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Title
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Permalink
https://escholarship.org/uc/item/05h1500f

Journal
Journal of Archaeological Science, 38(4)

ISSN
0305-4403

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Publication Date
2011-04-01

Peer reviewed

Emergence of Fusion/Fission Cycling and Self-Organized Criticality from a Simulation Model of Early Complex Polities

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Abstract

A spatial-temporal model of early complex polities is described in which cycles of consolidation and collapse emerged during simulations. Self-organized criticality (SOC) also was clearly observed. SOC is characteristic of simulations for iterative physical phenomena such as earthquakes and forest fires. Social scientists are interested in SOC as a theoretical framework to understand cyclical human cultural processes. In particular there has been considerable speculation that SOC underlies polity cycling. The current model is an attempt to move beyond speculation by demonstrating that: 1) the model unequivocally exhibits SOC, 2) there is a self-evident correspondence between the model's structure and actual polities as indicated in the archaeological record, 3) hierarchical settlement patterns emerge during simulations, and 4) simulated population distributions are consistent with empirical rank-size survey data typical of early complex polities.

Key words

1.0 Introduction

Cycles of consolidation and collapse, expansion and decline, rise and fall, growth and disintegration. Variants of these phrases can be found throughout the archaeological and historical literature to describe the fusion and subsequent fission of dynamic political entities (polities). When viewed at the appropriate macro-level, this iterative pattern has persisted over widely varying times, places, and scales: small polities fused into larger and more complex ones that over time eventually split into a number of simpler political aggregations. Subsequently they recombined in different configurations to begin another similar cycle.

Collapse has often been the focus when the life spans of complex polities were examined (Tainter 1988). However the argument has been made that the decline of all polities, regardless of size and complexity, could best be understood as the end of one cycle and the beginning of another (Chase-Dunn 2007). While embracing the spirit of that notion, the more modest objective here was to model a specific class of political entities, referring to them collectively as *early complex polities*.

1.1 Early Complex Polity Defined

Early complex polities can be characterized by their positions on a continuum of size and complexity. At the upper limit were archaic states: agrarian polities with nascent bureaucracies ruling from a primate center settlement through a multi-level hierarchy of secondary centers. Joyce Marcus's (1992) *dynamic model* originally described the cycling process of Maya archaic states, AD 400 to 1500. One of the largest was in the Tikal region which experienced cycles with 200 to 300 year periods. At the apogees of these cycles a single polity ruled on the order of 10,000 sq km. The dynamic model also has been applied to archaic states in five other world areas from the Andean highlands to Mesopotamia (Marcus 1998).

At the small and simple end of the size-complexity continuum considered here were typical Mississippian chiefdoms in the American Southeast during the period AD 1100 – 1550. These polities were composed of two to ten settlements, were no more than 40 km in breath, and had lifetimes not exceeding one or two human generations (Anderson 1996, Blitz 1999).

Nation states fall outside the defined size-complexity range. Consistent with this exclusion, section 2.0 presents robust emergent patterns that are indicative of early complex polities but are not typical for nation states. Extending the model to account for such differential behavior is touched on in 6.2.

1.2 Observing Self-Organized Criticality

There has been considerable speculation that polity cycling is a manifestation of self-organized criticality (SOC). These conjectures span multiple disciplines including: political science (Brunk 2002, Cederman 2003), geography (Coombes 2005), and archaeology (Brown 2003 p 1629). Processes exhibiting SOC were originally modeled by physicists (Bak 1988) to understand similarities observed in diverse physical phenomena such as earthquakes, landslides, and forest fires (Turcotte 1999). Social scientists became interested in SOC because of its promise as a

unifying theoretical framework to investigate a variety of social and economic processes characterized by cyclical collapse and recovery.

SOC is the quasi-steady state of a system in which the impact of small but regularly occurring inputs accumulate until there is a sudden collapse. Occurring at random intervals and with varying intensities each collapse is followed by new cycles of growth and decline. Self-organized refers to the ability of the system to repeatedly find its way into this state regardless of initial conditions (Turcotte 1999, Bak 1988). Criticality alludes to the system's persistent location in phase space on the cusp between stability and chaos (Brown 2003).

Published descriptions of SOC often use as an example the simplest form of the forest fire model (FFM) because it is very intuitive. The FFM is a cellular-automata (a special case of spatial agent-based models) that simulates wild fires and subsequent re-growth of trees (Henley 1993, Trucotte 1999). **Fig. 1**, left, shows a screen display from the FFM which was implemented in the NetLogo 4.0 modeling environment (Wilensky 2009). All the modeling and simulations reported here were developed and executed in NetLogo. (Model software is available from the author upon request).
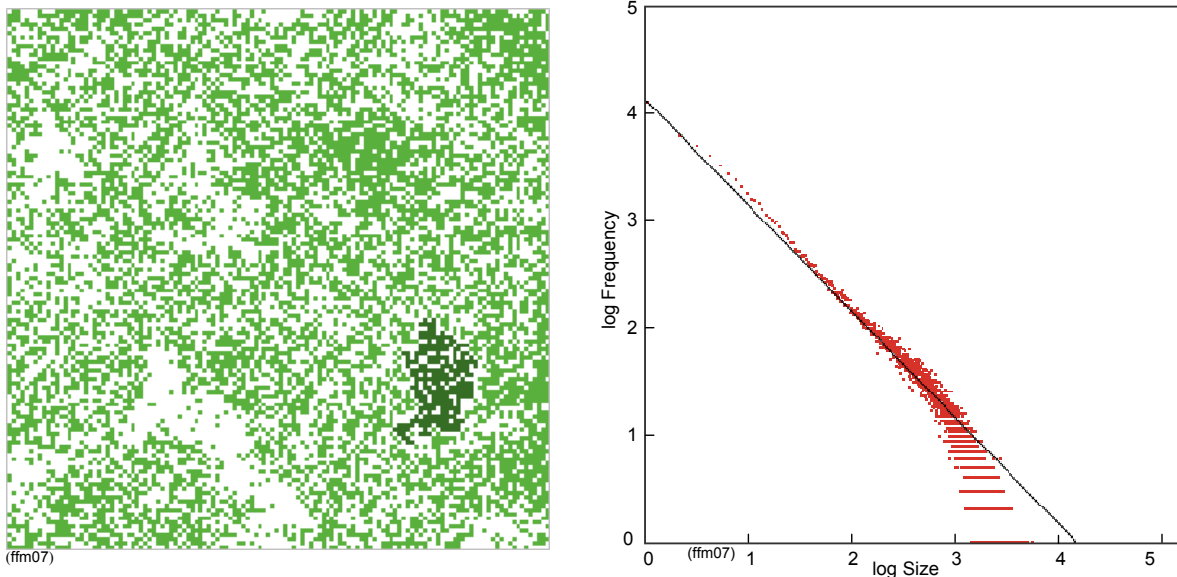
Fig. 1 (left) Snapshot of 128 x 128 grid display during single time step of forest fire model simulation. Healthy trees interspersed with empty areas (white). Cluster of burned trees in darker color. **(right)** Frequency-size distribution of simulated forest fires after 40 million time steps (128 x 128 grid, 150 step spark interval) overlaid by power law with exponent of -1

Referring to **Fig. 1,** left, a forest is represented by a grid of regularly spaced cells, 128 x 128 in the case shown. Each filled cell symbolizes a tree and the interspersed clumps of empty cells correspond to cleared areas within the forest. During a simulation step, one cell is selected at random and if empty, a new tree sprouts. Every T_S time steps ($T_S \gg 1$) a spark is randomly dropped somewhere in the forest grid. Nothing happens if the spark lands in an empty cell. If it lands on a tree, that tree is set ablaze and in the same step the fire spreads to all trees in the cluster of cells connected by the four immediate neighbors (left, right, up, down) shown as the darker color in **Fig. 1**. Just prior to the next step all cells in the burned cluster become empty spaces in which new trees can grow during subsequent steps.

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An important signature of SOC is illustrated on the right side of **Fig. 1** It is a log-log plot for the frequency of simulated fires f (number of occurrences) of a given size S (number of grid cells burned) after 40 million time steps of the same run as the grid display to the left. The plotted data was overlaid by a straight line that follows the power law function $f = f_0 S^a$, where f_0 is the observed frequency of the smallest fire (size of 1 for the simulation grid) and the exponent a = -1. Simulation results closely matched this power law over three orders of magnitude. (The deviation from the power law for large size fires is an artifact of the grid's finite dimensions. As grid dimensions increase so does the largest possible cluster of trees; see Trucotte (1999) for further discussion.)

Actual forest fires appear to have similar frequency-size distributions. For example, data was reported for 120 fires in the western United States during AD 1155–1960 based on tree ring growth. A log-log plot of frequency vs. size was well fitted by a straight line with slope of -1.3 over three orders of magnitude (Turcotte 1999, p 1403).

As with forest fires, empirical data for any naturally occurring phenomenon certainly lends credibility to hypotheses that SOC is present. However, comparable distributions have been observed in frequency-size data for any number of non-SOC processes (Solow 2005). A -1 power law fitted to either empirical data or simulation results should be viewed as a *necessary but not sufficient* condition. Alone, it cannot be taken as proof of the existence of SOC.

Despite its promise as a unifying framework, speculations that polity cycling is an SOC process have been unsatisfying to many archaeologists. That is because SOC describes the state of a system rather than a causal explanation of how this macro-level pattern could have emerged in actual polities (Bentley and Maschner 2008). What have been missing from these conjectures are clearly recognized correlates between the elements of an SOC model and micro-level constituents of actual polities. In contrast, the analogs of the FFM to forest fires in nature are self-evident: *Micro-level elements*: forests are composed of trees interspersed by empty space; sparks can occur anywhere in the forest. *Local interactions*: fire spreads to trees that are close by but does not easily jump empty space. Fires destroy trees and create empty space in which new trees will grow afterward. Stating the obvious makes the point of how clear the correspondence is between model and reality.

1.3 Objectives

Of the published cases that posit the operation of SOC in real-world processes, the most compelling are those presenting a constellation of three elements as exemplified by the FFM:

1. A model whose micro-level structure and local agent interactions clearly correspond to elements and behavior of the real-world system in question.

2. A model that meets the input/output requirements of an SOC system, and simulations in which the frequency-size distribution matches a power law with exponent of -1.

3. Empirical data from the real-world system being modeled that exhibit macro-level size distributions comparable to those emerging during simulations.

The objective of the work reported here was to develop a model of early complex polities that met all three of these criteria.

2.0 Fission/Fusion Model

It is proposed here that elements of the forest fire model (FFM) can be redefined as a fusion/fission model (also abbreviated FFM) of early complex polities. The result is a simple stylized model intended to capture the structural and dynamical essence of these political configurations. The following baseline assumptions have much in common with those at the core of the author's earlier polity model (Griffin and Stanish 2007, 2008).

Complex early polities were assumed to expand in size over time to accommodate population growth. Polities also expanded due to fusion with other polities. Fusion occurred when adjacent polities came into conflict when no empty land separating them remained for expansion. The net result was consolidation, even though many means to that end were likely employed by actual polities: overt conquest or intimidation, forming alliances, religious legitimization, rewarding loyalty, marriage, etc. Internally there was competition between factions within each polity. A faction could resist the ruler, which, if successful, caused the entire polity to collapse. (An alternative to the all-or-nothing model of collapse is considered in the Conclusions section.)

(ffm08)

Fig. 2: Grid display of 128x128 fusion/fission model showing polities represented by clusters of same colored cells.

Referring to **Fig. 2**, cell clusters now represented multi-settlement polities. To visually distinguish one settlement cluster from its neighbor each was randomly assigned a different color. The empty cells correspond to uncontrolled territory separating the polities. Initially all polities on the grid consisted of single cells representing isolated settlements. When simulations were run, multi-

settlement polities began to form as more settlements sprang up. When adjacent polities expanded to the point of touching they fused into larger consolidated polities. Any of these polities could collapse at any time leaving behind uncontrolled territory in which new settlements subsequently sprang up to begin another cycle.

Although a multitude of circumstances could have set the stage, the proximate cause of fission was assumed to be competition over leadership between internal factions (Stanish 2005). It also seemed reasonable to expect that the larger a polity the greater the number of internal factions and hence the more likely resistance would occur. This relationship was modeled earlier by assuming that the probability of resistance for any one settlement was constant, so the likelihood of resistance somewhere in a polity increased as the number of its settlements grew. The same effect was achieved in the current model by spatially uniform random occurrences of resistance, equivalent to the fire-starting sparks.

Assigning different colors to cell clusters was the only difference in the simulation software between the grid displays in **Fig. 1** and **2**. Thus all of the SOC characteristics of the forest fire model remained identical for the fission/fusion model, including its frequency-size distribution that matched a power law with -1 exponent.

Fig. 2 also illustrates two emergent patterns that are consistent with early complex polities while neither is typical of nation states. First, there are buffer zones of uncontrolled areas separating all polities. Second, the territories controlled by the simulated polities in the figure are notably noncontiguous. Early complex polities could be dendritic in shape as well as surround islands of uncontrolled territory. An example is Tiwanaku, one of the first pristine archaic states in the new world. Located in the Lake Titicaca basin that straddles the border of modern Peru and Bolivia, Tiwanaku was at its peak around AD 900. Its heartland was connected to distant settlements throughout the basin by ribbons of territory adjacent to the main trade routes (Stanish 2003). Similarly the areas controlled by the Maya polity Tikal were noncontiguous (Marcus 1998 p59) as were those of Tenochtitlan, capital of the Aztecs (Smith 2002 p 174).

3.0 Population Model

The baseline polity model described above was extended to account for changes in settlement populations due to fission/fusion and the simultaneous feedback on fission/fusion resulting from shifting population patterns. This was done in a way that assured preservation of the SOC exhibited before additions. The population for each grid cell was updated every time step during simulations as a function of: 1) births/deaths, and migration; and 2) the political allocation of resources, described below in 3.1 and 3.2 respectively.

Fig. 3 illustrates how population dynamics were visualized on the grid display by a color scale, where a higher level was indicated by a darker color. In the example shown levels were visible in cells between polities, and populations of settlements within them could be viewed by hiding the colors identifying distinct polities. The locations of polity centers were marked by white dots in the figure, important for determining relative political strength discussed in 3.2.

(ffm15)
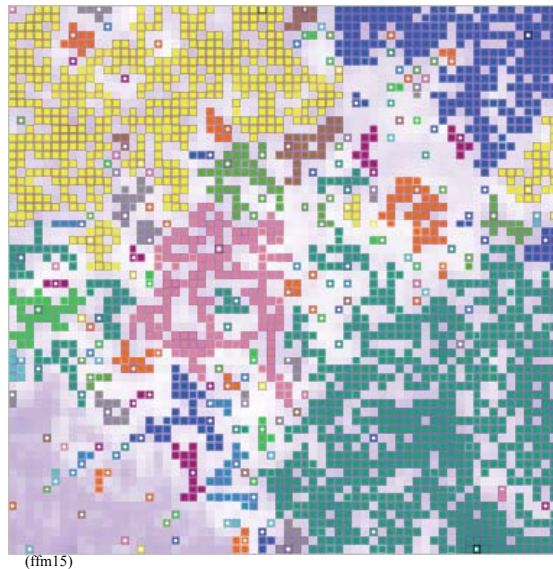
Fig. 3: Each polity composed of settlements (square grid cells) with same color. Polities overlay population color scale where higher density shown as darker color. White dots indicate polity centers (64 x 64 grid).

The settlement population variable was added for two reasons. The first was to achieve a more realistic correspondence to early complex polities as inferred from the archaeological record. This was reflected in basic modeling assumptions concerning population growth and resource allocation. **Table 1** summarizes these assumptions along with those for the baseline model as argued in section 2.0 above. This section presents the model structure and dynamics that manifest these underlying assumptions. (See the Appendix for discussion of an interesting, albeit not as apt, alternative for modeling population dynamics.)

Table 1: Modeling assumptions define micro-level correlates to early complex polities

Population Growth and Migration
In the absence of food shortages and migration, population increased in proportion to its then current level.
People migrated away from economic need over the shortest possible distance without regard to polity boundaries.

Fusion
Single settlement polities continually formed in uncontrolled areas.
Competition occurred when polities came into contact while expanding into uncontrolled areas separating them.
Competition between polities was ultimately resolved by consolidation, although the means were historically contingent.
When polities consolidated, the strongest center settlement remained primary and the weaker ones became subordinated secondary centers.

Fission
There was competition between factions within each multi-settlement polity.
A faction could resist its polity's center. The circumstances leading to and the means of resistance were historically contingent.
Successful resistance ultimately caused the entire polity to fission leaving uncontrolled areas.

Center's Strength
A polity center's strength was its overall potential to influence the population and other centers, be it the ability to: persuade, befriend, reward, organize, protect, intimidate, attack, defend, etc., or any combination thereof.

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Each center attempted to maximize its strength, which was:
    directly related to the population size and resources of its own polity, and
    diminished as the distance increased to the target of influence.

Resource Allocation
Polity leader organized labor to achieve economies of scale which increased productivity.
Polity leader suppressed overt conflict between settlements increasing safety which also raised productivity.
Resources produced by agricultural settlements were extracted by ruling elites to finance their leadership. Producers submitted to extraction in exchange for increased productivity and safety.
Amount extracted from a settlement close to the polity center was greater than from a more distant settlement.
Secondary centers subordinated by consolidation continued to extract resources from local settlements as before but in turn paid tribute to the primary center.

The second motivation for the extension was to allow use of readily available empirical settlement rank-size data for model validation. Variable settlement size was not part of the baseline FFM since settlements were represented by grid cells with constant uniform areas. (Even if the model included settlement area, it would only provide a proxy of population.) Rank-size distributions of the added population variable could then be compared to distributions of empirical size data described in Section 4.0.

3.1 Resource Limited Growth and Migration

Two fundamental observations underlie the population model: 1) there are only two ways settlement population (N) could change: 1) births/deaths and migration; and 2) population levels adjusted over time to carrying capacity (K). The model to account for both these dynamics was based on the Fisher-Skellam equation. This formulation has been widely used in theoretical population biology and other applications including analysis of human population changes during the Neolithic transition in Eurasia (Hazelwood 2004, Davidson 2006). In its basic form, the change in population over time is:

$$\partial N/\partial t = r_P N\,(1 - N/K) + c_F \nabla^2 N \qquad\qquad (1)$$

    N = population density
    r_P = intrinsic population growth rate (model parameter)
    K = carrying capacity
    c_F = diffusion constant (model parameter)
    ∇^2 = Laplacian operator

The second term on the right side of the equation accounts for spatial migration of N as a diffusion process proportional to constant c_F. The first term on the right models net births and deaths as logistic growth. Growth in population over time, $\partial N/\partial t$, is proportional to its current level N. The growth rate is initially high, close to the intrinsic rate of r_P, when population density N is low relative to the carrying capacity K, but decreases over time as N approaches the upper limit set by K. In the current model, K is a variable dependent on the political organization of the polity, which is detailed in the next subsection.

Difference equations implemented in the model software were a discrete version of the continuous differential above. Values of constants in the equations could be set during simulations as model parameters summarized in **Table 2**.

Table 2: Model Parameters

Parameter	Sweep**	Definition
girdSize	64, 128	One dimension of square grid (grid cell units)
radius	3 - 10	Radius of area surrounding center settlement within which localized rank-size was plotted (grid cell units)
T_S	50 - 300	Interval between occurrences of satellite settlement resistance to center's leadership (ticks*)
r_P	0, 0.0003 – 0.05	Intrinsic population growth rate (per tick*)
c_F	0, 0.003 – 0.3	Population diffusion constant (per tick*)
s_K	0, 1- 40	Factor by which satellite settlement's carrying capacity increased over uncontrolled area due to greater safety and scale efficiencies
x_K	0, 0.03 - 0.6	Portion of satellite settlement's carrying capacity extracted by center (prior to distance attenuation)
r_D	0, 0.02 – 0.2	Compounded rate at which center's influence on satellite is attenuated for each unit of grid distance between them

* tick = simulation time step
** Sweep= range of values during sweeps of parameter space

Population of each patch was updated each successive time increment by applying (2), (3), and (4) below. Expression (3) was added to prevent N from becoming negative when there was a sudden drop in K at the center of a polity when it collapsed, see the next subsection.

$$\Delta N = r_P N (1 - N/K) \quad \text{when } N < K \tag{2}$$

$$\Delta N = r_P N (K/N - 1) \quad \text{when } N >= K \tag{3}$$

$$N(t) = N(t-1) + \Delta N \tag{4}$$

In terms of the xy grid of patches, diffusion consists of a simultaneous two-way flow of population between neighboring patches. To be consistent with the fission/fusion model a neighborhood of the four nearest patches was assumed: left, right, up, and down. For the patch at grid location (x, y), c_F portion of its population N_{xy} flows equally to each neighbor during each time step, so that the incremental population change, ΔN_{xy}, and that of its neighbors, ΔN_{nbr} are given by

$$\Delta N_{xy} = - c_F N_{xy} , \text{ where } c_F = \text{diffusion constant (model parameter)} \quad (5)$$

$$\Delta N_{nbr} = - \Delta N_{xy} / 4, \text{ where nbr} = (x, y+1), (x, y-1), (x+1, y), (x-1, y) \quad (6)$$

At the same time there is a similar reverse flow into patch xy from its neighbors. The net effect is that population always moves away from areas of high density and towards a lower density. That is the appropriate behavior when N > K, in particular at the location of a former center after its

polity collapsed. However the model also must allow population to first concentrate in growing centers where N < K. Thus the diffusion term was made conditional so patch xy would forgo the out flow if it had a surplus carrying capacity i.e.,

$$\Delta N_{xy} = 0 \qquad \text{if } N_{xy} <= K_{xy} \qquad\qquad (7)$$

Rule (7) was recommended by its simplicity after experimenting with a more rigorous alternative: active infusion in combination with diffusion to implement pull as well as push forces found in traditional models of migration (Dorigo 1983). However the marginal impact on population flow did not warrant the added model complexity.

After (5), (6), and (7) were applied to each patch in the xy grid during a time increment, its population was updated by

$$N_{xy} = N_{xy} + \Sigma \Delta N_{xy} \qquad\qquad (8)$$

3.2 Resource Allocation

As in the earlier model (Griffin and Stanish, 2007, 2008), it was assumed that the allocation of resources within early complex polities was determined in large part by its political organization rather than a market economy. Ruling elites extracted resources from farmers to finance their leadership (Earle 1997, Johnson and Earle 2000). The extent of coercion for this purpose certainly varied greatly and its importance is still hotly debated. However the assumption was that commoner farmers submitted to elite leadership in exchange for improved security and increased productivity due to economies of scale (Stanish and Haley 2005). This reciprocal exchange was modeled as an increase in carrying capacity of a polity's satellite settlements (K_{sat}) accompanied by a transfer of some portion of the enhanced K to the polity's center (K_{ctr}). It was also assumed that the more distant a settlement was from the center, the less would have been extracted. Specifically:

When a new center settlement sprouted:

$$K_{ctr} = K_{sat} = K_{out}(1 + s_K) \qquad\qquad (9)$$

K_{out} = carrying capacity of an outland patch, i.e., not part of a polity, was initialized to a random value.

s_K (model parameter) = factor by which K_{out} increased due to improved security and economies of scale when patch became part of a polity, not including distance discount.

When fusion occurred at tick t:

For all exiting satellites of the fused polity's center, carrying capacity K_{sat} was unchanged:

$$K_{sat}(t) = K_{sat}(t-1) \qquad\qquad (10)$$

For all new satellites of the fused polity center, including former centers

$$R_{dis} = (1 - r_D)^D \qquad\qquad (11)$$

R_{dis} = portion remaining after attenuation over distance
D = distance from satellite to center, in grid units
r_D (model parameter) = compounded rate at which center's influence on satellite was attenuated for each unit of grid distance between them

$$K_{ext} = x_K \, R_{dis} \, K_{sat}(t\text{-}1) \qquad\qquad (12)$$

K_{ext} = carrying capacity extracted from satellite by center
x_K (model parameter) = portion of K_{sat} extracted by center (not including distance discounting)

$$K_{sat}(t) = K_{sat}(t\text{-}1) - K_{ext} \qquad\qquad (13)$$

For the center of fused polity, carrying capacity K_{ctr} after resources were extracted from all satellites

$$K_{ctr}(t) = K_{ctr}(t\text{-}1) + \Sigma \, K_{ext} \text{ from all new satellites in polity} \qquad (14)$$

During model execution, (9) was applied when a new center/settlement pair sprouted in a patch with no neighboring polities. Expressions (10) – (14) were used whenever polities fused, because the number of satellite settlements controlled by a single center grows, which increases the total K extracted by the center. For the same reason these expressions were used when a new settlement sprouted adjacent to an existing polity. If a polity collapsed during the current tick, K of each former settlement site reverted to K_{out}.

In order to evaluate the expressions above dependent on distance, the center's grid location must be known. That was determined during polity fusion and provided an important link between the carrying capacity expressions (9) – (14) above and the fission/fusion processes. That linkage operated as follows. Polities came into conflict when a settlement was added that bridged the gap between two or more polities. This corresponds to one or more of these neighboring polities attempting to expand into the buffer zone separating them. The center of the prevailing polity retained its current location and became the center of the newly constituted fused polity. The other competing centers became satellite settlements in the new larger polity.

The competition's winner was determined by comparing the effective strengths of two, three or four competing centers with the strongest being the winner. The assumption was that the strength of agrarian polities would have been determined by a combination of center's population size and resources discounted by distance. The form chosen for strength at a center was: sqrt($K_{ctr} \cdot N_{ctr}$), where N_{ctr} is the center's population and K_{ctr} is center carrying capacity determined by (14) above during the previous tick. (A simpler strength formulation, N_{ctr} alone, was considered but rejected for routinely allowing a large polity to be absorbed by a much smaller competitor on its distant fringe.) Strength at the respective centers was diminished by the distance it had to be projected to

the site of conflict, which was the point of fusion where the new settlement sprouted. Effective strength at the conflict site was the strength at center reduced by distance discounting, the same as in (11) above. This competition model was inspired by political scientist Lars-Erik Cederman's (1997) spatial model of emergent polarity.

It is important to note that the SOC exhibited by the baseline FFM was not altered by these population growth and resource allocation extensions. That is because the *occurrences* of both fusion and fission were not functions of carrying capacity or population, even though fusion *outcomes* were. That is, the relative strength of competing polities determined which one absorbed the others during fusion but did not explicitly influence when fusion occurred. To validate the preservation of SOC, the frequency-size distribution was compared from comparable simulation runs before and after extending the model. **Fig. 4** demonstrates that the distribution after extensions was the same as before.
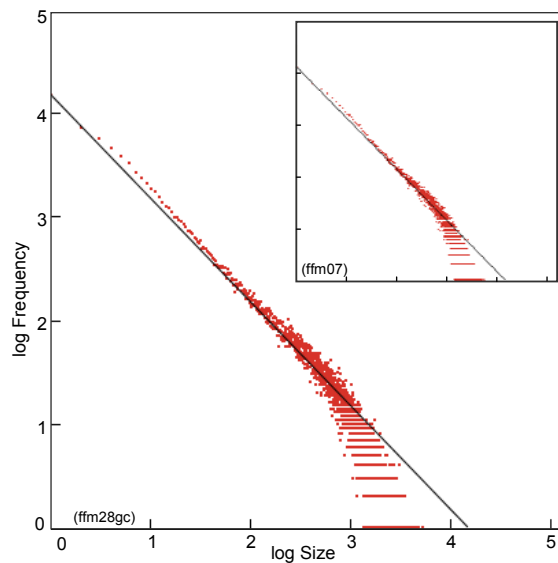
Fig. 4: Frequency-size distribution after model extended with population (N) and carrying capacity (K) variables. Insert is distribution from initial forest fire model (copy of Fig. 1). For both cases: grid size 128x128, resistance interval 125 steps, and 40 million step simulation length.

4.0 Empirical Data

Consider an empirical relationship based on the frequency (1/yrs per cycle) and size (km^2) of collapses for Mississippian chiefdom and Maya archaic state cited in the Introduction, 1.1. It is interesting to note that these two points (frequency, size) were reasonably well fitted by a power law with an exponent of -1.0 expected for an SOC process (-0.84 slope for a straight line through the two points plotted on log-log axes).

However these two data points were based on admittedly weak assumptions about the timing and magnitude of these prehistoric size variations. More daunting, there was a dearth of comparable data for polities in the size range between the two extremes.

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Estimating years per fission cycle also can be confounded by climate changes, such as evidence of a 200 year drought cycle in Maya regions (Hodell 2001). This ambiguity was also evident with Tiwanaku's decline around AD 1000 -1100. Although the collapse of Tiwanaku coincided with a major drought (Binford et al. 1997), it has also been argued that existing internal social tensions were the root causes of the state's demise (Janusek 2005).

As an alternative to problematic longitudinal data, employing spatial data was motivated by another characteristic of the FFM: the distribution of polity areas at one point in time mirrors the power law distribution of fission frequency over a long period of time. While not without issues, area measures suggested a comparatively reliable and abundant source of empirical spatial data. These are rank-size distributions of contemporaneous settlements in which the largest is ranked 1, next largest 2, and so on. Log-log plots of settlement rank-sizes have been published for a number of regional surveys world-wide.

Plots of settlement rank-size are characterized by their deviation from the straight line of a power law with an exponent of -1. Plots closely matching this line are said to obey Zipf's law (or rule) named after George Zipf, best remembered for the observation that modern city sizes were thusly distributed (Zipf 1949). (See the Appendix for discussion of why it is more appropriate to refer to this plot line as Zipf's law rather than *lognormal*, the convention often followed in the archaeology literature.)

Illustrated in **Fig. 5** (left) these plots from field surveys are classified as *convex* when the highest ranked settlement is slightly larger or same size as the second ranked and the remaining points are predominately above the line. It is classified as a *primate* pattern when points fall mostly below the line with the first ranked settlement much larger than the second largest. It has become common practice for archaeologists to use this classification to compare the settlement patterns from two or more time periods or geographic areas and infer differences in political and economic organization. The generally accepted interpretations of these two rank-size patterns are:

- Convex distributions indicate a dispersed and non-integrated region. However a convex pattern also can be expected from a survey area containing more than one primate center. Inferring a non-integrated area from this pooled settlement data would be erroneous (Johnson 1980, Smith and Schreiber 2006).

- Settlement size distributed as a primate pattern is indicative of a politically and otherwise culturally dominant center settlement surrounded by much smaller settlements articulated to that center but with little interaction between these satellite settlements (Johnson 1980).

Also shown in **Fig. 5** are values calculated for the A coefficient defined by Drennan and Peterson (2004) to quantify the deviation of rank-size distributions from Zipf's rule. This metric is one of several that have been proposed (Johnson 1980, Savage 1997, Griffin and Stanish 2007). The A coefficient was chosen for use here because of its intuitive correspondence to the difference in area enclosed by a rank-size plot above the Zipf line and the area below it. The area difference is normalized to range from -1.0 to 1.0. Hence A equals zero for an ideal Zipf distribution. Increasingly positive values for A indicate higher convexity, and more negative values signify an increasingly primate pattern. (A can be < -1.0 for very primate distributions.)
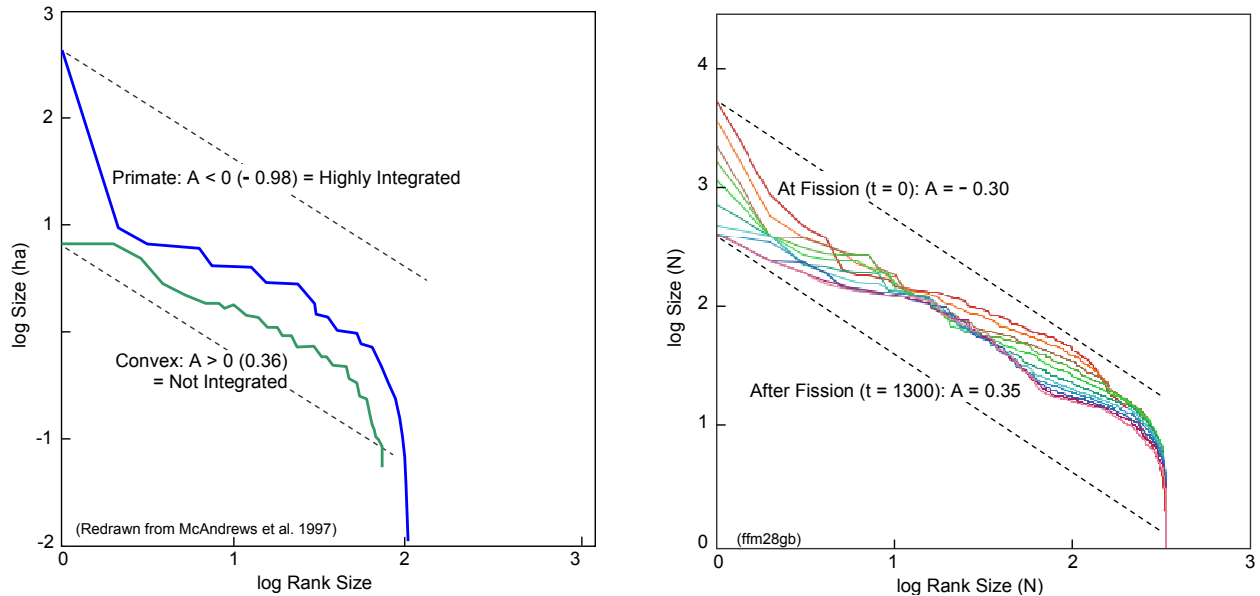
Fig. 5: Comparison of empirical and simulated rank-size settlement population (N) surrounding a primate center, Dashed lines show Zipf's rule. A is convexity metric. **(left)** Survey data from Tiwanaku valley (assumed N proportional to area). Convex lower plot, prior to urban center in Formative Period 1500 BC – AD 100. Primate upper plot, at height of urban density in Classic Period, AD 800 – 1000. **(right)** Typical simulated transition from primate to convex pattern following polity fission for largest center.

5.0 Simulation Results

Three macro-level patterns identified from empirical rank-size data were expected to emerge during simulations. All three of these patterns were consistently observed during numerous runs and were robust over the full range of parameter values listed in **Table 2**. The rationale and results for each of the three expected patterns are presented below.

Throughout these runs the plots shown in **Fig. 4,** along with other spatially related plots, were monitored to assure that the population model extensions had not distorted SOC confirmed for the baseline fusion/fission model.

5.1 Population rank-size distribution for an area surrounding a single dominant center will be primate immediately before fission and transition to convex thereafter.

The empirical basis for this expectation is illustrated in **Fig. 5** on the left, with survey results from the Tiwanaku Valley (McAndrews et al. 1997). The upper plot exhibits a primate rank-size pattern for a survey area, about 20 km across, containing the primate center of the Tiwanaku state at its height, i.e., a period of high integration. The convex lower plot covered the same survey area but was dated as preceding the location's growth as an urban center. Surveys after Tiwanaku's decline indicated that the entire area had reverted to a dispersed, non-integrated settlement pattern. Other instances of alternating primate-convex patterns, also localized around one dominant center, include the Valley of Oaxaca in present day Mexico 1500 BC to AD 1520 (Drennan and Peterson 2004) and the southern Levant during the early and middle Bronze Age (Savage and Falconer, 2003).

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The expected rank-size pattern seen in field survey data described above was consistently observed during simulations. During each time step the dominant center was chosen as the settlement with the largest population (N) over the entire grid. If the center's polity fissioned on the next time step a rank-size distribution of population was constructed and convexity measure A calculated for a circular region of grid cells centered on that dominant center. This was repeated at step intervals to create a time sequence of plots to trace the change in convexity following fission. **Fig. 5** (right) shows a typical sequence of such plots. The plot just prior to fission (tick = 0) was clearly primate, indicating a high degree of integration between the center and satellite settlements. Plots from the same grid region after the polity fissioned (tick = 1 ... 1300) became progressively convex due to population diffusing away from the polity's center. These results were robust for varying sizes of the surrounding grid region (controlled by radius parameter in **Table 2;** radius set to **7** grid units for the case shown). The range of variation illustrated in **Fig 6** indicated that these results were typical for simulated fission occurrences. Comparing **Fig 5** and **6**, the congruency between simulated and naturally occurring rank-size patterns was evident.
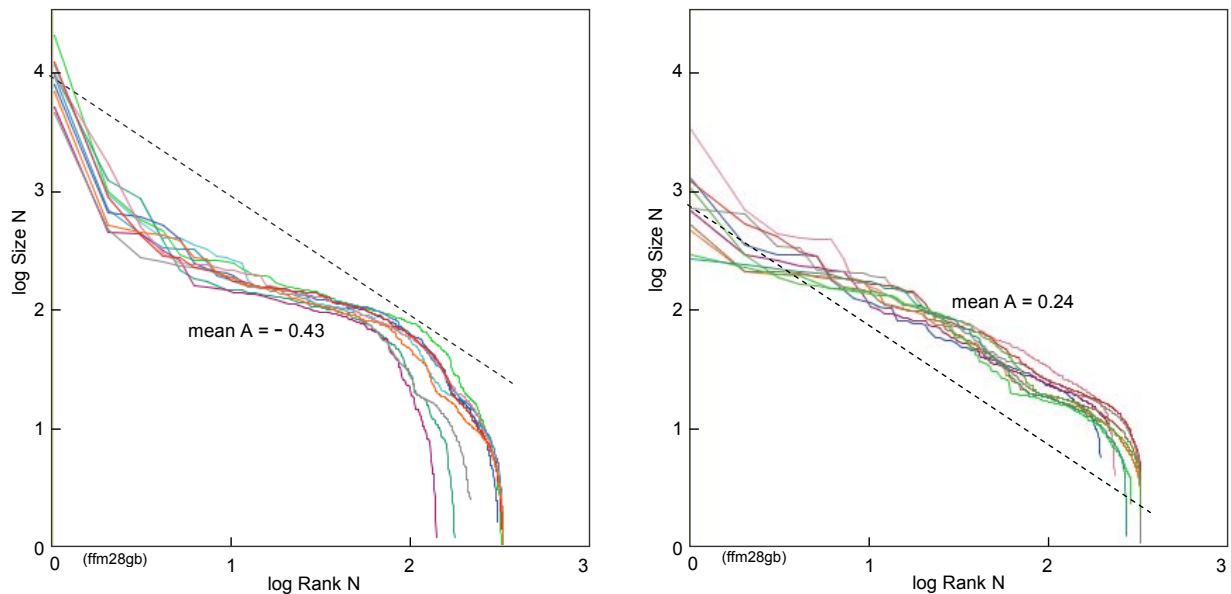
Fig. 6: Range of variation and mean convexity measure A. Simulated settlement population (N) rank-size from multiple collapses (of largest centers). At left each plot recorded at fission. Plots at right taken 1300 steps later, time for A to reach maximum. Dashed lines trace Zipf's rule, drawn at slope of -1 from mean of plot y-intercepts.

5.2 Strengthening each of four integrative processes by adjusting its associated parameter will decrease the time averaged rank-size convexity for the entire grid.

In more general terms this expected pattern could be restated as: The impact of changing model parameter values will be consistent with corresponding changes in real-world settlement systems integration as reflected in empirical rank-size distributions. Gregory Johnson (1980) defined the property of *systems integration* as the extent to which people, materials, and information flow within a system of settlements. He presented a number of case studies from historical and archaeological data in which rank-size convexity decreased when integration was increasing. These cases included the Susiana Plain in southwest Iran from 3800 to 3400 BC where the centralization of craft production and other micro-level diagnostics were proxies for increased

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integration. Similarly, rank-size convexity decreased in inverse proportion to internal shipping tonnage per-capita, another proxy for integration, in the colonial United States between 1790 and 1850. Each of these integration proxies was actually one of many micro-level processes that contributed to integration visible in the area wide rank-size distributions.

In like manner of the empirical rank-size cases above, four simulated integrative processes were defined. The strength of each was directly controlled by one model parameter as listed in **Table 3**. Consider population mobility as the exemplar for arguments made to define all these processes. By definition a more mobile population fosters increasing integration. According to model equations (5) and (6) in *3.1*, raising the diffusion constant parameter c_F increases the micro-level population flow between neighboring settlements. That change in turn was expected to lower the macro-level convexity measure A for the entire grid. Increasing the model diffusion parameter would be analogous to increasing shipping tonnage in the colonial US case above.

Table 3: Model parameter values used in each of seven simulation runs to progressively strengthen four associated micro-level processes contributing to area-wide settlement systems integration.

| Model Parameters(1)	Micro-level Integrative Processes						
	K Base-line (2)	More Surplus Extraction	More Scale Efficiency	Less Distance Attenuation	N Base-line (2)	Increasing Population Mobility	
c_F Population diffusion constant	-	-	-	-	**0.003** →	**0.03** →	0.3
r_D Distance attenuation rate	0.2	0.2	**0.2** →	**0.02**	0.02	0.02	0.02
s_K Scale/Safety Efficiency factor	1.0	**1.0** →	**10**	10	10	10	10
x_K Surplus extraction portion	**0.03** →	**0.3**	0.3	0.3	0.3	0.3	0.3

(1) Model parameters defined in Table 2.
(2) N = population, K = carrying capacity
(3) → marks a step change in parameter value to strengthen integrative process from one simulation run to the next.

Over a series of simulation runs each micro-level integrative process was progressively strengthened by stepping the value of its associated parameter while holding the others constant. Each column in **Table 3** lists the parameter values for each run. After a run reached steady state, rank-size and convexity metric A of either population (N) or carrying capacity (K) for the entire grid were sampled every 50 time steps for 1000 samples, from which mean values were calculated. (Rank-size plots appeared to reach stasis after 500 samples). N was sampled for the population mobility process. For the three other processes, differences in rank-size were more clearly observed by sampling K since the associated parameters were related directly to K through the equations in section *3.2*. Because N always tracked K, with a delay, there was no loss of generality over the sufficiently long time-averaging interval.

The expected pattern consistently emerged as the results in **Fig 7** indicate. When each integrative process was progressively strengthened, rank-size distribution became less and less convex. **Fig 7** also shows another characteristic seen in empirical rank-size. In all cases A was > 0, indicating a robust convex rank-size classification consistent with pooling expected for field survey areas containing more than one primate center.
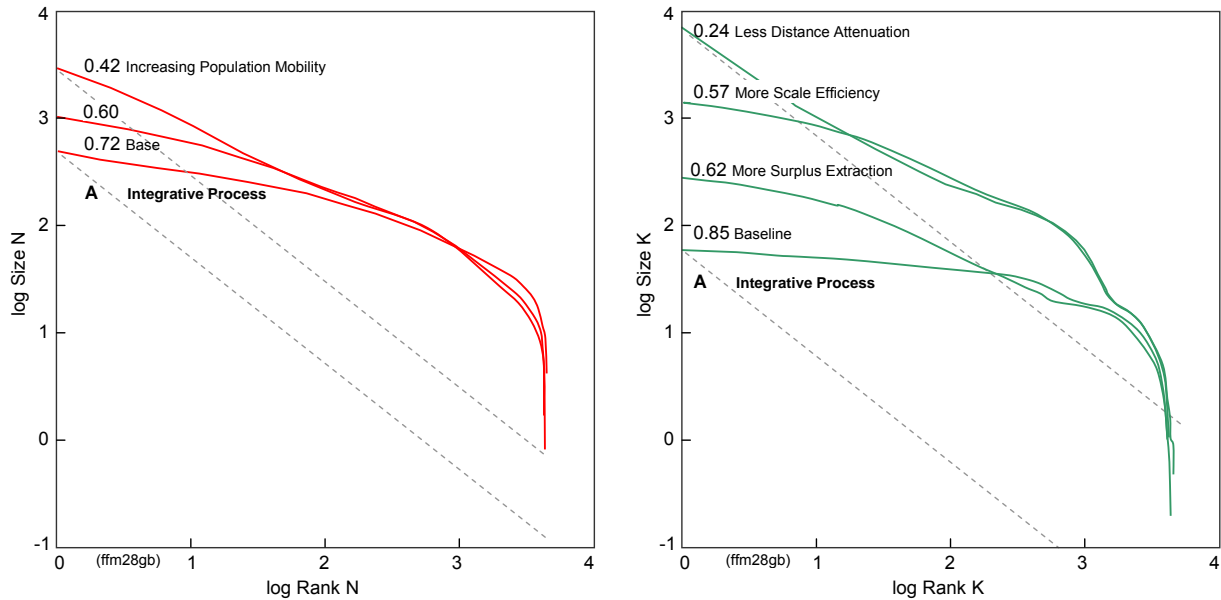
Fig. 7: Time averaged rank-size plots of population (N, on left) and carrying capacity (K, on right) for full grid (64 x 64). Plot and convexity measure A show results of strengthening integrative processes by adjusting associated parameters; see Table 3. Averaged from 1000 grid snapshots each taken every 50 steps (started after reaching steady state). Dotted lines indicate Zipf's rule for highest and lowest A.

5.3 Subordinate population centers articulated to a primate center or another subordinate center will be observed within polities.

By definition the existence of a complex polity is inferred from field surveys indicating a hierarchy of secondary centers. During simulations hierarchies of subordinate population centers consistently emerged. This was best visualized just after the collapse of the polity in question; an example is shown in **Fig. 8**. The relative sizes of the former population centers were visible as the extent of diffusion wave fronts streaming away from settlements of the collapsed polity.
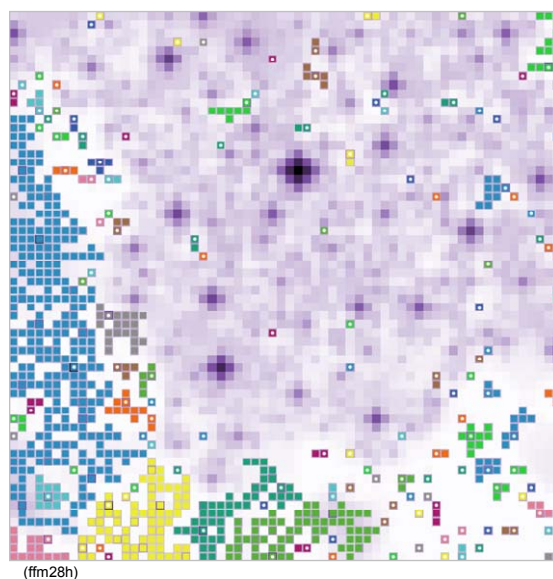
Fig. 8: Relative size and location of former primate center and secondary centers visualized by the "ghost" of large collapsed polity.

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The internal model dynamics responsible for this pattern occur during polity fusion. The settlement that was the center of the absorbed polity remains intact, albeit no longer a primate center. This former center retains the carrying capacity it had just prior to fusion less the amount extracted by the center of the newly consolidated polity. As a polity expands, each successive fusion thus adds another branch to an emerging virtual hierarchy of secondary population centers. The model contains no rules or structures to explicitly create or maintain this hierarchy; it is a side effect of applying expression (13) above to all settlements in an absorbed polity.

6.0 Conclusions

Social scientists have been intrigued by self-organized criticality (SOC) as a theoretical framework to understand a host of social processes including fusion/fission polity cycling. However speculations that this cycling is a manifestation of SOC have been unsatisfying for lack of clear correspondence between the elements of SOC simulation models and those of real-world polities. The polity model described here is intended to address that issue. Its micro-level structure and behavior have clear analogs to early complex polities, particularly the following processes:
- o population growth within and migration between settlements
- o allocation of resources between a polity's producing population and its ruling elites
- o internal competition between rival factions
- o external competition between expanding polities

Macro-level patterns have emerged during simulations that are consistent with the archaeological and historical records for early complex polities:
- o ongoing cycles of polity fusion and fission
- o noncontiguous polities separated by sparsely populated buffer areas
- o distribution of simulated settlement populations matching patterns seen in empirical rank-size data
- o hierarchy of secondary settlements articulated to a primate center

At the same time, the model described here unequivocally exhibited SOC. That is because it is structurally identical to the forest fire model, which is widely recognized for its SOC characteristics. In particular, simulations with the current model resulted in a fission frequency-size distribution closely matching a power law with exponent of -1.

Evaluating a model's usefulness requires knowing what has been left out in addition to what was included (Holland 1995). Two potentially significant features not included in the current model are discussed along with suggestions for addressing them in the future.

6.1 Gradual Collapse

Polities collapse suddenly and completely in the current model that is structurally equivalent to a special case of the forest fire model (FFM). Discrete events with catastrophic outcomes are typical

in models exhibiting SOC. Yet the archaeological record indicates that decline of early complex polities could be gradual and have many short troughs (Marcus 1998, Stanish 2003, Turchin 2003). How can this apparent contradiction be reconciled?

One could reasonably argue that over many trials the distribution of discrete yet temporally stochastic collapses is equivalent to a continuous gradual decline. Perhaps a less abstract approach would be to consider the more generalized form of the FFM. The special case of the model described up to this point specifies that fire always spreads from a burning tree to its healthy neighbors. However in the more general form there is another model parameter g, the probability that a living tree is immune to the spread of fire. In the special case g=0. When g > 0 the model becomes FFMIT, forest fire model with immune trees (Albano 1994).

For the corresponding polity cycling model, g proportion of settlements would be immune to the spread of resistance from neighbors. Resistance by one settlement would not necessarily cascade throughout a polity leading to its immediate collapse. Rather, the resistance could be extinguished after only a few settlements seceded. This could be implemented in the current model by making each new settlement immune with a probability of g. Simulation experiments could then determine the range of g values for which sustained polity cycling and SOC were observed.

6.2 Feedback from Population to Fission/Fusion

After extensions to include the impact of fission/fusion on carrying capacity (K) and population (N), simulations continued to exhibit SOC, a major objective for the current model. This persistence was assured by limiting the feedback on fission/fusion from changes in N and K to the outcome of fusion, i.e., which polity absorbed its competitor. However the probability of fission/fusion events remained independent of N and K.

In an earlier polity model (Griffin and Stanish 2007) the probability of both fusion and fission were modulated by more nuanced population feedback. The occurrences of fusion and fission were random functions of both the sizes and distance between opposing populations. It was straightforward to begin further extensions of the current model to complete these additional feedback loops. One effect observed in preliminary experiments was filling in empty spaces within polities due to a lower incidence of fusion and fission. Thus the dendritic shape typical of early complex polities transitioned to more contiguous areas separated by increasingly distinct borders, suggestive of early nation states. Further simulations are needed to observe collapse-size distributions as an indicator of changing SOC.

It is hoped that the model and simulations presented here suggest useful approaches for realizing the promise of self-organized criticality as a framework to understand polity cycling, particularly in relation to rank-size analysis of settlement survey data.

Appendix

A1. Comparing Zipf's Law and Lognormal Distributions

Zipf's law or rule is a rank-size distribution that follows a power law with an exponent of -1. It is so named for George Zipf, remembered for observing that a number of naturally occurring phenomena appeared to be distributed as such a power law, most notably the rank-size of cities (Zipf 1949). In the archaeological literature rank-size data that follows Zipf's law is often called *lognormal*. The more general usage of lognormal describes data whose log values have a normal distribution over their full range. Although lognormal data can appear linear on a log-log graph over a limited range of values it is quite distinct from a -1 power law distribution. In complexity theory distinction between the two is important, because the presence of one or the other implies very different underlying processes (Shalizi 2006).

The differences between Zipf and lognormal patterns also have been evident in the continuing debate over size distribution of modern cities, towns, and smaller settlements. Several international comparative studies have supported Zipf's law (Gabaix and Ioannides 2003) although in a recent analysis of multi-country data, the maximum likelihood estimates of power law exponents for cities were reported to be significantly different than -1.0. Further, when the smaller settlements were included the best fits were achieved with a lognormal rather than a power law function (Decker 2007). This is consistent with earlier observations that the rank-size plot for a number of urban systems displayed a downward curvature characteristic of a lognormal distribution (Parr 1976).

A2. Proportionate Random Growth

Proportionate random growth is a process in which each entity in a group grows at a random rate but in proportion to its current size. This was considered as the population growth mechanism for the current model. However, the causal correspondence to the assumed behavior of populations in early complex polities was judged as too shallow. Nonetheless, when combined with the basic fusion/fission model from 2.0, this growth process produced some intriguing simulation results.

A number of models based on proportionate random growth have received significant attention in regional economics and urban studies to explain why city sizes appeared distributed according to Zipf's law. (See (Corominas-Murtra and Sole 2010) for a recent compilation of proposed explanations for the universality of Zipf's law.) The theoretical underpinning of the random growth models was Gibrat's proportionate effect, which paradoxically results in a lognormal rather than a power law distribution (Eeckhout 2004, Batty 2001). Of particular interest here was the Yule-Simon growth model in which new cities of a minimum size were added to the system at a constant rate in addition to growth of existing cities by proportionate effect. (Gabaix and Ioannides, 2002 p 16; Parr 1976). The reported result was a Zipf distribution rather than a lognormal one as expected from proportionate effect alone.

A similar result was achieved by using polity fusion/fission as implemented in the current model to bound proportionate random growth. Collapsing polities effectively acted as an upper limit on

population. Similarly, formation of new settlements at a constant time interval set the lower limit. The resulting rank-size distribution closely matched a power law whose exponent varied over the domain of the only model parameter, variance of the random growth rate. Sweeping this parameter's value resulted in a stable plateau in the exponent's range at -1, the value corresponding to Zipf's law.

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