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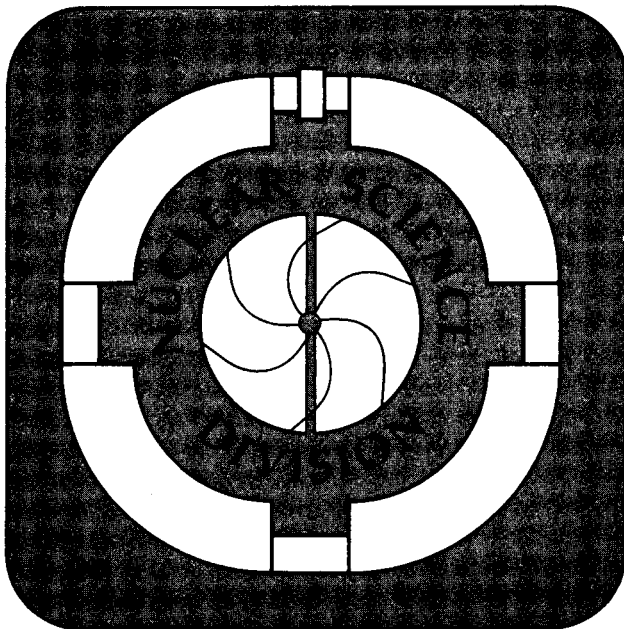
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January 1993



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COLOR, SPIN AND FLAVOR DIFFUSION IN QUARK-GLUON PLASMAS \*

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January, 1993

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# COLOR, SPIN AND FLAVOR DIFFUSION IN QUARK-GLUON PLASMAS

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In weakly interacting quark-gluon plasmas diffusion of color is found to be much slower than the diffusion of spin and flavor because color is easily exchanged by the gluons in the very singular forward scattering processes. If the infrared divergence is cut off by a magnetic mass,  $m_{mag} \sim \alpha_s T$ , the color diffusion is  $D_{color} \sim (\alpha_s \ln(1/\alpha_s) T)^{-1}$ , a factor  $\alpha_s$  smaller than spin and flavor diffusion. A similar effect is expected in electroweak plasmas above  $M_W$  due to  $W^\pm$  exchanges. The color conductivity in quark-gluon plasmas and the electrical conductivity in electroweak plasmas are correspondingly small in relativistic heavy ion collisions and the very early universe.

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Transport of color degrees of freedom in a quark-gluon plasma has recently been found to be infrared sensitive [1] and thus differ from other transport processes as viscous and thermal flow, stopping and electrical conduction [2, 3] as well as energy degradation [4]. Color does not flow easily due to the transfer of color (color-flip) in the exchange of a colored gluon in the very singular forward collisions. As will be discussed here, this suppression of the color flow also applies to the color diffusion and color conductivity, which are infrared sensitive like the quark and gluon quasiparticle relaxation rates [5, 6]. These results do not only apply to QCD plasmas consisting of quarks, antiquarks and gluons, as existing in the Early Universe and searched for in relativistic heavy ion collisions at Brookhaven (AGS and RHIC) and CERN (SPS and LHC). As will be shown below it is a general mechanism for most non-abelian gauge theories. For example, in an electroweak plasma consisting of leptons, photons and electroweak bosons at temperatures above  $M_W \simeq 80$  GeV, the exchange of  $W^\pm$  provides a charge exchange analogous to the color exchange which results in correspondingly poor electrical diffusion and conduction.

Earlier studies of transport processes in relativistic quark-gluon and electron-photon plasmas found that the effect of Landau damping effectively led to screening of transverse interactions and gave the characteristic relaxation rates in transport processes. Transport coefficients for weakly interacting electron-photon and quark-gluon plasmas for both thermal plasmas [2-4] as well as degenerate ones [7] were calculated to leading logarithmic order. Generally the transport relaxation rates have the following dependence on interaction strength

$$1/\tau_{ir} \sim \alpha_s^2 \ln(1/\alpha_s) T. \quad (1)$$

However, the quark and gluon quasiparticle damping rates,  $1/\tau_p$ , were not sufficiently screened by Landau damping for non-vanishing quasiparticle momentum,  $\mathbf{p}$ , and depends on an infrared cut-off,  $m_{mag} \simeq \alpha_s T$ , so that [5, 6]

$$1/\tau_p^{(g)} = 3\alpha_s \ln(1/\alpha_s) T, \quad (2)$$

to leading logarithmic order. Since the quasiparticle decay rates are not measurable transport coefficients the infrared sensitivity was not considered a serious problem. However, it was recently discovered [1] that diffusion of color in some abstract color space suffered from the same infrared divergence which led to the same color relaxation rates as the quasiparticle damping rates.

We will describe the two kinds of transport processes by calculating the flavor, spin, and color diffusion coefficients in a quark-gluon plasma within the Boltzmann kinetic equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{\mathbf{p}_1} \cdot \nabla_{\mathbf{r}} + \mathbf{F} \cdot \nabla_{\mathbf{p}_1}\right) n_1 = -2\pi\nu_2 \sum_{\mathbf{p}_2 \mathbf{p}_3 \mathbf{p}_4} \times [n_1 n_2 (1 \pm n_3)(1 \pm n_4) - n_3 n_4 (1 \pm n_1)(1 \pm n_2)]$$

$$\times |M_{12 \rightarrow 34}|^2 \delta_{\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_3 + \mathbf{p}_4} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4). \quad (3)$$

Here  $\mathbf{p}_i$  and  $\varepsilon_i$  are the quasiparticle momentum and energy respectively,  $n_i(\mathbf{p}_i)$  the quasiparticle distribution function, and  $\mathbf{F}$  the force on a quasiparticle. The r.h.s. is the collision integral for scattering particles from initial states 1 and 2 to final states 3 and 4, respectively with matrix element squared  $|M_{12 \rightarrow 34}|^2$  summed over final states and averaged over initial states. The  $(1 \pm n_i)$  factors correspond physically to the Pauli blocking of final states, in the case of fermions, and to (induced or) stimulated emission, in the case of bosons.  $\nu_2$  is the statistical factor,  $\nu_2 = 16$  for gluons and  $\nu_2 = 12N_f$  for quarks and antiquarks. For scattering of quarks of different flavor

$$|M_{12 \rightarrow 34}^{(qq')}|^2 = \frac{4}{9} g^4 \frac{u^2 + s^2}{t^2} \frac{1}{16\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4}; \quad (4)$$

quark-gluon and gluon-gluon interactions are just  $9/4$  and  $(9/4)^2$  times stronger respectively near forward scattering. In a medium this singularity is screened as given by the Dyson equation in which a gluon self-energy  $\Pi_{L,T}$  is added to the propagator

$$t^{-1} \rightarrow \omega^2 - q^2 - \Pi_{L,T}, \quad (5)$$

(we refer to [9] for details on separating longitudinal and transverse parts of the interaction) where the longitudinal and transverse parts of the self-energy in QED and QCD are for  $\omega, q \ll T$  given by

$$\Pi_L(\omega, q) = q_D^2 \left(1 - \frac{x}{2} \ln \frac{x+1}{x-1}\right), \quad (6)$$

$$\Pi_T(\omega, q) = q_D^2 \left[\frac{1}{2}x^2 + \frac{1}{4}x(1-x^2) \ln \frac{x+1}{x-1}\right], \quad (7)$$

where  $x = \omega/qv_p$  and  $v_p = c$  for the relativistic plasmas considered here. The Debye screening wavenumber in thermal QCD is  $q_D^2 = g^2(2N + N_f)T^2/6$  where  $N = 3$  is the number of colors,  $N_f$  is the number of quark flavors,  $T$  the plasma temperature and  $\mu_q$  the quark chemical potential. We refer to [8] for a detailed comparison to QED plasmas. In the static limit,  $\Pi_L(\omega = 0, q) = q_D^2$ , and the longitudinal interactions are Debye screened. For the transverse interactions the self-energy obeys the transversality condition  $q^\mu \Pi_{\mu\nu} = 0$ , which insures that the magnetic interactions are unscreened in the static limit,  $\Pi_T(\omega = 0, q) = 0$ . It has therefore been suggested that the transverse interactions are cut off below the "magnetic mass",  $m_{mag} \sim g^2 T$ , where infrared divergences appear in the plasma [10]. However, as was shown in [2, 3], dynamical screening due to Landau damping effectively screen the transverse interactions off in most transport problems at a length scale of order the Debye screening length  $\sim 1/gT$  as in Debye screening. Nevertheless, there are three important length scales in the quark-gluon plasma. For a hot plasma they are, in increasing size, the interparticle spacing  $\sim 1/T$ , the Debye

screening length  $\sim 1/gT$ , and the scale  $1/m_{mag} \sim 1/g^2T$  where QCD effects come into play. A weakly interacting QCD plasma and its screening properties is very similar to a QED plasma if one substitutes the fine structure constant  $\alpha_s = g^2/4\pi$  by  $\alpha = e^2/4\pi \simeq 1/137$ , the gluons by photons and the quarks by leptons with the associated statistical factors [8].

Let us first consider a quark-gluon plasma where the particle flavors have been separated spatially, i.e., the flavor chemical potential depends on position,  $\mu_i(\mathbf{r})$ . In a steady state scenario the quark flavors will then be flowing with flow velocity,  $\mathbf{u}_i$ . For simplicity we take the standard ansatz for the distribution functions (see, e.g., [7, 11])

$$n_i(\mathbf{p}_i) = \left( \exp\left(\frac{\varepsilon_{\mathbf{p}} - \mu_i(\mathbf{r}) - \mathbf{u}_i \cdot \mathbf{p}}{T}\right) \pm 1 \right)^{-1} \\ \simeq n_i^0 - \frac{\partial n_p}{\partial \varepsilon_p} \mathbf{u}_i \cdot \mathbf{p}. \quad (8)$$

The expansion is valid near equilibrium where  $\mu_i$  and therefore also  $\mathbf{u}_i$  is small. It gives two terms: the equilibrium distribution function  $n_i^0 = (\exp[(\varepsilon_{\mathbf{p}} - \mu_i(\mathbf{r})/T]) \pm 1)^{-1}$  and the deviation from that. In general the deviation from equilibrium has to be found self-consistently by solving the Boltzmann equation. However, as in the case of the viscosity [2], we expect the ansatz (8) to be good within few percent to leading logarithmic order.

The flavor diffusion coefficient,  $D_{flavor}$ , defined by:

$$\mathbf{j}_i = -D_{flavor} \nabla \rho_i, \quad (9)$$

is given in terms of the flavor current  $\mathbf{j}_i$  and the gradient of the number density  $\rho_i = \sum_{\mathbf{p}} n_i^0(\mathbf{p}_i) = \nu_i T^3 3\xi(3)/4\pi^2$  of a particular flavor  $i$ . From (8) we find

$$\mathbf{j}_i = \sum_{\mathbf{p}} n_{\mathbf{p},i} \mathbf{u}_i = \mathbf{u}_i \rho_i. \quad (10)$$

The density gradient,  $\nabla \rho_i = \nabla \mu_i \sum_{\mathbf{p}} (\partial n_i^0 / \partial \varepsilon_p)$  can be found by solving the Boltzmann equation. Linearizing in  $\mathbf{u}_i$  we obtain

$$\frac{\partial n_1}{\partial \varepsilon_1} \mathbf{v}_1 \cdot \nabla \mu_1 = 2\pi\nu_2 \sum_{\mathbf{p}_2 \mathbf{p}_3 \mathbf{p}_4} |M_{12 \rightarrow 34}|^2 \\ \times n_1^0 n_2^0 (1 - n_3^0) (1 \pm n_4^0) \\ \times (\mathbf{u}_1 \cdot \mathbf{p}_1 + \mathbf{u}_2 \cdot \mathbf{p}_2 - \mathbf{u}_3 \cdot \mathbf{p}_3 - \mathbf{u}_4 \cdot \mathbf{p}_4) \\ \times \delta_{\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_3 + \mathbf{p}_4} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4). \quad (11)$$

It is most convenient to choose the plasma center-of-mass system where one flavor is flowing with velocity  $\mathbf{u}_1$  and the others with velocity  $\mathbf{u}_2 = -\mathbf{u}_1/(N_f - 1)$ . The number of scatterers is then  $\nu_2 = 12(N_f - 1)$ . Equivalently, one can conveniently include the first flavor so that the number of scatterers is  $\nu_2 = 12N_f$  but  $\mathbf{u}_2 = 0$ . In steady state the gluons will not move in the c.m.s., i.e.  $\mathbf{u}_2 = 0$  for quark-gluon scattering. Since the flavor is unchanged in the collisions  $\mathbf{u}_3 = \mathbf{u}_1$  and  $\mathbf{u}_4 = \mathbf{u}_2$ .

To leading logarithmic order the singular interaction near forward scattering allows us to expand around  $\mathbf{q} \sim 0$ , where  $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_3 = \mathbf{p}_4 - \mathbf{p}_2$  is the momentum transfer in the collision. Multiplying both sides of (11) by  $\mathbf{p}_1$  and summing the Boltzmann equation reduces to

$$n_1 \nabla \mu_1 = -\mathbf{u}_1 \frac{\pi}{3} \nu_2 \sum_{\mathbf{q}, \mathbf{p}_1, \mathbf{p}_2} n_1^0 n_2^0 (1 - n_3^0) (1 \pm n_4^0) \\ \times |M_{12 \rightarrow 34}|^2 q^2 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4), \quad (12)$$

where we have used the antisymmetry of the r.h.s. by coordinate change  $\mathbf{p}_1 \rightarrow \mathbf{p}_3$  so that  $\mathbf{p}_1 \rightarrow \mathbf{q}/2$ . The r.h.s. collision integral of Eq. (12) is now straightforward to evaluate to leading logarithmic order when the screening is properly included (see also Refs. [2, 3, 7]). We find

$$D_{flavor}^{-1} \simeq \frac{\pi^5}{3^3 4 \xi(3)^2} (1 + N_f/6) \alpha_s^2 \ln(1/\alpha_s) T, \quad (13)$$

where  $\xi$  is the Riemann zeta-function. The term "1" arises from quark-gluon scatterings and the  $N_f/6$  from quark-quark scatterings. This result is similar to the viscous, thermal and momentum relaxation rates because the collision term contains the same factors of momentum transfer: the singular  $q^{-4}$  factor from the matrix element squared and the suppressing  $q^2$  factor because the quark flavors lose little momentum in forward scatterings. Including screening,  $q^{-4} \rightarrow (q^2 + \Pi_{L,T})^{-2}$ , where effectively  $\Pi_{L,T} \sim q_D^2$ , and integrating over momentum transfer,  $d^2q$ , gives the leading logarithmic term  $\ln(T^2/q_D^2) \simeq \ln(1/\alpha_s)$ .

Subsequently, let us consider the case where the particle spins have been polarized spatially by some magnetic field [11], i.e., the spin chemical potential depends on position,  $\mu_\sigma(\mathbf{r})$ . With the analogous ansatz to (8) for the distribution functions with  $\mu_\sigma$  instead of  $\mu_i$ , we find the spin current  $\mathbf{j}_\sigma = \mathbf{u}_\sigma n_\sigma$ . Linearizing the Boltzmann equation we find

$$\frac{\partial n_1}{\partial \varepsilon_1} \nabla \mu_{1,\sigma} \cdot \mathbf{v}_1 = 2\pi\nu_2 \sum_{\mathbf{p}_2 \mathbf{p}_3 \mathbf{p}_4} n_1^0 n_2^0 (1 - n_3^0) (1 \pm n_4^0) \\ \times \delta_{\mathbf{p}_1 + \mathbf{p}_2, \mathbf{p}_3 + \mathbf{p}_4} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \\ \times [|M_{12 \rightarrow 34}^{\uparrow\downarrow}|^2 (\mathbf{u}_1 - \mathbf{u}_2) \cdot (\mathbf{p}_1 - \mathbf{p}_2) \\ + |M_{12 \rightarrow 34}^{\uparrow\uparrow}|^2 (\mathbf{u}_1 - \mathbf{u}_2) \cdot \mathbf{q}], \quad (14)$$

where  $M^{\uparrow\downarrow}$  and  $M^{\uparrow\uparrow}$  are the amplitudes for interacting with and without spin-flip respectively. Without spin-flip the usual factor  $\mathbf{q}$  as in flavor diffusion appears. With spin-flip, however,  $\mathbf{u}_3 = \mathbf{u}_2$  and  $\mathbf{u}_4 = \mathbf{u}_1$  and the factor  $(\mathbf{p}_1 - \mathbf{p}_2)$  appears. Due to Galilei invariance both terms are necessarily proportional to the relative flow,  $(\mathbf{u}_1 - \mathbf{u}_2)$ .

The transition current can be decomposed into interactions via the charge and the magnetic moment by the Gordon decomposition rule

$$\mathbf{J}_\mu = \frac{g}{2m} \bar{u}_f [(p_f + p_i)_\mu + i\sigma_{\mu\nu} (p_f - p_i)^\nu] u_i, \quad (15)$$

where only the latter can lead to spin-flip. We notice that the spin-flip amplitude is suppressed by a factor  $p_f - p_i$

which leads to a spin-flip amplitude suppressed by a factor  $q^2$ . We then find that the spin-flip interactions do not contribute to collisions to leading logarithmic order and the collision integral is similar to that for flavor diffusion evaluated above. Consequently, the corresponding quark spin diffuseness parameter becomes

$$D_\sigma^{(q)} = D_{flavor}. \quad (16)$$

Gluon spin diffusion is slower by a factor 4/9, due to the stronger interactions, and by another factor 4/9, due to differences between Bose and Fermi distribution function, i.e.  $D_\sigma^{(g)} \simeq (4/9)^2 D_\sigma^{(q)}$ .

Finally, let us, like for the spin diffusion, assume that color has been polarized spatially given by a color chemical potential,  $\mu_c(\mathbf{r})$ . The basic difference to flavor and spin diffusion is that *quarks and gluons can easily flip color directions* in forward scattering by color exchanges, i.e., one does not pay the extra  $q^2$  penalty factor as in the case of spin-flip. Consequently, the color-flip interactions will dominate the collisions since they effectively reverse the color currents. The Boltzmann equation thus gives us an analogous result to Eq. (14) replacing spin by color where the color-flip amplitude now dominates. The flow velocity of the scatterers,  $\mathbf{u}_i$ ,  $i=1\dots4$ , depends on what color combination of the scattering quarks, antiquarks and gluons. However, in c.m.s. the scatterer has vanishing flow velocity,  $\mathbf{u}_2 = 0$ , on average. Likewise the final velocities will be zero on average. Multiplying both sides with  $\mathbf{p}_1$  and summing the Boltzmann equation reduces to (c.f., Eq.(14))

$$n_1 \nabla \mu_{1,c} = -\mathbf{u}_1 \nu_2 \sum_{\mathbf{q}, \mathbf{p}_1, \mathbf{p}_2} n_1^0 n_2^0 (1 - n_3^0) (1 \pm n_4^0) \times |M_{12 \rightarrow 34}|^2 (\mathbf{p}_1 - \mathbf{p}_2)^2 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4), \quad (17)$$

where we have used the antisymmetry by interchange of  $\mathbf{p}_1 \rightarrow \mathbf{p}_2$ . The matrix element entering in (17) is now averaged over all color combinations.

The transverse interactions actually diverge for small momentum and energy transfers even when integrating over energy transfers, i.e. dynamical screening is insufficient for obtaining a non-zero color diffusion coefficient like for the quasiparticle decay rates in QCD and QED plasmas (see [6]). Concentrating therefore on the leading contribution from transverse interactions for small  $\mu = \omega/q$ , where  $\Pi_T \simeq i(\pi/4)q_D^2 \mu$ , we find to leading order

$$n_1 \nabla \mu_{1,c} = -\mathbf{u}_1 \nu_1 \frac{11\pi^3}{3^3 5 \xi(3)^2} \alpha_s^2 T^7 \times \int_\lambda^{\sim T} q dq \int_{-1}^1 d\mu \frac{1}{q^4 + (\pi/4)^2 q_D^4 \mu^2} = -\mathbf{u}_1 \nu_1 \frac{22\pi^3}{3^3 5 \xi(3)^2} \alpha_s^2 \frac{T^7}{q_D^2} \ln\left(\frac{q_D^2}{\lambda^2}\right), \quad (18)$$

where we have introduced an infrared cutoff,  $\lambda$ . The upper limit on momentum transfers,  $\sim T$ , actually comes

from the distribution functions in Eq. (17) but it does not enter here because only  $q \lesssim q_D$  contribute to (18) to leading order. We find a color diffuseness parameter defined by  $\mathbf{j}_c = \mathbf{u}_1 \rho_c \equiv -D_{color} \nabla \rho_c$

$$D_{color}^{-1} = \frac{22\pi^6}{3^6 5 \xi(3)^2} \frac{1 + 7N_f/33}{1 + N_f/6} \alpha_s \ln(q_D^2/\lambda^2) T. \quad (19)$$

With  $\lambda \sim g^2 T$  the analogous result to Eq. (2) is obtained

$$D_{color}^{-1} \simeq 4.9 \alpha_s \ln(1/\alpha_s) T, \quad (20)$$

(where we have ignored the minor dependence on  $N_f$ ). Comparing Eqs. (16) and (20) we see that  $D_{color} \sim \alpha_s D_\sigma = \alpha_s D_{flavor}$ . The color-flip mechanism amplifies the forward collisions so the color cannot diffuse through the quark-gluon plasma as easily as spin or flavor.

The factor  $\ln(1/\alpha_s)$  in  $D_{color}$  has a completely different origin as the one in  $D_{flavor}$  or  $D_{spin}$ . In  $D_{color}$  the logarithm arises from an integral  $dq/q$  over momentum transfers from  $q \sim \lambda \sim g^2 T$  to  $q \sim q_D \sim gT$  as in the case of quark and gluon quasiparticle decay rates [6]. In  $D_{flavor}$  or  $D_{spin}$  and the transport coefficient discussed in [2, 3] a similar integral occurs, but with momentum transfers from  $q \sim q_D \sim gT$  to  $q \sim T$ . Thus the infrared cutoff does not enter these transport coefficients. In both cases the result is proportional to the logarithm of the ratio of the upper and lower limits on the momentum transfer, namely  $\ln(1/g)$ . The difference in the important range of momentum transfers in the two cases is due to the absence in the calculation of  $D_{color}$  of the extra factor  $\sim q^2/T^2$ . Therefore small momentum transfer processes have greater weight in the calculation of  $D_{color}$  and quasiparticle relaxation rates than they do for  $D_{flavor}$ ,  $D_{spin}$  and standard transport relaxation rates. In the latter cases the factor  $q^2/T^2 \sim q_D^2/T^2 \sim \alpha_s$ , also reduces the rates by a factor  $\alpha_s$ .

Other related transport coefficients are the electrical conductivity,  $\sigma_{el}$ , in QED and the corresponding color conductivity,  $\sigma_{color}$ , in QCD. Applying a color-electric field,  $\mathbf{E}_c$ , to the quark-gluon plasma generates a color current,  $\mathbf{j}_c$ . The color conductivity  $\sigma_{color} = -\mathbf{j}_c/E_c$  can thus be found by solving the Boltzmann equation analogous to the color diffusion process. We find

$$\sigma_{color} = \frac{2}{3} g^2 D_{color} \sum_{i,p} \nu_i \left( \frac{\partial n_i}{\partial \varepsilon_i} \right). \quad (21)$$

Here  $D_{color}$  plays the role of the color relaxation time. Eq. (21) is the standard result for a plasma except for the factor 2/3 which arises because only two-thirds of the colors contribute to the currents for a given color field. Inserting  $D_{color}$  from (20) we obtain

$$\sigma_{color} \simeq \frac{8\pi}{3} N_f \alpha_s D_{color} T^2 \simeq 1.7 N_f T / \ln(1/\alpha_s), \quad (22)$$

from quark currents alone. Gluon currents are slower due to stronger interactions and will reduce the conductivity slightly. This result differs from [1] by a numerical



factor only. For comparison the electrical conductivity below  $T \lesssim M_W$  is  $\sigma_{el} \sim T/\alpha \ln(1/\alpha)$  [3]. The different dependence on coupling constant arises because the exchanged photon does not carry charge whereas the exchanged gluon can carry color. The characteristic relaxation times for conduction are very different in QCD, where  $\tau_{color} \sim D_{color} \sim (\alpha_s \ln(1/\alpha_s)T)^{-1}$ , as compared to QED, where  $\tau_{el} \simeq (\alpha^2 \ln(1/\alpha)T)^{-1}$ . Consequently, QGP are much poorer color conductors than QED plasmas when  $T \ll M_W$  for the same coupling constant.

These surprising results for QCD are qualitatively in agreement with those found by Selikhov & Gyulassy [1] who have considered the diffusion of color in color space. They use the fluctuation-dissipation theorem to estimate the deviations from equilibrium and find the same color non-flip and color-flip terms, which they denote the momentum and color diffusion terms respectively, and they also find that the latter dominates being infrared divergent. Inserting the same infrared cut-off they find a color diffusion coefficient in color space equal to Eq. (2)

$$d_c = \frac{1}{\tau_1^{(g)}} = 3\alpha_s \ln(1/\alpha_s)T. \quad (23)$$

Note that this quantity is proportional to the inverse of  $D_{color}$  as given in Eq. (20).

The color-flip mechanism is not restricted to QCD but has analogues in other non-abelian gauge theories. In the very early universe when  $T \gg M_W \simeq 80$  GeV, the  $W^\pm$  bosons can be neglected and faces the same electroweak screening problems as QCD and QED. Since now the exchanged  $W^\pm$  bosons carry charge (unlike the photon, but like the colored gluon), they can easily change the charge of, for example, an electron to a neutrino in forward scatterings. Thus the collision term will lack the usual factor  $q^2$  as for the quasiparticle damping rate and the color diffusion. Since  $SU(2) \times U(1)$  gauge fields should have the same infrared problems as  $SU(3)$  at the scale of the magnetic mass,  $\sim e^2 T$ , we insert this infrared cutoff. Thus we find a diffusion parameter for charged electroweak particles in the very early universe of order

$$D_{el} \sim (\alpha \ln(1/\alpha)T)^{-1}, \quad (24)$$

which is a factor  $\alpha$  smaller than when  $T \ll M_W$ . The electrical conductivity will be smaller by the same factor as well,  $\sigma_{el} \sim T/\ln(1/\alpha)$ .

In summary, the flavor, spin and color diffusion coefficients have been calculated in QCD plasmas to leading order in the interaction strength. Color diffusion and the quark and gluon quasiparticle decay rates are not sufficiently screened and do depend on an infrared cut-off of order the magnetic mass,  $m_{mag} \sim g^2 T$ ; typically  $D_{color}^{-1} \sim \alpha_s \ln(q_D/m_{mag})T \sim \alpha_s \ln(1/\alpha_s)T$ . Flavor and spin diffusion processes are sufficiently screened by Debye screening for the longitudinal or electric part of the interactions and by Landau damping for the transverse or magnetic part of the interactions; typically  $D_{flavor}^{-1} = D_{spin}^{-1} \sim \alpha_s^2 \ln(1/\alpha_s)T$ . As a consequence, color diffusion

is slow and the QGP is a poor color conductor. In the very early universe when  $T \gg M_W$  exchanges of  $W^\pm$  provides charge exchange - a mechanism analogous to color exchange in QCD - and QED plasmas will also be poor electrical conductors.

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- [1] A. Selikhov and M. Gyulassy, *Phys. Lett.* **B316**, 316 (1993); and CU-TP-610/93.
  - [2] G. Baym, H. Monien and C. J. Pethick, Proc. XVI Int. Workshop on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, (ed. H. Feldmeier, GSI and Institut für Kernphysik, Darmstadt, 1988), p. 128; C. J. Pethick, G. Baym and H. Monien, *Nucl. Phys.* **A498**, 313c (1989); G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, *Phys. Rev. Lett.* **64**, 1867 (1990).
  - [3] G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, *Nucl. Phys.* **A525**, 415c (1991); G. Baym, H. Heiselberg, C. J. Pethick, and J. Popp, *Nucl. Phys.* **A544**, 569c (1992).
  - [4] M. Gyulassy and M. Thoma, *Nucl. Phys.* **B 351**, 491 (1991); E. Braaten and M. Thoma, *Phys. Rev.* **D44**, 1298, R2625 (1991)
  - [5] C. P. Burgess and A. L. Marini, *Phys. Rev.* **D45**, R17 (1992); A. Rebhan, *Phys. Rev.* **D48**, 482 (1992); R. D. Pisarski, *Phys. Rev.* **D47**, 5589 (1993).
  - [6] H. Heiselberg and C. J. Pethick, *Phys. Rev.* **D47**, R769 (1993);
  - [7] H. Heiselberg, G. Baym, and C. J. Pethick, *Nucl. Phys. B* (Proc. Suppl.) **24B** 144, (1991); H. Heiselberg and C. J. Pethick, *Phys. Rev.* **D48**, 2916 (1993).
  - [8] H. Heiselberg and C. J. Pethick, NBI-93-19, proceedings of "Plasma Physics", Les Houches, Feb. 2-11, 1993.
  - [9] H. A. Weldon, *Phys. Rev.* **D26**, 1394 (1982).
  - [10] A. D. Linde, *Phys. Lett.* **B96**, 289 (1980).
  - [11] G. Baym and C. J. Pethick, *Landau Fermi-liquid theory: concepts and applications* (Wiley, New York, 1991).

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