epsilon_0 \ , \tag{B.13}$$

where ϵ_0 is a constant spinor.

APPENDIX C

Vanishing of Scalar One-Point Functions from Supersymmetry

The scalar one-point function is given by

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \lim_{\epsilon \to 0} \left[\frac{1}{\epsilon^2} \left(z g_{\beta\bar{\alpha}} \partial_z \tau^\beta + \partial_{\bar{\alpha}} \mathcal{W} \right) \right] \ . \tag{C.1}$$

The derivative of the superpotential $\mathcal{W}$ simplifies to

$$\begin{aligned} \partial_{\bar{\alpha}} \mathcal{W} &= \partial_{\bar{\alpha}} \left(-\sqrt{2} e^{\mathcal{K}/2} |\xi_I Z^I| \right) \\ &= -\frac{1}{\sqrt{2}} e^{\mathcal{K}/2} \left(\sqrt{\frac{\xi_J Z^J}{\xi_J \bar{Z}^J}} \xi_I \partial_{\bar{\alpha}} \bar{Z}^I + (\partial_{\bar{\alpha}} \mathcal{K}) |\xi_I Z^I| \right) \ , \end{aligned} \tag{C.2}$$

where $|\xi_I Z^I|^2 = \xi_I \xi_J Z^I(\tau) \bar{Z}^J(\bar{\tau})$. For real scalars, we can choose a parameterization such that $\bar{Z}^I = Z^I$. This implies

$$\partial_{\bar{\alpha}} \mathcal{W} = -\frac{1}{\sqrt{2}} e^{\mathcal{K}/2} \left(\xi_I \partial_{\bar{\alpha}} \bar{Z}^I + (\partial_{\bar{\alpha}} \mathcal{K}) |\xi_I Z^I| \right) = -\frac{1}{\sqrt{2}} e^{\mathcal{K}/2} \xi_I \mathcal{D}_{\bar{\alpha}} \bar{Z}^I \ , \tag{C.3}$$

for $\xi_I Z^I > 0$ so that

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \lim_{\epsilon \to 0} \left[\frac{1}{\epsilon^2} \left(z g_{\beta\bar{\alpha}} \partial_z \tau^\beta - \frac{1}{\sqrt{2}} e^{\mathcal{K}/2} \xi_I \mathcal{D}_{\bar{\alpha}} \bar{Z}^I \right) \bigg|_{z=\epsilon} \right] \; . \tag{C.4}$$

The gaugino BPS variation in FG coordinates is

$$\left(z \gamma_3 \partial_z \tau^\beta - 2 i g e^{\mathcal{K}/2} \xi_I g^{\beta\bar{\alpha}} \mathcal{D}_{\bar{\alpha}} \bar{Z}^I \gamma_5 \right) \epsilon + O(z^3) \epsilon = 0 \; , \tag{C.5}$$

since $F_{ab} \sim 1/r^2 \sim O(z^2)$. At $O(z^2)$, the BPS equations imply

$$z \partial_z \tau^\beta = \pm 2 i g e^{\mathcal{K}/2} \xi_I g^{\beta\bar{\alpha}} \mathcal{D}_{\bar{\alpha}} \bar{Z}^I \; . \tag{C.6}$$

Without loss of generality, we can choose the upper sign by sending $g \to -g$ if necessary. After setting $g^2 = 1/8$ we have

$$\langle \bar{\mathcal{O}}_{\bar{\alpha}} \rangle = \langle \mathcal{O}_\alpha \rangle = 0 \; . \tag{C.7}$$

APPENDIX D

Matter-Coupled $D = 5, N = 4$ Supergravity Conventions and Calculations

The frame field for the metric

$$ds^2 = r^2(H_1H_2)^{1/3}\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\varphi^2\right) + \frac{f}{(H_1H_2)^{2/3}}d\theta^2 + \frac{(H_1H_2)^{1/3}}{f}dr^2,$$

is chosen to be

$$\begin{aligned} &e^0 = r(H_1H_2)^{1/6}\cosh\rho\, dt \qquad && e^1 = r(H_1H_2)^{1/6}d\rho, \qquad && e^2 = r(H_1H_2)^{1/6}\sinh\rho\, d\varphi, \\ &e^3 = \frac{f^{1/2}}{(H_1H_2)^{1/3}}d\theta, && e^4 = \frac{(H_1H_2)^{1/6}}{f^{1/2}}dr. && \end{aligned} \tag{D.1}$$

The spin connection is then given by

$$\begin{aligned}
\omega^{01} &= \sinh\rho\, dt,\\
\omega^{04} &= \frac{f^{1/2}}{(H_1H_2)^{1/6}}\frac{d}{dr}\left(r\,(H_1H_2)^{1/6}\right)\cosh\rho\, dt,\\
\omega^{12} &= -\cosh\rho\, d\varphi,\\
\omega^{14} &= \frac{f^{1/2}}{(H_1H_2)^{1/6}}\frac{d}{dr}\left(r\,(H_1H_2)^{1/6}\right)\, d\rho,\\
\omega^{24} &= \frac{f^{1/2}}{(H_1H_2)^{1/6}}\frac{d}{dr}\left(\frac{f^{1/2}}{(H_1H_2)^{1/3}}\right) d\theta.
\end{aligned} \tag{D.2}$$

All fermions satisfy the symplectic Majorana condition

$$\epsilon_a^* = B\Omega_{ab}\epsilon^b, \tag{D.3}$$

where B is related to the usual charge conjugation matrix C by $B = \gamma_0 C$. An explicit basis for the spacetime γ matrices in the signature $\eta_{ab} = \mathrm{diag}(-1,1,1,1,1)$ is

$$\begin{aligned}
\gamma_0 &= i\sigma_1 \otimes \mathbb{1},\\
\gamma_1 &= \sigma_2 \otimes \mathbb{1},\\
\gamma_2 &= \sigma_3 \otimes \sigma_1,\\
\gamma_3 &= \sigma_3 \otimes \sigma_2,\\
\gamma_4 &= \sigma_3 \otimes \sigma_3,\\
B &= \mathbb{1} \otimes \sigma_2.
\end{aligned} \tag{D.4}$$

A basis for the Euclidean Clifford algebra Γ is

$$\begin{aligned} \Gamma_1 &= \sigma_1 \otimes \mathbb{1}, \\ \Gamma_2 &= \sigma_3 \otimes \sigma_1, \\ \Gamma_3 &= \sigma_3 \otimes \sigma_3, \\ \Gamma_4 &= \sigma_2 \otimes \mathbb{1}, \\ \Gamma_5 &= \sigma_3 \otimes \sigma_2, \\ \Omega &= \sigma_1 \otimes \sigma_2. \end{aligned} \tag{D.5}$$

In the chosen gauging,

$$\begin{aligned} \zeta^{ij} &= -\frac{1}{2\sqrt{2}}\Sigma^2\Gamma_{45}^{ij}, \\ \zeta^{aij} &= 0, \\ \rho^{ij} &= \frac{1}{2\sqrt{2}}\frac{\cosh\phi_3}{\Sigma}\Gamma_{45}^{ij}, \\ \rho^{aij} &= -\frac{1}{2}\delta_1^a\frac{\sinh\phi_3}{\Sigma}\Gamma_{345}^{ij}. \end{aligned} \tag{D.6}$$

Using the explicit solution to the equations of motion, the dilatino and gaugino variations both lead to the projection condition

$$(\Gamma_{45})_i{}^j\,\epsilon_j = \frac{1}{r(H_1H_2)^{1/2}}\left(\gamma_{34}\Gamma_3 - i\sqrt{f}\gamma_4\right)_i{}^j\epsilon_j. \tag{D.7}$$

Substituting this projector into the $\mathrm{AdS}_3 \times S^1$ gravitino variations gives

$$\begin{aligned}
0 &= \left(\partial_t + \frac{1}{2}\sinh\rho\,\gamma_{01} - \frac{i}{2}\cosh\rho\,\gamma_{034}\Gamma_3\right)_i^{\ j}\epsilon_j, \\
0 &= \left(\partial_\rho - \frac{i}{2}\gamma_{134}\Gamma_3\right)_i^{\ j}\epsilon_j, \\
0 &= \left(\partial_\varphi - \frac{1}{2}\cosh\rho\,\gamma_{12} - \frac{i}{2}\sinh\rho\,\gamma_{234}\Gamma_3\right)_i^{\ j}\epsilon_j, \\
0 &= \left(\partial_\theta - \left(\mu_3 - \frac{1}{2}\right)\Gamma_{345}\right)_i^{\ j}\epsilon_j.
\end{aligned} \tag{D.8}$$

These equations can be integrated to

$$\begin{aligned}
\epsilon_i = \exp&\left(\theta\left(\mu_3 - \frac{1}{2}\right)\Gamma_{345}\right)_i^{\ j}\exp\left(\frac{i\rho}{2}\gamma_{134}\Gamma_3\right)_j^{\ k} \\
&\times\exp\left(\frac{it}{2}\gamma_{034}\Gamma_3\right)_k^{\ l}\exp\left(\frac{\varphi}{2}\gamma_{12}\right)_l^{\ m}\tilde{\epsilon}_m(r).
\end{aligned} \tag{D.9}$$

Anti-periodicity of ϵ_i under $\theta \to \theta + 2\pi$ requires the chemical potential to be quantized $\mu_3 \in \mathbb{Z}$. After multiplying by $\gamma_{34}\Gamma_3$, the projection condition can be expressed in the form

$$\left(1 + i\sqrt{f}\gamma_3\Gamma_3 + r\sqrt{H_1 H_2}\gamma_{34}\Gamma_{345}\right)_i^{\ j}\epsilon_j = 0. \tag{D.10}$$

Similarly multiplying by Γ_{45} leads to

$$\left(1 - i\frac{\sqrt{f}}{r\sqrt{H_1 H_2}}\gamma_4\Gamma_{45} + \frac{1}{r\sqrt{H_1 H_2}}\gamma_{34}\Gamma_{345}\right)_i^{\ j}\epsilon_j = 0. \tag{D.11}$$

Using these equations, the radial gravitino equation can be put into the form

$$\partial_r\epsilon_i = \left(a + b\gamma_{34}\Gamma_{345}\right)\epsilon_i. \tag{D.12}$$

The solution to equations of this form [149] is

$$\tilde{\epsilon}_i(r) = \frac{1}{r(H_1H_2)^{1/6}} \left(\sqrt{r\sqrt{H_1H_2}+1} + i\gamma_4\Gamma_{45}\sqrt{r\sqrt{H_1H_2}-1} \right)_i{}^j \times (1-\gamma_{34}\Gamma_{345})_j{}^k (\epsilon_0)_k \,, \tag{D.13}$$

for some constant symplectic Majorana spinor ϵ_0. It can be checked explicitly that the above Killing spinor satisfies the symplectic Majorana condition.

APPENDIX E

Euclidean 1/2-BPS Line Defect Solution

A 1/2-BPS solution describing a superconformal line defect can be constructed in the Euclidean version of pure Romans' supergravity. In the notation of [136], the supersymmetry variations are

$$\begin{aligned} \delta\psi_\mu &= D_\mu\epsilon - \frac{1}{12}\gamma_\mu W\hat{\sigma}_3\epsilon + \frac{i}{12}\left(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu\gamma^\rho\right)h_{\nu\rho}\epsilon, \\ \delta\chi &= -\frac{i}{2\sqrt{2}}\left(\gamma^\mu\partial_\mu\lambda + \partial_\lambda W\hat{\sigma}_3 + i\gamma^{\mu\nu}\partial_\lambda h_{\mu\nu}\right)\epsilon, \end{aligned} \tag{E.1}$$

with

$$\begin{aligned} W &= 2(2X + X^{-2}), \\ h_{\mu\nu} &= X^{-1}\left(F^i_{\mu\nu}\hat{\sigma}_3\sigma_i + B^+_{\mu\nu}\hat{\sigma}_- + B^-_{\mu\nu}\hat{\sigma}_+\right) - iX^2 f_{\mu\nu}, \\ X &= e^{-\lambda/\sqrt{6}}. \end{aligned} \tag{E.2}$$

The superconformal line defect preserves an SO(1,2)×SO(3) bosonic symmetry which can be realized by the ansatz

$$\begin{aligned} ds^2 &= f_1(y)^2 ds^2_{\mathbb{H}^2} + f_2(y)^2 d\Omega_2^2 + f_3(y)^2 dy^2, \\ B^- &= C_1(y)\mathrm{vol}_{\mathbb{H}^2} + C_2(y)\mathrm{vol}_{S^2}. \end{aligned} \tag{E.3}$$

A similar solution containing only these fields was analyzed in [136]. Imposing the projection condition $\hat{\sigma}_3\epsilon = \epsilon$, gives

$$\begin{aligned} \delta\psi &= D_\mu\epsilon - \frac{1}{2}\gamma_\mu\epsilon, \\ \delta\chi &= 0, \end{aligned} \tag{E.4}$$

which are the BPS equations describing AdS_5. Thus the tensor field B^- breaks half the supersymmetries and does not backreact on the metric. $C_1(y)$ and $C_2(y)$ are determined by the tensor field equation of motion

$$dB^- + *B^- = 0. \tag{E.5}$$

The full solution is

$$\begin{aligned} f_1 &= \cosh y, \\ f_2 &= \sinh y, \\ f_3 &= 1, \\ C_1 &= \frac{a}{\sinh y} + b\left(\frac{y}{\sinh y} + \cosh y\right), \\ C_2 &= \frac{a}{\cosh y} + b\left(\frac{y}{\cosh y} - \sinh y\right). \end{aligned} \tag{E.6}$$

Using the coordinates

$$
\begin{aligned}
ds^2_{\mathbb{H}^2} &= \frac{d\tau^2 + dx^2}{x^2}, \\
d\Omega_2^2 &= d\theta^2 + \sin^2\theta d\phi^2,
\end{aligned}
\tag{E.7}
$$

the solution can be mapped to Euclidean Poincaré coordinates

$$
ds^2 = \frac{1}{z^2}\left(d\tau^2 + dz^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right), \tag{E.8}
$$

through the coordinate transformation

$$
z = \frac{x}{\cosh y}, \qquad r = x\tanh y. \tag{E.9}
$$

In this coordinate system, the tensor field takes the form

$$
\begin{aligned}
B^- &= \tilde{C}_1 d\tau\wedge dr + \tilde{C}_2 d\tau\wedge dz + \tilde{C}_3\sin\theta d\theta\wedge d\phi, \\
r^{-1}\tilde{C}_1 = z^{-1}\tilde{C}_2 &= \frac{1}{(r^2+z^2)^{3/2}}\left[a\frac{z}{r} + b\left(\frac{z}{r}\sinh^{-1}\left(\frac{r}{z}\right) + \frac{\sqrt{r^2+z^2}}{z}\right)\right], \\
\tilde{C}_3 &= a\frac{z}{\sqrt{r^2+z^2}} + b\left(\frac{z}{\sqrt{r^2+z^2}}\sinh^{-1}\left(\frac{r}{z}\right) - \frac{r}{z}\right),
\end{aligned}
\tag{E.10}
$$

and the leading behavior of the tensor field at the boundary is

$$
B^- \sim \left(\frac{br}{z} + \frac{az}{r}\right)\frac{d\tau\wedge dr}{r^2} + \left(-\frac{br}{z} + \frac{az}{r}\right)\sin\theta\ d\theta\wedge d\phi, \tag{E.11}
$$

giving the source and vacuum expectation values of the dual $\Delta = 3$ operator. Since the spacetime is Euclidean AdS_5, the dual stress tensor vanishes

$$
\langle T_{ij}\rangle = 0. \tag{E.12}
$$

The solution can be uplifted to type IIB supergravity or $D = 11$ supergravity [144, 146], but the two-form fields become complex when Wick rotating back to Lorentzian signature.

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