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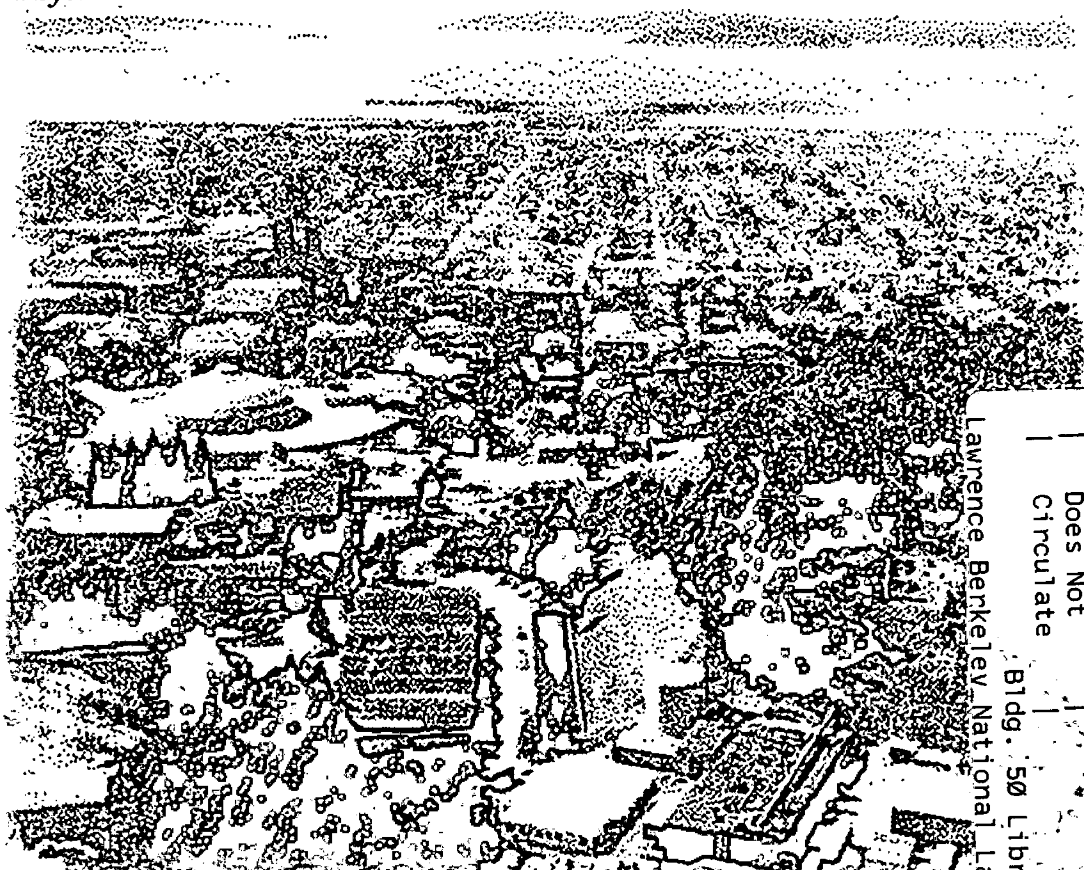

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ONE-LOOP PAULI-VILLARS REGULARIZATION OF SUPERGRAVITY I: CANONICAL GAUGE KINETIC ENERGY*

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Abstract

It is shown that the one-loop coefficients of on-shell operators of standard supergravity with canonical gauge kinetic energy can be regulated by the introduction of Pauli-Villars chiral and abelian gauge multiplets, subject to a condition on the matter representations of the gauge group. Aspects of the anomaly structure of these theories under global nonlinear symmetries and an anomalous gauge symmetry are discussed.

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1 Introduction and preliminaries

It was shown in [1] that Pauli-Villars regulation of the one-loop quadratic divergences of a general $N = 1$ supergravity theory is possible. This result was generalized [2] to the regularization of the one-loop logarithmic divergences of globally supersymmetric theories, including nonlinear sigma models, with canonical kinetic energy for Yang-Mills fields. It was further assumed that the theory was free of gauge and mixed gravitational-gauge anomalies. The purpose of the present paper is to generalize further these results.

In section 2 we give a full PV regularization of a general supergravity theory with canonical kinetic energy for the gauge fields and an anomaly-free gauge group. In section 3 we consider anomalies under Kähler transformations, and in section 4 we show how the regularization procedure must be modified in the presence of an anomalous $U(1)$ gauge group factor. Our results are summarized in section 5, and some calculational details, as well as corrections to [3, 4], are given in appendices.

We conclude this section with a brief review of the formalism used to evaluate the regularized Lagrangian. The one-loop effective action S_1 is obtained from the term quadratic in quantum fields when the Lagrangian is expanded about an arbitrary background:

$$\begin{aligned}\mathcal{L}_{quad}(\Phi,\Theta,c) &= -\frac{1}{2}\Phi^T Z^\Phi\left(\hat{D}_\Phi^2 + H_\Phi\right)\Phi + \frac{1}{2}\bar{\Theta}Z^\Theta\left(i\not{D}_\Theta - M_\Theta\right)\Theta \\ &\quad + \frac{1}{2}\bar{c}Z^c\left(\hat{D}_c^2 + H_c\right)c + O(\psi), \qquad (1.1)\end{aligned}$$

where the column vectors Φ, Θ, c represent quantum bosons, fermions and ghost fields, respectively, and ψ represents background fermions that we shall set to zero throughout this paper. The fermion sector Θ includes a C-odd Majorana auxiliary field α that is introduced to implement the gravitino gauge fixing condition. The full gauge fixing procedure used here is described in detail in [3], [4]. The one loop bosonic action is given by

$$\begin{aligned} S_1 &= \frac{i}{2}\mathrm{Tr}\ln\left(\hat{D}_\Phi^2 + H_\Phi\right) - \frac{i}{2}\mathrm{Tr}\ln\left(-i\,\not{D}_\Theta + M_\Theta\right) + \frac{i}{2}\mathrm{STr}\ln\left(\hat{D}_c^2 + H_c\right) \\ &= \frac{i}{2}\mathrm{STr}\ln\left(\hat{D}^2 + H\right) + T_-, \end{aligned} \tag{1.2}$$

where T_- is the helicity-odd fermion contribution which contains no quadratic divergences, and the helicity-even contribution is given by

$$\hat{D}_\Theta^2 + H_\Theta \equiv \left(-i\,\not{D}_\Theta + M_\Theta\right)\left(i\,\not{D}_\Theta + M_\Theta\right). \tag{1.3}$$

The background field-dependent matrices $H(\phi)$ and $\hat{D}_\mu(\phi) = \partial_\mu + \Gamma_\mu(\phi)$ are given in [3], [4], where the one-loop ultraviolet divergent contributions have been evaluated.

We regulate the theory by including a contribution from Pauli-Villars loops, regarded as a parameterization of the result of integrating out heavy (*e.g.*, Kaluza-Klein or string) modes of an underlying finite theory. The signature $\eta = \pm 1$ of a PV field determines the sign of its contribution to the supertrace relative to an ordinary particle of the same spin. Thus $\eta = +1(-1)$ for ordinary particles (ghosts). The contributions from Pauli-Villars fields with negative signature could be interpreted as those of ghosts corresponding to heavy fields of higher spin.

Explicitly evaluating (1.2) with an ultraviolet cut-off Λ and a massive Pauli-Villars sector with a squared mass matrix of the form

$$M_{PV}^2 = H^{PV}(\phi) + \begin{pmatrix} \mu^2 & \nu \\ \nu^\dagger & \mu^2 \end{pmatrix} \equiv H^{PV} + \mu^2 + \nu, \quad |\nu|^2 \sim \mu^2 \gg H^{PV} \sim H,$$

gives, with $H' = H + H^{PV}$:

$$\begin{aligned} 32\pi^2 S_1 &= -\int d^4x p^2 dp^2 \mathrm{STr}\ln\left(p^2 + \mu^2 + H' + \nu\right) + 32\pi^2\left(S_1' + T_-\right) \\ &= 32\pi^2\left(S_1' + T_-\right) - \int d^4x p^2 dp^2 \mathrm{STr}\ln\left(p^2 + \mu^2\right) \\ &\quad - \int d^4x p^2 dp^2 \mathrm{STr}\ln\left[1 + \left(p^2 + \mu^2\right)^{-1}\left(H' + \nu\right)\right]. \end{aligned} \tag{1.4}$$

S'_1 is a logarithmically divergent contribution that involves the operator $\hat{G}_{\mu\nu} = [\hat{D}_\mu, \hat{D}_\nu]$:

$$32\pi^2 S'_1 = \frac{1}{12}\int d^4x p^2 dp^2 \mathrm{STr}\frac{1}{(p^2+\mu^2)}G'_{\mu\nu}\frac{1}{(p^2+\mu^2)}G'^{\mu\nu}, \quad G'_{\mu\nu} = G_{\mu\nu}+G^{PV}_{\mu\nu}. \tag{1.5}$$

Finiteness of (1.4) when $\Lambda \to \infty$ requires

$$\begin{aligned}\mathrm{STr}\mu^{2n} &= \mathrm{STr}H' = \mathrm{STr}\left(2\mu^2 H' + \nu^2\right) = \mathrm{STr}\nu H' \\ &= \mathrm{STr}H'^2 + \frac{1}{6}\mathrm{STr}G'^2 + 2t'_- = 0,\end{aligned} \tag{1.6}$$

where t'_- is the coefficient of $\ln\Lambda^2/32\pi^2$ in $T_- + T_-^{PV}$. The vanishing of $\mathrm{STr}\mu^{2n}$ is automatically assured by supersymmetry. Once the remaining conditions are satisfied we obtain

$$S_1 = -\int \frac{d^4x}{64\pi^2}\mathrm{STr}\left[\left(2\mu^2 H' + \nu^2 + \mathrm{STr}H'^2 + \frac{1}{6}\mathrm{STr}G'^2 + 2t'_-\right)\ln\mu^2\right]. \tag{1.7}$$

2 Anomaly-free supergravity

We consider here a supergravity theory in which the Yang-Mills fields have canonical kinetic energy. We further assume that there are no gauge or mixed gauge-gravitational anomalies: $\mathrm{Tr}T^a = \mathrm{Tr}(\{T_a, T_b\}T_c) = 0$, where T_a is a generator of the gauge group.

To regulate chiral multiplet loops, we introduce Pauli-Villars chiral supermultiplets Z^I_α, that transform under gauge transformations like Z^I, Y^α_I, that transform according to the conjugate representation, and gauge singlets Y^0, Z^0. Additional charged fields X^A_β and U^β_A transform according to the representation R^a_A and its conjugate, respectively, under the gauge group factor $\mathcal{G}_a$, and V^A_β transforms according to a (pseudo)real representation that is traceless and anomaly-free. Their gauge couplings satisfy

$$\sum_{\beta,A}\eta^A_\beta C^a_A = \sum_i C^a_i \equiv C^a_M, \tag{2.1}$$

where

$$\mathrm{Tr}\left(T^aT^b\right)_R = \delta_{ab}C^a_R \tag{2.2}$$

for particles transforming according to the representation R (or $\bar{R}$), and the subscripts i, A, refer to the light fields and to X, U, V, respectively. For example, if the theory has $2N_f$ fundamental representations of $\mathcal{G}_a$, (as in supersymmetric extensions of the Standard Model) we can take PV fields in the fundamental and anti-fundamental representations with signatures that satisfy $\sum_\beta \eta^f_\beta = N_f$. If there are $2N_f + 1$ fundamental representations, one needs an anomaly-free (pseudo)real representation r for some V^A such that $C^a_r = (2m+1)C^a_f$. If no such representation exists, the theory cannot be regulated in this way.

To regulate gravity loops we introduce additional gauge singlets ϕ^γ, as well as $U(1)$ gauge supermultiplets W^α with signature η^α and chiral multiplets $Z^\alpha = e^{\theta^\alpha}$ with the same signature and $U(1)_\beta$ charge $q_\alpha\delta_{\alpha\beta}$, such that the Kähler potential $K(\theta,\bar{\theta}) = \frac{1}{2}\nu_\alpha(\theta+\bar{\theta})^2$ is invariant under $U(1)_\beta$: $\delta_\beta\theta_\alpha = -\delta_\beta\bar{\theta}_\alpha = iq_\alpha\delta_{\alpha\beta}$. The corresponding D-term:

$$\mathcal{D}(\theta,\bar{\theta}) = \mathcal{D}^\alpha_\theta\mathcal{D}^\theta_\alpha, \quad \mathcal{D}^\theta_\alpha = -i\sum_\beta K_\beta\delta_\alpha\theta^\beta = q_\alpha\nu_\alpha(\theta^\alpha+\bar{\theta}^\alpha), \tag{2.3}$$

vanishes in the background, but $(\theta^\alpha + \bar{\theta}^\alpha)/\sqrt{2}$ acquires a squared mass $\mu^2_\alpha = (2x)^{-1}q^2_\alpha\nu_\alpha$ equal to that of W^α, with which it forms a massive vector supermultiplet, where $x = g^{-2}$ is the inverse squared gauge coupling, taken here to be a constant.

Finally, to regulate the Yang-Mills contributions, we include chiral multiplets $\varphi^a_\alpha, \hat{\varphi}^a_\alpha$ that transform according to the adjoint representation of the gauge group.

We take the Kähler potential[1]

$$K_{PV} = \sum_\gamma\left[e^{\alpha^\phi_\gamma K}\phi^\gamma\bar{\phi}_\gamma + \frac{1}{2}\nu_\gamma(\theta_\gamma+\bar{\theta}_\gamma)^2 + \sum_A\left(|X^A_\gamma|^2 + |U^\gamma_A|^2\right)\right]$$

[1]This choice is by no means unique, only illustrative.

$$
\begin{aligned}
& +\sum_{\alpha,a}\left(e^K\varphi^a_\alpha\bar{\varphi}^\alpha_a+\hat{\varphi}^a_\alpha\hat{\bar{\varphi}}^\alpha_a\right)+\sum_\alpha\left(K^Z_\alpha+K^Y_\alpha\right)+\sum_{A\gamma}|V^A_\gamma|^2 \\
K^Z_\alpha &= \sum_{I,J=i,j}\left[K_{i\bar{j}}Z^I_\alpha\bar{Z}^{\bar{J}}_\alpha+\frac{b_\alpha}{2}\left\{\left(K_{ij}-K_iK_j\right)Z^I_\alpha Z^J_\alpha+\text{h.c.}\right\}\right]+|Z^0_\alpha|^2, \\
K^Y_{\alpha>3} &= \left[\sum_{I,J=i,j}K^{i\bar{j}}Y^\alpha_I\bar{Y}^\alpha_{\bar{J}}-a_\alpha\left(Y^\alpha_I\bar{Y}^0_\alpha K^i+\text{h.c.}\right)+|Y^\alpha_0|^2\left(1+a^2_\alpha K^iK_i\right)\right], \\
K^Y_{\alpha\leq 3} &= \sum_{I,J=i,j}\delta^{i\bar{j}}Y^1_I\bar{Y}^1_{\bar{J}}+|Y^1_0|^2, \quad K^i=K^{i\bar{m}}K_{\bar{m}}, \qquad (2.4)
\end{aligned}
$$

where $K^{i\bar{m}}$ is the inverse of the metric tensor $K_{i\bar{m}}$, the superpotential

$$
\begin{aligned}
W_{PV} &= \sum_{\alpha\beta}\left[\sum_I\mu^Z_{\alpha\beta}Z^I_\alpha Y^\beta_I+\mu^0_{\alpha\beta}Z^0_\alpha Y^\beta_0+\frac{1}{2}\sum_a\left(\mu^\varphi_{\alpha\beta}\varphi^a_\alpha\varphi^a_\beta+\mu^{\hat{\varphi}}_{\alpha\beta}\hat{\varphi}^a_\alpha\hat{\varphi}^a_\beta\right)\right] \\
&+\frac{1}{2}\sum_\gamma\left[\mu^\phi_\gamma(\phi^\gamma)^2+\mu^\theta_\gamma(\theta^\gamma)^2\right]+\sum_{A\gamma}\left(\mu^X_\gamma U^\gamma_A X^A_\gamma+\mu^V_\gamma(V^\gamma_A)^2\right) \\
&+\frac{1}{\sqrt{2}}\sum_{\alpha=4}\left(a_\alpha W_iZ^I_\alpha Y^\alpha_0+WZ^I_\alpha Y^\alpha_I\right)+\frac{1}{2}Z^I_1Z^J_1W_{ij} \\
&+\sqrt{\frac{2}{x}}\sum_{\alpha=4}\varphi^a_{\alpha-2}\left[Y^\alpha_I(T_aZ)^i+g_a\mathcal{D}_aY^\alpha_0\right]+\frac{1}{2}\sum_\alpha c_\alpha|Z^0_\alpha|^2W, \qquad (2.5)
\end{aligned}
$$

and gauge field kinetic functions

$$
f^{ab} = x\left(\delta^{ab}+2d_{\alpha\beta}\hat{\varphi}^a_\alpha\hat{\varphi}^b_\alpha\right), \quad f^{\alpha\beta}=\delta^{\alpha\beta}, \quad f^{a\alpha}=e^{\alpha\beta}\sqrt{2x}\varphi^a_\beta, \qquad (2.6)
$$

where the index a refers to the light gauge degrees of freedom. The function $K=K(Z,\bar{Z})$ is the Kähler potential for the light chiral multiplets $Z^i=(\bar{Z}^{\bar{\imath}})^\dagger$, $W=W(Z)$ is the superpotential, and

$$
K_i=\partial_iK=\frac{\partial}{\partial z^i}K, \quad K_{i\bar{m}}=\partial_i\partial_{\bar{m}}K, \quad K_{ij}=\partial_i\partial_jK, \quad etc. \qquad (2.7)
$$

Properties of the metric tensor for Y_I, Y_0, are given in Appendix A. The matrices $\mu_{\alpha\beta}, d_{\alpha\beta}, e_{\alpha\beta}$, are nonvanishing only when they couple fields of the same signature. The parameters μ, ν, play the role of effective cut-offs; they are constrained so as to eliminate logarithmically divergent terms of order

$\mu^2 \ln \Lambda^2$ in the integral (1.4). The parameters a, b, c, d, e, are of order unity, and are chosen to satisfy:

$$\begin{aligned} b_1 &= 1, \quad b_{\alpha\neq 1} = 0, \quad a \equiv \sum_{\alpha=4} \eta^Y_\alpha a^2_\alpha = -2, \quad a' \equiv \sum_{\alpha=4} \eta^Y_\alpha a^4_\alpha = +2, \\ \sum_\alpha e^2 &\equiv 2e = 4g - 2, \quad g \equiv \sum_{\alpha=4} (a_\alpha + g_\alpha)^2, \quad \sum_\alpha \eta^Z_\alpha c^2_\alpha = -N'_G. \end{aligned} \tag{2.8}$$

The signatures of the chiral PV multiplets satisfy

$$\begin{aligned} \sum_\alpha \eta^\varphi_\alpha &= 1, \quad \sum_\alpha \eta^{\hat\varphi}_\alpha = 2, \quad \sum_\alpha \eta^Z_\alpha = -1, \quad \eta^U_\alpha = \eta^X_\alpha, \\ \eta^Y_\alpha &= \eta^Z_\alpha, \quad \eta^\varphi_\alpha = \eta^Z_{\alpha+2}, \quad \eta^Z_1 = \eta^Z_2 = -\eta^Z_3 = -1. \end{aligned} \tag{2.9}$$

2.1 Quadratic divergences

In [1] it was shown how to regulate the quadratic divergences of supergravity that are proportional to[2]

$$\begin{aligned} \mathrm{STr}H &= -10V - 2M^2 + \frac{7}{2}r + 4K_{i\bar m}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar z^{\bar m} + 2\mathcal{D} + N_G\frac{r}{2} \\ &\quad + 2N\left(\hat V + M^2 - \frac{r}{4}\right) + 2x^{-1}\mathcal{D}_a D_i (T^a z)^i \\ &\quad - 2R_{i\bar m}\left(e^{-K}\bar A^i A^{\bar m} + \mathcal{D}_\nu z^i \mathcal{D}^\mu \bar z^{\bar m}\right), \end{aligned} \tag{2.10}$$

where N and N_G are the number of chiral and gauge supermultiplets, respectively, in the light spectrum. In these expressions, r is the space-time curvature, $R_{i\bar m}$ is the Ricci tensor associated with the Kähler metric $K_{i\bar m}$, $V = \hat V + \mathcal{D}$ is the classical scalar potential with $\hat V = e^{-K}A_i\bar A^i - 3M^2$, $\mathcal{D} = (2x)^{-1}\mathcal{D}^a\mathcal{D}_a$, $\mathcal{D}_a = K_i(T_a z)^i$, and $M^2 = e^{-K}A\bar A$ is the field-dependent squared gravitino mass, with

$$A = e^K W = \bar A^\dagger, \quad A_i = D_i A, \quad \bar A^i = K^{i\bar m}\bar A_{\bar m}, \quad etc., \tag{2.11}$$

[2]See Appendix D for corrections with respect to [3, 4]. Our conventions and notations are defined in the Appendices of these papers.

where D_i is the scalar field reparameterization covariant derivative.

In evaluating the effective one-loop action we set to zero all background Pauli-Villars fields; then the contribution of these fields to STrH is

$$\begin{aligned}\mathrm{STr}H^{PV} &= 2\sum_P \eta_\alpha\left[\frac{1}{x}\mathcal{D}^a D_P(T_a z)^P - R^P_{Pi\bar{m}}\left(\bar{A}^i A^{\bar{m}} e^{-K} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right)\right] \\ &\quad + 2\sum_P \eta_P\left(\hat{V} + M^2\right) - \left(\sum_P \eta_P - \sum_\alpha \eta^\theta_\alpha\right)\frac{r}{2}, \qquad (2.12)\end{aligned}$$

where P refers to all PV chiral multiplets, including θ^α. ¿From (2.1) we obtain for the relevant elements of the scalar reparameterization connection Γ and Riemann tensor R (see Appendix A):

$$\begin{aligned}D_I(T_a z_\alpha)^J &= D_i(T_a z)^j, \quad D_I(T_a y_1)^J = -(T_a)^i_j, \\ (R^{Z_\alpha})^I_{Jk\bar{m}} &= R^i_{jk\bar{m}}, \quad (R^{Y_1})^I_{Jk\bar{m}} = 0, \\ D_I(T_a y_\alpha)^J &= -D_j(T_a z)^i - a^2_\alpha K_j(T_a z)^i, \quad D_J(T_a y_\alpha)^0 = -a_\alpha(T_a z)^j, \\ D_0(T_a y_\alpha)^I &= a_\alpha\left(K_j D_i(T_a z)^j - K_{i\bar{m}}(T_a z)^{\bar{m}} + a^2_\alpha K_i \mathcal{D}_a\right), \quad D_0(T_a y_\alpha)^0 = a^2_\alpha \mathcal{D}_a, \\ (R^{Y_\alpha})^I_{Jk\bar{m}} &= -R^j_{ik\bar{m}} - a^2_\alpha \delta^j_k K_{i\bar{m}}, \quad (R^{Y_\alpha})^0_{0k\bar{m}} = a^2_\alpha K_{k\bar{m}}, \quad (R^{Y_\alpha})^0_{Jk\bar{m}} = 0, \\ (R^{Y_\alpha}_\alpha)^J_{0k\bar{m}} &= a_\alpha\left[K_i R^i_{jk\bar{m}} + a^2_\alpha\left(K_k K_{j\bar{m}} + K_j K_{k\bar{m}}\right)\right], \quad \alpha \neq 1, \\ D_C(T_a\phi)^D &= (T_a)^C_D + \delta^C_D \alpha_C \mathcal{D}_a, \quad R^C_{Dk\bar{m}} = \delta^C_D \alpha_C K_{k\bar{m}}, \quad \phi^{C,D} \neq Z, Y, \qquad (2.13)\end{aligned}$$

where $\alpha^\varphi = 1$, $\alpha^{\hat{\varphi}} = \alpha^\theta = 0$. Using these relations with (2.9) we obtain an overall contribution from heavy PV modes:

$$\begin{aligned}\mathrm{STr}H_{PV} &= -\frac{r}{2}\left(N' - N'_G\right) - 2\alpha\left(K_{i\bar{m}}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - 2\mathcal{D}\right) - 2x^{-1}\mathcal{D}_a D_i(T^a z)^i \\ &\quad + 2\hat{V}\left(N' - \alpha\right) + 2M^2\left(N' - 3\alpha\right) + 2R_{i\bar{m}}\left(e^{-K}\bar{A}^i A^{\bar{m}} + \mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right), \\ \alpha &= \sum_C \eta_C \alpha_C, \quad N' = \sum_P \eta_P - 2N, \quad N'_G = \sum_\gamma \eta^\theta_\gamma. \qquad (2.14)\end{aligned}$$

With (2.10) the finiteness condition STr$H' = 0$ imposes the constraints

$$N' = 7 - N, \quad N'_G = -N_G, \quad \alpha = 2. \qquad (2.15)$$

The vanishing of $\mathrm{STr}(\mu^2 H'+\nu^2)$ in (1.6) further constrains the parameters μ and ν. If, for example, we set[3] $\mu^P_{\alpha\beta} = \mu^P_\alpha \delta_{\alpha\beta}$, $q_a = 1$, $\mu^{P\neq\theta}_\alpha = \beta^P_\alpha \mu$, $\nu^\theta_\gamma = \left(\beta^\theta_\gamma\right)^2 |\mu|^2$, the finiteness constraint requires

$$\begin{aligned}
\sum_{\alpha=1}^{3} \eta^Z_\alpha \left(\beta^Z_\alpha\right)^2 &= \sum_{\alpha=4} \eta^Z_\alpha \left(a_\alpha \beta^Z_\alpha\right)^2 = N \sum_{\alpha=4} \eta^Z_\alpha \left(\beta^Z_\alpha\right)^2 + \sum_{C,\alpha^C=0} \eta_C \left(\beta_C\right)^2 = 0, \\
\sum_C \eta_C \left(\beta_C\right)^2 &= 0 \quad \text{for fixed } \alpha_C \neq 0, \quad C \neq Z^I, Y_I.
\end{aligned} \tag{2.16}$$

As explained in [1] the $O(\mu^2)$ contribution to $S_0 + S_1 = \int d^4x \, (\mathcal{L}_0 + \mathcal{L}_1)$ takes the form:

$$\mathcal{L}_0(g^0_{\mu\nu}, K) + \mathcal{L}_1 = \mathcal{L}_0(g_{\mu\nu}, K + \delta K), \quad g_{\mu\nu} = g^0_{\mu\nu}\,(1+\epsilon)$$

$$\begin{aligned}
\epsilon &= -\sum_P \frac{\lambda_P}{32\pi^2} e^{-K} A_{PQ} \bar{A}^{PQ} = \mathrm{Tr} \sum_P \frac{\lambda \Lambda^2}{32\pi^2} \zeta', \\
\delta K &= \sum_P \frac{\lambda_P}{32\pi^2} \left(e^{-K} A_{PQ} \bar{A}^{PQ} - 4\mathcal{K}_P\right) = \mathrm{Tr} \sum_P \frac{\lambda \Lambda^2}{32\pi^2} \zeta, \\
\mathcal{K}_P &= \delta_{P\theta_\gamma} K^{PV}_{\theta^\gamma \bar{\theta}^\gamma} \sum_\delta \delta_\delta \theta^\gamma \delta_\delta \bar{\theta}^\gamma = q^2_\gamma \nu_\gamma,
\end{aligned} \tag{2.17}$$

where [5]

$$\begin{aligned}
\lambda_{PQ} &= \delta_{PQ} \lambda_P, \quad \zeta_{PQ} = \delta_{PQ} \zeta_P, \\
\lambda_P &= 2 \sum_\alpha \eta^P_\alpha \left(\beta^P_\alpha\right)^2 \ln \beta^P_\alpha, \quad \zeta_{P\neq\theta} = \zeta'_{P\neq\theta} = 1, \quad \zeta_\theta = -4, \quad \zeta'_\theta = 0, \\
\left(\Lambda^2\right)^Q_P &= e^K K^{Q\bar{R}} K^{\bar{T}S} \mu_{PS} \bar{\mu}_{\bar{T}\bar{R}}, \quad P \neq \theta, \quad \Lambda^2_{\theta_\alpha \theta_\gamma} = \delta_{\alpha\gamma} |\mu_\theta|^2.
\end{aligned} \tag{2.18}$$

Λ^2 plays the role of the (matrix-valued) effective cut-off. As emphasized previously [1], if there are three or more terms in the sum over α, the sign of λ_P is indeterminate [5].

[3]The result is unchanged if the parameters $\mu \to \mu(z)$, $\nu \to \nu(z, \bar{z})$ depend on the light fields[1].

In the following we require only on-shell invariance,[4] so the quadratic divergences impose one less constraint than in (2.15). That is, we perform a Weyl transformation to write the one-loop corrected Lagrangian as

$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{L}_{tree}\left(g^R\right) - \frac{\Lambda^2}{32\pi^2}\mathrm{STr}H'^2 - \epsilon\left(\frac{r}{2} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} - 2V\right) \\ &\quad + O\left(\frac{\ln\Lambda^2}{32\pi^2}\right) + O\left[\left(\frac{\hbar}{16\pi^2}\right)^2\right] + \text{finite terms}, \\ g^R_{\mu\nu} &= (1+\epsilon)g_{\mu\nu}, \quad \epsilon = \frac{\Lambda^2}{32\pi^2}\left(N + N' - N_G - N'_G - 7\right), \end{aligned} \tag{2.19}$$

and we do not require ϵ to vanish. Then the finiteness conditions reduce to

$$N' = 3\alpha + 1 - N, \quad N'_G = \alpha - 2 - N_G. \tag{2.20}$$

In this case, the third finiteness condition in (1.6) becomes

$$\mathrm{STr}\left(2\mu^2 H' + \nu^2\right) = 2\mathrm{STr}\left(\mu_G^2 - \mu_\chi^2\right)\left(\frac{1}{2}r + K_{i\bar{m}}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - 2V\right) = 0. \tag{2.21}$$

The supertrace on the right hand side vanishes identically because the supertraces of the squared mass matrices μ^2_{PV} vanish separately in the chiral (μ^2_χ) and $U(1)$ gauge (μ^2_G) PV sectors.

2.2 Logarithmic divergences

¿From the results of [3, 4], if $\mathcal{L}(g, K)$ is the standard Lagrangian [6, 7] for $N = 1$ supergravity coupled to matter with space-time metric $g_{\mu\nu}$, Kähler potential K, and gauge kinetic function $f_{ab}(Z) = \delta_{ab}$, the logarithmically divergent part of the one loop corrected Lagrangian is

$$\mathcal{L}_{eff} = \mathcal{L}\left(g_R, K_R\right) + \frac{\ln\Lambda^2}{32\pi^2}\left(X^{AB}\mathcal{L}_A\mathcal{L}_B + X^A\mathcal{L}_A\right) + \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}L$$

[4]The off-shell divergences are prescription dependent; the extension of this regularization procedure beyond one loop may require a choice of prescription in which they can also be made finite.

$$
\begin{aligned}
L &= L_0 + L_1 + L_2 + L_3 + NL_\chi + N_G L_g, \\
\mathcal{L}_A &= \frac{\partial \mathcal{L}}{\partial \phi^A},
\end{aligned}
\tag{2.22}
$$

where ϕ^A is any light field, and[5]

$$
\begin{aligned}
L_0 &= 3C^a \delta_{ab}\left(\mathcal{W}^{ab} + \mathrm{h.c.}\right) - \frac{20}{3}\hat{V}^2 + \frac{10}{3}\hat{V}M^2 + 5M^4 + \frac{88}{3}\mathcal{D}M^2 \\
&+ \frac{47x}{6}\left[2x\mathcal{W}_{ab}\overline{\mathcal{W}}^{ab} - \left(F^a_{\rho\mu} - i\tilde{F}^a_{\rho\mu}\right)\left(F^{\rho\nu}_a + i\tilde{F}^{\rho\nu}_a\right)\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}}\right] \\
&- \frac{7i}{3}\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} \mathcal{D}^a F^{\mu\nu}_a + \frac{1}{3}\left(25\hat{V} + 10M^2\right) K_{i\bar{m}} \mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\mu z^i \\
&+ \frac{20}{3}\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a \mathcal{D}_b + 11\mathcal{D}K_{i\bar{m}}\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}} \\
&- \frac{14}{3}\mathcal{D}\hat{V} + 15\mathcal{D}_\mu z^j \mathcal{D}^\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} \mathcal{D}^\nu \bar{z}^{\bar{n}} K_{i\bar{n}} K_{j\bar{m}} \\
&- \frac{20}{3}\left(\mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\mu z^i K_{i\bar{m}}\right)^2 + \frac{20}{3}\mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{n}} \mathcal{D}^\nu z^j K_{i\bar{n}} K_{j\bar{m}},
\end{aligned}
\tag{2.23}
$$

$$
\begin{aligned}
L_\chi &= -\frac{x}{6}\left(F^a_{\rho\mu} - i\tilde{F}^a_{\rho\mu}\right)\left(F^{\rho\nu}_a + i\tilde{F}^{\rho\nu}_a\right)\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} \\
&+ \frac{1}{3}\left[x^2 \mathcal{W}_{ab}\overline{\mathcal{W}}^{ab} - \mathcal{D}\left(K_{i\bar{m}}\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}} + 2\hat{V} + 4M^2\right)\right] \\
&+ \frac{1}{3}\left(\hat{V} + 2M^2\right) K_{i\bar{m}} \mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\mu z^i - \frac{i}{3}\mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} \mathcal{D}^a F^{\mu\nu}_a \\
&+ \frac{2}{3}\hat{V}M^2 + M^4 + \frac{1}{3}\mathcal{D}_\mu z^j \mathcal{D}^\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} \mathcal{D}^\nu \bar{z}^{\bar{n}} K_{i\bar{n}} K_{j\bar{m}},
\end{aligned}
\tag{2.24}
$$

$$
\begin{aligned}
L_1 &= -\left[\mathcal{W}^{ab} D_i (T_b z)^j D_j (T_a z)^i + \mathrm{h.c.}\right] + \frac{2}{x}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} R^k_{i\bar{m}j} \mathcal{D}_a D_k (T^a \bar{z})^j \\
&+ \frac{2}{x}\mathcal{D}_a e^{-K} R^{k\,j}_{\,n\,i} A_k \bar{A}^n D_j (T^a z)^i + 2iF^a_{\mu\nu} D_j (T_a z)^i R^j_{i\bar{m}k} \mathcal{D}^\mu z^k \mathcal{D}^\nu \bar{z}^{\bar{m}} \\
&+ \mathcal{D}_\mu z^j \mathcal{D}^\mu \bar{z}^{\bar{m}} R^k_{j\bar{m}i} \mathcal{D}_\nu z^\ell \mathcal{D}^\nu \bar{z}^{\bar{n}} R^i_{\ell\bar{n}k} + \mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} R^k_{i\bar{m}j} \mathcal{D}^\mu z^\ell \mathcal{D}^\nu \bar{z}^{\bar{n}} R^i_{k\bar{n}\ell} \\
&- \mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} R^k_{i\bar{m}j} \mathcal{D}^\nu z^\ell \mathcal{D}^\mu \bar{z}^{\bar{n}} R^i_{k\bar{n}\ell} + 2e^{-K}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} R^k_{i\bar{m}j} R^{\ell\,j}_{\,n\,k} A_\ell \bar{A}^n \\
&+ e^{-2K} A_i \bar{A}^k R^{m\,i}_{\,n\,k} R^{n\,p}_{\,m\,q} A_p \bar{A}^q,
\end{aligned}
\tag{2.25}
$$

[5]See Appendix D for corrections with respect to [3, 4]

$$
\begin{aligned}
L_2 \;=\;& \frac{2}{3x}D_i(T_a z)^i \mathcal{D}_a\left(\mathcal{D}_\mu z^j \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{j\bar{m}} + \hat{V} + 3M^2\right) + \frac{2i}{3}\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} R_{i\bar{m}} \mathcal{D}^a F^{\mu\nu}_a \\
&+\frac{2}{3}D_i(T_a z)^i \left[\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\mathcal{D}_b + ixF^a_{\mu\nu}K_{\bar{m}j}\mathcal{D}^\mu z^j \mathcal{D}^\nu \bar{z}^{\bar{m}}\right] \\
&+\frac{4}{3}\mathcal{D}e^{-K}R^i_j A_i \bar{A}^j - \frac{2}{3}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\left[e^{-K}R^k_n A_k \bar{A}^n K_{i\bar{m}} + R_{i\bar{m}}\left(\hat{V} + 3M^2\right)\right] \\
&-\frac{2}{3}\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} R_{j\bar{n}}\left(\mathcal{D}^\mu z^j \mathcal{D}^\nu \bar{z}^{\bar{n}} - \mathcal{D}^\nu z^j \mathcal{D}^\mu \bar{z}^{\bar{n}}\right) - \frac{2}{3}e^{-2K}R^m_n A_m \bar{A}^n A_j \bar{A}^j \\
&-\frac{2}{3}\mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}} K_{i\bar{m}} \mathcal{D}^\mu z^j \mathcal{D}_\mu \bar{z}^{\bar{n}} R_{j\bar{n}} + \frac{4}{3}\mathcal{D}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} R_{i\bar{m}},
\end{aligned}
\tag{2.26}
$$

$$
\begin{aligned}
L_3 \;=\;& \mathcal{D}_\mu z^j \mathcal{D}^\mu z^i R^{k\;\;\ell}_{\;j\;i}\mathcal{D}_\nu \bar{z}^{\bar{n}} \mathcal{D}^\nu \bar{z}^{\bar{m}} R_{\bar{n}k\bar{m}\ell} \\
&+e^{-K}\left[\mathcal{D}_\mu z^i \mathcal{D}^\mu z^j \left(A_{ik\ell}\bar{A}^n R^{k\;\ell}_{\;n\;j} - R^{k\;\ell}_{\;j\;i}(A_{mk\ell}\bar{A}^m - A_{k\ell}\bar{A})\right) + \text{h.c.}\right] \\
&+\frac{e^{-K}}{x}\mathcal{D}_a\left[(T^a z)^i R_i{}^j{}_\ell{}^k \bar{A}^\ell A_{jk} + \text{h.c.}\right] + e^{-2K}\left(R^{j\;k}_{\;n\;i}A_{jk}\bar{A}^n A\bar{A}^i + \text{h.c.}\right) \\
&+e^{-K}\left(2\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} + e^{-K}\bar{A}^i A^{\bar{m}}\right) R^\ell_{j\bar{m}k} R^{j\;k}_{\;i\;n} A_\ell \bar{A}^n \\
&-\left(\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} + e^{-K}\bar{A}^i A^{\bar{m}}\right)\left[\mathcal{D}_i\left(e^{-K}R^k_{\ell\bar{m}j}A_k \bar{A}^{j\ell}\right) + \text{h.c.}\right],
\end{aligned}
\tag{2.27}
$$

$$
\begin{aligned}
L_g \;=\;& \frac{1}{3}K_{i\bar{m}}K_{j\bar{n}}\left(2\mathcal{D}_\mu z^i \mathcal{D}^\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} \mathcal{D}^\nu \bar{z}^{\bar{n}} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{n}} \mathcal{D}_\nu \bar{z}^{\bar{m}} \mathcal{D}^\nu z^j\right) \\
&-\frac{1}{3}\left(\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}}\right)^2 + x^2 \mathcal{W}_{ab}\overline{\mathcal{W}}^{ab} + \frac{1}{3}\left(\mathcal{W}_{ab} + \overline{\mathcal{W}}_{ab}\right)\mathcal{D}^a \mathcal{D}^b \\
&-\frac{1}{3}\hat{V}^2 + \frac{1}{3}\left(\hat{V} + \mathcal{D}\right)\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} - \frac{i}{3}\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} \mathcal{D}^a F^{\mu\nu}_a \\
&-\frac{2}{3}\hat{V}\mathcal{D} - \frac{x}{2}K_{i\bar{m}}\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\left(F^a_{\rho\mu} + i\tilde{F}^a_{\rho\mu}\right)\left(F^{\rho\nu}_a - i\tilde{F}^{\rho\nu}_a\right),
\end{aligned}
\tag{2.28}
$$

where $F^2 = F^a_{\mu\nu}F^{\mu\nu}_a$ with $F^a_{\mu\nu}$ the Yang-Mills field strength,

$$
\mathcal{W}_{ab} = \frac{1}{4}\left(F_a \cdot F_b + \tilde{F}_a \cdot F_b\right) - \frac{1}{2x}\mathcal{D}_a \mathcal{D}_b,
\tag{2.29}
$$

and

$$
\begin{aligned}
e^K D_i\left(e^{-K}R^\ell_{j\bar{m}k}A_\ell \bar{A}^{jk}\right) \;=\;& (D_i R^\ell_{j\bar{m}k})A_\ell \bar{A}^{jk} + R^\ell_{j\bar{m}k}A_{i\ell}\bar{A}^{jk} \\
&+2R^k_{i\bar{m}j}A_k \bar{A}^j + R^\ell_{j\bar{m}k}R^{j\;k}_{\;i\;n}A_\ell \bar{A}^n.
\end{aligned}
\tag{2.30}
$$

The renormalized Kähler potential is

$$\begin{aligned} K_R &= K + \frac{\ln\Lambda^2}{32\pi^2}\left[e^{-K}A_{ij}\bar{A}^{ij} - 2\hat{V} - 10M^2 - 4\mathcal{K}^a_a - 12\mathcal{D}\right], \\ \mathcal{K}^a_b &= \frac{1}{x}(T^a z)^i (T_b \bar{z})^{\bar{m}} K_{i\bar{m}}. \end{aligned} \tag{2.31}$$

The second term in the expression (2.24) for $\mathcal{L}_{eff}$ does not contribute to the S-matrix. Since we are only interested in on-shell finiteness, we can drop it. We have also dropped total derivatives, including the Gauss-Bonnet term which can readily be extracted from the results of [3, 4]:

$$\mathcal{L}_{eff} \ni \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}\frac{1}{48}\left(41 + N - 3N_G\right)\left(r^{\mu\nu\rho\sigma}r_{\mu\nu\rho\sigma} - 4r^{\mu\nu}r_{\mu\nu} + r^2\right), \tag{2.32}$$

in agreement with other calculations [8]. We similarly drop total derivatives in the logarithmically divergent PV contributions.

The Pauli-Villars contribution to (2.24) is, after an appropriate additional space-time metric redefinition,

$$\begin{aligned} \mathcal{L}_{PV} &= \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}\left[N'_G L_g + N' L_\chi + \sum_P \eta_P\left(L^P_1 + L^P_2\right) + L^Z_3 + L_{\mathcal{W}} + eL_e\right] \\ &\quad + \Delta_{K'}\mathcal{L}, \qquad K' = \frac{\ln\Lambda^2}{32\pi^2}e^{-K}\sum_{P,Q}\eta_P A_{PQ}\bar{A}^{PQ}, \end{aligned} \tag{2.33}$$

where

$$\begin{aligned} \frac{1}{\sqrt{g}}\Delta_F\mathcal{L} &= \Delta_F L = -F\hat{V} + \left(e^{-K}\bar{A}^i A^{\bar{m}} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right)\partial_i\partial_{\bar{m}}F \\ &\quad - \left\{\partial_i F\left[e^{-K}\bar{A}^i A + \frac{1}{2x}\mathcal{D}_a (T^a z)^i\right] + \text{h.c.}\right\}, \end{aligned} \tag{2.34}$$

is the shift in $\mathcal{L}/\sqrt{g}$ due to a shift $F(z,\bar{z})$ in the Kähler potential, and [see Appendix B and Eq. (B.38)]

$$\begin{aligned} L_{\mathcal{W}} &= x^2 \mathcal{W}_{ab}\overline{\mathcal{W}}^{ab}\left[2e^2 + (d-2e)^2\right], \\ L_e &= 2i\mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} \mathcal{D}^a F^{\mu\nu}_a + 4\mathcal{D}\left(3M^2 + \hat{V}\right) - 4x^2\mathcal{W}_{ab}\overline{\mathcal{W}}^{ab} \\ &\quad + x\left(F^a_{\rho\mu} - i\tilde{F}^a_{\rho\mu}\right)\left(F^{\rho\nu}_a + i\tilde{F}^{\rho\nu}_a\right)\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} \\ &\quad + 2\mathcal{D}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} - 4\Delta_{\mathcal{D}} L, \end{aligned} \tag{2.35}$$

are the contributions from the gauge kinetic terms given in (2.6), obtained by a straightforward generalization of the results of [4] to the case of a non-diagonal gauge kinetic function f_{ab} (see Appendix B).

To evaluate K' and L_3 we need the additional PV matrix elements (see appendix A):

$$\begin{aligned}
R^{Z_1}_{I\bar{m}J\bar{n}} &= R_{i\bar{m}j\bar{n}} + K_{i\bar{m}}K_{j\bar{n}} + K_{i\bar{m}}K_{j\bar{n}}, \quad A^{Z_1}_{IJ} = A_{ij}, \quad \bar{A}^{IJ}_{Z_1} = \bar{A}^{ij}, \\
A^{Y_\alpha,\varphi_{\alpha-2}}_{Ia} &= e^K\sqrt{\frac{2}{x}}(T_a z)^i, \quad \bar{A}^{Ia}_{Y_\alpha,\varphi_{\alpha-2}} = \sqrt{\frac{2}{x}}\left[(T^a\bar{z})^{\bar{m}}K_{i\bar{m}} + a_\alpha(a_\alpha+g_\alpha)K_i\mathcal{D}^a\right], \\
A^{Y_\alpha,\varphi_{\alpha-2}}_{0a} &= e^K\sqrt{\frac{2}{x}}g_\alpha\mathcal{D}_a, \quad \bar{A}^{0a}_{Y_\alpha,\varphi_{\alpha-2}} = \sqrt{\frac{2}{x}}(g_\alpha+a_\alpha)\mathcal{D}^a, \\
\sqrt{2}A^{Z_\alpha,Y_\alpha}_{IJ} &= \sqrt{2}A\delta^j_i, \quad \sqrt{2}\bar{A}^{IJ}_{Z_\alpha,Y_\alpha} = \delta^i_j\bar{A} + a^2_\alpha e^K K_j W\bar{A}^i, \quad \alpha\neq 1, \\
\sqrt{2}\bar{A}^{I0}_{Z_\alpha,Y_\alpha} &= a_\alpha\bar{A}^i, \quad \sqrt{2}A^{Z_\alpha,Y_\alpha}_{I0} = a_\alpha e^K W_i, \quad \alpha\neq 1, \\
A^\theta_{\alpha\beta} &= \delta_{\alpha\beta}\nu_\alpha W, \quad \bar{A}^{\alpha\beta}_\theta = \delta^{\alpha\beta}\nu^{-1}_\alpha\overline{W}, \\
A^{Z^0}_{\alpha\beta} &= \delta_{\alpha\beta}c_\alpha W, \quad \bar{A}^{\alpha\beta}_{Z^0} = \delta^{\alpha\beta}c_\alpha\overline{W},
\end{aligned} \tag{2.36}$$

where we have not included μ-dependent terms that are already contained in (2.17). Then, using (2.8-9) we obtain

$$K' = -\frac{\ln\Lambda^2}{32\pi^2}\left[e^{-K}A_{ij}\bar{A}^{ij} + 2\hat{V} + 2M^2 - 4\mathcal{K}^a_a - 4(e+1)\mathcal{D}\right]. \tag{2.37}$$

L_3 is determined by the expressions

$$\begin{aligned}
R^{Z_1}_{I\bar{m}J\bar{n}}\left(R^{Z_1}\right)^{I\ J}_{\ k\ \ell} &= R_{i\bar{m}j\bar{n}}R^{i\ j}_{\ k\ \ell} + 4R_{k\bar{m}\ell\bar{n}} + 2\left(K_{k\bar{m}}K_{\ell\bar{n}} + K_{\ell\bar{m}}K_{k\bar{n}}\right), \\
A^{Z_1}_{IJ}\bar{A}^{IJ}_{Z_1} &= A_{ij}\bar{A}^{ij}, \quad R^{Z_1}_{I\bar{m}J\bar{n}}\bar{A}^{IJ}_{Z_1} = R_{i\bar{m}j\bar{n}}\bar{A}^{ij} + 2\bar{A}_{\bar{m}\bar{n}},
\end{aligned} \tag{2.38}$$

giving

$$\begin{aligned}
L^Z_3 &= -L_3 + 4\Delta_{\hat{V}}L + 8\Delta_{M^2}L - \frac{2}{\sqrt{g}}e^{-K}\left(A_i\bar{A}\mathcal{L}^i + \text{h.c.}\right) \\
&\quad -4\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}\left(K_{i\bar{n}}K_{j\bar{m}} + R_{i\bar{m}j\bar{n}}\right) - 4M^2\left(2\hat{V} + 3M^2\right) \\
&\quad -4e^{-K}\left(2\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}} + e^{-K}\bar{A}^i A^{\bar{m}}\right)R^\ell_{i\bar{m}n}A_\ell\bar{A}^n - 8\mathcal{D}M^2,
\end{aligned} \tag{2.39}$$

where relations among operators given in Appendix B of [4] were used. L_2^P is obtained directly from (2.13):

$$\begin{aligned}\sum_P \eta_P L_2^P &= -L_2 - \frac{2}{3}\alpha L_\alpha,\\ L_\alpha &= \left(\hat{V}+3M^2\right)^2 - 4\mathcal{D}\left(K_{i\bar{m}}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} + \hat{V}+3M^2\right)\\ &\quad +2\left(\hat{V}+3M^2\right)K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i K_{i\bar{m}} + \left(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i K_{i\bar{m}}\right)^2\\ &\quad +\mathcal{D}^\mu z^i\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}}\left(\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}} - \mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}_\nu z^j\right)\\ &\quad -2i\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^a F_a^{\mu\nu} - \left(\mathcal{W}^{ab}+\overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a\mathcal{D}_b. \qquad (2.40)\end{aligned}$$

To evaluate L_1^P we need

$$\begin{aligned}D_I(T_a z_\alpha)^J D_J(T_b z_\alpha)^I &= D_i(T_a z)^j D_j(T_b z)^i,\\ D_I(T_a y_1)^J D_J(T_b y_1)^I &= -\delta_{ab}C_M^a,\\ (R^{Z_\alpha})^I_{Jk\bar{m}}(R^{Z_\alpha})^J_{I\ell\bar{n}} &= R^i_{jk\bar{m}}R^j_{i\ell\bar{n}},\\ (R^{Z_\alpha})^J_{Ik\bar{m}}D_J(T_b z_\alpha)^I &= R^i_{jk\bar{m}}D_j(T_b z)^i,\\ D_P(T_a y_\alpha)^Q D_Q(T_b y_\alpha)^P &= D_j(T_a z)^i D_i(T_b z)^j + a_\alpha^2 x\left(\mathcal{K}_{ab}+\mathcal{K}_{ba}\right),\\ (R^{Y_\alpha})^P_{Qk\bar{m}}(R^{Y_\alpha})^Q_{P\ell\bar{n}} &= R^j_{ik\bar{m}}R^i_{j\ell\bar{n}} - 2a_\alpha^2 R_{\ell\bar{n}k\bar{m}} + a_\alpha^4\left(K_{k\bar{m}}K_{\ell\bar{n}} + K_{k\bar{n}}K_{\ell\bar{m}}\right),\\ (R^{Y_\alpha})^P_{Qk\bar{m}}D_Q(T_b y^\alpha)^P &= R^j_{ik\bar{m}}D_i(T_b z)^j + a_\alpha^2 D_k(T_b z)^j K_{j\bar{m}}, \quad \alpha\neq 1,\\ D_C(T_a\phi)^D D_D(T_b\phi)^C &= C_C^a + \delta_D^C\alpha_C^2\mathcal{D}_a\mathcal{D}_b, \quad R^C_{Dk\bar{m}}R^D_{C\ell\bar{n}} = \delta_D^C\alpha_C^2 K_{k\bar{m}}K_{\ell\bar{n}},\\ R^C_{Dk\bar{m}}D_D(T_b\phi)^C &= \delta_D^C\alpha_C^2 K_{k\bar{m}}\mathcal{D}_b, \quad \phi^{C,D}\neq Z,Y. \qquad (2.41)\end{aligned}$$

Then using the constraints (2.8) and the results given in Appendix B of [3], we obtain (see Appendix A)

$$\begin{aligned}\sum_P \eta^P L_1^P &= -L_1 - 3C^a\delta_{ab}\left(\mathcal{W}^{ab}+\text{h.c.}\right) + \alpha' L_\alpha + L_1^Y, \quad \alpha' = \sum_C \eta_C\alpha_C^2,\\ L_1^Y &= 4\left[\Delta_{M^2}\mathcal{L} + M^2\left(2\hat{V}+3M^2+2\mathcal{D}\right)\right]\\ &\quad +8\Delta_{\mathcal{D}}L - \frac{2}{x\sqrt{g}}\left[\mathcal{D}_a(T^a z)^i\mathcal{L}_i + i\mathcal{D}_\mu\bar{z}^{\bar{m}}(T^a z)^i K_{i\bar{m}}\mathcal{L}_a^\mu + \text{h.c.}\right]\\ &\quad +4\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}\left(R_{i\bar{m}j\bar{n}} + K_{i\bar{n}}K_{j\bar{m}}\right)\\ &\quad +4e^{-K}\left(2\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}} + e^{-K}\bar{A}^i A^{\bar{m}}\right)R^\ell_{i\bar{m}n}A_\ell\bar{A}^n. \qquad (2.42)\end{aligned}$$

Adding the above, we get for the total PV contribution:

$$\begin{aligned}
\mathcal{L}_{PV} &= \frac{\ln\Lambda^2}{32\pi^2}\left(X^{AB}_{PV}\mathcal{L}_A\mathcal{L}_B + X^A_{PV}\mathcal{L}_A\right) + \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}L_{PV} + \Delta_{K^{PV}}\mathcal{L}, \\
K^{PV} &= K' + \frac{\ln\Lambda^2}{32\pi^2}\left[4\hat{V} + 12M^2 + 8\mathcal{D}\right] = -\left(K_R - K\right), \\
L_{PV} &= N'_G L_g + N' L_\chi - L_1 - L_2 - L_3 + L_{\mathcal{W}} + eL_e \\
&\quad + \left(\alpha' - \frac{2}{3}\alpha\right)L_\alpha. \qquad (2.43)
\end{aligned}$$

The renormalization of the Kähler potential is seen to be finite. Setting

$$2e^2 + (d - 2e)^2 = 2e, \qquad (2.44)$$

and using the constraints (2.20), we obtain for the remaining contributions

$$\begin{aligned}
L + L_{PV} &= -\left(6 + \alpha - \alpha'\right)\left[\hat{V}^2 + \mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^j\left(K_{i\bar{m}}K_{j\bar{n}} - K_{i\bar{n}}K_{j\bar{m}}\right)\right] \\
&\quad \left(2 - \alpha + 3\alpha'\right)\left(2\hat{V}M^2 + 3M^4 + 2M^2K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^iK_{i\bar{m}}\right) \\
&\quad +2\left(4 + \alpha'\right)\hat{V}K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^iK_{i\bar{m}} \\
&\quad +\left(14 + \alpha + \alpha'\right)\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}} \\
&\quad +4\left(7 + \alpha - 3\alpha' + 3e\right)\mathcal{D}M^2 \\
&\quad +\left(6 + \alpha - \alpha'\right)\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a\mathcal{D}_b \\
&\quad +2\left(7 + \alpha - e\right)x\left[x\mathcal{W}_{ab}\overline{\mathcal{W}}^{ab}\right. \\
&\qquad \left. -\frac{1}{2}\left(F^a_{\rho\mu} - i\tilde{F}^a_{\rho\mu}\right)\left(F^{\rho\nu}_a + i\tilde{F}^{\rho\nu}_a\right)\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}}\right] \\
&\quad -2\left(1 + \alpha' - e\right)\left(2\mathcal{D}\hat{V} + i\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^aF^{\mu\nu}_a\right) \\
&\quad +2\left(5 + \alpha - 2\alpha' + e\right)\mathcal{D}K_{i\bar{m}}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}. \qquad (2.45)
\end{aligned}$$

Finiteness is achieved by imposing

$$\alpha = -10, \quad \alpha' = -4, \quad e = -3. \qquad (2.46)$$

Once all the infinities have been removed, the Lagrangian takes the form (1.7), with the matrix-valued effective cut-off a function of the scalar fields.

In particular, the terms of order $\ln\mu$ are given by (2.22) with $\ln\Lambda^2$ replaced by the matrix $\sum_P \eta^P \ln(\mu_P^2)$.

3 Kähler anomalies

Classically, supergravity theories are invariant Kähler transformations that redefine the Kähler potential and the superpotential in terms of a holomorphic function $H(z)$:

$$K \to K + H + \bar{H}, \quad W \to e^H W, \tag{3.1}$$

and that shifts the the fermion axial $U(1)$ current:

$$\Gamma_\mu = \frac{i}{4}\left(\mathcal{D}^\mu z^i K_i - \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{\bar{m}}\right) \to \Gamma_\mu - \frac{1}{2}\partial_\mu \mathrm{Im} H. \tag{3.2}$$

This invariance is anomalous at the quantum level due to the conformal and chiral anomalies. Consider for example the one-loop correction to the Yang-Mills term:

$$\begin{aligned} \mathcal{L}_1^{YM} &= -\frac{1}{16\pi^2}\left(\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a - \frac{1}{2x}\mathcal{D}_a\mathcal{D}^a\right)\sum_P \eta_P C_P^a \ln\left(\Lambda_P^2\beta_P^2\right) + \cdots, \\ &= -\frac{1}{16\pi^2}\left(\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a - \frac{1}{2x}\mathcal{D}_a\mathcal{D}^a\right)\left[3C_G^a \ln\left(e^{K/3}\mu_\varphi^2\rho_\varphi\right) - C_M^a \ln\left(e^K\mu_Z^2\rho_Z\right)\right] \\ &\quad + \cdots, \end{aligned} \tag{3.3}$$

in the notation of (2.16), where the dots represent operators of higher dimension, and [5]

$$\ln\rho_\varphi = \sum_{\alpha,P=\varphi,\hat{\varphi}} \ln\left(\beta_\alpha^P\right)^2, \quad \ln\rho_Z = \sum_{\alpha,P=Z,X,V} \ln\left(\beta_\alpha^P\right)^2. \tag{3.4}$$

Under (3.1) the quantum correction (3.3) changes by

$$\delta\mathcal{L}_1^{YM} = -\frac{\mathrm{Re}H}{8\pi^2}\left(\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a - \frac{1}{2x}\mathcal{D}_a\mathcal{D}^a\right)\left(C_G^a - C_M^a\right), \tag{3.5}$$

Gauginos and chiral fermions have Kähler $U(1)$ weights $+1$ and -1, respectively, so the corresponding chiral anomaly

$$\delta_X \mathcal{L}_1^{YM} = -\frac{\mathrm{Im} H}{8\pi^2}\left(\frac{1}{4}F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a - \frac{1}{2x}\mathcal{D}_a\mathcal{D}^a\right)(C_G^a - C_M^a). \qquad (3.6)$$

combines with (3.5) to give the superfield expression

$$\delta \mathcal{L}_1^{YM} = -\frac{1}{8\pi^2}\int d^4\theta \frac{E}{8R} W_a^\alpha W_\alpha^a (C_G^a - C_M^a). \qquad (3.7)$$

The field dependence of the effective cut-offs was in fact determined in [15] by imposing the supersymmetric relation between the chiral and conformal anomalies associated with Kähler transformations; this in turn restricts the Kähler potential for charged PV fields.

Sigma-models coupled to supergravity are invariant under a group of nonlinear transformations $Z \to f(Z)$ that effect a Kähler transformation of the form (3.1), (3.2). This is in general a classical invariance, and an interesting question is under what circumstances this invariance, which we will refer to as modular invariance, can be respected at the quantum level. If modular invariance is broken at the quantum level, the resulting chiral and conformal modular anomalies must form a supermultiplet. We consider some examples below.

3.1 Nonlinear sigma-models

Consider first an ungauged supergravity theory with no superpotential and with a Kähler metric typically of the form

$$K = \sum_{A=1}^{m} K^A, \quad K^A = -\frac{1}{k_A}\ln\left(1 + \eta\sum_{i=1}^{n_A}|z_A^i|^2\right), \quad k_A = -\eta|k_A|, \qquad (3.8)$$

that is classically invariant under the infinitesimal nonlinear transformations

$$\delta z_A^i = \beta_A^i + \eta z_A^i \sum_j \bar{\beta}_A^{\bar{j}} z_A^j, \quad \delta K^A = F^A + \bar{F}^A, \quad F^A = \sum_j \bar{\beta}_A^{\bar{j}} z_A^j, \qquad (3.9)$$

where $\eta = +(-)1$ for a (non)compact symmetry group. Then the derivatives of the metric satisfy

$$
\begin{aligned}
K^A_{jk} &= k_A K^A_j K^A_k, \quad \Gamma^{Ai}_{jk} = k_A \left(\delta^{Ai}_j K^A_k + \delta^{Ai}_k K^A_j\right), \\
R^{Ai}_{jk\bar m} &= k_A \left(\delta^{Ai}_j K^A_{k\bar m} + \delta^{Ai}_k K^A_{j\bar m}\right), \quad \delta^{Ai}_j = \begin{cases} \delta^i_j & \text{if } K^A_i \neq 0 \\ 0 & \text{if } K^A_i = 0 \end{cases} .
\end{aligned} \tag{3.10}
$$

To regulate the theory, we need only include a subset of the chiral supermultiplets in (2.4). We take the Kähler potential

$$
\begin{aligned}
K_{PV} &= \sum_\gamma e^{\sum_A \alpha^A_\gamma K^A} \phi^\gamma \bar\phi_\gamma + \sum_{A,\alpha} K^Z_{A,\alpha} + K^Y_{A,\alpha}, \\
K^Z_{A,\alpha} &= \sum_{I,J=i,j} \left[K^A_{i\bar j} Z^I_{A,\alpha} \bar Z^{\bar J}_{A,\alpha} + \frac{b_\alpha}{2}\left(K^A_i K^A_j Z^I_{A,\alpha} Z^J_{A,\alpha} + \text{h.c.}\right)\right], \\
K^Y_{A,\alpha} &= e^{\sum_B \alpha^B_{A\alpha} K} \sum_{I,J=i,j} K^{i\bar j}_A Y^{A,\alpha}_I \bar Y^{A,\alpha}_{\bar J}, \quad \eta^Z_{A,\alpha} = \eta^Y_{A,\alpha} \equiv \eta^A_\alpha,
\end{aligned} \tag{3.11}
$$

and the superpotential

$$
W_{PV} = \sum_{I,A,\alpha\beta} \mu^Z_{A,\alpha\beta} Z^I_{A,\alpha} Y^{A,\beta}_I + \sum_{\alpha\beta} \mu^\phi_{\alpha\beta} \phi^\gamma \varphi^\beta, \tag{3.12}
$$

where $\mu_{\alpha\beta} = 0$ if $\eta_\alpha \neq \eta_\beta$.

Then (2.10) and (2.12) reduce to

$$
\begin{aligned}
\mathrm{STr}H &= 2\sum_A \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar z^{\bar m} K^A_{i\bar m}\left[2 - k_A(n_A + 1)\right] + \frac{r}{2}(7 - N), \\
\mathrm{STr}H_{PV} &= -2\sum_A \alpha_A \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar z^{\bar m} K^A_{i\bar m} - \frac{r}{2}N', \\
N &= \sum_A n_A, \quad N' = \sum_\gamma \eta_\gamma + 2n_A \sum_{\alpha,A} \eta^A_\alpha, \\
\alpha_A &= \sum_\alpha \eta^\phi_\alpha \alpha^A_\alpha + \sum_B \eta^B_\alpha \alpha^A_{B\alpha}.
\end{aligned} \tag{3.13}
$$

Cancellation of the on-shell quadratic divergences requires

$$
N + N' = 2\alpha_A + 2k_A(n_A + 1) + 3, \tag{3.14}
$$

and additional constraints on the parameters provide a cancellation of all one-loop ultraviolet divergences.

The PV Kähler potential (3.11) is invariant under the Kähler transformation (3.8), provided the PV superfields transform as

$$\begin{aligned}\delta Z^I_A &= \frac{\partial \delta z^i_A}{z^j_A} Z^J_A = \eta\left(Z^I_A F_A + z^i_A \sum_j \bar{\beta}^{\bar{j}}_A Z^J_A\right), \quad \delta\phi^\alpha = -\sum_A \alpha^A_\alpha F^A \phi^\alpha, \\ \delta Y^A_I &= = -\eta\left(Y^A_I F_A + \bar{\beta}^{\bar{i}}_A \sum_j z^j_A Y^A_J\right) - Y^A_I \sum_B \alpha^B_A F^B. \end{aligned} \qquad (3.15)$$

To obtain a fully invariant PV potential requires

$$\alpha^A_{B\alpha} = 1, \quad \mu^\phi_{\alpha\beta} = 0 \text{ if } \alpha^\phi_\alpha + \alpha^\phi_\beta \neq 1, \qquad (3.16)$$

in which case the superpotential (3.12) transforms under (3.8) as $\delta W_{PV} = -W_{PV}\sum_A F^A$, and the effective cut-offs Λ^2_{PQ} are constant. However in this case

$$\alpha_A = \frac{1}{2}N', \quad H_{PV} = -N'\left(\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} + \frac{r}{2}\right), \qquad (3.17)$$

which is removed by the Weyl transformation (2.19). Thus chiral supermultiplets with modular invariant masses do not contribute to quadratic divergences, nor do massive abelian gauge multiplets. Since modular invariance of their masses requires $\alpha^\theta = 0$, θ-loops contribute only to the space-time curvature term and exactly cancel the corresponding gauge loop contributions. Therefore, modular invariant regularization cannot be achieved unless the massless theory is free of quadratic divergences. This requires a constraint on the total massless spectrum. If it includes N_G gauge supermultiplets and N_q additional chiral supermultiplets ϕ^α with modular weights q^A_α, that is, with Kähler potential

$$K(\phi^\alpha, \bar{\phi}^\alpha) = \sum_\alpha |\phi^\alpha|^2 e^{\sum_A q^A_\alpha K^A}, \qquad (3.18)$$

the constraint reads

$$2\sum_\alpha q^A_\alpha - N_q - N + N_G + 3 + k_A(n_A + 1) = 0. \qquad (3.19)$$

If this constraint is satisfied, the Kähler potential is not renormalized, and the classical Bagger-Witten quantization condition [9, 10], which relates the pion decay constant to the Planck mass in a compact σ-model, is preserved at the quantum level. If this is not the case, one can still preserve the BW condition by imposing, in addition to (2.16), the additional constraints [see (2.17-18)] on the PV masses:

$$\sum_{\alpha\beta}\eta_\alpha\beta^2_{\alpha\beta}\ln\left(\beta_{\alpha\beta}\right)=0 \quad \text{for fixed} \quad \alpha_\alpha+\alpha_\beta\neq 1. \tag{3.20}$$

If the group of modular transformations is noncompact, a subgroup of the modular transformations (3.9) may be a classical invariance of the Lagrangian in the presence of a superpotential and of gauge interactions for a subset of the Z^i. An example is the Lagrangian for the "untwisted sector" of light fields in a class of orbifold compactifications of the heterotic string. The Kähler potential is (neglecting the dilaton)

$$K=\sum_{I=1}^{3}G^I, \quad G^I=-\ln\left(T_I+\bar{T}_I-\sum_{A=1}^{n-1}|\Phi^A_I|^2\right). \tag{3.21}$$

It is invariant under an $SL(2,R)$ group of modular transformations that leave K invariant, and the derivatives of K satisfy (3.10) with $K^A\to G^I$, $k_A\to k_I=1$. The superpotential has the form

$$W=\sum_{IJK,ABC}c_{ABC}|\epsilon_{IJK}|\Phi^A_I\Phi^B_J\Phi^C_K. \tag{3.22}$$

This model has the property that

$$A_{IA,JB}=0 \ \text{ if } \ I=J, \quad R^{i\bar{m}j\bar{n}}A_{ij}=0, \tag{3.23}$$

where the indices $i,j,\cdots$ run over all chiral fields z^i, and the logarithmically divergent contributions (2.22-28) simplify considerably. However, the ansatz (3.11) is insufficient to cancel logarithmic divergent terms proportional to $D_i(T^az)^jD_j(T_az)^i$ and $D_i(T^az)^jR^j_{ik\bar{m}}$, suggesting that modular invariant regularization is not possible for any choice of spectrum, although invariance of the $O(\mu^2)$ term can always be imposed by conditions analogous to (3.20).

3.2 String-derived supergravity

If the underlying theory is a superstring theory, there is generally invariance under a discrete group of modular transformations on the light superfields under which $K \to K + F(z) + \bar{F}(\bar{z})$, $W \to e^{-F(z)}W$, which cannot be broken by perturbative quantum corrections [11]. For example, in the class of orbifold compactifications mentioned above the Kähler potential, including twisted sector fields, takes the $SL(2,R)$ invariant form

$$\begin{aligned} K &= \sum_{I=1}^{3} g^I + f\left(e\Sigma q_A^I|\Phi^A|^2\right) = \sum_{I=1}^{3} g^I + e\Sigma q_A^I|\Phi^A|^2 + O\left(|\Phi^A|^4\right), \\ g^I &= -\ln\left(T_I + \bar{T}_I\right), \end{aligned} \tag{3.24}$$

which reduces to (3.21) when the twisted fields are set to zero. The general PV Kähler potential of (2.4) is modular invariant if the field Z_α^I has the same modular weight as Z^i and φ^C has modular weight α_C. The superpotential (2.5) can be made invariant under the discrete $SL(2,Z)$ subgroup of $SL(2,R)$ modular transformations, by an appropriate T_I-dependence of the PV masses: $\mu_\alpha \to \mu_\alpha(T_I) = \mu_\alpha \prod_I [\eta(T_I)]^{p_\alpha^I}$, where $\eta(T)$ is the Dedekind function. This modification of the effective cut-offs could be interpreted as threshold effects arising from the integration over heavy modes.

On the other hand, it is known that at least some of the modular invariance is restored by a universal Green-Schwarz counter term; this is in particular the case for the anomalous Yang-Mills coupling [12]–[15]. To study the conformal anomalies arising from the noninvariance of the effective cut-offs, consider the helicity-even part[6] of the one-loop action, given by

$$S_1 = \frac{i}{2}\mathrm{STr}\ln\left[D^2 + H(M_{PV})\right], \tag{3.25}$$

where M_{PV} is the PV mass matrix. Under a transformation on the PV fields, represented here by a column vector X^i, that leaves the tree Lagrangian, as

[6]The chiral anomaly can be obtained by a resummation [16] of the derivative expansion of the helicity-odd contribution T_-, which gives the standard results for the terms condsidered here.

well as the PV Kähler potential, invariant:

$$\begin{pmatrix} \bar{X}^{\bar{\imath}} \\ X^i \end{pmatrix} \to g_i \begin{pmatrix} \bar{X}^{\bar{\imath}} \\ X^i \end{pmatrix}, \quad M_i^{PV} = \begin{pmatrix} 0 & m_i \\ \bar{m}_i & 0 \end{pmatrix}, \quad M_{PV} \to M'_{PV}$$

$$\left(D^2 + H(0)\right)_i \to g_i \left(D^2 + H(0)\right)_i g_i^{-1}, \tag{3.26}$$

because all the operators in the determinant except M_{PV} are covariant, and the PV contribution to (3.25) changes by

$$\begin{aligned} (S_1)_{PV} &\to \frac{i}{2}\sum_i \eta_i \mathrm{STr} \ln \left\{ g_i \left[D_i^2 + H_i(g_i^{-1} M'_{PV} g_i) \right] g_i^{-1} \right\} \\ &= \frac{i}{2}\sum_i \eta_i \mathrm{STr} \ln \left[D_i^2 + H_i(g_i^{-1} M'_{PV} g_i) \right], \end{aligned} \tag{3.27}$$

where η_i is the signature, and the last equality holds if the integrals are finite. The PV Kähler potential $K_{PV} = k_{i\bar{m}} X^i X^{\bar{m}}$ is invariant provided $k_{i\bar{m}} \to g_i^{-1} k_{i\bar{m}} \bar{g}_m^{-1}$, $k^{i\bar{m}} \to g_i k^{i\bar{m}} \bar{g}_m$. If the PV mass is introduced via a superpotential term $W \ni \mu_{ij} X^i X^j$, $\mu =$ constant, the PV mass is

$$m_i^{\bar{m}} = e^{K/2} K^{j\bar{m}} \mu_{ij}, \quad m_i'^{\bar{m}} = e^{(K'-K)/2} \bar{g}_m K^{j\bar{m}} g_j K_{j\bar{n}} m_i^{\bar{n}}. \tag{3.28}$$

If the transformation is abelian: $g_i = e^{\phi_i}$, and the metric is diagonal: $K_{i\bar{m}} \propto \delta_{i\bar{m}}$, we just get

$$\begin{aligned} m_i'^{\bar{m}} &= e^{(K'-K)/2 + \bar{\phi}_m + \phi_i} m_i^{\bar{m}}, \quad g_i = \begin{pmatrix} e^{\bar{\phi}_i} & 0 \\ 0 & e^{\phi_i} \end{pmatrix}, \\ g_i^{-1} M_i g_i &= e^{(K'-K)/2} \begin{pmatrix} 0 & e^{2\phi_i} m_i \\ e^{2\bar{\phi}_i} \bar{m}_i & 0 \end{pmatrix}, \end{aligned} \tag{3.29}$$

if, *e.g.*, $\mu_{ij} \propto \delta_{ij}$.

If, following section 2, we introduce regulators X^A, X'_A for Φ^A with signature-weighted average modular weights $-q_I^A$, and X^a for the gauge fields with average weights $q_I^a = 1/3$, and the superpotential term

$$W_{PV} = \sum_A \mu_A X^A X'_A + \sum_a \mu_a X^a X_a, \quad m_i = e^{K/2 - \sum_I q_I^i g^I} \mu_i, \tag{3.30}$$

under a modular transformation we have

$$
\begin{aligned}
g_i &= \begin{pmatrix} e^{-\sum_I q_I^i \bar{F}^I} & 0 \\ 0 & e^{-\sum_I q_I^i F^I} \end{pmatrix}, \quad m_i' = e^{\sum_I (1-2q_I^i)\mathrm{Re}F^I} m_i, \\
g_i^{-1} M_i' g_i &= \begin{pmatrix} 0 & e^{-2i\sum_I q_I^i \mathrm{Im}F^I} m_i' \\ e^{2i\sum_I q_I^i \mathrm{Im}F^I} \bar{m}_i' & 0 \end{pmatrix}, \qquad (3.31)
\end{aligned}
$$

the contribution (3.3) shifts by

$$
\begin{aligned}
&-\frac{1}{64\pi^2}\delta\left\{F_a^2\left[3C_a \ln(|m_a^2|) - \sum_A C_a^A \ln(|m_A^2|)\right]\right\} + \cdots \\
&= -\frac{1}{32\pi^2}\sum_I \mathrm{Re}F^I F_a^2 \left[C_a - \sum_A C_a^A\left(1 - 2q_A^I\right)\right] + \cdots, \qquad (3.32)
\end{aligned}
$$

and the conformal anomaly matches the chiral anomaly arising from the axial currents

$$
A_\mu^\lambda = \Gamma_\mu = \frac{i}{4}\left(\mathcal{D}_\mu z^i K_i - \mathrm{h.c.}\right), \quad \left(A_\mu^\Phi\right)_B^A = -\Gamma_\mu + \frac{i}{2}\left(\mathcal{D}_\mu z^i \Gamma_{Bi}^A - \mathrm{h.c.}\right), \qquad (3.33)
$$

for gauginos and charged chiral fermions, respectively. The Casimirs and modular weights satisfy the sum rules:

$$
C^a - \sum_A (1 - 2q_A^I) C_A^a = C_{E_8} - b_a^I. \qquad (3.34)
$$

For orbifolds such as Z_3 and Z_7 that contain no N=2 supersymmetric twisted sector [17], $b_a^I = 0$, the anomaly (3.32) is completely cancelled by a Green-Schwarz term. For other models the residual anomaly is cancelled by string-loop threshold effects [12] that can be incorporated in the present formalism by making the φ^a masses moduli-dependent:

$$
\mu_\alpha^\varphi \to \prod_I [\eta(T_I)]^{b_a^I} \mu_\alpha^\varphi. \qquad (3.35)
$$

Note that since the masses are not modular invariant, additional conditions, analogous to (3.20), must be imposed to make the quadratically divergent terms anomaly free. Possibilities for cancelling the remaining modular anomalies will be studied elsewhere.

4 Anomalous $U(1)$

In this section we include an anomalous $U(1)_X$ gauge factor: $\mathrm{Tr}T_X, \mathrm{Tr}T_X^3 \neq 0$. To regulate a nonanomalous gauge theory we introduced heavy vector-like pairs of states with gauge invariant masses. Explicitly, under a gauge transformation $X^A \to g_A X^A$, $X'_A \to g_A^{-1} X_A$, $\bar{X}^{\bar{A}} \to g_{\bar{A}}^{-1}\bar{X}^{\bar{A}}$, $\bar{X}'_{\bar{A}} \to g_A \bar{X}_{\bar{A}}$, $M' = gMg^{-1}$, *i.e.*, the mass matrix (3.26) is covariant, and no anomaly is introduced by the regularization procedure.

However, the quadratically divergent piece contains the term

$$2x^{-1}\mathcal{D}_a D_i (T^a z)^i = 2x^{-1}\mathcal{D}_a \left(\mathrm{Tr}T^a + \Gamma^i_{ij}(T^a z)^j\right). \tag{4.1}$$

If $\mathrm{Tr}T_a \neq 0$, one cannot regulate the quadratic divergences[7] without introducing a mass term for PV states X^i with the *same* $U(1)_X$ charge q^i. As a consequence the effective cut-off is noninvariant, which gives the conformal anomaly counterpart to the chiral anomaly.

Thus, in addition to the PV regulators introduced in section 2, we introduce chiral fields X^i with signatures η_i that carry only $U(1)_X$ charge q_i:

$$K \to K + k^i, \quad k^i = f^i(Z^j, \bar{Z}^{\bar{m}})|X^i|^2 + O|X^i|^4, \quad W \to W + \mu^i (X^i)^2. \tag{4.2}$$

Their contribution to the chiral $U(1)_X$ anomaly vanishes; the explicit breaking through the mass terms cancels their contribution to the true anomaly.

We have been working with the covariant superspace formalism of [7], in which the vector potential[8] A_μ is introduced as the lowest component of an anti-hermetian one-form superfield, and matter superfields Φ are defined to be covariantly chiral:

$$\mathcal{D}_{\dot{\beta}}\Phi = 0, \quad \chi^\alpha = \mathcal{D}^\alpha \Phi|, \tag{4.3}$$

[7] In the context of renormalizable theories one can use dimensional regularization or reduction and the quadratic divergence never appears.

[8] $iA_\mu \to ia_m = \mathcal{A}_m|$ in the notation of [7].

where the covariant derivative $\mathcal{D}_M$ contains the gauge connection $\mathcal{A}_M$, and M is a coordinate index in superspace. Under a gauge transformation:

$$\mathcal{A}_M \to \mathcal{A}_M - g^{-1} D_M g, \quad \Phi^A \to g^{q_X^A} \Phi^A, \quad g^{-1} = g^\dagger. \tag{4.4}$$

The chiral Yang-Mills superfield W^α is obtained as a component of the two-form $\mathcal{F}_{MN}$, which is the Yang-Mills field strength in superspace. The authors of [7] point out that one can introduce the commonly used Yang-Mills superfield potential V_X such that

$$W_\alpha = -\frac{1}{4}\left(\bar{\mathcal{D}}^2 - R\right) \mathcal{D}_\alpha V_X, \tag{4.5}$$

where R is an element of the supervielbein and $\bar{\mathcal{D}}^2 - R$ is the chiral projection, but this field does not appear in the construction of the action which is invariant under an additional gauge transformation

$$V_X \to V_X' = V_X + \frac{1}{2}\left(\Lambda + \bar{\Lambda}\right), \tag{4.6}$$

that is independent of (4.4). Since the gauge invariant superpotential is invariant under the complex extension of the gauge group, there is no conflict between (4.4) and holomorphicity of the superpotential.

However, the superpotential (4.2) changes by a nonholomophic function under $U(1)_X$ if $X^i \to g^{q_i} X^i$. Therefore holomorphicity requires $X^i \to e^{-q_i \Lambda} X^i$, Λ holomorphic, under a $U(1)_X$ gauge transformation. To preserve gauge invariance of the Kähler potential, we take X^i chiral in the ordinary sense, that is, we define $\mathcal{D}_M X^i = D_M X^i$, where D_M contains no gauge connection, and modify the Kähler potential (4.2) to read

$$K \to K + k^i e^{2q_i V_X}. \tag{4.7}$$

As shown in Appendix C, one obtains the standard Lagrangian when this expression is evaluated in the Wess-Zumino gauge. This choice is not justified unless the full theory is gauge invariant. In fact, we are interested in the special case in which the $U(1)_X$ anomaly satisfies the "universality" condition

$$\frac{1}{3}\mathrm{Tr}T_X^3 = \mathrm{Tr}(T_X T_a^2) = \frac{1}{24}\mathrm{Tr}T_X = 8\pi^2\delta_X, \tag{4.8}$$

and – in string derived supergravity – is cancelled by a Green-Schwarz term [18]. Thus provided this term is included and evaluated in the WS gauge, there is no ambiguity.

Including the fields X^i we get a quadratically divergent contribution:

$$\mathrm{STr}H \ni 2g^2 d_X \left(\sum_A q_A^X + \sum_i \eta_i q_i\right). \tag{4.9}$$

where the first term is the light field contribution and $d_X = \sum_A K_A q_A^X \phi^A$, $\phi^A = \Phi^A\big|$. Finiteness requires

$$\sum_i \eta_i q_i = -\mathrm{Tr}T_X = -192\pi^2\delta_X, \quad \sum_i \eta_i q_i m_i^2 = 0. \tag{4.10}$$

Once all the infinities are cancelled one gets a finite contribution that grows with μ^2. Setting $\mu_i = \beta_i\mu$, we get a contribution of the form (2.17) with now

$$\delta K = \sum_i \eta_i m_i^2 \ln\beta_i, \quad m_i^2 = \beta_i^2 e^K K_{i\bar{i}}^{-2}|\mu_i|^2. \tag{4.11}$$

Taking, for example, the modular invariant form

$$k^i = e^{K/2}|X^i|^2, \quad \delta K = \frac{\mu^2}{32\pi^2}\sum_i \beta_i^2 \ln\beta_i e^{-4q_i V_X}, \tag{4.12}$$

the correction to the bosonic Lagrangian is [see (2.34) and Appendix C]

$$\begin{aligned}\Delta\mathcal{L} &= \sqrt{g}\frac{\mu^2}{32\pi^2}\left[\frac{1}{16}\mathcal{D}^\alpha\left(\bar{\mathcal{D}}^2 - R\right)\mathcal{D}_\alpha - \hat{V}\right]\delta K\Big| \\ &= \sqrt{g}\frac{\mu^2}{32\pi^2}\sum_i \beta_i^2\ln\beta_i\left[2g^2 q_i\left(d_X - q_i A_\mu A^\mu\right) - \hat{V}\right]. \end{aligned} \tag{4.13}$$

Note that a mass term is induced for the anomalous $U(1)_X$ gauge boson A_μ. Thus if the full quantum theory is not anomalous we must impose

$$\sum_i \eta_i q_i^2 \beta_i^2 \ln\beta_i = 0. \tag{4.14}$$

The logarithmically divergent contribution from X^i contains a term

$$\mathcal{L}_X \ni -\frac{1}{64\pi^2}\sum_i \eta_i \ln |m_i|^2 q_i^2 F_X^2 + \cdots. \tag{4.15}$$

Under $U(1)_X$, (4.6), $|m_i|^2 \to e^{-2q_i(\Lambda+\bar{\Lambda})}|m_i|^2$, so the quantum Lagrangian changes by

$$\delta\mathcal{L}_X \ni \frac{1}{32\pi^2}\sum_i \eta_i (\lambda + \lambda^*) q_i^3 F_X^2 + \cdots, \tag{4.16}$$

where $\lambda = \Lambda|$. The light fermion contribution gives the chiral anomaly:

$$\delta\mathcal{L}_X = \frac{i\delta_X}{2}(\ln g|)\sum_a F^a\tilde{F}_a + \cdots, \tag{4.17}$$

For $F^a = F_X$, the anomalies (4.16,4.17) form a supermultiplet if we take

$$g = e^{-\frac{i}{2}(\Lambda-\bar{\Lambda})}, \quad \sum_i \eta_i q_i^3 = 8\pi^2\delta_X. \tag{4.18}$$

To make the full anomaly determined by (4.8) supersymmetric, we must include PV fields with both $U(1)_X$ and the nonanomalous gauge charges. This can be accomplished by assigning the *same* $U(1)_X$ charge q_A to the previously introduced PV fields X^A, X'_A, defining the superspace derivative as $\mathcal{D}_M = D_M X + T_a A^a_M$, $A^a \neq A^X$, and setting

$$|X^A|^2 \to e^{2q_A V_X}|X^A|^2, \quad |X_A|^2 \to e^{2q_A V_X}|X_A|^2,$$

in the Kähler potential. The generalization of the Lagrangian of Appendix C to this case is tedious but straightforward. Once supersymmetry of the anomaly is imposed, with the appropriate constraints on the PV $U(1)_X$ charges, the full anomaly is cancelled by a Green-Schwarz term that gives the variation of the Lagrangian under the $U(1)_X$ transformation (4.6):

$$\begin{aligned} \delta\mathcal{L}^X_{GS} &= -\frac{\delta_X}{4}\int \frac{E}{R}\Lambda \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + \mathrm{h.c.} \\ &= -\frac{\delta_X}{2}\left(\mathrm{Re}\lambda \sum_a F^a F_a + \mathrm{Im}\lambda \sum_a F^a \tilde{F}_a\right) + \cdots. \end{aligned} \tag{4.19}$$

This mechanism introduces a D-term with a well-defined coefficient that has been used in many applications to phenomenology. Note that there is also a D-term in (4.13), that may be removed by an additional condition on the β_i. One needs further information on the underlying theory to determine whether or not this term is present.

5 Concluding remarks

We have shown that on-shell one-loop Pauli-Villars regularization is possible for supergravity theories with canonical kinetic energy for gauge superfields. The resulting Lagrangian depends on the PV masses μ that play the role of effective cut-offs. It remains an open question as to whether PV regularization remains possible at higher order without the addition of higher derivative terms. However since the chiral anomalies of the effective field theory are completely determined at one loop order, and their partner conformal anomalies are thereby fixed by supersymmetry – through constraints on the Pauli-Villars massess – at the same order, one loop calculations are sufficient to study the field theory anomalies.

We found that nonlinear sigma-model symmetries can be preserved at the quantum level only for ungauged theories with restricted particle spectra, such that there are no quadratic divergences. It is nevertheless possible to impose invariance of the $O(\mu^2)$ correction, thereby preserving the Bagger-Witten condition at the quantum level. Similarly, the $O(\mu^2)$ correction to an anomalous $U(1)$ gauge symmetry may be made gauge invariant. There is also an $O(\mu^2)$ D-term that does not automatically vanish when gauge invariance is imposed; further information on the underlying theory is needed to fix this term.

In string-derived supergravity a discrete subgroup of the sigma-model symmetry is preserved to all orders in perturbation theory; a study of the anomaly structure provides information on the type of counterterms that must be included to cancel the field theory anomalies. In these theories the gauge kinetic energy term is noncanonical, and is governed by couplings to a universal dilaton. The full loop corrections including the dilaton, and a more detailed study of supergravity theories based on orbifold compactifications of the heterotic string, will be presented elsewhere.

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Appendix

A. The metric tensor for Y

The metric tensor derived from $K^Y_{\alpha>3}$ in (2.4) is the inverse of that derived from the Kähler potential

$$k = \left[\sum_{I,J=i,j} Y^I_\alpha \bar{Y}^{\bar{J}}_\alpha \left(K_{i\bar{j}} + a^2_\alpha K_i K_{\bar{j}}\right) + a_\alpha \left(Y^I_\alpha \bar{Y}^0_\alpha K_i + \text{h.c.}\right) + |Y^0_\alpha|^2\right]. \quad (A.1)$$

It is straightforward to evaluate the derivatives of the metric $k_{P\bar{Q}}$, $P, Q = Y_I, Y_0$. Denoting by $\gamma^P_{Qi}, r^P_{Qi\bar{m}}$ the corresponding elements of the affine connection and Riemann tensor, respectively, we have

$$\begin{aligned} (T_a)^{Y_J}_{Y_I} &= -(T_a)^i_j, \quad D_{Y_I}(T_aY)_J = -D_j(T_az)^i, \\ (\Gamma^Y)^Q_{Pi} &= -\gamma^P_{Qi}, \quad (R^Y)^Q_{Pi\bar{m}} = -r^P_{Qi\bar{m}}, \end{aligned} \quad (A.2)$$

giving the results listed in (2.13) and (2.42). In addition we have,

$$A^Y_{PQ} = e^K W_{PQ}, \quad \bar{A}^{PQ}_Y = e^K K^{P\bar{P}'}_Y K^{P\bar{Q}'}_Y \bar{A}_{\bar{P}'\bar{Q}'} = e^K k_{P\bar{P}'} k_{Q\bar{Q}'} \bar{A}_{\bar{P}'\bar{Q}'},$$

$$A^Y_{P\varphi} = e^K W_{P\varphi}, \quad \bar{A}^{P\varphi} = e^K K^{P\bar{Q}} K^{\varphi\bar{\varphi}} \bar{A}_{\bar{Q}\bar{\varphi}} = k_{P\bar{Q}} \bar{A}_{\bar{Q}\bar{\varphi}}, \tag{A.3}$$

giving the results listed in (2.37).

B. Nondiagonal gauge kinetic function

Here we sketch the generalization of [4] to the case of a nondiagonal gauge kinetic function involving Pauli-Villars fields. Although in this paper, we assume a canonical kinetic energy term for the light gauge fields, we give the results here for the case of a universal dilaton. The case relevant to section 2 of this paper is recovered by setting $s =$ constant. With an arbitrary kinetic function $f_{ab}(Z)$, the Lagrangian for the auxiliary fields D_a of the Yang-Mills supermultiplets takes the form [7], upon solving for D_a,

$$\begin{aligned} \mathcal{L}_D &= \frac{1}{2}\left(\mathrm{Re}f\right)^{ab} D_a D_b - D_a \tilde{D}^a = -\frac{1}{2}\left[\left(\mathrm{Re}f\right)^{-1}\right]^{ab} \tilde{D}_a \tilde{D}_b, \\ \tilde{D}^a &= \mathcal{D}^a + \frac{i}{2}\left(f^{ab}_i \bar{\lambda}_b L \chi^i - \mathrm{h.c.}\right), \quad f^{ab}_i = \partial_i f^{ab}. \end{aligned} \tag{B.1}$$

Writing $f^{ab} = f_a \delta^{ab} + \epsilon^{ab}$, we may expand in ϵ to obtain

$$\mathcal{L}_D = -\frac{1}{2}\left(\mathrm{Re}f_a\right)^{-1}\left(\delta^{ab} - \frac{\mathrm{Re}\epsilon^{ab}}{\mathrm{Re}f_b} + \sum_c \frac{\mathrm{Re}\epsilon^{ac}\mathrm{Re}\epsilon^{cb}}{\mathrm{Re}f_b \mathrm{Re}f_c}\right) \tilde{D}_a \tilde{D}_b + \cdots. \tag{B.2}$$

Here we introduce a single Pauli-Villars abelian multiplet, denoted by 0, and take gauge kinetic functions of the form

$$\begin{aligned} f^{AB} &= \delta^{AB}\left(x_A + iy_A\right) + \epsilon^{AB}, \\ f^{ab} &= \delta^{ab} s + \frac{d}{2}\varphi^a \varphi^b, \\ f^{a0} &= e\varphi^a, \quad f_0 = 1, \quad K_{PV} = e^k \sum_a |\varphi^a|^2, \quad e^k = \frac{1}{2x}. \end{aligned} \tag{B.3}$$

In addition to scalar curvature terms,

$$R^a_{bs\bar{s}} = K_{s\bar{s}} \delta^a_b, \tag{B.4}$$

we have

$$D_s f^{a0}_{\varphi^b} = -\Gamma^{\varphi^c}_{s\varphi^b} f^{a0}_{\varphi^c} = -k_s \delta^a_b e = \frac{1}{2x}\delta^a_b e. \quad \text{(B.5)}$$

The relevant part of the tree Lagrangian [6], [7] is (setting all background fermions to zero)

$$\begin{aligned}
\frac{1}{\sqrt{g}}\mathcal{L}(\varphi^a, B^\alpha_\mu) &= e^k \mathcal{D}^\mu \varphi^a \mathcal{D}_\mu \bar{\varphi}^a - \frac{d}{8}\left[\varphi^a\varphi^b\left(F^b_{\mu\nu}F^{\mu\nu}_a - i\tilde{F}^{\mu\nu}_a F^b_{\mu\nu}\right) + \text{h.c.}\right] \\
&\quad -\frac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu}_\alpha - \frac{e_\alpha}{4}\left[\varphi^a\left(F^\alpha_{\mu\nu}F^{\mu\nu}_a - i\tilde{F}^{\mu\nu}_a F^\alpha_{\mu\nu}\right) + \text{h.c.}\right] \\
&\quad +\frac{i}{2}\bar{\lambda}^\alpha \not{D}\lambda_\alpha + ie^k\left(\bar{\chi}^a_L \not{D}\chi^a_L + \bar{\chi}^a_R \not{D}\chi^a_R\right) \\
&\quad -V - e_\alpha\left[i\bar{\lambda}^\alpha_R\left(\frac{1}{2x}\mathcal{D}_a + \frac{1}{4}\sigma_{\mu\nu}F^{\mu\nu}_a\right)\chi^a_L + \text{h.c.}\right],
\end{aligned}$$

$$V = -\frac{1}{8x^2}\mathcal{D}_a\mathcal{D}_b\left[d\left(\varphi^a\varphi^b + \bar{\varphi}^a\bar{\varphi}^b\right) - e^2(\varphi^a + \bar{\varphi}^a)(\varphi^b + \bar{\varphi}^b)\right]. \quad \text{(B.6)}$$

Following the procedure described in [19], we introduce off-diagonal connections in the bosonic sector so as to cast the quantum Lagrangian in the form

$$\begin{aligned}
\mathcal{L}_{\text{bose}} + \mathcal{L}_{gh} &= -\frac{1}{2}\Phi^T Z_\Phi\left(\hat{D}^2_\Phi + H_\Phi\right)\Phi + \frac{1}{2}\bar{c}Z_{gh}\left(\hat{D}^2_{gh} + H_{gh}\right)c, \\
\hat{D}^\Phi_\mu &= D_\mu + V_\mu, \quad (V_\mu)_{A\rho,B\sigma} = -\epsilon_{\rho\mu\sigma\nu}\frac{\partial^\nu y_{AB}}{2x}, \\
(V_\mu)_{A\nu,i} &= (V_\mu)_{i,A\nu} = \left[(V_\mu)_{\bar{\imath},A\nu}\right]^* = \frac{1}{4\sqrt{x_A x_B}}f^{AB}_i\left(\mathcal{F}_{B\mu\nu} - i\tilde{\mathcal{F}}_{B\mu\nu}\right) \\
&= \frac{e}{4}\left(F_{b\mu\nu} - i\tilde{F}_{b\mu\nu}\right), \quad \text{for} \quad i = \varphi^b, \quad A = A^0_\mu. \quad \text{(B.7)}
\end{aligned}$$

This introduces corresponding shifts in the background field-dependent "squared mass" matrices:

$$M^2_\Phi \to H_\Phi = M^2_\Phi - V_\mu V^\mu, \quad M^2_{gh} \to H_{gh} = M^2_{gh} - B_\mu B^\mu. \quad \text{(B.8)}$$

We have the following relations among derivatives of the kinetic function:

$$\begin{aligned}
f_a &= D_a f = e, \quad f^a = 2xe, \quad f_{sa} = D_s D_a f = \frac{e}{2x}, \\
f_{\bar{s}a} &= D_{\bar{s}} f_a = 0, \quad R^a_{s\bar{s}b} f_a X^{s\bar{s}} = -\frac{e}{4x^2} X^{s\bar{s}}, \\
D_\mu e^2 &= D_\mu \left(\frac{f_a^{b\alpha} \bar{f}^a_{b\alpha}}{2x} \right) \\
&= \frac{1}{2x} \left[\partial_\mu s \left(D_s f_a^{b\alpha} \right) \bar{f}^a_{b\alpha} + \text{h.c.} \right] - \frac{e^2}{2x} \partial_\mu x = 0,
\end{aligned} \tag{B.9}$$

In evaluating the matrix elements needed for PV loop contributions, we set background PV fields to zero and show explicitly only the terms involving the parameters e and d. The remainder of this Appendix closely parallels Appendix C of [4].

1. Matrix elements

The elements of H_{IJ}, $I, J = \varphi^a$, are

$$\begin{aligned}
H_{IJ} &= \hat{V}_{IJ} + R_{IJ} + \mathcal{D}_{IJ} + v_{IJ} - (V_\mu V^\mu)_{IJ}, \\
v_{i\bar{m}} &= v_{\bar{m}i} = (V_\mu V^\mu)_{i\bar{m}} = (V_\mu V^\mu)_{\bar{m}i} = 0, \\
(V_\mu V^\mu)_{\varphi^a \varphi^b} &= \frac{e^2}{8} \left(F_a^{\mu\nu} F^b_{\mu\nu} \mp i \tilde{F}_a^{\mu\nu} F^b_{\mu\nu} \right), \\
v_{\varphi^a \varphi^b} &= = \frac{d}{8} \left(F_a^{\mu\nu} F^b_{\mu\nu} \mp i \tilde{F}_a^{\mu\nu} F^b_{\mu\nu} \right),
\end{aligned} \tag{B.10}$$

where

$$\mathcal{D}_a^b = \frac{e^2}{2x} \mathcal{D}_a \mathcal{D}^b + \frac{1}{x} \mathcal{D}_c (T^c)^b_a, \quad \mathcal{D}_{ab} = \frac{1}{4x^2} \left(e^2 - d \right) \mathcal{D}_a \mathcal{D}_b. \tag{B.11}$$

The additional nonvanishing elements of $Z_\Phi H_\Phi$ are $-N_{\alpha\mu,\beta\nu}$ and $S_{\alpha\mu,a}$ with

$$\begin{aligned}
N_{\alpha\mu,\beta\nu} &= -\frac{x}{2} e_\alpha e_\beta \left(F^a_{\mu\rho} F_{a\nu}{}^\rho - \frac{1}{4} g_{\mu\nu} F^a_{\rho\sigma} F_a^{\rho\sigma} \right) + \delta_{\alpha\beta} r_{\mu\nu}, \\
S^0_{\alpha\mu,a} &= S^s_{\alpha\mu,a} = \frac{e_\alpha}{4} \mathcal{D}^\nu F_{a\nu\mu} + \frac{e_\alpha}{8x} F^\mp_{a\nu\mu} \partial^\nu \begin{pmatrix} s \\ \bar{s} \end{pmatrix} \\
&= \frac{e_\alpha}{4} \left[\hat{D}^\nu F_{a\nu\mu} - \frac{1}{2x} F^\pm_{a\nu\mu} \partial^\nu \begin{pmatrix} \bar{s} \\ s \end{pmatrix} \right] \quad a = \begin{cases} \varphi^a \\ \bar{\varphi}^a \end{cases}.
\end{aligned} \tag{B.12}$$

Finally we need

$$
\begin{aligned}
\hat{G}_{\mu\nu} &= (G_z + G_g + G_{gz})_{\mu\nu}, \\
\left(G^{z}_{\mu\nu}\right)^{a}_{b} &= \frac{\partial_\mu \bar{s} \partial_\nu s - \partial_\mu \bar{s} \partial_\nu s}{4x^2} \delta^a_b \pm i F^c_{\mu\nu} (T_c)^a_b, \quad a,b = \begin{cases} \varphi^{a,b} \\ \bar{\varphi}^{a,b} \end{cases}, \\
\left(G^{z}_{\mu\nu}\right)^{b}_{a} &= \frac{e^2 x}{4} \left(F_{a\mu\rho} F^{b\,\rho}_{\nu} \mp i \tilde{F}_{a\mu\rho} F^{b\,\rho}_{\nu} \right) - (\mu \leftrightarrow \nu), \quad a,b = \begin{cases} \varphi^a, \bar{\varphi}^b \\ \bar{\varphi}^a, \varphi^b \end{cases}, \\
\left(G^{gz}_{\mu\nu}\right)_{\alpha\rho,a} &= \left(G^{gz}_{\mu\nu}\right)_{a,\alpha\rho} \\
&= \frac{e_\alpha}{2} \mathcal{D}_\mu F^{\mp}_{a\nu\rho} + \frac{e_\alpha}{4x} \partial_\mu \begin{pmatrix} s \\ \bar{s} \end{pmatrix} F^{\mp}_{a\nu\rho} - (\mu \leftrightarrow \nu), \quad a = \begin{cases} \varphi^a \\ \bar{\varphi}^a \end{cases}, \\
\left(G^{g}_{\mu\nu}\right)_{\alpha\rho,\beta\sigma} &= \delta_{\alpha\beta} r_{\sigma\rho\mu\nu} + \frac{x}{4} e_\alpha e_\beta \left[F_{a\mu\rho} F^a_{\nu\sigma} + \tilde{F}_{a\mu\rho} \tilde{F}^a_{\nu\sigma} - (\mu \leftrightarrow \nu) \right]. \qquad \text{(B.13)}
\end{aligned}
$$

The matrix elements of M_Θ are given by

$$
\begin{aligned}
M^0_0 &= 0, \\
M^0_a &= m_a + M^{\mu\nu}_a \sigma_{\mu\nu}, \quad M^a_0 = \frac{1}{2} e^{-k} \left(m_a + M^{\mu\nu}_a \sigma_{\mu\nu} \right), \\
m_a &= m_a = \frac{ie}{2x} \mathcal{D}_a = m^*_a, \\
M^{\mu\nu}_a &= -M^{\mu\nu}_a = -\frac{ie}{8} \left(F^{\mu\nu}_a \mp i \tilde{F}^{\mu\nu}_a \right), \quad a = \begin{cases} \varphi^a \\ \bar{\varphi}^a \end{cases}, \qquad \text{(B.14)}
\end{aligned}
$$

with covariant derivatives as defined in [3, 4]:

$$
\begin{aligned}
\mathcal{D}_\rho M^{\mu\nu}_a &= -\mathcal{D}_\rho M^{\mu\nu}_a = -\left(\mathcal{D}_\rho \bar{M}^{\mu\nu}_a \right)^* = \left(\mathcal{D}_\rho \bar{M}^{\mu\nu}_a \right)^* \\
&= -\frac{ie}{8} \left(\mathcal{D}_\rho + \frac{\partial_\rho s}{2x} \right) \left(F_{a\mu\nu} - i \tilde{F}_{a\mu\nu} \right), \\
\mathcal{D}_\rho m_a &= \mathcal{D}_\rho m_a = (\mathcal{D}_\rho m_a)^* \\
&= -i \frac{\partial_\rho \bar{s}}{4x^2} e \mathcal{D}_a + \frac{ie}{2x} \left(K_{j\bar{m}} (T_a \bar{z})^{\bar{m}} \mathcal{D}_\rho z^j + \text{h.c.} \right). \qquad \text{(B.15)}
\end{aligned}
$$

The matrix elements of $G^\Theta_{\mu\nu}$ are

$$
\begin{aligned}
\left(G^{\pm}_{\mu\nu}\right)_{00} &= \pm \Gamma_{\mu\nu} + Z_{\mu\nu}, \\
\left(G^{\chi}_{\mu\nu}\right)^a_b &= \left(G^{z}_{\mu\nu}\right)^a_b + \delta^a_b \left(Z_{\mu\nu} \pm \Gamma_{\mu\nu} \right), \quad a,b = \begin{cases} \varphi^a, \varphi^b \\ \bar{\varphi}^a, \bar{\varphi}^b \end{cases}. \qquad \text{(B.16)}
\end{aligned}
$$

As in [4], we double the quantum fermions degrees of freedom and represent them as 8-component Dirac spinors. In the following **Tr** denotes the full trace of fermion mass and field strength ($G_{\mu\nu} = [D_\mu, D_\nu]$) which are $8n_1 \times 8n_2$ matrices, where n_i is the number of intrinsic fermion degrees of freedom: $n_i = N_G(N'_G)$ for $\chi^a(\lambda^\alpha)$. The explicit calculation given below is for just one nonvanishing e^α : $N'_G \to 1$.

2. Chiral multiplet supertrace

Defining

$$\frac{1}{2}\mathrm{STr}H_\chi^2 = H^i_j H^j_i + H_{ij}H^{ij} - \frac{1}{8}\mathbf{Tr}\left(H^{IJ}_\Theta H^\Theta_{IJ}\right), \quad h^\chi_{\bar{m}i} = (\bar{m}m)_{\bar{m}i}, \qquad \text{(B.17)}$$

we have

$$\begin{aligned}
\frac{1}{8}\mathbf{Tr}\,(H^\chi_1)^2 &= \mathrm{Tr}\;h^2_\chi + \frac{e^4}{32}\mathcal{D}_a\mathcal{D}^b F^a_{\mu\nu}F^{\mu\nu}_b, \quad (h^\chi)^b_a = \frac{e^2}{4x}\mathcal{D}_a\mathcal{D}^b, \\
H^b_a &= (h^\chi)^b_a + \delta^b_a\left(\hat{V} + M^2 - M^2_\lambda - \frac{\partial_\mu s\partial^\mu\bar{s}}{4x^2}\right) + \frac{e^2}{4x}\mathcal{D}_a\mathcal{D}^b + \frac{1}{x}\mathcal{D}_c(T^c)^b_a, \\
H_{ab} &= \frac{1}{2}\left(d - e^2\right)\mathcal{W}_{ab}. \qquad \text{(B.18)}
\end{aligned}$$

Thus:

$$\begin{aligned}
\frac{1}{8}\mathbf{Tr}\,(H^\chi_1)^2 &= \mathrm{Tr}\;h^2_\chi + \frac{e^4}{16}\left[\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a\mathcal{D}_b + 4\mathcal{D}^2\right], \\
\mathbf{Tr}\,(H^\chi_2)^2 &= 0, \quad \frac{1}{8}\mathbf{Tr}H^\chi_3 = O(N_G), \\
\frac{1}{8}\mathbf{Tr}\,(H^\chi_3)^2 &= \frac{1}{2}\mathrm{Tr}(T_aT^b)F^a_{\mu\nu}F^{\mu\nu}_b + O(N_G), \\
\frac{1}{4}\mathbf{Tr}H^\chi_3 H^\chi_1 &= -T^\chi_3 + \frac{r}{2}\mathrm{Tr}\;h^\chi, \qquad \text{(B.19)}
\end{aligned}$$

where

$$\begin{aligned}
\mathrm{Tr}\;h^\chi &= \frac{e^2}{2}\mathcal{D}, \\
T^\chi_3 &= \frac{ie^2}{4}\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^a F^{\mu\nu}_a + \frac{ie^2}{4x^2}\mathcal{D}^a F^{\mu\nu}_a\partial_\mu s\partial_\nu\bar{s} \\
&\quad + \frac{e^2}{4}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + e^2\mathcal{D}^2, \qquad \text{(B.20)}
\end{aligned}$$

and the chiral fermion contributions to the helicity-odd operator T_- (see [4]) are

$$\begin{aligned}
T^\chi &= T_3^\chi + T_4^\chi + T_r^\chi, \\
T_r^\chi &= -\frac{e^2}{12} x \left(r_\nu^\mu F_a^{\nu\rho} F_{\mu\rho}^a - \frac{1}{4} r F_a^{\mu\nu} F_{\mu\nu}^a \right), \\
T_4^\chi &= \frac{e^4 x^2}{384} \left[(F_{\mu\nu}^a F_b^{\mu\nu})^2 + (F_{\mu\nu}^a \tilde{F}_b^{\mu\nu})^2 \right] - \frac{e^4}{32} \mathcal{D}_a \mathcal{D}^b F_{\mu\nu}^a F_b^{\mu\nu}.
\end{aligned} \tag{B.21}$$

Then we obtain

$$\begin{aligned}
\mathrm{STr} H_\chi &= e^2 \mathcal{D} + O(N_G), \\
\frac{1}{2} \mathrm{STr} H_\chi^2 &= -T_3^\chi - \left(\mathcal{W}_{ab} + \overline{\mathcal{W}}_{ab} \right) \left[\mathrm{Tr}(T^b T^a) + \frac{e^4}{16} \mathcal{D}_a \mathcal{D}_b \right] - \frac{e^2}{4} r \mathcal{D} \\
&\quad + \frac{e^4}{2} \mathcal{D}^2 + 2e^2 \left(\hat{V} + M^2 - M_\lambda^2 \right) \mathcal{D} \\
&\quad - \frac{e^2}{2x^2} \mathcal{D} \partial^\mu \bar{s} \partial_\mu s + (d - e^2)^2 x^2 \mathcal{W}_{ab} \overline{\mathcal{W}}^{ab} \\
&\quad + \frac{ie^2}{2} \left(\frac{\partial_\mu s \partial_\nu \bar{s}}{x^2} + \mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{j\bar{m}} \right) \mathcal{D}^a F_a^{\mu\nu} \\
&\quad + \frac{e^2}{2} \mathcal{D}_a \mathcal{D}_b \left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab} \right) + 2e^2 \mathcal{D}^2 + O(N_G).
\end{aligned} \tag{B.22}$$

Finally we have

$$\left(G_{\mu\nu}^z \right)_{\varphi^a}^{\bar{\varphi}^b} \left(G_{\mu\nu}^z \right)_{\bar{\varphi}^b}^{\varphi^a} = 0, \tag{B.23}$$

so

$$\begin{aligned}
\frac{1}{12} \mathrm{STr} \hat{G}_{\mu\nu}^\chi \hat{G}_\chi^{\mu\nu} &= -T_r^\chi + \frac{e^2 x}{24} \left(r_\nu^\mu F_{\mu\rho}^a F_a^{\nu\rho} - \frac{r}{4} F_{\mu\nu}^a F_a^{\mu\nu} \right) \\
&\quad + \frac{e^4}{384} \left[\left(F_{\mu\nu}^a F_b^{\mu\nu} \right)^2 + \left(F_{\mu\nu}^a \tilde{F}_b^{\mu\nu} \right)^2 \right] + O(N_G).
\end{aligned} \tag{B.24}$$

3. Mixed chiral-gauge supertrace

For the bose sector we have $H_\Phi^{\chi g} = -S$, and, using (B.17) of [4],

$$\mathrm{Tr} S^2 = \frac{e^2 x}{4} \left(\hat{D}_\nu F_a^{\mu\nu} \right)^2 + \frac{e^2}{16x} F_a^{+\nu\mu} F_{\rho\mu}^{-a} \partial^\rho s \partial_\nu \bar{s} - \frac{e^2}{8} \left(F_{\nu\mu}^{-a} \partial^\nu s + \mathrm{h.c.} \right) \hat{D}_\rho F_a^{\rho\mu}$$

$$= \frac{e^2}{4x}\left[g^{-\frac{1}{2}}\mathcal{L}_{a\mu} - iK_{i\bar{m}}\left(\mathcal{D}_\mu\bar{z}^{\bar{m}}(T_a z)^i - \mathcal{D}_\mu z^i(T_a\bar{z})^{\bar{m}}\right)\right]^2 + \frac{e^2}{16x}F_a^{+\nu\mu}F_{\rho\mu}^{-a}\partial^\rho s\partial_\nu\bar{s}$$
$$-\frac{e^2}{8x}\left(F_{\nu\mu}^{-a}\partial^\nu s + \text{h.c.}\right)\left[g^{-\frac{1}{2}}\mathcal{L}_{a\mu} - i\left(K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}(T_a z)^i - \text{h.c.}\right)\right]. \qquad \text{(B.25)}$$

To evaluate the fermion matrix elements we use Eqs. (3.36) and (C.24-27) of [4]:

$$\frac{1}{8}\mathbf{Tr}\,(H_1^{\chi g})^2 = 0,$$
$$-\frac{1}{8}\mathbf{Tr}\,(H_2^{\chi g})^2 = 2\left(\mathcal{D}_\mu\bar{m}\right)_a^i\left(\mathcal{D}^\mu m\right)_i^a - 8\left(\mathcal{D}_\mu\bar{M}^{\mu\nu}\right)_a^i\left(\mathcal{D}^\rho M_{\rho\nu}\right)_i^a. \qquad \text{(B.26)}$$

with

$$8\left(\mathcal{D}_\mu\bar{M}^{\mu\nu}\right)_a^i\left(\mathcal{D}^\rho M_{\rho\nu}\right)_i^a = \frac{1}{2}\text{Tr}S^2,$$
$$2\left(\mathcal{D}_\mu\bar{m}\right)_a^i\left(\mathcal{D}^\mu m\right)_i^a = \frac{e^2}{4x^2}\left(\partial_\mu x\partial^\mu x + \partial_\mu y\partial^\mu y\right)\mathcal{D}$$
$$+\frac{e^2}{2x}\left\{K_{i\bar{n}}K_{j\bar{m}}\mathcal{D}^\mu z^j(T_a\bar{z})^{\bar{m}}\left[(T^a z)^i\mathcal{D}_\mu\bar{z}^{\bar{n}} + (T^a\bar{z})^{\bar{n}}\mathcal{D}_\mu z^i\right] + \text{h.c.}\right\}$$
$$-\frac{e^2}{2x^2}\partial^\mu x\mathcal{D}^a K_{j\bar{m}}\left[(T_a z)^j\mathcal{D}_\mu\bar{z}^{\bar{m}} + (T_a\bar{z})^{\bar{m}}\mathcal{D}_\mu z^j\right], \qquad \text{(B.27)}$$

and

$$T^{\chi g} = t_3^{\chi g} = -\frac{16}{3}\left(\mathcal{D}^\sigma\bar{M}_{\sigma\mu}\right)_a^i\left(\mathcal{D}_\rho M^{\rho\mu}\right)_i^a = -\frac{1}{3}\text{Tr}S^2. \qquad \text{(B.28)}$$

In addition we have

$$\text{Tr}\left(\hat{G}_\Phi^{\chi g}\right)^2 = 4\left(G_{\mu\nu}^{gz}\right)_{a\rho,i}\left(G_{\mu\nu}^{gz}\right)^{i,a\rho} = \frac{1}{2}\mathbf{Tr}\left(\hat{G}_\Theta^{\chi g}\right)^2$$
$$= 64\left(\mathcal{D}_\mu\bar{M}_{\nu\rho}\right)_a^i\left(\mathcal{D}^\mu M^{\nu\rho} - \mathcal{D}^\nu M^{\mu\rho}\right)_i^a = -4\text{Tr}S^2. \qquad \text{(B.29)}$$

Using the classical equations of motion (B.17) of [4], we obtain,

$$L_{\chi g} = \frac{1}{2}\text{STr}H_{\chi g}^2 + T_{\chi g} - \frac{1}{12}\text{STr}\hat{G}_{\chi g}^2$$
$$= -\frac{2e^2}{\sqrt{g}}\Delta_{\mathcal{D}}\mathcal{L} - \frac{e^2}{2gx}\mathcal{L}_{a\mu}\mathcal{L}^{a\mu} + \frac{e^2}{2x\sqrt{g}}\left(F_{\nu\mu}^a\partial^\nu x + \tilde{F}_{\nu\mu}^a\partial^\nu y\right)\mathcal{L}_{a\mu}$$

$$
\begin{aligned}
&+\frac{e^2}{x\sqrt{g}}\left[i\mathcal{L}^{a\mu}\left(K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}(T_az)^i-\text{h.c.}\right)+\mathcal{D}^a(T_az)^I\mathcal{L}_I\right]\\
&+2e^2\mathcal{D}\left(2M^2+2M_\lambda^2+2\text{Re}(M\bar{M}_\lambda)+\hat{V}\right)\\
&+\frac{5e^2}{4x^2}\mathcal{D}\partial_\mu s\partial^\mu\bar{s}-\frac{e^2}{8x^2}\left(F^{\nu\mu}-i\tilde{F}^{\nu\mu}\right)\left(F_{\rho\mu}+i\tilde{F}_{\rho\mu}\right)\partial_\nu s\partial^\rho\bar{s}\\
&-\frac{e^2}{2x^2}\left[(\partial^\mu x+2i\partial^\mu y)K_{j\bar{m}}(T_az)^j\mathcal{D}_\mu\bar{z}^{\bar{m}}+\text{h.c.}\right]\mathcal{D}^a\\
&-\frac{ie^2}{2x}\left(F^a_{\nu\mu}\partial^\nu x+\tilde{F}^a_{\nu\mu}\partial^\nu y\right)\left[K_{j\bar{m}}(T_az)^j\mathcal{D}_\mu\bar{z}^{\bar{m}}-\text{h.c.}\right]\\
= &-\frac{2e^2}{\sqrt{g}}\Delta_{\mathcal{D}}\mathcal{L}-\frac{e^2}{2gx}\mathcal{L}_{a\mu}\mathcal{L}^{a\mu}+\frac{e^2}{2x\sqrt{g}}\left(F^a_{\nu\mu}\partial^\nu x+\tilde{F}^a_{\nu\mu}\partial^\nu y\right)\mathcal{L}_{a\mu}\\
&+\frac{e^2}{x\sqrt{g}}\left[\mathcal{L}^{a\mu}\left\{i\left(K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}(T_az)^i-\text{h.c.}\right)+\frac{\partial^\mu x\partial^\nu y}{x}F^a_{\mu\nu}\mathcal{D}^a\right\}+\mathcal{D}^a(T_az)^I\mathcal{L}_I\right]\\
&+2e^2\mathcal{D}\left(2M^2+2M_\lambda^2+2\text{Re}(M\bar{M}_\lambda)+\hat{V}\right)\\
&+\frac{5e^2}{4x^2}\mathcal{D}\partial_\mu s\partial^\mu\bar{s}-\frac{e^2}{8x^2}\left(F^{\nu\mu}-i\tilde{F}^{\nu\mu}\right)\left(F_{\rho\mu}+i\tilde{F}_{\rho\mu}\right)\partial_\nu s\partial^\rho\bar{s}\\
&-\frac{e^2}{2x^2}\left[\left(ixF^{-a}_{\nu\mu}\partial^\nu s+\partial^\mu s\mathcal{D}^a\right)K_{j\bar{m}}(T_az)^j\mathcal{D}_\mu\bar{z}^{\bar{m}}+\text{h.c.}\right], \qquad \text{(B.30)}
\end{aligned}
$$

where in the last expression (C.76) of [4] was used with (B.9) above.

4. Yang-Mills supertrace

For the remaining bosonic contributions, we have $H^g_\Phi=-N$; we write $N_{\alpha\beta}=N'_{\alpha\beta}+\delta_{\alpha\beta}n$, and evaluate here only $N'_{\alpha\beta}\to N_{00}$.

$$
\begin{aligned}
\text{Tr}N &= N'_G n,\\
\text{Tr}N^2 &= N'_G n^2-e^2x\left(r^\mu_\nu F^a_{\mu\rho}F^{\nu\rho}_a-\frac{1}{4}rF^a_{\mu\nu}F^{\mu\nu}_a\right)\\
&\quad+\frac{x^2e^4}{16}\left[\left(F^a_{\mu\nu}F^{\mu\nu}_b\right)^2+\left(F^a_{\mu\nu}\tilde{F}^{\mu\nu}_b\right)^2\right], \qquad \text{(B.31)}
\end{aligned}
$$

where we dropped total derivatives and used (B.12–B.14) of [4], as well as the Yang-Mills Bianchi identity. Finally, writing $\left(\hat{G}^g_{\mu\nu}\right)^\alpha_\beta=\left(\hat{G}'_{\mu\nu}\right)^\alpha_\beta+\hat{g}_{\mu\nu}\delta^\alpha_\beta$, we have

$$\mathrm{Tr}\left(\hat{G}^g_\Phi\right)^2 = N'_G\hat{g}^2 + \frac{xe^2}{2}\left(4r^\nu_\mu F^{\mu\rho}_a F^a_{\nu\rho} - r^\mu_\nu F^a_{\mu\rho}F^{\nu\rho}_a\right) - \frac{x^2e^4}{8}\left[\left(F^a_{\mu\nu}F^{\mu\nu}_b\right)^2 + \left(F^a_{\mu\nu}\tilde{F}^{\mu\nu}_b\right)^2\right], \qquad (B.32)$$

where we used (B.12–14) of [4]. For the fermions we obtain:

$$\begin{aligned}
\frac{1}{8}\mathbf{Tr}H^g_1 &= \frac{N'_G}{4}\mathbf{Tr}h_1 + \frac{e^2}{2}\mathcal{D}, \\
\frac{1}{8}\mathbf{Tr}\,(H^g_1)^2 &= \frac{e^4}{4}\mathcal{D}^2 + \frac{e^4}{32}F^a_{\mu\nu}F^{\mu\nu}_b\mathcal{D}_a\mathcal{D}^b, \\
-\frac{1}{8}\mathbf{Tr}\,(H^g_2)^2 &= 0, \quad \frac{1}{8}\mathbf{Tr}H^g_3 = \frac{N'_G}{4}r, \quad \frac{1}{8}\mathbf{Tr}\,(H^g_3)^2 = \frac{N'_G}{4}\mathbf{Tr}h^2_3, \\
\frac{1}{4}\mathbf{Tr}\,(H_1H_3)^g &= \frac{N'_G}{2}\mathrm{Tr}\,(h_1h_3)^g + \frac{e^2}{4}r\mathcal{D} - \frac{ie^2}{4}\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^aF^{\mu\nu}_a \\
&\quad - \frac{e^2}{4}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) - e^2\mathcal{D}^2, \\
\frac{1}{2}\mathbf{Tr}\hat{G}^2_g &= N'_G\mathbf{Tr}\hat{g}^2 + xe^2\left(r^\mu_\nu F^a_{\mu\rho}F^{\nu\rho}_a - \frac{r}{4}F^a_{\mu\nu}F^{\mu\nu}_a\right) \\
&\quad - \frac{x^2e^4}{16}\left[\left(F^a_{\mu\nu}F^{\mu\nu}_b\right)^2 + \left(F^a_{\mu\nu}\tilde{F}^{\mu\nu}_b\right)^2\right].
\end{aligned} \qquad (B.33)$$

The nonvanishing contributions to $T^g = T^g_3 + T^g_4 + T^g_r$ are:

$$\begin{aligned}
T^g_3 &= \frac{ie^2}{4}\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^aF^{\mu\nu}_a \\
&\quad + \frac{e^2}{4}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + e^2\mathcal{D}^2 + N'_Gt_3, \\
T^g_4 &= T^\chi_4, \quad T^g_r = T^\chi_r.
\end{aligned} \qquad (B.34)$$

For the supertraces we obtain [using (B.17–20) of [4]]

$$\begin{aligned}
\mathrm{STr}H^g &= N'_G\mathrm{STr}h^g - e^2\mathcal{D}, \\
\frac{1}{2}\mathrm{STr}H^2_g &= \frac{1}{2}N'_G\mathrm{STr}h^2_g + e^4\left[x^2\mathcal{W}^{ab}\overline{\mathcal{W}}_{ab} + \frac{3}{16}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\right] \\
&\quad - T^g_3 - \frac{xe^2}{2}\left(r^\mu_\nu F^a_{\mu\rho}F^{\nu\rho}_a - \frac{1}{4}rF^a_{\mu\nu}F^{\mu\nu}_a\right) - \frac{e^2}{4}r\mathcal{D}
\end{aligned}$$

$$
\begin{aligned}
& +\frac{ie^2}{2}K_{j\bar{m}}\mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} \mathcal{D}^a F_a^{\mu\nu} + \frac{e^2}{2}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab}+\overline{\mathcal{W}}^{ab}\right) + 2e^2\mathcal{D}^2, \\
\frac{1}{12}\mathrm{STr}\hat{G}_g^2 \quad = \quad & \frac{1}{12}N'_G\mathrm{STr}\hat{g}^2 - \frac{1}{12}\mathrm{STr}\hat{G}_\chi^2 - T_4^g - T_4^\chi - T_r^g - T_r^\chi \\
& -\frac{e^4}{8}\left[4\mathcal{D}^2 + \mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab}+\overline{\mathcal{W}}^{ab}\right)\right] + O(N'_G). \qquad \text{(B.35)}
\end{aligned}
$$

The space-time curvature dependent terms in the supertraces evaluated above give a contribution $\mathcal{L}_r$ of the form (2.23) of [3] with

$$
\begin{aligned}
H_{\mu\nu} &= H^g_{\mu\nu} - \frac{\ln\Lambda^2}{32\pi^2}e^2x\left(F^a_{\mu\rho}F_{a\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}xF^a_{\rho\sigma}F_a^{\rho\sigma}\right), \\
\epsilon_0 &= \epsilon_0^g - \frac{\ln\Lambda^2}{32\pi^2}e^2\mathcal{D}, \\
\alpha &= O(N'_G), \quad \beta = O(N'_G). \qquad \text{(B.36)}
\end{aligned}
$$

The metric redefinition in (2.24–25) of [3] gives a correction

$$
\begin{aligned}
\Delta_r\mathcal{L} &= \frac{\ln\Lambda^2}{32\pi^2}\Delta_r L, \\
\Delta_r L &= O(N'_G) + e^2\left(\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}} - 2V\right)\mathcal{D} \\
& +e^2x\left(F^a_{\rho\mu}F_a^{\rho\nu}\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}} - \frac{1}{4}F^a_{\rho\sigma}F_a^{\rho\sigma}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}}\right) \\
& -2e^2\left[x^2\mathcal{W}^{ab}\overline{\mathcal{W}}_{ab} + \frac{1}{2}\left(\mathcal{W}^{ab}+\overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a\mathcal{D}_b + \mathcal{D}^2\right]. \qquad \text{(B.37)}
\end{aligned}
$$

The result for constant x, given in (2.35) of section 2, is obtained by setting $M_\lambda = 0$, $y = 0$, $s = x = g^{-2}$ constant in the above equations. In section 2 the fields $\hat{\varphi}^a_\alpha$ are taken to be canonically normalized. Combining the above results and evaluating $\mathcal{L}_1 - \mathcal{L}_r + \Delta_r\mathcal{L} - \Delta_K\mathcal{L} - \Delta_x\mathcal{L} - \mathcal{L}_A X^A - \mathcal{L}_A\mathcal{L}_B X^{AB}$ yields the results given in (2.35), with $\varphi^a \to \sqrt{2x}\hat{\varphi}^a_\alpha$ and

$$
\begin{aligned}
e^2 &\to \sum_{\beta\gamma} e^2_{\gamma\beta} \equiv 2e, \quad e^4 \to \sum_{\alpha\beta\gamma\delta} e_\alpha e_\beta e_\gamma e_\delta \equiv 4e^2, \\
(d-e)^2 &\to \sum_{\beta\gamma}\left(d_{\gamma\beta} - \sum_\alpha e_{\gamma\alpha}e_{\alpha\beta}\right)^2 \equiv (d-2e)^2. \qquad \text{(B.38)}
\end{aligned}
$$

C. Lagrangian with a vector potential superfield

In this appendix we follow the notation of [7]: Greek letters are used for two-component spinorial indices, Roman letters for tangent space and coordinate indices, and the metric is $(-+++)$, *i.e.* the negative of the one used elsewhere in the text. We include the chiral fields $X^x = \{X^i, Z^a\}$, where the X^i are PV regulator fields charged only under an anomalous $U(1)_X$, and Z^a are the physical, light fields of the effective low energy theory.

Defining, in analogy with the chiral superfield $X_\alpha = -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\mathcal{D}_\alpha K$ introduced in [7],

$$X'_\alpha = -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\mathcal{D}_\alpha\left(k^i e^{2q_i V_X}\right), \quad x_\alpha = -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\mathcal{D}_\alpha k^i, \tag{C.1}$$

the PV Lagrangian gets contributions (in WZ gauge)

$$\begin{aligned}
\mathcal{L}^i_{PV} \ni\ & -\frac{1}{4}\mathcal{D}^\alpha X'_\alpha\Big| + \frac{i}{2}\bar{\psi}_m\bar{\sigma}^m X'\Big| + \text{h.c.} = -\frac{1}{4}\mathcal{D}^\alpha x_\alpha\Big| + \frac{i}{2}\bar{\psi}_m\bar{\sigma}^m x\Big| \\
& + \frac{1}{2}k^i q_i\bar{\psi}_m\bar{\sigma}^m\lambda_X - i\frac{\sqrt{2}}{2}\bar{\psi}_m\bar{\sigma}^n\sigma^m\bar{\chi}^{\bar{x}}k^i_{\bar{x}}a_m - \frac{1}{2}q_i^2 k^i a_m a^m + iq_i a_m\mathcal{D}^m z^x k^i_x \\
& + \frac{1}{2}q_i k^i D_X + q_i\sqrt{2}\chi^x\lambda_X k^i_x + \frac{1}{2}q_i a_m k^i_{x\bar{y}}\bar{\chi}^{\bar{y}}\sigma^m\chi^x + \text{h.c.} \\
=\ & -\frac{1}{4}\mathcal{D}^\alpha x_\alpha\Big| + \frac{i}{2}\bar{\psi}_m\bar{\sigma}^m x\Big| + \frac{1}{2}d^i\bar{\psi}_m\bar{\sigma}^m\lambda_X - i\frac{\sqrt{2}}{2}\bar{\psi}_m\bar{\sigma}^n\sigma^m\bar{\chi}^{\bar{x}}K'_{i\bar{x}}(T_X x)^i a_m \\
& - \frac{1}{2}(T_X x)^i(T_X\bar{x})^{\bar{\imath}}K'_{i\bar{\imath}}a_m a^m + ia_m\mathcal{D}^m z^x(T_X\bar{x})^{\bar{\imath}}K'_{x\bar{\imath}} + \frac{1}{2}d^i D_X \\
& + \sqrt{2}\chi^x\lambda_X(T_X\bar{x})^{\bar{\imath}}K'_{x\bar{\imath}} + \frac{1}{2}q_i a_m k^i_{x\bar{y}}\bar{\chi}^{\bar{y}}\sigma^m\chi^x + \text{h.c.},
\end{aligned} \tag{C.2}$$

where $K' = K + k^i$ and the last equality follows because

$$q_i k^i = (T_X x)^i K'_i = d^i, \quad q_i k^i_{\bar{x}} = (T_X z)^i K'_{i\bar{x}}, \quad q_i^2 k^i = (T_X x)^i(T_X\bar{x})^{\bar{\imath}}K'_{i\bar{\imath}}. \tag{C.3}$$

The first two terms are the contributions to $\tilde{\mathcal{D}}_{\mathcal{M}}$ of [7] quadratic in X^i without the gauge connections for X^i, and

$$k^i_i = \frac{\partial k^i}{x^i}, \quad k^i_a = \frac{\partial k^i}{z^a}, \quad etc., \quad x^i = X^i\Big|, \quad a \neq i, \quad x, y = i, a. \tag{C.4}$$

The remaining terms covariantize $\partial_m x^i$ and give the correct ψ, λ_X, D_X terms. All fermion derivatives include the Kähler $U(1)$ connection that has a piece:

$$
\begin{aligned}
A_a| &\ni \frac{1}{16}\bar{\sigma}^{\dot{\beta}\alpha}\left[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\beta}}\right]\left(k^i e^{2q_i V_X}\right)\Big| = \frac{1}{16}\bar{\sigma}^{\dot{\beta}\alpha}\left[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\beta}}\right] k^i\Big| + \frac{i}{2} q_i k^i v_a \\
&\ni \frac{1}{4} K'_i \left(\partial_a + i q_i a_a\right) x^i - \text{h.c.}. \qquad (C.5)
\end{aligned}
$$

In other words A_a includes the gauge connection for $U(1)_X$. The fully covariant derivative for the fermions contains the additional gauge connection terms:

$$
\begin{aligned}
D_m \chi^x &\ni i a_m \left[(T_X \chi)^x + (T_X z)^y \Gamma'^x_{yz} \chi^z\right], \\
D_m \chi^i_X &\ni i a_m q_i \left(\chi^i_X + x^i \Gamma'^i_{ia} \chi^a\right) + O(X^3) = i a_m q_i \left(\chi^i_X + \frac{k^i_{a\bar{\imath}}}{k^i_{i\bar{\imath}}} \chi^a\right) + O(X^3) \\
D_m \chi^a &\ni i a_m q_i x^i \Gamma'^a_{ib} \chi^b + O(X^4) = i a_m q_i K^{a\bar{b}} \left(k^i_{\bar{b}b} - \frac{k^i_{\bar{b}i} k^i_{b\bar{\imath}}}{k^i_{\bar{\imath}i}}\right) \chi^b + O(X^4), \quad (C.6)
\end{aligned}
$$

where we used the fact that

$$
K'^{a\bar{\imath}} = -K^{a\bar{b}} \frac{k^i_{\bar{b}i}}{k^i_{i\bar{\imath}}} + O(X^3), \qquad (C.7)
$$

So the fully covariant kinetic energy term contains the terms:

$$
-\frac{i}{2}\left(D_m \chi^x\right) \chi^{\bar{y}} K'_{x\bar{y}} + \text{h.c.} \ni q_i a_m k^i_{x\bar{y}} \bar{\chi}^{\bar{y}} \sigma^m \chi^x + \text{h.c.} + O(X^4), \qquad (C.8)
$$

which is just the last term in (C.2). Thus we get the standard form of the tree Lagrangian, and loop corrections from X^i are also of standard form. Converting to the notation used previously (*e.g.*, $a_m a^m \to -A_\mu A^\mu$), we obtain the results (4.9,4.13,4.15) given in section 4, where we used the classical equation of motion $D_X = -g^2 d_X$. The right hand side of (4.13) is given by the RHS of (C.2) with fermion fields set to zero and $k^i \to \mu^2 =$ constant.

D. Errata

Here we list corrections to [3, 4].

1. The term $+\frac{1}{8}\left(g_{\mu\rho}r_{\nu\sigma}+g_{\nu\rho}r_{\mu\sigma}+g_{\mu\sigma}r_{\nu\rho}+g_{\nu\sigma}r_{\mu\rho}\right)$ is missing from the expression for $X_{\mu\nu,\rho\sigma}$ in (2.22) and (B3) of [3]. As a consequence (B6) should read

$$\mathrm{Tr}X=-20V+2r,\quad \mathrm{Tr}X^2=40V^2-24rV+22r_{\mu\nu}r^{\mu\nu}-2r^2+\text{total derivative},$$

the following replacements should be made in (B20):

$$\frac{N+1}{12}r^2\to\frac{N-7}{12}r^2,\quad -5Vr\to-13Vr,\quad r_{\mu\nu}r^{\mu\nu}\to 8r_{\mu\nu}r^{\mu\nu},$$

the first three equations in (B22) should read:

$$\begin{aligned}\alpha &= -2\frac{\ln\Lambda^2}{32\pi^2},\quad \beta=\frac{N+89}{6}\frac{\ln\Lambda^2}{32\pi^2},\\ \epsilon_0 &= -\frac{\ln\Lambda^2}{32\pi^2}\left\{e^{-K}\left(A_{ij}\bar{A}^{ij}-\frac{2}{3}R^i_jA_i\bar{A}^j\right)+\frac{2N+68}{3}\hat{V}+\frac{2N+16}{3}M^2_\psi\right\},\end{aligned}$$

and (B23) (as well as footnote 23 of [4]) should read:

$$\begin{aligned}\frac{1}{\sqrt{g}}\Delta_r\mathcal{L}=\frac{\ln\Lambda^2}{32\pi^2}\Bigg[&\left\{-2e^{-K}\left(A_{ki}\bar{A}^{ik}-\frac{2}{3}R^k_nA_k\bar{A}^n\right)-\frac{3N+95}{3}\hat{V}-\frac{4N+32}{3}M^2_\psi\right\}\hat{V}\\ &+\left[K_{i\bar{m}}\left\{\frac{N+55}{3}\hat{V}+e^{-K}\left(A_{ki}\bar{A}^{ik}-\frac{2}{3}R^k_nA_k\bar{A}^n\right)+\frac{2N+16}{3}M^2_\psi\right\}+\frac{4}{3}R_{i\bar{m}}\hat{V}\right]\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}\\ &-\left\{\frac{2}{3}\left(R_{i\bar{m}}+16K_{i\bar{m}}\right)\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}g_{\mu\nu}-\frac{N+113}{6}\left(\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}+\mathcal{D}_\nu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}\right)K_{i\bar{m}}\right\}\mathcal{D}^\mu z^j\mathcal{D}^\mu\bar{z}^{\bar{n}}K_{i\bar{n}}\end{aligned}$$

In addition, in Eq. (C.55) of [4], the replacements

$$xF^a_{\mu\nu}F^{\mu\nu}_a r\to 5xF^a_{\mu\nu}F^{\mu\nu}_a r,\quad +2r^\mu_\nu xF^a_{\mu\rho}F^{\nu\rho}_a\to-12r^\mu_\nu xF^a_{\mu\rho}F^{\nu\rho}_a,$$

should be made in the expression for $\mathrm{Tr}X^2$, the replacements

$$-\frac{3x}{4}rF^a_{\mu\nu}F^{\mu\nu}_a\to+\frac{5x}{4}rF^a_{\mu\nu}F^{\mu\nu}_a,\quad +2r^\mu_\nu xF^a_{\mu\rho}F^{\nu\rho}_a\to-5r^\mu_\nu xF^a_{\mu\rho}F^{\nu\rho}_a,\quad -5r\mathcal{D}\to-13r\mathcal{D},$$

should be made in the second equation of (C.62), the first two equations of (C.63) should read:

$$\begin{aligned}H_{\mu\nu} &= H^0_{\mu\nu}+H^g_{\mu\nu}-x\left(10+x^2\rho_i\rho^i\right)\frac{\ln\Lambda^2}{32\pi^2}\left(F^a_{\mu\rho}F_{a\nu}{}^\rho-\frac{1}{4}g_{\mu\nu}F^a_{\rho\sigma}F^{\rho\sigma}_a\right),\\ \epsilon_0 &= (\epsilon_0)_0+\epsilon^g_0-\frac{\ln\Lambda^2}{32\pi^2}\left\{\frac{70}{3}\mathcal{D}+2x^2\rho_i\rho^i\mathcal{D}+\frac{2}{3x}\mathcal{D}_aD_i(T^az)^i\right\},\end{aligned}$$

and (C.64) should read:

$$\Delta_r\mathcal{L} = (\Delta_r\mathcal{L})_0 + \Delta_{rg}\mathcal{L} + \frac{\ln\Lambda^2}{32\pi^2}\left\{\frac{N-99}{3}\mathcal{D}^2 - \frac{2N+194}{3}\mathcal{D}\hat{V} - \frac{4N+32}{3}\mathcal{D}M_\psi^2\right.$$
$$+\left(\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}} - 2V\right)\left[2x^2\rho_i\rho^i\mathcal{D} + \frac{2}{3x}\mathcal{D}_aD_i(T^az)^i\right]$$
$$-2\mathcal{D}e^{-K}\left(A_{ij}\bar{A}^{ij} - \frac{2}{3}R^i_jA_i\bar{A}^j\right) + \frac{1}{3}\mathcal{D}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\left[4R_{i\bar{m}} - (N-57)K_{i\bar{m}}\right]$$
$$+\left(\frac{N+29}{6} - x^2\rho_i\rho^i\right)\left[2x^2\mathcal{W}^{ab}\overline{\mathcal{W}}_{ab} + \left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right)\mathcal{D}_a\mathcal{D}_b + 2\mathcal{D}^2\right]$$
$$+\left(\frac{N+71}{3} - x^2\rho_i\rho^i\right)\frac{x}{4}F^a_{\rho\sigma}F^{\rho\sigma}_a\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}}$$
$$\left.-\left(\frac{N+71}{3} - x^2\rho_i\rho^i\right)xF^a_{\rho\mu}F^{\rho\nu}_a\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K_{i\bar{m}}\right\}.$$

2. The sign of the last term in the expression for $D^2 + H_{Gh}$ in (2.12) of [3] and in (C.14) of [4] is incorrect. As a consequence, $-18\Gamma_{\mu\nu}\Gamma^{\mu\nu}$ in footnote 22 of [4] and $-6\Gamma_{\mu\nu}\Gamma^{\mu\nu}$ in (B18) of [3] should both be replaced by $-2\Gamma_{\mu\nu}\Gamma^{\mu\nu}$ in (C.61).

3. In the expressions for $[D_\mu, D_\nu]$ for fermions, $\Gamma_{\mu\nu} \to \Gamma_{\mu\nu} - \frac{i}{2}F^a_{\mu\nu}D_a$. As a consequence of this and the above item, the coefficient -24 should be replaced by $+2$ in $\mathrm{Tr}H^2_{Gh}$, Eq. (C.61) of [4], and the coefficient of $\mathcal{D}_a\mathcal{D}^bF^a_{\mu\nu}F^{\mu\nu}_b$ should be $\frac{1}{2}$ instead of 2 in the same equation. In addition the final results (4.6-8) and (5.2) of [4] are modified by the addition of the terms

$$-\frac{1}{3}(N+7+N_G)\left[i\mathcal{D}^aF^{\mu\nu}_a\mathcal{D}_\mu z^iK_{i\bar{m}}\mathcal{D}_\nu\bar{z}^{\bar{m}} + \frac{1}{2}\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + 2\mathcal{D}^2\right]$$
$$+\frac{2}{3}\left[i\mathcal{D}^aF^{\mu\nu}_a\mathcal{D}_\mu z^iR_{i\bar{m}}\mathcal{D}_\nu\bar{z}^{\bar{m}} + D_i(T_az)^i\left\{\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + \frac{2}{x}\mathcal{D}\mathcal{D}_a\right\}\right]$$

from contributions proportional to $[D_\mu, D_\nu]^2$ from fermion loops and $\frac{1}{6}\mathbf{Tr}G^2_{Gh}$, the term

$$+2x^2\rho^j\rho_j\left[\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + 4\mathcal{D}^2\right]$$

from $-\frac{1}{4}\mathbf{Tr}H^\chi_1H^\chi_3 + t_\chi - \frac{1}{4}\mathbf{Tr}H^g_1H^g_3 + T_g$, Eqs. (C.34,35,59) of [3], and an identical contribution from and an additional term

$$-2\left[\mathcal{D}_a\mathcal{D}_b\left(\mathcal{W}^{ab} + \overline{\mathcal{W}}^{ab}\right) + 4\mathcal{D}^2\right]$$

from $-\frac{1}{4}\mathbf{Tr}H_1^g H_3^g$. In addition the contribution of $R_{\mu\nu}$ was neglected in the calculation of $2t_\chi$; this gives an additional contribution

$$-2i\mathcal{D}_\mu z^k \left[x\mathcal{D}_\nu \bar{z}^{\bar{m}}\rho_{\bar{m}jk} + \rho_{jk}\left(\partial_\nu x - i\partial_\nu y\right)\right]\left[x\rho^j\mathcal{D}^a F_a^{\mu\nu} + 2(T_a z)^j\left(F_a^{\mu\nu} - i\tilde{F}_a^{\mu\nu}\right)\right]$$
$$+2\rho^j\rho_j\partial_\mu x\partial_\nu y\mathcal{D}^a F_a^{\mu\nu} + \text{h.c.},$$

which does not contribute to (2.22), and only the last term contributes when the string dilaton is present.

4. The coefficient of $\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}R_{j\bar{n}}\left(\mathcal{D}^\mu z^j\mathcal{D}^\nu\bar{z}^{\bar{n}} - \mathcal{D}^\nu z^j\mathcal{D}^\mu\bar{z}^{\bar{n}}\right)$ in footnotes 6, 13 and 21 and the coefficient of

$$\frac{1}{3}\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\sum_\alpha\left(N_\alpha + 1\right)K_{j\bar{n}}^\alpha\left(\mathcal{D}^\mu z^j\mathcal{D}^\nu\bar{z}^{\bar{n}} - \mathcal{D}^\nu z^j\mathcal{D}^\mu\bar{z}^{\bar{n}}\right)$$

in footnote 8 of [4] should be multiplied by -2.

5. The last term in brackets in the expression for $\mathbf{Tr}(H_3^\chi)^2$ in (C.33) of [4] should be multiplied by $\frac{1}{2}$, and the last term in (C.38) should be multiplied by -2, with corresponding changes in (C.36) and the final results.

6. There are errors in the coefficients of the the expressions following $-T_4^{\chi g}$ in the second equality for $\frac{1}{8}\mathbf{Tr}\left(H_1^{\chi g}\right)^2$, Eq. (C.41), and in similar terms in the other traces. For the canonical gauge kinetic energy case considered here the corrections to amount to the changes: $-2\mathcal{D}\hat{V} - 6\mathcal{D}M^2$ in (C.41), $-28\mathcal{D}M^2$ in the expressions for $\frac{1}{2}\mathrm{STr}H_\chi^2$, Eq.(C.36), $+8\mathcal{D}M^2$ and $-8\mathcal{D}M^2$ in $\frac{1}{8}\mathbf{Tr}\left(H_1^{\chi G}\right)^2$, and $\frac{1}{2}\mathrm{STr}\left(H_1^{\chi G}\right)^2$, respectively, Eqs. (C.50,51), and $+4\mathcal{D}M^2$ in $\frac{1}{8}\mathbf{Tr}\left(H_1^{g+G}\right)^2$, Eq.(C.58).

7. In (3.33) the expression for T_3 is missing a term:

$$T_3 \to T_3 - \frac{i}{3p^2}r_\nu^\mu\mathrm{Tr}\left(\tilde{M}_{\mu\rho}\bar{M}^{\nu\rho} - \tilde{\bar{M}}_{\mu\rho}M^{\nu\rho}\right),$$

the last line of $\mathrm{Tr}\mathcal{R}\mathcal{R}_5$ in (3.35) has the wrong sign, and the last term in the second line of the RHS of (3.36) should be multiplied by $-2/3$. As a consequence, $\frac{1}{8} \to -\frac{1}{12}$ in T_3^χ, (C.35), and in T_3^g, (C.59); $\frac{7}{8} \to \frac{13}{12}$ in the fourth line of (C.62). In addition $\frac{1}{4} \rightarrow \frac{1}{6}$ in the second line of $\mathrm{STr}\hat{G}^2_{g+G}$ in (C.62), and the terms proportional to $x^3\rho_i\rho^i$ in $H_{\mu\nu}$ in (C.63) and in the last three terms in (C.64) should be multiplied by two.

8. The following are misprints in [4]:
   The second line of (B.20) should be multiplied by x^{-1}.
   $\mathbf{Tr}\left(\hat{G}^{\chi g}_{\Theta}\right)^2$ should be multiplied by $\frac{1}{2}$ in the first line of (C.46); the sign of the last term in footnote 23 is incorrect.
   $(N+5)/r^2 \rightarrow 5/r^2$ in (C.58).
   In addition, a factor $\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i$ is missing from the coefficient of $2K_{i\bar{m}}\left(\hat{V}+2M^2_{\psi}\right)$ in the expression for $\frac{1}{4}\mathrm{Tr}|D_\mu M_\theta|^2$ in (B12) of [3].

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