Title

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Abstract

The results reported in this paper reveal that the Black version of the CAPM is not robust with respect to the change in the construction of the Consumer Price Index that occurred in January 1983. Specifically, both the measure of inflation risk and the market price of inflation risk change substantially when a reconstructed version of the new CPI is substituted for the old CPI. This finding raises the question of whether macroeconomic data are sufficiently precise to permit the development of meaningful tests of competing asset pricing models.

1. Introduction

Incorporating inflation into asset pricing models has proved to be difficult, both theoretically and empirically. From a theoretical standpoint, it is generally accepted that inflation is a monetary phenomenon, but it is not clear how to include money in an asset pricing model. As Lucas (1983) notes, asset pricing theory has been developed using barter models. Inflation is then grafted onto these models by assuming a given stochastic process for the numeraire. Examples of this procedure include Breeden (1979,1983), Brock (1982), Friend, Landskroner and Losq (1976), and Long (1974).

Wherther such an electic approach is acceptable is, in part, an empirical question: Do the hybrid theories accurately price financial assets under uncertain inflation. The question is difficult to answer using assets like common stock, for which the variance of the nominal return is an order of magnitude higher than the variance of the inflation rate, so research has focused primarily on short-term Treasury bills. If the maturity of the bill eqauls the observation interval, then the random component of the return is due solely to inflation.

From an empirical standpoint, a critical issue is whether the price index data is "good enough" to test the competing theories. In general, Consumer Price Index, CPI, has been used to measure inflation.[1] Aside from

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[1] Examples include Fama (1975), Fama and Gibbons (1982, 1984), Mishkin (1982), and Wakeman and Bhagat (1984). In a study related to this paper, Huizinga and Mishkin (1984) examine the impact of using different indexes to measure inflation.

the fact that an unambiguous price index cannot be constructed unless all individuals have homothetic preferences, the CPI has numerous practical problems as well. One of these problems, the treatment of residential housing, was considered to be so severe that in January 1983 the Bureau of Labor Statistics began reporting a new CPI in which a "rental equivalent" measure replaced the residential housing component.[2] In light of this change, an obvious questions is whether tests and applications of asset pricing models are robust with respect to changes in the CPI. This paper provides a partial answer to that question by extending the work of Wakeman and Bhagat (1984).

Wakeman and Bhagat employ Black's (1972) two period asset pricing model to estimate the risk premium incorporated in Treasury bill returns. The most important conclusion they draw from their analysis is that:

> Montly returns on U.S. Treasury bills are consistent with the Black model of asset pricing under uncertain inflation in that the real return on the bills is linearly related to the covariance between the real return on bills and the real return on the market portfolio.

The major finding of this study is that the these conclusios is not robust with respect to the way the CPI is constructed. The implications of this fact extend beyond the Wakeman-Bhagat paper. Unlike the original CAPM, the first order conditions derived from modern intertemporal asset pricing

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[2] See Samuelson and Swamy (1974) for a discussion of the theoretical issues involving the construction of price indexes. For more information on the CPI and its revision see, CPI Detailed Report (1983).

models generally involve macroeconomic variables such as real consumption.[3] If tests of these models are sensitive to the construction of the price index, and if data for the "theoretically correct" index is not available, then it is difficult to to determine whether a test rejects a given model because the model is faulty or whether it rejects the model because an incorrect price index was used.

The remainder of the paper is organized as follows. The next section presents a description of the change in the CPI in January 1983 and demonstrates why this change may alter the covariance between the real return on Treasury bills and the real return on the market portfolio. In section three a proxy for the new CPI is computed backwards in order to reproduce the Wakeman-Bhagat study using the alternative index. The final section contains a summary of the conclusions and a brief discussions of their implications.

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[3] See, for example, Breeden (1979) or Brock (1982).

2. The CPI: Old and New

Under the old method of computation, the CPI homeownership component consisted of five subcomponents: the net price of homes purchased, the amount of mortgage interest expected to be paid over one-half the stated life of a home mortgage, property taxes, property insurance, and home maintenance and repairs. As such, it reflected investment elements, largely related to the purchase price and the mortgage interest subcomponents, as well as consumption components. To eliminate these investment components, and to reduce the sensitivity of the index to fluctations in mortgage rates, the Bureau of Labor Statistics switched to a "rental equivalent" method of computing housing costs in January 1983. In the new CPI the homeowner's cost consists of the owner's equivalent rent and household insurance. The owner's equivalent rent is defined to be the income the homeowner foregoes by choosing not to rent his house. As a result of the change in methodology, the overall weight of homeownership in the CPI declined from about 26 percent to 14 percent.

The most important aspect of the change in the CPI, from the standpoint of asset pricing under uncertain inflation, is that the measure of inflation is now much less dependent upon changes in interest rates. A simple example illustrates the potential significance of this alteration. Defining the "true" inflation rate, using the new index, to be I^*, and the inflation rate measured by the old CPI to be I, we can write,

$$I = I^* + f(dr) \ ,$$

where f(dr) represents a monotonic function of the change in the mortgage rate. In addition, define R to be the <u>nominal</u> return on a one-month Treasury bill and M to be the nominal return on the market portfolio. (All rates are continuously compounded.) In their application of the Black model, Wakeman and Bhagat use cov(R-I,M-I) to estimate the risk of holding a Treasury bill, when in fact the correct measure is cov(R-I*,M-I*). Because the covariance operator is linear, it is easy to show the the "error" is given by

$$\text{cov}(R-I^*,M-I^*) - \text{cov}(R-I,M-I) = \text{cov}(f(dr),f(dr) - M - R + 2I^*) \quad (1)$$

There is no reason why this error is necessarily small relative to cov(R-I*,M-I*) or cov(R-I,M-I).

3. Data and Empirical Results

When the Bureau of Labor Statistics introduced the new CPI it did not compute the index backwards. However, the new series was calculated back to January 1954 by Robert Dennis of the Congressional Budget Office. His series, which I refer to as the CPIX, is used in this study.[4] Since the old CPI was replaced in January 1983, the sample period runs from January 1954 to December 1982.

The Treasury bill data are continuously compounded returns observed on the last business day of the month for the bill maturing closed to the end of the next month. This is the same data set employed by Fama and Gibbons (1984) and Wakeman and Bhagat (1984).[5] As in Wakeman and Bhagat the CRSP value weighted index is used as a proxy for the market portfolio.

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[4] I thank John Huizinga for giving me the data.

[3] The data was made available by Michael Gibbons.

It should be noted that there is a timing problem created by the sampling methods used in constructing the price indexes. The problem is that the price components in the indexes are sampled at different times over the course of the month. Thus it is not clear that the inflation rate is measured over the same interval as the security returns. This problem, however, is common to all studies that use monthly data including Fama (1975) and Fama and Gibbons (1982, 1984), as well as Wakeman and Bhagat.

There are two basic ways to construct time series for the covariance. The first is to employ a rolling estimation technique where this period's covariance is estimated by the sample covariance over the prevous n months. This method has the advantage of allowing period-by-period updating, but it has a potential defect. If there is a missing variable in the pricing equation which is highly autocorrelated, then by construction the covariance term can proxy for the unobserved variable. The second approach is to divide the data into non-overlapping blacks and estimate the covariance for each block. Wakeman and Bhagat use both approaches and find that their results are not sensitive to the choice of estimation technique. That conclusion is confirmed by this study, therefore, only the block estimation results are presented in order to conserve space.

Summary statistics for the price indexes, inflation rates and covariance measures are reported in Table 1. As one would expect the levels of the two indexes indexes are nearly perfectly correlated, since both reflect the same long-run trend. If the mean nonstationarity is removed by considering monthly inflation rates, however, the correlation falls from 0.99 to 0.72. More importantly, the correlation between cov(R-I*,M-I*) and cov(R-I,M-I) is <u>-0.11</u>, when 12 month blocks are used to estimate the covariance. Because this result is so surprising, the individual block covariance estimates are also reported in Table 1.

TABLE 1

SUMMARY STATISTICS FOR PERIOD JAN 1954 - JUNE 1971, JAN 1974 - DEC 1982

	Mean	Standard deviation
CPI	142.8	61.6
CPIX	131.8	54.3
I	0.3644	0.3654
I*	0.3334	0.3169
cov(R-I,M-I)	0.1039	0.1877
cov(R-I*,M-I*)	0.0162	0.2128

CORRELATION MATRIX

	CPI	CPIX	I	I*	COV(R-I,M-I)	COV(R-I*,M-I*)
CPI	1.00	0.99	0.57	0.59	0.56	-0.28
CPIX		1.00	0.56	0.62	0.57	-0.29
I			1.00	0.72	0.20	-0.29
I*				1.00	0.22	-0.28
cov(R-I,M-I)					1.00	-0.11
cov(R-I*,M-I*)						1.00

BLOCK COVARIANCE ESTIMATES

Date	cov(R-I,M-I)	cov(R-I*,M-I*)
1954	-0.0566	0.2224
1955	-0.0107	0.1028
1956	-0.2492	0.2745
1957	-0.3249	-0.0332
1958	0.0165	0.2287
1959	0.0870	0.1842
1960	-0.0515	-0.1512
1961	0.0921	0.0048
1962	0.1880	-0.2245
1963	0.1980	-0.1763
1964	-0.0346	0.0454
1965	0.2745	0.1433
1966	0.1495	0.1871
1967	0.1989	0.1354
1968	0.1044	-0.0033
1969	0.0466	0.0373
1970	0.1234	0.0992
1971	-0.0546	-0.0656
1972	0.0569	-0.0345
1973	0.8980	-0.2715
1974	0.0176	-0.1639
1975	0.4500	-0.1964
1976	0.3394	0.1760
1977	0.0056	0.3456
1978	-0.0002	-0.2531
1979	0.0723	-0.2454
1980	0.2646	-0.4145
1981	0.4789	0.4327
1982	0.4016	-0.2958

The findings reported in Table 1 reveal that although the inflation rate is not particularly sensitive to the construction of the CPI, the same is not true for the covariance between the inflation rate and the real return on the market portfolio.[6] Apparently the mortgage interest and homeownership components of the original CPI are important determinants of the covariance.

Correlation results alone are insufficient to determine the effect of the change in the index on the estimation of the Black model. Assuming that the real rate of return on a riskless portfolio (riskless in real terms) is constant, Wakeman and Bhagat show that applying Black's model to the Fisher relation and moving the inflation rate to the left-hand side as suggested by Fama yields the equation,

$$I_t = a + bR_t + c(\mathrm{cov}(R-I, M-I)_t) + u_t \; . \qquad (2)$$

The sensitivity of the model to the measure of inflation can be gauged by estimating equation (2) using each price index.

The regression results using the block covariance estimates are reported in Table 2 for the CPI and Table 3 for the CPIX. To facilitate comparison with Wakeman and Bhagat, the full period is divided into several subperiods which correspond, as closely as possible, with their sample periods.

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[6] There are brief intervals, however, during which the two inflation rates were different. In the late 1970's, for example, sharply rising interest rates and housing prices pushed the CPI up relative to the CPIX. During this period the ex-post real rate calculated using the CPI was generally negative, but the real rate calculated using the CPIX remained positive in most cases. See Huizinga and Mishkin (1984) for a further discussion.

TABLE 2

Estimation Results for the CPI Inflation Measure[a]

$$I_t = a + bR_t + c(cov(R-I,M-I)_t)$$

I. January 1954 - June 1971, January 1974 - December 1981: 318 Observations

a	b	c	R^2	p_1	p_2	p_3
-.016	.929	----	.472	.40	.32	.26
(-0.62)	(17.61)			(.06)	(.06)	(.06)
-.030	1.044	-.382	.516	.38	.27	.21
(-1.15)	(17.60)	(-4.34)		(.06)	(.06)	(.06)

II. January 1954 - June 1971: 210 Observations

a	b	c	R^2	p_1	p_2	p_3
-.063	.964	----	.298	.09	.14	-.03
(-2.07)	(9.42)			(.07)	(.07)	(.07)
-.077	1.052	-.235	.321	.06	.10	-.08
(-2.52)	(9.87)	(-2.60)		(.07)	(.07)	(.07)

III. January 1976 - December 1981: 72 Observations

a	b	c	R^2	p_1	p_2	p_3
.295	.595	----	.301	.57	.36	.21
(3.45)	(5.50)			(.12)	(.12)	(.12)
.299	.786	-.736	.443	.45	.24	.11
(3.88)	(7.30)	(-4.19)		(.12)	(.12)	(.12)

IV. January 1974 - December 1982: 108 Observations

a	b	c	R^2	p_1	p_2	p_3
.343	.499	----	.140	.60	.32	.30
(3.80)	(4.15)			(.10)	(.10)	(.10)
.430	.636	-.816	.316	.47	.16	.15
(5.22)	(5.74)	(-5.20)		(.10)	(.10)	(.10)

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[a] The number in parentheses below the coefficient estimates are t-statistics, the numbers below the autocorrelations coefficients are standard errors.

TABLE 3

Estimation Results for the CPIX Inflation Measure

$$I^*_t = a + bR_t + c(cov(R-I^*,M-I^*)_t)$$

I. January 1954 - June 1971, January 1974 - December 1981: 318 Observations

a	b	c	R^2	p_1	p_2	p_3
.011 (0.49)	.776 (16.59)	----	.466	.37 (.06)	.30 (.06)	.18 (.06)
.029 (1.21)	.740 (15.44)	-.175 (-2.79)	.479	.39 (.06)	.32 (.06)	.20 (.06)

II. January 1954 - June 1971: 210 Observations

a	b	c	R^2	p_1	p_2	p_3
-.035 (-1.20)	.796 (8.18)	----	.243	.02 (.07)	.20 (.07)	-.08 (.07)
-.038 (-1.25)	.799 (8.16)	.030 (0.34)	.244	.02 (.07)	.20 (.07)	-.09 (.07)

III. January 1976 - December 1981: 72 Observations

a	b	c	R^2	p_1	p_2	p_3
.405 (5.70)	.352 (3.91)	----	.179	.57 (.12)	.24 (.12)	.02 (.12)
.441 (6.85)	.306 (3.74)	-.313 (-4.23)	.348	.40 (.12)	.01 (.12)	-.24 (.12)

IV. January 1974 - December 1982: 108 Observations

a	b	c	R^2	p_1	p_2	p_3
.427 (6.09)	.295 (3.16)	----	.086	.57 (.10)	.25 (.10)	.21 (.10)
.425 (6.16)	.280 (3.03)	-.189 (-2.19)	.126	.53 (.10)	.19 (.10)	.15 (.10)

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[a] The number in parentheses below the coefficient estimates are t-statistics, the numbers below the autocorrelations coefficients are standard errors.

Looking first at the January 1954 to June 1971 period, the CPI results are very similar to those reported by Wakeman and Bhagat for the period January 1953 to June 1971. The coefficient of the covariance term is negative, the coefficient of the nominal interest rate is almost identically 1.0, and the residuals are nearly white noise. The results for the period January 1976 to December 1981 are also close, but not identical, to those reported by Wakeman and Bhagat. As they note the model deteriorates during this period in the sense that the interest rate coefficient is significantly less than one and the residuals are highly autocorrelated. The autocorrelation of the residuals indicates the model is misspecified during the period. This finding is consistent with results reported by Huizinga and Mishkin (1984) which indicate that the variance of the real rate increased significantly in the late 1970's, so that the assumption of a constant real rate on a riskless portfolio is no longer viable. Adding the covariance term does increase the interest rate coefficient and reduce the autocorrelation of the residuals, but the risk adjusted model is still inconsistent with the standard interpretation of the Fisher equation. The picture remains unchanged when the sample period is extended to January 1974 to December 1982. Finally, the results for the full sample, excluding the period of price controls, conform largely to the story told by Wakeman and Bhagat. When a covariance term is added its coefficient is significantly negative and the nominal interest rate coefficient moves toward one.

The results for the CPIX are noticeably different. For the January 1954 to June 1971 period, the covariance coefficient is <u>positive</u>, though insignificant. In the post 1974 samples, the covariance term has a negative coefficient, but its addition does not "improve" the results in the sense of pushing the nominal interest rate coefficient toward one or reducing the residual autocorrelation. For example, the coefficient on the nominal interest rate falls from .295 to .280 when the covariance term is added during the period January 1974 to December 1982. In addition, the residuals remain highly autocorrelated indicating the misspecification of the model in this period is not due to the use of the CPI rather than the CPIX. For the full period, the covariance coefficient is signficantly negative, but its value is less than half that reported in the comparable CPI regression. Furthermore, the interest rate coefficient is signficantly less than one in all the CPIX regressions.

Overall, the results clearly show that the Black model is not robust with respect to the choice of price index when pricing Treasury bills under uncertain inflation. Both the measure of inflation risk and the market price of inflation risk change substantially when the CPIX is substituted for the CPI. If we know that the CPIX is the correct inflation measure, then this finding can be dismissed as little more than further evidence that the old CPI is misleading. Unfortunately, the CPIX is not the theoretically correct measure either. Whether the differences between the CPIX and the correct measure are important remains an empirical question, but until it is answered it will be difficult to test competing theories of asset pricing under uncertain inflation.

4. Concluding Comments

Though asset pricing models typically employ strong assumptions such as homogenous beliefs and perfect markets, these assumptions are rationalized on the grounds that tests should be based on a model's predictions, not its assumptions. This assumes, however, the data are good enough to distinguish between competing theories. If minor revisions in the data lead to different conclusions, then any tests will be inconclusive. For instance, Roll (1977) shows that tests of the original CAPM are not robust with respect to the choice of a proxy for the market portfolio. The data problem becomes even more troublesome when analyzing more recent intertemporal models, because macroeconomic data are required. This paper presents a simple test designed to determine the Black CAPM is robust with respect to the change in the CPI that occurred in January 1983. It turns out that both the measure of inflation risk and the market price of inflation risk are sensitive to the construction of the CPI.

In conclusion, the paper may best be interpreted as a warning. A great deal of intellectual effort has gone into to the development of asset pricing models. Nearly equal effort has been expended analyzing the econometric problems connected with testing the theories. Unfortunately, much less time has been spent examining whether the macroeconomic data is sufficiently precise to let us choose between existing theories, yet alone new models that might be developed. The results presented here indicate that this final problem is not insignficant.

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