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Authors
Fox, Geoffrey C.
Freedman, Daniel Z.

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January 15, 1969

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Geoffrey C. Fox and Daniel Z. Freedman

January 15, 1969

COMPTON SCATTERING SUM RULES AND THEIR SATURATION*

Geoffrey C. Fox

Lawrence Radiation Laboratory
University of California
Berkeley, California 92720

and

Daniel Z. Freedman

Institute of Theoretical Physics
State University of New York
Stony Brook, L. I., New York 11790

January 15, 1969

ABSTRACT

Theoretical features of the many (twenty-six) fixed momentum transfer dispersive sum rules which can be written for generalized nucleon Compton scattering amplitudes (retarded products of vector currents) are surveyed and the sum rules put to experimental test. Theoretical attention is focused on the occurrence of right signature fixed poles in the angular momentum plane, such as the $j = 1$ fixed poles whose couplings are related to electromagnetic form factors by current algebra. Unitarity is used to estimate the sum rule integrands in terms of data for the photoproduction processes $\gamma N \to \pi N$ and $\gamma N \to \pi \Delta$. Data limitations require that the sum rules be cutoff at photon lab energy $E_{lab} = 1.12$ GeV.

The main results are as follows

(a) Reasonable evidence is presented that two time component current algebra sum rules involving the electric and magnetic isovector form factors $G_E^V(t)$ and $G_M^V(t)$ are correct for small spacelike $-t$. If they are also to be correct for $-t \gtrsim -0.6\ (\mathrm{GeV}/c)^2$ then the ρ Regge pole must choose nonsense at $\alpha = 0$ and the associated wrong signature fixed pole

there must be multiplicative. A time-space current algebra sum rule probably fails.

(b) The separate isotopic components of the Drell-Hearn sum rule are investigated. Those with $I = 0$ exchange in the t channel seem very successful whereas the $I = 1$ exchange sum rule clearly fails. The failure indicates an important contribution of a hitherto unsuspected $J^P(I^G) = 1^+ (1^-)$ fixed pole.

(c) Detailed results on wrong signature anti-algebra sum rules, on Regge-pole sum rules (FESR's) and on sum rules testing conspiracy are presented.

I. INTRODUCTION

Many fixed momentum transfer dispersive sum rules can be written for nucleon Compton amplitudes. These sum rules test various assumptions about high-energy behavior and about the equal-time algebra of vector current components. In this paper we survey theoretical aspects of these sum rules and report on a systematic attempt to saturate them, at several t values, using presently available experimental data. Within the limit set by the extent and accuracy of this data, our goal is to milk from the sum rules all the theoretical interesting information they contain.

Since there is very little data on the Compton scattering process itself, we use the unitarity condition to express the integrands of the sum rules in terms of amplitudes for the photoproduction of hadronic states. We include the contributions of the πN and, in cruder form, the $\pi\pi N$ intermediate states. Specifically we use the multipole analyses of $\gamma N \to \pi N$ by Berends, Donnachie, and Weaver[1] and by Walker,[2] and a modified Stichel-Scholz[3] model for the process $\gamma N \to \pi\Delta$. This gives us a description of the sum rule integrand which seems reasonably accurate up to the laboratory energy $E_{lab} = 1.12$ GeV (c.m. energy $\sqrt{s} = 1.73$ GeV), and we cut off our sum rules at this value.

Because of spin and isospin complexity there are 26 independent amplitudes for the generalized Compton scattering process, and the use of photoproduction data decomposed into definite angular momentum and isospin components allows us to study sum rules for all of them. We study the sum rules derived from current algebra,[4] as well as superconvergence relations[5] and finite energy sum rules[6] which give information on Regge pole parameters and on the question of conspiracy. We are mainly interested in theoretical questions involving the presence of fixed j-plane poles.

Finite energy sum rules have been much used recently to study meson-baryon scattering[6,7] where there are two important advantages. First good partial wave analyses exist,[8] at least for πN scattering, up to the c.m. energy $\sqrt{s} = 2.19$ GeV; and second there is considerable high energy data with which to compare Regge pole predictions. In our case the low energy data are unfortunately crude, and there are no high energy experiments. However, because we study photon amplitudes with the possibility of double helicity flip, many of our sum rules are more convergent than their analogues in meson-baryon scattering. Further we remark that the analysis of Dolen, Horn, and Schmid[6] at cutoff $\sqrt{s} = 1.73$, identical to ours, gave reasonable results for the couplings of the ρ trajectory, and we therefore have reason to hope for good results at this cutoff in the Compton case.[9]

The plan of the paper is the following. For the benefit of readers primarily interested in the results, a summary of the most important results is given in Section II together with references to that part of the text where specific sum rules are discussed. The kinematics of Compton scattering is presented in Section III. Theoretical questions pertaining to the sum rules are discussed in Section IV. In Section V we explain our treatment of the experimental data, and in Section VI we present and discuss the results of our attempt to saturate the sum rules. Section VII is reserved for some final methodological comments, while some necessary technical questions are treated in Appendices.

II. MAIN RESULTS

Our main results are summarized here, although we would caution that a wrong impression of the strength of our conclusions could well be gained without some study of the quantitative behavior of the sum rules. The quickest way to proceed would be via Section VI.A, in which the graphical format of the results is given, to the point of Section VI where the specific questions are discussed and the appropriate graphs presented.

Regge Pole Sum Rules: (VI.B)

From sum rules for amplitudes in which the P, P', and A_2 trajectories couple to photons with helicity flip 2, we find the following results. There is no particular evidence for important contributions to the sum rule from right signature fixed poles at $j = 0$.[10] Factorization tests give values of the ratio of the nucleon flip and nucleon nonflip couplings of the trajectories which agree with the values deduced from meson-nucleon scattering, although there is an uncertainity of about a factor of two in this comparison. Our results are consistent with the nonsense choosing mechanism for the A_2 at $\alpha_{A_2}(t) = 0$.

Current Algebra Sum Rules[11]: (VI.C)

Two well-known sum rules can be obtained by studying the equal-time commutators of time components of the isovector current, taken between states with nucleon helicity nonflip and flip (measured in the t channel c.m. system). The nonflip sum rule, whose right hand side involves the electric form factor $G_E(t)$, coincides at $t = 0$ with the sum rule of Cabibbo and Radicati.[12] The flip sum rule similarly tests the magnetic form factor $G_M(t)$ and seems to have been first written down by Muzinich.[13]

Our results indicate good agreement with current algebra predictions near $t = 0$. At large momentum transfer $(t \approx -0.6)$ there is some evidence for a possible violation of current algebra, although we prefer an interpretation in which current algebra is valid. In this interpretation the ρ trajectory chooses nonsense at $\alpha_\rho(t) = 0$ and has a singular coupling to the currents there. Because of the singular coupling the nonsense dips[14] associated with ρ exchange in hadronic processes are not present in the Compton amplitude.

Both these sum rules receive important contributions at low energies from nonresonating multipoles; a fact which suggests that theoretical models[15] in which saturation occurs purely with resonances may be unrealistic. We give some idea of the relative magnitude of resonant and nonresonant contributions to the sum rules in Section VI.I.

A sum rule involving the commutator of the time and space components of the isovector current has been written down by Beg[16] and further studied by Adler and Dashen.[4] This sum rule has some peculiar features,[4] and it is perhaps not surprising that our numerical results show that it is probably violated.

Anti-Algebra Sum Rules: (VI.D)

We use this name (see Section IV.B) for sum rules[17] sensitive to wrong signature fixed poles. We find evidence for wrong signature fixed poles (at $j = 1$) which couple strongly to Pomeranchuk and A_2 exchange. The theoretical significance of such fixed poles has been recently studied.[18]

Drell-Hearn Sum Rules: (VI.E)

Here we refer to sum rules for three different isospin symmetric amplitudes with t-channel photon helicity flip, antisymmetrized in the nucleon helicity indices. The sum rules are superconvergence relations (SCR's) which follow from the assumption that $j = 1$ fixed poles are absent in these amplitudes. At $t = 0$, the sum of our three SCR's coincide with the original sum rule written by Drell and Hearn[19] for the anomalous magnetic moment of the proton.

Our results indicate that the two sum rules involving isoscalar exchange are very well satisfied, but that the sum rule involving isovector exchange is badly violated. This last result was a surprise to us, and seems to indicate an important contribution from a $J^{PG} = 1^{+-}$ fixed pole.

One negative result which may be of some interest is that neither of two sum rules sensitive to A_1 exchange showed any evidence for this Regge pole with an intercept near zero. (See also Section VI.G.)

Conspiracy Sum Rules: (VI.F)

By using a sum rule of Pagels[20] which relates the π^o lifetime to an integral involving a Compton amplitude we infer that the effective π conspirator trajectory residue function $\beta_{c_\pi}(t)$ in Compton scattering is a smooth function of momentum transfer near $t = 0$. Unlike the photoproduction case we cannot write a sum rule sensitive to the t dependence of the pion residue function itself. However, comparison of the $t = 0$ value obtained from the conspiracy condition with the value at the π pole (known from the π^o lifetime) suggests a zero in β_π near $t = -m_\pi^2$. The behavior of both the pion and its conspirator are consistent with that found in strong interactions.

III. KINEMATICS

Using covariantly normalized states

$$\langle p_2 \mid p_1 \rangle = 2p_0\, \delta^3(\underline{p}_2 - \underline{p}_1) \tag{1}$$

we define transition amplitudes for all two-body reactions

$$\langle p_2\ k_2 \,|S|\, p_1\ k_1 \rangle = \langle p_2\ k_2 \mid p_1\ k_1 \rangle + i(2\pi)^{-2}\, \delta^4(p_2+k_2-p_1-k_1) T(p_2,k_2;\ p_1,k_1) \ . \tag{2}$$

Differential cross sections are given by, ignoring the spin summation

$$\frac{d\sigma}{d\Omega} = |f(E,\theta)|^2$$
$$f = (8\pi\sqrt{s})^{-1}\,(p_f/p_i)^{1/2}\ T \tag{3}$$

where $s = (p_1 + k_1)^2$ and p_i and p_f are the center-of-mass momenta of the initial and final states.

Compton scattering amplitudes are related to retarded products of currents by the formula

$$T(p_2,k_2;\ p_1,k_1) = (2\pi)^3\, e^2\, \epsilon_2^{*\mu}\, \epsilon_1^{\nu}\, i \times \int d^4x\, e^{ik_2\cdot x} \langle p_2|\theta(x_0)[J_\mu^{em.}(x),\ J_\nu^{em.}(0)]|p_1\rangle \tag{4}$$

where $e^2/4\pi = 1/137$. We do not write explicitly the polynomial terms which may be required on the right hand side of (4) to ensure covariance.

The electromagnetic current operator $J_\mu^{em.}(x)$ can be decomposed into isotopic singlet and triplet parts

$$J_\mu^{em.}(x) = J_\mu^{I=0}(x) + J_\mu^{I=1,M=0}(x) . \tag{5}$$

In general we are led to consider covariant amplitudes formed as in Eq. (4) from the individual pieces J_μ^{0} and $J_\mu^{1,M}$ with $M = \pm 1, 0$ and construct these amplitudes according to the following isospin conventions.

First we construct amplitudes $T_I(I'_\gamma, I_\gamma)$ describing transition between normalized states of total s-channel isospin I built up from nucleons and isoscalar $(I_\gamma = 0)$ or isovector $(I_\gamma = 1)$ photons. There are five independent amplitudes. Each $T_I(I'_\gamma, I_\gamma)$ gives rise to a scattering

$$T_I(I'_\gamma, I_\gamma)\, C(\tfrac{1}{2}, I'_\gamma, I; M'_N, M'_\gamma) C(\tfrac{1}{2}, I_\gamma, I; M_N, M_\gamma) \tag{6}$$

in states specified by third component of isospin for the nucleon (M_N) and photon (M_γ). The C's are standard Clebsch-Gordan coefficients. Our sum rules are written for the following combinations of the $T_I(I'_\gamma, I_\gamma)$ formed by symmetrizing or antisymmetrizing in the (t-channel) photon isospin labels:

$$\begin{aligned}
T^1 &= 2\, T_{1/2}(0, 0) \\
T^2 &= (2/3)[T_{1/2}(1, 1) + 2\, T_{3/2}(1, 1)] \\
T^3 &= (2\sqrt{3}/3)[T_{1/2}(0, 1) + T_{1/2}(1, 0)] \\
T^4 &= (2/3)[T_{3/2}(1, 1) - T_{1/2}(1, 1)] \\
T^5 &= (2\sqrt{3}/3)[T_{1/2}(0, 1) - T_{1/2}(1, 0)]
\end{aligned} \tag{7}$$

Amplitudes 1 and 2 carry isospin 0 in the t channel while amplitudes 3, 4, and 5 carry isospin 1. The Compton scattering amplitudes of physical photons are related to ours by the equations

$$\begin{aligned} T(\gamma p \to \gamma p) + T(\gamma n \to \gamma n) &= T^1 + T^2 \\ T(\gamma p \to \gamma p) - T(\gamma n \to \gamma n) &= T^3 . \end{aligned} \tag{8}$$

To relate our amplitude T^4 to that of the current algebra literature[5] we observe that T^4 is given by Eq. (4) with the commutator replacement

$$[J_\mu^{\,em.}(x), J_\nu^{\,em.}(0)] \longrightarrow [J_\mu^{(+)}(x), J_\nu^{(-)}(0)] - [J_\mu^{(-)}(x), J_\nu^{(+)}(0)]. \tag{9}$$

Physical Compton scattering data cannot be used to resolve the individual contribution of T^1 and T^2 in Eq. (8), or to determine the amplitudes in isospin segments 4 and 5. The real parts of T^2 can conceivably be measured only in neutrino processes. However the imaginary parts of all amplitudes are related unambiguously by unitarity to experimentally measurable photoproduction processes. Isospin segment 5 has very peculiar kinematics, discussed below, and does not seem to have been mentioned in the literature.

We always express our sum rules in terms of regularized t-channel parity-conserving helicity amplitudes,[21] which are advantageous for us because they have simple analyticity and crossing properties and definite t-channel quantum numbers. Direct channel helicity amplitudes $M_{\lambda_3\lambda_4;\lambda_1\lambda_2}$ can be defined from Eq. (4) by choosing nucleon states and photon polarization vectors according to standard conventions.[22] We take the nucleon as "particle 1." We define t-channel helicity amplitudes through the crossing relations[23]

$$A^{i}_{\lambda_3\lambda_1;\lambda_4\lambda_2} = -i(-1)^{\lambda_3-\lambda_1} \sum_{\lambda'_3\lambda'_1} d^{1/2}_{\lambda'_1\lambda_1}(\pi - X)\, d^{1/2}_{\lambda'_3\lambda_3}(X)\, M^{i}_{\lambda'_3-\lambda_4;\lambda'_1\lambda_2} \qquad (10)$$

where

$$\cos X = \frac{(s + m^2)}{(s - m^2)} \left(\frac{-t}{4m^2 - t}\right)^{1/2} \qquad (11)$$

$$\sin X = \frac{2m}{(s - m^2)} \left[\frac{(s - m^2)^2 + st}{4m^2 - t}\right]^{1/2}.$$

and the superscript i indicates a definite isospin amplitude formed according to Eq. (7).

For physical photons, the kinematic singularities of the amplitudes $A_{\lambda_3\lambda_1;\lambda_4\lambda_2}$ have recently been obtained.[23,24] The analysis of Reference 23 depended on a simplification of the crossing relation (9) using the time reversal constraint

$$M_{\lambda_3\lambda_4;\lambda_1\lambda_2} = (-1)^{\lambda_{12}-\lambda_{34}} M_{\lambda_1\lambda_2;\lambda_3\lambda_4} \qquad (12)$$

where $\lambda_{ij} = \lambda_i - \lambda_j$. An identical condition holds for our isospin amplitudes 1 - 4 , and for these amplitudes the results of Reference 23 apply completely with the single exception that the $s - u$ crossing properties of isospin 4 amplitudes are opposite to those of isospin 1 -3 because of photon antisymmetry.

We give here the exact definition of the amplitudes for which our sum rules are written in isospin segments 1 - 4. In terms of reduced t-channel helicity amplitudes,

$$\hat{A}^{i}_{\lambda_3\lambda_1;\lambda_4\lambda_2} = (\cos\tfrac{1}{2}\theta_t)^{-|\lambda_{42}+\lambda_{31}|}(\sin\tfrac{1}{2}\theta_t)^{-|\lambda_{42}-\lambda_{31}|} A^{i}_{\lambda_3\lambda_1;\lambda_4\lambda_2} \qquad (13)$$

we take the following combinations which are kinematic singularity free in both s and t.

$$B_1{}^{i} = i\, t^{-1}(4m^2 - t)^{-1/2} \hat{A}^{i}_{\frac{1}{2}\frac{1}{2};1-1}$$

$$B_2{}^{i} = -t^{-1}(t - 4m^2)^{-1}\left[\hat{A}^{i}_{\frac{1}{2}-\frac{1}{2};1-1} - \hat{A}^{i}_{-\frac{1}{2}\frac{1}{2};1-1}\right]$$

$$B_3{}^{i} = (-t)^{-3/2}(4m^2 - t)^{-1/2}\left[\hat{A}^{i}_{\frac{1}{2}-\frac{1}{2};1-1} + \hat{A}^{i}_{-\frac{1}{2}\frac{1}{2};1-1}\right].$$

$$B_4{}^{i} = \tfrac{1}{2}\, i\, (-t)^{-1/2}\left[\hat{A}^{i}_{\frac{1}{2}\frac{1}{2};11} - \hat{A}^{i}_{-\frac{1}{2}-\frac{1}{2};11}\right]$$

$$B_5{}^{i} = i\, t^{-1}(4m^2 - t)^{1/2}\left[\hat{A}^{i}_{\frac{1}{2}\frac{1}{2};11} + \hat{A}^{i}_{-\frac{1}{2}-\frac{1}{2};11}\right]$$

$$B_6{}^{i} = -2t^{-1}\hat{A}^{i}_{\frac{1}{2}-\frac{1}{2};11} \qquad (14)$$

The $B_j{}^{i}(s,t)$ are independent except for the constraint condition at $t = 0$

$$\lim_{t \to 0} \left\{ B_4^{\ i}(s,t) + (8m)^{-1}(s - u)\, B_6^{\ i}(s,t) \right\} = 0 \qquad (15)$$

and other constraints at $t = 4m^2$ which are not relevant for our analysis. In Appendix A we express the $B_j^{\ i}(s,t)$ in terms of s-channel helicity amplitudes and Hearn-Leader[25] invariant amplitudes.

In isospin segment 5 the situation is different. Because of antisymmetry an extra minus sign must be inserted in the time reversal condition (12), and this means that there are only two independent nonvanishing s-channel helicity amplitudes which we take to be $M^5_{\frac{1}{2}1;-\frac{1}{2}1}$ and $M^5_{\frac{1}{2}-1;\frac{1}{2}1}$. There is an analogous restriction, due to charge conjugation invariance to two nonvanishing t-channel amplitudes $A^5_{\frac{1}{2}\frac{1}{2};1-1}$ and $A^5_{\frac{1}{2}-\frac{1}{2};11}$. The crossing relations simplify to

$$A^5_{\frac{1}{2}\frac{1}{2};1-1} = i\, M^5_{\frac{1}{2}1;-\frac{1}{2}1}$$

$$A^5_{\frac{1}{2}-\frac{1}{2};11} = i\, M^5_{\frac{1}{2}-1;\frac{1}{2}1} \,. \qquad (16)$$

The kinematic singularities are easily obtained and we choose the following singularity-free amplitudes

$$B_7^{\ 5} = i(-t)^{-1/2}(m^4 - us)^{-1}\, A^5_{\frac{1}{2}\frac{1}{2};1-1}$$

$$B_8^{\ 5} = -i\, t^{-1}\, (m^4 - us)^{-1/2}\, A^5_{\frac{1}{2}-\frac{1}{2};11} \,. \qquad (17)$$

The amplitudes $B_j^{\ i}$ satisfy dispersion relations in the variable $\nu = \frac{1}{2}(s - u)$ which we write in the form

IV. THEORETICAL MATTERS

A. Analyticity and Asymptotic Behavior

The sum rules which we study test both the analyticity properties and the high-energy behavior of Compton scattering amplitudes. Although the necessary analyticity--that underlying the dispersion relations (18) --can be proved rigorously from the axioms of quantum field theory, there is very little rigorous information on the asymptotic behavior. We review briefly here the types of asymptotic behavior which our present incomplete theoretical knowledge suggests.

For purely hadronic processes there are some rigorous asymptotic bounds on scattering amplitudes, such as the Froissart bound[26] which can be derived using analyticity and (s-channel) unitarity. For most applications this information is insufficient, and it is customary to assume that asymptotic behavior is determined by the singularities in the angular momentum variable of analytically continued t-channel partial wave amplitudes. This hypothesis, called "Analyticity of the Second Kind" by Chew,[27] effectively means an asymptotic structure of moving Regge poles and cuts.

In theories with analyticity of the second kind, t-channel unitarity plays an important role in determining asymptotic behavior. Its role is reviewed in the discussion of this subsection and references to the original literature are given. Further details, important in understanding our results are precented in subsection C.

Fixed poles in hadronic amplitudes are severely restricted by the t-channel unitarity condition;[28] they are allowed only at angular momentum values for which the unitarity cut is shielded by Regge cuts.

Our present knowledge of this shielding mechanism[29] indicates that fixed poles occur because of the third double spectral function present in relativistic amplitudes and occur at wrong signature nonsense values of angular momentum. These fixed poles do not contribute directly to asymptotic behavior, although they may modify the behavior of Regge-pole residues in an observable way. Schwarz sum rules[17] can be used to test for the presence of these fixed poles.

Compton scattering amplitudes are an example of the general class of "weak" amplitudes--those which never appear bilinearly in a unitarity relation. Because of the absence of bilinear unitarity in the s channel, the Froissart bound cannot be proved in the usual way, and there is at present no rigorous information on high-energy behavior. Further the loss of bilinear unitarity in the t channel means that fixed poles in the angular momentum plane are no longer restricted.

Nevertheless, it is intuitively attractive to assume Regge asymptotic behavior for weak processes, and this was done in most early work on Compton scattering[30] and on more general weak amplitudes.[31] This Regge pole picture led to puzzling features in the Pomeranchuk contribution to physical Compton scattering[30] and in the interpretation of current algebra sum rules.[32] Fixed poles (and Kronecker delta terms[33]) provided the solution to these puzzles.

The known mechanisms for fixed poles in doubly weak amplitudes are discussed in References 18 and 32, and we summarize them here. By doubly weak we mean four-point amplitudes with two hadrons and two currents on external lines. In general such amplitudes will have the j-plane behavior of their Born terms because this behavior is not smoothed by the weak unitarity condition. In particular doubly weak amplitudes will have fixed poles at nonsense integers of both signatures. Usually the strong interactions--i.e. higher-order graphs--modify the residues of the fixed poles so that they differ from their Born values. Modification can be expected for both right and wrong signature fixed poles even if the third double spectral function (dsf) vanishes, although the third dsf mechanism will also contribute to wrong signature fixed poles of weak amplitudes.

In general, therefore, the theory tells us the locations of fixed poles but is not powerful enough at present to predict their residues which depend on the details of strong interactions. Sum rules, as we will see, can be used to evaluate the fixed pole residues directly from the experimental data.

There are two exceptions in which the general theory does give information about the fixed pole residues. The first occurs in Compton scattering[18] where, because of photon masslessness, the Born terms of some amplitudes have a singular coefficient of t^{-1}. This may be observed in Table I for amplitudes B_1, B_3, B_5, B_7, and B_8. Since other contributions to the amplitude are regular at $t = 0$, the residue of the fixed pole at the highest nonsense point is also singular at $t = 0$ and is determined there by the Born term. This mechanism works in other kinematical configurations also.[34] Unfortunately, the corresponding sum rules reduce to simple identities at $t = 0$, to which only the Born terms contribute, and are thus devoid of interest.

The second exception in which theory actually predicts the fixed pole residue as a function of t concerns current algebra. It has been shown[32] that the well-known (and variously credited) Adler-Dashen-Fubini-Gell-Man sum rules imply that the sum rule amplitudes have fixed poles at $j = 1$ and that the residues are given in terms of vector and axial vector hadronic form factors. An observed failure of the sum rules would imply either (1) that the underlying algebra of currents must be modified, or (2) that the assumptions necessary to derive the sum rule from the algebra are incorrect,[4] or (3) both (1) and (2) are true. It may also be possible to relate the residues of fixed poles at $j = 0$ and $j = -1$ to properties of the current algebra.[11,16,35]

We have stressed that the basic mechanism which permits fixed poles in weak amplitudes is the breakdown of bilinear unitarity. Linear or weak unitarity still requires factorization for Regge-pole couplings to weak amplitudes as we show in subsection C. One effect of fixed poles is usually to make Regge-pole residues more singular at nonsense integers than they would otherwise be. This effect will be seen clearly through our sum rules.

B. Sum Rules and Fixed Poles

The preceding arguments motivate us to assume that the typical asymptotic behavior of the $B(\nu,t)$ amplitudes is (with η denoting the crossing phase),

$$B(\nu,t) \sim - \sum_r G_r(t)(e^{-i\pi\alpha_r(t)} + \tau_r)(\sin \pi\alpha_r(t))^{-1} \nu^{\alpha_r(t)-\lambda} - \sum_{k=1}^{\infty} F_k(t)\nu^{-k} [1 + \eta(-1)^k] + \sum_{m=0}^{M} D_m(t)\, \nu^m[1 + \eta(-1)^m] \tag{21}$$

corresponding to Regge poles (of leading signature $\tau = \eta(-)^{\lambda}$), right signature fixed poles (at $j = \lambda - k$) and Kronecker deltas (at $j = \lambda + m$). Wrong signature fixed poles manifest themselves in (21) only in their effect on the $G_r(t)$. We ignore possible Regge cuts because our sum rules are not accurate enough to distinguish between poles and cuts. Nonleading Regge pole terms (20) can easily be included in spin types 2 and 3.

The sum rules we use can now be derived very easily. The functions $\nu^n B(\nu,t)$ are analytic in the cut ν plane and therefore satisfy

$$\frac{1}{2\pi i} \oint_C d\nu \quad \nu^n B(\nu,t) = 0 \tag{22}$$

where C is the contour of Fig. 1. We evaluate the integral over the semicircular portions approximately by using the asymptotic form (21) and taking ν_c as the radius of the semicircle. We collapse the contour to the cut, separate out the Born contribution and obtain the resulting sum rule

$$-\nu_B^{\ n} C(t) + \frac{1}{\pi} \int_{\nu_0}^{\nu_c} d\nu \ \nu^n \ \mathrm{Im}\ B(\nu,t) = \frac{1}{\pi} \sum_r G_r(t) \ \frac{(\nu_c)^{\alpha_r(t)+n-\lambda+1}}{\alpha_r(t) + n - \lambda + 1}$$

$$+ F_{n+1}(t) \tag{23}$$

for n satisfying $(-)^{n-\lambda} = -\tau$, and a trivial identity for $(-)^{n-\lambda} = +\tau$. We remind the reader of our notation $\nu = \frac{1}{2}(s - u)$, $\nu_B = \frac{1}{2} t$, and $\nu_0 = 2m\mu + \mu^2 + \frac{1}{2} t$.

Notice that the nth moment sum rule is sensitive only to the fixed pole at $j = \lambda - n - 1$, and totally insensitive to possible Kronecker delta terms. The latter, as we shall see, can be tested using the dispersion relations (18) in which experimental values of the real part of the amplitude can be inserted.

Wrong signature sum rules[17] can be similarly derived by considering an artificial amplitude $\overline{B}(\nu,t)$ with the same right-hand cut and the negative left-hand cut of the corresponding $B(\nu,t)$. Wrong signature fixed poles manifest themselves in the asymptotic behavior of $\overline{B}(\nu,t)$. The sum rule is derived by considering the integral of $\nu^n\, \overline{B}(\nu,t)$ over the contour C. For n satisfying $(-)^{n-\lambda} = -\tau$ the result is a trivial identity, and for $(-)^{n-\lambda} = +\tau$ we obtain a sum rule identical in form to (23) with $F_{n+1}(t)$, as the asymptotic coefficient of the wrong signature fixed pole term at $j = \lambda - n - 1$. Therefore we can understand Eq. (23) as valid for all integer n and testing right (wrong) signature fixed poles for $(-)^{n-\lambda} = \mp\tau$.

Using an intermediate state expansion of the retarded product (4), it is easy to see that only the second term of the commutator contributes to the left-hand cut of the amplitudes $B(\nu,t)$. It is therefore amusing to note that the corresponding signatured amplitude $\overline{B}(\nu,t)$ is formally given by an anticommutator expression, and its fixed pole residues are formally determined by equal time anticommutators. We refer to this situation as anti-algebra.

The operation of the fixed pole mechanisms discussed above can be clearly seen in Eq. (23). For amplitudes with singular Born term $C(t)$ the left side of the $n = 0$ sum rule is singular at $t = 0$. This singularity must be matched on the right side either by the fixed pole residue $F_1(t)$ or by the contribution of a Regge pole satisfying $\alpha(0) = \lambda - 1$. In nonvacuum channels, there is no indication of the existence of Regge trajectories with the necessary properties,[36] and we must expect a fixed pole at the highest nonsense point $j = \lambda - 1$ with residue singular at $t = 0$. In vacuum channels, the Pomeranchuk trajectory has the required intercept and the Born singularity can be matched either by the singular Pomeranchuk term on the right side of (23) or by a wrong signature fixed pole at $j = 1$. The sum rules can be used to distinguish between these alternatives.

We also observe that if a Regge trajectory passes through the nonsense integer $\alpha(t_0) = \lambda - n - 1$ for some t_0 and if $G(t_0) \neq 0$, the Regge pole term in nth moment sum rule has a pole at $t = t_0$. This pole is not present on the left side of Eq. (23), because we are dealing with a nonsense or unphysical point, and it must therefore be cancelled by a similar pole in the fixed pole residue $F_n(t)$. Current algebra amplitudes, where $F_n(t)$ is a form factor with the ρ-meson pole, are an example of this mechanism.

Curiously enough the fixed-pole residue function can have poles at spacelike t values. If $G_\rho(t_0) \neq 0$ (or $G_{A_2}(t_1) \neq 0$) at the negative t value t_0 (or t_1) where $\alpha_\rho(t_0) = 0$ (or $\alpha_{A_2}(t_1) = 0$), the $j = 0$ wrong (or right) signature fixed-pole residue develops a pole at t_0 (or t_1) corresponding to the nonsense ghost state on the trajectory. In the wrong signature case this is clearly a triumph of anti-algebra.

C. Unitarity, Factorization, and Fixed Poles

In this subsection we discuss two principle results, both essentially known. First we show that t-channel unitary requires factorization for the couplings of Regge poles to both weak and hadronic channels. Proof of the absence of fixed poles requires a more stringent form of unitary satisfied only in hadronic amplitudes. Second we show that fixed double poles should be expected at nonsense integers in four-point amplitudes with all lines weak. Knowledge of this fact is required to understand our results for the current algebra sum rules.

To prove factorization we generalize the argument given by Oehme,[37] which requires only the analytically continued partial wave unitarity condition and the existence of a nondegenerate two-particle threshold preferably involving stable hadrons. In $\tau P = +1$ amplitudes the necessary threshold is provided by the $\pi\pi$ $(G = +1)$ or $K\bar{K}$ $(G = \pm 1)$ channels. For $\tau P = -1$ amplitudes the $N\bar{N}$ threshold is nondegenerate. We will use the $K\bar{K}$ channel for $\tau P = +1$ since it is present for both G parities.

First let us introduce the $\tau P = +1$ partial wave amplitudes $a_{jk}(t,J)$ (evaluated on the physical sheet) where j and k denote channel indices, according to the following assignment: $j = 1, K\bar{K}$; $j = 2, N\bar{N}$ helicity nonflip; $j = 3$, $N\bar{N}$ helicity flip; $j = 4$, $\gamma\gamma$ helicity nonflip; and $j = 5$, $\gamma\gamma$ helicity flip. Our argument applies to all weak channels; although we restrict ourselves, for definiteness, to the $\gamma\gamma$ channel which is doubly degenerate.

Denote the analytic continuation of $a_{jk}(t,J)$ onto the sheet reached by continuing through the $K\bar{K}$ threshold by $\overset{II}{a}_{jk}(t,J)$. Since

$a_{jk}(t,J)$ can be chosen symmetric in the channel indices, the requirement of unitarity can be written

$$a_{jk}(t,J) - a_{jk}^{II}(t,J) = 2i\rho(t)\, a_{j1}(t,J) a_{k1}^{II}(t,J) \qquad (24)$$

where

$$\rho(t) = \left(\frac{t - 4m_K^{\,2}}{t} \right)^{1/2}$$

and then reexpressed as the set of equation

$$a_{11}(t,J) = a_{11}^{II}(t,J) + \left[\frac{2i\, \rho \left(a_{11}^{II}(t,J)\right)^2}{1 - 2i\rho\, a_{11}^{II}(t,J)} \right], \qquad (25)$$

$$a_{j1}(t,J) = a_{j1}^{II}(t,J) + \left[\frac{2i\, \rho\, a_{j1}^{II}(t,J)\, a_{11}^{II}(t,J)}{1 - 2i\, \rho\, a_{11}^{II}(t,J)} \right], \text{ for } j \neq 1, \qquad (26)$$

$$a_{jk}(t,J) = a_{jk}^{II}(t,J) + \left[\frac{2i\, \rho\, a_{j1}^{II}(t,J)\, a_{1k}^{II}(t,J)}{1 - 2i\, \rho\, a_{11}^{II}\,(t,j)} \right], \text{ for } j,k \neq 1. \qquad (27)$$

Regge poles occur in the following way. There is an analytic trajectory function $J = \alpha(t)$ for which $1 - 2i\, \rho\, a_{11}^{II}(t,\alpha(t)) \equiv 0$. It is easily seen that, as far as the second terms in Eqs. (25)-(27) are concerned, the Regge pole appears in all amplitudes of the coupled channel system, and that its residues factor. Therefore factorization could be spoiled only if the second sheet function $a_{jk}^{II}(t,J)$ contained the moving pole at $J = \alpha(t)$. This exceptional circumstance, corresponding to a zero of the multichannel D function of rank higher than one, cannot be ruled out. In the language of this proof it means that there is really a second Regge pole miraculously with the same trajectory $\alpha(t)$ which does not couple to the $K\overline{K}$ state but couples to higher mass hadronic channels. In this sense factorization is the normal case. A similar proof of factorization for $\tau P = -1$ ples can be constructed using an even simpler set of channels.

One important aspect of this argument is that weak and strong channels enter on equivalent footing. Proofs of the absence of fixed poles in multichannel systems require the existence of an intermediate state threshold for each external channel considered[38] and therefore apply only to hadronic channels.

Let j be a weak channel and k be a hadronic channel. It is clear that Eqs. (26) and (27) permit the presence of fixed poles in doubly weak amplitudes, and we have reviewed in subsection A several arguments showing that fixed poles actually are present at nonsence integers. If we take both j and k to be weak channels, Eq. (27) strongly suggests the presence of fixed <u>double</u> poles at nonsense integers.

Our interest in this last point is the following. First we observe, using (26) and (27), that fixed poles in doubly weak amplitudes at integer j_0 generally induce $[\alpha(t) - j_0]^{-1}$ factors in the Regge residues of those amplitudes. Similarly fixed double poles lead to $[\alpha(t) - j_0]^{-2}$ factors in the Regge residues of completely weak four point amplitudes. Our study of the current algebra sum rules indicates that the ρ-meson

Regge pole coupling in smooth and nonvanishing near $t = -0.6\ (\mathrm{GeV}/c)^2$ whereas hadronic amplitudes generally exhibit the well-known nonsense zero (dip) there.[14] This situation is consistent with factorization only because singular $\gamma\gamma$ couplings, corresponding to a fixed double pole at $j = 0$ in the $\gamma\gamma \to \gamma\gamma$ amplitude, are allowed.

D. Conspiracy

We turn our attention now to the conspiracy condition Eq. (15) which relates at $t = 0$ the amplitude B_4 containing $\tau P = -1$ trajectories in the t channel to the amplitude B_6 containing $\tau P = +1$. We suppress the isospin superscripts in this discussion. Since Eq. (15) holds identically in s, it imposes constraints on the residues at $t = 0$ of these trajectories. Either the couplings $G_4(t)$ and $G_6(t)$ vanish at $t = 0$ (evasion) or there exists pairs $\alpha_-(t)$ and $\alpha_+(t)$ of negative and positive τP trajectories satisfying the conditions (of conspiracy)

$$\alpha_-(0) = \alpha_+(0)$$

$$G_4(0) = -\frac{1}{4m}\, G_6(0). \tag{28}$$

Sum rules for amplitudes B_4 and B_6 can, in principle, be used to investigate possible conspiracies for the π (isospin segment 3) and η (isospin segments 1 and 2). One would simply explore the sum rules as functions of t for several moments to obtain a parameterization of the trajectories and residues. Although this technique has recently been used to investigate π conspiracy in the process $\gamma N \to \pi N$,[39] it does not seem possible to use it for Compton scattering, at least with presently available data. First the B_4^3 sum rule has $\lambda = 0$ and $n_{min} = 1$; it diverges badly asymptotically, emphasizing the most inaccurately known part of the data. Second the B_6^3 sum rule, although more accurate, is useful only near $t = 0$ for determining the parameters of conspirator trajectories because important contributions from non-conspiring trajectories (such as A_2) mix in away from that point.

Hence the only number which can be determined from the B_4 and B_6 sum rules and associated with the parameters of a single Regge trajectory with relative confidence is the value of the B_6 sum rule at $t = 0$. However even this number provides an interesting test of conspiracy, through a sum rule of Pagels,[20] which we rederive here to incorporate recent clarification of the questions of conspiracy and of the relation between asymptotic behavior and subtractions.

We start with the $n = 0$ sum rule for $B_6^3(\nu,t)$ assuming domination by a single Regge pole and a right signature fixed pole at $j = 0$:

$$-C_6^3(t) + \frac{1}{\pi} \int_{\nu_0}^{\nu_c} d\nu \; \mathrm{Im}\, B_6^3(\nu,t) = G(t) \frac{(\nu_c)^{\alpha(t)}}{\pi\alpha(t)} + F(t). \tag{29}$$

Now set $t = 0$, evaluate the Born term using Table I, and reexpress the continuum contribution using the conspiracy condition (15):

$$-e^2[2\kappa_p + \kappa_p^2 - \kappa_n^2] - \frac{4m}{\pi} \int_{\nu_0}^{\nu_c} \frac{d\nu}{\nu} \; \mathrm{Im}\, B_4^3(\nu,0) = G(0) \frac{\nu_c^{\alpha(0)}}{\pi\,\alpha(0)} + F(0). \tag{30}$$

We proceed with the derivation under two different assumptions.

1. Pure Reggeism

We assume that the π meson lies on a Regge trajectory $\alpha_\pi(t)$ which couples to the B_4^3 amplitude with strength $G_\pi(t)$. If $G_\pi(0) \neq 0$ then there is a conspirator $\alpha_c(t)$ which couples to B_6^3 with strength $G_c(t)$, and these functions may be identified with the Regge functions in Eq. (30). We set $F(t) \equiv 0$.

The amplitude B_4^3 has a pole at $t = m_\pi^2$ corresponding to the π^0 intermediate state in the t channel. The residue of the pole is closely related to the lifetime of the π^0. Using $B_4(\nu,t) = A_3(\nu,t)$ where A_3 is the Hearn-Leader amplitude, and comparing the residue of the pole in the π^0-Regge pole term defined in (21) with Eqs. (2.8) and (2.12) of Reference 20 we find

$$\frac{-2\, G_\pi(m_\pi^2)}{\pi\, \alpha'_\pi(m_\pi^2)} = g_{\pi N}\; m_\pi^2\, F_\pi(m_\pi^2)$$

$$F_\pi^2(m_\pi^2) = \frac{64\,\pi}{m_\pi^3\,\tau} \tag{31}$$

where τ is the π^0 lifetime and $g_{\pi N}$ is the $\pi N\bar{N}$ coupling constant. We assume that the π Regge pole coupling $G_\pi(t)$ varies slowly with t so that

$$G_\pi(m_\pi^2) \approx G_\pi(0)\ . \tag{32}$$

We use (24) to rewrite (30) (with $F(t) = 0$) as

$$\ell\text{.h.s. of (30)} = -G_\pi(0)[4m\pi\alpha_\pi(0)]^{-1} \nu_c^{\alpha_\pi(0)}$$

$$\approx G_\pi(m_\pi^2)[4m\pi\, m_\pi^2\, \alpha'_\pi(m_\pi^2)]^{-1} \nu_c^{\alpha_\pi(0)} . \qquad (33)$$

Using (31) we then obtain

$$\frac{e^2}{4m}(2\kappa_p + \kappa_p^2 - \kappa_n^2) + \frac{1}{\pi}\int_{\nu_0}^{\nu_c} \frac{d\nu}{\nu}\, \text{Im}\, B_4^3(\nu,0)$$

$$= \frac{4\, g_{\pi N}}{m_\pi^2}\left(\frac{\pi\, m_\pi}{\tau}\right)^{1/2} \nu_c^{\alpha_\pi(0)} \qquad (34)$$

which is the form of Pagels' sum rule appropriate for pure Regge behavior.

2. Elementary π

Here we assume that Regge pole terms are unimportant on the right side of Eq. (30), and that the sum rules evaluates the residue of the $j = 0$ right signature fixed pole. If $F(0) \neq 0$, as our numerical result shows, then the conspiracy condition requires a Kronecker δ_{j0} term[33] in the amplitude B_4^3 with asymptotic coefficient $D_0(0) = -4mF(0)$ at $t = 0$. We assume that the Kronecker δ_{j0} coefficient has a pole at $t = m_\pi^2$ corresponding to the elementary π^0 meson and that this pole term dominates at $t = 0$. We then can write

$$-8m\, F(0) \approx g_{\pi N}\, F_\pi(m_\pi^2) \qquad (35)$$

and the sum rule (30) becomes

$$\frac{e^2}{4m}\left(2\kappa_p + \kappa_p^2 - \kappa_n^2\right) + \frac{1}{\pi}\int_{\nu_0}^{\nu_c} \frac{d\nu}{\nu}\,\mathrm{Im}\,B_4^3(\nu,0)$$

$$= \frac{4\,g_{\pi N}}{m_\pi^2}\left(\frac{\pi\, m_\pi}{\tau}\right)^{1/2} . \qquad (36)$$

At present practicable cutoff energies one cannot distinguish between (34) and (36), and therefore one cannot directly probe the Regge pole nature of the pion in Compton scattering. The sum rule does provide a check on the overall strength of the asymptotic structure corresponding to the π meson and on the assumption of smooth variation of the effective π-pole residue. The sum rule for amplitude B_7^5 in which the π trajectory can be exchanged although $j = 0$ is a nonsense point also provides some information on conspiracy.

Sum rules similar to (34) and (36) can be written for the η meson. We refer the reader to Section VI for further discussion of our results on conspiracy.

E. Polarizability and Kronecker Deltas

We finally discuss a possible test for the presence of Kronecker delta terms in physical Compton scattering amplitudes.

The amplitude $B_1^i(\nu,\ t{=}0)$, in isospin segments 1 - 3, satisfies the dispersion relation

$$B_1{}^i(\nu,0) = -\frac{a^i\, me^2}{\nu^2} + \frac{1}{\pi}\int_{\nu_0}^{\infty} d\nu' \, \frac{2\nu' \, \mathrm{Im}\, B_1{}^i(\nu,0)}{\nu'^2 - \nu^2} + c^i + d^i\, \nu^2 \tag{37}$$

where we have included contributions of Kronecker deltas at $j = 2$ and $j = 4$. The nucleon-pole coefficient is $a_1 = a_2 = \frac{1}{2}\, a_3 = 2$. Using the crossing relations (10) at $t = 0$, we find

$$\begin{aligned} \nu^2\, B_1{}^i(\nu,0) &= m(M_{\frac{1}{2}1;\frac{1}{2}1} + M_{\frac{1}{2}-1;\frac{1}{2}-1}) \\ &= 4\, m^2\, f_1{}^i(\nu) \end{aligned} \tag{38}$$

where $f_1(\nu)$ is the forward spin-averaged Compton scattering amplitude of the classical era of dispersion relations.[40] A power series expansion about $\nu = 0$ gives

$$f_1{}^i(\nu) = -\frac{a^i\, e^2}{4m} + b^i\, \nu^2 + O(\nu^4)\,. \tag{39}$$

The parameter b^i is related quite simply to the energy derivative at threshold of the forward unpolarized Compton scattering differential cross section,[41, 42] and to the sum of electric and magnetic polarizabilities of the nucleon[43] by $4m^2\, b^i = 4\pi(\alpha^i + \beta^i)$. Combining (37) - (39) and using the optical theorem, we find for the polarizability sum:

$$(\alpha + \beta)^i = \frac{m}{\pi^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2}\, \sigma_t{}^i(\nu) + \frac{c^i}{4\pi}\,. \tag{40}$$

This sum rule has long been known in the form with $c^i = 0$ (no Kronecker delta) and has been used to constrain a two-parameter fit to low energy Compton scattering.[44] Drell[42] has recently emphasized the importance of using Eq. (40) to test for the presence of the δ_{j2} term in the asymptotic behavior of proton Compton scattering.[45] In this case the total photo absorption cross section is known up to 6 GeV, and the rapidly convergent integral term can be quite accurately estimated from the data. Ironically it is the polarizability sum, which could be determined in low energy (20 - 80 MeV) Compton scattering experiments, which is unknown. Thus we have here a situation in which measurement of a single low-energy parameter can answer an important question in high energy physics, and we join Drell in urging active consideration of low energy proton Compton-scattering experiments.

Our contribution to the question of the Kronecker delta term consists of the evaluation of the integral term in Eq. (40) in isospin segments 1 - 3.

V. TREATMENT OF EXPERIMENTAL DATA

The most conspicuous feature of the nucleon Compton process is the lack of direct experimental data. Since the sum rules (23) involve only the imaginary parts of Compton amplitudes, we use unitarity to express the integrands bilinearly in terms of hadronic photoproduction amplitudes.

The unitarity condition is shown schematically in Fig. 2. One must sum the contributions from all intermediate states that are energetically allowed. Experimentally the quasi-elastic (πN) intermediate state dominates[46] up to photon lab energies (E_{lab}) of 0.5 GeV, and between 0.5 and 1.1 GeV the inelastic contribution is dominated by the $\pi\pi N$ state in the configuration $\pi\Delta$.

In studies of sum rules for the processes $\pi N \to \pi N$, $KN \to KN$,[7] and $\gamma N \to \pi N$,[39,47] there is "experimental data" available for both real and imaginary parts of the amplitudes. This leads to two advantages which we do not enjoy. First continuous moment sum rules, involving real parts, can be used. Second inelasticity is automatically incorporated, and one need not treat individually the contributions of different intermediate states.

A. πN Intermediate State

There have been many theoretical and phenomenological attempts to describe low-energy photoproduction,[1,2,48-50] $\gamma N \to \pi N$. Only two of these are sufficiently complete for our purposes, since we require a description of photoproduction amplitudes which is accurate as to phase, helicity and isospin dependence. The multipole analysis of Walker[2] is a direct fit to the experimental data, up to photon energies of 1.2 GeV, using a Born term, Breit-Wigner terms for known resonances, plus correction terms. Berends, Donnachie, and Weaver[1] (BDW) have given a more theoretical treatment, based on dispersion relations, which extends only to $E_{lab} = 0.5$ GeV. Their results do not fit the data as well as Walker but probably contain a better estimate of the helicity and isospin structure of the background.

In our estimate of the πN contribution to Im B we calculate the integral upto 0.5 GeV using both BDW and Walker and compare the two evaluations. Above this energy we use Walker's analysis. In the low energy region there is often serious discrepancies between the BDW and Walker multipoles, particularly for isoscalar photons. When one calculates the experimental $d\sigma/dt$ for photoproduction this difference shows up most clearly in $\gamma n \to \pi^- p$ where BDW predicts a much flatter t distribution than Walker for the energy range $0.4 \leq E_{lab} \leq 0.5$. The data used by Walker would appear to agree with his own analysis[2] and not BDW[1]!

To illustrate the importance of this difference we plot in Figs. 3-5 the values of $\frac{1}{\pi}$ Im B at $t = 0$ for three sum rules of particular interest. One $(B_3^{\ 4})$, the helicity flip current algebra amplitude, has a small discontinuity at 0.5 GeV between the BDW and Walker analyses. The Drell-Hearn amplitude involving two isoscalar photons $(B_2^{\ 1})$ is badly discontinuous while the discontinuity of $B_2^{\ 3}$, the Drell-Hearn amplitude in which isoscalar and isovector photons interfere, is intermediate between these two extremes.

Both the BDW and Walker data are essentially given directly in terms of multipoles. To calculate Im B for our sum rules, we use Eq. (A.2) expressing the B_j^i in terms of s-channel helicity amplitudes and then decompose into partial waves. Then the partial wave unitarity equation[22,25] enables us to express the Compton scattering partial wave amplitudes in terms of photoproduction multipoles.

There is, unfortunately, a technical difficulty in this approach in that the box diagram of Fig. 6, leads to a divergence of the partial wave series for $t \lesssim -0.28$. This was countered by calculating (in a way too inelegant to reveal) the divergent part of Fig. 6 and subtracting its partial wave decomposition from the divergent series produced by the photoproduction multipoles.

B. Inelastic Intermediate States

We must now turn to the insertion of inelastic intermediate states in our unitarity sum. In the energy range of interest $\pi\pi N$ is the most important inelastic state and this is predominantly produced in the quasi-two body state $\pi\Delta$.[3,46] Thus at 0.7 GeV $\pi\Delta$ is essentially 100% of the inelasticity while at $E_{lab} = 1.1$ GeV it is more like 50 → 70%.

In order to describe $\pi N \to \pi\Delta$ we use the Stichel-Scholz[3] model which approximates[51] the amplitude by the s-channel nucleon Born term and the u channel Δ pole of Fig. 7. We chose to calculate these graphs by fixed t-dispersion relations utilizing the known residues at the poles. Then by gauge invariance the t channel one π exchange term (Fig. 8) is automatically included. This model fits the data well near $t = 0$ both in $d\sigma/dt$ and the density matrix elements ρ_{33}, ρ_{31}, ρ_{3-1} describing the decay of the Δ.

This calculation ignores the magnetic moments of the N and the Δ which are important away from $t = 0$. Other obvious omissions are the higher s-channel resonances, which can be estimated, and the u-channel resonances, which cannot, due to the unknown $\gamma\Delta \to N^{**}$ coupling. One effect of these omitted terms is to destructively interfere with the Born terms of Fig. 7, and reduce the calculated cross section. They also introduce of course nonzero values into helicity and isospin states not populated in the model of Fig. 7. The relative size of these effects may be estimated by examining $\gamma N \to \pi N$ at large $|t|$ where the mass difference of N and Δ becomes negligible and we have similar kinematics. However, we contented ourselves with taking the amplitude of Fig. 7 and multiplying it by a form factor $F(t)$ determined so as to fit the experimental values of $d\sigma/dt$ for $\pi N \to \pi\Delta$. This simulates the destructive interference at large $|t|$ of the omitted terms but not the population of new helicity and isospin states. The helicity structure thus obtained is essentially the same as that given by an absorption model calculation based on the one-pion exchange graph (Fig. 8). Thus Fig. 7, with form factor, already contains the most important effects given by absorptive corrections. A typical $F(t)$ at $E_{lab} = 0.85$ GeV was given by:

$$F^2(t) = 0.66 \exp(-2.9t - 12t^2) .$$

It may be worth noting that in our modified Stichel-Scholz model for $\gamma N \to \pi\Delta$, the amplitudes involving isoscalar photons vanish. We expect the isoscalar photon contribution to be small (because there is no π exchange pole) and of the same order as many omitted effects in the isovector part. Such effects are difficult to estimate.

In order to find the contribution to Im B of the $\pi\Delta$ state we follow the same procedure as for πN. Namely we decompose $\pi N \to \pi\Delta$ into partial waves and use partial wave unitarity.[22] We note that the diagram of Fig. 9, does not cause a divergence of the partial wave series until $t \approx -1.2$ and so we need no special action like that necessary for Fig. 6.

In order to describe the inelasticity not produced in the $\pi\Delta$ intermediate state we add incoherently the contributions of higher resonances as in Fig. 10 multiplied by the factor

$$\frac{\Gamma_{inel} - \Gamma_{\pi\Delta}}{\Gamma_{tot}}$$

so as to get the fraction not already included in the πN and $\pi\Delta$ states.

Since we must use both the $\gamma N \to \pi N$ multipole analyses[2] and the $\pi N \to \pi N$ phase shifts[8] in order to extract the $\gamma N \to N^{**}$ coupling by factorization, the incoherent resonance contribution is ambiguous because of differences in the resonance mass and width parameters in Refs. 2 and 8. There are further ambiguities due to our inaccurate knowledge[52] of the $\pi\Delta$ partial widths $\Gamma_{\pi\Delta}$ and because of defects in the treatment of resonances in our model for $\gamma N \to \pi\Delta$. These ambiguities are taken into account in our error analysis.

Finally we would like to record a possibly more fundamental objection to the simulation of inelastic effects in weak amplitudes using a resonance dominance model. In hadronic amplitudes large t-channel contributions (such as our π exchange in $\gamma N \to \pi\Delta$) violate the unitarity bound in the s channel and usually lead to an s-channel resonance which can give an alternate description of the t-channel phenomenon. In weak processes such as photoproduction and Compton scattering, there is no unitarity bound and there is less reason to believe that t-channel exchanges can be reasonably described by s-channel resonances. We realize that vector dominance relates Compton scattering to strong processes (e.g. $\rho N \to \rho N$) but this only depends the mystery.[53]

C. Errors in Evaluation of Sum Rule

We assigned errors to our sum rules by the following method. Divide the contribution to the sum rule into ten pieces. Seven of these coming from the πN intermediate state (namely Walker's 6 resonant partial waves S_{11}, P_{11}, P_{33}, D_{13}, D_{15}, F_{15} plus the sum of nonresonant partial waves) plus one piece each for the $\pi\Delta$ and non-$\pi\Delta$ inelastic contributions. The last contribution is the nucleon form factor needed for the fixed pole in the current algebra sum rules $B^{4}_{1\to 3}$. The error in the last is estimated from the dispersion in the various fits of Ref. 54. The first 9 quantities were assigned preset errors ranging from 10% for well determined isovector photon couplings to 100% for some isoscalar couplings. The size of the discontinuity between BDW and Walker at 0.5 GeV was a help in judging these errors. The total error is found by adding the above as uncorrelated errors to an error estimated from assuming the discontinuity at 0.5 GeV propagated over an s range chosen as 0.3 GeV^2.

Although this arbitrary method cannot be trusted to give more than a rough indication of the error in any given sum rule, we might hope that it does give an accurate picture of the relative errors of the sum rules for different isospins, helicities and t values.

VI. ANALYSIS OF SUM RULES

A. General Properties

We finally come to a description of our evaluation of the sum rules (23). We have calculated the left-hand side of (23) for t varying between 0 and -0.9 and for all 26 sum rules corresponding to the various spin and isospin states. We have also taken different values of n in the range 0 to 3, thus obtaining information about both right and wrong signature fixed poles in (23). We have selected from these the most interesting sum rules and present our results graphically in Figs. 11-28. Before commenting on the significance of these results, we will describe the meaning of the sundry quantities plotted in the figures.

The integrals $I_j^{\,i}(n)$ are defined to be the left-hand side of (23) evaluated in units such that $\hbar = c = \text{GeV} = 1$. Thus

$$I_j^{\,i}(n) = -(t/2)^n\, C_j^{\,i}(t) + \frac{1}{\pi}\int_{\nu_0}^{\nu_c} d\nu\, \nu^n\, \text{Im}\, B_j^{\,i}(\nu,t) \tag{41}$$

where the first term is the Born contribution.

Here the cutoff ν_c corresponds to a photon lab energy of 1.12 above which the published data[55] on $\sigma_{total}(\gamma p)$ shows our model for $\text{Im}\, B_j^{\,i}$ to be undoubtedly wrong.

In the graphs ◇ represents the integrals $I_j^{\,i}(n)$ with errors estimated as described in Sec. VC. The integrals are evaluated using the BDW multipole analysis[1] from the threshold to 0.5 GeV and Walker's analysis[2] above that energy. All the sum rules have also been evaluated with Walker's multipoles for the whole energy range, eliminating BDW. Usually the difference between these evaluations is smaller than our estimated errors but where they differ significantly we also graph the pure Walker evaluation of $I_j^i(n)$ which we denote by $\Box$.

The Born term contribution to $I_j^i(n)$ is represented by a solid line where in the current algebra sum rules $I^4_{1,2,3}$ this also includes the fixed pole contribution. In $I_2^4(1)$ the dotted line indicates the Born term without the fixed pole.

The lowest value $n = n_{min}$ (0 or 1) such that (23) is a right signature sum rule is given in Table 1. In theory one may use the value of $I_j^i(n_{min})/I_j^i(n_{min} + 2)$ to estimate a value for the intercept α of the Regge pole assumed to saturate both sum rules. However the presence of unknown fixed poles in $I_j^i(n_{min} + 2)$ renders this dubious in our case. Instead for sum rules $I_j^i(n \neq n_{min})$ we plot the quantity (denoted by $\triangle$ on the graph)

$$Q_j^i(n) = \frac{\alpha - \lambda + n_{min} + 1}{\alpha - \lambda + n + 1} \nu_c^{n-n_{min}} I_j^i(n_{min}) \tag{42}$$

where for α we put the values already known from the analysis of strong interactions. We include generous errors in our knowledge of α in the plotted errors of $Q_j^i(n)$. If $Q_j^i(n)$ and $I_j^i(n)$ differ significantly it may indicate the presence of a fixed pole.

In spin type 2 we indicate with X an estimate of the non-asymptotic parts of P, P′, ρ and A_2 exchange calculated from (19), (20) and Appendix C as

$$N_2^i(n) = -\frac{(2-\alpha)t}{2\alpha} \frac{(\alpha + n_3 - 1)}{(\alpha + n - 2)} \nu_c^{n-n_3-1} I_3^i(n_3) \tag{43}$$

where n_3 is the value of n_{min} for spin type 3 and the same isospin i.

Finally in the conspiracy sum rules (spin type 6) we indicate with a ∇ symbol an estimate of the nonconspiring contribution calculated from factorization as

$$I_6^{\,i}(0) = \frac{-t}{2\nu_c^{\,2}} \; \frac{\alpha + 2}{\alpha} \; I_5^{\,i}(1) \; I_3^{\,i'}(1)/I_1^{\,i'}(1) \quad , \qquad (44)$$

and we restrict to $i = 1, 2, 3$ as $i = 4$ has a (known) fixed pole. For $i = 3$ we have $i' = 3$, while for $i = 1, 2$ we take $i' = 2$ as being more reliable than $i' = 1$ (because isovector photon couplings are more accurately determined than isoscalar).

The main tools in the analysis of our results are the sum rule graphs just described. Perhaps the most important thing we are interested in, is to discriminate between Regge pole and fixed pole contributions to the sum rules. For higher moment sum rules this can be done through the quantity $Q_j^{\,i}(n)$ of Eq. (42). For some lowest moment $(n = n_{min})$ sum rules we exploit the factorization property of Regge residues (this has already been used in obtaining (44)). For example, the amplitudes $\text{Im}\, B_1^{\,i}$ and $\text{Im}\, B_3^{\,i}$ are dominated at high energy by, respectively, the nucleon helicity nonflip and nucleon helicity flip couplings of the <u>same</u> Regge pole. If there are no fixed pole contributions to the sum rules $I_1^{\,i}$ and $I_3^{\,i}$, then factorization (see Appendices B and C) requires

$$I_1^{\,i}/I_3^{\,i} = N_n/2N_f = \frac{(4m^2 - t)A'}{2\nu_c\, B} \qquad (45)$$

where A' and B are the conventional nonflip and flip residues used to describe πN and KN scattering.[56] If the sum rule ratio agrees with the value calculated from hadronic processes, then we have evidence suggesting that the fixed pole contribution to these sum rules is unimportant.

Another quantity which is sensitive to fixed pole contributions to the sum rules is the effective trajectory $\alpha_j^i(n)$ (which is also a function of t) defined numerically by

$$\alpha_j^i(n) = (\lambda - n - 1) + \frac{\nu^{n+1} \text{ Im } B_j^i(\nu,t)}{\pi \quad I_j^i(n)} \qquad (46)$$

where we average the numerator over energies E_{lab} between 0.88 and 1.12 GeV. This quantity is the trajectory $\alpha(t)$ whose Regge term (as in (21)) both saturates the sum rule $I_j^i(n)$ and fits the imaginary part data averaged over the upper end of our integration range. By examing Eqs. (21) and (23) one can see the following. If $\alpha_j^i(n)$ comes out reasonably close in shape to the trajectory known to couple to the amplitude B_j^i, then this indicates that the fixed pole in that amplitude is weak. However, if $\alpha_j^i(n)$ turns out closer to the fixed pole value $(\lambda - n - 1)$ to which the sum rule $I_j^i(n)$ is sensitive, then we have evidence for a strong fixed pole which contributes to the denominator in (46) but not to the numerator since a fixed pole term is purely real.

Graphs of the quantity $\alpha_j^i(n)$ are used whenever their accuracy allows useful information to be extracted. The plotted errors in the graphs include those of $I_j^i(n)$ and the dispersion obtained by varying the numerator in (46) over the energy range 0.88 to 1.12 GeV. Unfortunately $\alpha_j^i(n)$ is rather sensitive to errors in the parameterization of the data near 1 GeV and depends on the dubious assumption of the validity of Regge behavior at this low energy. For this reason evidence from the effective α graphs must be taken with a healthy grain of salt.

B. Regge Pole Sum Rules: $I_{1,3}^{1,2,3}$

Although right signature fixed poles can be present in the amplitudes $B_{1,3}^{1,2,3}$ there is no compelling theoretical reason, such as would follow from the mechanisms discussed in Sec. IVA, for them to be present. Therefore we might expect the right signature sum rules $(n = 1,3)$ for these amplitudes to be dominated by the P, P', and A_2 Regge poles. Further we should expect reasonable answers from these sum rules, because they are at least as convergent as the corresponding low moment sum rules in πN and KN scattering.[9]

If there are no $j = 0$ fixed poles, then the $n = 1$ sum rules $I_{1,3}^{1,2,3}(1)$ should directly measure the photon (helicity flip) couplings of the P, P', and A_2, and the quotient $I_1^{\,i}(1)/I_3^{\,i}(1)$ should reveal, through Eq. (45), the same nonflip/flip nucleon coupling ratio obtained by analyzing πN, KN, and NN elastic scattering. The current models[7,56,57] for these amplitudes would lead us to believe that near $t = 0$

$$\begin{aligned} &\frac{A'}{\nu B} \qquad \text{for P and P'} \sim \frac{1}{2} \\ &\frac{A'}{\nu B} \qquad \text{for } \rho \text{ and } A_2 \sim \frac{1}{20} \end{aligned} \tag{47}$$

remembering our definition of ν is $2m$ larger than the usual $(s - u)/(4m)$.

There is also some evidence that the amplitude A' has an additional zero for P' and A_2 near $t \sim -0.5$ over and above that needed to erase the ghost. The evidence for this zero comes from a photoproduction FESR[47] for the A_2 while for P' the zero is indicated by πN FESR's[7] and also by the structure in $\bar{p}p$ elastic scattering near $t \sim -0.5$.[58] The work of Refs. 47 and 58 was claimed to be evidence for the so-called no compensation mechanism for the P' and A_2. This has an extra zero in both the flip and nonflip couplings but in fact their analysis was most sensitive to the nonflip zero and for the A_2, at least, one can rule out the flip zero from high energy data for $\pi N \to \eta N$, and $\pi N \to \eta \Delta$. If this zero is a true effect of the leading Regge trajectory, and not due to interference with secondary trajectories, our sum rules should reproduce it.

1. P and P' Exchange Sum Rules: $I^{1,2}_{1,3}(1,3)$

Here there are two possible isospin states, 1 and 2, corresponding to isoscalar and isovector photons and one may expect the latter to be more reliable. Thus in general the amplitudes involving isoscalar photons will have rather small Im B because the resonance couplings of Walker are larger for isovector than isoscalar photons and because our model for the inelasticity has a very small isoscalar part. Thus isospin 1 sum rules tend to be dominated by their Born terms which are not always small. Under such circumstances Eq. (46) predicts that the effective α will be nearer the fixed pole value $\lambda - n - 1$ than the intercept of the hoped for Regge pole. One should however note that BDW and Walker are not in quantitative agreement (cf. Fig. 3) and such sum rules have a large discontinuity at $E_{lab} = 0.5$ GeV. In Fig. 11a we have plotted the results of using Walker from threshold rather than BDW and as expected this leads to results showing a smaller deviation of $I_j^i(n)$ from its Born value.

The nicest sum rule of this section is $I_1^{\,2}(1)$ shown in Fig. 12a. The corresponding α (Fig. 12b) estimated as in (46) is in agreement with an expected average $P + P'$ intercept while even the higher moment sum rule $I_1^{\,2}(3)$ (Fig. 12c) shows agreement with $I_1^{\,2}(1)$. Both results suggest that there is no important $j = 0$ fixed pole.

The corresponding flip sum rule $I_3^{\,2}(1)$ (Fig. 14a) is not so spectacular with both $\alpha_3^{\,2}(1)$ (Fig. 14b) and $I_3^{\,2}(3)$ (not shown) showing less agreement with the $P + P'$ and preferring a lower intercept.

The isoscalar photon sum rules $I_1^{\,1}(1)$ and $I_3^{\,1}(1)$ (Figs. 11 and 13) do not provide striking evidence for or against a fixed pole at $j = 0$.

From Eq. (45) we find at $t = 0$

$$\frac{A'}{\nu B} \text{ for } P + P' \text{ from isospin 1} \sim 0.6 \qquad\qquad (48)$$
$$\text{from isospin 2} \sim 0.3$$

which agree reasonably with the πN result of 0.5. Of course it is quite possible that the ratio of P and P' is very different in πN and Compton scattering (and again it may differ here in the two isospin states). However this does not affect the above argument too much as high energy data on $\pi^{\pm}p$ polarization suggest[56] $A'/\nu B$ is similar for both P and P'.

In fact[59,60] one may attempt to calculate the relative amount of P and P' in our amplitudes by using at $t = 0$ the linear combination $\frac{1}{2}(I_1^{\,1}(1) + I_1^{\,2}(1) + I_1^{\,3}(1)) \sim \frac{1}{2} I_1^{\,2}(1)$ which only involves σ_{total} data for the γp state and combine it with the σ_{total} data known upto 7.5 GeV.[55] If you fit the latter to $A\nu^{\alpha_P - 1}(1 + c\,\nu^{\alpha_{P'} - \alpha_P})$ subject to the constraint provided by the finite-energy sum rule one finds

$\alpha_P = 1 \quad \alpha_{P'} = 0.65 \quad$ gives $\quad c = 5.7 \pm 5.0$

$\alpha_P = 1 \quad \alpha_{P'} = 0.5 \quad$ gives $\quad c = 2. \pm 0.9$.

Thus the closeness of α_P and $\alpha_{P'}$ makes it difficult to disentangle their separate contributions but in any case there is a good simultaneous fit to the FESR and the σ_{total} data. This is in agreement with our rougher estimates $\alpha_1{}^2(1)$, $I_1{}^2(3)$ which also indicate there is no necessity for a large $j = 0$ right signature fixed pole.

Our work also agrees with that of Costa et al.[60] and Creutz et al.[59]. The latter authors stress the importance of looking for a $j = 0$ fixed pole but it is strange that they should use a sum rule (namely $I_1{}^1(3) + I_1{}^2(3) + I_1{}^3(3)$), sensitive to $j = -2$ fixed poles, as part of their investigation.

2. A_2 Exchange Sum Rules: $I^3_{1,3}(1)$

Our results are given in Figs. 15 and 16 and both the sum rules and the effective α plots appear to be consistent with A_2 exchange. At $t \approx -0.5$ we expect a zero in $I_1{}^3(1)$ and none in $I_3{}^3(1)$ which is not inconsistent with our graphs. At $t = 0$ we find from (45)

$$\frac{A'}{\nu B} = \frac{1}{7 \rightarrow 15}$$

which is not ridiculous compared with (47). (However see our comment in VIE.)

On the basis of an argument involving F/D ratios, factorization and a crude evaluation (Born term only) of the $I_1{}^3(1)$ sum rule for the nucleon and its SU(3) partners Σ and Ξ , Gross and Pagels[10] have suggested that there is an important $j = 0$ fixed pole in this sum rule. From our more complete saturation of the nucleon sum rule and the associated effective α plot (Fig. 15b) we find no evidence for a large fixed pole (particularly if the BDW isoscalar photon multipoles are correct). However our method is not very sensitive to this because of the closeness of the A_2 intercept to zero. If our findings are to be compatible with Gross and Pagels then their fixed pole must couple predominantly to the strange baryons.

C. Current Algebra Sum Rules: $I^4_{1,2,3}$

1. Time-Time Sum Rules: $I^4_{1,3}$

Here we study the sum rules obtained by taking matrix elements of the equal time commutator of time components of the isovector current between nucleon states with helicity nonflip, $I_1{}^4(0)$, and helicity flip, $I_3{}^4(0)$. Although these sum rules are well known[4,12,13] previous evaluations[61] seem to have been solely concerned with $I_1{}^4(0)$ at $t = 0$ where it coincides with the Cabibbo-Radicati sum rule.[12]

These sum rules have Born contributions which are infinite at $t = 0$ and require the existence of a $j = 1$ fixed pole to produce a finite answer. (See Sec. IVB.) Current algebra, after the usual technical assumptions,[4] predicts that the fixed pole residues (as defined in Eq. (23)) are

$$\begin{aligned} F_1(t) &= -(2me^2/t)\, G_E{}^V(t) \qquad \text{in} \qquad I_1{}^4(0) \\ F_1(t) &= -(2e^2/t)\, G_M{}^V(t) \qquad \text{in} \qquad I_3{}^4(0) \end{aligned} \tag{49}$$

where $G_E^V(t)$ and $G_M^V(t)$ are the usual electric and magnetic isovector form factors of the nucleon normalized to $G_E^V(0) = 1$ and $G_M^V(0) = 1 + \kappa_p - \kappa_n$.

For our test of these sum rules we first note that the ratio of couplings of the ρ Regge pole at $t = 0$ can be estimated from πN scattering as $A'/\nu B \approx 1/20$, a number which is reduced by a factor of $2 \to 3$ from its value at the ρ pole $t = m_\rho^2$. If factorization holds we must have for all t (See Appendix B and Eqs. (23), (45), and (49).)

$$\frac{A'}{\nu_c B} = \frac{2}{4m^2 - t} \frac{I_1^4(0) + (2me^2/t)\, G_E^V(t)}{I_3^4(0) + (2e^2/t)\, G_M^V(t)} . \tag{50}$$

In Figs. 17 and 18a the sum rules $I_1^4(0)$ and $I_3^4(0)$ are plotted with the fixed poles of Eq. (49) subtracted off. If current algebra has supplied us with the correct value of the fixed poles then the resulting sum rules are superconvergent[5] and for high energy cutoffs the data points should lie very near to the zero line of the figures. Thus one is somewhat comforted that the data points lie in between their generalized Born terms and zero.

Since the form factors have been subtracted off, the plotted points of Figs. 17 and 18a correspond exactly to the numerator and denominator of the last factor of Eq. (50) and determine the ρ couplings through Eq. (23). We see from the figures that the general character of the sum rules is given by the Born minus fixed pole contributions. At $t = 0$ we have for the (finite part of) these contributions

$$\text{Born minus fixed pole of } I_1^{\,4}(0) = -0.0244(21.1 - 7.05 \frac{d}{dt} G_E^{\,V}(0))$$

$$\text{" " " " "} \quad I_3^{\,4}(0) = -0.026(13.7 - 7.05 \frac{d}{dt} G_M^{\,V}(0)) \; . \tag{51}$$

Since $\frac{d}{dt} G_E^{\,V}(0) \approx 3.3$ and $\frac{d}{dt} G_M^{\,V}(0) \approx 13.$, $I_1^{\,4}(0)$ exhibits a large cancellation between the finite part of the Born term and the derivative of the form factor. In $I_3^{\,4}(0)$ this cancellation does not occur. Therefore the smallness of the nonflip/flip ratio of the ρ Regge couplings at $t = 0$ is qualitatively realized by the Born minus fixed pole contributions to the sum rules. Note that in the ρ dominance model for the form factors the ratio of the fixed pole contributions at <u>$t = 0$</u> is essentially the value $A'/\nu B$ at the <u>ρ pole</u>. The exact value of the right hand of (50) is in agreement, within the errors, with the πN scattering value at $t = 0$. Of course one expects factorization to hold only to the extent that a ρ' contribution[62] is unimportant.

The agreement at $t = 0$ extends to nonzero t for $B_1^{\,4}$ as $I_1^{\,4}(0)$ remains small for all t . In this sum rule we expect the Höhler zero[14] at $t \approx -0.2$ and this is exhibited in Fig. 17, while at $\alpha_\rho(t) = 0$ we expect a zero in the sum rule (for ρ choosing either sense or nonsense) if the $\rho \rightarrow \gamma\gamma$ coupling is regular and no zero if it is singular. (See Sec. IVC.) In our opinion the data slightly favors the latter alternative. Unfortunately the effective α calculation (Eq. (46)) for this sum rule is of no use, because the sum rule is so small. We would be dividing by a small number with large errors in (46). Incidentally, at $t = 0$, $I_1^{\,4}(0)$ is in agreement with earlier work[61] both as to the value of the sum rule and the relative size of individual multipole contributions (see Table 3).

In $I_3^4(0)$ the situation is not so good at large t. The effective trajectory $\alpha_3^4(0)$ (Fig. 18b) shows little agreement with the expected ρ shape and the large value $\alpha \gtrsim 1$ for $t < -0.5$ would seem to indicate that we should have subtracted off a form factor of larger modulus than $(-2e^2/t)\, G_M^V(t)$. Taken at face value this is a violation of current algebra. However it hinges on a rather delicate feature of the data. Thus $\mathrm{Im}\, B_3^4(\nu,t)$, for $E_{lab} \approx 1$ GeV, changes sign near $t = -0.5$ due to the fact that the dominant resonant contribution $(\frac{5}{2}^+(1688))$ vanishes,[63] and this sign change forces $\alpha_3^4(0)$, calculated from Eq. (46), above the fixed pole value. Although the vanishing of the resonance contribution is perhaps expected[64] it does mean that the resultant amplitude depends delicately on the more uncertain parameters of Walker's analysis,[2] as well as our own dubious analysis of the inelastic contribution. This, together with our theoretical bias, makes us prefer to ignore this apparent violation of current algebra.

Therefore, assuming that the current algebra prediction of the fixed pole is correct, we note the interesting point that $I_3^4(0)$ has no zero near $\alpha_\rho(t) = 0$. If the ρ chooses sense at $\alpha = 0$ we expect a double zero if $\rho \to \gamma\gamma$ is nonsingular and a single zero if it is singular. The ρ choosing nonsense predicts one less zero than the above. Thus our sum rule predicts ρ choosing nonsense with a singular $\rho \to \gamma\gamma$ coupling. If current algebra were wrong, the larger fixed pole necessary to produce a better $\alpha_3^4(0)$ could also produce a zero in the ρ coupling at $\alpha_\rho(t) = 0$.

Finally we show the sum rule $I_3^{4}(1)$ and its associated $\alpha_3^{4}(1)$ in Figs. 18c and 18d. The sum rule is sensitive to a wrong signature fixed pole at $j = 0$ which is needed, if our interpretation of the ρ in $I_3^{4}(0)$ is correct, with a singular residue at $\alpha_\rho(t) = 0$ in order to cancel the pole of the Regge term. It is evident from Fig. 18c that something, presumably the fixed pole, has nicely cancelled the singularity in the Δ contribution, Eq. (42), and has produced a sum rule with a smooth variation in t. The effective $\alpha_3^{4}(1)$ suggests ρ exchange at small $|t|$ and, somewhat dubiously, since the sign change mentioned in connection with $I_3^{4}(0)$ also occurs here, suggests the fixed pole value at large $|t|$. Therefore $I_3^{4}(1)$ is certainly not inconsistent with an interpretation that current algebra is correct for $I_3^{4}(0)$, but one must admit $I_3^{4}(1)$ is hardly a stringent test of that interpretation. We do favor the interpretation that current algebra is correct. However, it is rather remarkable, although hopefully coincidental, that $I_3^{4}(1)$ and $\alpha_3^{4}(1)$ are consistent with no $j = 0$ wrong signature fixed pole and a ρ with a single zero in its residue function. Unfortunately as we have seen such a ρ is inconsistent with the $n = 0$ sum rule unless you increase the $j = 1$ fixed pole from its current algebra value (49).

We cannot claim on the basis of this work to have definitely confirmed or refuted current algebra although we do favor the former alternative. First both the sum rules appear to be converging and secondly, we obtain agreement near $t = 0$ with the hypothesis of ρ dominance of the sum rules once the form factor terms are subtracted off.

At large t, assuming current algebra is right, we obtain the interesting prediction that ρ chooses nonsense with a singular $\rho \to \gamma\gamma$ coupling which eliminates the zeros found in ρ couplings to hadronic processes. In this picture of the ρ couplings the wrong signature fixed pole at $j = 0$ plays very different roles in weak and strong processes. In the strong case this fixed pole seems to be purely "additive,"[29] giving zeros in the ρ-Regge term but spoiling the Schwarz[17] sum rules. In the weak case it is "multiplicative" and fills in the zeros.

2. Time-Space Sum Rules: $I_2^{\,4}(1)$

Using low-energy theorems and the assumption of an unsubtracted dispersion relation Beg[16] obtained a sum rule for the amplitude $B_2^{\,4}(\nu,t)$ at $t = 0$. This sum rule was rederived and extended to all t by Adler and Dashen[4] using the equal-time commutator of the time and space components of the isovector current and the infinite momentum limit. One interesting property of this sum rule is that it is invalid in a field theory of free nucleons, because the infinite momentum damping assumptions fail in that theory. On the basis of Regge theory (Appendix D) the fixed pole (effectively at $j = 0$) of $B_2^{\,4}(\nu,t)$ can be calculated to be

$$F_2(t) = e^2 G_M^{\,V}(t) + H(t) \tag{52}$$

where the first term is the nonasymptotic contribution of the $J^{PG} = 1^{-+}$ fixed pole of $B_3^{\,4}$, and the second term is the contribution of a possible $J^{PG} = 0^{-+}$ fixed pole. If the current algebra derivations of the sum rule are correct, then $H(t) \equiv 0$.

We show $I_2^{\,4}(1)$ in Fig. 19a and $\alpha_2^{\,4}(1)$ in Fig. 19b. The X's denote the nonasymptotic contributions of the ρ trajectory which Regge theory permits us to calculate from $I_3^{\,4}(0)$ (see Appendices C and D). This contribution is meaningless near $\alpha_\rho(t) = 0$ because its singularity there must be cancelled by a compensating trajectory.[65] The current algebra fixed pole residue $e^2\, G_M^{\,V}(t)$ is subtracted off and the combined Born minus fixed pole is plotted as the solid line in Fig. 19a. The Born term alone is plotted as the dashed line to show the dominant effect of the $e^2\, G_M^{\,V}(t)$ term.

If the current algebra fixed pole was correct then, at least for the mythical high energy cutoff, the data points $\diamond$ would be expected to lie near the zero line in Fig. 19a. Since the data points have a sign opposite to the ρ nonasymptotic term (near $t = 0$ where the latter might be trusted) and even lie on the wrong side of the generalized Born term, Fig. 19a suggests that the current algebra prediction is wrong and that $H(t) \approx -e^2\, G_M^{\,V}(t)$.

However $\alpha_2^{\,4}(1)$ does not support this interpretation near $t = 0$ and indicates an effective intercept consistent with an X trajectory ($\tau^{PG} = (+)^{-+}$) with $\alpha_X(0) \approx -0.5$, instead of the fixed pole value of zero. Although the sum rule results are presumably more reliable than the effective α determination at our low cutoff energy, we speculate further on the X trajectory. If $\alpha_X(t)$ stays one unit below the ρ upto $t \approx -0.6$ it could well be the necessary compensator, a possibility which is supported by the fact that the X coupling apparently has opposite sign to the ρ nonasymptotic term. The wild behavior of $\alpha_2^{\,4}(1)$ for $-t > 0.4$ could be due to a complicated cancellation between the ρ

and its compensator. On the timelike side if $\alpha_X(t)$ were roughly parallel to α_ρ one would expect a 0^{-+} meson at reasonably low mass, for which the lowest threshold decay channels are 4π and $K\bar{K}\pi$. Further if $I_2^{\,4}(1)$ is satisfied by an X trajectory, not a 0^{-+} fixed pole, this Regge pole will contribute via its nonasymptotic term (see Appendix D) to $I_3^{\,4}(0)$. This effect is quite large (~ 25% of $I_3^{\,4}(0)$) at $t = 0$ but negigible at the crucial larger $|t|$ values.

In summary, although the sum rule $I_2^{\,4}(1)$ seems to show that the current algebra prediction is incorrect, and that the fixed pole value is much nearer the free field theory value of zero, the effective $\alpha_2^{\,4}(1)$ plot allows us to explain this on the basis of a large X trajectory contribution.

D. Antialgebra Sum Rules: $I_{1,3}^{1\to 3}(0)$, $I_7^{\,5}(0)$

Current algebra purports to associate right signature $j = 1$ fixed poles with the equal-time commutators of currents satisfying pretty algebraic properties. In Sec. IVD we anticipated the proposal of a fundamental algebra of anticommutators to describe wrong signature fixed poles at $j = 1$. Of particular interest are those sum rules which share with $I_{1,3}^{4}(0)$ the property of having Born terms which are singular at $t = 0$. In the current algebra case this normalization condition on the fixed pole, in terms of the Born singularity, corresponds to current conservation.

Because the singular Born term mechanism (discussed in Ref. 18 and our Sec. IVA) applies, the sum rules $I_{1,3}^{3}(0)$ and $I_7^{\,5}(0)$ are guaranteed to exhibit wrong signature $j = 1$ fixed poles with singular coupling strength at $t = 0$ fixed by the Born term. Since isoscalar photons with small continuum contributions are involved, we also expect that the fixed pole couplings at large $|t|$ follows the shape of the Born term. In the case of right signature $j = 1$ fixed poles (if current algebra is correct) this is not true because the fixed pole couplings display the marked t dependence of the form factors, Eq. (49).

As a typical example we show $I_1^{3}(0)$ in Fig. 20. It is clear that the data points ◇ follow the Born term (solid line) and lie far from the △ points calculated, Eq. (42), assuming no wrong signature fixed pole. Because the continuum contribution is small, $\alpha_1^{3}(0)$ would clearly support the fixed pole interpretation.

Because the Pomeranchuk pole (with $\alpha_P(t) = 1$ at $t = 0$) is present, the sum rules $I_{1,3}^{1,2}(0)$ need not have a wrong signature $j = 1$ fixed pole but can be satisfied by the Pomeranchuk Regge pole term with singular coupling at $t = 0$. ($I_1^{2}(0)$ is presented in Fig. 21.) The lack of correspondence between the sum rule points ◇ and the $P + P'$ contribution △ calculated from $I_1^{2}(1)$ definitely shows the existence of a strong $j = 1$ fixed pole, and this interpretation is supported by $\alpha_1^{2}(0)$ (not shown).

The interesting behavior of $I_1^{2}(0)$ at large $|t|$ should be noted. Comparison of the data points ◇ with the Regge contribution △ shows that the sum rule is dominated by the fixed pole term even for $|t| \geq 0.6$. The fact that the wrong signature fixed pole couplings do not decrease rapidly with increasing $-t$ may be related to the presence of left-hand cuts in the wrong signature couplings not present in the right signature case.

The formula

$$\sigma_T = 2\pi \; e^2 \, \alpha'_P(0)[\, \frac{1}{4} Y^2 \; + \; \frac{1}{3} I(I+1)] \,, \qquad (53)$$

for the total photon cross section on hadron targets of hypercharge Y and isospin I, was derived in Ref. 18 assuming pure Pomeranchuk pole dominance. Existence of the wrong signature $j = 1$ fixed pole invalidates this formula, at least for nucleons. Equation (53) is very dubious on other grounds, since, using factorization, one can derive from it clearly erroneous results for the ratio of asymptotic total cross sections for any strongly interacting system. Neither our sum rules nor the factorization argument directly invalidates the weaker hypothesis--namely absence of the $j = 1$ fixed pole in $\gamma\pi \rightarrow \gamma\pi$ only--used by Mueller and Trueman.[66]

E. Drell-Hearn Sum Rules: $I_2^{1-3}(0)$

These sum rules are sensitive to right signature $j^P = 1^+$ fixed poles in the amplitudes $B_2^{1-3}(\nu,t)$. If conventional theory is correct, the fixed poles are absent and, since we have helicity flip $\lambda = 2$, the amplitudes satisfy superconvergence relations.[5,67] In explanation of the phrase "conventional theory," we cite two facts. First, the assumption of superconvergence for $B_2(\nu,t)$ is, at $t = 0$, equivalent[68] to the assumption, used in the original derivation[19] of the Drell-Hearn sum rule, of low-energy theorem plus unsubtracted dispersion relation for the forward spin flip Compton amplitude $f_2(\nu)$. Second, it would seem that the superconvergent sum rules follow from the conventional algebra of the time component of the appropriate isospin part of the electromagnetic current plus the usual technical assumptions of the infinite momentum method.[4]

Drell and Hearn considered only the proton sum rule obtained by adding $\frac{1}{2}(I_2^{1}(0) + I_2^{2}(0) + I_2^{3}(0))$, but, at the cost of using the more uncertain isoscalar photon data, we investigate all three sum rules. Normal parity contributions to $B_2(\nu,t)$ are suppressed by one power of energy,[69] and we therefore consider the abnormal parity trajectories D and E as well as the normal P and P' in isospins 1 and 2 (I = 0 exchange) and the abnormal A_1 and normal A_2 in isospin 3 (I = 1 exchange). We write schematically

$$
\begin{aligned}
I_2^{1,2}(0) &\sim \nu_c^{\alpha_{D,E}(t)-1} + \nu_c^{\alpha_{P,P'}(t)-2} \\
I_2^{3}(0) &\sim \nu_c^{\alpha_{A_1}(t)-1} + \nu_c^{\alpha_{A_2}(t)-2}
\end{aligned}
\qquad (54)
$$

indicating the asymptotic powers of the Regge pole contributions.

On the basis of the expected intercepts of these Regge poles, all three sum rules should superconverge at large cutoff energy. However some doubt has been expressed[66] concerning the convergence of the I = 0 exchange sum rules on the basis of Regge cut theory. If there are important abnormal parity components of the two-Pomeranchuk Regge cut, then to within logarithms we would expect $\mathrm{Im}\, B_2^{1,2}(\nu,t) \sim \nu^{-1}$ and the corresponding sum rules would diverge. Note that a fixed pole would make $\mathrm{Re}\, B_2 \sim \nu^{-1}$ and the sum rule integral would still converge.

Our results are presented in Figs. 22-24. If the superconvergence assumptions (rapid falloff of $\mathrm{Im}\, B_2$, absence of fixed pole in $\mathrm{Re}\, B_2$)

are satisfied, then at sufficiently high cutoff the data points should lie right on the zero line in the graphs. The value of $I_2^1(0)$ (Fig. 22) is very small and seems quite satisfactory[20] within the large errors (see Fig. 3 for the disturbing picture of $\mathrm{Im}\, B_2^1$). The sum rule $I_2^2(0)$ (Fig. 23) shows an impressive cancellation[19,20] between the Born term and the continuum for all t. The data points are consistent with zero (within errors) even at our low cutoff energy, and the sum rule must be deemed a success.

In $I_2^3(0)$, Fig. 24a, on the other hand, continuum and Born term reinforce, for both the pure Walker and the BDW plus Walker evaluations, and produce a sum rule which gives no hint of the expected superconvergence. This judgment is based on relative size of sum rule and Born contribution rather than on the absolute size of the former. Although the rule of thumb that the scale of a convergent sum rule is set by its Born term has proven quite reasonable, it is not clear a priori that it should be true, and it therefore becomes important to compute $\alpha_2^3(0)$.

In the context of this sum rule, the question answered by the effective α calculation can be rephrased as follows. What is the trajectory shape $\alpha(t)$ whose Regge term fits our observed sum rule result at cutoff 1.12 GeV, but would hopefully make the sum rule superconverge to the zero line at higher energies? It is clear from Fig. 24b that $\alpha_2^3(0)$ exceeds even the Froissart bound for small t (it could not produce superconvergence) and lies much higher than the expected α_{A_1} or $\alpha_{A_2} - 1$ trajectories. Therefore the only way we can interpret these results is to say that there is an important axial vector ($J^{PG} = 1^{+-}$) fixed pole contribution.

This is our most surprising result. The Drell-Hearn sum rule fails in the isospin segment where one would have least expected failure. Such a fixed pole would invalidate either the usual current algebra or the technical assumptions necessary to derive the covariant sum rule $I_2^3(0)$ from the antecedent equal time commutator.

Although this miserable fixed pole seriously challenges our theoretical ideas, it seems to have one beneficial effect on our sum rule results as follows. As shown in Appendix D, an axial vector fixed pole with coupling $A(t)$ to the amplitude B_2^3 also contributes nonasymptotically to B_3^3. We take $A(t)$ from $I_2^3(0)$, and assume that its nonasymptotic effect in B_3^3 is not modified by a possible 0^+ fixed pole there ($S(t)$ in Eq. (D.14)). We then recalculate at $t = 0$ the nonflip/flip ratio (Eq. 45) for the A_2 Regge pole (assuming domination of $I_3^3(1)$ by the A_2 and the fixed pole). This give a decreased value in better agreement, with the expected $A'/\nu B$ of strong interaction, than the previous value (calculated assuming $A(t) = S(t) = 0$).

We close this section by reminding any remaining readers that the Drell-Hearn proton sum rule, obtained by adding our three isotopic components, agrees with the original analysis[19] within errors.

F. Conspiracy Sum Rules: $I_6^{1\to3}(0)$, $I_7^5(1)$

We have discussed the theory of these sum rules in Sec. IVD.

As pointed out by Pagels[20] there is cancellation in $I_6^2(0)$ between the continuum and the Born terms, with the result that both $I_6^{1,2}(0)$ (Figs. 25 and 26) are consistent with zero at $t = 0$. Thus we have evidence against a large conspiring pole with vacuum quantum numbers. Correspondingly there is no hope of using these sum rules to obtain information on the $\eta \to 2\gamma$ coupling.

For the pion conspirator sum rule $I_6^3(0)$ (Fig. 27a) we confirm Pagels' result[20] at $t = 0$ but the flatness in t of $\alpha_6^3(0)$ (Fig. 27b) bears more resemblance to a right signature $j = 0$ fixed pole than a pion conspirator Regge trajectory. In fairness it must be said there is little reliable information from purely strong interactions on the slope of the conspirator and recent[70] photoproduction data suggest that the intercept is essentially zero upto $-t = 2\ \text{GeV}^2$.

We note that determination of the $\pi^0 \to 2\gamma$ coupling through the Pagels sum rule critically involves the assumption of smooth extrapolation to $t = 0$ of the π-pole term. In similar kinematic configurations involving π exchange (e.g. $\gamma p \to \pi^+ n$, $np \to pn$), the π exchange amplitude is more consistent with the rapidly varying form $(2m_\pi^2)^{-1}(t + m_\pi^2)(t - m_\pi^2)^{-1}$ near $t = 0$ rather than the smooth pole form $(t - m_\pi^2)^{-1}$ taken by Pagels. It is not clear whether the rapidly varying form should apply to doubly weak Compton scattering since the success of the absorptive model for π exchange suggests that the rapid variation is connected with the strong interaction unitarity condition.

From our numerical result for $I_6^3(0)$ at $t = 0$, we obtain through Eq. (36) the prediction $\tau_{\pi^0} = 2.5 \times 10^{-16}$ secs on the basis of a smooth π-pole residue which would become a factor of 4 smaller if the rapidly varying term above were used. These two values[20,52] quite closely enclose the possible range of experimental values, although the second possibility, rapidly varying pole term, would seem to be preferred on the basis of the wallet card value.[52]

In principle we can test whether the zero at $t = -m_\pi^2$ of the rapidly varying term is the factorable zero of a π-Regge pole residue by studying the sum rule $I_7^5(1)$, Fig. 28a, to which the π-Regge trajectory should couple although there is no π pole at $t = m_\pi^2$ because of photon helicity flip. The sum rule shows no hint of a zero. However any attempt to use this fact to speculate about π meson Reggeization would be thwarted by the fact that $\alpha_7^5(1)$, Fig. 28b, suggests an effective trajectory somewhat lower than π. Although the zero in question is suggested by simple π conspiracy models for $np \to pn$ and $\gamma p \to \pi^+ n$,[71] there is ample evidence from strong processes that[72] the zero does not factorize.

G. Other Sum Rules (Spin Segments 4, 5, and 8)

Spin types 4 and 5 are too divergent for useful information to be obtained from our low cutoff. We tried to use spin type 5 to predict the nonconspiring contribution to spin type 6, through Eq. (44), and obtained only untrustworthy and useless results. The sum rule $I_5^4(0)$ has an unknown fixed pole at $j = -1$ necessary to cancel the singular Born term.

Unfortunately $I_4^4(0)$ and $I_6^4(1)$ have the same continuum but different Born terms. Thus we need a fixed pole in one or both of them. It is presumably in $I_6^4(1)$ because this has $\tau P = +$ and it would then be the spinflip analogue of the $I_5^4(0)$ fixed pole. However the sum rules (not shown), if anything, prefer the assignment of a fixed pole to $I_4^4(0)$.

Finally $I_8^5(1)$ (not shown) appears to exhibit a fixed pole at $j = -1$ rather than the hoped for A_1 Regge pole. We remember the A_1 was also somewhat elusive in $I_2^3(0)$.

H. Polarizabilities

On integrating (40) upto $E_{lab} = 1.12$ GeV we find (assuming $c_i = 0$) the results given in Table 2. Here the column headed Walker uses his analysis from threshold onwards while that headed BDW uses the analysis of Ref. 1 from 0.15 to 0.5 GeV and Walker thereon. The last row contains the proton's polarizability and is half the sum of the first 3 lines. As described in Sec. IVE this and row 3 (isospin 3 which is the difference between the proton and neutron) may be hoped to be measured experimentally.

From the published data[55] on sigma total for photons on protons we may estimate the contribution of the integral from 1.12 to ∞ for the proton as follows. We get from 1.12 to 5.5, 0.9×10^{-43} cm^3 (error ~ 20%) and from 5.5 onwards $\lesssim 0.2 \times 10^{-43}$ cm^3. The former comes from direct integration and the latter from assuming sigma total does not increase after 5.5 GeV.

On may try to estimate the integral from 1.12 to ∞ for isospin 1 and 3 by assuming it to be dominated by the Regge pole saturating $I_1^{\,1}(1)$ and $I_1^{\,3}(1)$ respectively. The result obtained is an order of magnitude smaller than the difference between the two determinations of the integral upto 1.12.

I. Relative Importance of Different Intermediate States

In our graphical results we have only given the total integral over Im B in (41). So as one may judge the relative importance of the contributions of various intermediate states we give in Table 3 the break-up of $\frac{1}{\pi} \int_{\nu_0}^{\nu_c} d\nu\, \nu^n$ Im B for various sum rules. The columns headed P_{33}, D_{13}, D_{15}, and F_{15} give the separate contributions of the πN intermediate state in these spin and isospin quantum numbers. This isolates the important resonances in our energy range. The remaining contribution of the πN state is in the rest column while further columns give the inelastic and Born contributions to (41). The resonant S_{11} and P_{11} contributions to the rest column are small and this column thus represents nonresonant background which near threshold gets large contributions from the photoproduction Born terms. Both the total and πN columns are evaluated using the BDW analysis upto 0.5 GeV and Walker thereafter.

We would like to warn the reader that the first four πN columns include the total contribution of these states integrated over the whole energy range and not just the resonant portion. Thus in $I_3^{\,4}(0)$ the resonant F_{15} is much bigger than the resonant D_{15} state but this latter entry is large in Table 3 due to low energy contributions of these quantum numbers.

VII. METHODOLOGICAL COMMENTS

We discuss here some of the features, both desirable and undesirable, of our analysis and make suggestions for possible improvements and related future work.

For tests of the Drell-Hearn and current algebra sum rules, which derive from theoretical features particular to Compton amplitudes (e.g. algebraic properties of conserved currents), it would be desirable to relax the close dependence of our analysis on the Regge pole model of high-energy behavior. Although model-independent statements concerning the validity of the sum rules could presumably be easily obtained if the cutoff were sufficiently high, at the present cutoff we can say only the following. Adopting the phenomenological criterion that the scale of a convergent[73] sum rule is set by its generalized Born term (Born minus theoretically predicted fixed pole) it is clear from the figures that the $I = 0$ exchange Drell-Hearn sum rule $I_2^{\,2}(0)$ and the time component current algebra sum rules $I_1^{\,4}(0)$ and $I_3^{\,4}(0)$ must be regarded as successful, while the $I = 1$ Drell-Hearn sum rule $I_2^{\,3}(0)$ and the Beg sum rule $I_2^{\,4}(1)$ seem to be failures. To strengthen these statements we have been forced, at this low cutoff energy, to explore the consistency of our results with the Regge-pole parameters which have been obtained from high-energy data and FESR calculations on hadronic processes. Actually the exploration of the Regge pole model enriches our understanding of high-energy behavior. For example, we regard our results concerning the lack of nonsense zeros in ρ Regge coupling to the Compton amplitude as one of the more interesting facts which this analysis has revealed.

Our study has been handicapped by the lack of generally accurate estimates of the imaginary parts of Compton amplitudes. In this situation it becomes crucial to study as many sum rules as possible in order to obtain some feeling for the reliability of the results. For example, if one studies five equally convergent sum rules and finds that four of them go according to theoretical expectations and the fifth contains a surprise, it is then rather difficult to explain away the surprise on the basis of poor data.

It is, of course, distressing that we were forced to cutoff our integrals at the dubiously asymptotic value of $E_{lab} = 1.12$ GeV. In spin segments 2 and 3 this low cutoff was reflected in the quantitative importance of the nonasymptotic terms in the Regge formalism, suppressed by a factor $1/\nu$ from the leading terms.

Unfortunately it appears very hard to extend our integrals beyond $E_{lab} = 1.12$ GeV as long as we use unitarity to estimate the imaginary part. Thus above our cutoff a multitude of inelastic states become important and one would have to make models of the spin and isospin structure of all of these to find the imaginary part of the general Compton amplitude. Hence to extend our cutoff we would need data on Compton scattering itself but even this would not allow us to probe the general isospin state.

It follows that in the foreseeable future the main improvement in the evaluation of our sum rules must come from an improved treatment of the region upto 1.12 GeV, and here the elastic (πN) intermediate state is dominant (see Sec. VI.I).

It is rather disconcerting that different multipole analyses of low-energy photoproduction experiments, and perhaps even different experiments, are inconsistent. An obvious approach which would hopefully lead to an improved multipole analysis would be to combine the theoretical treatment of BDW and the phenomenological method of Walker. Thus one could formulate the dispersion theory with parameters, representing its weakest points, to be determined from a fit to the data. Such a treatment would at least have the virtue of incorporating elementary theoretical constraints such as Watson's theorem[74] on the phase of multipole amplitudes, which is not obeyed in purely phenomenological analyses. It is also possible that the use of theoretical models for the inelastic reactions $\gamma N \to \pi \Delta$ and $\pi N \to \pi \Delta$ would permit an approximate incorporation of unitarity for photoproduction above the BDW cutoff energy of $E_{lab} = 0.5$ GeV.

Since the greatest discrepancy between BDW and Walker is in the isoscalar photon multipoles, it would be very useful to study the FESR's for isoscalar photoproduction to determine whether the size of the predicted Regge pole terms is compatible with the isoscalar component of high-energy photoproduction which can be estimated from recent data.[70] Such an analysis could determine whether isoscalar photon multipoles were underestimated in Walker's analysis.

Since there is an experiment underway at CEA to measure the proton Compton scattering differential cross section in the 4-5 GeV energy range, it would be interesting to use the sum rules to work up a Regge pole prediction for this quantity. This could be done very easily with our existing computer programs.

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APPENDIX A

We give here the relation of our amplitudes defined by Eqs. (14) and (17) to the invariant amplitudes A_k of Hearn and Leader[25] and reduced s-channel amplitudes defined analogously to (13) by

$$\hat{M}_{\lambda_3\lambda_4;\lambda_1\lambda_2} = (\cos\tfrac{1}{2}\theta_s)^{-|\lambda_{34}+\lambda_{12}|}(\sin\tfrac{1}{2}\theta_s)^{-|\lambda_{34}-\lambda_{12}|} M_{\lambda_3\lambda_4;\lambda_1\lambda_2} . \tag{A.1}$$

We now list the expressions for the amplitudes $B_j^{\,i}$ in which for clarity we have omitted the isospin index i .

$$B_1 = -\tfrac{1}{2}(us - m^4)^{-1}\{(A_2 - A_1)(4m^2 - t) + (A_4 - A_5)\, m(s-u)\}$$

$$= (s - m^2)^{-2}\{s^{-1/2}(s + m^2)\sin^2\frac{\theta_s}{2}\ \hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}$$

$$+ m\Big(\hat{M}_{\frac{1}{2}1;\frac{1}{2}1} + \cos^2\frac{\theta_s}{2}\ \hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}\Big)\}$$

$$B_2 = (us - m^4)^{-1}\{A_6(s-u) + \tfrac{1}{2}t(A_5 - A_4)\}$$

$$= (s - m^2)^{-2}\{\hat{M}_{\frac{1}{2}1;\frac{1}{2}1} + 2m\, s^{-1/2}\sin^2\frac{\theta_s}{2}\ \hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}$$

$$- \Big(1 - m^2\, t(s - m^2)^{-2}\Big)\hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}\}$$

$$B_3 = (m^4 - us)^{-1}\{A_6(4m^2 - t) + \tfrac{1}{2}(s-u)(A_4 - A_5)\}$$

$$= (s - m^2)^{-4}\{(s - m^2)^2\,\hat{M}_{\frac{1}{2}1;\frac{1}{2}1} - 2m\sqrt{s}\,(s-u)\,\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}$$

$$+ \Big(m^2 t + (s - m^2)(s + 3m^2)\Big)\hat{M}_{-\frac{1}{2}1;-\frac{1}{2}1}\} \tag{A.2}$$

$$B_4 = A_3$$

$$= +\frac{1}{2}\sqrt{s}\;(s - m^2)^{-1}\,\{\,\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1} + \sin^2\frac{\theta_s}{2}\;\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1}\,\}$$

$$B_5 = t^{-1}\,\{(A_1 + A_2)(4m^2 - t) - (A_4 + A_5)m(s - u)\}$$

$$= +s(s - m^2)^{-2}\,\{+s^{-1/2}\,(s + m^2)(\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1} - \sin^2\frac{\theta_s}{2}\;\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1})$$

$$- 4m\cos^2\frac{\theta_s}{2}\;\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}\,\}$$

$$B_6 = A_4 + A_5$$

$$= -(s - m^2)^{-2}\,\{\,2m\sqrt{s}\;(\hat{M}_{-\frac{1}{2}-1;\frac{1}{2}1} - \sin^2\frac{\theta_s}{2}\;\hat{M}_{\frac{1}{2}-1;-\frac{1}{2}1})$$

$$+ 2(s + m^2)\sin^2\frac{\theta_s}{2}\;\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}\,\}\;.$$

$$B_7 = -\sqrt{s}\,(s - m^2)^{-3}\quad\hat{M}_{\frac{1}{2}1;-\frac{1}{2}1}$$

$$B_8 = s(s - m^2)^{-3}\quad\hat{M}_{\frac{1}{2}-1;\frac{1}{2}1}$$

APPENDIX B

In our study of the sum rules in Sec. VI we will need to know the exact predictions that factorization of the Regge couplings makes for our singularity-free amplitudes $B_1 \to B_8$. In this appendix we outline a derivation of these conditions while in Appendix C we give the resultant expressions for $G_j(t)$, $H_j(t)$ (defined in (19), and (20)) in terms of singularity free vertex functions. These latter we will denote by P_n P_f for the photon-photon coupling in nonflip (n) and spinflip (f) states and N_n N_f for the corresponding nucleon-antinucleon couplings. We will add a superscript c if the pole conspires.[75]

First we write our t-channel helicity amplitudes

$$A_{\lambda_3\lambda_1:\lambda_4\lambda_2} = -\frac{(e^{-i\pi\alpha} + \tau)}{2 \sin \pi\alpha} \exp[i\pi(\lambda_{13} - \lambda_{24})/2] \cdot \gamma'_{\lambda_1\lambda_3} \gamma'_{\lambda_2\lambda_4} \nu^{\alpha} . \tag{B.1}$$

While to include terms of order $\alpha - 1$ it is necessary to multiply the resultant form (B.1) gives to the reduced amplitudes (13) by

$$1 - \frac{\lambda_{min}(\lambda - \alpha)}{2\alpha\nu} [-t(4m^2 - t)]^{1/2} \tag{B.2}$$

where $\lambda = \max(|\lambda_{13}|, |\lambda_{24}|)$, $\lambda_{min} = \min(|\lambda_{13}|, |\lambda_{24}|) \times \text{Sign}(\lambda_{13} \lambda_{24})$.

We will need (B.2) to derive the form of $H(t)$ defined in Eq. (20). (This is considered in greater detail in Appendix D.)

We must now remove the kinematic singularities from γ' which we do first for the $\gamma - \gamma$ coupling by defining

$$\gamma'_{11} = t\, P_n$$

$$\gamma'_{1-1} = P_f \tag{B.3}$$

if the particle evades at $t = 0$ while if it conspires we put:

$$\gamma'_{11} = i \sqrt{-t} \quad P_n^{\,c}$$

$$\gamma'_{1-1} = i \sqrt{-t} \quad P_f^{\,c} \, . \tag{B.4}$$

For the $N\bar{N}$ coupling we must consider separately $\tau P = +$ and $\tau P = -$.

(a) $\tau P+$ (nonconspiring) $\quad \gamma'_{\frac{1}{2}\frac{1}{2}} = i\, N_n/(4m^2 - t)^{1/2}$

$$\gamma'_{\frac{1}{2}-\frac{1}{2}} = -\sqrt{-t} \quad N_f/(4m^2 - t)^{1/2} \tag{B.5}$$

(b) $\tau P+$ (conspiring) $\quad \gamma'_{\frac{1}{2}\frac{1}{2}} = -\sqrt{-t} \;\; N_n^{\,c}/(4m^2 - t)^{1/2}$

$$\gamma'_{\frac{1}{2}-\frac{1}{2}} = i\, N_f^{\,c}/(4m^2 - t)^{1/2} \tag{B.6}$$

(c) $\tau P-$ (nonconspiring)

$(\pi) \qquad \gamma'_{\frac{1}{2}\frac{1}{2}} = i \sqrt{-t} \quad N_n$

$(A_1) \qquad \gamma'_{\frac{1}{2}-\frac{1}{2}} = N_f \qquad$ (B.7)

(d) $\tau P-$ (conspiring)

$(\pi) \qquad \gamma'_{\frac{1}{2}\frac{1}{2}} = N_n^{\,c} \, . \qquad$ (B.8)

Substituting (B1 → 8) into Eqs. (14) and (17) we get the results given in Appendix C.

We will wish to compare our ratio of spin nonflip to spinflip couplings for P, P', ρ, and A_2 exchange with those obtained from analyzing strong interactions. However it is conventional[56] to analyze πN and KN elastic scattering in terms of invariant amplitudes A' and B which are related to our formalism by

$$N_f/N_n = \frac{\nu B}{(4m^2 - t)A'} \quad . \tag{B.9}$$

The behavior of $N_{n,f}$ and $P_{n,f}$ near $\alpha = 0$ for various sense-nonsense mechanism is given in Table 4.

APPENDIX C

Here we give the expansion of the functions $G_j^{\,i}(t)$ and $H_j^{\,i}(t)$ of Eqs. (19) and (20) in terms of the factorized vertex functions of Appendix B. We omit the isospin index i in all these results.

(i) $\tau P = +$ Contributions

$$G_2 = G_4 = G_7 = G_8 = 0 \qquad H_1 = H_3 = H_4 = H_5 = H_6 = H_7 = H_8 = 0$$

$$G_1 = \frac{1}{2}\,[N_n\,P_f + t\,N_n^{\,c}\,P_f^{\,c}]$$

$$H_2 = -\frac{(2-\alpha)}{2\alpha}\,t\,[N_f\,P_f + N_f^{\,c}\,P_f^{\,c}]$$

$$G_3 = [N_f\,P_f + N_f^{\,c}\,P_f^{\,c}]$$

$$G_5 = -\,[N_n\,P_n + N_n^{\,c}\,P_n^{\,c}]$$

$$G_6 = [t\,N_f\,P_n + N_f^{\,c}\,P_n^{\,c}] \,. \tag{C.1}$$

(ii) $\tau P = -$ Contributions

$$G_1 = G_3 = G_5 = G_6 = 0 \qquad H_1 = H_2 = H_4 = H_5 = H_6 = H_7 = H_8 = 0$$

$$G_2 = N_f\,P_f$$

$$H_3 = (4m^2 - t)[(2-\alpha)/2\alpha]\;N_f\,P_f$$

$$G_4 = -\frac{1}{2}\,[t\,N_n\,P_n + N_n^{\,c}\,P_n^{\,c}]$$

$$G_7 = -\frac{1}{2}\,[N_n\,P_f + N_n^{\,c}\,P_f^{\,c}]$$

$$G_8 = \frac{1}{2}\,N_f\,P_n \,. \tag{C.2}$$

APPENDIX D

The j-Plane Jungle in Spin Segments 2 and 3

Although the direct connection established in Sec. IIIB between asymptotic terms of the amplitudes $B(\nu,t)$ (Eq. (21)) and contributions to the sum rules (Eq. (23)) is sufficient to understand most of the physics contained in the sum rules, for some features it is necessary to go farther into the Reggeization of parity conserving helicity amplitudes. This is especially necessary for spins 2 and 3 because Regge poles of both parities contribute and because we have the additional complication of a large nonsense interval in the j plane.

Since the terrifying but straightforward details of Reggeization are known[65, 76, 77] for hadronic amplitudes, we concentrate here on effects of fixed poles and on matters directly connected with the interpretation of our sum rules such as the nonasymptotic Regge contributions (Eq. (20)) and compensators.

We study the amplitudes

$$A_{\pm}(\nu,t) = \hat{A}_{\frac{1}{2}-\frac{1}{2};1-1} \pm \hat{A}_{-\frac{1}{2}\frac{1}{2};1-1} \qquad (D.1)$$

which differ from $B_{2,3}$ by the kinematic factors of Eq. (14), and the definite parity partial wave amplitudes

$$a_{\pm}^{j}(t) = a^{j}_{\frac{1}{2}-\frac{1}{2};1-1}(t) \pm a^{j}_{-\frac{1}{2}\frac{1}{2};1-1}(t) \qquad (D.2)$$

defined in the usual way.[22] After defining signatured partial wave amplitudes, introducing rotation functions of the second kind[78] and performing the Mandelstam-Sommerfeld-Watson contour shift we obtain the representation

$$A_{\pm}(\nu,t) = \frac{1}{8\pi i} \sum_{\tau=\pm} \int_{\frac{3}{2}-i\infty}^{\frac{3}{2}+i\infty} dj \, \frac{(2j+1)}{\cos \pi j} \, (\tau + e^{-i\pi j})$$

$$\times \left\{ a_{\pm}^{j\tau}(t) \, E_{21+}^{j}(z) + a_{\mp}^{j\tau}(t) \, E_{21-}^{j}(z) \right\} \quad . \qquad (D.3)$$

We take $t \lesssim 0$ so that Regge poles satisfy $\mathrm{Re}\,\alpha(t) < \frac{3}{2}$ and do not explicitly appear in (D.3). We have ignored a discrete sum over half-integral j values because its terms are asymptotically (in ν) weaker than those we are interested in and becuase they cancel out when further shifts of the integration contour are made. The angular functions appearing in (D.3) are given by

$$E_{\lambda\mu\pm}^{j}(z) = \left\{ [(1-z)/2]^{1/2} \right\}^{-|\lambda-\mu|} \left\{ [(1+z)/2]^{1/2} \right\}^{-|\lambda+\mu|} e_{-\lambda-\mu}^{-j-1}(z)$$

$$\pm \left\{ [(1-z)/2]^{1/2} \right\}^{-|\lambda+\mu|} \left\{ [(1+z)/2]^{1/2} \right\}^{-|\lambda-\mu|}$$

$$\times \, e_{-\lambda\mu}^{-j-1}(z) \qquad (D.4)$$

and the e functions differ from those of Andrews and Gunson[78] by the factor $(-)^{\lambda-\mu}$. The scattering cosine z is given by

$$z = -\frac{2\nu}{[-t(4m^2 - t)]^{1/2}} \quad . \qquad (D.5)$$

For Compton amplitudes with definite crossing, the signature, parity and isospin are all correlated. See Table 1 . For given τ and P from the table the $a_{\pm}^{j\tau}$ with subscript $(-\tau P)$ vanish.

The E functions have the asymptotic behavior (for $\lambda \geq |\mu| \geq 0$)

$$
\begin{aligned}
E^{j}_{\lambda\mu+}(z) &\sim f(j)\, z^{j-\lambda} \left\{ 1 + \frac{g(j)}{j} z^{-2} + O(z^{-4}) \right\} \\
E^{j}_{\lambda\mu-}(z) &\sim f(j)\, \frac{\mu(\lambda - j)}{j}\, z^{j-\lambda-1} \left\{ 1 + \frac{h(j)}{j-1} z^{-2} + O(z^{-4}) \right\}
\end{aligned}
\tag{D.6}
$$

where $g(j)$ and $h(j)$ are regular (albeit zero for some λ and μ) at integer values and $f(j)$ has the following behavior

$$
\begin{aligned}
f(j) &\sim (j - j_0)^{-1} && \text{near} \quad j_0 = \lambda,\ \lambda+1,\ \lambda+2,\ \cdots \\
&\sim (j - j_0)^{-1/2} && \text{near} \quad j_0 = |\mu|,\ |\mu|+1,\ \cdots\ \lambda - 1 \\
&\sim \text{regular} && \text{near} \quad j_0 = 0,\ 1,\ \cdots,\ |\mu| - 1 \\
&\sim (j - j_0)^{-1} && \text{near} \quad j_0 = -|\mu|,\ -|\mu|+1,\ \cdots,\ -1 \\
&\sim (j - j_0)^{-1/2} && \text{near} \quad j_0 = -\lambda,\ -\lambda+1,\ \cdots,\ -|\mu| - 1 \\
&\sim \text{regular} && \text{near} \quad j_0 = -\lambda - 1,\ -\lambda - 2,\ \cdots .
\end{aligned}
\tag{D.7}
$$

Although the leading term in the asymptotic series is regular near a positive nonsense-nonsense integer, subsidiary terms may be singular, as is crudely shown in (D.6). The exact relation between the singular parts of the E functions at reflected integers in the nonsense-nonsense interval is

$$\lim_{j \to j_0} (j - j_0)\, E^{j}_{\lambda\mu\pm}(z) = -(-)^{\lambda-\mu} \mathrm{Sign}(\lambda\mu) \lim_{j \to j_0} (j - j_0) E^{-j-1}_{\lambda\mu\mp}(z) \;. \tag{D.8}$$

If fixed poles are present, then the partial wave amplitudes $a^{j\tau}_{\pm}(t)$ are expected to have the j-plane behavior of their Born terms, namely

$$\begin{aligned} a^{j\tau}_{\pm}(t) &\sim \text{regular} && \text{near} \quad j_0 = 2, 3, 4 \\ &\sim (j - j_0)^{-1/2} && \text{near} \quad j_0 = 1 \\ &\sim (j - j_0)^{-1} && \text{near} \quad j_0 = 0, -1 \\ &\sim (j - j_0)^{-1/2} && \text{near} \quad j_0 = -2 \\ &\sim (j - j_0)^{-1} && \text{near} \quad j_0 = -3, -4, \cdots \end{aligned} \tag{D.9}$$

where we have again specialized to the particular helicity values, $\lambda = 2$, $\mu = 1$, we are interested in. In the absence of fixed poles the expected behavior is a factor of $(j - j_0)$ smoother at all nonsense points $(j_0 \leq 1)$.

The singular parts of the partial wave amplitudes at the reflected nonsense-nonsense integers $j_0 = 0$ and $j_0 = -1$ are related by:

$$\lim_{j \to j_0} (j - j_0)\, a^{j\tau}_{\pm}(t) = \lim_{j \to j_0} (j - j_0)\, a^{-j-1(-\tau)}_{\mp}(t) \;. \tag{D.10}$$

This condition expresses the absence of fixed double poles at $j_0 = -1$ and follows formally from the Froissart-Gribov definition, and a mathematical relation, similar to (D.8), for the rotation functions. Equation (D.10) implies that fixed poles occur in pairs at $j = 0$ and $j = -1$ with residues satisfying (D.10) and that for every Regge trajectory passing through $\alpha(t) = 0$ with nonvanishing residue, there is a compensating trajectory[79] of opposite parity and signature passing through $\alpha'(t) = -1$.

All of this technicality is necessary to understand what happens in (D.3) when the vertical contour of integration is shifted to the line $\mathrm{Re}\, j = -\frac{3}{2}$. The double poles encountered do not contribute asymptotically and obnoxious terms such as fixed powers in the imaginary part of the amplitude cancel between the $j = 0$ and $j = -1$ contributions because of the phenomenon of compensation expressed by (D.8) and (D.10). The net result is a set of relatively simple expressions for the asymptotic terms of the amplitudes $A_\pm(\nu,t)$ or $B_{2,3}(\nu,t)$ which we proceed to give.

The current algebra amplitudes $B^4_{2,3}(\nu,t)$ have asymptotic contributions from isovector right signature fixed poles at $J^{PG} = 1^{-+}$ and 0^{-+} and from the ρ-Regge trajectory and a mythical X trajectory with $\tau^{PG} = (+)^{-+}$. We find

$$B_3{}^4(\nu,t) \approx \frac{2e^2}{t}\, G_M{}^V(t)\, \nu^{-1} - H'(t)\, \nu^{-3} - G_\rho(t)\, \frac{(-1 + e^{-i\pi\alpha_\rho(t)})}{\sin \pi\, \alpha_\rho(t)}\, \nu^{\alpha_\rho(t)-2}$$

$$- G_X(t)(4m^2 - t)\, \frac{2 - \alpha_X(t)}{2\, \alpha_X(t)}\, \frac{(1 + e^{-i\pi\alpha_X(t)})}{\sin \pi\, \alpha_X(t)}\, \nu^{\alpha_X(t)-3}, \tag{D.11}$$

$$B_2^{\,4}(\nu,t) \approx -e^2\, G_M^{\,V}(t)\, \nu^{-2} - H(t)\, \nu^{-2}$$

$$+ \ t\, G_\rho(t)\, \frac{2 - \alpha_\rho(t)}{2\alpha_\rho(t)}\, \frac{(-1 + e^{-i\pi\alpha_\rho(t)})}{\sin \pi\, \alpha_\rho(t)}\, \nu^{\alpha_\rho(t)-3}$$

$$- \ G_X(t)\, \frac{1 + e^{-i\pi\alpha_X t}}{\sin \pi\, \alpha_X(t)}\, \nu^{\alpha_X(t)-2} \ . \qquad \text{(D.12)}$$

We have used current algebra to relate the residue of the 1^{-+} fixed pole to the isovector magnetic form factor. Here $H(t)$ is the coupling of a hypothetical 0^{-+} pole, and $H'(t)$ is a kinematic singularity free function which expresses the net contribution of the nonasymptotic term of the 1^{-+} fixed pole, and the 0^{-+} fixed pole and its compensator at $J^P = (-1)^-$.

We have not included explicitly the effects of compensating Regge trajectories near $\alpha = -1$ which are necessary to cancel the singularity at $\alpha_\rho(t) = 0$ in $B_2^{\,4}$ and the possible singularity at $\alpha_X(t) = 0$ in $B_3^{\,4}$.

Notice that the $I_3^{\,4}(0)$ sum rule is sensitive only to the 1^{-+} fixed pole, while the $I_2^{\,4}(1)$ sum rule has contributions from both $G_M^{\,V}(t)$ and $H(t)$. If current algebra is correct and the infinite-momentum method is valid for the commutator of one time and one space component, then the resulting Beg sum rule predicts[80] that $H(t) \equiv 0$. From the standpoint of current algebraists, failure of the Beg sum rule would mean that either current algebra or the infinite momentum method is wrong.[4] However, from the standpoint of Reggeologists, success of the $I_3^4(0)$ sum rule and failure of the Beg sum rule would indicate the existence of a 0^{-+} fixed pole. However in assessing the $I_2^4(1)$ sum rule one must be careful to take into account the possible effect of an X trajectory contribution.

The isospin symmetric amplitudes $B^i_{2,3}(\nu,t)$, $i = 1,2,3$, have asymptotic contributions from possible fixed poles at $J^P = 1^+$ and 0^+. We explicitly treat $B^3_{2,3}(\nu,t)$, to which the A_2 and A_1 Regge trajectories contribute. Letting $A(t)$ and $S(t)$ denote the couplings of the 1^+ and 0^+ fixed poles, we find the asymptotic expressions

$$B_2^3(\nu,t) \approx -2\,A(t)\nu^{-1} + 2S'(t)\nu^{-3} + t\,G_{A_2}(t)\,\frac{2-\alpha_{A_2}(t)}{2\,\alpha_{A_2}(t)}\,\frac{1+\exp[-i\pi\alpha_{A_2}(t)]}{\sin\pi\,\alpha_{A_2}(t)}\,\nu^{\alpha_{A_2}(t)-3} - G_{A_1}(t)\,\frac{-1+\exp[-i\pi\alpha_{A_1}(t)]}{\sin\pi\,\alpha_{A_1}(t)}\,\nu^{\alpha_{A_1}(t)-2}, \qquad (D.13)$$

$$B_3^3(\nu,t) \approx -(4m^2-t)A(t)\nu^{-2} - 2S(t)\,\nu^{-2} - G_{A_2}(t)\,\frac{1+\exp[-i\pi\alpha_{A_2}(t)]}{\sin\pi\,\alpha_{A_2}(t)}\,\nu^{\alpha_{A_2}(t)-2} - (4m^2-t)G_{A_1}(t)\,\frac{2-\alpha_{A_1}(t)}{2\,\alpha_{A_1}(t)}\,\frac{-1+\exp[-i\pi\alpha_{A_1}(t)]}{\sin\pi\,\alpha_{A_1}(t)}\,\nu^{\alpha_{A_1}(t)-3}. \qquad (D.14)$$

Here we have a situation opposite to that of the current algebra segment. The Drell-Hearn sum rule $I_2^3(0)$ is sensitive to the axial vector fixed pole only, while the sum rule $I_3^3(1)$ detects the combined effect of the axial vector and scalar fixed poles. In a derivation of these sum rules based on quark model current algebra and the infinite momentum limit, both fixed poles are absent. See Sections VIB (2) and VIE for our experimental results on this question.

FOOTNOTES AND REFERENCES

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80. See Ref. 4. We thank Professor Roger Dashen, who has independently worked out the j-plane analysis given in this appendix, for helpful discussions concerning the Beg sum rule.

Tables 1A and 1B. Vital Statistics of the Amplitudes $B_j^{\,i}(t)$

The meaning of the various quantities in Tables 1A and 1B are as follows. $B_j^{\,i}(t)$ are defined in Eqs. (14) and (17). $\eta_j^{\,i}$ and $C_j^{\,i}(t)$ are defined by Eq. (18). In the $C_j^{\,i}(t)$ column κ_p and κ_n are the anomalous magnetic moments of the proton and neutron respectively. λ is defined after Eq. (19). n_{min} is the lowest value of n in Eq. (23) for the latter to be a right signature sum rule. τ, P, and G are the signature, parity, and G parity of the allowed Regge pole exchanges. Plausible candidates for the latter are listed in the next column; here we have taken the meson quantum numbers from the customary bible (Ref. 52). Further in this column c_X denotes the $\tau P = +$ partner of an $m = 1$ conspiracy (Ref. 75) with the $\tau P = -\ X_c$ trajectory of X quantum numbers. ($X = \eta, \pi, B$) . X by itself means non-conspiring.

Table 1A

Amplitude	$\eta_j^{\,i}$	λ	n_{min}	τ	P	G	Regge Pole
$B_1^{\,1}$	+	2	1	+	+	+	P, P' $c_\eta(\sim t)$
$B_1^{\,2}$	+	2	1	+	+	+	P, P' $c_\eta(\sim t)$
$B_1^{\,3}$	+	2	1	+	+	-	A_2 $c_\pi(\sim t)$
$B_1^{\,4}$	-	2	0	-	-	+	ρ $c_B(\sim t)$
$B_2^{\,1}$	-	2	0	-	+	+	D, E(?)$(\sim s^{\alpha-2})$
				+	+	+	P, P', $c_\eta(\sim t\ s^{\alpha-3})$
$B_2^{\,2}$	-	2	0	-	+	+	D, E(?)$(\sim s^{\alpha-2})$
				+	+	+	P, P', $c_\eta(\sim t\ s^{\alpha-3})$
$B_2^{\,3}$	-	2	0	-	+	-	A_1 $(\sim s^{\alpha-2})$
				+	+	-	A_2, $c_\pi(\sim t\ s^{\alpha-3})$
$B_2^{\,4}$	+	2	1	+	-	+	?
				-	-	+	ρ, c_B $(\sim t\ s^{\alpha-3})$
$B_3^{\,1}$	+	2	1	+	+	+	P, P', c_η $(\sim s^{\alpha-2})$
				-	+	+	D, E(?) $(\sim s^{\alpha-3})$

Table 1A. (Cont.)

Amplitude	η_j^i	λ	n_{min}	τ	P	G	Regge Pole
B_3^2	+	2	1	+	+	+	P, P', c_η ($\sim s^{\alpha-2}$)
				-	+	+	D, E(?) ($\sim s^{\alpha-3}$)
B_3^3	+	2	1	+	+	-	A_2, c_π ($\sim s^{\alpha-2}$)
				-	+	-	A_1 ($\sim s^{\alpha-3}$)
B_3^4	-	2	0	-	-	+	ρ, c_B ($\sim s^{\alpha-2}$)
				+	-	+	?
B_4^1	+	0	1	+	-	+	η ($\sim t$) η_c
B_4^2	+	0	1	+	-	+	η ($\sim t$) η_c
B_4^3	+	0	1	+	-	-	π ($\sim t$) π_c
B_4^4	-	0	0	-	+	+	B ($\sim t$) B_c
B_5^1	+	0	1	+	+	+	P, P', c_η
B_5^2	+	0	1	+	+	+	P, P', c_η
B_5^3	+	0	1	+	+	-	A_2, c_π
B_5^4	-	0	0	-	-	+	ρ, c_B

Table 1A. (Cont.)

Amplitude	η_j^i	λ	n_{min}	τ	P	G	Regge Pole
B_6^1	-	1	0	+	+	+	P, P' ($\sim$ t) c_η
B_6^2	-	1	0	+	+	+	P, P' ($\sim$ t) c_η
B_6^3	-	1	0	+	+	-	A_2 ($\sim$ t) c_π
B_6^4	+	1	1	-	-	+	ρ ($\sim$ t) c_B
B_7^5	+	2	1	+	-	-	π , π_c
B_8^5	+	1	1	-	+	-	A_1

Table 1B

Amplitude	Born Residue $c_j^{\,i}(t)$
$B_1^{\,1}$	$-\frac{2me^2}{t} + \frac{e^2}{4m}[1 - (1+\kappa_p+\kappa_n)^2]$
$B_1^{\,2}$	$-\frac{2me^2}{t} + \frac{e^2}{4m}[1 - (1+\kappa_p-\kappa_n)^2]$
$B_1^{\,3}$	$-\frac{4me^2}{t} + \frac{e^2}{2m}[1 - (1+\kappa_p)^2 + \kappa_n^{\,2}]$
$B_1^{\,4}$	$\frac{2me^2}{t} - \frac{e^2}{4m}[1 - (1+\kappa_p-\kappa_n)^2]$
$B_2^{\,1}$	$-\frac{e^2}{4m^2}(\kappa_p + \kappa_n)^2$
$B_2^{\,2}$	$-\frac{e^2}{4m^2}(\kappa_p - \kappa_n)^2$
$B_2^{\,3}$	$-\frac{e^2}{2m^2}(\kappa_p^{\,2} - \kappa_n^{\,2})$
$B_2^{\,4}$	$\frac{e^2}{4m^2}(\kappa_p - \kappa_n)^2$
$B_3^{\,1}$	$-\frac{2e^2}{t}(1+\kappa_p+\kappa_n) - \frac{e^2}{4m^2}(\kappa_p+\kappa_n)^2$

Table 1B. (Cont.)

Amplitude	Born Residue $C_j^{\,i}(t)$
$B_3^{\,2}$	$-\frac{2e^2}{t}(1+\kappa_p-\kappa_n) - \frac{e^2}{4m^2}(\kappa_p-\kappa_n)^2$
$B_3^{\,3}$	$-\frac{4e^2}{t}(1+\kappa_p) - \frac{e^2}{2m^2}(\kappa_p^{\,2}-\kappa_n^{\,2})$
$B_3^{\,4}$	$\frac{2e^2}{t}(1+\kappa_p-\kappa_n) + \frac{e^2}{4m^2}(\kappa_p-\kappa_n)^2$
$B_4^{\,1}$	$\frac{1}{2}me^2(1+\kappa_p+\kappa_n)$
$B_4^{\,2}$	$\frac{1}{2}me^2(1+\kappa_p-\kappa_n)$
$B_4^{\,3}$	$me^2(1+\kappa_p)$
$B_4^{\,4}$	$-\frac{1}{2}me^2(1+\kappa_p-\kappa_n)$
$B_5^{\,1}$	$\frac{4m^3e^2}{t} - \frac{me^2}{2}[1+(1+\kappa_p+\kappa_n)^2]$
$B_5^{\,2}$	$\frac{4m^3e^2}{t} - \frac{me^2}{2}[1+(1+\kappa_p-\kappa_n)^2]$

Table 1B. (Cont.)

Amplitude	Born Residue $C_j^{\,i}(t)$
$B_5^{\,3}$	$\frac{8\, m^3\, e^2}{t} - me^2\,[(1 + \kappa_p)^2 - \kappa_n^{\,2} + 1]$
$B_5^{\,4}$	$- \frac{4\, m^3\, e^2}{t} + \frac{1}{2}\, me^2\,[1 + (1 + \kappa_p - \kappa_n)^2]$
$B_6^{\,1}$	$\frac{1}{2}\, e^2\,[(1 + \kappa_p + \kappa_n)^2 - 1]$
$B_6^{\,2}$	$\frac{1}{2}\, e^2\,[(1 + \kappa_p - \kappa_n)^2 - 1]$
$B_6^{\,3}$	$e^2\,[(1 + \kappa_p)^2 - 1 - \kappa_n^{\,2}]$
$B_6^{\,4}$	$- \frac{1}{2}\, e^2\,[(1 + \kappa_p - \kappa_n)^2 - 1]$
$B_7^{\,5}$	$- e^2\, \kappa_n/(mt)$
$B_8^{\,5}$	$-e^2\, \kappa_n/t$

Table 2. The polarizability in the various isospin states (see Sec. VIH). The units are 10^{-43} cm^3 .

Isospin state	Walker	BDW
1	0.2	0.4
2	25.6	25.5
3	-1.	-2.
proton	12.4	12.

Table 3. The break up of $I_j^{\,i}(n)$ (defined in (41)), at $t = 0$, into the contributions of various intermediate states as defined in VI.I.

				πN Intermediate State				
	Total	Born	Inelastic	P_{33}	D_{13}	D_{15}	F_{15}	Rest
$I_1^{\,2}(1)$	1.25	0.086	0.27	0.39	0.1	0.01	0.04	0.36
$I_1^{\,3}(1)$	0.12	0.17	0.008	0.	-0.01	0.002	0.02	-0.07
$I_1^{\,4}(0)$	0.02	0.044	-0.004	0.31	-0.09	-0.01	-0.02	-0.20
$I_2^{\,2}(0)$	0.06	0.358	-0.05	-0.43	-0.09	-0.006	-0.02	0.31
$I_2^{\,3}(0)$	-0.09	-0.024	-0.005	0.	0.01	-0.002	-0.01	-0.06
$I_3^{\,4}(0)$	1.2	2.04	-0.18	0.45	-0.32	-0.1	-0.06	-0.65
$I_6^{\,2}(0)$	0.08	-0.97	-0.15	0.53	-0.02	0.01	-0.002	0.69
$I_7^{\,5}(1)$	-0.04	-0.094	0.005	0.	0.02	-0.002	-0.002	0.02

Table 4. $\alpha = 0$ Sense-Nonsense Factors

Here we give the dependence at $\alpha = 0$ of the nonflip (n) and flip (f) residue P_n, P_f, N_n, N_f defined in Appendix B. The columns headed "No Fixed Pole" apply to the hadronic $N\bar{N}$ vertex, and the "Fixed Pole" column applies to the weak $\gamma\gamma$ vertex. We do not give the dependence, applicable for negative signature poles, corresponding to a strong interaction fixed pole and a fixed double pole in Compton scattering.

		No Fixed Pole		Fixed Pole	
Nomenclature	Signature	Nonflip	Flip	Nonflip	Flip
Choosing sense	-	1	α	1	1
Choosing nonsense	+ or -	$\sqrt{\alpha}$	$\sqrt{\alpha}$	$1/\sqrt{\alpha}$	$1/\sqrt{\alpha}$
Chew's mechanism	+	$\sqrt{\alpha}$	$\alpha\sqrt{\alpha}$	$1/\sqrt{\alpha}$	$1/\sqrt{\alpha}$
No-compensation mechanism	+	α	α	1	1

FIGURE CAPTIONS

Fig. 1. The contour C of Eq. (22).

Fig. 2. Unitarity condition in Compton scattering.

Fig. 3. The value of $\frac{1}{\pi}\,\mathrm{Im}\,B_2^{\;1}$ plotted against photon lab energy. The dotted line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

Fig. 4. The value of $\frac{1}{\pi}\,\mathrm{Im}\,B_2^{\;3}$ plotted against photon lab energy. The dotted line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

Fig. 5. The value of $\frac{1}{\pi}\,\mathrm{Im}\,B_3^{\;4}$ plotted against photon lab energy. The dotted line is the prediction of BDW (Ref. 1) and the solid line that of Walker (Ref. 2).

Fig. 6. A diagram causing a divergence of the partial wave series in the (s,t) region of interest.

Fig. 7. The diagrams considered in the Stichel-Scholz model (Ref. 3) of $\gamma N \to \pi\Delta$.

Fig. 8. The one pion exchange contribution to $\gamma N \to \pi\Delta$.

Fig. 9. A diagram NOT causing a divergence of the partial wave series in the (s,t) region of interest.

Fig. 10. A diagram representing our treatment of inelasticity NOT due to the $\pi\Delta$ state.

Fig. 11. Pomeranchuk exchange nonflip sum rule (isoscalar photons). See VIA for the graphical notation and VIB for comments.

(a) The $n = 1$ sum rule $I_1^{\;1}(1)$.

(b) The corresponding effective α.

Fig. 12. Pomeranchuk exchange nonflip sum rule (isovector photons). See VIB for comments.

(a) The $n = 1$ sum rule $I_1^2(1)$.

(b) The effective α corresponding to (a).

(c) The $n = 3$ sum rule $I_1^2(3)$.

Fig. 13. Pomeranchuk exchange flip sum rule (isoscalar photons). See VIB for comments.

(a) The $n = 1$ sum rule $I_3^1(1)$.

(b) The corresponding effective α .

Fig. 14. Pomeranchuk exchange flip sum rule (isovector photons). See VIB for comments.

(a) The $n = 1$ sum rule $I_3^2(1)$.

(b) The corresponding effective α .

Fig. 15. A_2 exchange nonflip sum rule. See VIB for comments.

(a) The $n = 1$ sum rule $I_1^3(1)$.

(b) The corresponding effective α .

Fig. 16. A_2 exchange flip sum rule. See VIB for comments.

(a) The $n = 1$ sum rule $I_3^3(1)$.

(b) The corresponding effective α .

Fig. 17. Nonflip current algebra sum rule $I_1^4(0)$. See VIC(1) for comments.

Fig. 18. Spinflip current algebra sum rule. See VIC(1) for comments.

(a) The $n = 0$ sum rule $I_3^4(0)$.

(b) The effective α corresponding to (a).

(c) The $n = 1$ wrong signature sum rule $I_3^4(1)$.

(d) The effective α corresponding to (c) .

Fig. 19. The time-space current algebra sum rule. See VIC(2) for comments.

(a) The $n = 1$ sum rule $I_2^{\,4}(1)$.

(b) The corresponding effective α .

Fig. 20. A_2 exchange nonflip wrong signature sum rule $I_1^{\,3}(0)$. See VID for comments.

Fig. 21. Pomeranchuk exchange nonflip wrong signature sum rule $I_1^{\,2}(0)$. See VID for comments.

Fig. 22. Drell-Hearn sum rule $I_2^{\,1}(0)$ (isoscalar photons). See VIE for comments.

Fig. 23. Drell-Hearn sum rule $I_2^{\,2}(0)$ (isovector photons). See VIE for comments.

Fig. 24. Drell-Hearn sum rule (isovector exchange). See VIE for comments.

(a) The $n = 0$ sum rule $I_2^{\,3}(0)$.

(b) The corresponding effective α .

Fig. 25. η conspirator sum rule $I_6^{\,1}(0)$ (isoscalar photons). See VIF for comments.

Fig. 26. η conspirator sum rule $I_6^{\,2}(0)$ (isovector photons). See VIF for comments.

Fig. 27. π conspirator sum rule. See VIF for comments.

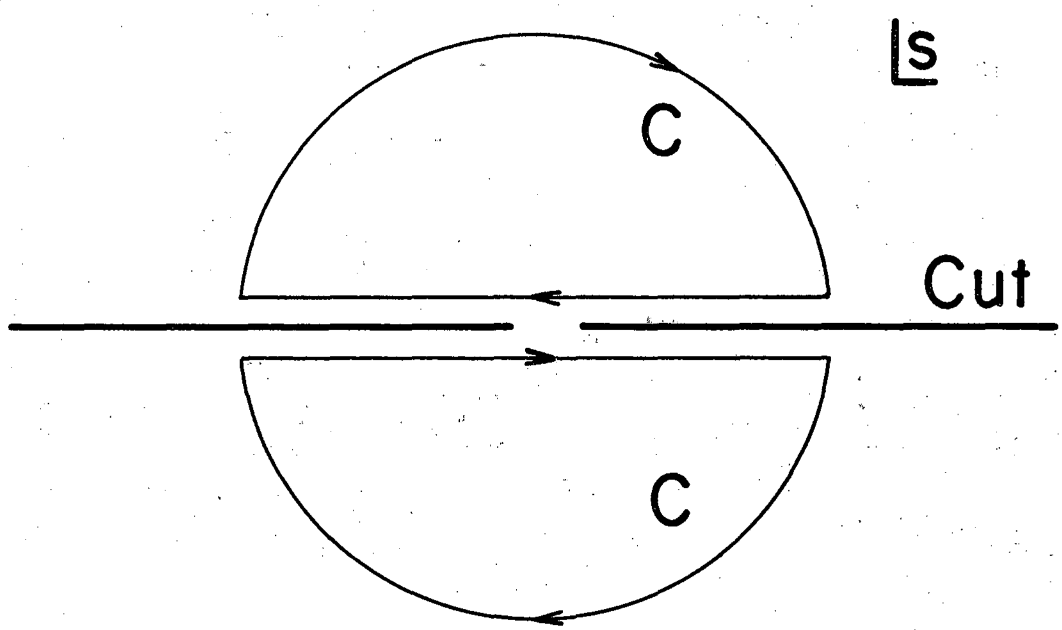
(a) The $n = 0$ sum rule $I_6^{\,3}(0)$.

(b) The corresponding effective α .

Fig. 28. π spinflip sum rule. See VIF for comments.

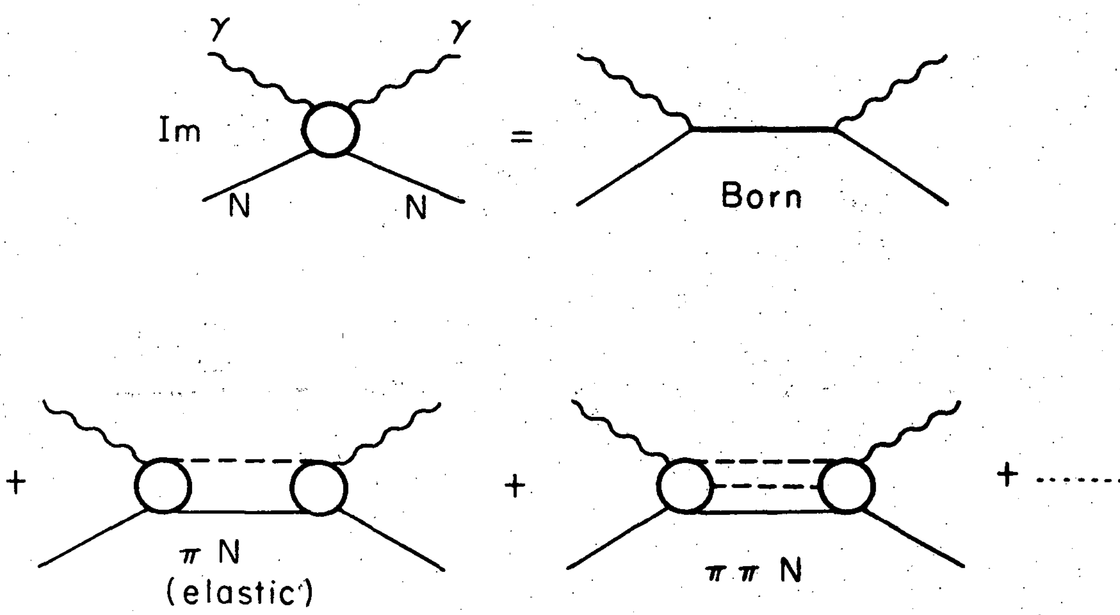
(a) The $n = 1$ sum rule $I_7^{\,5}(1)$.

(b) The corresponding effective α.



XBL6812-7472

Fig. 1



XBL6812-7473

Fig. 2

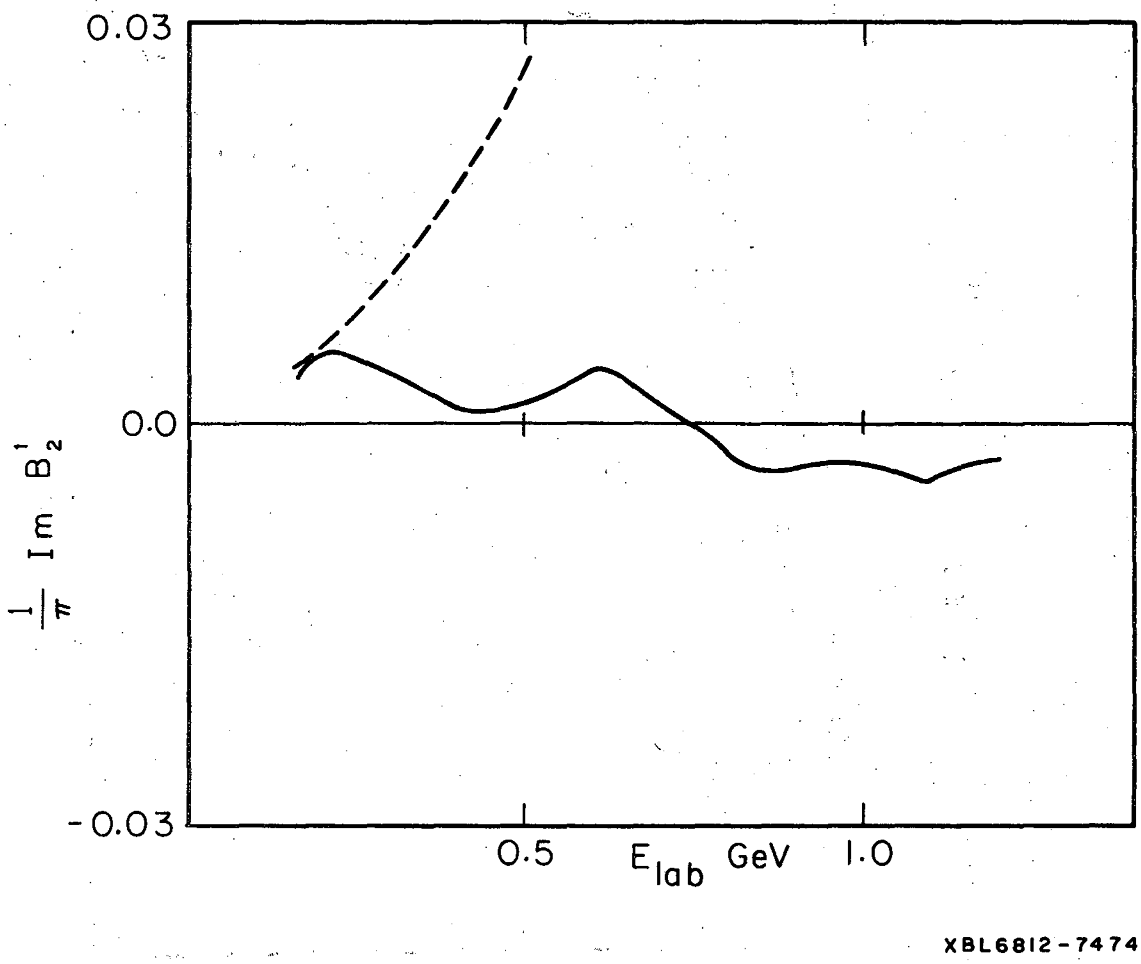
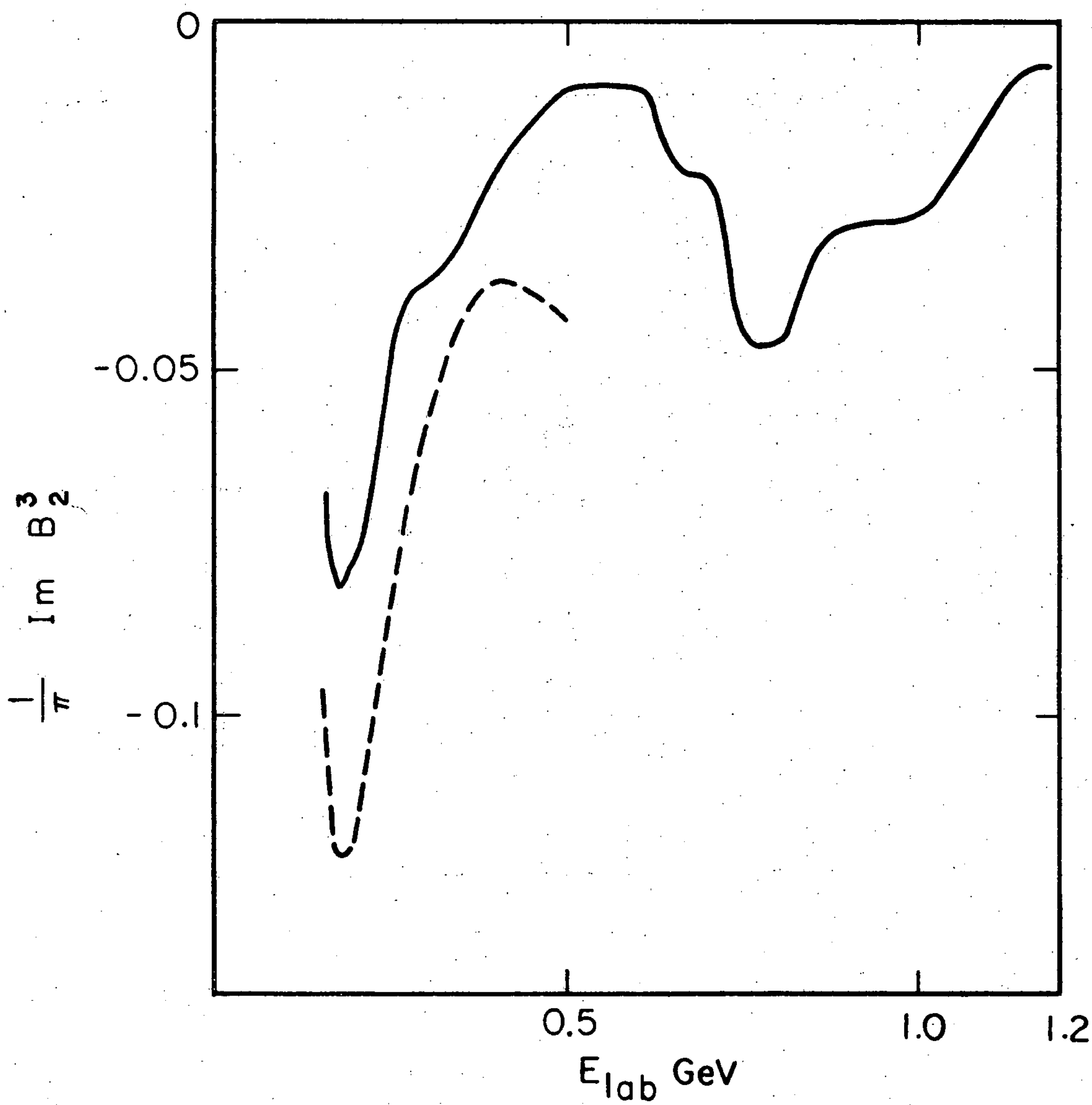


Fig. 3



XBL6812-7475

Fig. 4

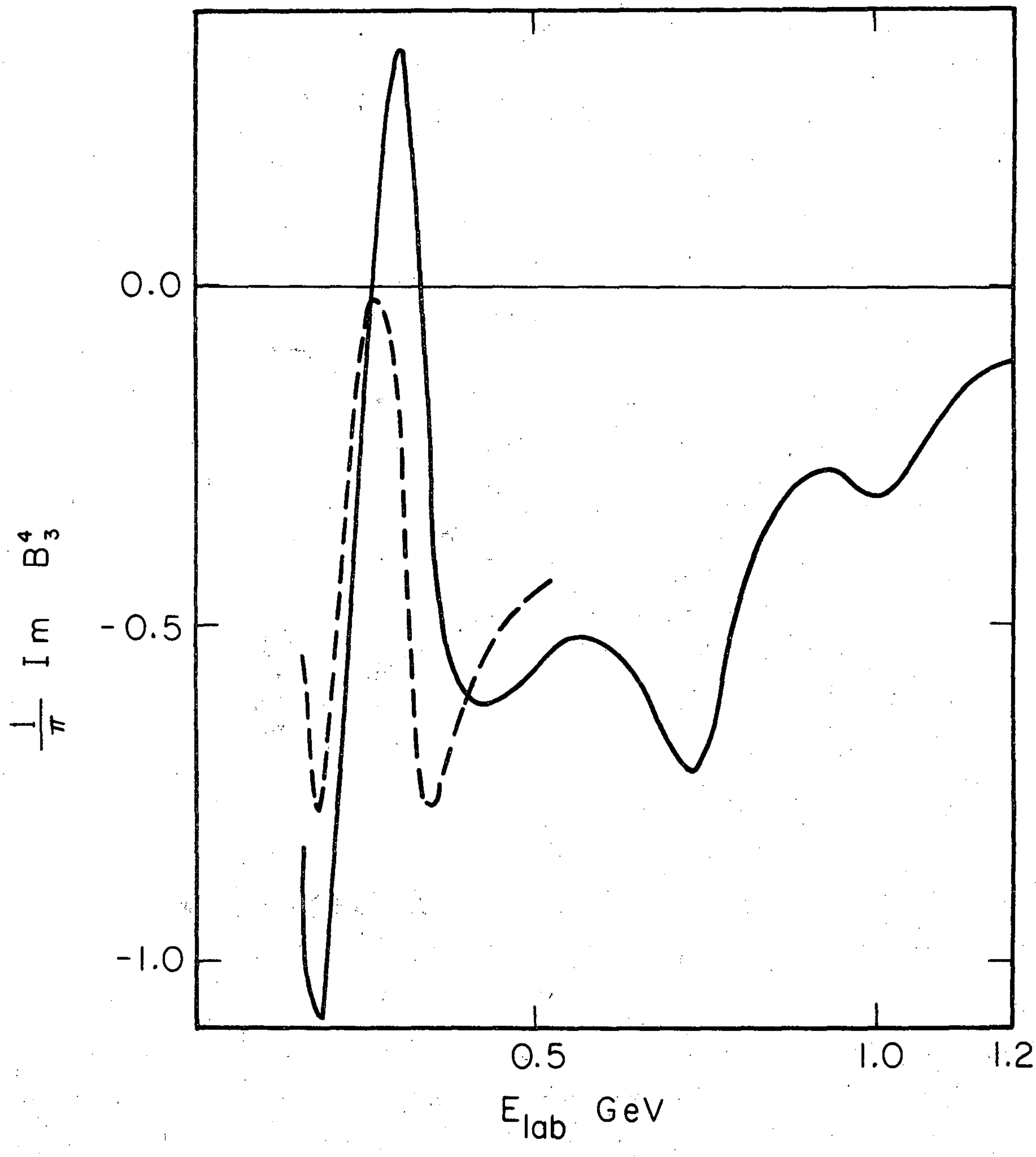
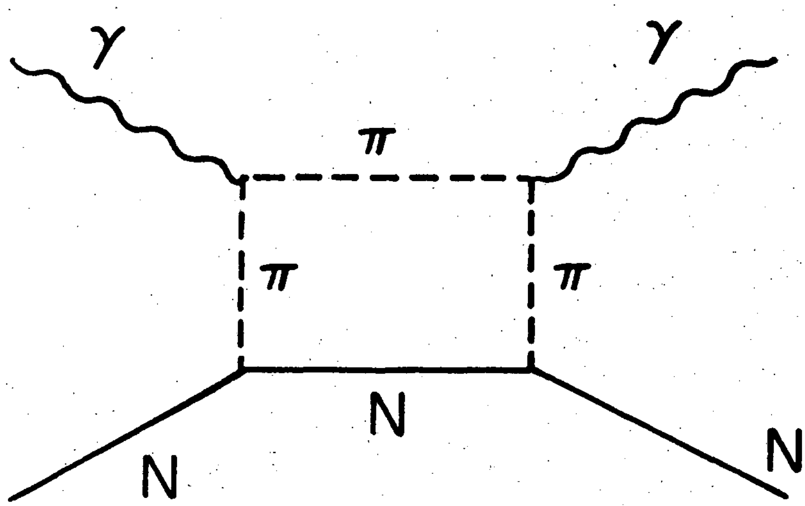
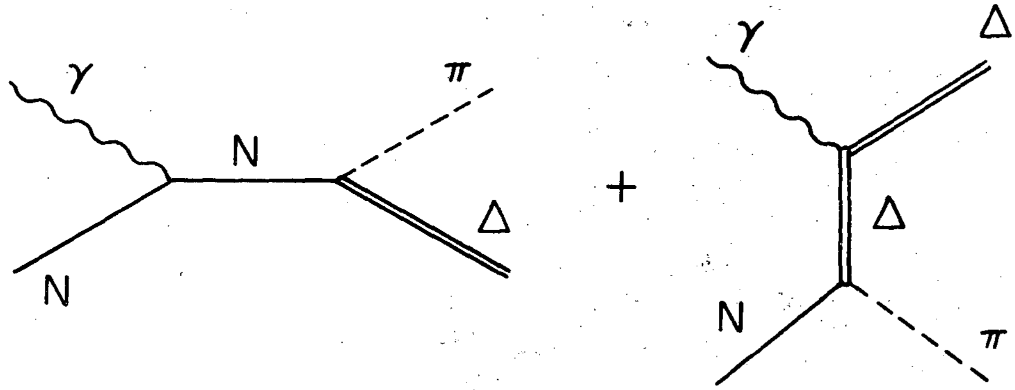


Fig. 5



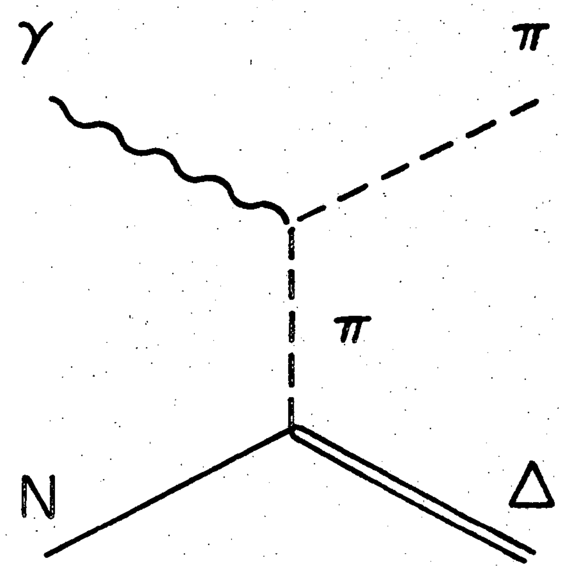
XBL6812-7477

Fig. 6



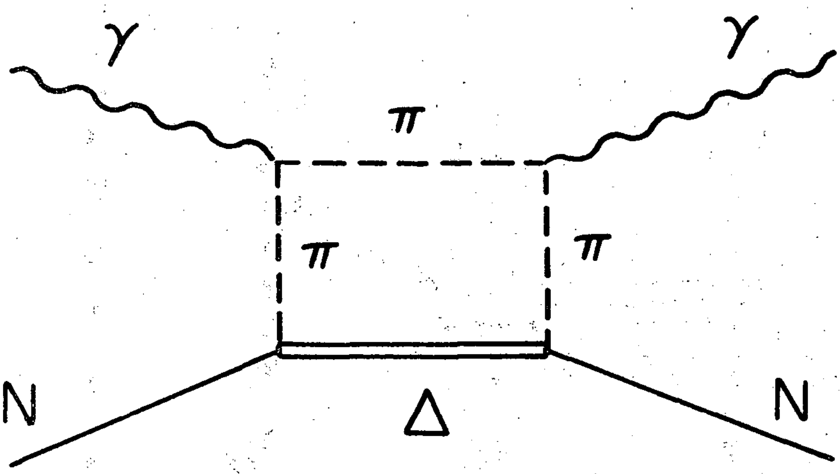
XBL6812-7478

Fig. 7



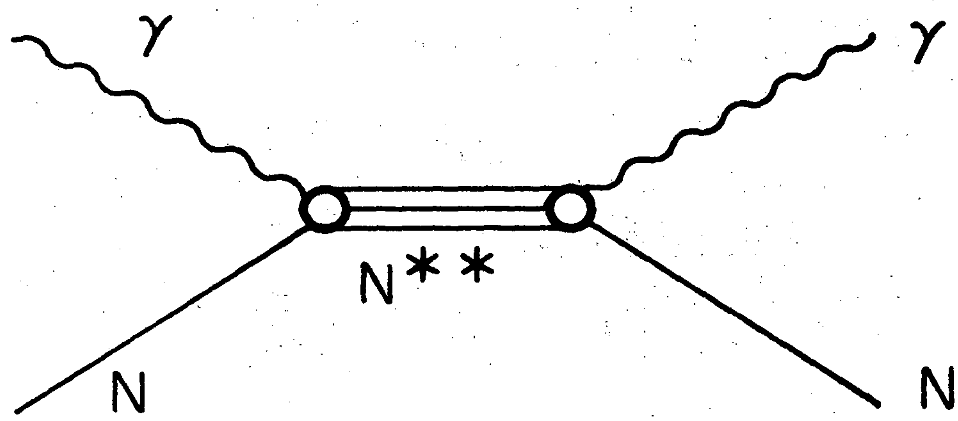
XBL6812-7479

Fig. 8



XBL6812-7480

Fig. 9



XBL6812-7481

Fig. 10

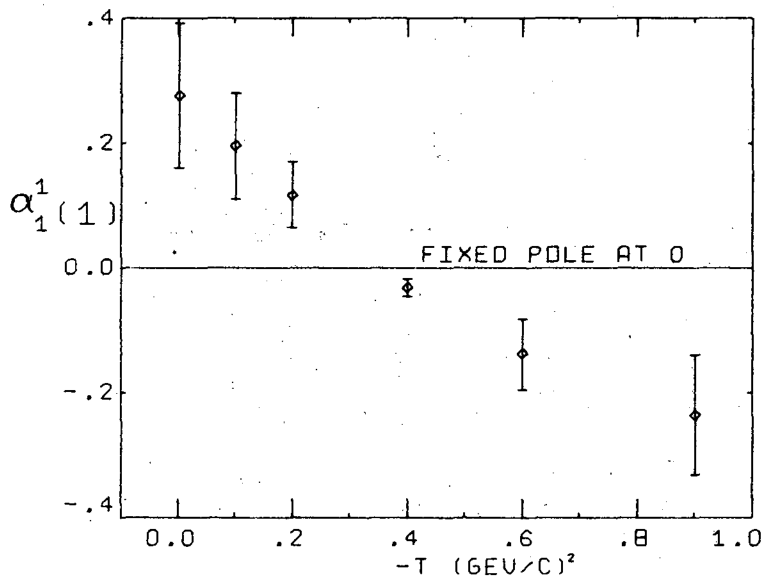
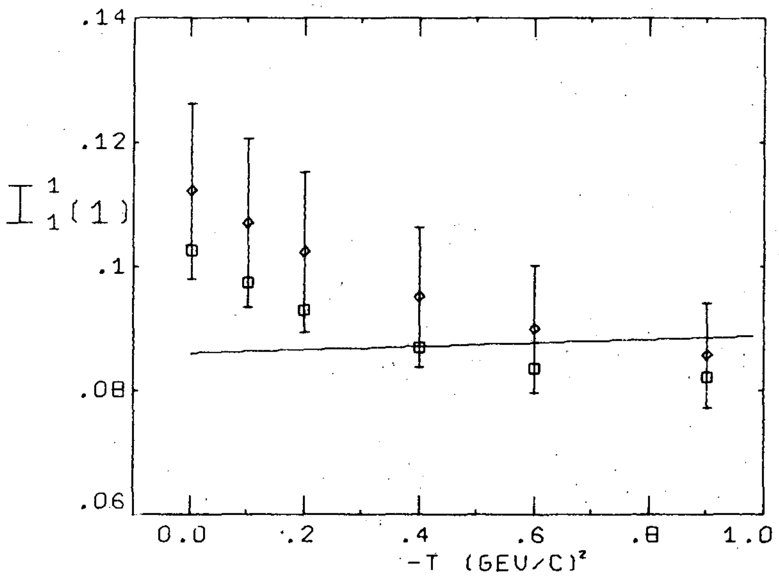


Fig. 11b



XBL 6812-6413

Fig. 11a

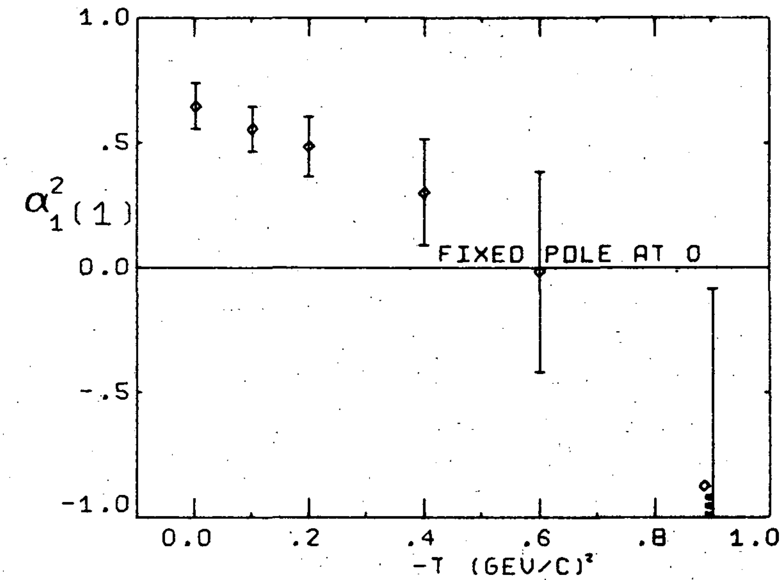
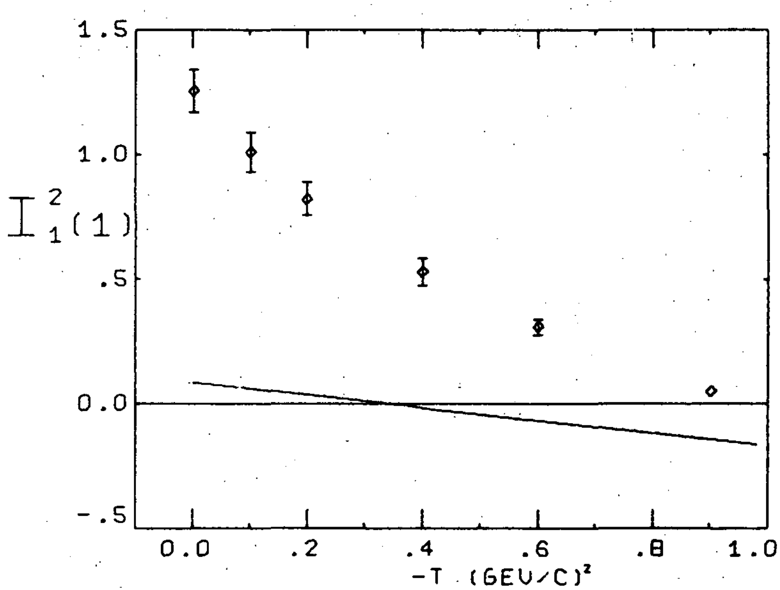
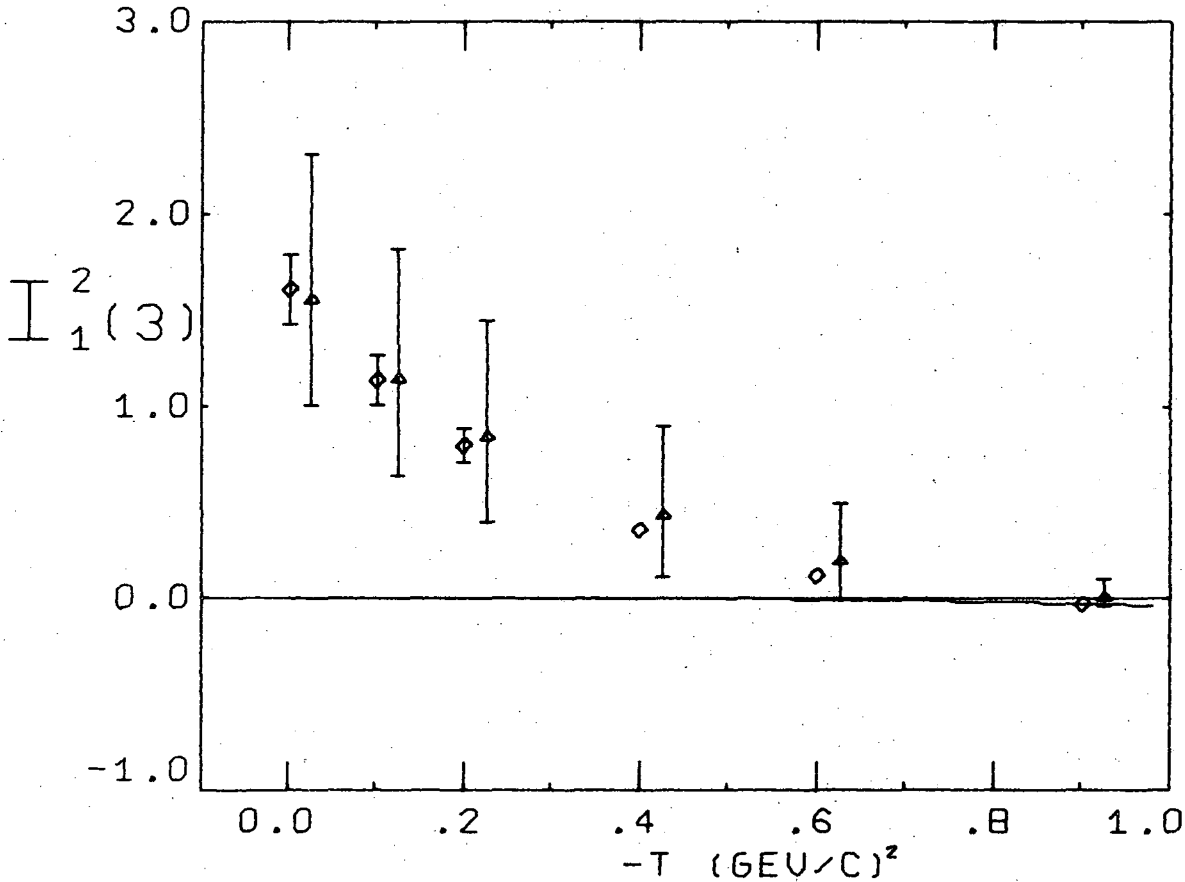


Fig. 12b



XBL 6812-6414

Fig. 12a



XBL 6812-6415

Fig. 12c

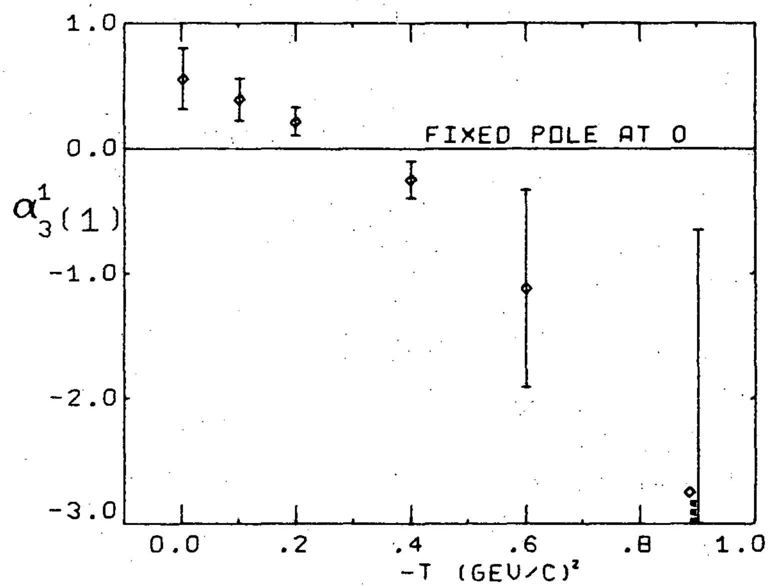
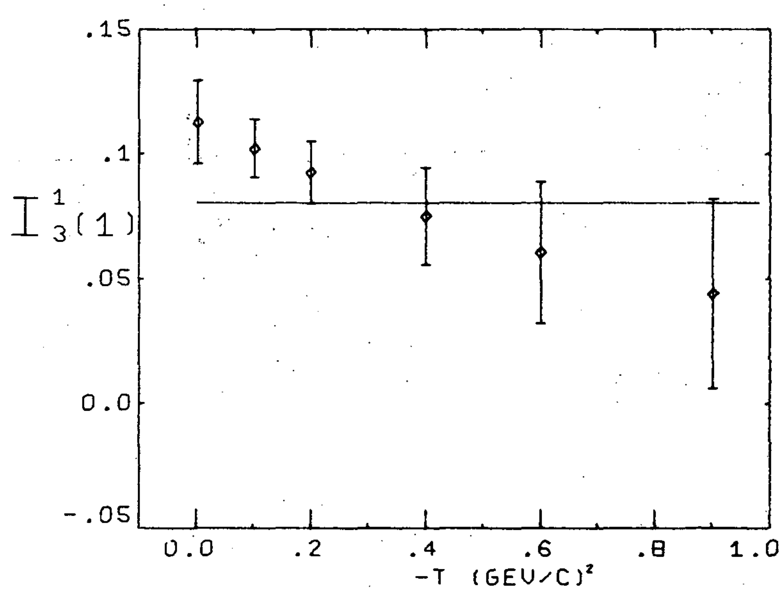


Fig. 13b



XBL 6812-6416

Fig. 13a

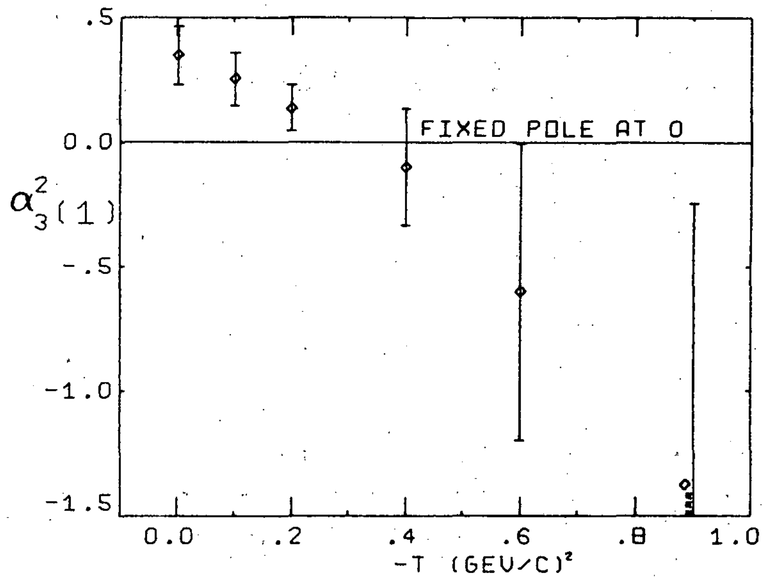
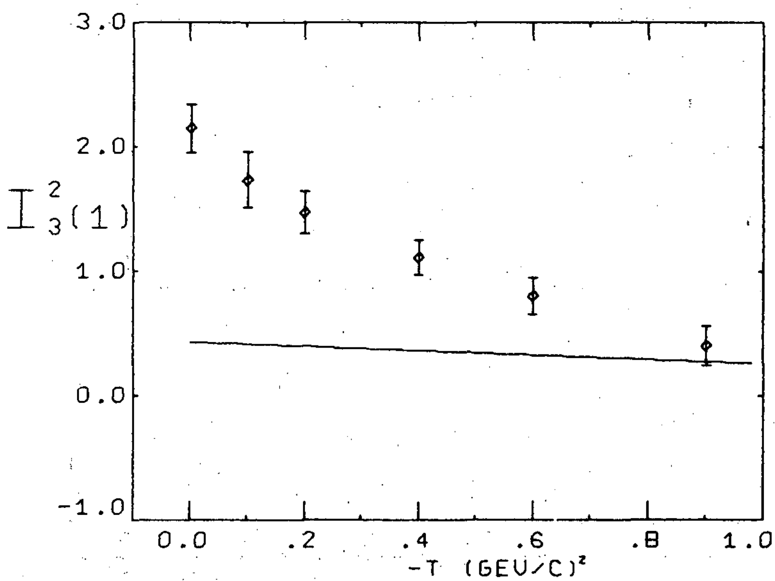


Fig. 14b



XBL 6812-6417

Fig. 14a

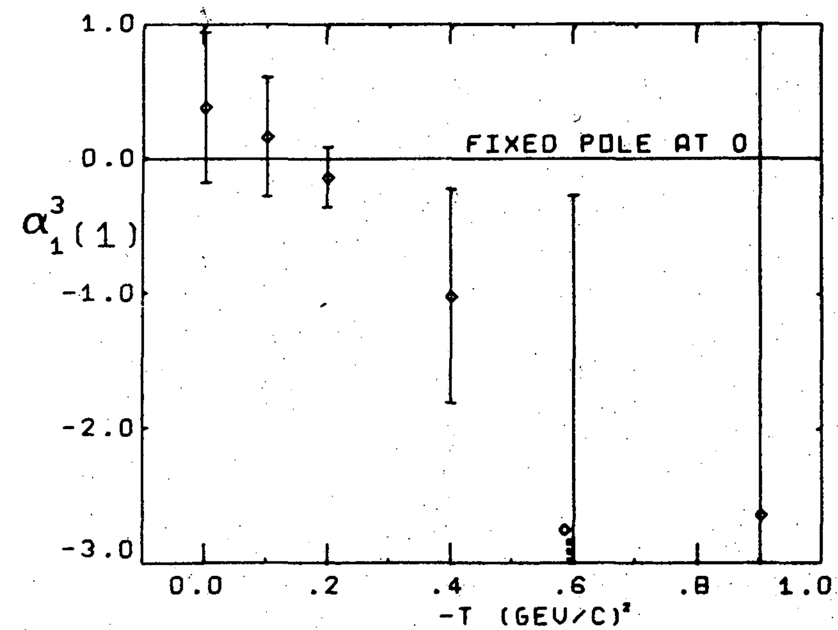
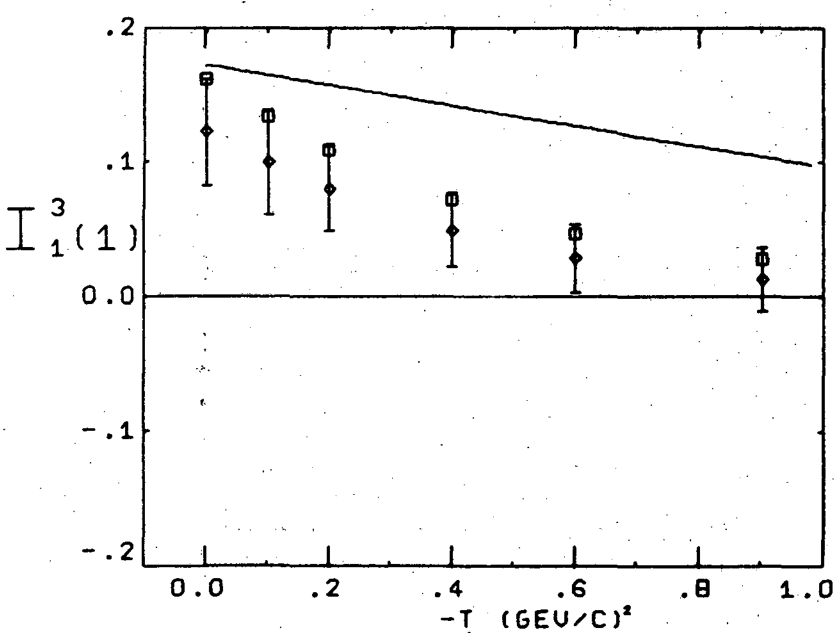


Fig. 15b



XBL 691-122

Fig. 15a

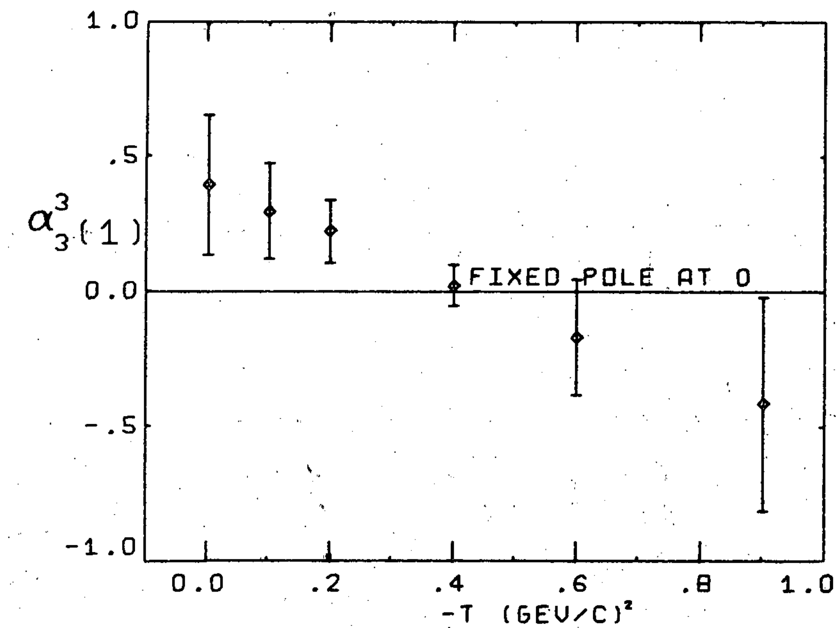
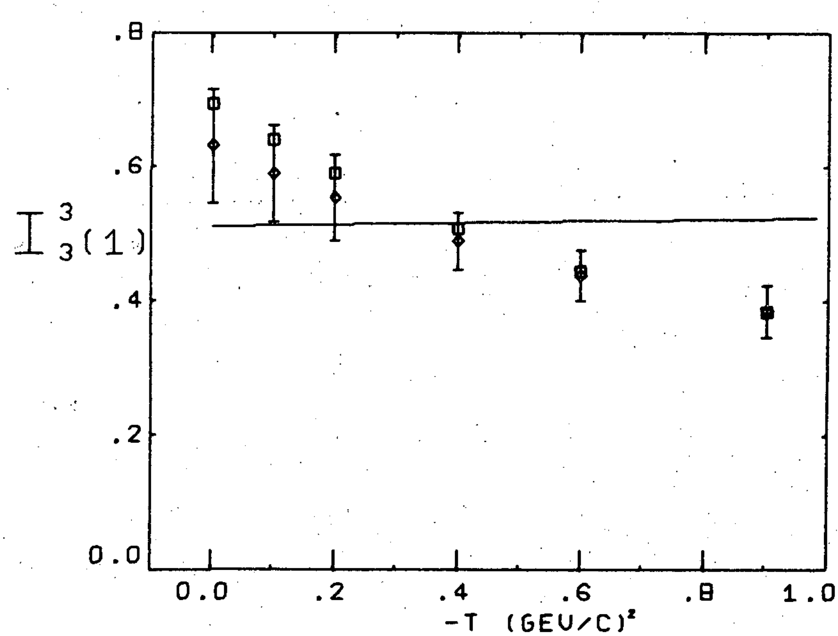
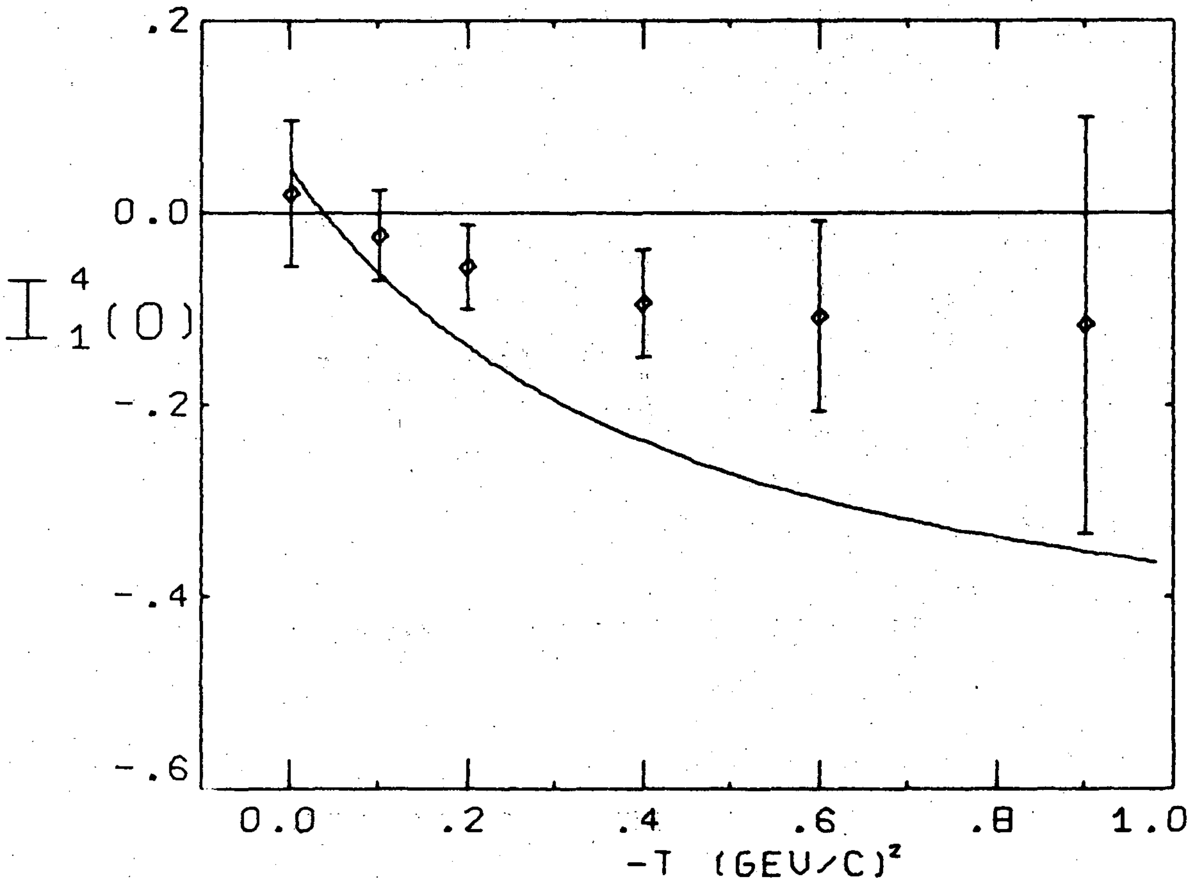


Fig. 16b



XBL 691-124

Fig. 16a



XBL 6812-6420

Fig. 17

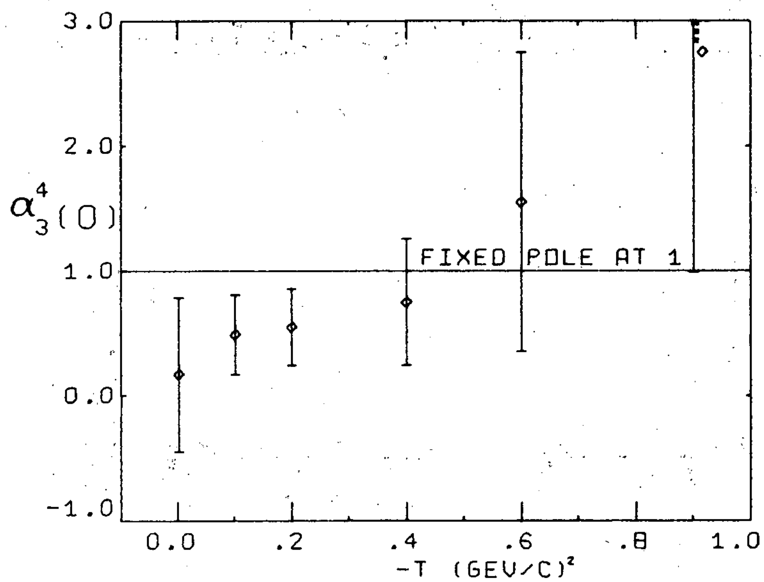
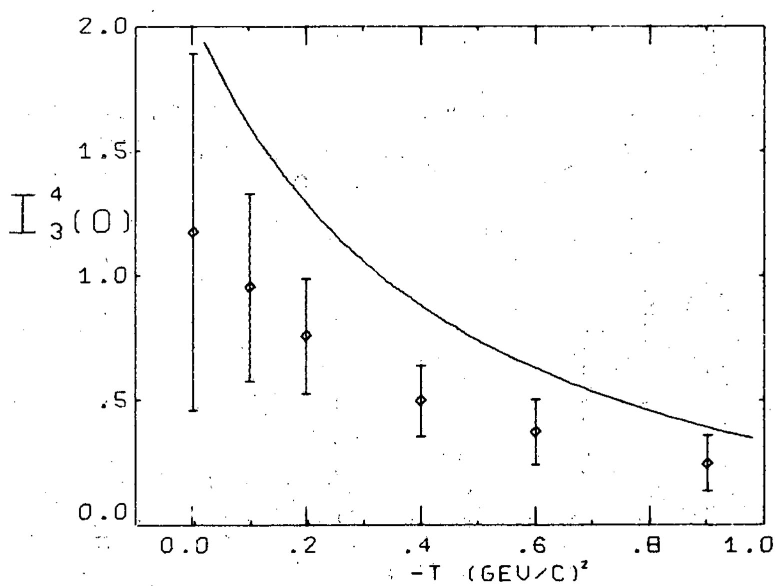


Fig. 18b



XBL 6812-6421

Fig. 18a

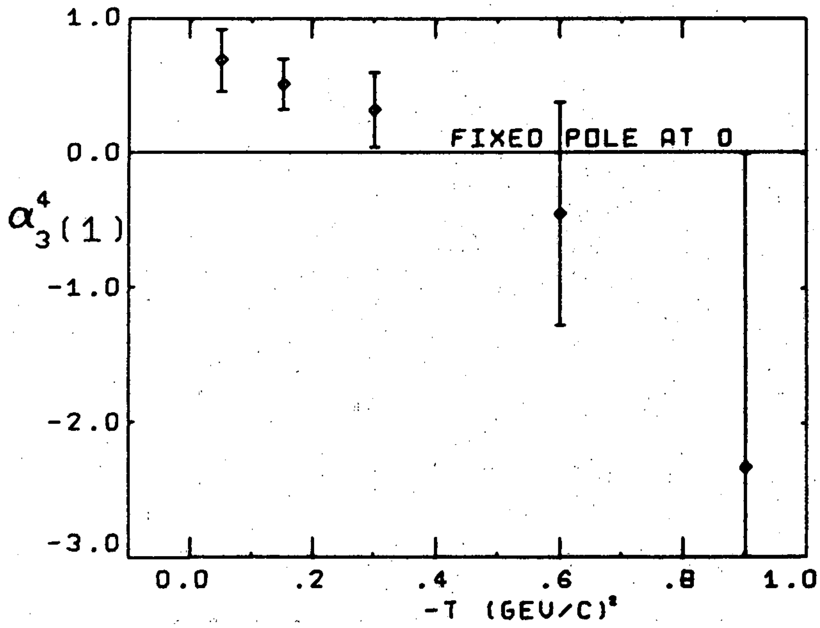
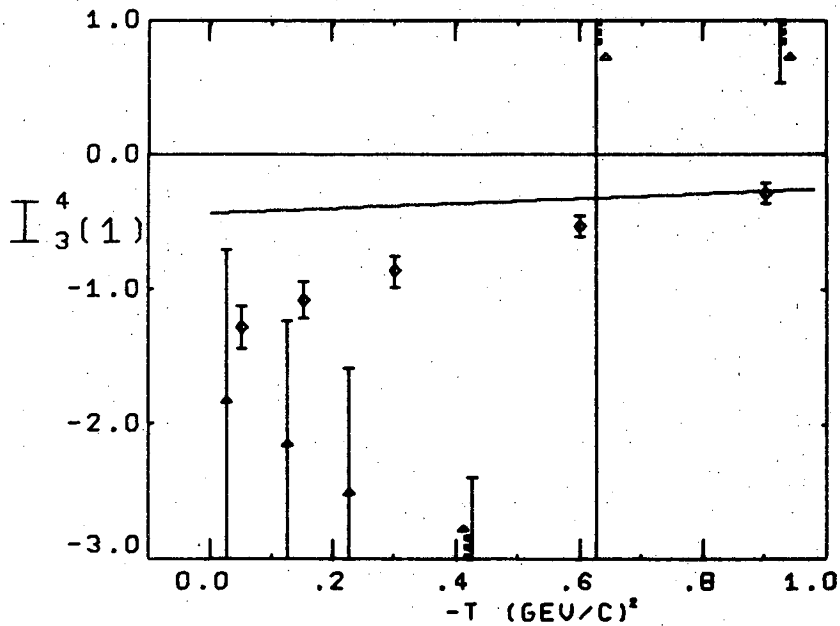


Fig. 18d



XBL 691-125

Fig. 18c

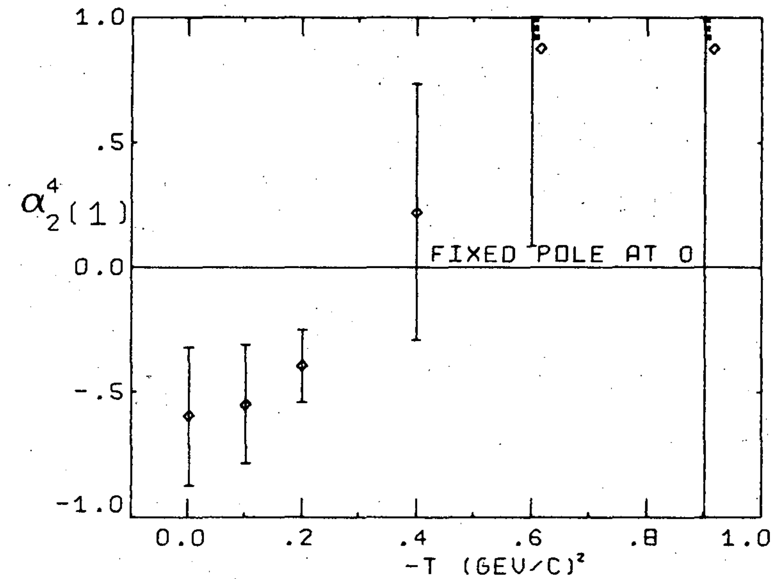
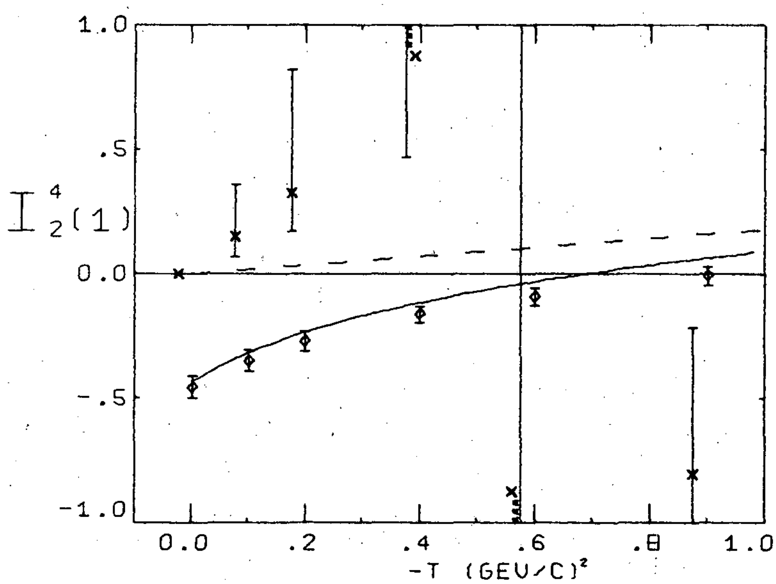
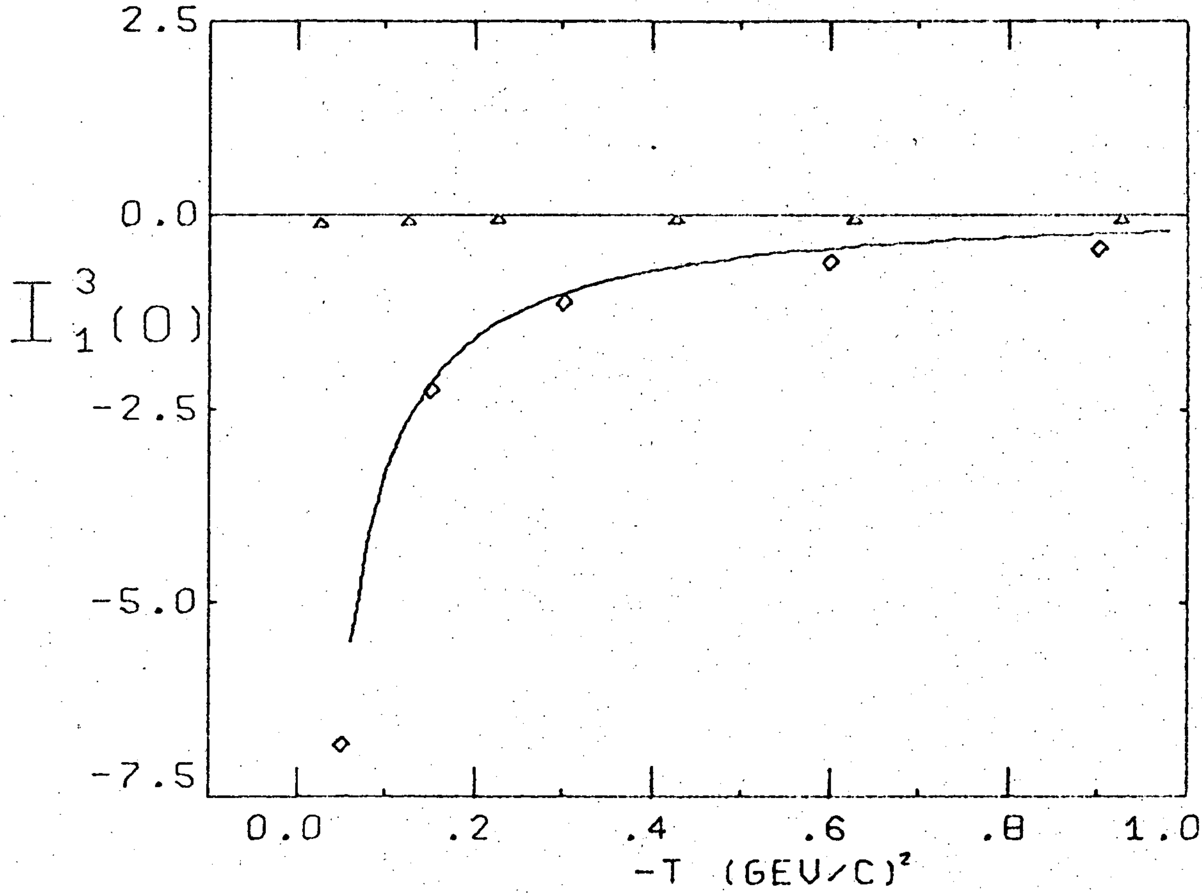


Fig. 19b



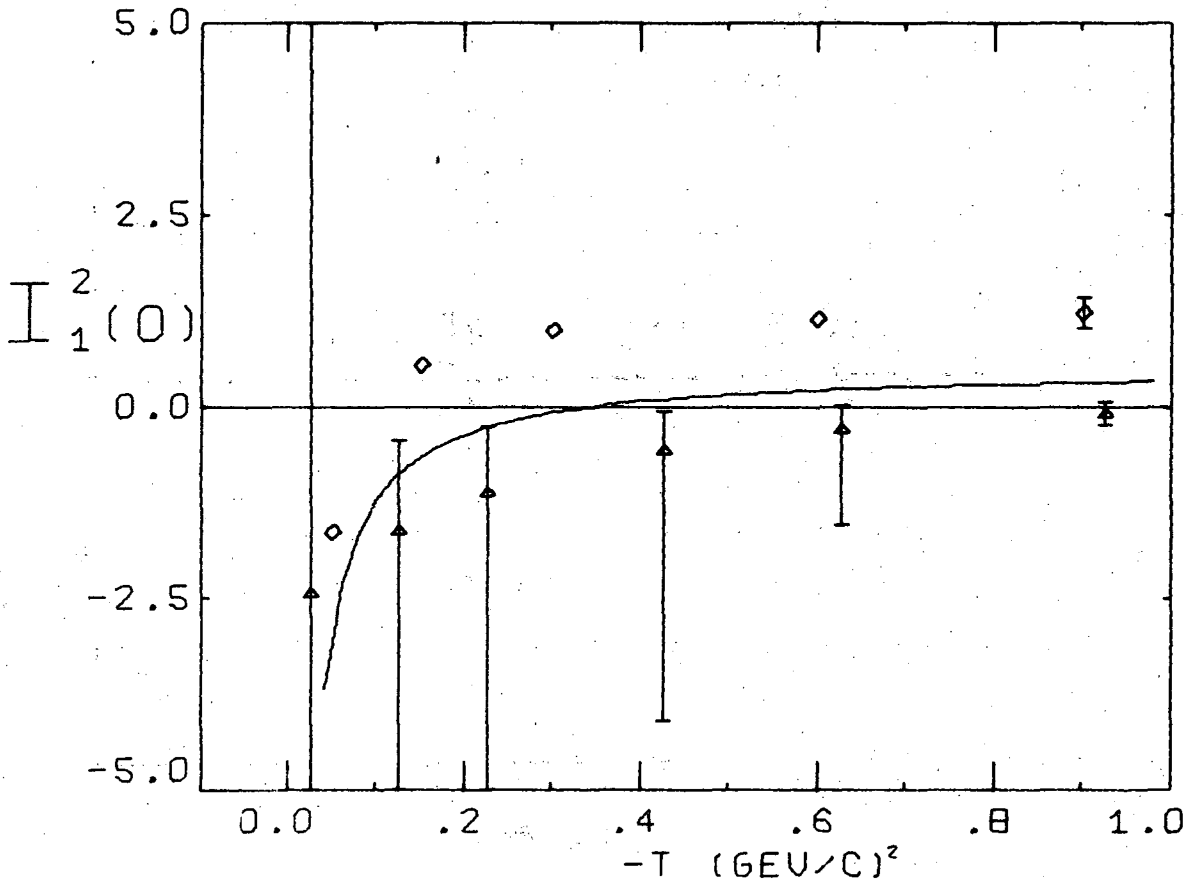
XBL 6812-6423

Fig. 19a



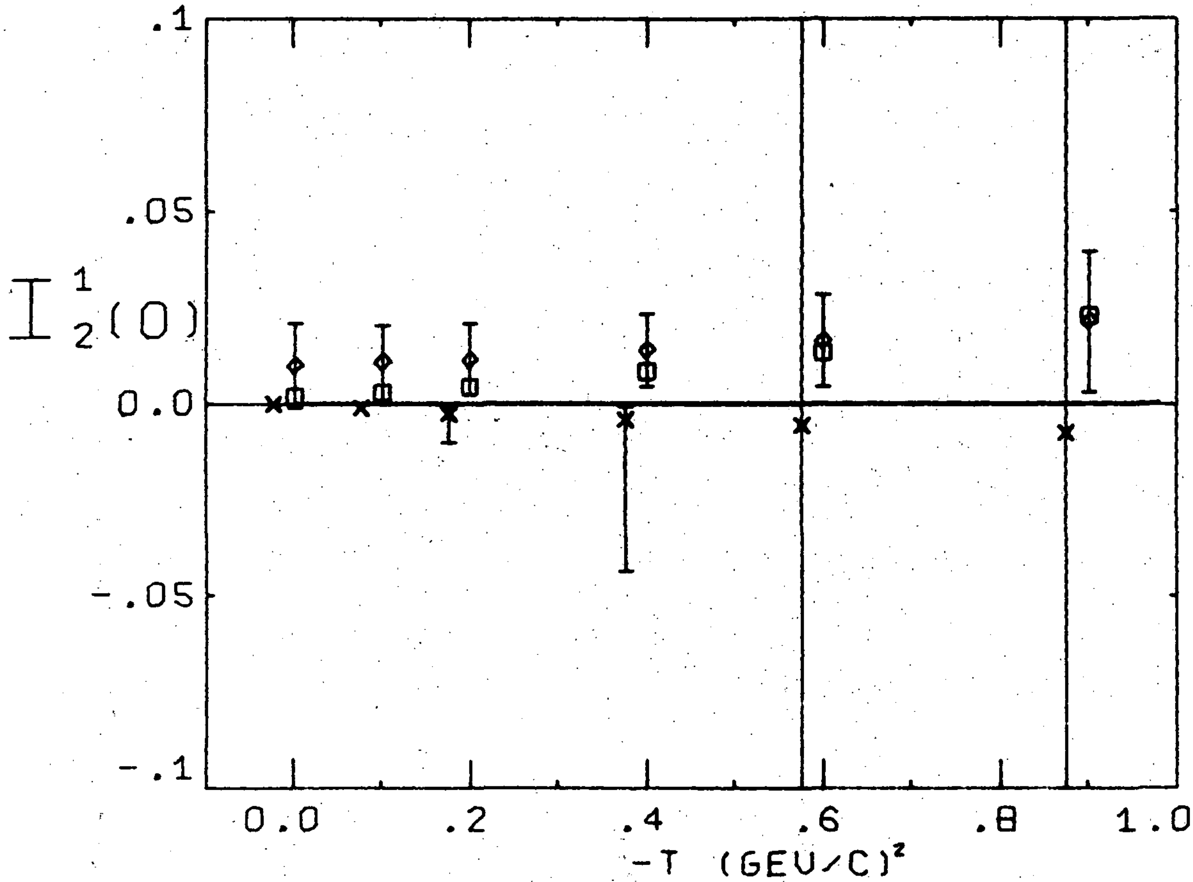
XBL 6812-6424

Fig. 20



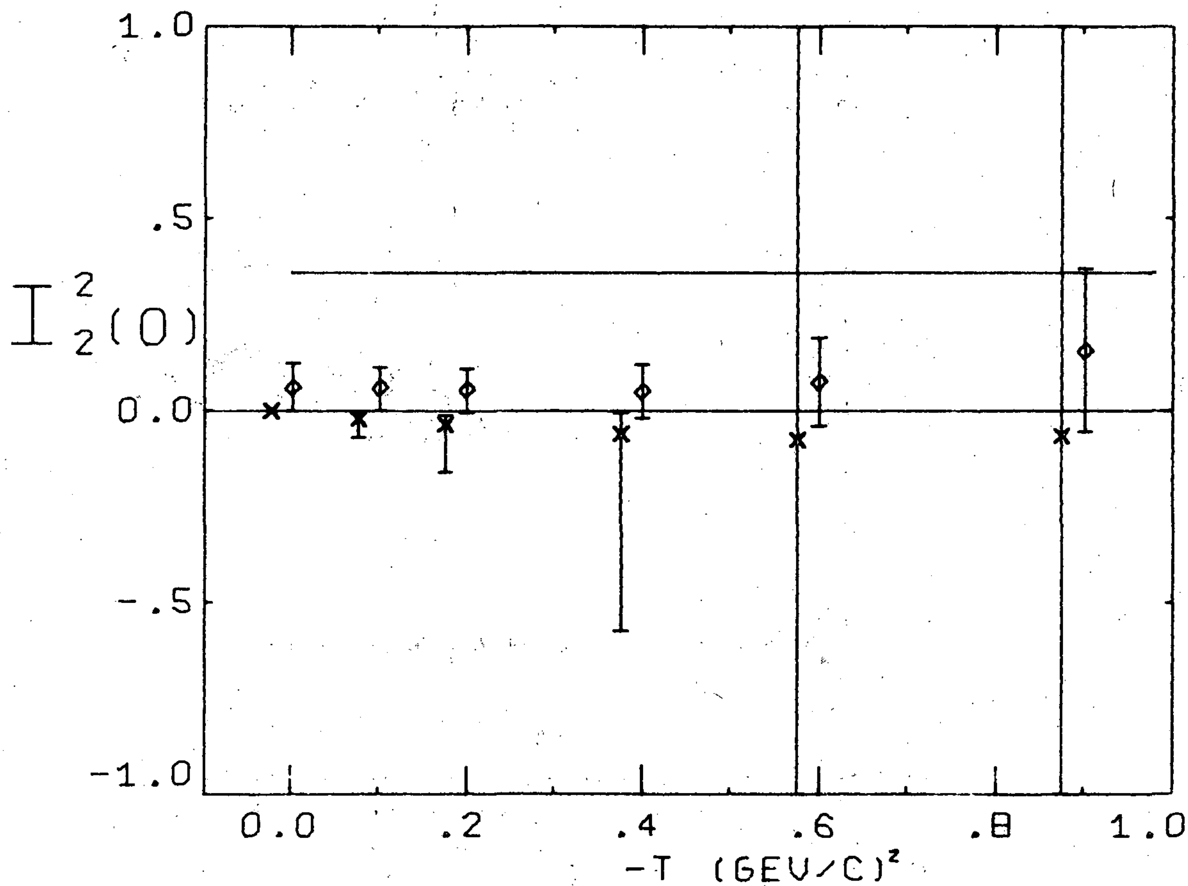
XBL 6812-6425

Fig. 21



XBL 6812-6426

Fig. 22



XBL 6812-6427

Fig. 23

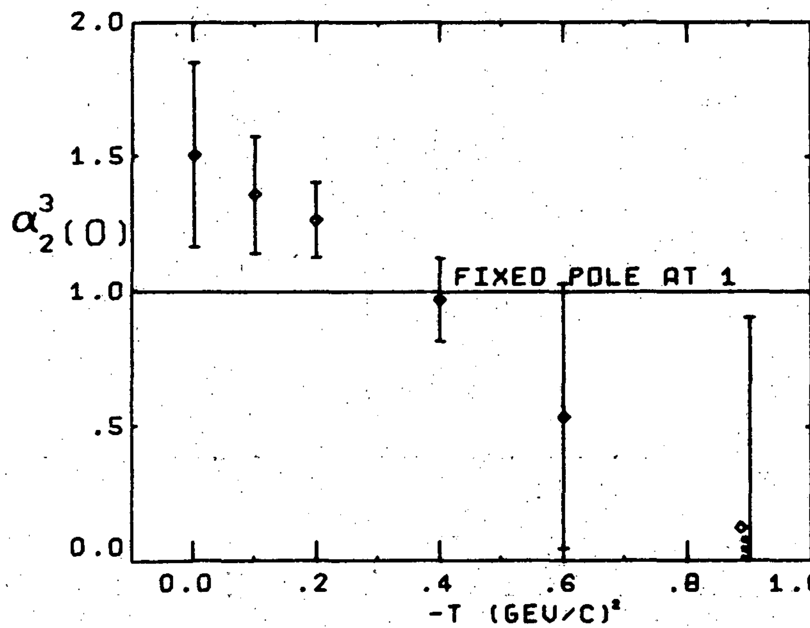
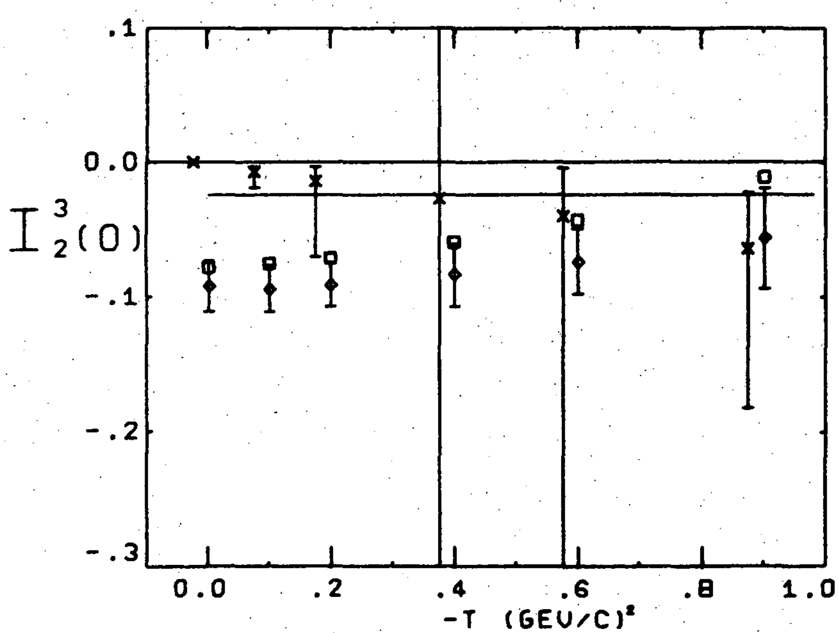
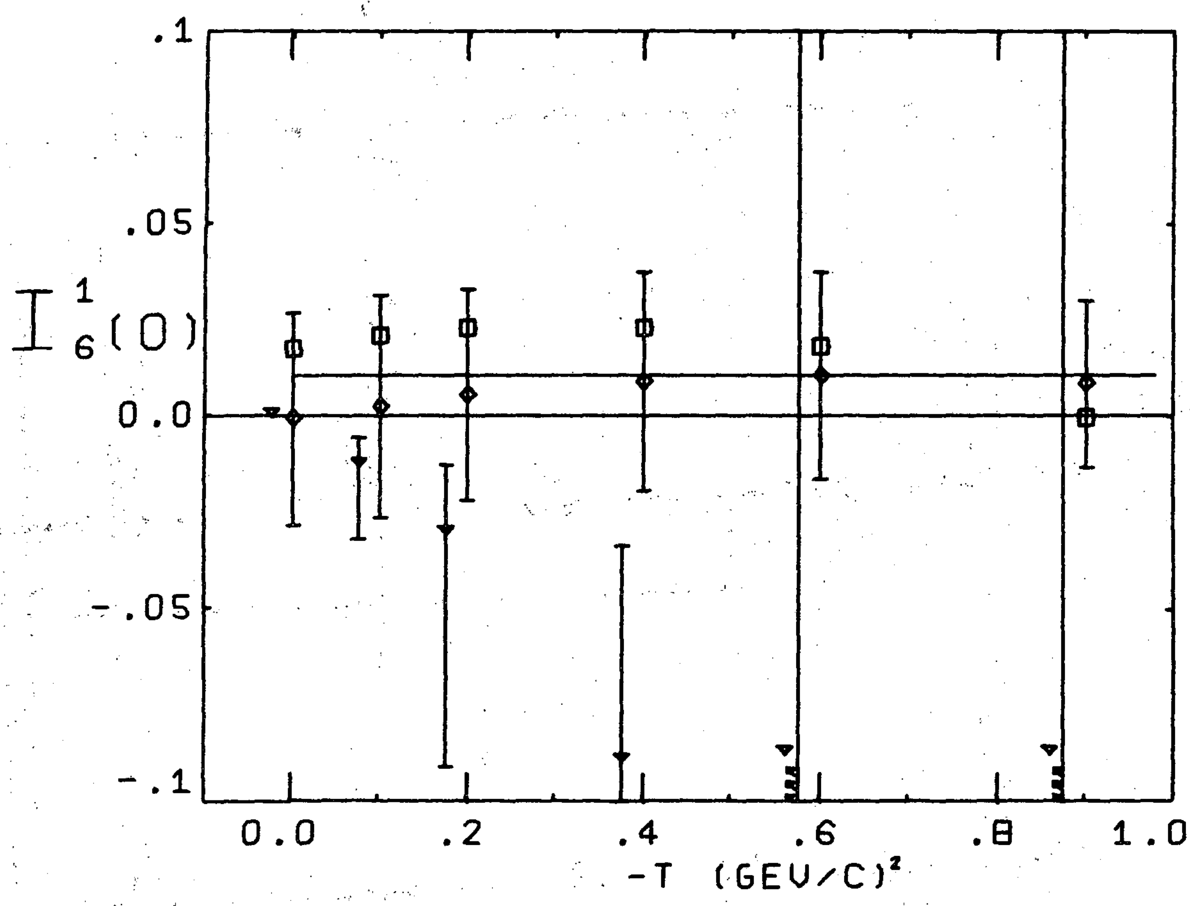


Fig. 24b



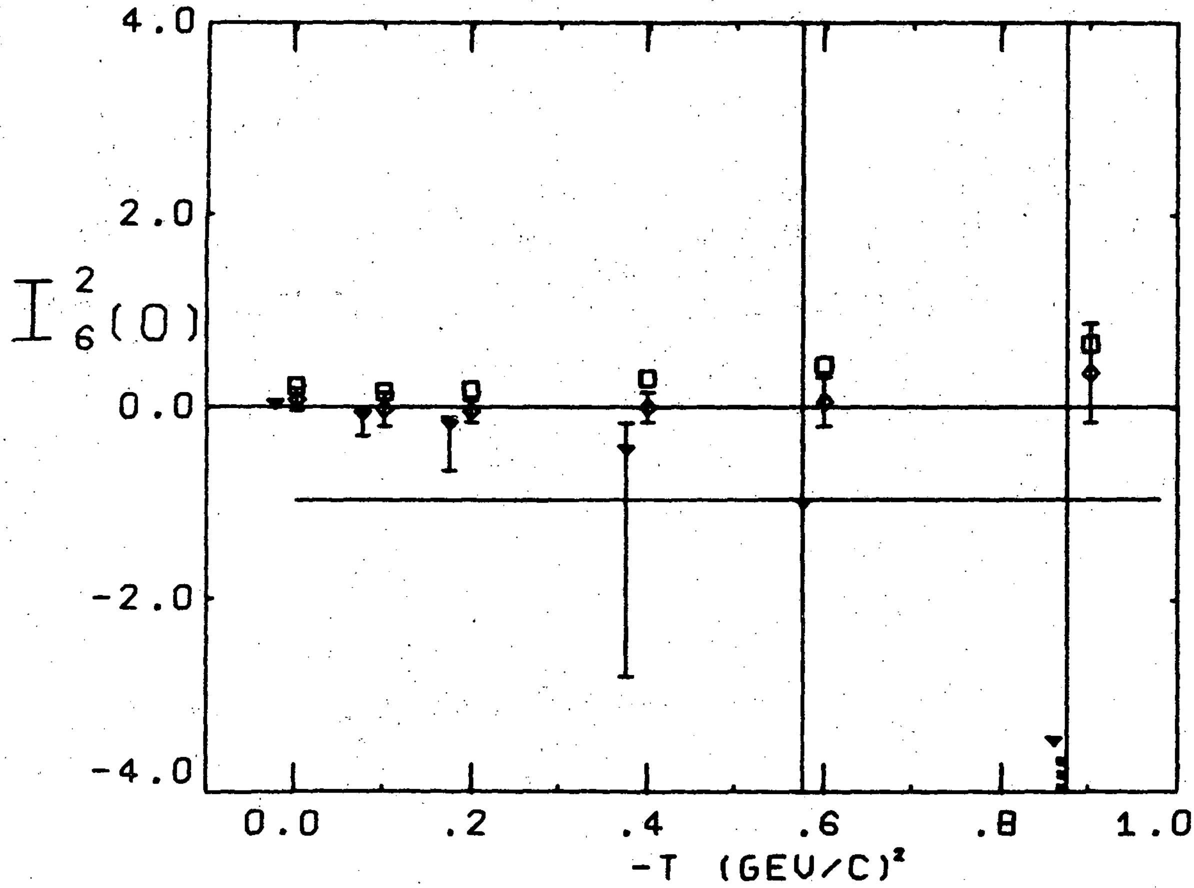
XBL 691-123

Fig. 24a



XBL 691-126

Fig. 25



XBL 691-127

Fig. 26

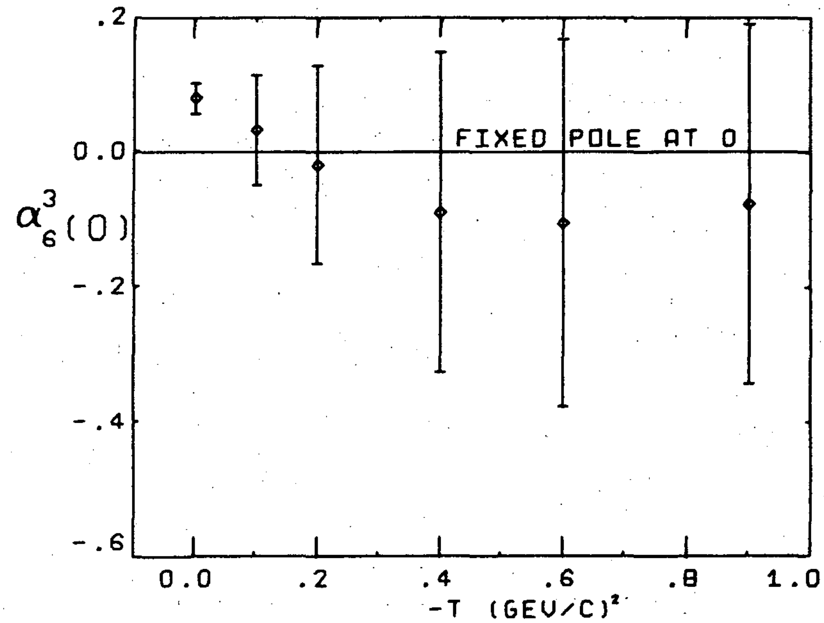
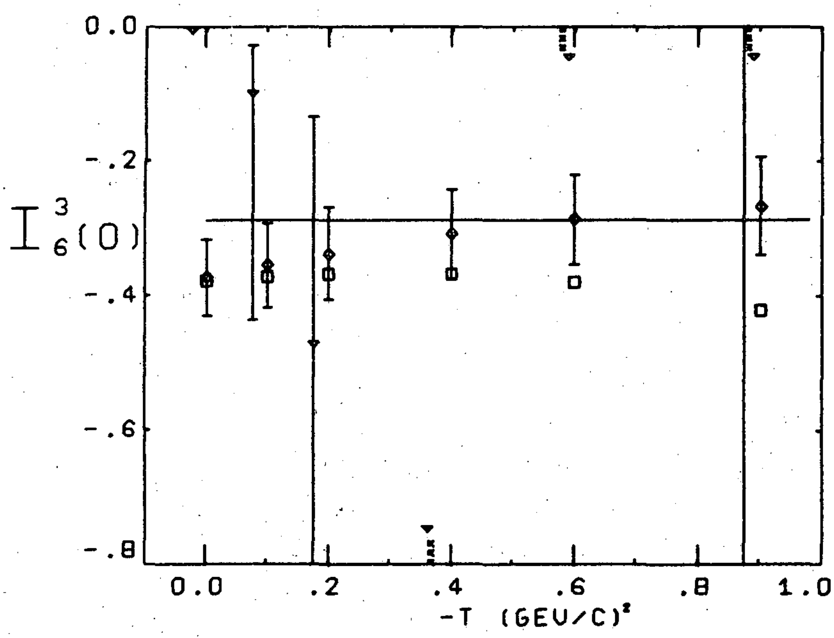


Fig. 27b



XBL 691-128

Fig. 27a

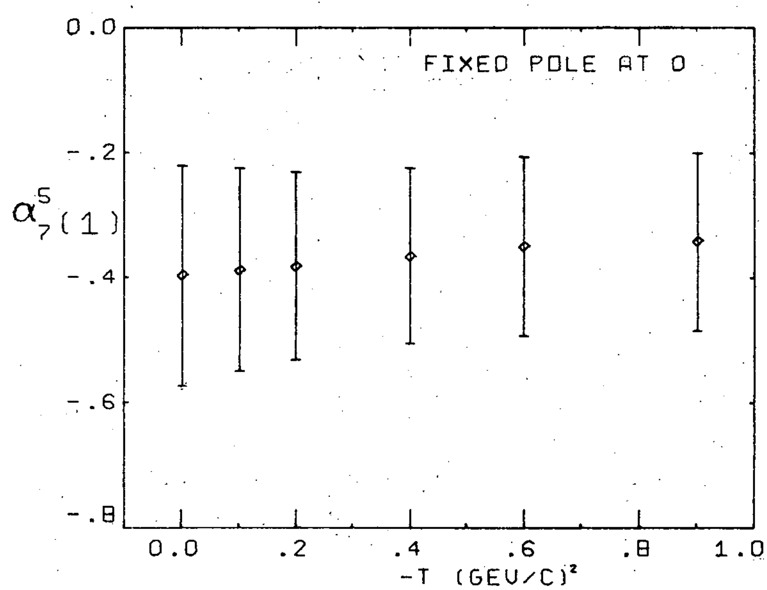


Fig. 28b

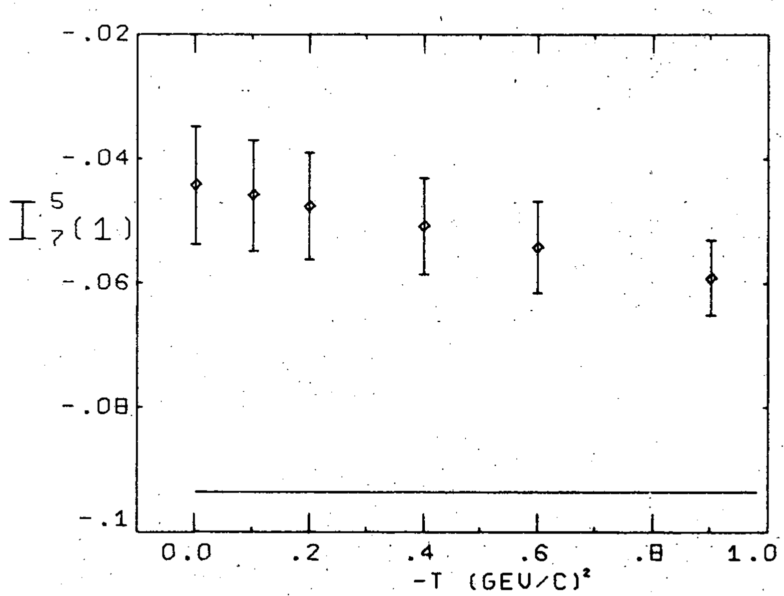


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Fig. 28a

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TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720