Title
The Subarray MVDR Beamformer: A Space-Time Adaptive Processor Applied to Active Sonar

Permalink
https://escholarship.org/uc/item/03n7w8g9

Author
Bezanson, Leverett Guidroz

Publication Date
2013-01-01

Peer reviewed|Thesis/dissertation
The Subarray MVDR Beamformer: A Space-Time Adaptive Processor Applied to Active Sonar

A thesis submitted in partial satisfaction of the requirements for the degree Master of Science in Electrical Engineering (Signal and Image Processing) by Leverett Guidroz Bezanson

Committee in charge:

William Hodgkiss, Chair
Gerald D'Spain
Truong Nguyen

2013
The thesis of Leverett Guidroz Bezanson is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

__________________________________________

Chair

University of California, San Diego

2013
DEDICATION

This thesis is dedicated to my wife Carly Marie Bezanson and to my daughter Zoë Marie Bezanson.
# TABLE OF CONTENTS

Signature Page ................................................................. iii
Dedication ................................................................. iv
Table of Contents ...................................................... v
List of Figures ........................................................... vi
List of Tables ............................................................. vii
Acknowledgements ....................................................... viii
Abstract of the Thesis ................................................... ix

1 Introduction ............................................................... 1
   1.1 Problem ........................................................... 1
   1.2 Background ...................................................... 1

2 Method ................................................................. 5
   2.1 Active Sonar Processing Chain ................................... 5
   2.2 Match Filter Bank ................................................. 6
   2.3 Beamforming .................................................... 8
      2.3.1 Conventional DAS Beamforming ....................... 9
      2.3.2 Adaptive MVDR Beamforming ....................... 12
      2.3.3 Subarray MVDR Beamforming ....................... 15
   2.4 Doppler Cube Processing ..................................... 20
      2.4.1 Resolution ............................................. 21
      2.4.2 Performance Metrics and SNR Calculations ......... 21

3 Results ................................................................. 24
   3.1 Simulation ....................................................... 24
      3.1.1 Snapshot Number(L) .................................. 24
      3.1.2 Subarray Selection(N) ................................ 25
      3.1.3 Spatial Shading ....................................... 25
   3.2 Experiment ....................................................... 26
      3.2.1 Beampatterns and Beamformer Output .............. 29
      3.2.2 2-Dimensional Doppler Cube Visualizations ....... 30
      3.2.3 SNR Performance Metrics ........................... 33

4 Conclusions and Future Work ..................................... 40

Bibliography ............................................................ 42
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Bistatic sonar diagram</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Block diagram of processing chain used in this thesis</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>Matched Filter Analysis</td>
<td>7</td>
</tr>
<tr>
<td>2.3</td>
<td>Conventional beamformer schematic</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>Conventional DAS beamformer simulation example</td>
<td>11</td>
</tr>
<tr>
<td>2.5</td>
<td>MVDR beamformer simulation</td>
<td>14</td>
</tr>
<tr>
<td>2.6</td>
<td>Diagram of the Subarray MVDR beamformer’s first stage of beamforming</td>
<td>17</td>
</tr>
<tr>
<td>2.7</td>
<td>Subarray MVDR beampattern derivation</td>
<td>18</td>
</tr>
<tr>
<td>2.8</td>
<td>Subarray MVDR beamformer vs conventional beamformer simulation</td>
<td>19</td>
</tr>
<tr>
<td>2.9</td>
<td>Doppler data cube</td>
<td>22</td>
</tr>
<tr>
<td>3.1</td>
<td>Subarray MVDR simulation with varied N subarrays</td>
<td>26</td>
</tr>
<tr>
<td>3.2</td>
<td>Subarray MVDR simulation with varied window functions</td>
<td>27</td>
</tr>
<tr>
<td>3.3</td>
<td>CMRE experiment diagram</td>
<td>28</td>
</tr>
<tr>
<td>3.4</td>
<td>Simulation of CMRE Subarray MVDR beamformer</td>
<td>30</td>
</tr>
<tr>
<td>3.5</td>
<td>Beamformer output, and beampatterns of the CMRE experiment</td>
<td>31</td>
</tr>
<tr>
<td>3.6</td>
<td>Geo-referenced range and bearing plots</td>
<td>34</td>
</tr>
<tr>
<td>3.7</td>
<td>Doppler vs bearing and range for 1033 UTC</td>
<td>35</td>
</tr>
<tr>
<td>3.8</td>
<td>Doppler vs Bearing and range zoomed in for 1033 UTC</td>
<td>36</td>
</tr>
<tr>
<td>3.9</td>
<td>Doppler vs bearing and Range for 1024 UTC</td>
<td>37</td>
</tr>
<tr>
<td>3.10</td>
<td>Doppler vs bearing and Range for 1049 UTC</td>
<td>38</td>
</tr>
<tr>
<td>3.11</td>
<td>SNR comparison of entire experiment for all 36 datasets</td>
<td>39</td>
</tr>
</tbody>
</table>
Table 3.1: Parameters of the Subarray MVDR beamformer used on the CMRE sea trial datasets. 29
ACKNOWLEDGEMENTS

I would like to express my sincer gratitude to many people who have supported me in the pursuit of this thesis and in my master of science degree. I would first like to thank my thesis advisor Professor William Hodkgkiss. He not only helped me define and follow through with the thesis, but has also taught me a large portion of what I will take away from UCSD through our discussions and taking his classes.

Dr. Kevin LePage and Robert Been from CMRE deserve many thanks. This thesis would not have happened without their continued help and support. Their contributions to the underwater research world are significant and cutting edge. I was very fortunate to work with them and all of the other researchers at CMRE.

I have worked at Progeny systems for 7 years and I would like to thank my supervisor John Thornton for allowing me the time to pursue this degree over several of those years. I would also like to thank him for introducing me to ocean engineering and hiring me as a undergraduate intern to take on huge taskings.

My brother Derrick Bezanson also deserves special thanks. We have taken classes together starting at the community level all the way to the advanced DSP classes for this degree. It has been a pleasure to work along side him. Other thanks go out to Alexes Allegra who rounds out the UCSD DSP working stiff study group.

Chapter 3, in part, was the basis for the material presented in The Subarray MVDR Beamformer for Active Littoral Sonar Systems. L. Bezanson; K. LePage; and R. Been; IEEE Oceans 2013. The thesis author, Leverett Bezanson was the primary investigator and author of this paper.
ABSTRACT OF THE THESIS

The Subarray MVDR Beamformer: A Space-Time Adaptive Processor
Applied to Active Sonar

by

Leverett Guidroz Bezanson

Master of Science in Electrical Engineering (Signal and Image Processing)

University of California, San Diego, 2013

William Hodgkiss, Chair

The research for this thesis was mainly performed at the NATO Underwater Research Center, now named the Center for Maritime Research and Experimentation (CMRE). The purpose of the research was to improve the detection of underwater targets in the littoral ocean when using active sonar. Currently these detections are being made by towed line arrays using a delay and sum beamformer for bearing measurements and noise suppression. This method of beamforming has can suffer from reverberation that commonly is present in the littoral environment. A proposed solution is to use an adaptive beamformer which can attenuate reverberation and increase the bearing resolution. The adaptive beamforming algorithms have existed for a long time and typically are not used in the active case
due to limited amount of observable data that is needed for adaptation. This deficiency is caused by the conflicting requirements for high Doppler resolution for target detection and small time windows for building up a full-rank covariance estimates. The algorithms also are sensitive to bearing estimate errors that commonly occur in active sonar systems. Recently it has been proposed to overcome these limitations through the use of reduced beamspace adaptive beamforming [1]. The Subarray MVDR beamformer is analyzed, both against simulated data and against experimental data collected by CMRE during the GLINT/NGAS11 experiment in 2011. Simulation results indicate that the Subarray MVDR beamformer rejects interfering signals that are not effectively attenuated by conventional beamforming. The application of the Subarray MVDR beamformer to the experimental data shows that the Doppler spread of the reverberation ridge is reduced, and the bearing resolution improved. The signal to noise ratio is calculated at the target location and also shows improvement. These calculated and observed performance metrics indicate an improvement of detection in reverberation noise.
1 Introduction

1.1 Problem

Active sonar systems are challenged by noise, reverberation, and clutter, especially for low Doppler targets. The theoretical developments in beamspace adaptive beamforming have made the application of high resolution processing feasible in active sonar systems. Adaptive sonar algorithms historically have been plagued by errors in the wave field estimation and snapshot deficiency, which limited the effectiveness. The goal of the work presented in this thesis is to analyze the performance and limitations of an adaptive space-time processor when applied to active sonar. The Subarray Minimum Variance Distortionless Response (MVDR) algorithm is evaluated in simulation and when applied to underwater acoustic datasets. The performance metric used is the increase in detectability of an underwater target.

1.2 Background

Detection of underwater targets in the ocean environment is accomplished by the monitoring of sound fields. Hydrophones are used in arrays to detect specific acoustic signatures, thus creating a sonar system. Sonar systems have existed for over a century as a way to detect targets in the ocean. When the ambient noise in the ocean is too loud to hear these targets an active system can be used. An active sonar system will transmit pulses to illuminate a target and create a reflection. In-between transmissions a receiver will listen for these reflections and analyze the properties of the reflected pulse leading to a localization of the target. Much of
the early developments in this field can be found in [2] which provides a thorough guide to sonar systems and underwater sound propagation.

A monostatic active sonar system is one type of sonar system where the source and receiver are co-located. When the receive array and source are spatially separated then the system is bistatic, and if there are multiple sources and/or receivers the system is multistatic [3]. There are many types of geometries for sonar receivers and transmitters. Typically they are chosen as a function of the environment they are deployed in and the target they wish to detect. This thesis will evaluate a littoral bistatic active sonar system with an azimuthally omni-directional transmitter and a uniformly spaced horizontal towed line array receiver. The properties that are of interest in this active sonar system are the range, bearing, and velocity of the target. These properties are used to locate the target in relation to the receiver and track it over time. In order to detect a target’s bearing the array must be focused or beamformed into candidate target directions. In the case of the uniformly spaced line array the detectable space is 180 degrees from bow to stern. The omni-directional nature of the hydrophones in the line geometry creates a left/right or conical ambiguity. Range is computed by the time it takes for the transmitted pulse to travel from the source transmitter to the target, reflect and travel to the receiver. Since the target, receiver, and source are all in different locations and potentially moving the source and receiver must be synchronized in time and their locations must be known to measure the travel time and by extension the range. The velocity of the target is measured by the frequency shift of a transmitted pulse. This shift is referred to as the Doppler effect. There are several different types of pulses that are used in active sonar. This thesis will analyze the continuous wave (CW) pulse. It should be noted that the Subarray MVDR has been successfully applied to other waveforms in [1] and [4].

An example of a bistatic sonar system geometry is shown in Figure 1.1. Note the range is in the form of an equal travel time ellipse. This is one dimension of the localization information and the bearing is another. The bearing in this case has a cone of ambiguity which leads to a left right target ambiguity in the littoral environment with respect to the array axis. Also shown in the diagram
Figure 1.1: A bistatic source-target-receiver example that shows the constant range ellipse that expands outward as a function of time from the pulse transmission. A potential source of reverberation is shown as the shoreline.

is a potential noise source. In the littoral environment the shoreline can be a large source of reverberation noise. As shown in Figure 1.1 the shoreline can reflect energy in the form of reverberation equal to the time of a target return. The Doppler shift difference between the target and reverberation will allow the active system localize the target vs the noise. For active systems reverberation is the main source of noise against which a detector must operate. Reverberation increases with source level, and as beamwidths increase reverberation increases burying the target in self noise.

In passive sonar, adaptive beamforming algorithms traditionally have been used to improve beamforming performance. There are many forms of passive adaptive beamformers, but most of them build upon the Minimum Variance Distortionless Response beamformer (MVDR) or Capon beamformer [5]. An adaptive
beamformer will null out correlated noise sources such as ships and increase the resolution of the target bearing. The MVDR beamformer adaptively forms element weights based on the signal+noise covariance estimated from observed data. These weights contain the nulls that are steered in the direction of correlated noise that is arriving from directions other than the steered direction. The increased bearing resolution will also aid in reverberation rejection not only in bearing but also in Doppler. As described in [6] Doppler frequency shift is not only a function of the relative velocity of the target, but it is also a function of bearing. The high resolution bearing output of the beamformer will localize the reverberation to a smaller region of bearing, because of this relationship.

Adaptive beamforming has not traditionally been performed in active sonar because the transmitted signal generally is transient, limiting the amount of time that can be used to estimate the signal covariance. Another problem with MVDR beamforming which is common to both passive and active processing is that the signal model must be quite accurate in order not to form nulls in the direction of the steered bearing angle. There are many ways to make the MVDR beamformer robust against these errors such as: diagonal loading, beamspace processing, spatial averaging, etc. [5].

The method chosen to increase the robustness of the MVDR beamformer for active sonar is to pursue a reduced beamspace method. The Subarray MVDR beamformer conventionally beamforms smaller aperture arrays in the direction of interest and then adaptively combines the resulting beams to increase performance. This method of beamforming reduces the integration (observed) time needed for subarray covariance estimation and increases robustness by spatially averaging out element level signal model errors.

This thesis describes an ongoing research project at the Centre for Maritime Research and Experimentation (CMRE) (Formerly NATO Undersea Research Centre) in 2012. The Subarray MVDR beamformer is compared to the conventional beamformer for simulated data and real active sonar data. Experimental data was collected by a 32 element array, towed by an AUV during the GLINT/NGAS11 Sea Trial. Earlier and similar experiments are described in [3] and [7].
2 Method

2.1 Active Sonar Processing Chain

In order to determine the benefits of using the Subarray MVDR beamformer, a processing chain was setup to properly analyze the detection capability of the typical conventional beamformer and the proposed adaptive beamformer. This processing chain is shown in Figure 2.1. The data is received as raw sampled time series and is then transformed into the frequency domain during the Match Filtering stage. Match Filtering is a common technique used when the signal that is being detected is a known signal. In this case the signal is a CW pulse. The match filter stage will be a filter bank of offset CW pulses corresponding to Doppler shifted velocity cells of resolution. The output of each filter is processed in parallel by the beamforming stage.

In the beamforming stage, data from all of the hydrophones are combined and beamformed to give the bearing information. In this study there will be two beamformers compared: the conventional delay and sum beamformer (DAS) and the Subarray MVDR beamformer. The simulation that also analyzes the merits of the Subarray MVDR Beamformer represents this stage by itself. The output of the beamformer stage contains bearing, and Doppler frequency shift measurements for a given integration time.

The beamforming and match filter stage can be done in either order because they are linear. The end result of the match filter bank and beamformer stage is a Doppler data cube for each transmission and return dataset. A dataset is defined as a period of time that contains the initial transmission and a predetermined quiet period. These Doppler cube datasets are processed typically with a normalizer,
detector and tracker. These stages can be areas of research all on their own. In this thesis the normalizer stage was created to analyze the effectiveness of the different beamformers.

![Diagram](image)

**Figure 2.1:** Block diagram of processing chain used in this thesis.

## 2.2 Match Filter Bank

When attempting to detect a signal in Gaussian noise the optimum filtering method is to use a match filter or replica correlator. This method is commonly used in communication, radar and sonar for detection purposes. The derivation and performance predictions of the match filter is found in Chapter 4 of [8] and Chapter 6 in [9]. As stated earlier, the transmitted active sonar signal that is being analyzed is the CW pulse. The pulse is shaded by a window function so that the high frequency harmonics are reduced [10] The corresponding match filter would in turn be a CW sinusoid pulse at the same center frequency shaded by the same window. The implementation of the match filter can be done through cross correlation in time or through multiplication in the frequency domain. Raw acoustic data is received at the element level in the sampled time domain. To transform it to the frequency domain a Discrete Fourier Transform is performed by multiplying the time data by complex sinusoids. It is common to use the Fast Fourier Transform (FFT) algorithm to accomplish this transform. Because this algorithm uses sinusoids to perform the transform the match filter reduces to the FFT of equal length and shading. The derivation of this process can be found in [9] and [8]. Since the pulse return is offset from the transmission frequency a match filter bank has to be constructed of these off center frequencies to be able to resolve the Doppler velocities. The transmitted CW pulse was 1 second in duration. The top panel of Figure 2.2 shows the filter bank constructed to
Figure 2.2: Matched Filter Bank Visualisation: The Top panel shows the frequency response of the offset Doppler shifted CW pulses. Note that at -3dB bandwidth the filters are 50% overlapped. The middle and bottom panels show the raw and filtered spectrograms from a single filter for a single hydrophone with a -7 Hz match filter output.

detect the Doppler return in the frequency domain. Note the line at the -3dB. This represents the bandwidth of each filter. The offset frequencies were chosen so that there is 50% overlap for this particular window and pulse length. This method will ensure that the frequency is not attenuated by a poorly designed filter bank. The lower two panels of Figure 2.2 show the raw and filtered data. The bandwidth of the transmission CW pulse corresponds to the reciprocal of the pulse length in time (T), which is 1Hz. Since the pulse is shaded the bandwidth is now 1.2Hz. The relationship between Doppler shifted frequency and target velocity is [11]:

\[ f_d = \frac{f_t}{c} \frac{d}{dt}(R_{ST} + R_{RT}) \]  
(2.1)
where $f_d$ is the Doppler shifted frequency, $c$ is the speed of sound in the ocean, $f_t$ is the transmitted frequency, $R_{ST}$ and $R_{RT}$ are the distances from source to target and receiver to target respectively. The derivative with respect to time is taken from these distances to find the velocity shifts. The target in the dataset being analyzed is known not to exceed a Doppler shift of 10Hz, therefore a search area of 21Hz corresponding to -10Hz to +10 Hz was selected. This corresponds to 31 offset match filters. This process is linear and can be done before or after beamforming. High resolution beamforming requires many more beams than the 31 filters of the matched filter bank. Additionally beamforming is ideally accomplished in the frequency domain. Therefore the first stage of the processing chain is the match filter bank with an FFT followed by beamforming.

2.3 Beamforming

A transmitter and receiver can consist of a single element or an array of elements configured into an array. The array allows beamforming to be possible in transmission and reception. In the case of littoral active sonar, receive arrays are used to control the array response and increase the SNR from the target echo. This increase in SNR is called array gain. Beamforming is a method of spatially processing propagating waves in a medium. An acoustic transmitted wave of frequency $f$ has a propagation speed of $c \approx 1500 \text{ m/s}$ which has a wavelength of $\lambda = c/f$. When processing in time the Nyquist criterion dictates that you need to sample a signal’s period more than 2 times in time, in other words the sampling frequency must be greater than 2 times the sampled frequency of interest. This is related to spatial processing where element to element spacing can be thought of as a sampling frequency for a wave. Acoustic waves will propagate spherically, but are considered as plane waves when the transmission source is in the far field of the receiver. A signal model of a propagating wave field is described in [5] as

$$s(t, q) = Ae^{j(\omega t - \tau_q)}.$$  \hspace{1cm} (2.2)

where $A$ is the amplitude of the signal, $\omega = 2\pi f$ where $f$ is in radians, $\tau_q = kq/c$ is the time advance of the signal $k$ is the wavenumber vector and $q$ is the spatial
position.

In an array of M elements each element records a signal \( x(t) \) that contains the direct blast \( s_0(t) \), the contact signal along with all reverberations signals \( s_q(t) \) and the noise from each sensor \( n(t) \). The small signal model of each \( m^{th} \) sensor may be written as:

\[
x_m(t) = s_0(t - \tau_{m,0}) + \sum_{q=1}^{Q} s_q(t) \ast \delta(t - \tau_{m,q}) + n_m(t) \tag{2.3}
\]

where \( q \) is a reflection to the \( m^{th} \) sensor, \( \delta \) is an impulse function, \( \tau_{m,q} \) is the delay in time from reflector \( q \) to sensor \( m \), and \( (\ast) \) represents convolution. There are \( Q + 1 \) reflections including the reflection off of the target. The entire wave field is represented as a vector of all \( x_m(t) \) elements in a \( M \times 1 \) vector \( X(t) \) [5] [12].

### 2.3.1 Conventional DAS Beamforming

A conventional delay and sum (DAS) beamformer will delay signal inputs in time so that an array of sensors can focus in a certain direction which is called the beam angle. The summation of the sensors results in a signal to noise ratio (SNR) array gain against noise arriving from other directions. An illustration shown in Figure 2.3 shows a conventional beamformer schematic operating as a receiver [13]. Along with the delay and summation of the signal, element weights \( w_m \) are also shown in the schematic, these are applied to form a window. In conventional beamforming these weights are pre-defined and data independent. The weights can be uniform (boxcar, rectwin) or defined by a window function. On a uniformly spaced line array the window function serves a similar purpose to the temporal window of applied to the FFT time snapshots. Instead of energy leaking into frequency bins the energy will leak into other beamformer outputs steered into other angles. This energy is commonly referred to as the side lobes or back beam.

The frequency domain representation of the delay and sum beamformer output is expressed as:

\[
B(\omega) = \frac{1}{\alpha} \sum_{m=0}^{M-1} w_m e^{-j\omega \tau_m} x_m(\omega) = W^H X(\omega) \tag{2.4}
\]
Figure 2.3: A block diagram of the DAS beamformer for a four element array with a single impinging wavefield. Once the delays are applied the array is weighted to control sidelobes, and then combined to form the beamformer output $B(\omega)$ [13]

where $w_m$ is the element weighting for the spatial window, $\alpha$ is the sum of the weights, $W$ is the $M \times 1$ steering vector with the desired weights and phase shift. The power at the beamformer output is found by taking the expectation of the beamformer output.

$$P(\omega) = E[|B(\omega)|^2]$$

The beamformer output will contain the energy at a particular angle and each beam will be a resolution bin. In addition to the beamformer output, the arrays response to far field sources is of interest. Beam angles $\beta(\theta)$ are formed in the available search area of $0^\circ$ to $180^\circ$ and the corresponding energy levels displayed make up the beampattern. The beampattern of the array and can be defined for the DAS beamformer as:

$$\beta(\theta) = \sum_{m=0}^{M-1} w_m^H e^{j\tau_{\theta,m}}$$

Figure 2.4 is an example of a beamformer simulation of a line array with 32 elements uniformly spaced at $\lambda/2$. The top panel shows a scenario where 2 signals of SNR 10dB arrive at a receiver from $55^\circ$ and $125^\circ$. The top panel shows the output of the beamformer $P(\omega)$ for all angles and the bottom panel shows the beampattern $\beta(\theta)$ for the 90 degree look angle. The beampattern and simulated beamformer output reveal several characteristics of the conventional beamformer. The resolution of the beamformer is the -3dB level of the main lobe of the beampattern [13]. The side lobe levels show how much signal energy arriving from off
angles is being attenuated by the beamformer. The valleys of these sidelobes are considered nulls. The effects of the spatial shading windows can be seen by the overlaid plots in Figure 2.4. When observing the application of the Hamming window the resolution of the beamformer is notably reduced, but the off beam energy caused by higher sidelobes also is reduced [13]. In the bottom panel the arrival angels of the signals are drawn as dashed lines onto the beampattern. This type of simulation will be used to describe the other beamformers explored in this thesis. The advantages of the DAS beamformer is the simplicity of the implementation,

![Figure 2.4](image_url)

**Figure 2.4:** A simulation example of the DAS Beamformer output $P(\omega)$ (Top) and the Beampattern $\beta(\theta)$ (Bottom). The incoming signals are represented as dashed lines on the beampattern.

and the robustness to estimation errors. Some negative attributes of the beamformer are that the resolution can be poor for some applications. The wide main lobe and the side lobes can be susceptible to strong correlated interference from
off beam angle directions also.

2.3.2 Adaptive MVDR Beamforming

Adaptive beamforming consists of changing the array element weights to be functions of the measured wavefield as opposed to the predefined sensor weights of the DAS beamformer. For each section of integration time the beamformer may have a different set of weights that are optimized by the algorithm criterion. Adaptive algorithms for active sonar come in many types [14] [6] [13] [1] [15] [16]. The criterion that is commonly sought is the noise rejection and improved resolution when compared to a DAS beamformer. The price that is paid for this improvement is an increased complexity and susceptibility to errors in the impinging wavefield direction estimation causing instability.

The MVDR beamformer forms weights in a way that will attempt to maintain unity gain of the beamformer in the beam angle direction while steering nulls in the direction of high energy interference. Where there is more interference energy the beamformer will apply a deeper null through the adaptive process. In directions where there is low noise the beamformer will allow the sidelobes to balloon higher in some cases even larger than the beam angle direction. The MVDR beamformer computes the weights by minimizing the power of the beamformer output. With reference to Equation 2.4, the beamformer output power is:

\[ P(\omega) = E[|B(\omega)|^2] = W^H R(\omega) W \]  

where \( R(\omega) \) is the covariance matrix or cross-spectral density matrix. In practice an FFT is taken from a snapshot in time and averaged with other overlapping snapshots to form the sampled covariance matrix. For L snapshots the sampled covariance matrix is:

\[ \hat{R}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} X_l(\omega)X_l^H(\omega). \]  

The weights are calculated by solving the following minimization equations with unity gain restraint.

\[
\begin{align*}
\text{minimize} & \quad W^H(\omega)\hat{R}(\omega)W(\omega) \\
\text{subject to} & \quad W^H(\omega)d = 1 \\
\end{align*}
\]  

(2.9)
where \( \mathbf{d} \) is the steering vector that focuses the array in the desired direction represented by \( \tau_m \) for each element \( m \): (see Eq. 2.4)

\[
d_m = e^{j\omega \tau_m}.
\]  

(2.10)

The above equations are solved by using Lagrange multipliers [5] and have the solution:

\[
\mathbf{W}_o(\omega) = \hat{\mathbf{R}}^{-1}(\omega)\mathbf{d} \mathbf{d}^H \hat{\mathbf{R}}^{-1}(\omega) \mathbf{d}.
\]  

(2.11)

The result is the MVDR beamformer output

\[
P_o(\omega) = \mathbf{W}_o^H(\omega)\hat{\mathbf{R}}(\omega)\mathbf{W}_o(\omega).
\]  

(2.12)

and the corresponding MVDR beampattern

\[
\beta_o(\theta) = \sum_{m=0}^{M-1} w_{o,m}^H e^{j\tau_{\theta,m}}
\]  

(2.13)

There are a few problems with this method. Nulls will be created in the direction of any correlated noise that is not in the direction of the steered beam. If phase errors exist between the the desired component of the data \( \mathbf{X}(\omega) \) and the steering vector \( \mathbf{d} \), the MVDR beamformer potentially will null out the desired signal that the beamformer is trying to detect. There are many methods for making the beamformer less sensitive to signal model errors[5]. Typically the MVDR beamformer will have to give up resolution or nulling capability to achieve robustness. The most common of the techniques is to diagonally load the covariance matrix. This is could be considered similar to adding spatial white noise and will widen the steered main lobe of the beamformer output. Another method is the reduced beamspace processing method. This is a method of reducing the degrees of freedom so that the number of snap shots \( L \) required is reduced, while spatially averaging element level errors. The Subarray MVDR beamformer is one type of this method.

Similar to the DAS conventional beamformer, the MVDR capabilities are best displayed with a simulation that shows the beamformer output and the beampattern of the adaptive weights. Figure 2.5 simulates a line array of 16 elements uniformly spaced at \( \lambda/2 \). There are three signals in this simulation with SNRs of
16 elements, $\lambda/2$ spacing, 31 snapshots, source angles = $[45^\circ, 90^\circ, 135^\circ]$, source SNR = $[-10,0,10]$

**Figure 2.5**: A simulation of the performance of the MVDR beamformer. The eigenvalue decomposition of the covariance matrix (Top) which is used to compute the beamformer output (Middle) and the beampatterns steered at various beam angles. (Bottom)

-10 dB, 0 dB, and 10 dB corresponding to directions of arrival (DOAs) of 45°, 90°, and 135°, respectively. The top panel in Figure 2.5 shows the eigenvalues of the eigenvalue decomposition of $\hat{R}$. Each element in the array represents a degree of freedom and a corresponding eigenvalue. By using the inverse of this matrix the signals with higher SNR will create deeper nulls and result in higher resolution. These effects are viewed in the middle and bottom panels of Figure 2.5 which
correspond to the MVDR beamformer output and the beampatterns, respectively. For comparison a conventional beamformer power output is overlaid on the beamformer output. When looking at the 10dB instance, the resolution is nearly 1 degree. This makes it very susceptible to phase errors caused by poor array element localization, waveguide effects, or electronic errors[17]. Another problem with the unconstrained MVDR is the number of snapshots required to make the covariance matrix full rank[5]. To be full rank covariance matrix requires the number of snapshots to be greater than or equal to the number of degrees of freedom (number of elements in this case). As discussed in [5], using the minimum number of snapshots will result in a poorly represented wavefield. A common rule of thumb is to use twice the number of degrees of freedom equal to the number of snapshots to create the covariance matrix. In the active sonar case, this is not an option because of the dynamic nature of the target, reverberation and receiver.

In active sonar all of the targets, and interfering signals are moving in time. This means that if there are 16 to 32 snapshots of time needed to satisfy the statical requirement of the simulated example the snapshots must be very short. However in active sonar scenario we desire to maximize Doppler resolution which requires large snapshot sizes. If we choose to use too many snapshots for the estimation of the array covariance matrix we will begin to integrate over time periods without a target return, reducing the gain of the beamformer. Additionally, some sources of reverberation (e.g. coastal reflections in our case) will change their directions over time further reducing the ability to form nulls in that direction. If we choose a snapshot size that is too short we will surrender Doppler resolution, which likewise impacts the target SNR. A solution to these problems comes through the use of the Subarray MVDR Beamformer[1].

### 2.3.3 Subarray MVDR Beamforming

The Subarray MVDR beamformer is a type of reduced beamspace adaptive beamformer. The reduced beamspace beamformer will form conventional DAS beams in a reduced search space, say 90 degrees instead of 180 degrees. Then it will use the output of these beams to form a covariance matrix with a reduced
number of degrees of freedom assuming that there are less beams than there are elements. Not only does this method reduce the number of degrees of freedom in the covariance matrix, it also spatially averages out element level phase errors that can plague the MVDR beamformer. The benefits of using a reduced beamspace beamformer in the presence of phase errors can be found in Section 6.9 of [5] and in great detail in [1] as it pertains specifically to the Subarray MVDR beamformer. The Subarray MVDR beamformer will reduce the number of degrees of freedom by sectioning the array into identical independent subarrays. These subarrays are beamformed conventionally into the steering direction from the subarray’s perspective and independent of each other. These beams are then adaptively processed via the MVDR method using the phase centers of the subarrays to steer the adaptive weights into the same focused direction of the DAS subarray beams with respect to the full array geometry. The subarrays are identical in this case, but the method can be applied to non-identical subarray geometries. Additionally this thesis will limit the scope of the study to subarrays along a uniformly spaced line array that keep the original spacing of the line array to form the sub arrays. The subarrays will use either completely separate sensors or be 50% overlapped with each other.

To derive the Subarray Beamformer let \( N \) is the number of subarrays of \( K \) elements each, with \( n \) the subarray index and \( k \) the subarray element. The corresponding subarray beamformer output is:

\[
B_n(\omega) = \frac{1}{\alpha} \sum_{k=0}^{K-1} w_k e^{-j\omega \tau_k} x_k(\omega) = W_n^H X_n(\omega).
\]  

(2.14)

From the output of the conventional subarray beamformers a new array is created from the centers of the subarrays in space as they relate to the overall array. The new steering vector consists of \( N \) phase delays. The subarray steering vector element is described as [1]:

\[
c_n = e^{j\pi \omega \tau_n}.
\]  

(2.15)

Figure 2.6 shows an array with 3 subarrays conventionally beamformed. A covariance matrix is formed from the subarray beams, creating a reduced beamspace covariance matrix of \( N \times N \) elements. As before when creating a sample covariance matrix, snapshots in time are overlapped and averaged. \( L \) snapshots in time are
Figure 2.6: A diagram of the first stage of the Subarray MVDR beamformer is shown with N=3. The array positions are shown in the top diagram and their corresponding DAS beams are shown in the bottom diagram.

used to build up statistics from separate beamformer outputs instead of frequency bins. Creating the beam vector \( \mathbf{B}_l = [B_0, B_1, \ldots, B_{N-1}]^H \), the covariance matrix is defined as:

\[
\hat{\Gamma}(\omega) = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{B}_l(\omega)\mathbf{B}_l^H(\omega) \tag{2.16}
\]

Lastly the MVDR weights are calculated as:

\[
\mathbf{H}_o(\omega) = \frac{\hat{\Gamma}^{-1}(\omega)\mathbf{c}}{\mathbf{c}^H\hat{\Gamma}^{-1}(\omega)\mathbf{c}}. \tag{2.17}
\]

The derivation of the beampattern for the subarray MVDR beamformer is more complicated than the previous two beamformers. The beampattern derived from \( \mathbf{H}_o \) will yield only the MVDR components of the beamformer. Similarly the beampattern of the subarray weights \( \mathbf{W}_n \) will only yield the DAS beampattern of a K length array. Since the subarrays are identical and the beampattern transform has a multiplicative property the resulting beampattern can be visualized by multiplying their respective beampatterns together.
Figure 2.7: The Subarray MVDR beamfomer consists of subarray beampattern and MVDR beampattern (Top) multiplied together to create the Subarray MVDR beampattern.(Bottom) The beampatterns are steered toward a beam angle of $130^\circ$ and a null is created in the main lobe against the high energy interference.

$$
\Psi(\theta) = \left( \sum_{k=0}^{K-1} w_k^H e^{j\tau_{\theta,k}} \right) \left( \sum_{n=0}^{N-1} h_{\alpha,n}^H e^{j\tau_{\theta,n}} \right)
$$

(2.18)

The simulation that illustrates this property is shown in Figure 2.7. This beam pattern was simulated using a 32 element array uniformly spaced at $\lambda/2$. Three identical subarrays were formed with 16 elements each and they were placed with 50% overlap across the whole array. The conventional subarrays had Hamming windows applied to them. The top panel contains the MVDR beampattern and the subarray DAS beampattern overlaid. The beam angle was steered toward $130^\circ$. The two interfering signals have an SNR of 10dB and were purposely placed
Figure 2.8: This simulation shows the comparison of the Subarray MVDR beamformer output vs the conventional beamformer output (Top) and a comparison of their beampatterns when the beampatterns are steered toward 130°. (Bottom) This example shows the potential increase in bearing resolution offered by the Subarray MVDR beamformer.

inside the main lobe and on the highest side lobe at 80° and at 135°. The number of snapshots \((L)\) creating \(\hat{\Gamma}\) is 10. The positions of the subarray beams violate the spacial sampling criterion and the aliased beams are clearly present. The signal that arrives from 80° is rejected in the subarray sidelobes and the MVDR beamformer creates a null only at the higher energy beam inside the main beam lobe. The Subarray MVDR beamformer only can only form \(N-1\) nulls [5].

The power output of the Subarray MVDR beamformer is:

\[
P_s(\omega) = H_o^H(\omega)\hat{\Gamma}(\omega)H_o(\omega) = (c^H\hat{\Gamma}^{-1}(\omega)c)^{-1}
\]  

(2.19)
Figure 2.8 shows a comparison between the beamformer outputs and the beam-patterns of a conventional beamformer with Hamming weights and the Subarray MVDR beamformer. In the top panel for each angle the power was calculated by applying Equation 2.19 for the subarray beamformer power $P_s(\omega)$ and Equation 2.7 for the conventional beamformer $P(\omega)$. The bottom panel compares the beampatterns of each beamformer. The conventional DAS beamformer uses Equation 2.6 to calculate $\beta(\theta)$ and the Subarray MVDR beamformer uses Equation 2.18 to calculate $\Psi(\theta)$. The conventional beamformer sidelobe level is lower than the Subarray MVDR beamformer. However, the Subarray MVDR beamformer forms a null in the main lobe thereby increasing the resolution of the beamformer output in the top panel. This increase in resolution is due to the interference signal being at a high SNR. The simulation accurately represents the active sonar scenario where transmissions typically are much higher than the ambient Gaussian noise. This simulation is an example of one configuration of the of the Subarray MVDR beamformer and is the basis for determining the best parameters to use for analyzing the experimental dataset gathered at sea. The parameters varied are: $N, K, L$ and the conventional beamformer window function. The results of the simulation study are discussed in the Chapter 3.1.

### 2.4 Doppler Cube Processing

The output of the beamformer and match filter stage is a Doppler cube of Bearing angles, Doppler Frequencies and range/time slices. These range/time slices individually represent on instance of integration time. The integration time is a function of $L$. Time/range cells are spaced independent of integration time and are relatively much shorter than the snapshot size. The data sets contain 12 seconds of acoustic data and are separated by one minute time intervals before the next transmission. The receive array was a uniformly spaced array of 32 elements separated by $0.7\lambda/2$. One element was used for timing and contained energy spikes at close intervals in time. This element was replaced with zeros. No other adjustments were made to compensate for the non-operational element. A visual
representation of the Doppler data cube can be found in Figure 2.9

### 2.4.1 Resolution

In the Doppler cube each dimension is made up of resolution cells. The data cells are the: a frequency shifted filter of the match filter bank, samples that represent target velocity, bearing, and range information, respectively. The choices of these cell sizes were dictated by the physics of the transmitted pulse, ocean environment, and receiver characteristics. The Doppler resolution already has been discussed and is 1.2 Hz which is the inverse of the transmitted pulse length in time with shading. The beamformer resolution depends on the receiver geometry and the shading scheme used. The beam width can be considered the -3dB down bearing off of the main lobe. In the case of the conventional beamformer the resolution is 12 degrees. The Subarray MVDR beamformer will be dependent upon the energy of the pulse. The SNR of the target echo is known to be relatively high. Therefore beam angles are formed in one degree increments. The range resolution is defined by the transmission pulse and has a length of $cT/2$ which equates to 750 meters in this case. To ensure that the entire pulse is captured, a time/range progression is chosen as 200ms/300m for each cell.

### 2.4.2 Performance Metrics and SNR Calculations

Evaluating performance enhancements of reverberation rejection is not a trivial task due to the fact that the target and reverberation will change as a function of relative velocity difference, bearing, and time. Additionally the detection process can be an area of study on its own. Typically, the 3 dimensional Doppler data cube would be processed by a normalizer, detector and tracker. The metrics will be limited to only the normalizer. From [8] there is an increase of detectability if there is an increase in SNR from the normalizer output. This will be the numerical performance metric that we will use to evaluate the beamformers. The normalizer created is a cubic split window normalizer. It consists of an inner cubic
window that matches the size of the target echo and an outer cubic shell representing
the noise energy. The SNR is calculated by taking the mean energy found in
the target cube and dividing it by the mean of the energy in the outer cubic shell.
An illustration of the cubic split window normalizer is shown in Figure 2.9. The
cubic split window normalizer commonly will contain a guard band separating the
inner target cube from the noise cubic shell (not shown in Figure 2.9).

Figure 2.9: One transmission and wait time creates a Doppler data cube (a). To
evaluate the SNR a cubic split window normalizer is used (b) where the red is the
signal cube and the orange is the cubic shell that represents the noise.

The performance of the Subarray MVDR beamformer was obtained numerically by comparing its SNR compared to the conventional beamformer’s SNR.
These values were calculated by a cubic split window normalizer with no guard
band at the location of the target. The guard band was not used because the re-
verberation was close in proximity to the target in each dimension and the added
complexity of calculating its size. The target cube dimensions typically will match
the size of the expected return as described previously. However target cube was
chosen to be larger in each dimension of cube. One reason for this is the inte-
gration time is larger than the pulse length which has a smearing effect in time.
Additionally the larger target cube allows the placement of the window to be less
precise than if a tight window was used with a guard band. The location of the
target was found by using non-acoustic sensors that were on the AUV towing the receive array, the source, and on the target. These sensors provided GPS locations of the source, target, and receiver. The velocity information was extracted by the position change between samples. This did not always result in the exact target location so a search algorithm was used to seek out the target confined within a small search area centered on the estimated target location. The search algorithm and large target window resulted in the capture of the target in each dataset and was verified empirically. The SNR given from this method does not provide the full picture of how much better an adaptive beamformer will perform over the conventional beamformer. However it will give an indication if the adaptive beamformer is outperforming the conventional beamformer at all. This indication should inspire further research into the performance metrics.

The Doppler cube datasets were evaluated empirically to ensure that the SNR calculations were not biased by unforeseen phenomena. Two dimensional slices of the cube were extracted and plotted as mosaics. The slices were Doppler vs Bearing, Doppler vs Range, and Range vs Bearing. These 2D mosaics were taken from the center of the inner cubic window representing the target. In these plots the bearing resolution increase and the reduced reverberation were evaluated. Some of these two dimensional plots are presented in this thesis, but all of the two dimensional slices were evaluated for the experiment.

The Subarray MVDR beamformer will increase the bearing resolution and possibly reject reverberation not fully attenuated by the subarray DAS beamforming in the same Doppler dimension. The Subarray MVDR beamformer also will increase the Doppler separation and reduce the reverberation outside of the Doppler frequencies containing the target energy. This is due to the bearing dependent nature of reverberation the shoreline, bottom, and surface. This dependency and how adaptive beamformers can be used to reject Doppler reverberation and is described in [6] [4] [18].
3 Results

3.1 Simulation

The Subarray MVDR simulation was studied to view the effects of varying certain parameters of the Subarray MVDR algorithm when compared to the conventional beamformer. The beamformer outputs are examined as well as the beampatterns. The simulation was made by mimicking the conditions of the experiment that will be processed. The three main parameters that required investigation were the Subarray number $N$ (and by extension $K$), the spatial shading, and the snapshot number $L$. These parameters are varied and analyzed by displaying the beamformer outputs and beampatterns similar to Figure 2.8. These parameter studies will lead to parameter choices and a specific Subarray MVDR processor ideal for this experiment.

3.1.1 Snapshot Number ($L$)

From the point of view of the beamformer, the integration time should equal the transmission pulse length. By doing this, each snapshot would contain a somewhat stationary signal for detection and stationary reverberation that can be nulled out. As a result, the longer integration time will decrease the Doppler resolution. Lower resolution is an unacceptable trade off in many cases including the case of the CMRE experiment. Keeping the requirement that 1 snapshot equate to 1 transmission length in time the new desire would be to keep this integration time as short as possible. This will also maximize the SNR by ensuring most of the snapshots contain signal and noise energy and not just noise. To keep the snapshot
requirement to a minimum, a Subarray number $N$ would naturally be selected as 2. As stated prior a good rule of thumb is to double the snapshots in reference to the degrees of freedom. As a compromise between the desire for $L=2$ to minimize the integration time and the desire for $L=4$ to build up a representative covariance matrix $L$ was chosen to be 3. A follow on area of research could be to study the effects of varying integration time on the various beamformers.

3.1.2 Subarray Selection ($N$)

The number of subarrays ($N$) can be thought of a sliding parameter that makes the array behave more like a MVDR beamformer rather than a conventional beamformer. Where a subarray choice of $N=1$ would be a conventional beamformer and a subarray selection of $N=32$ would be a MVDR beamformer. Figure 3.1 shows the beampatterns and power output of the simulation as $K$ and $N$ are varied from $N=1,3,7,15,31$ and $K=32,16,8,4,2$, respectively. All elements were spaced at $\lambda/2$. This simulation has a 50% spatial overlap and Hamming window applied to the subarray conventional beamformers.

While the beampatterns change drastically it should be noted that the final beamformer power output keeps the high resolution when $N$ is small, the only difference is a few dB of gain. The beampatterns plotted illustrate how the sidelobes start by relying on the conventional beamformer to relying on nulls as $N$ is increased. The subarray choice made for active sonar is kept to $N=2$ subarrays to reduce the number of snapshots needed.

3.1.3 Spatial Shading

As shown in Chapter 2.3.1, using a window function on a DAS conventional beamformer will reduce the sidelobe levels and reduce energy seen at off angle directions. It would follow that the DAS should always be shaded in the Subarray MVDR beamformer. When a window is applied to the DAS subarray beamformer the main lobe width will widen. The result is a scenario where the combination of the heavily aliased MVDR beampattern can not be attenuated by the Subarray
DAS beampattern when no overlap exists and a window function is applied. This phenomenon is shown in Figure 3.2. Energy will will leak into other beams as an effect of this shading. To combat this effect there must be another overlapping subarray or there must be no spacial shading of the subarrays.

From the results of the simulation, the conclusions drawn are that the parameters that will work best on the experimental data set are N=2, L=3, K=16, window=none, overlap=0%.

3.2 Experiment

In September 2011 the GLINT/NGAS11 multistatic sonar experiment was conducted by CMRE (NURC at the time) in littoral Italian waters. The experi-
Figure 3.2: The negative effects of applying a window function to the DAS when there are no overlapping elements. Spatial aliasing is observed in the top panel. To avoid this one should either use 50% overlap or no window function if there are no overlapping subarrays.

ment consisted of a sound source deployed from the NRV Alliance in station keeping mode, an echo repeater deployed from the CRV Leonardo, and two towed arrays deployed from AUVs. The data analyzed in this thesis comes from only one of the two receive arrays, making it a bistatic geometry. During this run, the receiver was subjected to reverberation from a nearby shoreline. This caused detections of echo repeater returns of the Doppler sensitive CW signal to become engulfed by noise and therefore undetectable in many instances. The data was taken as the receiver moved closer to shore, experiencing high reverberation along the way. The geometry of the positions of the source, receiver and array are shown in Figure 3.3. The geometry data was extracted from GPS positions recorded by non-acoustic
sensors. Note the proximity to the shore line. The circles indicate the end position of the AUV and the echo-repeaters and the dot indicates their starting points.

**Figure 3.3:** The CMRE experiment track data is shown in the top panel with the coast position. The end points are marked as "o" and the beginning are solid circles. The Doppler for the target and reverberation were 5Hz and -2 Hz respectively and were fairly steady during the run.

The receive array was 32 elements; one element was used for timing and set to 0. The array was spaced at 0.7λ/2 with respect to the center frequency of the transmitted pulse. The source was omni-directional azimuthally and relatively stationary. The CW transmission waveform was a 1 second shaded pulse. The dataset included the direct blast from the source and the echo return from a towed echo repeater. Each dataset was 12 seconds in length and there was a new dataset observed every minute. There were 36 datasets analyzed to test the beamformer performance. The parameters chosen for the subarray MVDR beamformer were derived in the previous section and listed in Table 3.1
Table 3.1: Parameters of the Subarray MVDR beamformer used on the CMRE sea trial datasets.

<table>
<thead>
<tr>
<th>Subarray Number (N)</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subarray Elements (K)</td>
<td>16</td>
</tr>
<tr>
<td>Snapshots (L)</td>
<td>3</td>
</tr>
<tr>
<td>Spacial Shading</td>
<td>none</td>
</tr>
<tr>
<td>Subarray Overlap</td>
<td>0%</td>
</tr>
<tr>
<td>Integration Time</td>
<td>2s</td>
</tr>
</tbody>
</table>

3.2.1 Beampatterns and Beamformer Output

In order to determine whether the simulations are representative of the experiment when applied to the dataset, the beamformer output and beampatterns are examined along the slice of time and Doppler shift of a target. The simulation of the experiment is shown in Figure 3.4. Due to the fact that there is no shading of the DAS, higher sidelobes that result in the energy spikes seen in the beamformer output are observed. The beampattern creates nulls from the targets high SNR and it yields the desired high resolution effect. For comparison Figure 3.5 shows the beamformer output and 2 beampatterns extracted from the experiment at 1049 UTC. Referring back to Figure 3.3, it is noted that the dataset is toward the end of the track where the source, receiver, and target are closer to together thereby increasing the target strength and reverberation noise. The results do indicate what the simulation predicted. In the top panel of Figure 3.5 high resolution is achieved. In the middle and lower panels the beampatterns are shown when steered in the direction of the target and when steered 7 degrees off the target. Since there are only two degrees of freedom we do observe that when the beamformer is steered in the direction of the target the main lobe and side lobes are significantly higher than when it is steered off target. In the bottom panel, a null is observed in the direction of the target. There are no notable nulls in the direction of the reverberation shown as high energy between 20° and 60°. The DAS subarray beamformer appears to have more or less successfully attenuated that region.
Figure 3.4: Simulation of the Subarray MVDR Beamformer used to process the CMRE experiment datasets. The beamformer output (top) shows some sign of higher sidelobes from the beampattern (bottom) as off beam energy spikes.

3.2.2 2-Dimensional Doppler Cube Visualizations

The two dimensional slices were extracted for each of the 36 transmissions that were analyzed in this thesis. Presented in this section are 3 of those transmissions. One transmission from the beginning of the dataset at 1024 UTC, one from the middle of the dataset 1033 UTC, and one toward the end 1049 UTC. The experiment shown in Figure 3.3 starts with the source, receiver, target and shoreline relatively far from each other and as time progresses they all move closer together ending at the indicated position. The three time slices show the performance in three distinctly different scenarios. The dataset from 1033 UTC is analyzed in greater detail first. The Doppler cube slices consist of bearing vs range, Doppler
The beampattern of the Subarray MVDR beamformer is extracted from the experiment database for time 1033 UTC. The power output $P(\omega)$ and $P_S(\omega)$ are shown in the top panel and the beampattern of $\beta(\theta)$ and $\Psi(\theta)$ are in the bottom two panels.

The dataset from 1033 UTC is ideal to analyze in detail, because it can be considered a middle of the road case, where there reverberation does exist but it is not dominating the dataset as it does later in the experiment. Figures 3.6 show geo-referenced range vs bearing plots of the Doppler Cube at the estimated Doppler frequency. The source and the target are indicated by a circle and plus marker. The source is at approximately [6500 8500] meters, the receiver is located...
at [8500 4500] meters, and the target is located at [11000 45000] meters. The energy is measured in dB and a colorbar is used for a sense of relative scale. A distinct increase in bearing resolution with the Subarray MVDR can be easily seen by comparing these two figures. Note also the proximity of the shoreline. This dataset clearly indicates an increase in bearing resolution, but it is ambiguous as to whether any improvement in reverberation rejection was achieved.

Figure 3.7 displays the Doppler shift in frequency vs time and bearing for both beamformers for the dataset 1033 UTC. The top two panels show Doppler vs bearing with the conventional beamformer on the top panel and the Subarray MVDR beamformer below it. The bottom two panels display the Doppler shift vs range with the beamformers arranged in the same manner. The dashed lines represent the target parameter estimates made from the non-acoustic sensors. The inner rectangle represents the boundaries of the inner split level cubic window that encompasses the target while the outer rectangle represents the boundary for the outer cubic shell window that estimates the noise level. It clearly is observed in this figure that the Subarray Beamformer does increase the bearing resolution on the target and the split level window does encompass the target even though the non-acoustic data did not quite estimate the location exactly. Furthermore it can be seen that the reverberation ridge has less areas of red and more areas of blue when the Subarray MVDR beamformer is applied. This is the result of the high resolution nature of the MVDR beamformer and the dependency of Doppler shift on beam angle.

Figure 3.8 is a close up of the split level window that is used to estimate the SNR vs time for all 36 datasets. The panels are set up in the same fashion as in Figure 3.7. The solid rectangle lines are placed in middle of the resolution cells. Therefore the cells on the lines are included in the inner cube average. It can be determined that the physical estimates for the resolution do include most of the observed signal and stand as acceptable metrics to evaluate whether or not the Subarray MVDR beamformer increases the performance SNR metrics and if the bearing resolution is increased.

Figure 3.9 shows the same information in Figure 3.7 for 1024 UTC. In this
example the target energy was weaker than most of the transmissions. Due to the weak target signal the Subarray MVDR beamformer did not improve the bearing resolution or SNR over the conventional beamformer. The Subarray MVDR beamformer will work better with a higher SNR target.

Figure 3.10 shows the same information in Figure 3.7 for 1049 UTC. This example illustrates when the Subarray MVDR beamformer is the most effective. There is a large amount of reverberation noise completely engulfing the signal when the DAS is used. When the Subarray MVDR beamformer is applied, the target has a higher SNR and is separated from the reverberation as indicated by the yellow cells in the split window outer cube. Overall, the reverberation is reduced as observed by the color scale in the case of Doppler vs bearing and Doppler vs range. This indicates that if high reverberation is expected in a given active sonar application, the Subarray MVDR clearly will outperform the DAS beamformer.

3.2.3 SNR Performance Metrics

In an attempt to quantify the overall performance of the Subarray MVDR beamformer vs the DAS beamformer the cubic split level window normalizer was applied and for each dataset the mean of the inner cube window representing the signal was divided by the mean of the outer cubic shell window representing the noise for each of the beamformers. This metric represents the SNR. The gain was calculated by dividing the SNR of the Subarray MVDR beamformer by the SNR of the DAS beamformer. In Figure 3.11 all three calculations are plotted vs UTC. The mean gain for the entire experiment was 6.17 dB.

Chapter 3, in part, was the basis for the material presented in The Subarray MVDR Beamformer for Active Littoral Sonar Systems. L. Bezanson; K. LePage; and R. Been; IEEE Oceans 2013. The thesis author Leverett Bezanson was the primary investigator and author of this paper.
(a) Conventional Beamformer Source=S, Receiver=R, Target=T

(b) Subarray MVDR Beamformer

Figure 3.6: Geo-Referenced range vs bearing at target Doppler frequency for 1033 UTC. Source = [6500 8500]m. Receiver = [8500 4500]m Target = [11000 45000]m
Figure 3.7: Doppler cube data slices for 1033 UTC. Doppler vs Bearing (upper two panels) and Doppler vs Range (lower two panels). The conventional beamformer is placed above the Subarray MVDR beamformer for comparison. The estimated target parameters are indicated by dashed lines and the split level cubic window normalizer boundaries are indicated by solid lines.
Figure 3.8: Detailed look at Doppler cube data slices for 1033 UTC zoomed in. Doppler vs bearing (upper two panels) and Doppler vs range (lower two panels). The conventional beamformer is placed above the Subarray MVDR beamformer for comparison. The estimated target is indicated by dashed lines and the split level cubic window normalizer boundaries are indicated by solid lines.
Figure 3.9: Doppler cube data slices for 1024 UTC. Doppler vs Bearing (upper two panels) and Doppler vs Range (lower two panels). The conventional beamformer is placed above the Subarray MVDR beamformer for comparison. The estimated target parameters are indicated by dashed lines and the split level cubic window normalizer boundaries are indicated by solid lines.
**Figure 3.10:** Doppler cube data slices for 1049 UTC. Doppler vs Bearing (upper two panels) and Doppler vs Range (lower two panels). The conventional beamformer is placed above the Subarray MVDR beamformer for comparison. The estimated target parameters are indicated by dashed lines and the split level cubic window normalizer boundaries are indicated by solid lines.
Figure 3.11: The average SNR gain for each dataset is plotted over UTC time (hours) creating a metric of improvement of the Subarray MVDR beamformer over the DAS beamformer in this experiment.
4 Conclusions and Future Work

The detection of underwater targets continues to be an area of research. This thesis looked at the active sonar detection problem in a littoral environment. An area of research is to determine if a reduced beamspace adaptive beamformer can increase the detectability of a target that is subjected to reverberation noise when the delay and sum beamformer can not. The Subarray MVDR beamformer shows potential to raise the probability of detection in a littoral environment for a given probability of false alarm when using Doppler sensitive waveforms in active sonar. The beamformer successfully increased the bearing resolution and reduced the area over which the sidelobe reverberation interference was integrated. The beamformer has maintained the benefits of the adaptive beamformer while overcoming the challenges that typically reduce its performance significantly. Results were shown both for simulation and for experimental data collected during CMRE’s GLINT/NGAS11 sea trial. The results were in the form of SNR calculated at the target location with a cubic split level window normalizer. Empirical results were evaluated from 2D slices of the Doppler data cube and showed the reduction of reverberation noise.

To complete the full study of this beamformer, a full processing chain should be developed and the receiver operator characteristic curves (ROC) should be evaluated for the Subarray MVDR beamformer and compared to the conventional DAS beamformer. Only then will the full detectability of this algorithm will be known. Parameters of the Subarray MVDR beamformer can be optimized for this and other applications with the development of this full processing chain. Other experiment datasets can also be evaluated to show the full potential. At CMRE this chain of events is already underway with the addition of research on different
geometries that can be applied to the adaptive beamformer.

Not only should this algorithm be further researched to fully understand its benefits, but other algorithms should be compared to it to see if there are better algorithms for different applications. These algorithms can be applied to more applications than the underwater detection problem. The algorithm also can be used in sonar imaging, ultrasound imaging for medical devices and for radar applications. All of these areas have the potential to benefit from the Subarray MVDR algorithm or other adaptive beamforming algorithms.
Bibliography


