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FOR THE OPTICS OF HIGH-ENERGY PARTICLE BEAMS

Thomas J. Devlin

September 15, 1961

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ABSTRACT

A program has been developed for the IBM 709 digital computer to handle many of the problems encountered in the optical design of high-energy particle beams. The program employs a six-dimensional vector space that takes into account the position and direction in both planes perpendicular to the beam direction, chromatic aberrations and particles of different mass. The quadrupole problem with one or two degrees of freedom (i. e. , one or two singlets, a doublet, or a symmetric triplet) is solved for the field gradients necessary to yield given beam conditions. The vectors are evaluated at the end of each element in the system. Effects higher than first-order, with the exception of second-order chromatic aberrations in the quadupoles, are not included. The effect of apertures is calculated in terms of the limitations on the phase-space volume that can be transmitted by the system.

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I. INTRODUCTION

A number of excellent treatments of the problems encountered in the design of high-energy particle beams are to be found in the literature.^{1,2,3} Therefore we will not attempt to deal with these problems in any more detail than necessary to provide a background for the use of OPTIK.

In beam design the following elements are usually employed:

- (a) Field-free regions,
- (b) Bending magnets (either wedge-shaped or rectangular regions of homogeneous magnetic field),
- (c) Quadrupole lenses with one or two degrees of freedom (singlets, doublets, or symmetric triplets),
- (d) Velocity spectrometers (crossed homogeneous electric and magnetic fields perpendicular to the beam line).

In this paper we refer to any one of the above four as an "element" or "beam element." The collection of such elements from the source of particles to the point where their interactions are studied is called "the beam." The path of a particle that traverses the axes of all quadrupoles in the beam is referred to as "the beam line," or "the central ray." Thus the beam line is a straight line through all elements except bending magnets, where it follows a circular path. A "monochromatic beam" is one in which all particles have the same momentum within small limits.

II. DISCUSSION

A. The Six-Dimensional Vector Space

We define a six-dimensional vector space as

$$(x, x', y, y', dP/P, M),$$

where x is the horizontal displacement from the beam line in inches, x' is the horizontal direction with respect to the beam line in radians, y and y' are the corresponding quantities in the vertical direction. We write $dP/P = (P' - P)/P$, where P is the desired momentum for which the system is tuned, and P' is the true momentum of the particle under consideration. M is a projection operator, which accounts for particles of different mass, and thus different velocities in a monochromatic beam. It is used directly only by the velocity spectrometer. It takes a value 0 for the particle that is to pass undeflected through the spectrometer, and 1 for the particle to be deflected.

The action of each element on the vector defined above may be represented, to first order in the components of the vector, as a 6-by-6 matrix. We will discuss the matrix for each of the four elements.

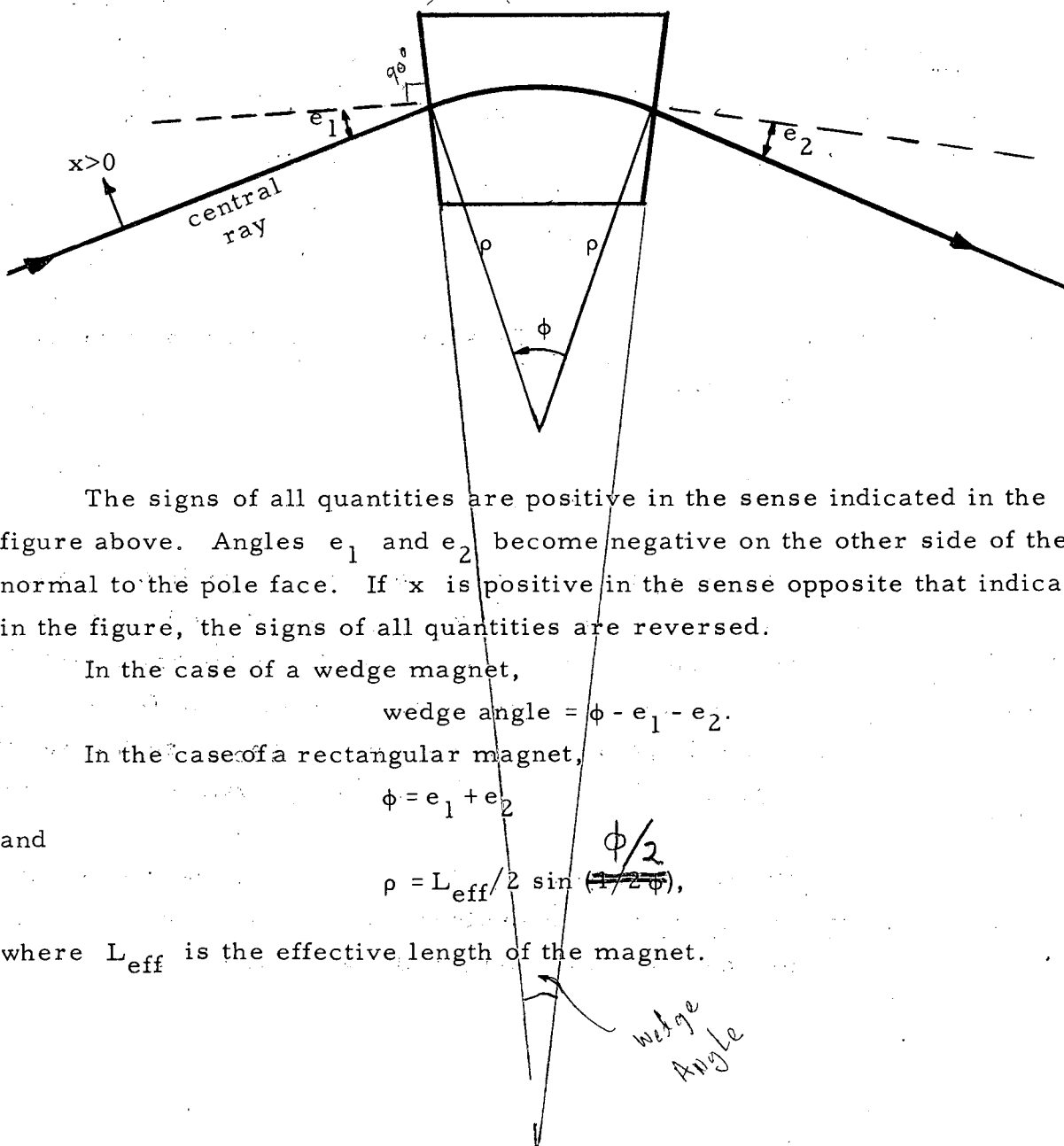
B. Field-Free Regions

A field-free region is characterized by only one parameter, its length. The matrix for a field-free region is

$$F = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

C. Bending Magnets

A bending magnet is characterized by four parameters, ϕ , e_1 , e_2 , and ρ . The ϕ is the total angle of bend, i. e., the angle between the central ray at the entrance to the field region of the magnet and the central ray at the exit; e_1 is the angle the central ray makes with the normal to the entrance face of the magnet; e_2 is the angle the central ray makes with the normal to the exit face of the magnet; and ρ is the radius of curvature of the central ray in the homogeneous field of the magnet.



The signs of all quantities are positive in the sense indicated in the figure above. Angles e_1 and e_2 become negative on the other side of the normal to the pole face. If x is positive in the sense opposite that indicated in the figure, the signs of all quantities are reversed.

In the case of a wedge magnet,

$$\text{wedge angle} = \phi - e_1 - e_2.$$

In the case of a rectangular magnet,

$$\phi = e_1 + e_2$$

and

$$\rho = L_{\text{eff}} / 2 \sin \left(\frac{\phi}{2} \right),$$

where L_{eff} is the effective length of the magnet.

wedge
Angle

From the four parameters we can determine the matrix B for a bending magnet as follows:

$$B_{11} = \cos(\phi - e_1) / \cos e_1,$$

$$B_{12} = \rho \sin \phi,$$

$$B_{15} = 2\rho \sin^2(1/2\phi),$$

$$B_{21} = (-1/\rho) \sin(\phi - e_1 - e_2) / (\cos e_1 \cos e_2),$$

$$B_{22} = \cos(\phi - e_2) / \cos e_2,$$

$$B_{25} = 2 \sin(1/2\phi) \cos(e_2 - 1/2\phi) / \cos e_2,$$

$$B_{33} = 1 - \phi \tan e_1,$$

$$B_{34} = \rho \phi,$$

$$B_{43} = (-1/\rho) (\tan e_1 + \tan e_2 - \phi \tan e_1 \tan e_2),$$

$$B_{44} = 1 - \phi \tan e_2,$$

$$B_{55} = 1,$$

$$B_{66} = 1.$$

All other matrix elements are zero. These expressions have been derived from the equations given by Cross.⁴

D. Quadrupoles

A single quadrupole section acts like a converging lens in one plane transverse to the beam line, and like a diverging lens in the other plane. For convenience, we will refer to a section converging in the horizontal plane as C , and to the one diverging in the horizontal plane as D . A combination of two or more quadrupole sections may be operated in such a way that there is a net convergence in both planes. The most prevalent combinations are

- (a) the doublet: two sections of equal length, one C , and one D ; and
- (b) the symmetric triplet: two sections of length L , converging in the same plane, separated by a section of length $2L$, which is converging in the other plane.

Various requirements imposed on the beam determine whether a doublet is operated CD (entrance section converging horizontally), or DC (entrance section diverging horizontally). Similarly, the symmetric triplet may be operated in either the CDC mode or the DCD mode.

Three parameters characterize the focusing action of a quadrupole section: its length L , its field gradient dB/dr , and the particle momentum P . The matrices for the two modes of operation are:

$$C = \begin{pmatrix} \cos \theta & (L/\theta) \sin \theta & 0 & 0 & 0 & 0 \\ (-\theta/L) \sin \theta & \cos \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh \theta & (L/\theta) \sinh \theta & 0 & 0 \\ 0 & 0 & (\theta/L) \sinh \theta & \cosh \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

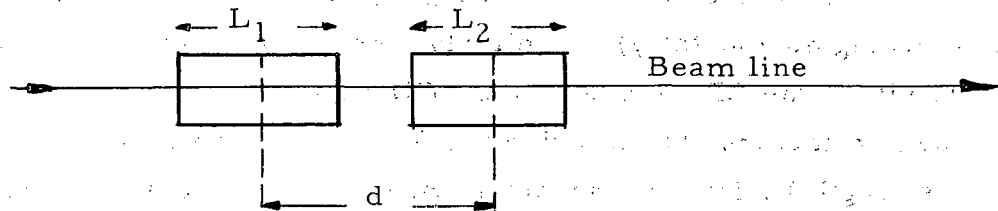
and

$$D = \begin{pmatrix} \cosh \theta & (L/\theta) \sinh \theta & 0 & 0 & 0 & 0 \\ (\theta/L) \sinh \theta & \cosh \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \theta & (L/\theta) \sin \theta & 0 & 0 \\ 0 & 0 & (-\theta/L) \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

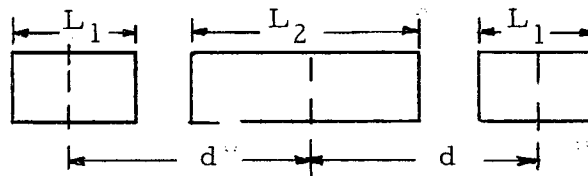
where

$\theta^2 = L^2 (dB/dr) / (1313.2 P)$. Here L is the length of the section in inches, dB/dr is the field gradient in gauss/inch, and P is the momentum in Mev/c.

We may fully specify all the first-order properties of a doublet with the following quantities: and identification number specifying whether it is CD or DC; L_1 and L_2 , the effective lengths of the first and second sections, respectively; dB_1/dr and dB_2/dr , the field gradients; and d , the distance from the midpoint of the first section to the midpoint of the second section along the beam line.



The same set of quantities determines the properties of a symmetric triplet, where subscript 1 now refers to both the first and third sections, and d is the distance from the midpoint of the center section to the midpoint of either end section.



For a given magnet, either doublet or triplet, and a given momentum beam, two parameters remain for adjustment of the focal properties, that is the two field gradients. Within limits, the field gradients can be adjusted to give any combination of focal lengths in the horizontal and vertical planes.

E. Velocity Spectrometers

The properties of a velocity spectrometer may be specified by the following quantities: the potential difference between the two electrodes, V ; the gap, or spatial separation between the electrodes, D ; the length of the separator, L ; and the magnetic field B . In most applications L is fixed for a given spectrometer, and there is a maximum value of V that can be applied to the plates without a serious sparking problem. The D and B are variable, and must be determined by the results desired.

The separator is operated in such a way that the net force produced by the electric and magnetic fields on the desired particle is zero:

$$F_u = e(E + v_u B) = 0, \quad (E = V/D).$$

The value of B is therefore fixed by the velocity of the unseparated particle v_u and the value of the electric field E:

$$B = -E/v_u.$$

The net force on the undesired, or separated, particle is

$$F_s = eE(1 - v_s/v_u).$$

If we calculate the trajectory of such a particle through the separator, we find that the separation may be represented exactly by an angular deflection introduced halfway along the length of the separator. Since the separators are usually operated in such a way that the deflection is introduced in the vertical plane, we call this deflection dy' . Its value is given by

$$dy' = (V/Pc) (L/D) (1/\beta_u - 1/\beta_s),$$

where V is the potential difference in millions of volts, and Pc is the momentum in Mev. L and D must be expressed in the same units. The β_u and β_s are the velocities of the unseparated and separated particles in units of the velocity of light.

The matrix representing the action of the separator may then be represented as the product of three matrices, $(L/2)$, (S) and $(L/2)$, where $(L/2)$ is a field-free matrix (see Sec. II-B) of length $L/2$, and S is given by:

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dy' \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In practice, it is usually convenient to assume the action of the separator to be represented by S at the midpoint of the field region along the beam line,

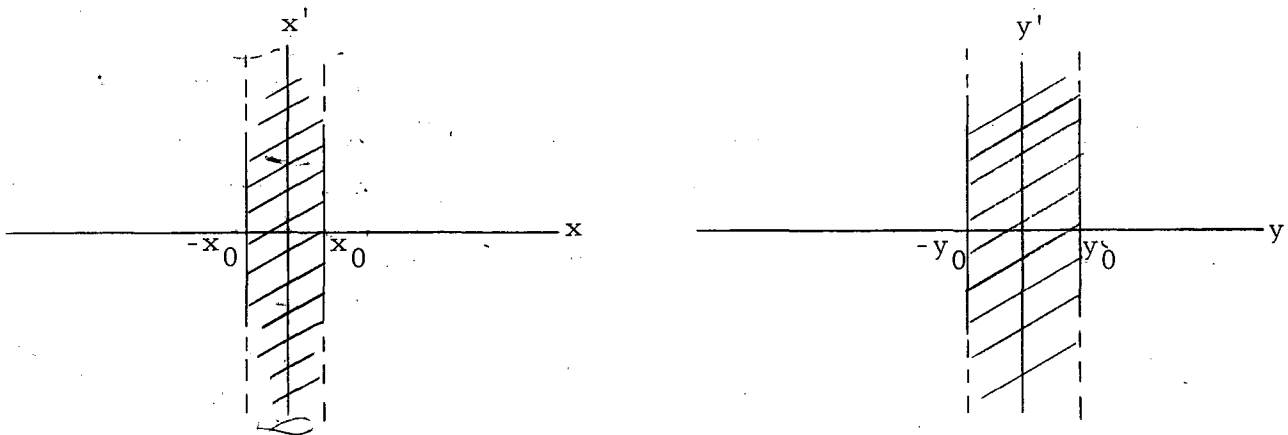
and to extend the field-free regions before and after the separator to its midpoint.

We see that the matrix S has an effect only on those vectors having the value of the sixth component different from zero; this is why the sixth component is 0 for the desired particle and 1 for a given undesired particle.

F. Apertures

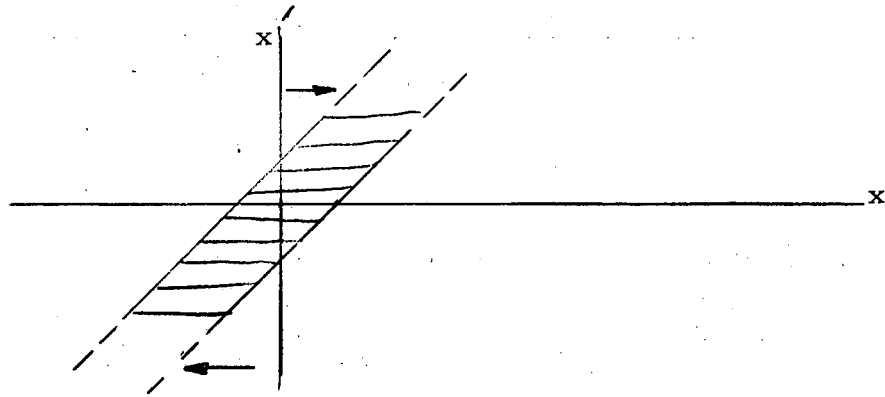
The number of particles transmitted by a system of the type under consideration here is proportional to the volume in phase space determined by the apertures of the system. The phase space is four-dimensional (two spatial and two angular components), and the volume has the dimensions inches-squared-steradians. In first-order theory, with a horizontal plane of symmetry, the effects in the horizontal and vertical planes are independent of one another. Therefore we may treat the two subspaces independently as graphical plots of angle vs spatial position.

The phase space at a source of particles can be represented in the plots

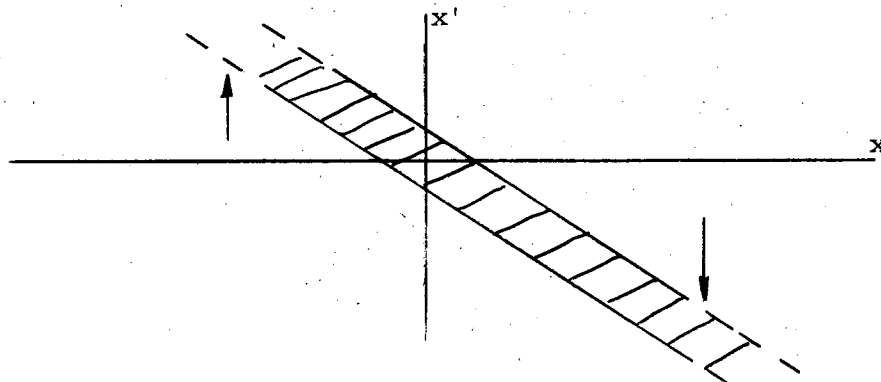


where $2x_0$ and $2y_0$ are the physical dimensions of the source. Particles are emitted in all directions, so there are no limits on extent in the directions of x' and y' . For small angles, the population of the phase space by particles is uniformly dense. From this point on, we will deal only with the x plane, since the discussion is identical for both planes.

Traversal of a field-free region causes the phase-space volume to shear to the right above the x axis and to the left below the axis:

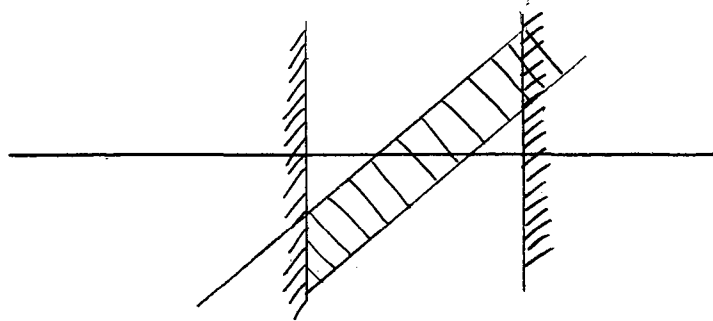


When the particles pass through a lens with positive focal length, the phase-space volume moves generally down to the right of the x' axis and up to the left of the axis.



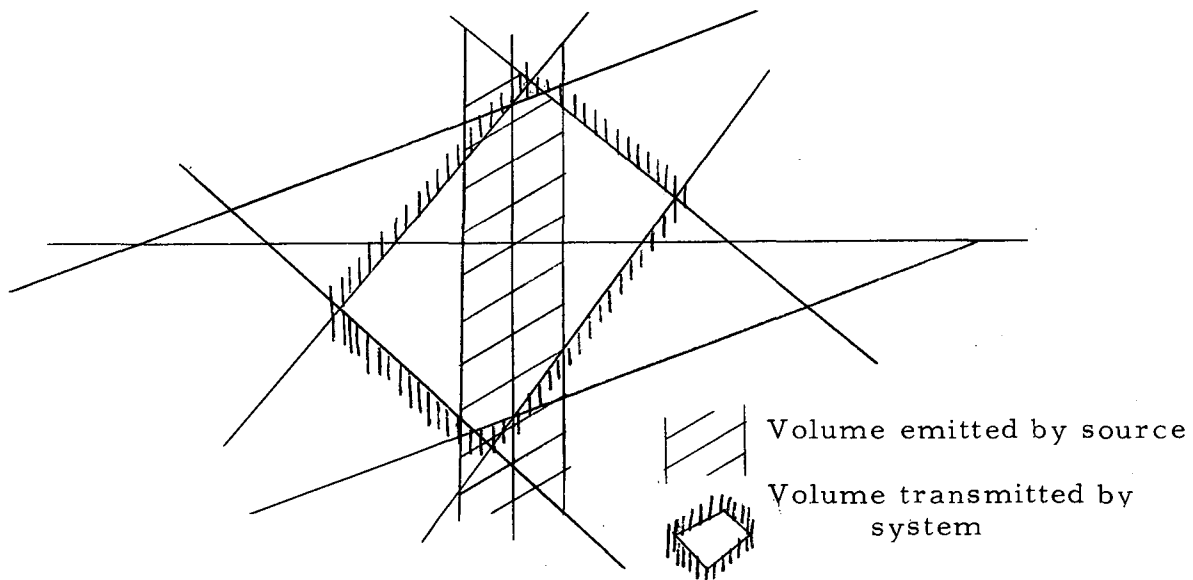
During all these transitions, the phase-space volume remains constant (a consequence of Liouville's theorem reflected in the fact that the determinant of the transfer matrix is always unity).

The effect of an aperture is to limit the volume in phase space which is transmitted through it. If we make a phase-space plot at the position of the aperture, the stops may be represented as vertical lines in phase space:



That part of the phase-space volume falling outside the stops is lost.

In order to determine the effect of each stop in the system, it would be necessary to make a phase-space plot at the position of each aperture, make the appropriate adjustments in the volume, and then transmit it to the next aperture. It is much more convenient, however, to take the vertical line in phase space which represents the effect of the stop and propagate it back through the system to the source of particles. Then all the effects of all apertures may be shown on one graph. This has been done in OPTIK. The effect of an aperture at the end of each beam element is given in terms of the intercepts with the x axis and the x' axis in phase space at the source of particles. A similar calculation is carried out in the vertical plane. In addition, a third calculation is made in the vertical plane for those particles affected by velocity separators, since the limitations on the phase-space volume are different for the deflected and undeflected particles. Dispersion by the bending magnets and chromatic aberrations are included in these calculations. A sample of this type of plot is shown below.



Calculation of the intercepts is made by solving the following equations for x_{int} and x'_{int} :

$$\begin{pmatrix} h_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = (M) \begin{pmatrix} x_{int} \\ 0 \\ 0 \\ 0 \\ \Delta P/P \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} h_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = (M) \begin{pmatrix} 0 \\ x'_{int} \\ 0 \\ 0 \\ \Delta P/P \\ 0 \end{pmatrix}$$

where h_x is the position in the horizontal plane of the aperture, M is the full matrix from the source of particles to the position of the aperture, and x_{int} and x'_{int} are the intercepts in phase space at the source of the line that represents the effect of the aperture. The calculation is the same in the vertical plane for the particles undeflected by the velocity separator. In the case of the deflected particles, the sixth component of the vector on the right is set equal to unity.

DESCRIPTION OF THE PROGRAM

A. General Description

OPTIK consists of a main control program and nine primary subroutines. In addition, there are 12 control and calculational subroutines, most of which are employed in the solution of the quadrupole problem.

The main program routes control, on the basis of an identification number in the data, to one of the following primary subroutines:

- (1) START - Reads the particle momentum and N initial vectors into the program ($N = 1$ to 9), and forms an 6-by-N matrix composed of these N-column vectors. It writes these vectors on the output tape, and then transfers control back to the main program at a point where all variables are reset for the start of a new problem.
- (2) FREE - Reads the length of a field-free region and forms a field-free matrix.
- (3) QUADU - Solves the quadrupole problem where the field gradients are to be determined. It reads ID numbers to determine whether the magnet is a doublet or symmetric triplet, and whether the entrance section is C or D. This subroutine has access to all other subroutines on the same level except itself, SING, and START. We will discuss this subroutine in more detail below.
- (4) BEND - Reads the values of ϕ , e_1 , e_2 , and ρ , and forms the matrix for a bending magnet. It also calculates and writes the value of the magnetic field and $\int H dL$ through the magnet.
- (5) SEP - Reads the masses of the unseparated and separated particles, the value of the potential difference between the electrodes, the length of the separator, and the gap width. It calculates the angular separation, the value of the magnetic field, the velocities of the two particles, and then forms the separator matrix.
- (6) CONDTN - Reads-in the subscripts of a single matrix element upon which a condition is to be placed, and also the desired value of the element. It is used mainly by QUADU and SING in specifying conditions to be satisfied by the quadrupole focusing. When under control of the main program, it writes the subscripts and desired value of the matrix element that is to be specified, and also writes the whole matrix for comparison. Examples of beam conditions that can be specified in this manner are given in Appendix C.

- (7) QUADK - Forms the matrix for a doublet or triplet with a known solution. Its output looks the same as that of QUADU, but it does not go through the iteration procedure to determine the solution.
- (8) ODD - Reads-in an arbitrary matrix calculated outside the program.
- (9) SING - Solves the quadrupole problem for one or two singlets (similarly to QUADU).

After each of these subroutines has returned control to the main program, it employs a matrix multiplication subroutine, MATMPY, to determine the full-product matrix from the source of particles up to and including the element under consideration. It also calculates the new values of the vectors at this point. It writes this information and then returns to the point where it will read the identification number for the next element in the beam. The problem is terminated when no more data is available, or when subroutine START is called, for the beginning of a new problem.

B. Subroutines QUADU and SING

QUADU iterates the values of the two θ 's by a gradient method in such a way as to satisfy the conditions put upon the beam focusing. This is done in the following manner:

- (1) Prod is the matrix from the source to the quadrupole entrance.
- (2) $Q(\theta_1, \theta_2)$ is the matrix for the quadrupole itself. It can be either doublet or triplet, with entrance plane either C or D; it is formed at a lower level of subroutines in the program on the basis of identification numbers. The iteration procedure is the same for all four cases.
- (3) Qmat 1 and Qmat 2 are the matrices from the quadrupole exit up to the first and second beam conditions, respectively. Both conditions are needed in order to fix the values of the two θ 's.

- (4) The matrix products

$$(Qmat\ 1) \times (Q) \times (Prod)$$

and

$$(Qmat\ 2) \times (Q) \times (Prod)$$

are formed, and the appropriate matrix elements, E_1 and E_2 , required by the beam conditions to take on a specified value, are examined for the trial values of θ_1 and θ_2 specified in the data.

- (5) An increment is added to θ_1 with θ_2 fixed, and E_1 and E_2 are examined again. Similarly, θ_2 is varied with θ_1 fixed, and we obtain a set of finite difference equations:

$$\Delta E_1 = \frac{\partial E_1}{\partial \theta_1} \Delta \theta_1 + \frac{\partial E_1}{\partial \theta_2} \Delta \theta_2$$

and

$$\Delta E_2 = \frac{\partial E_2}{\partial \theta_1} \Delta \theta_1 + \frac{\partial E_2}{\partial \theta_2} \Delta \theta_2$$

Since we know the values of the E 's produced by the trial values of the θ 's, and the desired values of the E 's, by inverting the above equations and inserting the desired changes in the E 's, we can solve for the changes in the θ 's necessary to produce the desired change.

- (6) The percentage change in the E 's is tested. If it is greater than 0.01%, the iteration is repeated with the new values of the θ 's as the trial values. If it is less than 0.01% for both, the iteration is stopped and the values are taken as the final solution. When this point has been reached, the subroutine proceeds to calculate the field gradients from the θ 's, and forms the quadrupole matrix for momentum $P(1+\Delta P/P)$. The results are written on the output tape, and control is returned to the main program.

Several remarks are appropriate. First, the program, in order to form Qmat 1 and Qmat 2, must search through subsequent elements in the beam until it reaches a beam condition. The subroutines, corresponding to each of the elements following the quadrupole up to and including the two beam conditions, are advised of the fact that they are under control of QUADU instead of the main program, and, no output is written. Second, once the quadrupole problem has been solved, we must continue with the first element following it. Instead of storing all the information about the elements between the quadrupole and the beam conditions, OPTIK backspaces the input tape to the element immediately following the quadrupole, and continues normal operation. Third, an element that calls QUADU or SING with an unknown solution must not occur between an undetermined quadrupole and the beam conditions that it uses for a solution, i. e., only one unknown quadrupole may be solved at one time. A quadrupole calling on QUADK or SING with a known solution may be included in this region.

SING operates in a manner similar to QUADU, but has the following differences:

- (1) SING is capable of handling either one or two singlets with other beam elements placed between them.
- (2) It can also operate in a mode wherein the solution is known, thus also performing the function of QUADK. An identification number must be given, to determine which mode of operation is employed.
- (3) The ID number that calls SING may be used more than once before the beam conditions that determine the solution are encountered. However, it can deal with only two singlets at a time with undetermined solutions. No restriction is placed on calling SING when the solution is already known.

C. Capabilities of the Program

- (1) Except for the restrictions on QUADU and SING, any number of elements and in any order may be included in a beam to be handled by OPTIK.
- (2) Up to nine initial vectors may be propagated through the beam and evaluated at the end of each element. Thus, some degree of ray tracing may be performed (though not detailed trajectories inside field regions), and phase-space volumes may be calculated at various points along the beam.
- (3) The matrix for each element is evaluated, along with the product matrix from the source of particles up to and including that element.

D. Limitations of the Program

- (1) In solving the quadrupole problem, the program can deal with, at most, only two degrees of freedom.
- (2) The only conditions that can be placed upon the focusing properties are those explicitly placing a value on some matrix element in the product matrix from the source of particles up to the point under consideration. Thus, a field lens may not be included in a calculation of the full beam. Such problems may be dealt with, however, by running OPTIK

for different parts of the beam which do not include the true source as the start of the problem, and then including the solutions as known elements in a calculation of the full beam (See appendix D).

(3) inhomogeneous fields, such as the Bevatron field, are not explicitly included in the program. However, the effect of such a field may be included as a series of wedge magnets, or its full effect included by the use of the subroutine ODD which feeds-in a full matrix calculated externally. Such a procedure involves a prior solution of the effect of the field, e. g., "bug" orbits, or the use of the IBM 650 program ~~DAVID~~ ³ ETHELBERT. ⁵

(4) Beam elements other than the ones listed above are not included in the program explicitly. Such elements may be included in the calculation by the use of ODD in those cases in which a matrix can be calculated outside the program.

(5) Only the quadrupole parameters are varied by OPTIK in order to satisfy beam conditions. None of the other parameters are treated in this manner. If, for example, one wished to determine the proper angle of bend in a bending magnet in order to produce an energy focus, it would be necessary to run OPTIK once for each of several bending angles. The product matrix at the point where the energy focus is desired can be examined to determine which angle causes the (1, 5) matrix element to be zero.

(6) Only one value of $\Delta P/P$ may be involved in a given problem.

It is possible that new subroutines might be added to OPTIK, but such a procedure would involve substantial revision of the existing program, in particular the main program, QUADU, and SING. Also, the input format for the data would have to be changed, since the one digit format for the ID number which calls the various subroutines is already used to full capacity. It is hoped that the present flexibility of the program is sufficient, and that judicious use will enable the user to overcome its limitations.

The author has tested the program under a fairly wide variety of input conditions in an attempt to eliminate as many "bugs" as possible. At the time of this writing all errors which have occurred in the use of OPTIK have been the result of incorrect data input or attempts to exceed the limitations stated above. If use of the program leads to incorrect results which cannot be

ascribed to either of these sources of error, the author would appreciate being advised of the details of the problem involved.

APPENDICES

A. General: Input Data for OPTIK

The input data consist solely of information necessary to call and use the seven primary subroutines. START must always be the first subroutine called, and may not be called again until the beginning of a complete new problem. The others must be called in the order in which they occur in the beam subject to the limitations stated on QUADU.

1. Input for START

- First Card Format (I1, F9.5, 3F10.5):
 1, H_{Left}, H_{Right}, H_{up}, H_{Down} (ID number by which the main program calls START, and dimensions of source).
- Second Card Format (I1):
 N (Number of initial vectors to be propagated through the system, $1 \leq N \leq 9$).
- Third Card Format (2F10.5):
 P, $\Delta P/P$ (Momentum and momentum increment).
- Fourth Card Format (6F10.5):
 $x_1, x'_1, y_1, y'_1, \Delta P/P_1, M_1$ (First vector, x and y in inches, \dot{x} and \dot{y} in radians).
- Fifth Card Format (6F10.5):
 $x_2, x'_2, y_2, y'_2, \Delta P/P, M_2$ (Second vector)
-
- (N + 3)th Card Format (6F10.5)
 $x_N, x'_N, y_N, y'_N, \Delta P/P, M_N$ Nth vector, $N \leq 9$).

77+4 = SCALE (HORIZONTAL) 200

2. Input for FREE

- First Card Format (I1, F9.5, 3F10.5):
 2, H_L, H_R, H_U, H_D (ID number by which the main program calls FREE, and apertures at end of region).
- Second Card Format (F10.5):
 L (Length of the field-free region in inches).

3. Input for QUADU

First Card

Format (I1, F9.5, 3F10.5):

3, H_L , H_R , H_U , H_D (ID number by which the main program calls QUADU and apertures at exit).

Second Card

IDQ, IDP

Format (2I1):

IDQ = $\begin{cases} 1 & \text{(for a doublet)} \\ 2 & \text{(for a symmetric triplet)} \end{cases}$

IDP = $\begin{cases} 1 & \text{(for entrance plane C)} \\ 2 & \text{(for entrance plane D).} \end{cases}$

Third Card

Format (5F10.5):

L_1 , L_2 , d , θ_1 , θ_2 (As explained above; lengths in inches, angles in radians. θ_1 and θ_2 are trial values of the solution, which need be only very rough guesses.)

4. Input for BEND

First Card

Format (I1, F9.5, 3F10.5):

4, H_L , H_R , H_U , H_D (ID number by which the main program calls BEND, and apertures at exit).

Second Card

Format (5F10.5):

e_1 , e_2 , ϕ , ρ (As explained above; angles in degrees, ρ in inches).

5. Input for SEP

First Card

Format (I1):

5, H_L , H_R , H_U , H_D (ID number by which the main program calls SEP and aperture at center of separator).

Second Card

Format (5F10.5):

M_u , M_d , V , L , D (M_u and M_d are the masses in Mev of the undeflected and deflected particles, respectively; V is in millions of volts, and L and D are in inches.)

6. Input for CONDTN:

First Card

Format (I1, F9.5, 3F10.5):

6

(ID number by which the main program calls CONDTN).

*1 = COME
HARRY*

Second Card Format (2I1, F10.5):
 I, J, Cond (Subscripts of the matrix element which is to be specified, and its value).

7. Input for QUADK

First Card Format (I1, F9.5, 3F10.5):
 7, H_L , H_R , H_U , H_D (ID number by which the main program calls QUADK, and exit apertures).

SECOND CARD SAME AS QUADU

THIRD
Second Card Format (5F10.5):

L_1 , L_2 , D, θ_1 , θ_2 (As in QUADU).

8. Input for ODD

First Card Format (I1, F9.5, 3F10.5):
 8, H_L , H_R , H_U , H_D (ID number by which the main program calls ODD, and apertures at end of this element).

Second Card Format (6F10.5):

✓ M_{11} , M_{12} , M_{13} M_{16}

Third Card Format (6F10.5):

M_{21} , M_{22} , M_{26}

Seventh Card Format (6F10.5):

M_{61} , M_{62} , M_{66}

9. Input for SING

First Card Format (I1, F9.5, 3F10.5):
 9, H_L , H_R , H_U , H_D (ID number by which the main program calls SING and exit apertures).

Second Card Format (2I1):

ISOLVE, ICONV

ISOLVE	{	1.(look for correct θ)
		2 (correct θ is given)
ICONV	{	1 (converging horizontally)
		2 (diverging horizontally).

Third Card Format (2F10.5):

L, θ (as explained above).

B. Subroutines used by OPTIK

OPTIK consists of a main program, OPTIK, and 21 subroutines. Nine of these have already been discussed.

- | | |
|-----------|------------|
| (1) START | (5) SEP |
| (2) FREE | (6) CONDTN |
| (3) QUADU | (7) QUADK |
| (4) BEND | (8) ODD |
| (9) SING | |

In addition to these, 12 other subroutines are used in various part of the program. These are:

- (10) MATMPY - A matrix multiplication routine. This is not the ordinary version of MATMPY in common use. The ordinary version of MATMPY will return all zeroes whenever it is used by SELECT. The version in use has been adapted to special conditions encountered here, and has the feature that the product matrix can be returned under the same symbol as one of the factor matrices.
- (11) SELECT - This subroutine selects the proper channel for forming the quadrupole matrix on the basis of IDQ and IDP.
- (12) CDOUB - Calls on the proper combination of CONV, DIV, QFREE, and MATMPY, appropriate to a doublet with entrance section C, and forms its matrix.
- (13) DDOUB - Forms the matrix for a doublet with entrance section D.
- (14) CTRIP - Forms the matrix for a symmetric triplet with entrance section C.
- (15) DTRIP - Forms the matrix for a symmetric triplet with entrance section D.
- (16) CONV - Forms the matrix for a single quadrupole section that is converging horizontally.
- (17) DIV - Forms the matrix for a single quadrupole section which is diverging horizontally.
- (18) QFREE - Forms the matrix for a field-free region. This is different from FREE in that it obtains the length of the region from a calculation in the program, rather than reading the input tape.

- (19) QTRY - This subroutine forms the product matrices $(Qmat\ 1) \times (Q) \times (Prod\ 1)$ and $(Qmat\ 2) \times (Q) \times (Prod\ 1)$, and returns the value of the matrix element upon which a condition is to be placed.
- (20) S1TRY - Like QTRY, for one singlet.
- (21) S2TRY - Like QTRY, for two singlets.

Other subroutines are used by OPTIK, but they are included in the FORTRAN library. It is useful, however, to include the complete FORTRAN library as part of the program deck in order to save the time used to search through the FORTRAN tape for the appropriate subroutines.

C. Specification of Beam Conditions

Subroutine CONDTN is called by the ID number 6, as the first card of two used by it. The second card has the indices of the appropriate matrix element in the first two columns, and the desired value of that element in the next ten columns. Some examples of the format for the second data card for CONDTN are:

- (1) 120.0 The source is brought to an image in the horizontal plane at this point.
- (2) 340.0 Image in the vertical plane.
- (3) 220.0 All rays from a point source are required to be parallel to each other in the horizontal plane at this point.
- (4) 440.0 Parallel rays in the vertical plane.

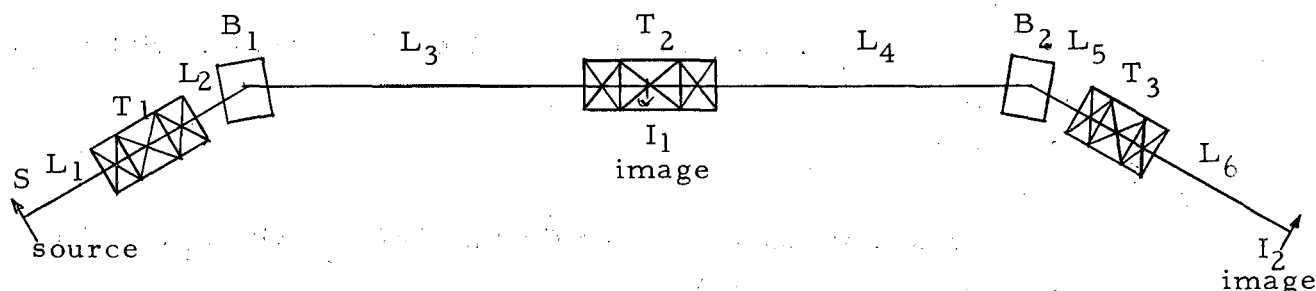
These are the most frequently used beam conditions, but others might be used also, e. g., physical limitations on the position and currents of a particular quadrupole might make it incapable of producing parallel rays from a point source. In this case, one might relax the conditions and require only that the divergence of the beam be small, say $1/10$ of its divergence at the source. Then this condition, in the horizontal plane would be 220.1. Experience in using the matrix formulation of the problem will enable the reader to specify a wide variety of conditions.

It is not possible for the quadrupoles to fully specify matrix elements with subscripts 5 and 6. For example, an energy focus corresponds to 150.0, but the quadrupoles alone cannot accomplish this, so a beam condition should never try to specify any matrix elements with subscripts greater than 4.

Trial and error running of OPTIK, varying parameters of other elements, will enable the user to specify almost any desired beam condition.

D. An Example

Consider the following beam to be put into OPTIK:



A source at S is to be focused by a symmetric quadrupole triplet T_1 to an image at the center of T_2 . This image is to be brought again to a focus at I_2 by quadrupole T_3 . Quadrupole T_2 is a field lens operated in such a manner as to focus the exit aperture of T_1 onto the entrance aperture of T_3 . The B_1 and B_2 are identical bending magnets. The beam from I_1 to I_2 is a mirror image of the beam from S to I_1 , so that there will be an energy focus at I_2 .

We must first determine the solution to T_2 . This may be done by running the following elements through OPTIK:

- (a) START (initial vectors and the momentum of the particles).
- (b) FREE (L_2).
- (c) BEND (the angles and radius of curvature for B_1).
- (d) FREE (L_3).
- (e) QUADU (physical dimensions and trial values of the θ 's for a triplet, T_2).
- (f) FREE (L_4).
- (g) BEND (angles and radius of curvature for B_2).
- (h) FREE (L_5).
- (i) CONDTN (120.0; matrix element (1,2) must be zero at this point for an image of the aperture of T_1 in the horizontal plane).

(j) CONDTN (340.0; the same for vertical plane).

This set of data run through OPTIK will determine the θ 's for T_2 . Then we may think of T_2 as two identical doublets back to back and run it in this manner for the full beam.

Given the following data, OPTIK will solve for the field gradients of T_1 and T_2 , and the problem is completed:

- (a) START.
- (b) FREE (L_1).
- (c) QUADU (physical dimensions and trial values of the θ 's for triplet T_1).
- (d) FREE (L_2).
- (e) BEND (B_1).
- (f) FREE (L_3).
- (g) QUADK (physical dimensions of the first half of T_2 , with θ_1 and $\frac{1}{2}\theta_2$ as the focusing parameters).
- (h) CONDTN (120.0; horizontal focus at this point).
- (i) CONDTN (340.0; vertical focus at this point).
- (j) QUADK (physical dimensions of the second half of T_2 , with $\frac{1}{2}\theta_2$ and θ_1 as the focusing parameters).
- (k) FREE (L_4).
- (l) BEND (B_2).
- (m) FREE (L_5).
- (n) QUADU (physical dimensions and trial values of the θ 's for T_3).
- (o) FREE (L_6).
- (p) CONDTN (120.0 horizontal focus).
- (q) CONDTN (340.0 vertical focus).

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