Lawrence Berkeley National Laboratory

Recent Work

Title

Generating one-column grids with fractal flow dimension

Permalink

<https://escholarship.org/uc/item/0561p1p9>

Author

Doughty, C

Publication Date

2017-11-01

DOI

10.1016/j.cageo.2016.11.010

Copyright Information

This work is made available under the terms of a Creative Commons Attribution-NonCommercial-NoDerivatives License, available at <https://creativecommons.org/licenses/by-nc-nd/4.0/>

Peer reviewed

GENERATING ONE-COLUMN GRIDS WITH FRACTAL FLOW DIMENSION

Christine Doughty

Energy Geosciences Division

Lawrence Berkeley National Laboratory

#1 Cyclotron Rd, Mailstop 74/316C

Berkeley, California, USA 94720

email: CADoughty@lbl.gov

ABSTRACT

The grid generation capability built into the numerical simulator TOUGH for multi-phase fluid and heat flow through geologic media can create one-column grids with linear or radial geometry, corresponding to one-dimensional or two-dimensional radial flow, respectively. The integral-finite-difference-method that TOUGH employs for spatial discretization makes it very simple to generalize the grid-generation algorithm from integer to non-integer (fractal) flow dimension. Here the grid-generation algorithm is generalized to create one-column grids with fractal flow dimension ranging from less than 1 to 3. The fractal grid generation method is verified by comparing numerical simulation results to an analytical solution for a generalized Theis solution for integer and non-integer flow dimensions between 0.4 and 3. It is then applied to examine gas production decline curves from hydraulically fractured shale that is modeled as a fractal-dimensioned fracture network with flow dimensions between 0.25 and 3. Grids with fractal flow dimension are useful for representing flow through fracture networks or highly heterogeneous geologic media with fractal geometry, and may be particularly useful for inverse methods.

- 24 Keywords: grid generation, fractal geometry, TOUGH, integral-finite-difference method, fractal
- 25 dimension, flow dimension

1. INTRODUCTION

The TOUGH family of codes for multi-phase fluid and heat flow through geologic media (Pruess, 2004) use the integral-finite-difference (IFD) method (Narasimhan and Witherspoon, 1976) for spatial discretization. In this method, the volumes of grid blocks, the areas of interfaces between grid blocks, and the distances between nodal points and the interfaces are specified, without reference to a global coordinate system (Figure 1). This feature enables creation of one-column grids such as those shown in Figure 2 that can represent

- linear flow geometry (all grid blocks have equal interface areas),
 - radial flow geometry (interface area is proportional to r_i),
 - spherical flow (interface area is proportional to r_i^2),
- where r_i is the distance from the origin of the grid to the i th grid block.

Figure 1. Geometry data in the IFD method: V_m and V_n are the volumes of grid blocks m and n , respectively; A_{mn} is the area of the interface between them; and D_m and D_n are the distances from the nodal points to the interface.

Figure 2. Schematic view of one-column grids for linear flow, radial flow, and spherical flow.

One-column linear and radial grids are familiar to most TOUGH users and can be created automatically using the MESHMAKER module (Pruess et al., 1999). The present paper describes how the MESHMAKER grid generation algorithm can be generalized to non-integer flow dimensions (i.e., fractal flow dimensions). For a flow dimension d ($0 < d \leq 3$), one can create a one-column grid to represent d -dimensional flow simply by making interface area proportional to r_i^{d-1} and integrating to obtain grid-block volume proportional to $r_{i+1}^d - r_i^d$, where r_{i+1} and r_i are the distances from the origin of the grid to two adjacent grid blocks. The use of non-integer dimension grids should be applicable to any numerical simulator based on the IFD method, not just the TOUGH family (i.e., TOUGH2 (Pruess et al., 2012), iTOUGH2 (Finsterle, 1999), T2VOC (Falta et al., 1995), TMVOC (Pruess and Battistelli, 2002), and TOUGHREACT (Xu and Pruess, 2001)).

This paper is organized as follows. First, the concept of fractal flow dimension is reviewed and illustrated in the context of flow through fracture networks or heterogeneous porous media. Next, the fractal grid generation method is described, then two examples of its use are presented. The first example compares numerical simulation results to an analytical solution for a generalized Theis solution (Barker, 1988) and serves to verify that the created grid does possess the desired

fractal flow dimension. The second example considers gas production from hydraulically fractured shale, and illustrates how the early-time slope of the pressure decline curve can be related to the fractal flow dimension.

2. THE CONCEPT OF FRACTAL FLOW DIMENSION FOR GEOLOGIC MEDIA

A fractal dimension is defined here as a dimension that may have a non-integer value. The usual concept of a spatial domain being 1-, 2-, or 3-dimensional can be generalized to allow d -dimensional spaces, where d need not be an integer. Since the concept of fractal geometry (Mandelbrot, 1983) was introduced into the Earth Sciences in the 1980's, numerous authors have used fractals to describe heterogeneous porous and fractured media and investigated the resulting flow and transport behavior. Barker (1988) and Acuna and Yortsos (1995) looked at pressure-transient well testing; Wheatcraft and Tyler (1988), Neuman (1990), and Doughty and Karasaki (2002) considered tracer transport; Tyler and Wheatcraft (1990) studied soil water retention in the vadose zone; Hewett (1986) and Emanuel et al. (1987) examined oil reservoir performance. An early review paper by Sahimi and Yortsos (1990) presented many authors' work. More recently, interest in analyzing the production decline curves from hydraulically fractured tight shale formations (Silin and Kneafsey, 2012; Patzek et al., 2013) motivated the present application of fractal geometry.

In the present paper, the intrinsic fractal dimension of a geologic medium, d_i , is distinguished from the fractal flow dimension, d , that describes flow through the medium to a particular source or sink, with the latter being the relevant dimension for grid generation. To understand the intrinsic fractal dimension, which ranges from 1 to 3, it is helpful to first consider the usual integer dimensions. Two examples of media with $d_i = 3$ are a thick body of uniform sand and a

highly connected fracture network (Figure 3a). In contrast, extensive thin sand bodies over- and underlain by low permeability clay layers and isolated fracture planes (Figure 3b) are examples of media with $d_i = 2$. A set of unconnected tube-like channels of high permeability (Figure 3c) is an example of a medium with $d_i = 1$. An intrinsic fractal dimension between 2 and 3 can be useful to describe thin sand bodies separated by discontinuous clay lenses and anisotropic fracture networks with great connectivity in two directions and sparse connectivity in the other direction (Figure 3d). A sparsely connected network of tubes (Figure 3e) could be described by a fractal dimension d_i between 1 and 2. Altogether, a variety of geologic media can be represented with intrinsic fractal dimensions ranging from 1 to 3.

Figure 3. Schematic views of geologic media with a range of intrinsic fractal dimension d_i . Shaded regions represent high-permeability media. In frames (b) and (c) there is no connection between individual high-permeability zones, whereas in frames (d) and (e) there is.

However, when considering the dimension d that the fluid flow field will have in these media, we need to consider not only the intrinsic fractal dimension of the medium d_i but also the source or sink of fluid. The simplest case is a point source or sink (Figure 4). In this case, flow

dimension d equals intrinsic dimension d_i . For a point source or sink, fractal flow dimension can range from 1 to 3, with $d = 3$ corresponding to a uniform medium surrounding the point (Figure 4a), $d = 2$ corresponding to a planar feature surrounding the point (Figure 4b), $d = 1$ representing a tube-like channel containing the point (Figure 4c), and all values of d between 1 and 3 possible to represent intermediately connected structures (Figures 4d and 4e). Figure 4 illustrates that when $d = 1$, flow geometry is linear, neither converging nor diverging towards the point sink, and that increasing values of d represent increasingly converging flow to the point sink.

Figure 4. Schematic views of geologic media with a range of intrinsic fractal dimension d_i and a point source or sink with typical flow lines for fractal flow dimension d .

Next, consider the case of a line source or sink, the familiar representation of a well fully penetrating a formation (Figure 5). For a uniform medium with $d_i = 3$, the flow dimension becomes $d = 2$, representing radial flow toward the well (Figure 5a). For a uniform planar feature with $d_i = 2$, flow dimension becomes $d = 1$, representing linear flow toward the well (Figure 5b). For integer values of d_i containing a line source or sink, flow dimension d is given by $d = d_i - 1$. For non-integer values of d_i , the relationship between d_i and d may not be so

simple, but generally for a line source, $d < d_i$. Thus, for intrinsic fractal dimension $2 < d_i < 3$, flow dimension is $1 < d < 2$ (Figure 5c), and as for the point sink case, increasing values of d represent increasingly converging flow toward the sink. In contrast, if the medium has intrinsic fractal dimension $d_i < 2$, then $d < 1$, which represents flow that must diverge from the medium to the well (Figure 5d).

Figure 5. Schematic views of geologic media with a range of intrinsic fractal dimension d_i and a line source or sink with typical flow lines for fractal flow dimension d .

Finally, consider a plane source or sink (Figure 6). For a uniform medium with $d_i = 3$, the flow dimension becomes $d = 1$, representing linear flow toward the plane (Figure 6a), and we see that

$d = d_i - 2$. If the medium has intrinsic fractal dimension $d_i < 3$, then $d < 1$ is certainly possible, representing flow diverging from the medium to the plane (Figure 6b).

Figure 6. Schematic views of geologic media with a range of intrinsic fractal dimension d_i and a plane source or sink with typical flow lines for fractal flow dimension d .

For integer dimensions, the relationships between flow dimension and intrinsic dimension can be generalized to $d = d_i - d_s$, where d_s is the dimension of the source or sink: 0 for a point, 1 for a line, and 2 for a plane. For non-integer dimensions, this equality may not hold, but generally d decreases as d_s increases. Moreover, just as d and d_i need not be integers, neither does d_s . A value of $0 < d_s < 1$ corresponds to a source or sink consisting of one or more short line segments, which could represent a partially penetrating well. A value of $1 < d_s < 2$ represents a source or sink consisting of one or more finite areas within a plane, which could represent a heterogeneous hydraulic fracture plane. The smaller d_s is, the more flow will converge toward the sink, and the larger will be d .

Note that for Figures 5d and 6b, the sink intersects multiple flow paths or fractures, resulting in flow that must diverge from the medium to the sink ($d < 1$). If the sink were located where it intersected only one flow path or fracture, this would effectively reduce the dimension of the

sink (it would become a point source for Figure 5d and a line source for Figure 6b), and result in $d > 1$, signifying flow converging toward the sink.

3. FRACTAL GRID GENERATION METHODOLOGY

The essential concept to create a one-column grid with fractal dimension d is to set grid-block volume proportional to r^d and interface area proportional to r^{d-1} , where r is the distance from the origin of the grid to a given grid block. An additional feature to consider is how to define the volume and interface area of the first block of the grid, which represents the fluid source or sink. For simplicity in comparing grids with different dimensions d , the interface area of the first grid block, which is the source/rock interface and is denoted A_w , can be specified to be the same for all values of d (a variation on this option will be discussed for the second example problem, described in Section 5). A more elegant approach (Barker, 1988) is based on the definition of the surface area of a unit sphere in d dimensions, $2\pi/\Gamma(d/2)$, which yields

$$A_w = \frac{2\pi^{d/2}}{\Gamma(d/2)} b^{3-d} r_w^{d-1}, \quad (1)$$

where r_w is the source radius, b is the thickness of the layer penetrated by the source, and Γ is the gamma function. Note that Equation (1) produces the expected values of A_w for integer values of d , as shown in Table 1, and is easily generalized to yield a relationship for A as a function of r for all values of d :

$$A(r) = \frac{2\pi^{d/2}}{\Gamma(d/2)} b^{3-d} r^{d-1} \quad (2)$$

Note that when $d > 1$, $A(r)$ increases as r increases, providing a converging geometry for flow toward a sink, and when $d < 1$, $A(r)$ decreases with increasing r , enabling diverging flow. In the discrete notation of a grid, for the i th grid block, with outer radius r_i

$$A_i = \frac{2\pi^{d/2}}{\Gamma(d/2)} b^{3-d} r_i^{d-1} \quad (3)$$

To determine grid-block volume, V_i , the expression for A_i is integrated from r_{i-1} to r_i , yielding

$$V_i = \frac{2\pi^{d/2}}{d\Gamma(d/2)} b^{3-d} (r_i^d - r_{i-1}^d). \quad (4)$$

In addition to grid block volumes and interface areas, the distances between nodal points and interfaces must be specified (D_m and D_n in Figure 1). For a one-column grid, this amounts to determining the r coordinate of the each nodal point, denoted R_i . Often, nodal points are placed mid-way between adjacent interfaces

$$R_i = (r_i + r_{i-1})/2. \quad (5)$$

A more rigorous algorithm is to locate them at the center of mass of the grid block. Thus, for a d -dimensional grid

$$R_i = \frac{\int_{r_{i-1}}^{r_i} r r^{d-1} dr}{\int_{r_{i-1}}^{r_i} r^{d-1} dr} = \left[\frac{d}{d+1} \right] \frac{(r_i^{d+1} - r_{i-1}^{d+1})}{(r_i^d - r_{i-1}^d)} \quad (6)$$

Note that for $d = 1$, Equation (6) simplifies to Equation (5), and for other values of d , the nodal point locations obtained from Equations (5) and (6) do not differ very much, except close to $r = 0$.

Table 1. Source/rock interface areas A_w for integer values of d , from Equation (1).

d	$\Gamma(d/2)$	A_w
1	$\pi^{1/2}$	$2b^2$
2	1	$2\pi r_w b$
3	$\pi^{1/2}/2$	$4\pi r_w^2$

4. VERIFICATION AGAINST BARKER'S GENERALIZED THEIS SOLUTION

Barker (1988) developed analytical solutions for the pressure-transient response to hydraulic tests in fractured rock for any flow dimension $d > 0$, including non-integer dimensions. Figure 7 shows a schematic view of his conceptual model for integer dimensions 1, 2, and 3.

For a constant volumetric injection rate Q through an infinitesimal source into an infinite medium, Barker found that the pressure change may be expressed as

$$\Delta P = \frac{Q\mu r^{2\nu}}{4\pi^{1-\nu}kb^{3-d}}\Gamma(-\nu, u) \quad (7)$$

where $\Gamma(-\nu, u)$ is the complementary incomplete gamma function, $\nu = 1 - d/2$, and

$$u = \frac{\phi C\mu r^2}{4kt}. \quad (8)$$

The dimensionless parameter u is related to dimensionless time t_D as $u = 1/(4t_D)$. Medium properties are thickness b , porosity ϕ , permeability k , and compressibility C , and μ is fluid viscosity.

Figure 8 shows dimensionless pressure change Γ as a function of dimensionless time t_D for d values ranging from 0.4 to 3. For $d < 2$, for long times the solution becomes linear with slope ν , where $\nu = 1 - d/2$. For $d = 2$, $\Gamma(0, u)$ is identical to the exponential integral $E_1(u)$ of the Theis (1935) solution. For $d \geq 2$, the long-time slope approaches zero.

Figure 7. Schematic well tests for (a) $d = 1$ with a plane source, (b) $d = 2$ with a line source, and (c) $d = 3$ with a point source. Modified from Barker (1988).

Figure 8. Lines show Barker (1988) analytical solution for generalized Theis problem, where d is fractal flow dimension. For $d < 2$, late-time slope is $v = 1 - d/2$. Solid symbols show TOUGH2 results for a grid block at $r \approx 0.5$ m and open symbols show TOUGH2 results for the grid block representing the source with $r_w = 0.1$.

Figure 8 also shows the results of isothermal TOUGH2 simulations using EOS1 (Pruess et al., 1999) for single-phase water at 20°C, initially at 1 bar, with constant-rate injection of water, for one-column grids created with values of d ranging from 0.4 to 3. Material properties and numerical parameters used for the model are shown in Table 2. The grids all have thickness $b = 1$ m, begin with a source with $r_w = 0.1$ m, and contain 100 elements with steadily increasing radial increments (increase factor 1.1), to allow good near-well resolution, but a grid that extends far enough from the well for the medium to be infinite-acting. Injection rate is kept small enough so that density does not change significantly, to conform to the assumptions for the analytical solution. TOUGH2 simulation results shown as solid symbols show time series of

dimensionless pressure change obtained for a grid block close to the source ($r \approx 0.5$ m); they match the analytical solution very well.

Note that $\Gamma(-v, u)$ depends on r and t only through the similarity variable u (Equation 7). Hence, TOUGH2 results for various r and t combinations that yield the same u should overlie one another. Time series of $\Gamma(-v, u)$ for various grid blocks and snapshots of $\Gamma(-v, u)$ at various times mostly show excellent invariance, except at very early time, when the assumption of an infinitesimal source is not met by the finite source ($r_w = 0.1$ m) used for the numerical model, as illustrated by the open symbols in Figure 8. Additionally, for $d > 2.5$, profiles of $\Gamma(-v, u)$ versus t for r_w are slightly offset from $\Gamma(-v, u)$ versus t for all other values of r , again reflecting the inaccuracy inherent in representing an infinitesimal source with a finite grid block. A rule of thumb suggested by the present simulations is that when representing an infinitesimal source by an element with extent r_w , an accurate pressure response can be obtained for all grid blocks with $r_i > 3r_w$.

232 Table 2. Material properties and numerical parameters used for the TOUGH2 simulations of
 233 Barker's generalized Theis problem.

Material Properties	
Permeability	50 mD
Porosity	0.2
Compressibility (rock + water)	$10^{-8} + 4.5 \cdot 10^{-10} \text{ (Pa}^{-1}\text{)}$
Water viscosity	0.001 (Pa s)
Grid Specifications	
b	1 (m)
r_w	0.1 (m)
Flow dimensions d considered	0.4 - 3
Assignment of A_w	Equation (1)
Assignment of D_m and D_n	Equation (6)
Number of grid blocks	100
Grid spacing	Variable, with radial increments increasing by a factor of 1.1
Operating conditions	
Injection Rate Q	$10^{-5} \text{ m}^3/\text{s} \approx 0.01 \text{ kg/s}$
Initial pressure	1 bar
Initial temperature	20°C

234

235 To gain some insight into what it means for a medium to be represented with a fractal flow
 236 dimension, Figure 9 compares the Theis solution ($d = 2$ curve from Figure 8) to the
 237 dimensionless pressure change obtained with a radial dual-porosity model (i.e., high-

permeability fractures embedded in a low-permeability matrix). Note that the pressure responses
 shown in Figure 8 are very smoothly varying for all values of d . For $d = 2$ the smooth curve on
 the log-log plot of Figure 8 becomes a straight line on the log-linear plot of Figure 9. In contrast,
 the dual-porosity pressure response shows a transition between $t_D = 10^3$ and $t_D = 10^5$, from an
 early-time linear trend, when it represents the fracture response only, to a late-time linear trend,
 when it represents both the fracture and matrix responses. Such transitions, or bends, are
 common for heterogeneous media when the pressure pulse passes through distinct domains with
 different flow properties. But no such transitions occur for fractal flow dimension models,
 because there are no distinct domains with different flow properties. A single flow dimension
 represents the entire domain, from early times (when the pressure pulse is near the well), to late
 times (when the pressure pulse has moved far away). This invariance with scale, or self-
 similarity, is a well-known characteristic of fractals.

Figure 9. Comparison of Theis solution to dual-porosity formulation.

5. APPLICATION TO GAS PRODUCTION FROM HYDRAULICALLY FRACTURED SHALE

A typical conceptual model for gas production from hydraulically fractured shale is illustrated in Figure 10, which shows several fracture stages along a horizontal well (Figure 10a). Each fracture stage (Figure 10b) consists of a stimulated reservoir volume (SRV) of thickness $2D$, consisting of a planar primary fracture of areal extent A_p perpendicular to the wellbore and a network of smaller secondary fractures orthogonal to the primary fracture. It is assumed that for a tight gas reservoir, production only occurs from the SRV, which is a rectangular prism of volume $2DA_p$. Additionally, it is assumed that the permeability of the primary fracture is so great that the pressure in the primary fracture is uniform and equals the pressure in the well, making the well effectively a plane sink of areal extent A_p . The fluid is assumed to be single-phase, constant-compressibility gas, which flows according to Darcy's law, and the system is assumed to remain at constant temperature. Initially, the gas in the reservoir and the well is at a constant pressure, p_R ; at time zero the pressure at the well is dropped to p_W , where it is held constant during the production process.

Although the fracture network making up the SRV will likely be irregular (as illustrated schematically in Figure 10b), it is assumed that flow through it toward the primary fracture can be modeled with a single value of fractal flow dimension. If the SRV is composed of a highly connected, isotropic secondary fracture network and a uniformly high-permeability primary fracture, then $d_i = 3$ and $d_s = 2$, yielding a flow dimension $d = d_i - d_s = 1$. The early-time production decline (before the effect of the no-flow boundary a distance D away from the primary fracture is felt) is then linear on a log-log plot, with a slope $-1/2$ (Silin and Kneafsey, 2012). The assumption that $d = 1$ implies that gas flows uniformly through the SRV toward a

planar primary fracture with uniform high permeability, as illustrated schematically in Figure 11b. Gas flow to the primary fracture has linear flow geometry, and this flow is uniform over the entire area of the fracture.

Figure 10. Schematic of idealized hydraulic fracture problem (modified from Silin and Kneafsey, 2012).

However, if one considers a non-uniform primary fracture, with localized regions of high permeability, then $d_s < 2$, yielding $d > 1$. Flow from the SRV converges to the high-

permeability portions of the fracture (Figures 11c and 11d). In the extreme case of just one point
 on the primary fracture providing high permeability, then $d_s = 0$, $d_i - d_s$ yields $d = 3$, and
 spherically symmetric flow from the SRV enters the fracture at that one point. If quasi-linear
 regions of the primary fracture provide high permeability, then $d_s = 1$, $d_i - d_s$ yields $d = 2$, and
 radial flow from the SRV enters the fracture along those lines (Figure 11d). Different patterns of
 localized high permeability in the primary fracture could produce non-integer values d_s , resulting
 in $1 < d < 2$ (Figure 11c). On the other hand, if the primary fracture has uniform, high
 permeability ($d_s = 2$), but flow paths through the SRV are limited due to a sparse or poorly
 connected fracture network ($d_i < 3$), then $d < 1$ is also possible (Figure 11a), with flow diverging
 from the fracture network to the primary fracture. Various combinations of $d_i < 3$ and $d_s < 2$
 could produce diverging, linear, or converging flow. Thus, the goal is to analyze SRVs in which
 the fracture network has a fractal flow dimension ranging from less than one to more than two.
 The essential difference between $d < 1$ and $d > 1$ is that for $d < 1$, there is a diverging geometry
 for the flow from the fracture network to the primary fracture, and for $d > 1$, there is a
 converging geometry.

Figure 11. Schematic diagrams of flow from fracture network (thin lines) to high-permeability
 regions of primary fracture (thick lines), showing (a) diverging geometry for $d < 1$, (b)
 linear geometry for $d = 1$, (c) slightly converging geometry for $1 < d < 2$, and (d)
 strongly converging geometry for $d = 2$. Plots show cross-section perpendicular to
 plane of primary fracture.

A series of numerical simulations was done with TOUGH2 using the equation of state module EOS7C (Oldenburg et al., 2004) to investigate gas production from hydraulically fractured medium, for a range of flow dimensions for the network of fractures making up the SRV and the primary hydraulic fracture. Material properties and numerical parameters used for the model are shown in Table 3. Grids with $d = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 2, 2.5$, and 3 were created to model gas production from the SRV. One end of the one-column grid ($r = 0$) is the planar primary fracture and the opposite end ($r = D$) is a no-flow boundary, to represent the outer limit of the SRV, beyond which permeability is assumed to be negligible. The first grid block represents the primary fracture and has thickness $r_w = 0.5$ m. The length of the column ($D = 150$ m) and the number of grid blocks (300) are the same for each grid.

Unlike the model used in Section 4 for the generalized Theis solution, where the area of the source element varied with flow dimension (Equation 1), here the area of the source is chosen in keeping with the conceptual model illustrated in Figure 11. For $d \leq 1$, the area of the sink (A_w) is equal to the area of the entire primary fracture (A_p), and interface area decreases as r increases, to represent a sparse fracture network, according to Equation (2). In contrast, for $d > 1$, the area at $r = D$ is set to A_p , and area decreases as r decreases, to represent flow converging to the heterogeneous primary fracture, yielding

$$A_w = \left(\frac{r_w^{d-1}}{D^{d-1}} \right) A_p. \quad (9)$$

Figure 12 shows interface area A as a function of r , for the grids generated for nine values of d , illustrating the key feature of A increasing with r when $d > 1$ and decreasing with r when $d < 1$. The constant area for $d = 1$ and the linearly increasing area for $d = 2$ are familiar to TOUGH users of MESHMAKER modules XYZ and RZ2D, respectively

Figure 12. Interface area A scaled by entire primary fracture area A_p as a function of distance r from the primary fracture. For $d < 1$, $A = A_p$ at the primary fracture ($r = r_w = 0.5$ m). For $d > 1$, $A = A_p$ at the outer limit of the SRV ($r = D = 150$ m).

Simulation results (Figure 13) indicate that for $d = 1$, the early-time production rate is linear with a slope of $-1/2$ on a log-log plot, which is consistent with many studies of shale-gas production (e.g., Silin and Kneafsey, 2012; Patzek et al., 2013; Lunati and Lee, 2014). The time when the outer boundary of the SRV is felt can be identified by the production rate dropping below the linear trend. For $d \leq 1$, Figure 13a indicates that early-time production rate is linear with a slope $-\nu$, where $\nu = 1 - d/2$, as for the Barker problem. This should come as no surprise, because for early times, before the outer boundary of the SRV is felt, the SRV can be assumed to be of infinite radial extent. Then the only differences from the Barker problem are that the sink is modeled as constant pressure rather than constant flow rate, and the flowing fluid is a methane rather than water. For $1 < d < 2$ (Figure 13b), the outer boundary of the SRV is felt before the slope $-\nu$ can be observed, but additional simulations with models of large radial extent (black dashed lines in Figure 13) show slopes of $-\nu$ for $d < 2$, and slopes approaching zero for $d > 2$.

Using different relationships between A_w and A_p for $d < 1$ and $d > 1$ does not affect the slopes of the production decline curve, it just translates the curves up and down, to reflect the conceptual model of limited flow within the primary fracture for $d > 1$.

Table 3. Material properties and numerical parameters used for the TOUGH2 simulations of shale gas production from a hydraulic fracture.

Material Properties	
Permeability	0.0126 mD
Porosity	0.12
Compressibility (rock + methane*)	$3 \cdot 10^{-8} + 7.96 \cdot 10^{-6} \text{ (Pa}^{-1}\text{)}$
Methane* viscosity	$2.05 \cdot 10^{-5} \text{ Pa s}$
Grid Specifications	
r_w source extent	0.5 (m)
A_p area of primary hydraulic fracture	31416 m^2
D half-thickness of SRV	150 m
Flow dimensions d considered	0.25 – 3
Assignment of A_w	$A_w = A_p$ for $d \leq 1$; $A_w = (r_w^{d-1}/D^{d-1})A_p$ for $d > 1$
Assignment of D_m and D_n	Equation (5)
Number of grid blocks	300
Grid spacing	0.5 m
Operating Conditions	
Production pressure p_w	175 bars
Initial reservoir pressure p_R	200 bars
Initial temperature	27.85°C

*water is also present at residual saturation (0.15), but methane is the only flowing fluid.

Figure 13. TOUGH2 simulation results showing production rate versus time for hydraulic fracture problem. Symbols show simulation results; colored lines show linear fits to large symbols. Black dashed lines show linear fits to late-time simulation results for models with large radial extent. Q is volumetric production rate at standard conditions, obtained by dividing the mass flow rate TOUGH provides by methane density at $T = 273 \text{ K}$, $P = 1.01 \text{ bars}$, 0.717 kg/m^3 .

Production operations in the field may not be as idealized as the present analysis, especially at early times when the assumptions of constant flowing pressure and single-phase gas flow are not likely to be met, but a linear trend can often be observed in production decline data after the first few months (Figure 14). The linear trends for different wells show a range of slopes $-v$, not just $-\frac{1}{2}$, suggesting that flow from the SRV to the primary fracture can be described by a range of fractal flow dimensions d , where $d = 2(1 - v)$. The transition from linear production decline to more rapid decline that occurs when the outer boundary of the SRV is felt is usually easy to identify. Thus, numerical simulations using fractal-dimension grids can be compared to production data to gain insights into SRV flow dimension, hydrologic properties, and extent, as well as make predictions for future production decline. Compared to most TOUGH models, the one-column fractal grids are extremely simple: they require a small number of grid blocks, have only two connections per grid block, and connection distances and interface areas change gradually from one grid block to the next. Such models run very efficiently, making them well suited for use in inverse methods. However, it must be pointed out that a model with fractal flow dimension is not the only model that yields production decline curves with early-time slopes different from $-\frac{1}{2}$. Other authors (e.g., Olorode et al., 2012; Cinco-Ley and Samaniego, 1981) have hypothesized entirely different, non-fractal geometries for the primary fracture/secondary fracture network components of the SRV and also obtained early-time slopes different from $-\frac{1}{2}$.

Figure 14. Production decline data (symbols) from shale-gas wells (Texas Railroad Commission data). Log-log slopes of $-1/3$, $-1/2$, and $-2/3$ are also shown (lines).

6. CONCLUSIONS

The integral-finite-difference formulation of TOUGH makes it straightforward to create one-column grids with fractal (non-integer) flow dimension. The essential feature of these grids is that interface area between adjacent grid blocks increases in proportion to r^{d-1} , where r is the distance from the origin to the interface and d is the fractal flow dimension. For integer values of d , the method produces the usual linear, radial, or spherical geometry grids. Non-integer-dimension grids can be used to study fluid flow through geologic media with fractal dimension, and may be particularly useful in inverse methods, where pressure transient or production rate data are available, in order to infer the fractal dimension and flow properties of the geologic medium.

ACKNOWLEDGMENT

The insightful reviews of John Barker and an anonymous reviewer are gratefully acknowledged. This work was performed at Lawrence Berkeley National Laboratory of the US department of Energy (DOE) under Contract No. DE-AC02-05CH11231. It was funded by the Research

Partnership to Secure Energy for America (RPSEA) through the Ultra-Deepwater and Unconventional Natural Gas and Other Petroleum Resources program authorized by the US Energy Policy Act of 2005. RPSEA is a non-profit corporation whose mission is to provide a stewardship role in ensuring the focused research, development, and deployment of safe and environmentally responsible technology that can effectively deliver hydrocarbons from domestic resources to the citizens of the US. RPSEA, operating as a consortium of premier US energy research universities, industry, and independent research organizations, manages the program under a contract with DOE's National Energy Technology Laboratory.

REFERENCES

Acuna, J.A. and Y.C. Yortsos, 1995. Application of fractal geometry to the study of networks of fractures and their pressure transient, *Water Resources Research*, 31(3), 527 - 540.

Barker, J.A., 1988. A generalized radial flow model for hydraulic tests in fractured rock, *Water Resources Research*, 24(10), 1796-1804.

Cinco-Ley, H. and F. Samaniego, 1981. Transient pressure analysis for fractured wells, SPE 7490, *Journal of Petroleum Technology*, 33, 1749-1766.

Doughty, C. and K. Karasaki, 2002. Flow and transport in hierarchically fractured rock, *Journal of Hydrology* 263, 1-22.

Emanuel, A.S., G.K. Alameda, R.A. Behrens, and T.A. Hewett, 1987. Reservoir performance prediction methods based on fractal geostatistics, SPE 16971, presented at the 1987 SPE Annual Technical Conference and Exhibition, Dallas, 27-30 September, 1987.

413 Falta, R.W., K. Pruess, S. Finsterle, and A. Battistelli, 1995. T2VOC User's Guide, Rep. 36400,
 414 Lawrence Berkeley National Laboratory, Berkeley, CA.

415 Finsterle, S.A., 1999. ITOUGH2 User's Guide, Rep. LBNL-40040, Lawrence Berkeley National
 416 Laboratory, Berkeley, CA.

417 Hewett, T.A., 1986. Fractal distributions of reservoir heterogeneity and their influence on fluid
 418 transport, SPE 15386, presented at 61st Annual Technical Conference and Exhibition of the SPE,
 419 New Orleans, LA, 5-8 October, 1986.

420 Lunati, I. and S.H. Lee, 2014. A dual-tube model for gas dynamics in fractured nanoporous
 421 shale formations, *Journal of Fluid Mechanics*, 757, 943-971.

422 Mandelbrot, B.B., 1983. *The Fractal Geometry of Nature*, Freeman, San Francisco.

423 Narasimhan, T.N. and P.A. Witherspoon, 1976. An integrated finite difference method for
 424 analyzing fluid flow in porous media, *Water Resources Research*, 12(1), 57 – 64.

425 Neuman, S.P., 1990. Universal scaling of hydraulic conductivities and dispersivities in geologic
 426 media, *Water Resources Research*, 26(8), 1749 – 1758.

427 Oldenburg, C.O., G.J. Moridis, N. Spycher, and K. Pruess, 2004. EOS7C Version 1.0: TOUGH2
 428 module for carbon dioxide or nitrogen in natural gas (methane) reservoirs, Rep. LBNL-56589,
 429 Lawrence Berkeley National Laboratory, Berkeley, CA.

430 Olorode, O.M., C.M. Freeman, G.J. Moridis, and T.A. Blasingame, 2012. High-resolution
 431 numerical modeling of complex and irregular fracture patterns in shale gas and tight gas

432 reservoirs, SPE 152482, presented at SPE Latin American and Caribbean Petroleum Engineering
 433 Conference, Mexico City, Mexico, 16-18 April 2012.

434 Patzek, T.W., F. Male, and M. Marder, 2013. Gas production in the Barnett Shale obeys a
 435 simple scaling theory, *Proceedings, National Academy of Science*, 110(49), 19731-19736.

436 Pruess, K., 2004. The TOUGH codes - A family of simulation tools for multiphase flow and
 437 transport processes in permeable media, *Vadose Zone Journal*, 3(3), 738–746.

438 Pruess, K. and A. Battistelli, 2002. TMVOC, A numerical simulator for three-phase non-
 439 isothermal flows of multicomponent hydrocarbon mixtures in saturated-unsaturated
 440 heterogeneous media, Rep. LBNL-49375, Lawrence Berkeley National Laboratory, Berkeley,
 441 CA.

442 Pruess K., C. Oldenburg, and G. Moridis, 2012. TOUGH2 Users Guide, Ver. 2.0, Rep. LBNL-
 443 43134, Lawrence Berkeley National Laboratory, Berkeley, CA.

444 Sahimi, M. and Y.C. Yortsos, 1990. Applications of fractal geometry to porous media: A
 445 review, SPE 20476, presented at 1990 Annual Fall Meeting of the SPE, New Orleans, LA, 23-26
 446 September, 1990.

447 Silin, D. and T. Kneafsey, 2012. Shale gas: nanometer-scale observations and well modelling,
 448 SPE 149489, *Journal of Canadian Petroleum Technology*, November, 464-475.

449 Theis, C.V., 1935. The relation between the lowering of the piezometric surface and the rate and
 450 duration of discharge of a well using groundwater storage, *Am. Geophys. Union Trans.*, 16, 519-
 451 524.

452 Tyler, S.W. and S.W. Wheatcraft, 1990. Fractal processes in soil water retention, *Water*
453 *Resources Research*, 26(5), 1047 – 1054.

454 Wheatcraft, S.W. and S.W. Tyler, 1988. An explanation of scale-dependent dispersivity in
455 heterogeneous aquifers using concepts of fractal geometry, *Water Resources Research*, 24(4),
456 566 – 578.

457 Xu, T., E. Sonnenthal, N. Spycher, and K. Pruess, 2001. TOUGHREACT User's Guide: A
458 simulation program for nonisothermal multiphase reactive geochemical transport in variably
459 saturated geologic media, Rep. LBNL-55460, Lawrence Berkeley National Laboratory,
460 Berkeley, CA.