Lawrence Berkeley National Laboratory

Recent Work

Title

RECOIL TECHNIQUES IN NUCLEAR REACTION AND FISSION STUDIES

Permalink

<https://escholarship.org/uc/item/0378g4c5>

Author

Harvey, B.G.

Publication Date

1960-04-01

UNIVERSITY OF
CALIFORNIA

Ernest O. Lawrence

*Radiation
Laboratory*

BERKELEY, CALIFORNIA

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

For pub. Annual Review of Nuclear Science.

UCRL-9151

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No- W-7405-eng-48

RECOIL TECHNIQUES IN NUCLEAR REACTION AND FISSION STUDIES

B. G. Harvey

April 1960

RECOIL TECHNIQUES IN NUCLEAR REACTION AND FISSION STUDIES

B. G. Harvey

Lawrence Radiation Laboratory
University of California, Berkeley, California

April 1960

INTRODUCTION

Nuclear reactions are traditionally studied by measurement of reaction cross sections, differential cross sections, angular distributions and energy spectra of emitted particles or gamma rays and more recently by measurements of the polarization of emitted particles and angular correlations. Except for radiochemical determinations of reaction cross sections, all these techniques involve the observation of emitted light particles. The momenta and angular distributions of the recoiling product nuclei are connected to the momenta and angular distributions of the emitted particles by the requirements of momentum and energy conservation. Thus the study of the recoiling nuclei may in some cases be an excellent method for obtaining information about a nuclear reaction.

Because of their low velocities, the recoil nuclei have short ranges, and scatter very badly. For these reasons, measurements based on emitted light fragments and particles are usually more accurate, but some kinds of information

cannot be obtained in this way. Such cases arise when the nature of the nuclear reaction leading to a specific product is the subject of interest. For example, measurement of the spectrum and angular distribution of neutrons emitted from Au^{197} under bombardment with 50 Mev helium ions would not easily yield information about the reaction $\text{Au}^{197}(\alpha, 2n)\text{Tl}^{199}$, because only an insignificant fraction of the total yield of neutrons would arise from this reaction. However, it is possible that the information could be obtained by studying the kinetic energy and angular distribution of the recoiling Tl^{199} atoms. The essential point is that the Tl^{199} could be distinguished from the other products by means of its chemical and radioactive decay properties, whereas the neutrons from the $(\alpha, 2n)$ reaction are of course indistinguishable from those of the $(\alpha, 3n)$ and higher reactions.

Recoil studies are particularly useful when a large number of different nuclear reaction products are formed simultaneously, for example in the study of fission and of high energy spallation reactions. The results which have been obtained are usually only semi-quantitative, for a variety of reasons. First the formation of a wide range of products in a single bombardment necessarily

implies a low cross section for each individual species. Second, the high energy accelerators produce beams of small intensity, and long bombardment periods are not easily obtained. For these reasons, it has been necessary for most workers to use thick targets in order to obtain measurable yields of the products of interest, and the interpretation of thick target results is far from straightforward. Third, the fundamental processes of scattering and stopping of heavy recoils are understood only semi-quantitatively. In particular, the conversion of experimental ranges into kinetic energies has often been accomplished with the crudest of range - energy relationships, and the effects of scattering have been neglected. Fourth, scattering becomes particularly severe when the recoil energies are low. Hence the advantages of the high beam intensities which are available from low energy accelerators are offset by the necessity of using extremely thin targets to minimize scattering. An ideal application of recoil methods seems to be in the study of nuclear reactions in which many particles are emitted, since coincidence counter measurements of the light particles are limited by low counting rates.

The present review will be restricted to a discussion of the techniques that have been used in experiments involving the measurement of angular distributions and ranges of recoils which were identified by their chemical and radioactive properties. The behaviour of recoiling fission fragments was reviewed in 1957 by Walton (1). The fission process was reviewed from a less specialized point of view in 1959 by Halpern (2). High energy nuclear reactions were reviewed in 1959 by Miller and Hudis (3) and Harvey (4).

EXPERIMENTAL METHODS IN FISSION AND SPALLATION STUDIES

The quantities which can be measured are the angular distributions and ranges of recoil atoms. Detailed angular distributions of fission fragments have been obtained in several experiments, but rarely have they been measured in spallation reactions because of the intensity problems discussed above. More commonly, the fractions of the recoils escaping from a thick target into the forward, backward, and perpendicular directions were measured. Kinetic energies were almost invariably measured by range determinations. Measurements of momenta by magnetic analysis are possible, but they are complicated by the difficulty that a given nuclear reaction product is usually formed in a wide range of charge states. For example, the most probable

charge carried by atoms of mass 97 formed in the fission of U^{235} was found to be +21 (5), but charge states +20 and +22 were also present in sufficient abundance to interfere seriously with the momentum analysis.

The different experimental methods may be roughly divided into two categories. In the first, the target layer is made very thin with respect to the range of the recoils of interest. The emerging recoils therefore preserve the range distribution and angular distribution with which they were formed, and these distributions can be measured. The disadvantage of this method is that very thin targets produce very small yields of recoils, and hence radio-chemical identification of the products may be difficult. In practice, the thin target method is most suitable for the study of products which are very easily identified, especially α -emitters, for which accurate yield measurements may be made even with low counting rates.

In the second method, the target is of a thickness which is large with respect to the range of the recoils. By measuring the fractions of a given product which recoil from the front and back faces of the target, it is possible to measure an average recoil range in the target material, and the quantity η , where η is the ratio v/V , and v is the average velocity (in the

laboratory system) imparted to the recoil by the impact of the incident particle, and V is the average velocity (in the system of the moving recoil) imparted to the struck nucleus by the particles which are emitted. Figure 1 shows the relationship between η and the vectors v and V . Thus the results obtained by the thick target method are less direct, but many experimental difficulties are avoided.

Thin target method : ranges

For heavy element targets bombarded with low energy particles, such as 20 Mev deuterons, ranges of recoils may be as low as $20 \mu\text{g}/\text{cm}^2$ (6). Hence "thin" targets must be $1 \mu\text{g}/\text{cm}^2$ or less, which corresponds to a monatomic layer of a heavy element. Suitable targets may be prepared by standard methods of vacuum evaporation or by electrodeposition. Under bombardment with charged particles, thin targets deteriorate rather rapidly. Yellow or brown deposits form on the target surface; they appear to be caused by radiation decomposition of vacuum pump oil deposits. Clean vacuum techniques minimize this difficulty.

Ranges of recoils have been measured in stacks of metallic catcher foils or plastic films, or in gases. The plastic films suffer severe radiation

damage and become extremely fragile; their only advantage is that they can be made as thin as $1 \mu\text{g}/\text{cm}^2$. Films of the plastics known as VYNS were used by Harvey, Wade, and Donovan (7). (Methods for making VYNS films and for measuring their thickness have been described by Pate and Yaffe (8).) Such films were found to withstand bombardment with a few tenths of a microampere of helium ions for several hours, provided that the beam area was small compared with the area of the films. A beam spot of 1/4-inch diameter with films 1 inch in diameter was found to be satisfactory. Following a bombardment long enough to generate a sufficient number of recoil nuclei, differential range curves were obtained by analysis of the catcher foils for the required nuclear reaction products. The ranges were, however, not very reproducible. When the ranges to be measured were sufficiently great, metal foils (usually aluminum) were preferred by most workers.

If a thin source which is emitting recoils isotropically is pressed close to a stack of absorber foils, then the range of the recoils in the material of the foils may be measured by analysis of the foils in the stack. The number of recoils A_t penetrating the stack to a depth t is given by $A_t = \text{const.} (R - t)$

where R is the range. This method was first used by Segrè and Wiegand (9) and later by Finkle, Hoagland, Katcoff, and Sugarman (10), Douthett and Templeton (11) and many others, to measure the ranges of fission fragments. The advantages of the method are that collimation of the beam of recoils is not required, and that the source may be of any area.

Katcoff, Miskel and Stanley (12) measured the ranges in air of several products from the fission of Pu^{239} with slow neutrons. The recoiling fission products passed through a stack of very thin ($15 \mu\text{g}/\text{cm}^2$) plastic films, between which there were air gaps. Virtually all the energy loss occurred in the air gaps, but the fragments were caught in or on the plastic film closest to the end of their range. Analysis of the films gave excellent differential range curves. The method was found not to be suitable for the study of fission products which contained a rare gas of half life greater than 20 seconds among their precursors; diffusion of the gas before decay distorted the differential range curve. The results given by these authors are ranges in air at 760 mm pressure and 15°C temperature, and not at S.T.P.

Integral range curves in gases have been measured by placing a source of recoils at a fixed distance from a "catcher" and varying the gas pressure, or by keeping the gas pressure constant and varying the distance (13,14).

Another method for the measurement of differential ranges in gases was developed by Ghiorso and Sikkeland (15). A collimated beam of recoils passes through a gas, and the stopped ions are collected on a catcher plate by the application of a small electrostatic field normal to the recoil direction. The apparatus is represented schematically in Figure 2. After a suitable period of collection, the catcher plate may be cut into strips, and the pieces analyzed for individual recoil products. A differential range curve obtained in this way is shown in Figure 3, as well as curves obtained by Katcoff, Miskel and Stanley (12).

The recoil atoms must remain ionized to the very end of their range if they are to be collected electrostatically. Ghiorso was able to collect gross fission fragments from the spontaneous fission of Cf^{252} in high yields, using air as the stopping gas. Valyocsik (16) collected Th^{226} and Ra^{224} recoils stopped in H_2 , D_2 , He , N_2 , Ne or A , but Valyocsik, Donovan, and Morton (17) were unable to obtain a high yield of At or Po stopped in He . Collecting fields of about 100 volts per centimeter were used in all cases.

Using ~ 0.1 Mev Ra^{224} recoils arising from the α -decay of Th^{228} , Valyocsik found that, when the collimator (Figure 2) was made of polystyrene, all the recoils appeared on the leading edge of the catcher plate regardless of the gas pressure. Substitution of a metal collimator removed the difficulty. It appears likely that the polystyrene acquired surface charges at a sufficiently high potential (10^5 volts or more) to stop the recoil ions.

The electric collecting potential will have little effect upon the trajectory of the recoils until their kinetic energies are extremely low. The path of the recoils should therefore break sharply downwards, and the final position of a recoil on the catcher plate should be vertically below the end of its range. Valyocsik (16) considered the possibility that the differential range curve is broadened by diffusion of the ions in their relatively slow drift down the electric field lines. On the assumption that the recoil atoms are always charged, he was able to show that the range straggling introduced in this way should be negligible by comparison with the true range straggling even in hydrogen (where range straggling should be minimal). However, if the drifting ions spend an appreciable part of the time in an uncharged state, then the diffusion might be much greater. In

this case, however, it seems unlikely that a high collection efficiency would be achieved.

The electrostatic collection method shows great promise, but further experiments are needed to establish it as a reliable technique. The ranges of Ra^{224} recoils measured by Valyòcsik agree reasonably well with the integral range measurements in gases of Baulch and Duncan (13), but the range stragglings which he obtained do not agree with theoretical estimates in the lighter gases (see below), and it seems possible that the electrostatic method introduced some constant broadening to the differential range curves.

Alexander and Gazdik (18) used targets that were not of negligible thickness, but they were able to obtain accurate ranges of fission fragments. Most workers have added on to the measured ranges a correction for the energy loss in the target material, and have tried to minimize the correction by keeping the targets as thin as possible while still obtaining a measurable yield of recoils. Alexander and Gazdik showed that the fraction F_t of the total yield of a given recoil species which succeeds in passing through a thickness t of external absorber is given by

$$F_t = 1/2 (1 - t/\bar{R} - cW/2\bar{R}) \quad (1)$$

where W is the target thickness and $\bar{R}$ is the average reciprocal range in the external absorbers, (i.e. $\bar{R} = (\langle 1/R \rangle)^{-1}$). Equation 1 assumes that the recoils are emitted isotropically from a stationary nucleus, that the target is in close contact with the external absorber stack (50% geometry), and that the rate of velocity loss in the target material is proportional to the rate of velocity loss in the external absorbers: - $(dV/dR)_{\text{absorber}} = c (dV/dR)_{\text{target}}$. From Equation 1, $(\partial F_t / \partial W)_t = -c/4\bar{R}$, and the constant c can be evaluated by measuring the change of F_t with respect to the target thickness W . This method is equivalent to measuring a range as a function of target thickness, and extrapolating the results to zero thickness.

Suzor (19) measured ranges of individual fission products in aluminum and gold foils, using an external absorber stack. He found that the thickness of a uranium oxide layer formed by electrodeposition on a copper surface was substantially greater than merely the weight of the oxide layer; presumably the electrodeposition process attacked the copper. Layers deposited on nickel were more satisfactory.

Alexander and Gazdik (18) made measurements in which one of the aluminum foils adjacent to the target was replaced by a gold foil. They found that some

backscattering of the recoils occurred in the gold foil, but they were able to make a correction for this effect and to obtain the ranges of selected fission fragments in gold. Serious backscattering will undoubtedly occur in the target material, especially when it is a heavy element. Until this problem is more completely understood, figures quoted for the accuracy of range measurements should be viewed skeptically.

Thin target method : angular distributions

Detailed angular distributions of the products of spallation reactions have been obtained by only two groups of investigators (6,7,20). Donovan et al. (6) found that scattering of the recoils in the target material was a severe problem. To obtain accurate angular distributions of At^{211} recoils generated by the reaction $\text{Bi}^{209}(\alpha, 2n)\text{At}^{211}$ with helium ions between 20 and 48 Mev, it was necessary to use bismuth targets with a surface density of only about 1 $\mu\text{g}/\text{cm}^2$, which is approximately a monolayer of bismuth.

Ball, Fairhall, and Halpern (20) studied the reactions $\text{Ni}^{58}(\alpha, \gamma)\text{Zn}^{62}$ and $\text{Co}^{59}(\alpha, 2n)\text{Cu}^{61}$ at 30 Mev. Since their targets were of lower atomic number, the recoil products were of higher velocity, and scattering was less severe; they

were able to use target thicknesses as high as $200 \mu\text{g}/\text{cm}^2$. Further, they were mainly interested in recoils emitted almost normal to the target surface, and these should scatter less than those emitted obliquely, since they travel through a smaller thickness of target material.

Both groups allowed the recoils to travel through an evacuated space until they were intercepted by a plane aluminum foil normal to the cyclotron beam, and thick enough to stop all the recoils. The foil was subsequently cut into concentric rings which were analyzed for the various radioactive products of the nuclear reactions.

Thick target methods

In many nuclear reactions, especially those induced by heavy incident particles, the momentum imparted to the target nucleus by the incident particle is greater than the combined momenta of all the emitted secondary particles. No matter in what direction the secondary particles are emitted, the final recoiling nucleus will always have a momentum component in the direction of the incident particle. Thus all recoil nuclei will leave a target from the front (down beam) face. If the target is thick, only those product nuclei formed

sufficiently close to its front face will be able to escape: nuclei formed at greater depths will come to rest within the target. An average range $\langle R \rangle$ of a particular reaction product in the material of the target is obtained from

$$\langle R \rangle = N_c T / (N_c + N_t) \quad (2)$$

where N_c is the number of recoils which escape from the target, N_t is the number which remain in the target, and T is the target thickness.

In other reactions, the combined momentum of the emitted particles may exceed the momentum imparted by the incident particle, so that recoil nuclei may leave a target from both the front and back faces, depending on the relative orientation of the two momenta. By sandwiching a target of thickness W between two catcher foils thick enough to stop the recoil nuclei, the fraction of the total number of product nuclei which recoil in the forward and backward directions may be measured. Measurements of this kind have been made by Sugarman, Campos, and Wielgoz (21), Porile and Sugarman (22), Fung and Perlman (23), and Porile (24). Sugarman, Campos, and Wielgoz showed that the fractions R_f and R_b of the total number of product nuclei recoiling in the forward and backward directions respectively were related to the average range of the recoils in the target material by the equation

$$\begin{aligned}\langle R \rangle &= 4 W R_f / (1 + \eta)^2 \\ &= 4 W R_b / (1 - \eta)^2\end{aligned}\tag{3}$$

The quantity η was defined above. It is related to R_f and R_b by the equation

$$\eta = \frac{(R_f/R_b)^{1/2} - 1}{(R_f/R_b)^{1/2} + 1}\tag{4}$$

In the derivation of these equations, it was assumed that the momentum due to the emission of secondary particles is isotropic, that the range of recoil atoms is proportional to their velocity, that the recoil paths are straight, and that the velocities involved are non-relativistic. Porile and Sugarman (22) and Winsberg (25) discussed the situation for distributions of momentum of secondary particles of the forms $(a + b \cos^2 \theta)$ and $(a + b \sin^2 \theta)$. Winsberg also derived the equations which arise when the range of the recoil atoms is proportional to their energy rather than to their velocity: this situation may arise when the velocities are below about 2×10^8 cm/sec.

Porile and Sugarman (22) measured the amounts of various fission products (from the bombardment of bismuth and tantalum with 450 Mev protons) which were projected in the forward, backward, and perpendicular directions relative to the

incident particles. From these measurements, they were able to extract the average kinetic energies of the various fragments (using an approximate range-energy relationship), the average energy deposited in the struck nucleus by the incident particle for the formation of the various products, and the ratio of the parameters b/a in the angular distributions. It was possible to distinguish between the $(a + b \cos^2 \theta)$ and $(a + b \sin^2 \theta)$ distributions by comparison of the average forward - backward ranges R_{FB} with the perpendicular ranges R_P . The former distribution gives $R_{FB} > R_P$, and the latter $R_{FB} < R_P$. Experiments of this kind require the greatest possible accuracy in the estimation of the yields of recoils stopped in the various foils, and it is not clear to which coordinate system the angular distributions belong.

Wolke and Gutman (26) bombarded with 450 Mev protons a brass sphere coated with a 15 mg/cm^2 thick layer of bismuth, and caught the recoiling fission fragments on a conical catcher which was subsequently sectioned and analyzed for the individual species. A section of the cone was cut away to permit passage of the beam. They were able to obtain the angular distribution of each species in the center of mass system, making the assumption that the

fissioning nucleus was moving in the direction of the incident beam at the moment of fission. (There was some evidence in their work that this was not a good assumption.) Ramaniah and Sugarman (31) re-investigated the method used by Wolke and Gutmann, and found that it gave incorrect results when the intensity of the cyclotron beam at the target varied radially, because the missing sector of the catcher prevented proper averaging of the yield of recoils over the azimuthal angle. Angular distributions of the fission fragments using catcher foils placed at a few angles around the target have been measured by Cohen and co-workers (27), Winhold and Halpern (28), Coffin and Halpern (29) and Meadows (30).

RANGE-ENERGY RELATIONS

Theory

The theory of the penetration of atomic particles into matter has been studied by several workers (32-39), but especially by Bohr (39), and the main features of the various types of interaction are reasonably well understood. At sufficiently high velocities, the charged moving particle interacts with the atomic electrons of the stopping medium, causing ionization and excitation. Collisions with electrons cause the moving particle to lose those electrons whose

orbital velocities are less than the velocity of the particle. Its ionization, and hence rate of energy loss, will therefore be greater at the beginning of its range when its velocity is greatest (40). Very roughly, the charge of the recoil, Z^* , is related to its velocity V and atomic number Z by $Z^* = Z^{1/3} V/V_0$, where $V_0 = e^2/\hbar^2 = 2.2 \times 10^8$ cm per sec. (Bohr (39)). The equation predicts charges of about +20 for fission fragments at the beginning of their range, and experimental values in agreement with this estimate have been obtained by Perfilov (41), Lassen (42), and Cohen, Cohen, and Coley (5). No reliable theoretical estimates have been made for the total range of particles as a function of their initial velocity, but very approximately, the range should be a linear function of the initial velocity (39).

At velocities below V_0 , ionization and excitation become a relatively small part of the energy loss mechanism, and energy loss by elastic collisions (so-called nuclear stopping) becomes increasingly important. This does not mean that slow particles produce no ionization. In fact the density of ionization remains high, as cloud chamber, counter and nuclear emulsion studies show (Bøggild, Brostrøm, and Lauritsen (43), Lassen (44), Demers (45), Mathieu and

Demers (46)). However, the average energy loss per ion pair formed is greater for slow than for fast particles (47).

In the nuclear stopping region, the behaviour of the recoils is dependent on the ratio of the masses of the moving and stopping atoms, M_1/M_2 . If the ratio is large, then the average energy loss per collision will be small, and the recoils will follow a path which does not deviate seriously from a straight line. However, if the ratio is unity or less, the moving particles can be deflected through large angles; if the ratio is close to unity the particles can lose a large fraction of their energy in a single encounter. It therefore becomes rather meaningless to use the concept of a range; when $M_1 < M_2$ the process should be discussed in terms of diffusion theory (37).

Bohr (39) assumed that the interaction of two atoms in the nuclear stopping region was governed by a potential of the form $P(r) = Z_1 Z_2 e^2 a / e r^2$, where Z_1 and Z_2 are the atomic numbers of the moving and stopping atoms, e is the electronic charge, e is the base of natural logarithms, and r is the distance between the two atoms. The screening parameter a is equal to $a_0 / (Z_1^{1/3} + Z_2^{1/3})^{1/2}$ where a_0 is the "radius" of the hydrogen atom, $\hbar^2 / \mu e$. With this

potential, the relationship between the velocity of a particle and its mean range R_0 was found to be given by:-

$$R_0 = \frac{e^2}{\pi^2 a_0^2} \cdot \frac{M_1 + M_2}{2\mu N} \cdot \frac{V^2}{V_0^2} \cdot \frac{\sqrt{Z_1^{2/3} + Z_2^{2/3}}}{Z_1 Z_2} \quad (5)$$

where N is the volume density of the atoms of the stopping medium. The equation reduces to the form

$$R_0 \text{ (micrograms/cm}^2\text{)} = kE \cdot \frac{M_2 (M_1 + M_2)}{M_1} \cdot \frac{\sqrt{Z_1^{2/3} + Z_2^{2/3}}}{Z_1 Z_2} \quad (6)$$

where E is the kinetic energy of the recoil, and k is a constant whose value is 602 when E is expressed in Mev.

The distribution of ranges about the mean range R_0 is expected to be given by the Gaussian expression

$$W(R) = \frac{1}{(2\pi)^{1/2} \rho R_0} \cdot \exp \left[- (R_0 - R)^2 / 2 \rho^2 R_0^2 \right] \quad (7)$$

and Lindhard and Scharff (38) gave the following expression for ρ , the range straggling parameter:-

$$\rho^2 = 2 M_1 M_2 / [3 (M_1 + M_2)^2] \quad (8)$$

A similar expression, differing only by a factor of two, was obtained by Bohr (39).

Gaussian range distributions were indeed observed in several experiments (16,48,49) but not in others (12,50). Range straggling parameters measured by

Valyocsik (16) in argon and by Alexander and Winsberg (48) are in better agreement with Equation 8 than with the equation given by Bohr.

The mean range given by Equations 5 and 6 is in fact the total distance travelled by a moving particle in coming to rest, but the experimental ranges are the vector sums of the projections on a fixed direction of the paths of the moving particle between collisions. For $M_1 \gg M_2$, the path should be nearly straight, so that the true range and the experimental range should be nearly identical. However, for $M_1 \approx M_2$, the projected (experimental) range should be less than the true range, and the constant k of Equation 6 should be some function of M_1 and M_2 .

In the high energy (electronic stopping) region, Bohr (39) discussed range straggling. He concluded that the observed straggling should arise almost entirely from the elastic collision processes occurring at the end of the range of the particle. The straggling will therefore depend on M_1 and M_2 through Equation 8, and on the fraction of the range which lies in the nuclear stopping region.

In the following sections, an attempt has been made to discuss the more reliable range-energy and straggling measurements, first those in which the

recoil energies lie in the elastic collision region, and then those at higher energies. The results of the low energy experiments are summarized in Table I; the high energy experiments are summarized in Table II.

Experimental results : low energy region

Davies (50) has reported preliminary values for the ranges of 30 kev Na^{24} , Rb^{86} , and Cs^{137} ions in aluminum and for Cs^{137} ions in germanium. He allowed the electrostatically accelerated ions to fall upon an aluminum surface, and then obtained differential range curves by anodic oxidation of the surface to a known depth, after which the oxide film was stripped and analyzed. Repeated application of this step gave the differential range curves. The curves were not symmetrical about the most probable range. Ranges in germanium were measured by allowing the ions to fall upon a thin germanium layer on an aluminum backing. The germanium was subsequently dissolved off and analyzed.

Recoil atoms resulting from the formation and decay of a compound nucleus should have an average momentum nearly equal to the momentum of the incident particle. The evaporation of particles will cause the recoil momentum to be modified, but close to the threshold of the reaction, the particles must be

Table I. Ranges of Monoenergetic Ions in the
Nuclear Stopping Region

Moving Ion	Energy (Mev)	Stopping Medium	Mean Range ($\mu\text{g}/\text{cm}^2$)	Straggling Parameter		k	M_1/M_2	Ref.
				Exp.	Calc.			
Cs ¹³⁷	0.004	Ge	1.5	--	--	1200	1.89	50
"	0.020	"	8.0	--	--	1280	"	"
"	0.040	"	11	--	--	882	"	"
Na ²⁴	0.030	Al	15.0	--	--	386	0.89	"
Rb ⁸⁶	0.030	"	8.4	--	--	930	3.18	"
Cs ¹³⁷	0.030	"	7.6	--	--	1250	5.07	"
Ra ²²⁴	0.097	H ₂	3.9	0.136	0.0543	725	224	16
"	0.097	D ₂	6.6	0.145	0.0765	656	112	"
"	0.097	He	6.6	0.162	0.107	644	56	"
"	0.097	N ₂	7.0	0.238	0.192	619	16	"
"	0.097	Ne	7.3	0.264	0.224	624	11.2	"
"	0.097	A	9.4	0.315	0.293	624	5.6	"
Po ²¹⁸	0.101	Air	8.27	--	--	674	15.2	13
Tl ²⁰⁸	0.116	H ₂	5.0	--	--	777	208	"

(continued)

Table I. (continued)

Moving Ion	Energy (Mev)	Stopping Medium	Mean Range ($\mu\text{g}/\text{cm}^2$)	Straggling Parameter		k	M_1/M_2	Ref.
				Exp.	Calc.			
Tl ²⁰⁸	0.116	Air	8.66	--	--	598	14.5	13
"	0.116	A	10.5	--	--	547	5.2	"
At ²¹⁰	0.676	Bi	100	--	--	382	1.00	7
At ²⁰⁹	0.676	"	121	--	--	380	1.00	"
Th ²²⁶	0.725	D ₂	32	--	--	429	113	16
"	0.725	He	30	--	--	394	56.5	"
"	0.725	N ₂	38	--	--	456	16.2	"
"	0.725	A	40	--	--	365	5.65	"
At ²⁰³	2.80	Ag	476	0.40	0.389	382	1.88	49
At ²⁰⁵	2.20	"	434	0.39	0.389	448	1.88	"
Tb ¹⁴⁹	4.0	Al	350	0.28	0.294	497	5.52	48
At ²⁰³	4.0	"	275	0.28	0.263	498	7.53	"
"	4.0	Au	510	0.43	0.408	358	1.04	"
Au ¹⁹⁶	0.014 ^a	"	5.8	--	--	1100	0.99	51
Ag ^{106m}	0.022 ^a	Ag	7.7	--	--	690	0.98	"

(continued)

Table I (continued)

Moving Ion	Energy (Mev)	Stopping Medium	Mean Range ($\mu\text{g}/\text{cm}^2$)	Straggling Parameter		k	M_1/M_2	Ref.
				Exp.	Calc.			
Mo ⁹¹	0.026 ^a	Mo	7.3	--	--	540	0.95	51
Cu ⁶²	0.040 ^a	Cu	12.6	--	--	482	0.98	"
Cu ⁶⁴	0.040 ^a	"	13.6	--	--	510	1.01	"
Zn ⁶³	0.040 ^a	Zn	9.1	--	--	364	0.97	"
Ti ⁴⁵	0.053 ^a	Ti	14.7	--	--	353	0.94	"

a. These recoils were not monoenergetic. They are included because there are so few data in this energy range.

emitted with very low energies. Thus the recoils should be monoenergetic, with well-known energies. Valyocsik (16) measured differential range curves in various gases for 0.725 Mev Th^{226} recoils produced by the reaction $\text{Ra}^{226}(\alpha, 4n)\text{Th}^{226}$. In these experiments, the straggling in range was partly caused by the momentum carried away by the four neutrons, but if the neutrons were emitted isotropically, then the most probable range should have been very little affected. Valyocsik also studied the ranges of monoenergetic Ra^{224} atoms produced by the α -decay of Th^{228} .

Ranges of (monoenergetic) recoils generated by the α -decay process have been measured by a large number of workers. The results have been summarized by Baulch and Duncan (13), who also measured integral range curves for Tl^{208} and Po^{218} recoils in various gases.

Harvey, Wade, and Donovan (7) measured the ranges in bismuth of At^{209} and At^{210} recoils from the reactions $\text{Bi}^{209}(\alpha, 3n)\text{At}^{210}$ and $\text{Bi}^{209}(\alpha, 4n)\text{At}^{209}$, using the thick target method. Near the reaction thresholds, where the neutrons must be emitted with low energies, the recoils should have been monoenergetic.

Alexander and Winsberg (48) generated recoils from heavy ion-induced compound nucleus reactions such as $\text{Pr}^{141}(\text{C}^{12}, 4n)\text{Tb}^{149}$ and $\text{Au}^{197}(\text{C}^{12}, 6n)\text{At}^{203}$.

They were able to measure both ranges and range straggling in aluminum and gold foils at several energies. The emission of the neutrons was shown to cause only a negligible increase in the measured range straggling of recoils from heavy-ion induced reactions, and the straggling parameters are in good agreement with Equation 8. Their results for 4 Mev recoils are shown in Table I. Similar results were obtained by Leachman and Atterling (49) for the ranges of At^{203} and At^{205} recoils in aluminum and silver. However, their bombardments were made with a continuous energy spectrum of C^{12} ions, and the actual particle energies responsible for the formation of the astatine products were not accurately known. The energy of Leachman and Atterling's recoils may be obtained from their ranges in aluminum combined with Winsberg and Alexander's range-energy curve for astatine recoils in aluminum. The energies and ranges in silver may then be used to define two points on a range energy curve for At^{203} or At^{205} in silver. Again the range straggling parameters agree well with Equation 8.

Schmitt and Sharp (51) studied the ranges of several recoil species resulting from (γ, n) reactions. The momentum of the recoils derived almost entirely from the evaporated neutrons and neither the neutrons nor the recoils were mono-energetic.

The results which they obtained for recoils moving in elementary matrices are shown in Table I.

All the ranges shown in Table I are in the energy region for which range is expected to be proportional to energy, and Equations 6 and 8 should apply, at least when $M_1 \gg M_2$. Column 7 of Table I shows the values of k (Equation 6) calculated from the experimental ranges; they lie within the interval 320 to 1280, and many of them are close to the value 600. Column 8 of Table I shows the values of M_1/M_2 , and Figure 4 shows the variation of k with the mass ratio. As expected, k increases with increasing mass ratio, but it is also a function of the initial energy of the moving particles. At the lowest energies, k exceeds the theoretical value; at higher energies, k is substantially less than the theoretical value, even for large values of M_1/M_2 . This effect is perhaps due to the greater contribution of electronic stopping at the higher energies. Unfortunately, there are not yet sufficient data on ranges as a function of E , M_1 , and M_2 to establish a universal empirical range-energy relationship.

Except for the results obtained by Valyocsik for the light gases, Table I shows that there is good agreement between the experimental range straggling

parameters and the values calculated from Equation 8. Alexander and Winsberg (48) studied the variation of ρ with recoil energy. Figure 5 shows their results; ρ is plotted against the reciprocal of the range of the recoil expressed in mgm/cm^2 . In the nuclear stopping region at the right of the figure, where Equation 8 is expected to apply, ρ is independent of recoil range.

Experimental results : high energy region

As the velocity of the recoils approaches V_0 , gradually increasing departures from Equation 5 should be observed, and the ranges should become more nearly proportional to velocity than to energy (velocity squared). For recoils of mass 200, the velocity V_0 corresponds to a kinetic energy of 5 Mev. Except for a few recent experiments in which recoils were generated by bombardment with heavy ions, all the experimental results in the high energy region were obtained from the fission process, and the energies of the recoils are very far above the critical region around V_0 . The ranges of gross fission products or of the light and heavy groups have been measured many times (9,44-46,56-66). In several cases, the ranges of individual fission products were obtained (12,18,19,52-55,62,67), and the results of some of these experiments are summarized in Table II. The only

TABLE II. Ranges of Fission and Spallation Products

U^{235} + thermal neutrons, in the Electronic Stopping Region Ranges in Aluminum				U^{235} + thermal neutrons, Ranges in Uranium			
Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.	Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.
87	96.9	3.98	52	77	--	12.9	53
89	96.6	4.05	"	86	97.2	10.5	"
		4.12	18	89	96.6	11.5	"
91	96.5	4.02	"	90	96.5	11.9	"
97	97.8	4.16 } 3.99 }	19	91	96.5	11.5	"
99	98.5	4.16	"	93	96.7	11.3	"
111	--	3.51	18	95	97.1	11.4	"
115	--	3.33	"	97	97.8	11.4	"
131	85.6	3.37	"	99	98.5	11.2	"
132	75.2	3.49	19	103	95.7	11.2	"
137	68.8	3.21	52	106	91.5	10.9	"
140	64.9	2.98	18	109	--	10.1	"
				111	--	9.74	"

(continued)

Table II. (continued)

U ²³⁵ + thermal neutrons, Ranges in Gold				U ²³⁵ + thermal neutrons, Ranges in Uranium (continued)			
Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.	Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.
89	96.6	10.8	18	112	--	9.61	53
97	97.8	10.6	19	115	--	9.52	"
111	--	9.0	18	125	--	9.14	"
115	--	8.6	"	127	76.6	9.58	"
131	75.6	8.6	"	129	76.4	9.75	"
140	64.9	8.0	"	132	75.2	9.63	"
U ²³⁵ + thermal neutrons, Ranges in Zirconium*				136	70.2	8.36	"
89	96.6	6.88	54	137	68.8	9.18	"
99	98.5	6.12	"	140	64.9	8.74	"
138	67.5	5.51	"	141	64.0	8.55	"
140	64.9	4.78	"	143	62.1	8.42	"
141,144	64.0,61.0	4.20	"	144	61.1	8.37	"
* Containing 0.05% uranium.				147	58.1	8.07	"
				153	--	7.43	"
				156	--	7.1	"

Table II. (continued)

Pu ²³⁹ + thermal neutrons,				Spontaneous Fission Cf ²⁵² ,			
Ranges in Air				Ranges in Air			
Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.	Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.
83	--	3.25	12	91	104.5	3.13	55
91	101	3.12	"	97	104.3	2.99	"
92	101	3.12	"	99	104.0	2.99	"
93	101	3.10	"	111	100.5	2.84	"
94	101	3.09	"	112	100.0	2.83	"
97	99.2	3.06	"	115	98.8	2.77	"
99	98.8	3.04	"	121	95.6 ^a	2.69	"
105	99.0	2.96	"	127	91.4 ^a	2.56	"
109	96.6	2.88	"	132	87.0	2.54	"
112	91.4	2.74	"	139	80.0	2.47	"
117	--	2.55	"	140	79.0	2.43	"
127	82.9	2.56	"	141	78.0	2.38	"
129	81.9	2.56	"	143	76.0	2.33	"
(continued)				(continued)			

Table II. (continued)

Pu ²³⁹ + thermal neutrons, Ranges in Air (continued)				Spontaneous Fission Cf ²⁵² , Ranges in Air (continued)			
Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.	Mass Number	Energy (Mev)	Range (mg/cm ²)	Ref.
132	79.2	2.51	12	147	71.7	2.34	55
133	78.1	2.50	"	153	65.4	2.30	"
134	76.5	2.50	"	156	62.4	2.28	"
140	69.8	2.35	"	160	58.3	1.99	"
143	66.8	2.32	"	^a Only at these two energies do Refs. 68 and 69 differ by more than 3%.			
149	60.8	2.23	"				
157	--	2.20	"				

Table II. (continued)

Tb ¹⁴⁹ from Heavy Ion Reactions, Ranges in Aluminum (Ref. 48)		At ²⁰³ from Heavy Ion Reactions, Ranges in Aluminum (Ref. 48)	
Energy (Mev)	Range (mg/cm ²)	Energy (Mev)	Range (mg/cm ²)
4.17	0.367	3.98	0.281
4.43	0.371	4.76	0.318
5.82	0.459	4.91	0.318
8.44	0.657	6.38	0.411
9.42	0.708	5.86	0.410
9.24	0.730	9.12	0.598
10.24	0.765	10.55	0.670
13.06	0.921	10.66	0.635
21.42	1.323	12.03	0.732
23.84	1.421	12.89	0.735
26.25	1.510	13.31	0.771
28.59	1.557	14.73	0.807

Table II. (continued)
At²⁰³ from Heavy Ion Reactions,
Ranges in Gold (Ref. 48)

<u>Energy (Mev)</u>	<u>Range (mg/cm²)</u>
4	0.51
5	0.65
6	0.81
7	0.96
8	1.11
9	1.30

* Values read from smooth curve.

fission experiments included in the Table are those for which the energy of the fragment can be obtained from measurements independent of range measurements, in practice from velocity measurements. The kinetic energies of the fragments shown in the Table were obtained from the velocity measurements of Stein (68) (U^{235} and Pu^{239}) or of Milton and Fraser (69) (Cf^{252} spontaneous fission). The Cf^{252} spontaneous fission fragment velocities were also measured by Stein and Whetstone (70), but their measurements do not agree with Milton and Fraser's in the region of symmetrical fission. Here Stein and Whetstone found a large decrease in the total kinetic energy (which is well established in the thermal neutron fission of U^{235} , Pu^{239} , and U^{233}). The preliminary range measurements of Miskel (55), which were made by the method of Katcoff, Miskel, and Stanley (12) described earlier, appear to support the results of Milton and Fraser, so that the energies shown in Table II were calculated from the velocity measurements of these authors.

Actually, with the exception of the two points indicated in the Table, the results of Stein and Whetstone agree within 3% with the results of Milton and Fraser.

In Table II, the kinetic energies are the kinetic energies of the fragments after the evaporation of neutrons. It was assumed that the average velocity is not

altered by the neutron evaporation process, but that the kinetic energy decreases in proportion to the mass decrease.

Alexander and Gazdik (18) used their own results and the more reliable published data to obtain range-velocity curves for the median light and median heavy fission product groups in air, aluminum, and gold. The experimental ranges were obtained from their own results and from the experiments of Katcoff, Miskel, and Stanley (12), Leachman and Schmitt (63), and Fulmer (66), and Suzor (19). Rates of velocity loss were obtained from the papers of Leachman and Schmitt, and Fulmer. The resulting range-velocity curves could be fitted equally well with the empirical equations $R = kV - \Delta$ and $R = KE^{2/3}$, where the quantities k , Δ , and K are functions of the mass numbers of the recoil atoms and the atoms of the stopping medium. The following relations between range and velocity and energy were found to agree with the experimental results to within 0.5 Mev, but these equations should not be used for fragment energies below about 25 Mev, or for fragments whose mass-to-charge ratio is different from that of a fission product.

$$R (\text{air}) = (5.44 \times 10^{-3} M_1 + 2.253) V - \Delta (\text{air}) \quad (9)$$

$$R (\text{Al}) = (2.84 \times 10^{-3} M_1 + 3.206) V - \Delta (\text{Al}) \quad (10)$$

In these equations, the velocity V is equal to $(2E(\text{Mev})/M_1)^{1/2}$, where M_1 is the mass number of the fission product. Values of Δ (air) and Δ (Al) as a function of M_1 are given in Figure 6. Values of K for aluminum and air are given as a function of M_1 in Figure 7, in units of $\text{mgm}/(\text{cm}^2 \text{ Mev}^{2/3})$.

Unfortunately, calculation of the kinetic energies of fragments from the spontaneous fission of Cf^{252} , using Equation 9 and the ranges quoted in Table II, give results which differ by an average of 8.3% from the kinetic energies calculated from Milton and Fraser's velocity measurements. Obviously more range-velocity measurements are badly needed.

In the transition region between nuclear and electronic stopping, the ranges of Tb^{149} in aluminum and At^{203} in aluminum and gold were measured by Winsberg and Alexander (48), using both differential and integral methods.

Measurements were made over a wide range of energies by generating the recoils in different nuclear reactions, for example $\text{Pr}^{141}(\text{C}^{12}, 4n)\text{Tb}^{149}$, $\text{La}^{139}(\text{O}^{18}, 8n)\text{Tb}^{149}$ and so on. There are too many points in their range-energy curve for At^{203} stopping in gold to make it possible to include them all in Table II. Therefore, the Table contains a few values read from the smooth range-energy curve. Application of Equation 10 to the Tb^{149} data in Table II gives calculated ranges which

are about 15% greater than the experimental values, but Tb^{149} is on the neutron deficient side of stability, while Equation 10 was established for neutron-excess fission products. The charge carried by Tb^{149} atoms is likely to be different from the charge carried by a fission fragment of the same mass, so that it is not surprising that somewhat different stopping properties are observed.

CALCULATIONS

Conversion of a laboratory system recoil kinetic energy or angular distribution into the center of mass system requires a knowledge of the reaction Q values. Since recoil measurements are invariably made with nuclear reactions in which particles are emitted with a continuous distribution of kinetic energies, it is clear that conversion to the center of mass system is not straightforward. There is no unique Q value for such reactions, but rather an unknown distribution of values. The problem would be no simpler if the emitted light particles were studied instead of the recoiling nuclei. However, there are many reactions in which the velocities of the light particles are much greater than the center of mass velocity, and then their kinetic energy and angular distributions are nearly the same in both systems of coordinates. Except in fission studies, such a circumstance is not likely to occur for the heavy recoil fragments.

If, from some model of the nuclear reaction, it is possible to construct a probability distribution of Q , then the problem of conversion can be solved. Gonzalez-Vidal (71) attempted an analytical conversion from laboratory to center of mass system for angular distributions of recoil nuclei from reactions in which one or two neutrons were evaporated with an energy spectrum of the form $P(E) = Ee^{-E/T}$, where T is a nuclear temperature, but he found that the two neutron problem involved equations which required computer programming. An alternative approach was made by Donovan, Harvey, and Wade (6,7), who applied the Monte Carlo method to reactions in which up to four neutrons were evaporated with the (assumed) energy spectrum mentioned above. Angular distributions and kinetic energies of light particles have been calculated by the Monte Carlo method both for high energy reactions (72) and evaporation reactions (73). These results may be used to calculate recoil kinetic energies and angular distributions which can then be compared with experimental results. The disadvantage of this approach is that when calculation and experiment fail to agree, it is necessary to repeat the calculations with a modified model, and it may not be obvious in which way the model requires changing. Even when agreement is achieved, it is not obvious that the chosen model is the only one which would lead to agreement.

The precise meaning of ranges and values of η obtained by the forward-backward ratio method with the aid of Equations 3 and 4 has been discussed by Porile and Sugarman (22) and Winsberg (25). Regardless of the form of the range-energy relationship, these equations yield, in the limit for small values of η , $\langle 1/R \rangle^{-1}$ and $\langle \eta \rangle$ when the target is thin, but when the target thickness exceeds $W_{\max}$ the measured quantities become $\langle R \rangle$ and $\langle R\eta \rangle / \langle R \rangle$. $W_{\max}$ depends on the range-velocity relationship: if this can be expressed in the form $R = kV^N$, then $W_{\max} = R(1 \pm \eta)^N$. Naturally it is important to remember the kind of averaging which has taken place when ranges are converted into energies which are subsequently used in dynamical calculations.

SUMMARY

The study of recoiling nuclei is likely to be of increasing importance in the nuclear reaction field. The techniques are improving very rapidly, but it is clear from the discussion above that better range-energy relationships and a better understanding of scattering are urgently needed. Until this has been accomplished, the most promising applications will perhaps be in uncovering

new phenomena, and making semi-quantitative measurements in relatively unexplored fields such as the study of complex nuclear reactions in which many particles are emitted, or of reactions for which the cross sections are small.

ACKNOWLEDGMENTS

Many studies of recoil ranges were in process of completion as this review was compiled. At the risk of including data which will not stand the test of time, the author has persuaded several authors to permit him to quote information which is about to be published. For this cooperation, thanks are due to Drs. Davies, Schmitt, Niday, Miskel, Alexander, and Winsberg, and Mrs. Gazdik. Particular thanks are due to Drs. Alexander and Winsberg for many valuable discussions.

LITERATURE CITED

1. Walton, G. N., Progress in Nuclear Physics, 6, 192 (O. R. Frisch, Editor, Pergamon Press, London, 297 pp., 1957)
2. Halpern, I., Annual Reviews of Nuclear Science, 9, 245 (Emilio Segrè and Leonard I. Schiff, Co-editors, Annual Reviews, Inc., Palo Alto, California, 625 pp., 1959)
3. Miller, J. M. and Hudis, J., Annual Reviews of Nuclear Science, 9, 159 (Emilio Segrè and Leonard I. Schiff, Co-editors, Annual Reviews, Inc., Palo Alto, California, 625 pp., 1959)
4. Harvey, B. G., Progress in Nuclear Physics, 7, 89 (O. R. Frisch, Editor, Pergamon Press, London, 326 pp., 1959)
5. Cohen, B. L., Cohen, A. F., and Coley, C. D., Physical Review, 104, 1046 (1956)
6. Donovan, P. F., Harvey, B. G., and Wade, W. H., U. S. Atomic Energy Commission Document UCRL-9060 (1960); Physical Review (in press)
7. Harvey, B. G., Wade, W. H., and Donovan, P. F., U. S. Atomic Energy Commission Document UCRL-9061 (1960); Physical Review (in press)

8. Pate, B. D., and Yaffe, L., Canadian Journal of Chemistry, 33, 15 (1954)
9. Segre, E., and Wiegand, C., Physical Review, 70, 808 (1946)
10. Finkle, B., Hoagland, E. J., Katcoff, S., and Sugarman, N., Radiochemical Studies: The Fission Products, National Nuclear Energy Series, Division IV, Vol. 9, Book 1, 463, 471 (McGraw-Hill Book Company, Inc., New York, 516 pp., 1951)
11. Douthett, E. M., and Templeton, D. H., Physical Review, 94, 128 (1954)
12. Katcoff, S., Miskel, J. A., and Stanley, C. W., Physical Review, 74, 631 (1948)
13. Baulch, D. L., and Duncan, J. F., Australian Journal of Chemistry, 10, 112 (1957)
14. Good, W. M., and Wollan, E. O., Physical Review, 101, 249 (1956)
15. Ghiorso, A., and Sikkeland, T., Private Communication
16. Valyocsik, E. W., U. S. Atomic Energy Commission Document UCRL-8855 (1959)
17. Valyocsik, E. W., Donovan, P. F., and Morton, J. R., Unpublished Work (1958)
18. Alexander, J. M., and Gazdik, M. F., U. S. Atomic Energy Commission Document UCRL-8978-Revised (1960); Physical Review (to be published)

19. Suzor, F., Compt. rend., 224, 1155 (1947); Compt. rend., 226, 1081 (1948);
Annales de physique, 4, 269 (1949)
20. Ball, J. B., Fairhall, A. W., and Halpern, I., Physical Review, 114, 305
(1959)
21. Sugarman, N., Campos, M., and Wielgoz, K., Physical Review, 101, 388 (1956)
22. Porile, N. T. and Sugarman, N., Physical Review, 107, 1410 (1957); with
corrections in Physical Review, 111, 1747 (1958)
23. Fung, Si-Chang and Perlman, I., Physical Review, 87, 623 (1952)
24. Porile, N. T., Physical Review, 108, 1526 (1957)
25. Winsberg, L., U. S. Atomic Energy Commission Document UCRL-8618, 44 (1959)
26. Wolke, R. L., and Gutmann, J. R., Physical Review, 107, 850 (1957)
27. Cohen, B. L., Jones, W. H., McCormick, G. M., and Ferrell, B. L., Physical
Review, 94, 625 (1954); Cohen, B. L., and Neidigh, R. V., Rev. Sci.
Instruments, 25, 255 (1954); Cohen, B. L., Ferrell-Bryan, B. L., Coombe,
D. J., and Hullings, M. K., Physical Review, 98, 685 (1955)
28. Winhold, E. J., and Halpern, I., Physical Review, 103, 990 (1956)
29. Coffin, C. T., and Halpern, I., Physical Review, 112, 536 (1958)

30. Meadows, J. W., Physical Review, 110, 1109 (1958)
31. Ramaniah, M. V., and Sugarman, N., University of Chicago Report EFINS-59-60
(1959); Physical Review (to be published)
32. Brunnings, J. H. M., Knipp, J. K., and Teller, E., Physical Review, 60, 657
(1941)
33. Knipp, J. K., and Teller, E., Physical Review, 59, 659 (1941)
34. Lamb, W. E., Physical Review, 58, 696 (1940); Physical Review, 59, 687 (1941)
35. Bell, G. I., Physical Review, 90, 548 (1953)
36. Bohr, N., and Lindhard, J., Danske Matematisk-fysiske Meddelelser, 28, No. 7
(1954)
37. Nielsen, K. O., Electromagnetically Enriched Isotopes and Mass Spectrometry,
68 (M. L. Smith, Editor, Academic Press, Inc., New York, and Butterworth and
Company, London, 272 pp., 1956)
38. Lindhard, J., and Scharff, M., quoted by Leachman and Atterling (Ref. 49)
39. Bohr, N., Danske Matematisk-fysiske Meddelelser, 18, No. 8 (1948)
40. Lassen, N. O., Physical Review, 69, 137 (1946)
41. Perfilov, N. A., Comptes rendus (Doklady) de l'académie des Sciences de
l'U.R.S.S., 28, 426 (1940)

42. Lassen, N. O., Danske Matematisk-fysiske Meddelelser, 23, No. 2 (1945);
Physical Review, 68, 142 (1945); Danske Matematisk-fysiske Meddelelser, 26
No. 5 (1951); Danske Matematisk-fysiske Meddelelser, 26, No. 12 (1951)
43. Bøggild, J. K., Brostrøm, K. J., and Lauritsen, T., Danske Matematisk-fysiske Meddelelser, 18, No. 4 (1940)
44. Lassen, N. O., Danske Matematisk-fysiske Meddelelser, 25, No. 11 (1949);
Physical Review, 69, 137 (1946); Physical Review, 70, 577 (1946)
45. Demers, P., Canadian Journal of Physics, 31, 78 (1953)
46. Mathieu, R., and Demers, P., Canadian Journal of Physics, 31, 97 (1953)
47. Madsen, B. S., Danske Matematisk-fysiske Meddelelser, 23, No. 8 (1945)
48. Alexander, J. M., and Winsberg, L., U. S. Atomic Energy Commission Document
UCRL-8997 (1960); Physical Review (to be published)
49. Leachman, R. B., and Atterling, H., Arkiv för Fysik, 13, 101 (1957)
50. Davies, J. A., Private Communication (March, 1960); Canadian Journal of
Chemistry (to be published)
51. Schmitt, R. A., and Sharp, R. A., Physical Review Letters, 1, 445 (1958);
Schmitt, R. A., and van Lint, V. A. J., General Atomic Company Report
GA-1193 (San Diego, California, 1960)

52. Sugarman, N., Journal of Chemical Physics, 15, 544 (1947)
53. Niday, J. B., Private Communication (March, 1960)
54. Smith, E. R., and Frank, P. W., Bettis Atomic Power Laboratory Report
WAPD-TM-198 (Pittsburgh, Pennsylvania, November, 1959)
55. Miskel, J. A., Private Communication (March, 1960)
56. Booth, E. T., Dunning, J. R., and Slack, F., Physical Review, 55, 982 (1939)
57. Corson, D. R., and Thornton, R. L., Physical Review, 55, 509 (1939)
58. McMillan, E. M., Physical Review, 55, 510 (1939)
59. Green, L. L., and Livesey, D. L., Nature, 158, 272 (1946)
60. San-Tsiang, T., Zah-Wei, H., Chastel, R., and Vigneron, L., Journal de
physique et Le radium, 8, 165 (1947)
61. Bøggild, J. K., Arrøe, O. H., and Sigurgeirson, T., Physical Review, 71
281 (1947)
62. Bøggild, J. K., Minhagen, L., and Nielsen, O. B., Physical Review, 76, 988
(1949)
63. Leachman, R. B., and Schmitt, H. W., Physical Review, 96, 1366 (1954)
64. Shamov, V. P., and Lozhkin, O. V., Journal of Experimental and Theoretical
Physics (U.S.S.R.), 29, 286 (1955); English Translation, Consultants Bureau,

- New York, Journal of Experimental and Theoretical Physics, 2, 111 (1956)
65. Lozhkin, O. V., Perfilov, N. A., and Shamov, V. P., Journal of Experimental and Theoretical Physics (U.S.S.R.), 29, 292 (1955); English Translation, Consultants Bureau, Journal of Experimental and Theoretical Physics, 2, 116 (1956)
66. Fulmer, C. B., Physical Review, 108, 1113 (1957)
67. Freedman, M. S., Metcalf, R. P., and Sugarman, N., Radiochemical Studies: The Fission Products, National Nuclear Energy Series, Division IV, Vol. 9, Book 1, 459 (McGraw-Hill Book Company, Inc., New York, 516 pp., 1951)
68. Stein, W. E., Physical Review, 108, 94 (1957)
69. Milton, J. C. D., and Fraser, J. S., Physical Review, 111, 877 (1958)
70. Stein, W. E. and Whetstone, S. L., Physical Review, 110, 476 (1957)
71. Gonzalez-Vidal, J., Private Communication (1958)
72. Metropolis, N., Bivins, R., Storm, M., Miller, J. M., Friedlander, G., and Turkevich, A., Physical Review, 110, 204 (1958); Metropolis, N., Bivins, R., Storm, M., Turkevich, A., Miller, J. M., and Friedlander, G., Physical Review, 110, 185 (1958)

73. Dostrovsky, I., Fraenkel, Z., Rabinowitz, P., Proceedings of the Second

International Conference on the Peaceful Uses of Atomic Energy, Paper 1615

(United Nations, Geneva, 1958); Dostrovsky, I., Rabinowitz, P., Bivins, R.,

Physical Review, 111, 1659 (1958); Dostrovsky, I., Fraenkel, Z., Friedlander,

G., Physical Review, 116, 683 (1959); Dostrovsky, I., Fraenkel, Z., and

Winsberg, L., Physical Review (to be published); Dostrovsky, I., Fraenkel, Z., and

Rabinowitz, P., Physical Review (to be published)

MU - 19651

Fig. 1. Relationship between vectors $\mathbf{v}$ and $\mathbf{V}$ and the quantity η used in analysis of thick target experiments.

MU-19652

Fig. 2. Apparatus for differential range measurements by electrostatic collection.

MU-19657

Fig. 3a. Differential range curves for fission products stopping in air. (Katcoff, Miskel, and Stanley, Ref. 12).

MU-18166

Fig. 3b. Differential range curve for 96.8 keV Ra^{224} recoils stopping in deuterium (Valyocsik, Ref. 16).

Fig. 4. Variation of k (Equation 6) with mass ratio M_1/M_2 in the nuclear stopping region. Δ 0.026-0.053 Mev; $\square$ ~0.1 Mev; $\odot$ ~0.7 Mev; ∇ 2.2-4.0 Mev.

MU-19654

Fig. 5. Variation of range straggling parameter ρ with range (Ref. 48). Closed points are for At and Po recoils, open points for Tb^{149} . The recoils were made by Heavy Ion Bombardment as follows:- C^{12} $\square$, N^{14} $\diamond$, O^{16} $\triangle$, O^{18} ∇ , Ne^{20} $\odot$. Theoretical values from Equation 8 are shown by dashed lines on the right of the figure.

MU - 19655

Fig. 6. The quantities $\Delta(\text{air})$ and $\Delta(\text{Al})$ of Equations 9 and 10, as functions of mass number M_1 . (Ref. 18.)

MU - 19656

Fig. 7. The parameter K of the range-energy equation $R(\text{mg}/\text{cm}^2) = K E^{2/3}$, as a function of mass number M_1 , for stopping in aluminum (upper curve) and air (lower curve). (Ref. 18.)

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.