The Design of Random Surfaces with Specified Scattering Properties: Surfaces that Suppress Leakage

A. A. Maradudin[a], I. Simonsen[a,b], T. A. Leskova[c], E. R. Méndez[d]
[a]*Department of Physics and Astronomy and Institute for Surface and Interface Science, University of California, Irvine, CA 92697, U.S.A.*
[b]*Department of Physics, The Norwegian University of Science and Technology, N-7491 Trondheim, Norway*
[c]*Institute of Spectroscopy, Russian Academy of Sciences, Troitsk 142092, Russia*
[d]*División de Física Aplicada, Centro de Investigación Científica y de Educación Superior de Ensenada, Apartado Postal 2732, Ensenada, Baja California 22800, México*

We present a method for generating a one-dimensional random metal surface of finite length L that suppresses leakage, i.e. the roughness-induced conversion of a surface plasmon polariton propagating on it into volume electromagnetic waves in the vacuum above the surface. Perturbative and numerical simulation calculations carried out for surfaces generated in this way show that they indeed suppress leakage.

I. INTRODUCTION

The great majority of theoretical studies of scattering from randomly rough surfaces have been devoted to the direct problem, in which the surface profile function and its statistical properties are specified, and it is the angular distribution of the scattered intensity, and its polarization properties, that are sought. Comparatively little attention has been devoted to the inverse problem. In the usual formulation of this problem scattering data are provided by experimentalists, and the surface profile, or some statistical properties of it, are sought. This kind of inverse problem can be called a passive inverse problem, because the theorist has no control over the experimental data provided. In contrast, in the present work we consider a different type of inverse problem, one that we can characterize as an active inverse problem. In this type of problem, the angular distribution of the scattered intensity is specified, and the problem is to design, and ultimately to fabricate, a surface that generates that angular distribution.

This active type of inverse problem has been studied even less than the passive type, yet it arises in a variety of contexts of both a technical and a basic physics nature. In this paper, we present an illustrative example of the design of a random surface with specified scattering properties that is prompted by the observation that as a surface plasmon polariton propagates across a randomly rough metal surface it continuously loses energy through its roughness-induced conversion into volume electromagnetic waves in the vacuum above the random surface, that propagate away from the surface. This leakage, as it is called, interferes with the determination of the Anderson localization length of the surface plasmon polariton by means of numerical simulation calculations, or experimental measurements, of its transmissivity as a function of the length of the random segment of the surface [1-3]. The use of a random surface in such studies that suppresses leakage therefore facilitates the investigation of the strong localization of surface plasmon polaritons by random surface roughness.

In the approach to the suppression of leakage taken by Sornette and his colleagues [1,2], it was assumed that the random surface was not planar on average, but periodic, so that the dispersion curve of the surface plasmon polaritons supported by the mean surface displays a gap at the boundary of the one-dimensional first Brillouin zone defined by the period of the mean surface. Leakage should then either vanish or substantially decrease for the surface plasmon frequency at the band edge. However, this was not observed in the numerical simulation calculations of leakage carried out in Refs. 3 and 4.

In this work we present an approach to designing a one-dimensional random surface that suppresses the leakage of a surface plasmon polariton as it propagates across it that differs from that proposed by Sornette *et al.* [1,2]. Although the power spectrum of the resulting surface is nonzero in a narrow range of wave numbers, that surface is not periodic on average. However, as with the surface proposed by Sornette *et al.*, our surface is specific to the frequency of the surface plasmon polariton propagating across it: if that frequency is changed, a new surface has to be designed.

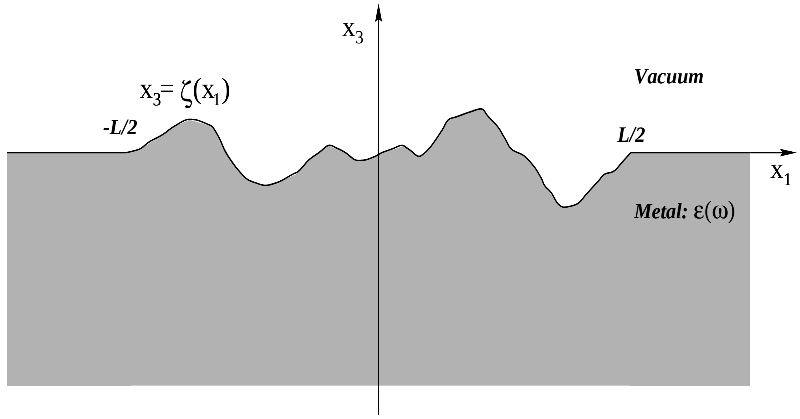

FIG. 1. The system studied in this paper.

II. THE SCATTERED FIELD

We study the scattering of a p-polarized surface plasmon polariton of frequency ω propagating in the x_1-direction that is incident on a segment of a one-dimensional randomly rough surface defined by the equation $x_3 = \zeta(x_1)$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is nonzero only in the interval $-L/2 < x_1 < L/2$ (Fig. 1). The region $x_3 > \zeta(x_1)$ is vacuum; the region $x_3 < \zeta(x_1)$ is a metal characterized by an isotropic, frequency-dependent, complex dielectric function $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$. We are interested in the frequency range in which $\epsilon_1(\omega) < -1, \epsilon_2(\omega) > 0$, within which surface plasmon polaritons exist. We assume that the surface roughness is sufficiently weak that the surface profile function $\zeta(x_1)$ satisfies the conditions for the validity of the Rayleigh hypothesis [5]. In this case the single nonzero component of the magnetic field in the vacuum region $x_3 > \zeta(x_1)_{max}$ can be written as the sum of the fields of the incident and scattered waves

$$H_2^{>}(x_1, x_3|\omega) = \exp[ik(\omega)x_1 - \beta_0(\omega)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R^{>}(q,\omega) \exp[iqx_1 + i\alpha_0(q,\omega)x_3], \tag{2.1}$$

while in the region of the metal , $x_3 < \zeta(x_1)_{min}$,

$$H_2^{<}(x_1, x_3|\omega) = \exp[ik(\omega)x_1 + \beta(\omega)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R^{<}(q,\omega) \exp[iqx_1 - i\alpha(q,\omega)x_3]. \tag{2.2}$$

In Eqs. (2.1)-(2.2) $k(\omega)$ is the surface plasmon polariton wave number,

$$k(\omega) = \frac{\omega}{c} \left[\frac{\epsilon(\omega)}{\epsilon(\omega)+1} \right]^{\frac{1}{2}} = k_1(\omega) + ik_2(\omega), \tag{2.3}$$

with

$$k_1(\omega) = \frac{\omega}{c} \left(\frac{|\epsilon_1(\omega)|}{|\epsilon_1(\omega)| - 1}] \right)^{\frac{1}{2}} > 0 \tag{2.4a}$$

$$k_2(\omega) = \frac{1}{2} \frac{\omega}{c} \left(\frac{|\epsilon_1(\omega)|}{|\epsilon_1(\omega)| - 1} \right)^{\frac{1}{2}} \frac{\epsilon_2(\omega)}{|\epsilon_1(\omega)|(|\epsilon_1(\omega)| - 1)} > 0, \tag{2.4b}$$

while the functions

$$\beta_0(\omega) = \frac{\omega}{c} \left[\frac{-1}{\epsilon(\omega)+1} \right]^{\frac{1}{2}} \tag{2.5a}$$

$$\beta(\omega) = -\epsilon(\omega) \frac{\omega}{c} \left[\frac{-1}{\epsilon(\omega)+1} \right]^{\frac{1}{2}} \tag{2.5b}$$

characterize the exponential decay of the field of the surface plasmon polariton with increasing distance from the interface into the vacuum and the metal, respectively. The functions $R^{>}(q,\omega)$ and $R^{<}(q,\omega)$ are the scattering amplitudes of the surface plasmon polariton in the vacuum and in the metal, respectively, and

$$\alpha_0(q,\omega) = \left(\frac{\omega^2}{c^2} - q^2\right)^{\frac{1}{2}} \qquad Re\alpha_0(q,\omega) > 0, Im\alpha_0(q,\omega) > 0 \tag{2.6a}$$

$$\alpha(q,\omega) = \left(\epsilon(\omega)\frac{\omega^2}{c^2} - q^2\right)^{\frac{1}{2}} \qquad Re\alpha(q,\omega) > 0, Im\alpha(q,\omega) > 0. \tag{2.6b}$$

The scattering amplitude $R^{>}(q,\omega)$ satisfies the reduced Rayleigh equation [6]

$$R^{>}(p,\omega) = -\frac{\epsilon(\omega)-1}{\epsilon(\omega)\alpha_0(p,\omega)+\alpha(p,\omega)}\left\{\frac{J(\alpha(p,\omega)-i\beta_0(\omega)|p-k(\omega))}{\alpha(p,\omega)-i\beta_0(\omega)} \times[pk(\omega)+i\alpha(p,\omega)\beta_0(\omega)] + \int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{J(\alpha(p,\omega)-\alpha_0(q,\omega)|p-q)}{\alpha(p,\omega)-\alpha_0(q,\omega)}[pq+\alpha(p,\omega)\alpha_0(q,\omega)]R^{>}(q,\omega)\right\}, \tag{2.7}$$

where

$$J(\gamma|Q) = \int_{-\infty}^{\infty} dx_1 e^{-iQx_1}(e^{-i\gamma\zeta(x_1)}-1). \tag{2.8}$$

This equation will be solved perturbatively and numerically.

III. THE TOTAL SCATTERED POWER

From Eq. (2.1) we see that the scattered field in the vacuum region can be written in the form

$$H_2^{>}(x_1,x_3|\omega)_{sc} = \int_{-\infty}^{\infty}\frac{dq}{2\pi}R^{>}(q,\omega)e^{iqx_1+i\alpha_0(q,\omega)x_3}. \tag{3.1}$$

The total scattered power, normalized by the total power in the incident surface plasmon polariton, is

$$S(\omega) = \frac{P_{sc}}{P_{inc}}, \tag{3.2}$$

where

$$P_{sc} = L_2\frac{\omega}{16\pi^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta_s \cos^2\theta_s \left|R^{>}\left(\frac{\omega}{c}\sin\theta_s,\omega\right)\right|^2 \tag{3.3a}$$

$$P_{inc} = L_2\frac{c^2}{16\pi\omega}\frac{\epsilon^2(\omega)-1}{(-\epsilon(\omega))^{3/2}}, \tag{3.3b}$$

with L_2 the length of the surface along the x_2-axis. The scattering angle θ_s, measured clockwise from the x_3-axis, is related to the wavenumber q by $q = (\omega/c)\sin\theta_s$. We therefore find that

$$S(\omega) = \frac{1}{\pi}\frac{\omega^2}{c^2}\frac{(-\epsilon(\omega))^{3/2}}{\epsilon^2(\omega)-1}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta_s \cos^2\theta_s \left|R^{>}\left(\frac{\omega}{c}\sin\theta_s,\omega\right)\right|^2. \tag{3.4}$$

Since the integrand in Eq. (3.3a) (and in Eq. (3.4)) is non-negative, we see that the only way in which leakage can be suppressed, i.e. the only way in which P_{sc} can be made to vanish, is to design a one-dimensional random surface for which the amplitude $R^{>}(q,\omega)$ is identically zero for $-(\omega/c) < q < (\omega/c)$. In the next section we introduce one such surface.

IV. THE RANDOM SURFACE

We write the surface profile function $\zeta(x_1)$ in the form

$$\zeta(x_1) = T(x_1)s(x_1), \tag{4.1}$$

where $s(x_1)$ is a single-valued function of x_1 that is differentiable and constitutes a stationary, zero-mean, Gaussian random process defined by

$$\langle s(x_1)\rangle = 0 \tag{4.2a}$$
$$\langle s(x_1)s(x_1')\rangle = \delta^2 W(|x_1 - x_1'|)\rangle \tag{4.2b}$$
$$\langle s^2(x_1)\rangle = \delta^2, \tag{4.2c}$$

where the angle brackets denote an average over the ensemble of realizations of $s(x_1)$. The function $T(x_1)$ serves to restrict the nonzero values of $s(x_1)$ to the interval $-L/2 < x_1 < L/2$. One form $T(x_1)$ can have is

$$T(x_1) = \theta(\frac{L}{2} + x_1)\theta(\frac{L}{2} - x_1), \tag{4.3}$$

where $\theta(x_1)$ is the Heaviside unit step function. A smoother, differentiable version of $T(x_1)$ is provided by

$$T(x_1) = \frac{1 + \cosh\frac{1}{2}\beta L}{\cosh\beta x_1 + \cosh\frac{1}{2}\beta L}, \tag{4.4}$$

where the parameter β controls the range of x_1 values over which $T(x_1)$ decreases from 1 to 0. A value of β given by $100/L$ cuts off $s(x_1)$ smoothly.

The power spectrum of $s(x_1), g(|Q|)$, is defined by

$$g(|Q|) = \int_{-\infty}^{\infty} dx_1 e^{-iQx_1} W(|x_1|). \tag{4.5}$$

A surface that suppresses leakage is defined by the power spectrum (Fig. 2)

$$g(|Q|) = \frac{\pi}{2\Delta k}[\theta(Q - k_{min})\theta(k_{max} - Q) + \theta(-Q - k_{min})\theta(k_{max} + Q)], \tag{4.6}$$

where

$$k_{min} = 2k_1(\omega) - \Delta k \tag{4.7a}$$
$$k_{max} = 2k_1(\omega) + \Delta k, \tag{4.7b}$$

and Δk must satisfy the inequality

$$\Delta k < k_1(\omega) - (\omega/c). \tag{4.8}$$

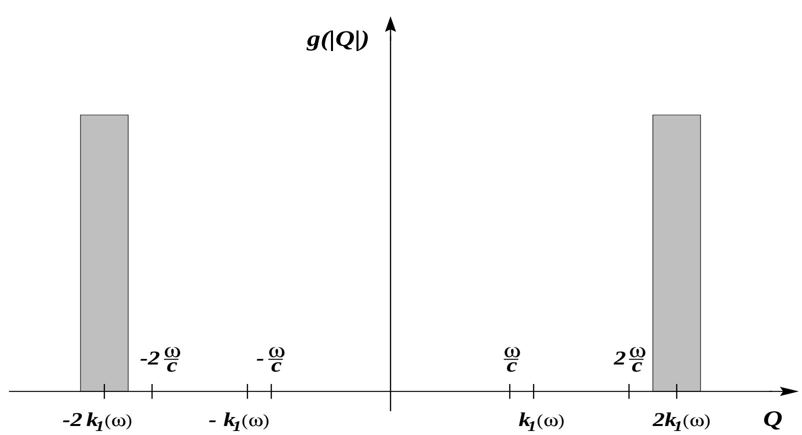

FIG. 2. The power spectrum of the surface roughness assumed in this work

That a surface characterized by the power spectrum (4.6) suppresses leakage can be seen from the following argument. The incident surface plasmon polariton has a wave number whose real part is $k_1(\omega)$. After its first interaction with the surface roughness the real part of its wave number will lie in the two intervals $(3k_1(\omega) - \Delta k, 3k_1(\omega) + \Delta k)$ and $(-k_1(\omega) - \Delta k, -k_1(\omega) + \Delta k)$. This is because the wave numbers in the spectrum of the surface roughness with which $k_1(\omega)$ can combine lie in the intervals $(2k_1(\omega) - \Delta k, 2k_1(\omega) + \Delta k)$ and $(-2k_1(\omega) - \Delta k, -2k_1(\omega) + \Delta k)$. For the same reason, after its second interaction with the surface roughness the real part of the wave number of the surface plasmon polariton will lie in the three intervals $(5k_1(\omega) - 2\Delta k, 5k_1(\omega) + 2\Delta k)$, $(k_1(\omega) - 2\Delta k, k_1(\omega) + 2\Delta k)$, and $(-3k_1(\omega) - 2\Delta k, -3k_1(\omega) + 2\Delta k)$. After three interactions with the surface roughness the real part of its wave number will lie in the four intervals $(7k_1(\omega) - 3\Delta k, 7k_1(\omega) + 3\Delta k)$, $(3k_1(\omega) - 3\Delta k, 3k_1(\omega) + 3\Delta k)$, $(-k_1(\omega) - 3\Delta k, -k_1(\omega) + 3\Delta k)$, and $(-5k_1(\omega) - 3\Delta k, -5k_1(\omega) + 3\Delta k)$, and so on. Thus, for example, if $-k_1(\omega) + 3\Delta k < -(\omega/c)$, so that $\Delta k < \frac{1}{3}(k_1(\omega) - (\omega/c))$, after three scattering processes the surface plasmon polariton will not have been converted into volume electromagnetic waves. In general, if we wish the surface plasmon polariton to scatter n times from the surface roughness without being converted into volume electromagnetic waves, we must require that $\Delta k < \frac{1}{n}(k_1(\omega) - (\omega/c))$.

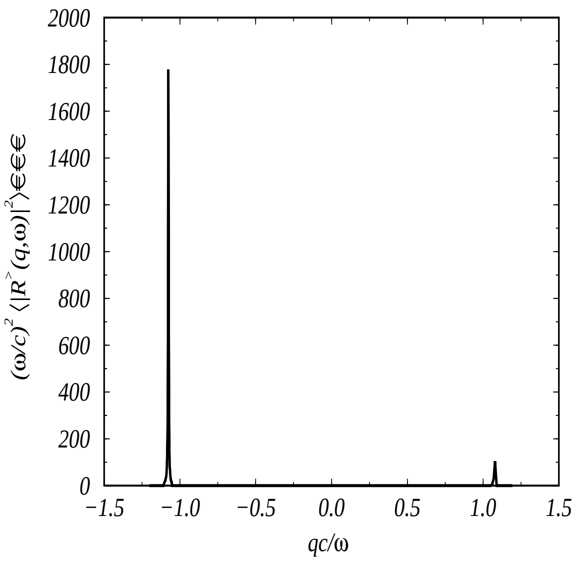

FIG. 3. A perturbative result for $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ as a function of (cq/ω) for a surface plasmon polariton, whose frequency corresponds to a vacuum wavelength $\lambda = 457.9$ nm, propagating across a random silver surface ($\epsilon(\omega) = -7.5 + i0.24$) of length $L = 20\lambda$, defined by the power spectrum (4.6) with $\Delta k = 0.3(k_1(\omega) - (\omega/c))$, $\delta = 10$ nm, and Eq. (4.3).

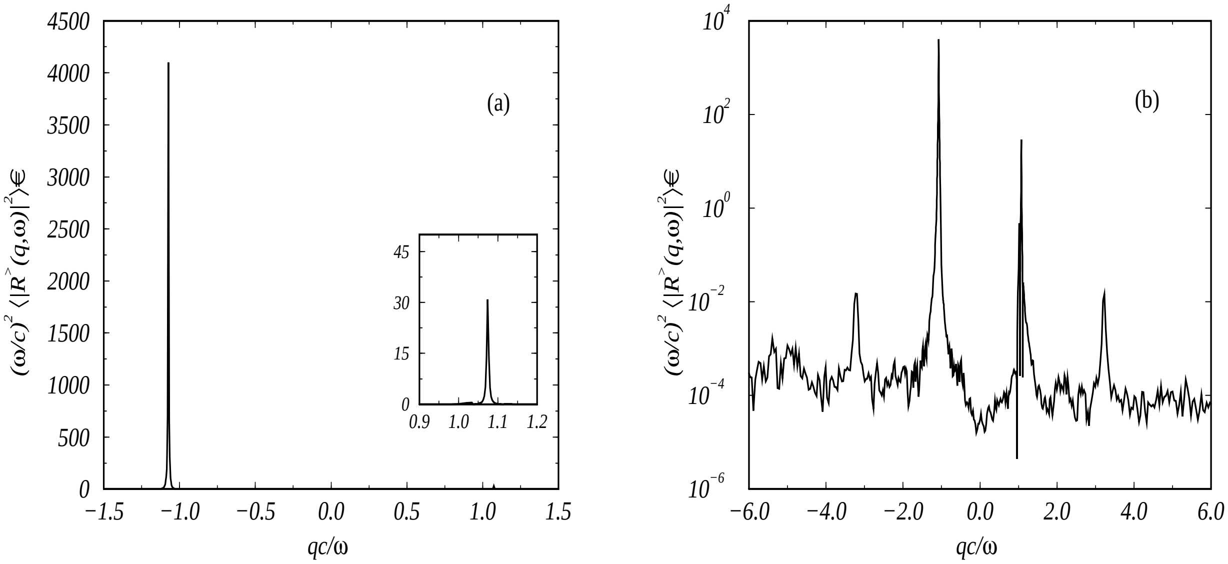

FIG. 4. A computer simulation result for $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ as a function of (cq/ω), plotted on (a) a linear-linear scale and (b) a linear-log scale, for a surface plasmon polariton, whose frequency corresponds to a vacuum wavelength $\lambda = 457.9$ nm, propagating across a random silver surface ($\epsilon(\omega) = -7.5 + i0.24$) of length $L = 20\lambda$, defined by the power spectrum (4.6) with $\Delta k = 0.3(k_1(\omega) - (\omega/c))$, $\delta = 30$ nm, and Eq. (4.4). The results for 50 realizations of the surface profile function were averaged to obtain the results plotted in this figure.

To show that this approach to suppressing leakage works, in Fig. 3 we present a plot of $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ as a function of cq/ω for a silver surface characterized by the power spectrum (4.6) with $\Delta k = 0.3(k_1(\omega) - (\omega/c))$ and $\delta = 10$ nm. The frequency of the surface plasmon polariton corresponds to a vacuum wavelength of $\lambda = 457.9$ nm, and the dielectric function of silver at this frequency is $\epsilon(\omega) = -7.5 + i0.24$. The calculation of $\langle|R^>(q,\omega)|^2\rangle$ was carried out as an expansion in powers of the surface profile function through terms of fourth order. For this $R^>(q,\omega)$ was calculated as an expansion in powers of $\zeta(x_1)$ through terms of third order (there is no zero order term). The expression (4.3) was used for $T(x_1)$, and the length of the random segment of the metal surface was $L = 20\lambda$. The peak in $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ for $q < -(\omega/c)$ arises from the terms in the expansion of $R^>(q,\omega)$ of first and third orders in $\zeta(x_1)$; that for $q > (\omega/c)$ arises from the second-order term. We see from this figure that $\langle|R^>(q,\omega)|^2\rangle$ indeed vanishes for $-(\omega/c) < q < (\omega/c)$. With our choice of $\Delta k = 0.3(k_1(\omega) - (\omega/c))$, a calculation of $R^>(q,\omega)$ to, say, fourth order in $\zeta(x_1)$ would have yielded a small nonzero contribution to $\langle|R^>(q,\omega)|^2\rangle$ in the radiative region $-(\omega/c) < q < (\omega/c)$, but even that could be suppressed by choosing $\Delta k < 0.25(k_1(\omega) - (\omega/c))$.

In Fig. 4 we show that the same result is obtained if we increase the roughness of the surface by increasing δ from 10 nm to 30 nm, keeping the same values for the remaining material and roughness parameters that were used in obtaining the results plotted in Fig. 3. In this case a computer simulation approach based on the numerical solution of Eq. (2.7) was used. The expression (4.4) was used for $T(x_1)$, and the results for 50 realizations of the surface profile function were averaged to obtain the results plotted in this figure. in Fig. 4b four peaks are easily seen. They correspond to the real parts of the wavenumbers of the scattered surface plasmon polaritons resulting from the scattering of an incident surface plasmon polariton of wavenumber $k(\omega) = k_1(\omega) + ik_2(\omega) = (1.0741 + i0.0026)\omega/c$. Earlier in this section we predicted that the real parts of these wavenumbers should be in the vicinity of $\pm k_1(\omega)$, $\pm 3k_1(\omega)$, etc. These predictions fit very well with what can be observed from Fig. 4b, even though the peaks at $\pm 5k_1(\omega)$, and higher, cannot be distinguished from the noisy background due to the low number of samples on which this figure is based. It should be observed that the peak at $k_1(\omega)$ is stronger then the one at $3k_1(\omega)$, even though the latter is of first order (in perturbation theory) while the former is of second order. We believe that this is related to the fact that both the surface plasmon polaritons resulting from a single interaction with the rough surface can be scattered into surface polaritons of (real) wavenumber $k_1(\omega)$, which in this sense gives it a multiplicity of two. The surface polaritons at $3k_1(\omega)$ can, in addition to the single scattering contribution, only be reached by scattering processes of order three or higher, which are all weak for the roughness assumed here.

Thus, the method presented in this work provides an effective way of producing random surfaces that suppress leakage.

V. CONCLUSIONS

In this paper we have presented an approach to generating a one-dimensional random surface that suppresses leakage. We have shown by perturbative and numerical simulation calculations that surfaces generated in this way indeed possess this property.

ACKNOWLEDGMENTS

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